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Investigation of pressure gradient aware wall modeling in LES

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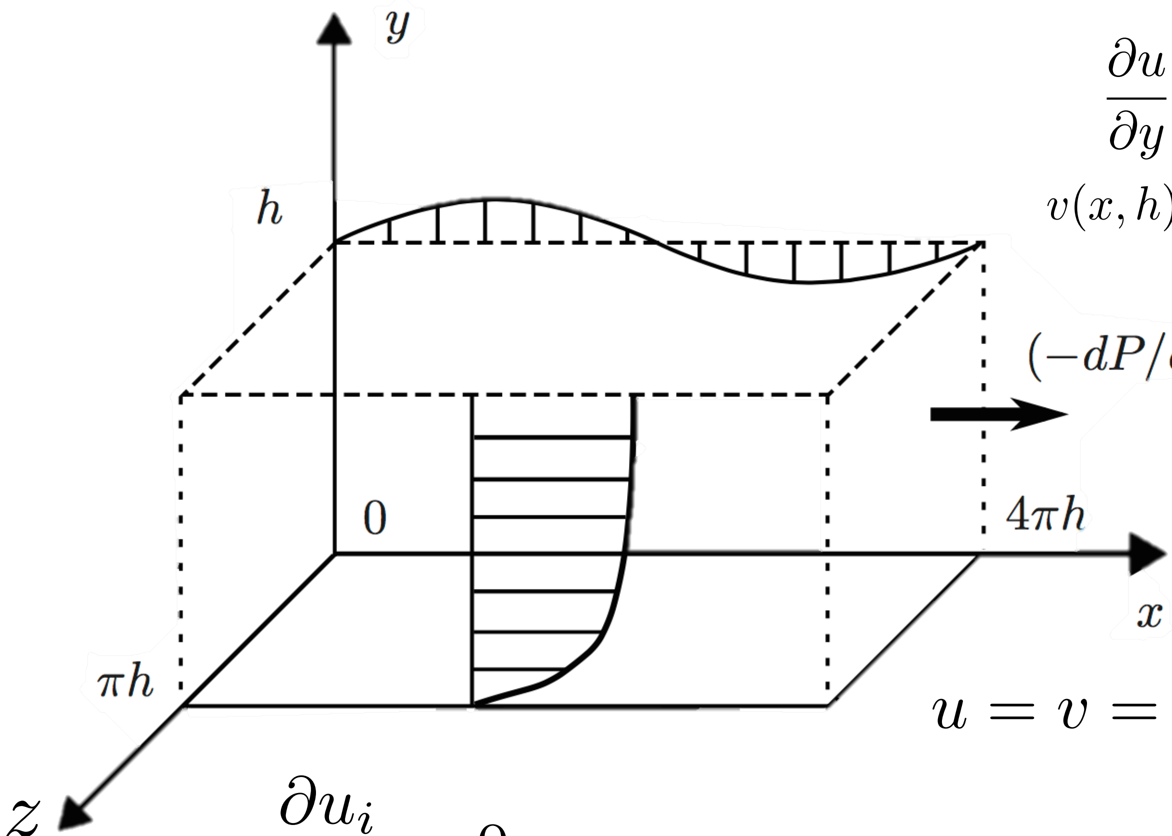
Channel flow with mild pressure gradient, obtained by blowing and suction



$$\frac{\partial u}{\partial y} = \frac{\partial w}{\partial y} = 0$$

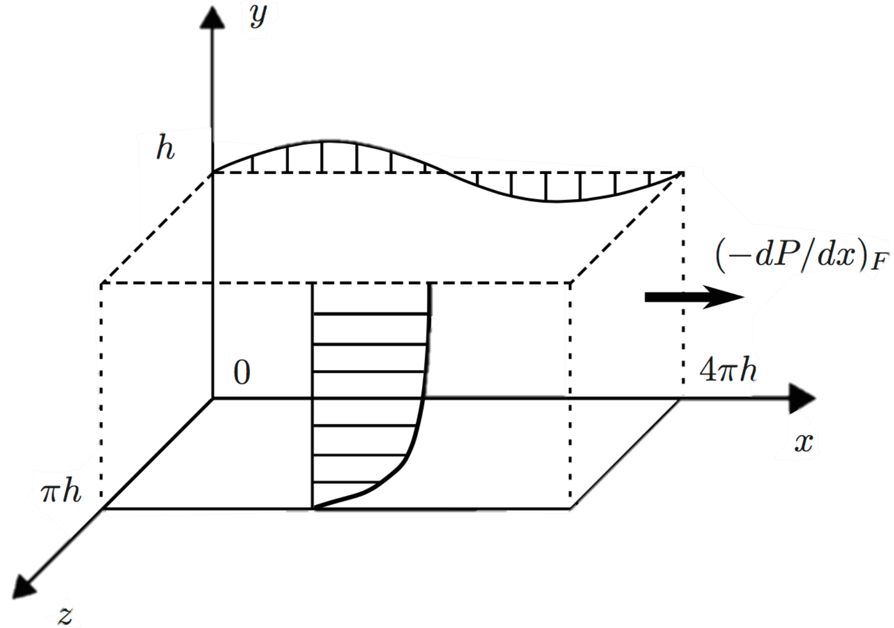
$$v(x,h) = v_{max} \sin\left(\frac{2\pi x}{L}\right)$$

$$u = v = w = 0$$

$$\frac{\partial u_i}{\partial x_i} = 0$$

$$\frac{\partial u_i}{\partial t} + \frac{\partial u_i u_j}{\partial x_j} = -\left(\frac{dP}{dx_i}\right)_F - \frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_j}\left(2\nu S_{ij} + \sigma_{ij}^{sgs}\right)$$

Wall-resolved LES



- Define forcing velocity scale

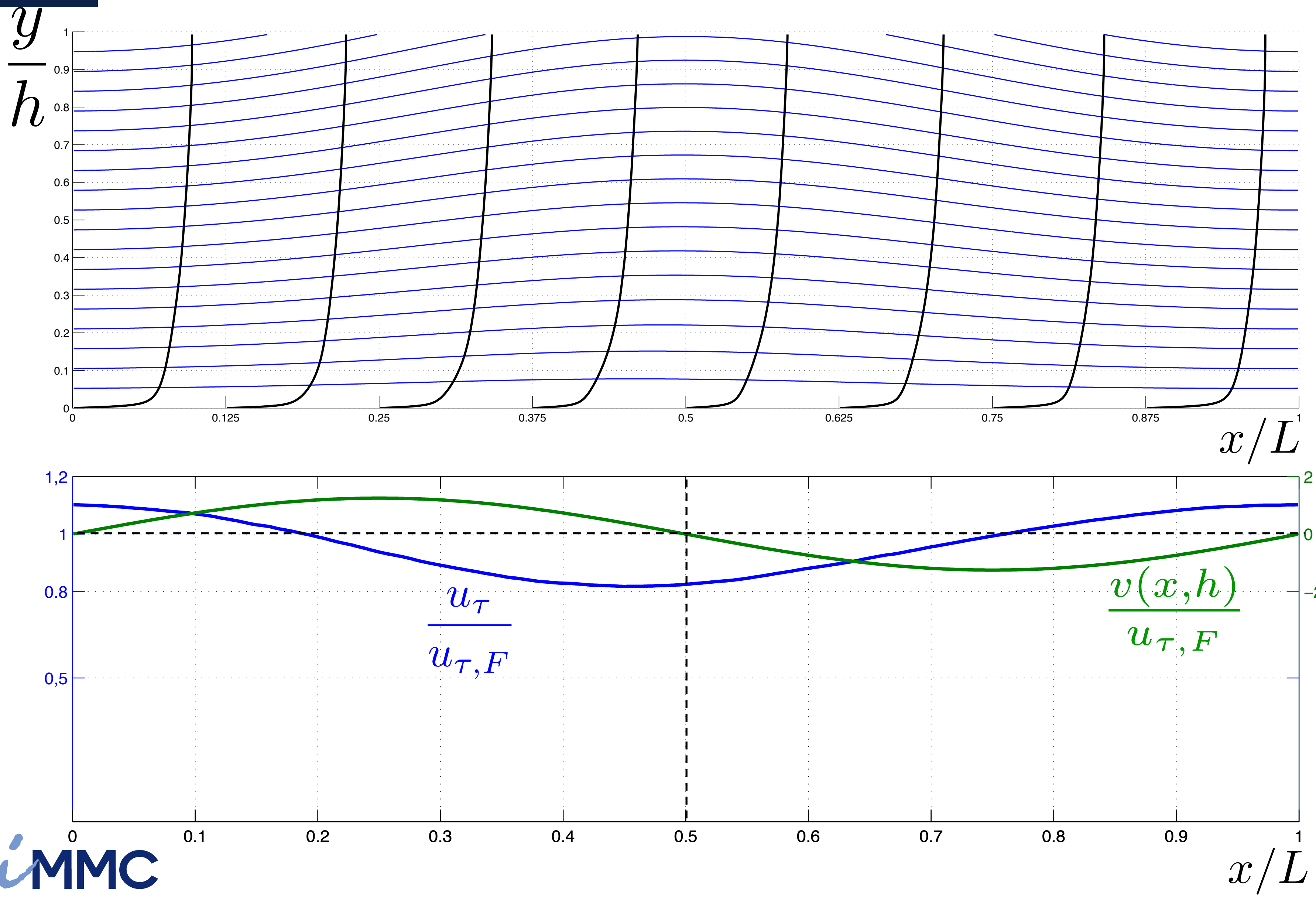
$$u_{\tau,F} = \left(h \left(\frac{dP}{dx} \right)_F \right)^{1/2}$$

$$Re_{\tau,F} = \frac{u_{\tau,F} h}{\nu} = 2000$$

- Mild blowing intensity $\quad v^*_{max} = \frac{v_{max}}{u_{\tau,F}} = 1.25$

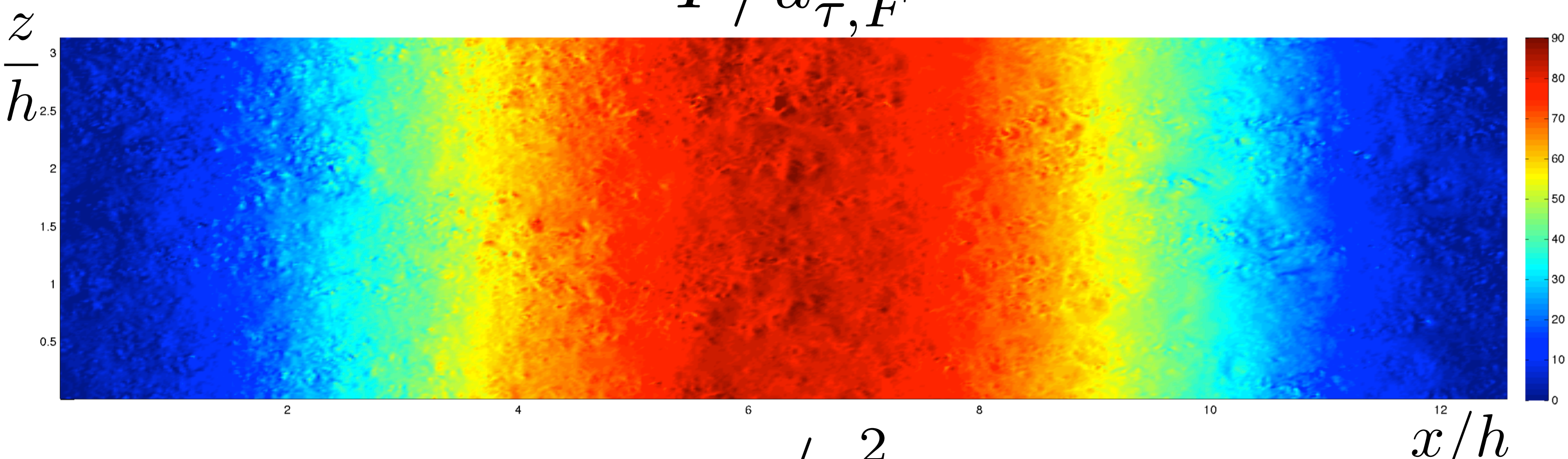
- $N_x \times N_y \times N_z$ = 640 x 192 x 256
- Δx^* = 39 ; Δy^* = 0.5 ; Δz^* = 24

Mean streamlines and velocity profiles

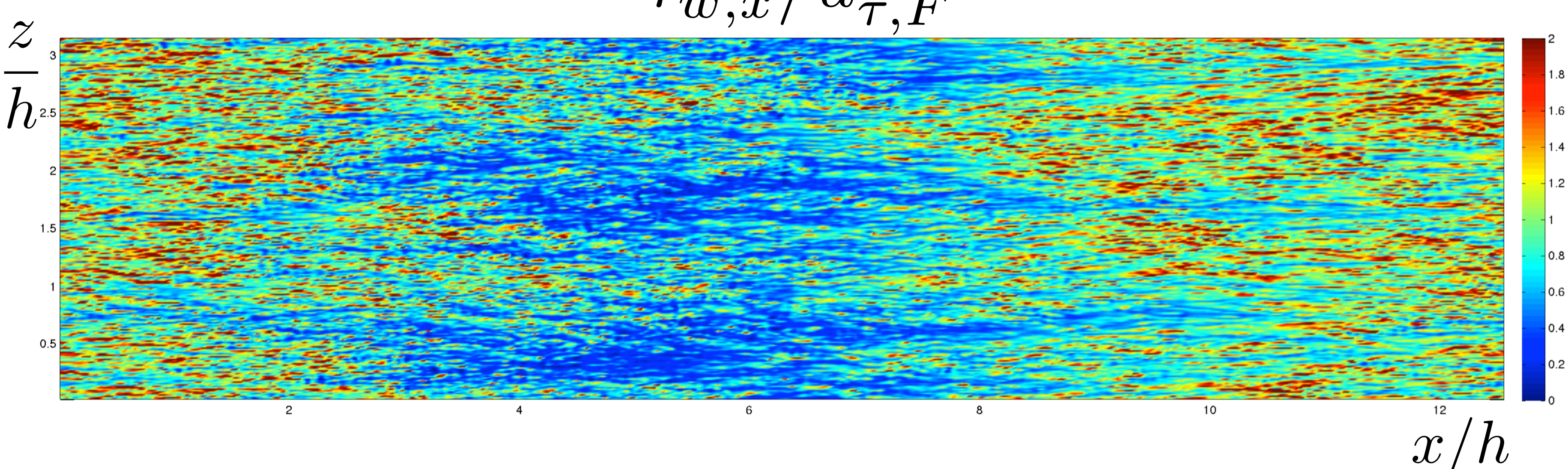


Instantaneous wall pressure and shear stress

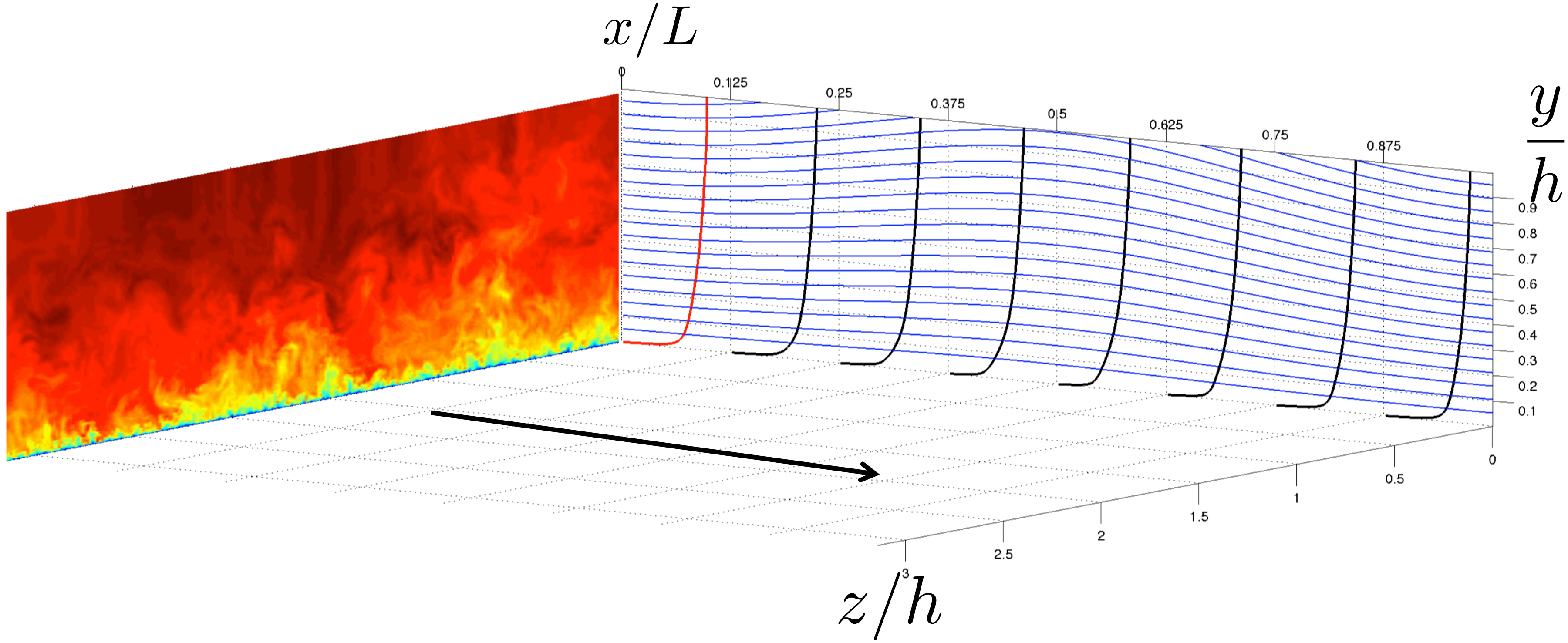
$$P/u_{\tau,F}^2$$



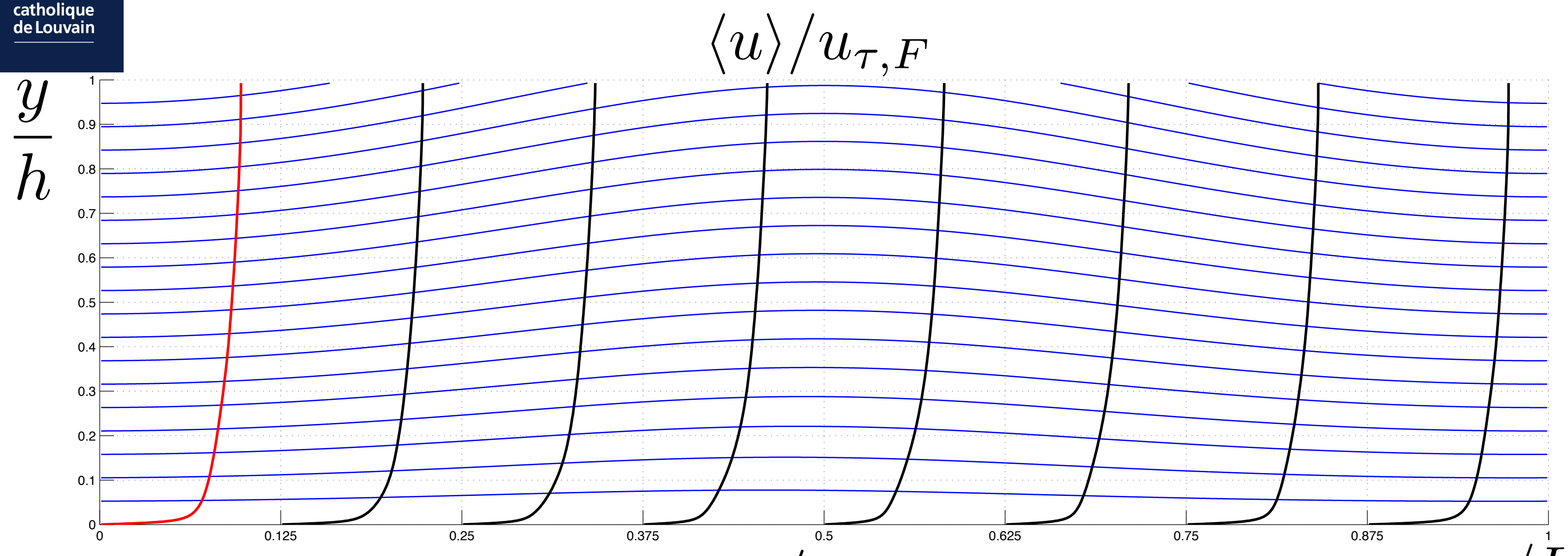
$$\tau_{w,x}/u_{\tau,F}^2$$

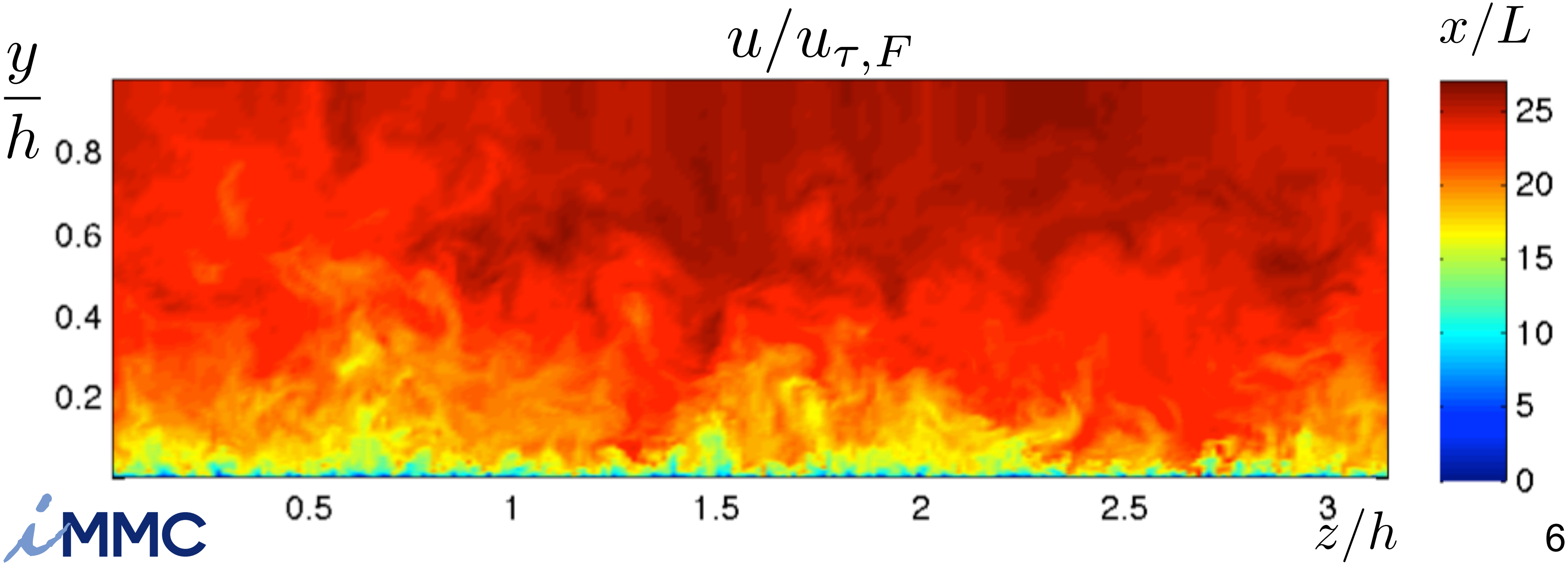


Instantaneous fields: $u/u_{\tau,F}$

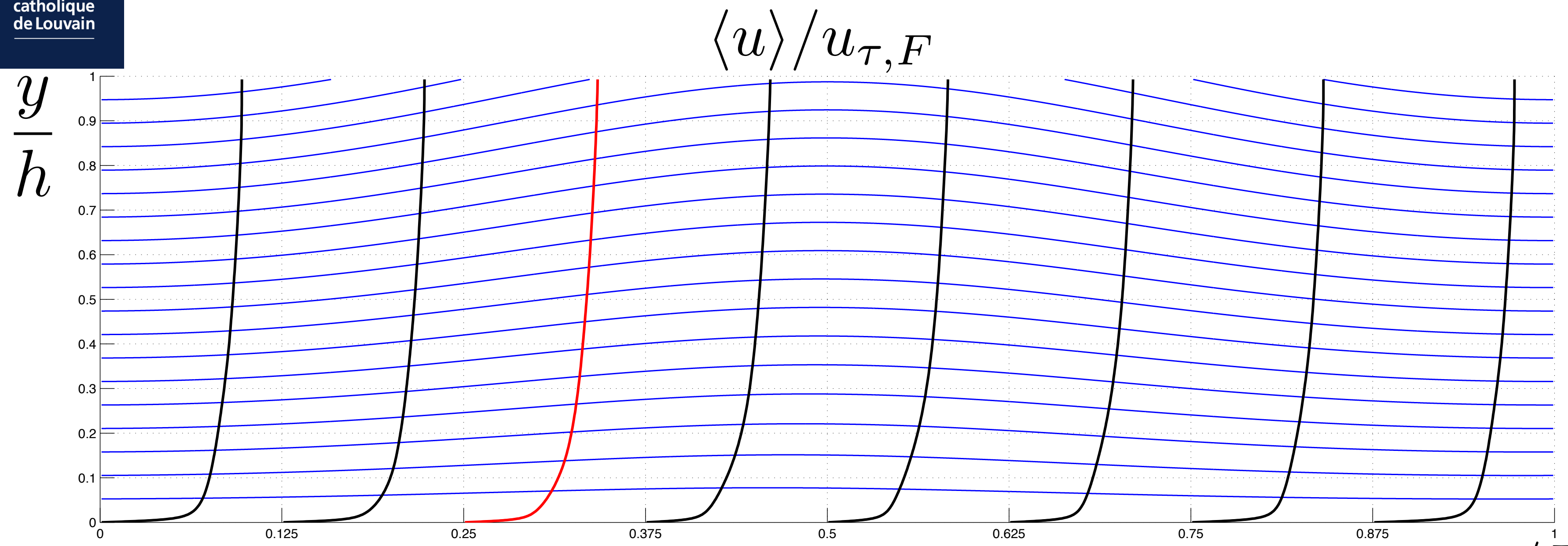


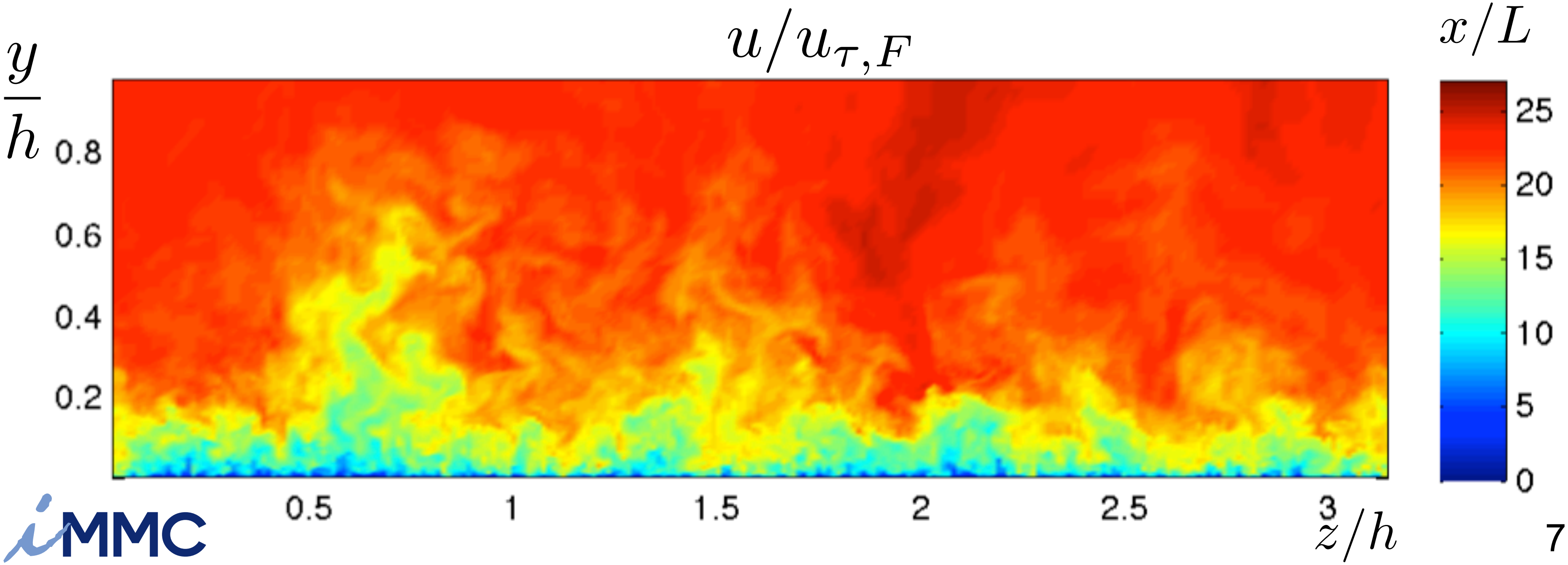
Streamwise velocity at x/L = 0



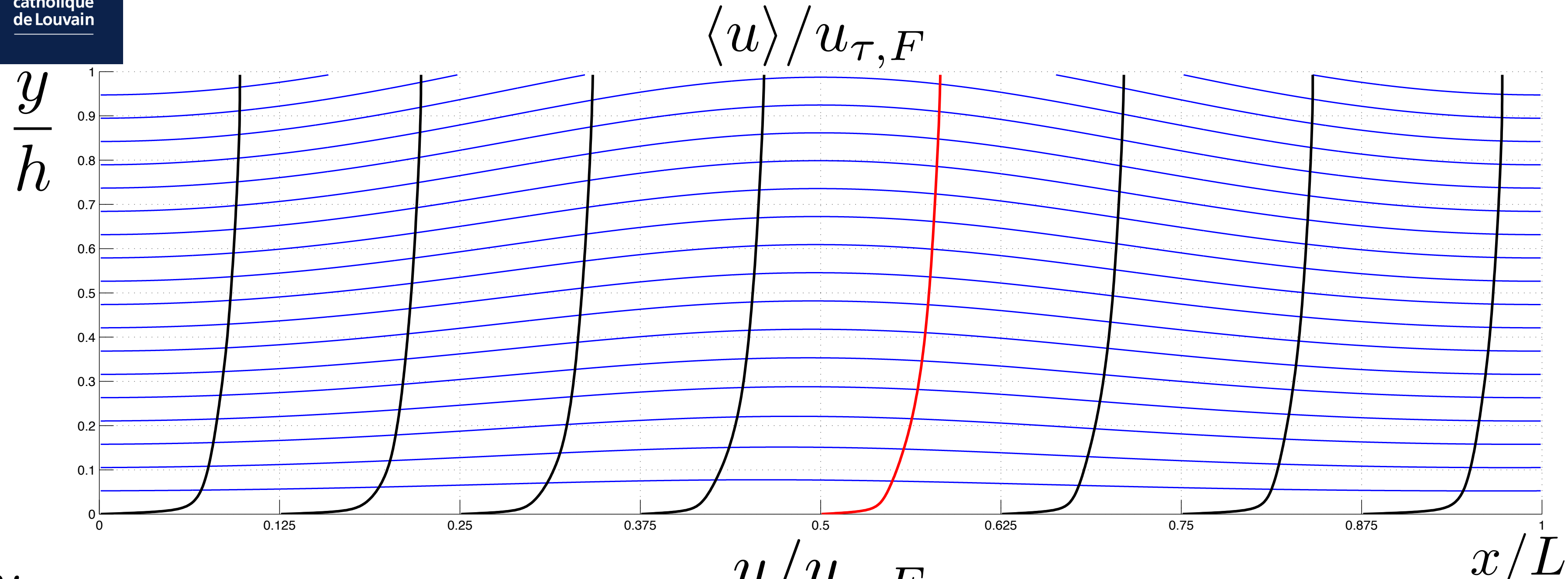


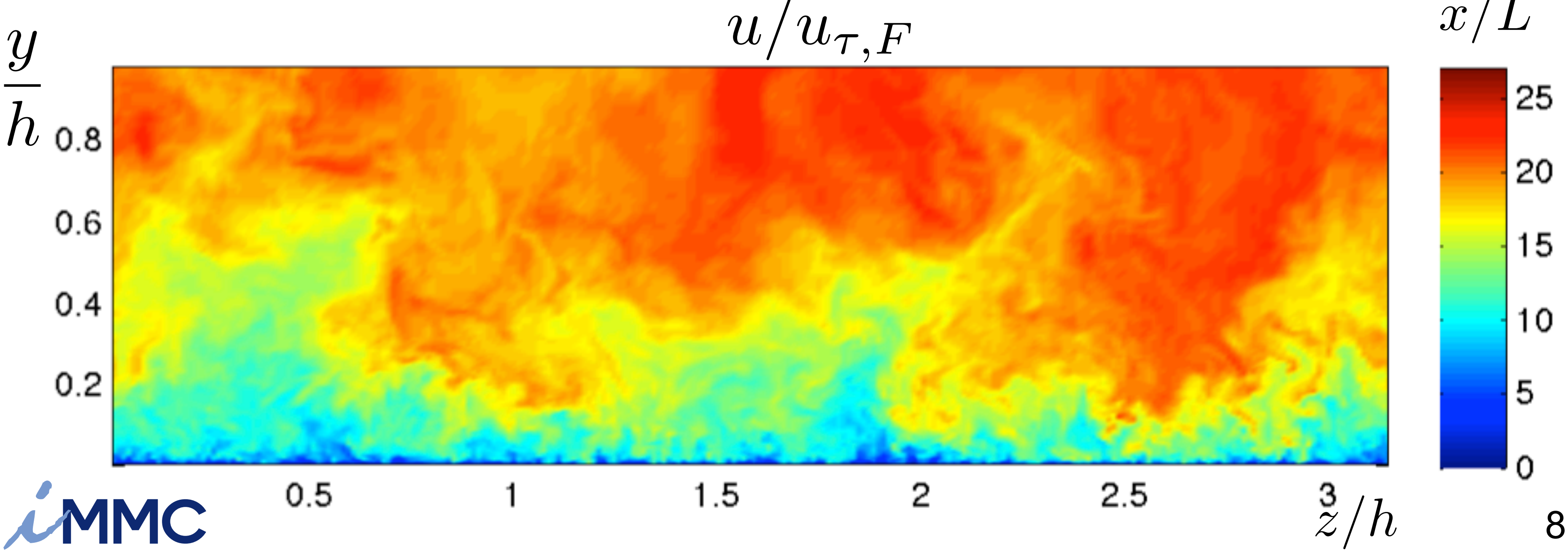
$u/u_{\tau,F}$ at $x/L = 0.25$; maximum suction



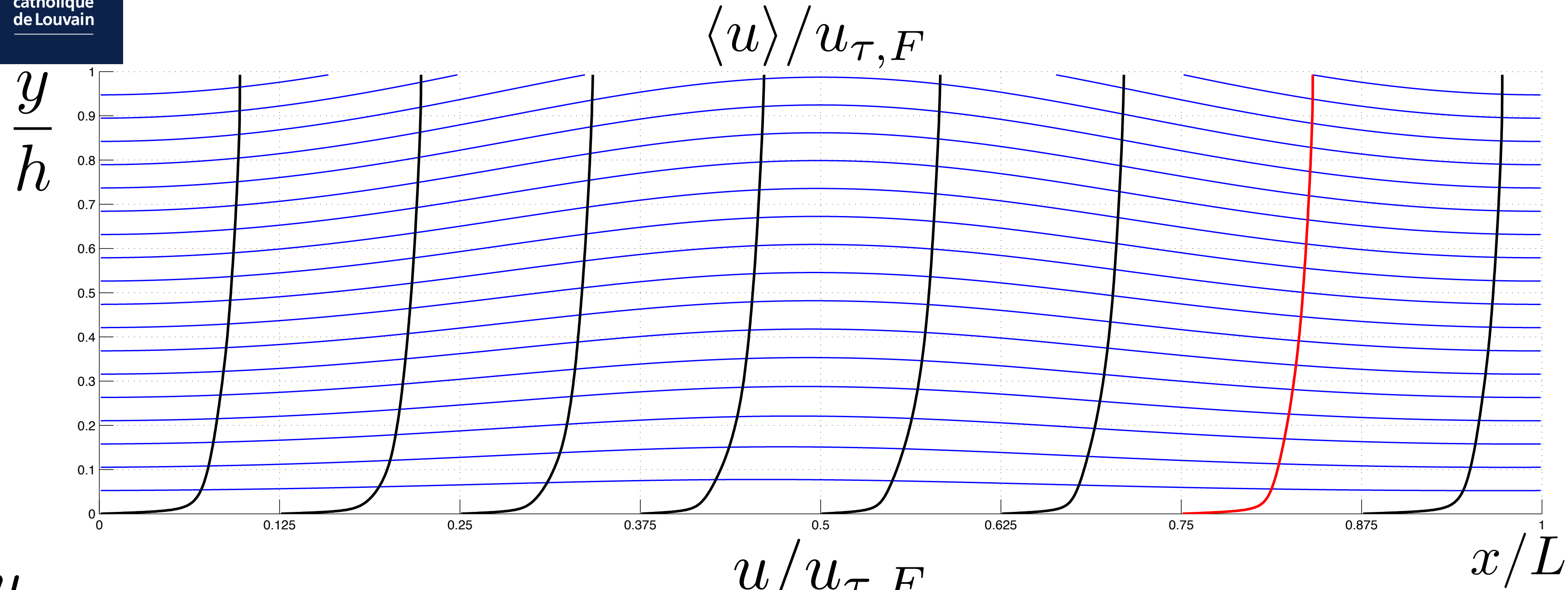


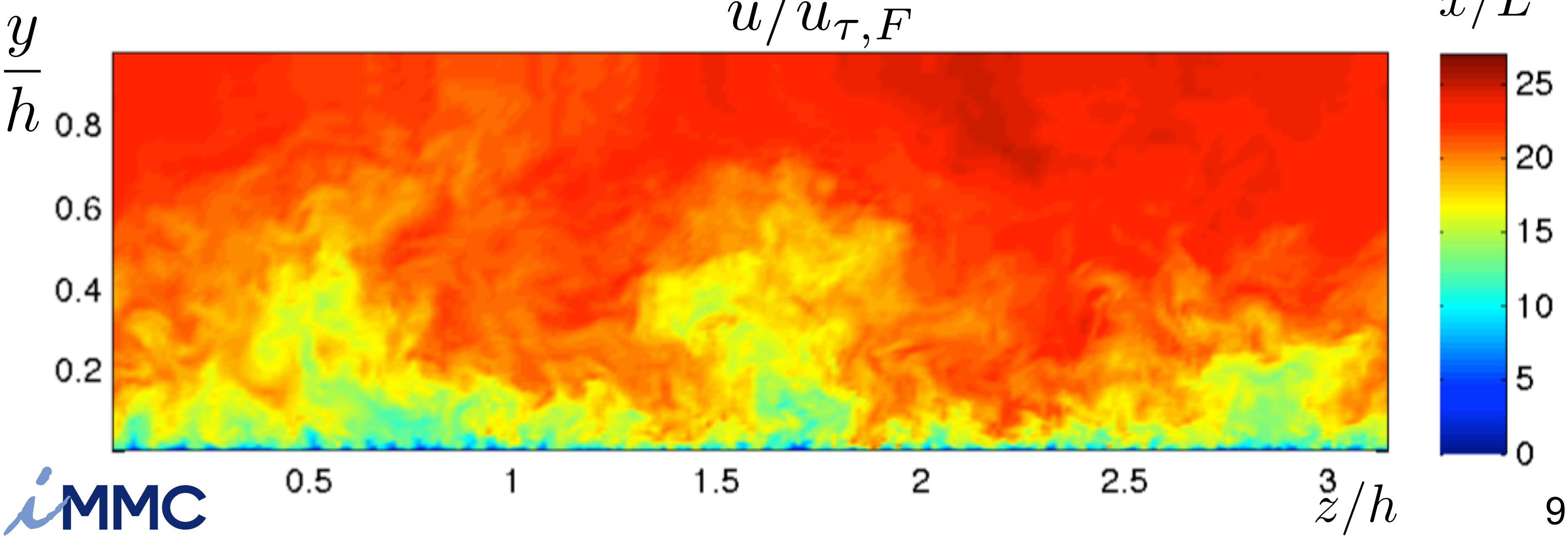
$u/u_{\tau,F}$ at x/L = 0.5





$u/u_{\tau,F}$ at x/L = 0.750; maximum blowing

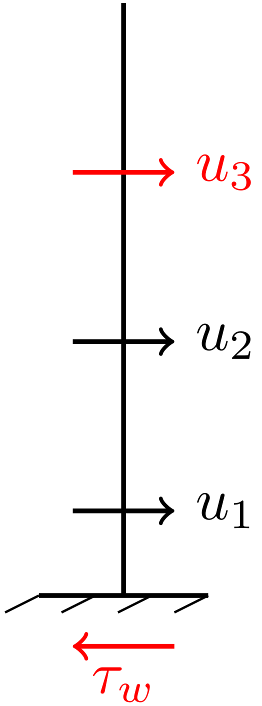




Algebraic wall models

- Friction velocity is obtained from a prescribed *law of the wall* and *averaged* (here a 5x5 window) flow information at point y_3

$$\langle u_\tau \rangle = \frac{|\langle u \rangle|}{f\left(y_3 \langle u_\tau \rangle / \nu\right)}$$



- Wall friction is computed from *averaged* and *local* values

$$\tau_w = \rho \frac{\mathbf{u}|\mathbf{u}|}{|\langle \mathbf{u}|\mathbf{u}| \rangle|} \langle u_\tau \rangle^2$$

Pressure gradient effects

- Velocity scale $$u_p = \left(\nu \left|\frac{dP}{d\mathbf{x}}\right|\right)^{1/3}$$

$$p^+ = \frac{\nu}{u_\tau^3}\frac{dP}{dx}$$

- Shih wall function:

$$\mathbf{u} = \mathbf{u_1} + \mathbf{u_2}$$

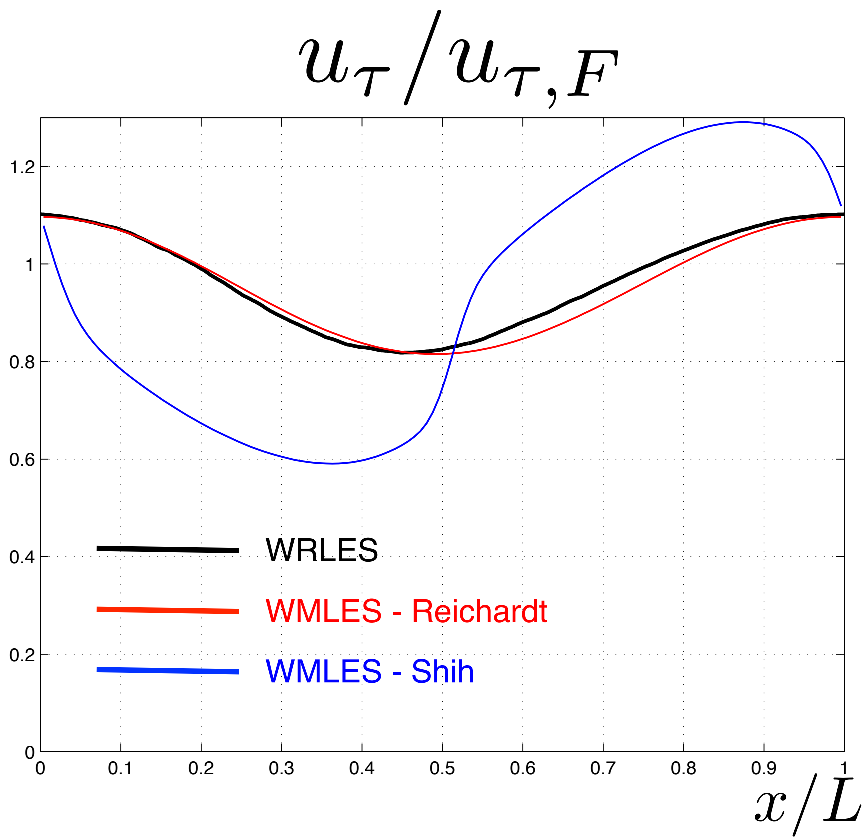
Due to wall shear stress

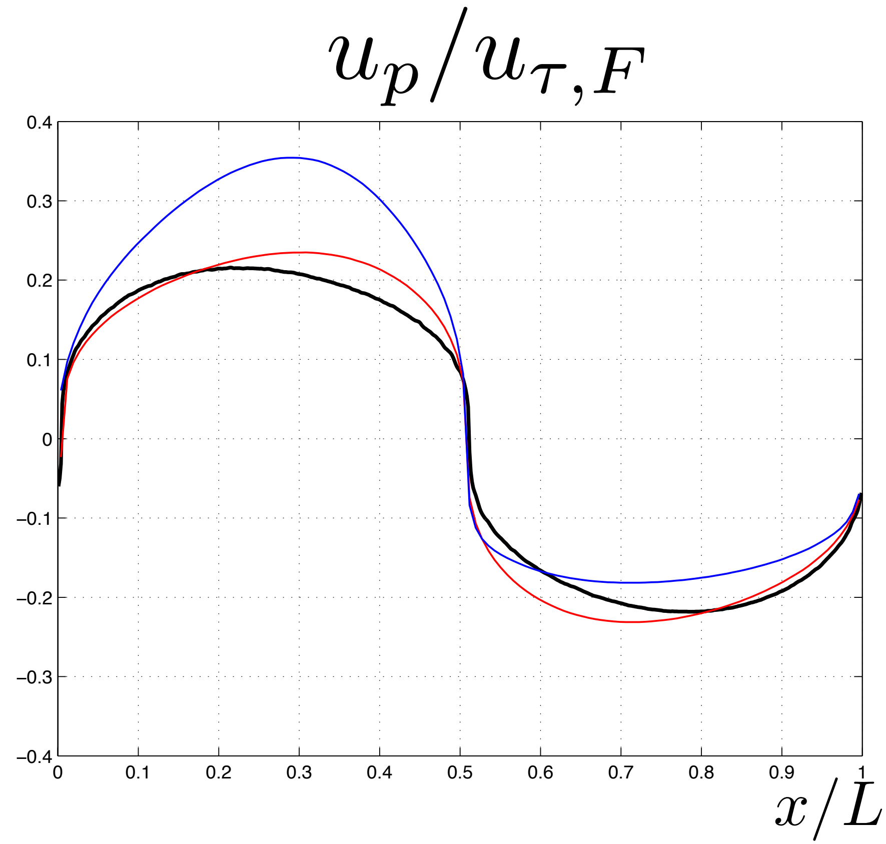
Due to pressure gradient

$$\frac{\mathbf{u_1}}{u_\tau} = f\left(\frac{yu_\tau}{\nu}\right)\hat{\mathbf{n}}_\tau \qquad \frac{\mathbf{u_2}}{u_p} = g\left(\frac{yu_p}{\nu}\right)\hat{\mathbf{n}}_\mathbf{p}$$

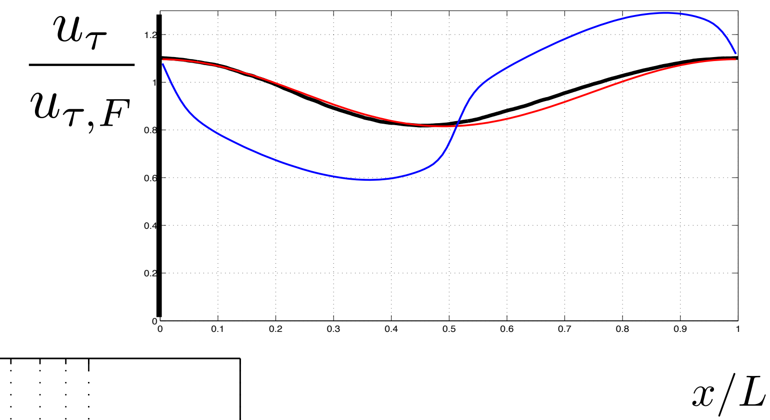
Shih, Povinelli, Liu, Chen, Generalized wall function for complex turbulent flows, AIAA 2000

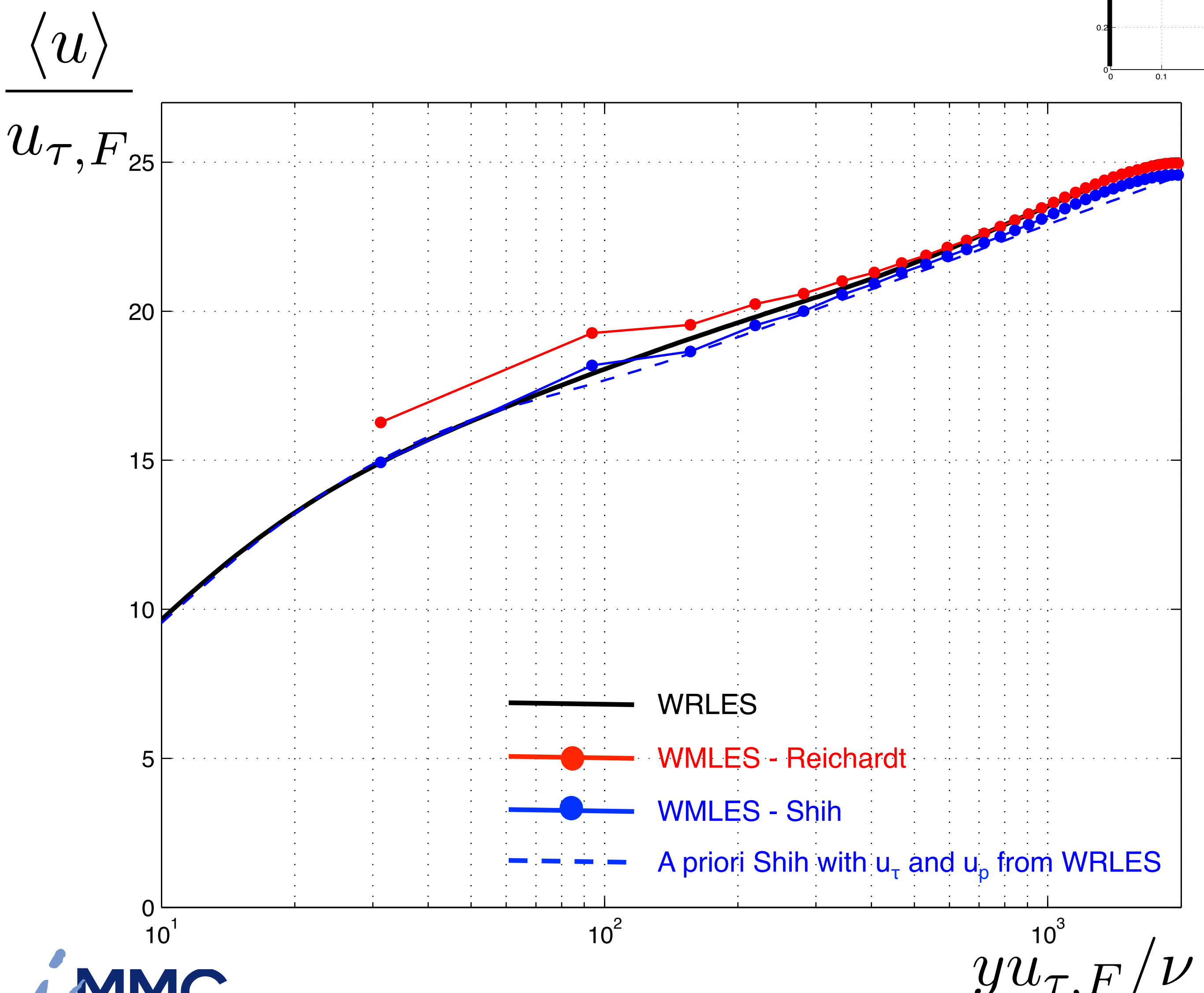
Algebraic models: Wall shear stress and wall pressure



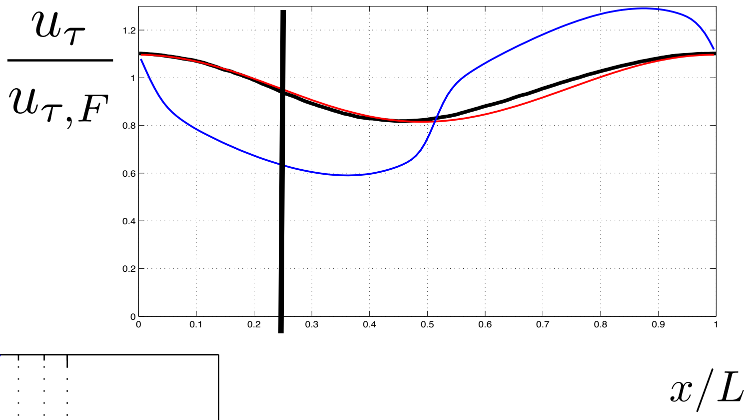


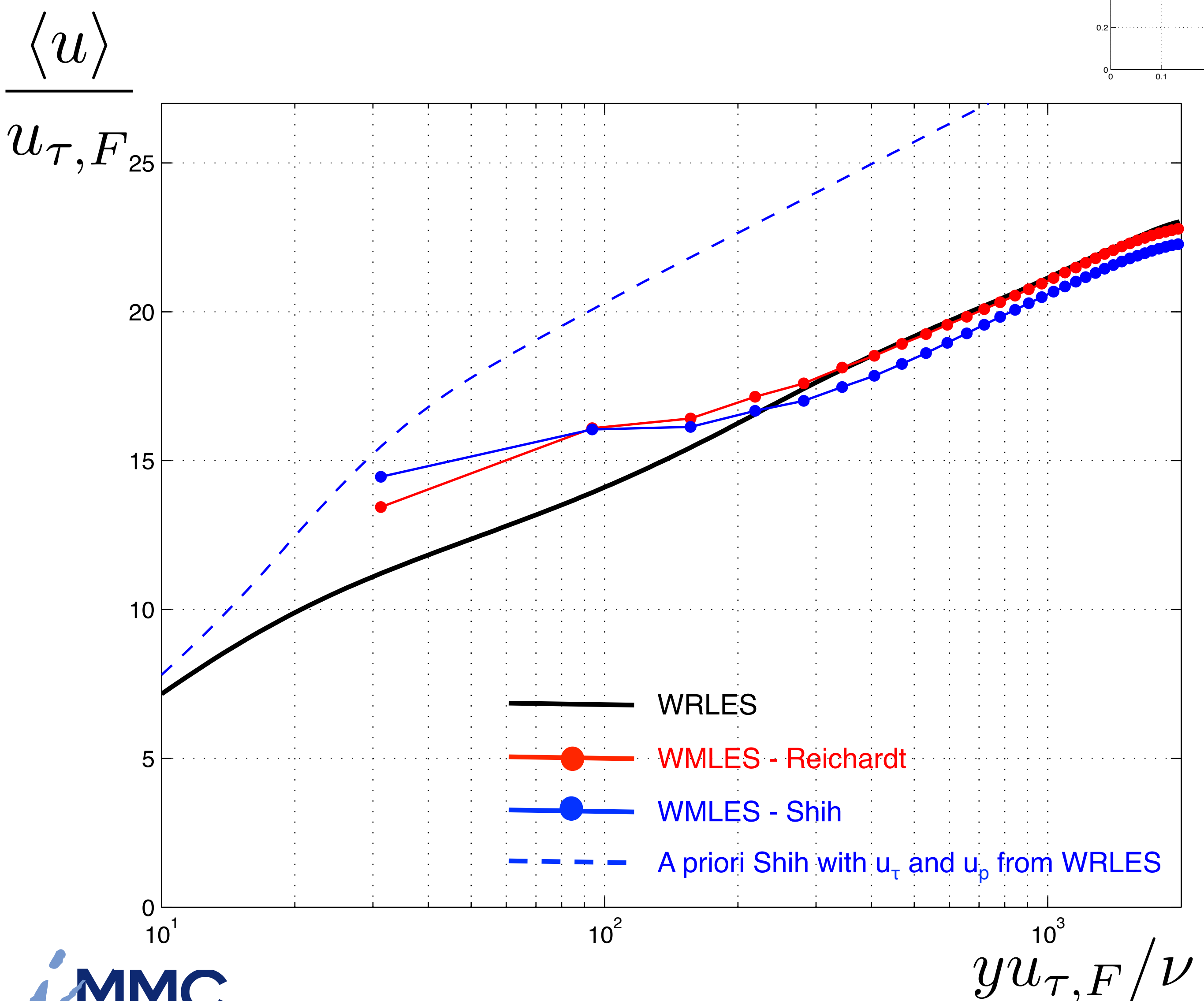
Algebraic models: Velocity profiles



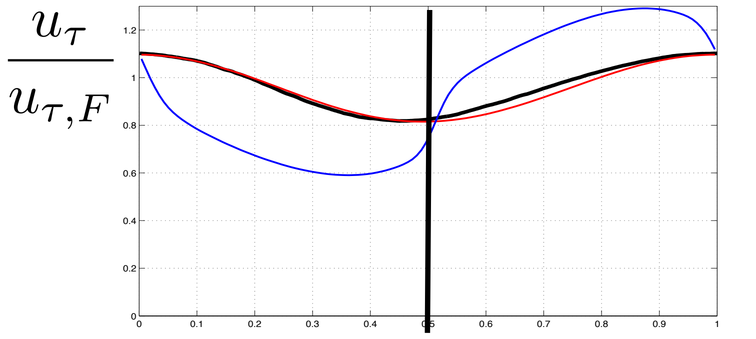


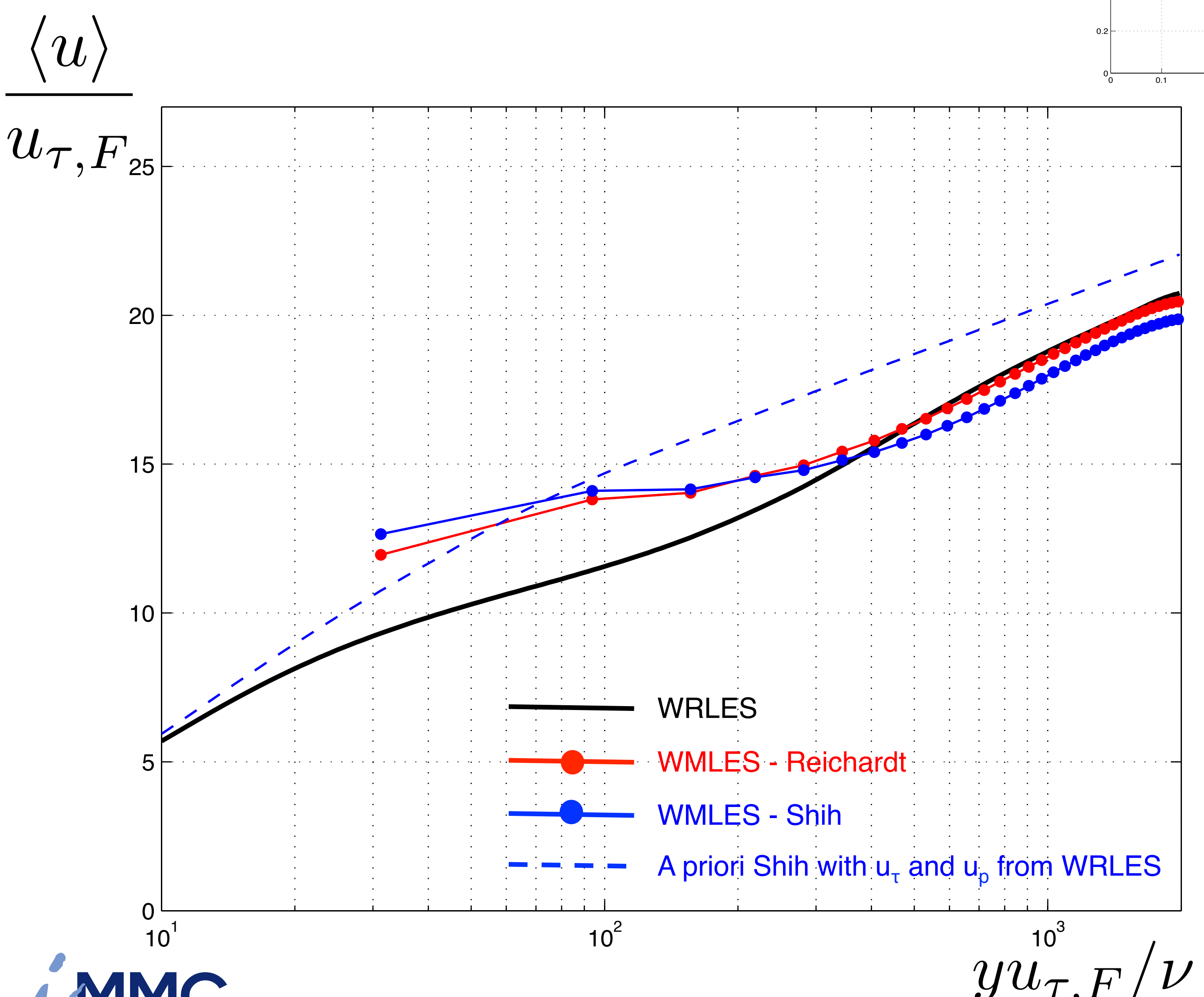
Algebraic models: Velocity profiles



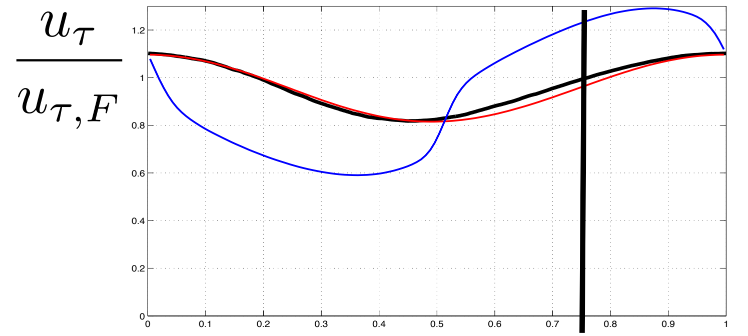


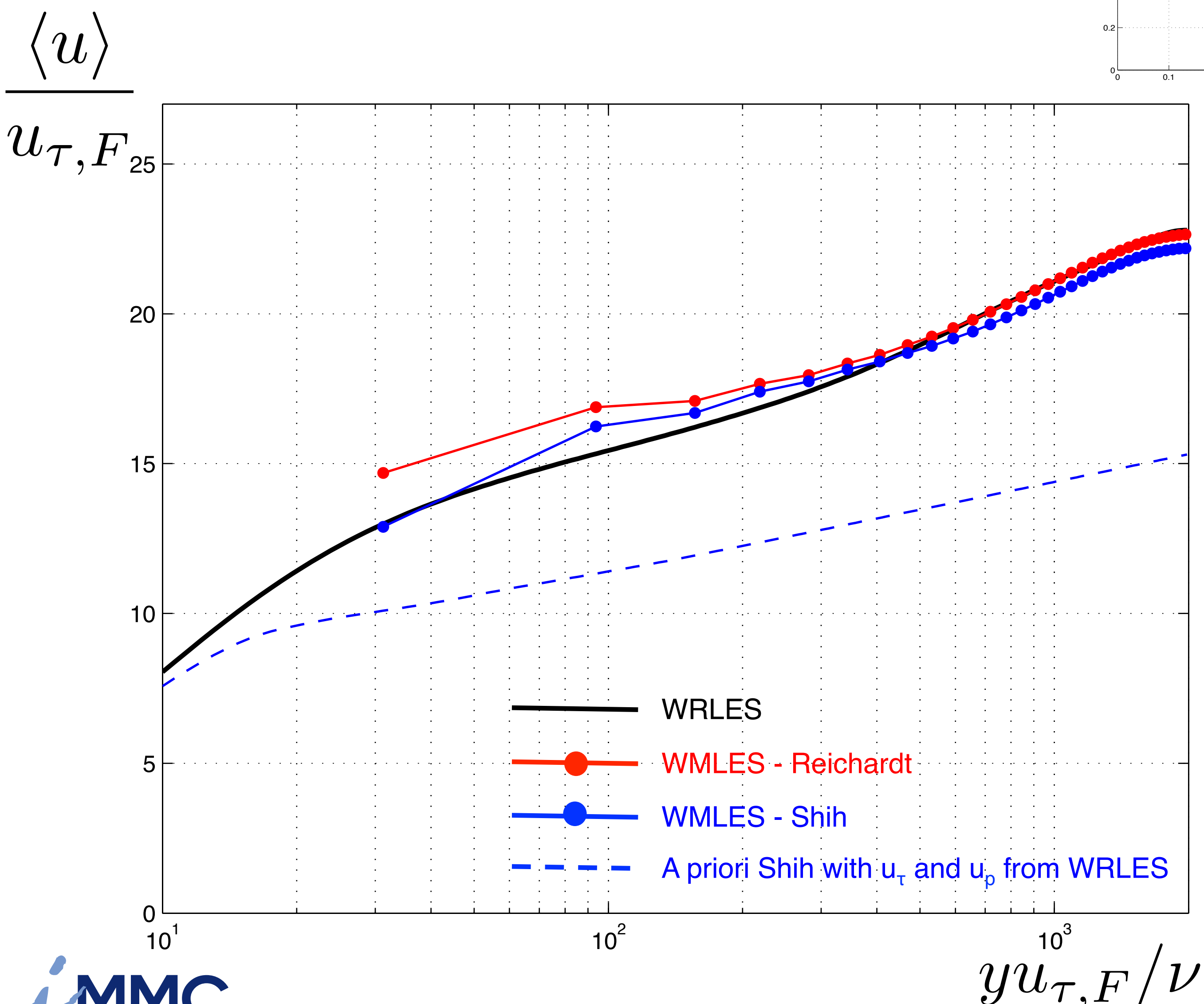
Algebraic models: Velocity profiles



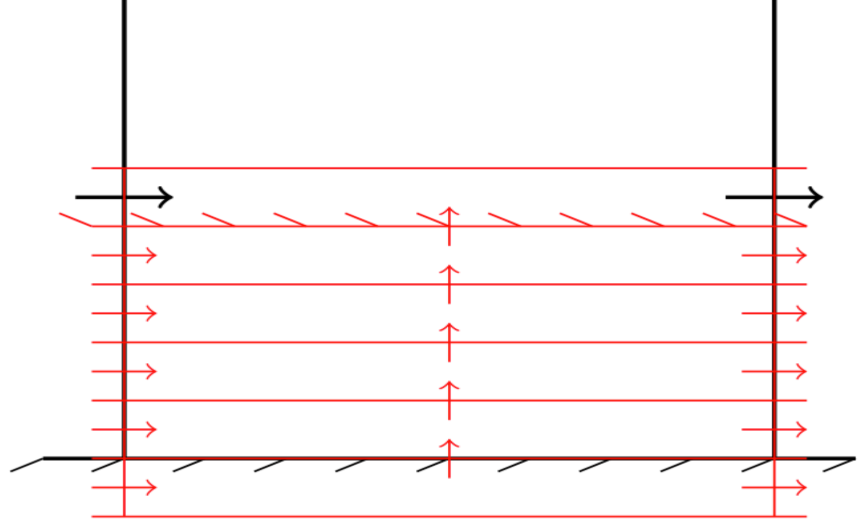


Algebraic models: Velocity profiles





RANS sub-layer models



- TBL equations with a RANS model

Convective terms   Pressure terms

$$\frac{\partial u_i}{\partial t} + \frac{\partial}{\partial x_j}(u_i u_j) = -\frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_2}\left((\nu + \nu_t)\frac{\partial u_i}{\partial x_2}\right), \qquad i = 1, 3$$

$$u_2 = -\int_0^{x_2} \left(\frac{\partial u_1}{\partial x_1} + \frac{\partial u_3}{\partial x_3}\right) dx_2'$$

- Cabot and Moin (2000)

$$\nu_t = \kappa y u_\tau \left(1 - e^{-\frac{y^+}{17}}\right)^2$$

RANS models with pressure gradient effects

- Duprat (2011)

$$\frac{\nu_t}{\nu} = \kappa y^* \left(\alpha + y^*(1-\alpha)^{3/2}\right)^\beta \left(1 - e^{-\frac{y^*}{1+A\alpha^3}}\right)^2$$

$$u_{\tau p} = \sqrt{u_\tau^2 + u_p^2} \qquad \alpha = \left(\frac{u_\tau}{u_{\tau p}}\right)^2$$

$$y^* = \frac{y u_{\tau p}}{\nu}$$

- Granville (1989)

$$\nu_t = \tau_t \left(\kappa y \left(1 - e^{-\frac{y^+\sqrt{1+b\;a\;p^+}}{26}}\right)\right)^2 |S|$$

$$\tau_t = 1 + a\,p^+ y^+$$

Correcting the effective viscosity in the RANS

- Some fluctuations are resolved on the RANS mesh

- Get ν_t from RANS model and correct to account for those fluctuations:

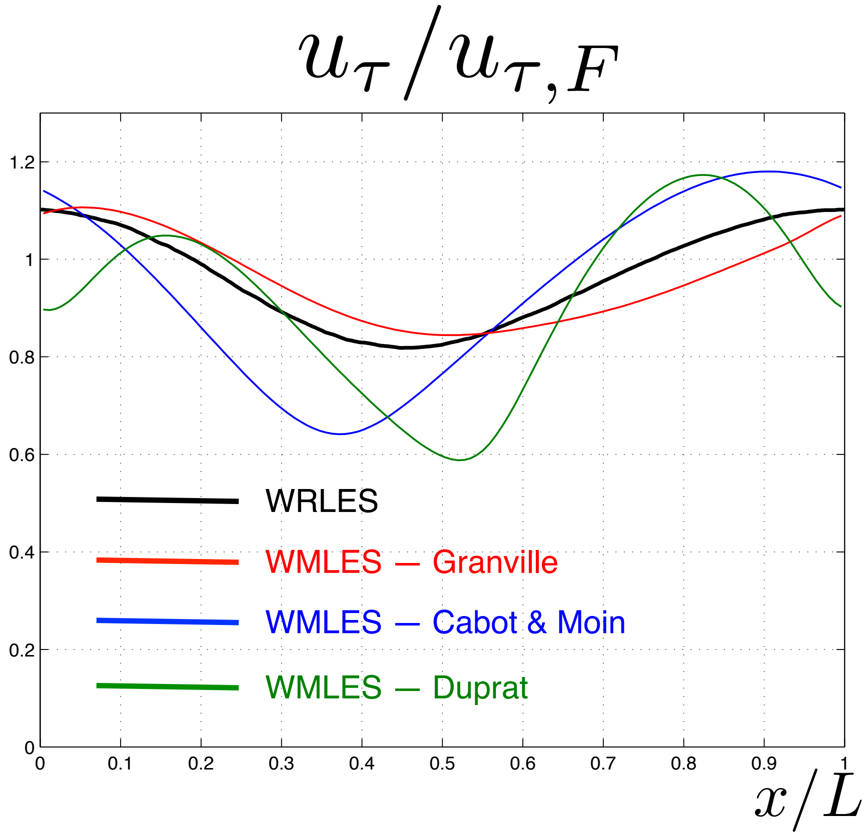
$$\nu_{t,corr} = \nu_t + \frac{\overline{uv}}{\frac{\partial \overline{u}}{\partial y}}$$

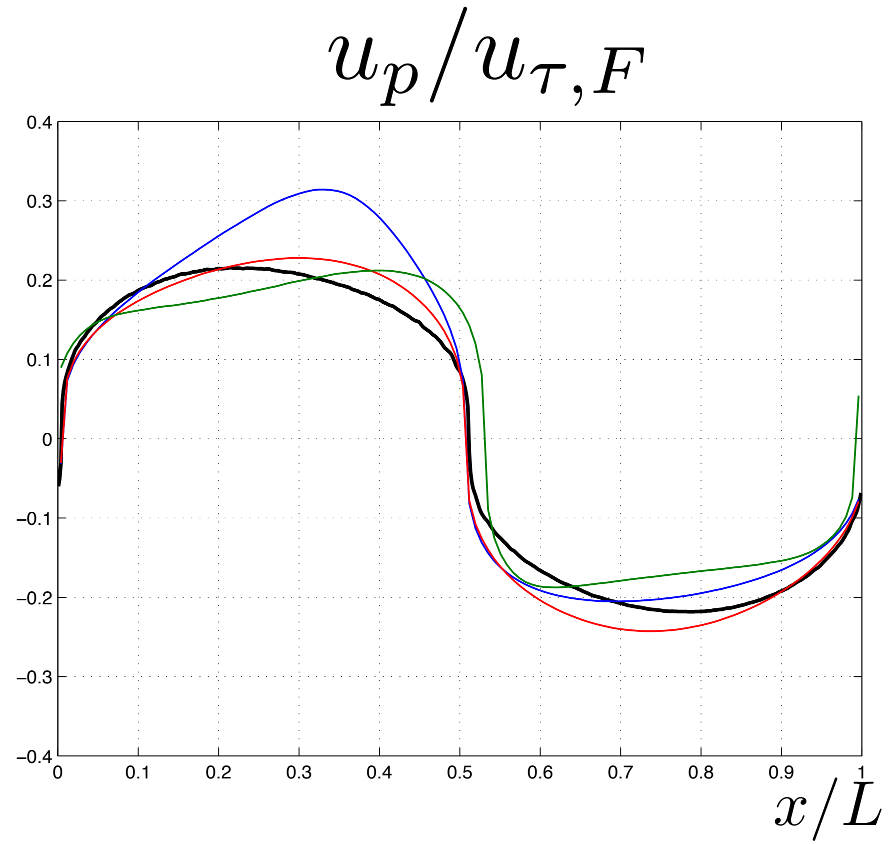
RANS averaging

Medic, Daeninck, Templeton, Kalitzin, A framework for near-wall RANS/LES coupling, CTR Annual Research Brief 2005

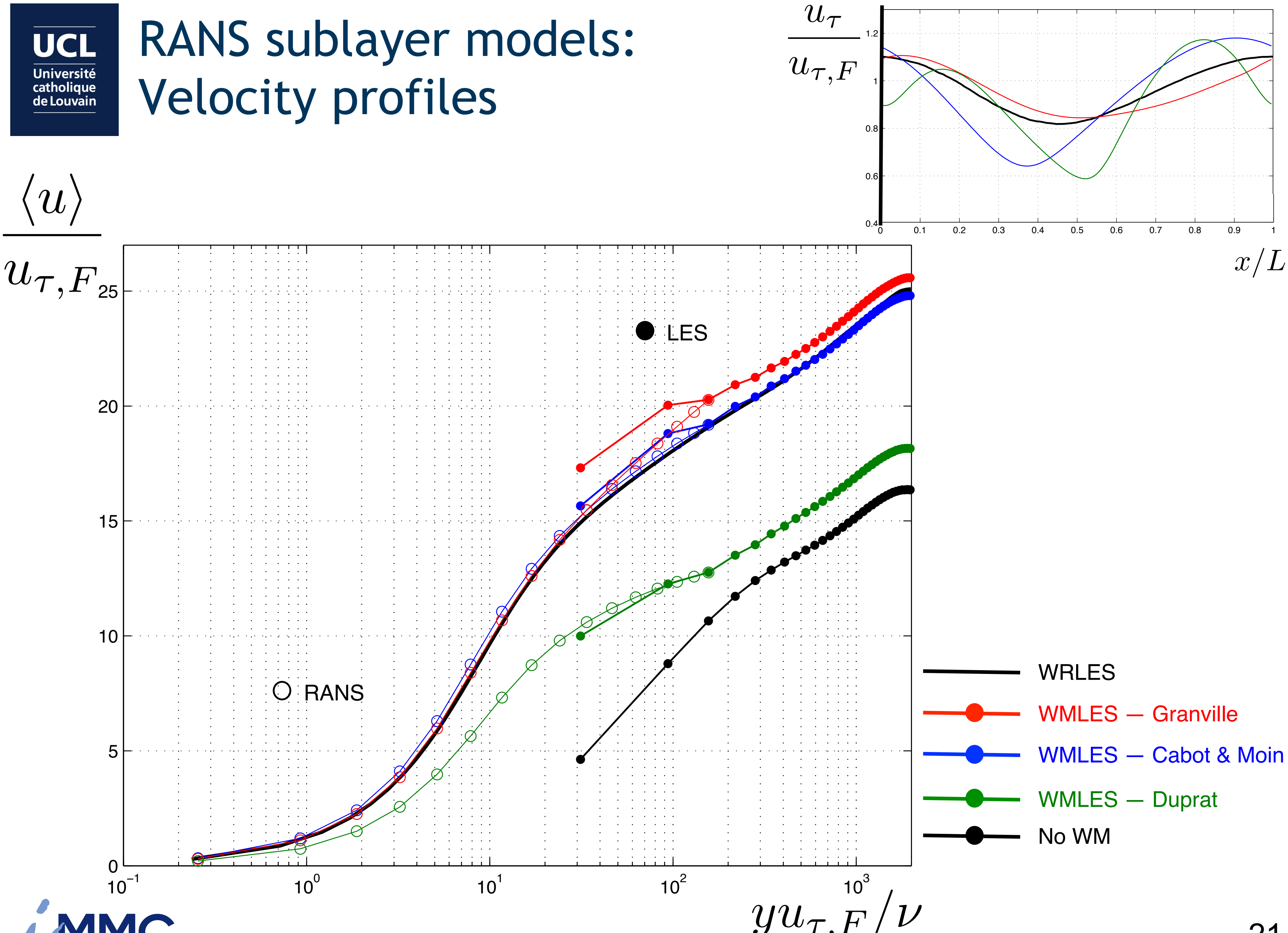
Medic, Templeton, Kalitzin, A formulation for near-wall RANS/LES coupling, International Journal of Engineering Science 44 (2006)

RANS sublayer models: Wall shear stress and wall pressure

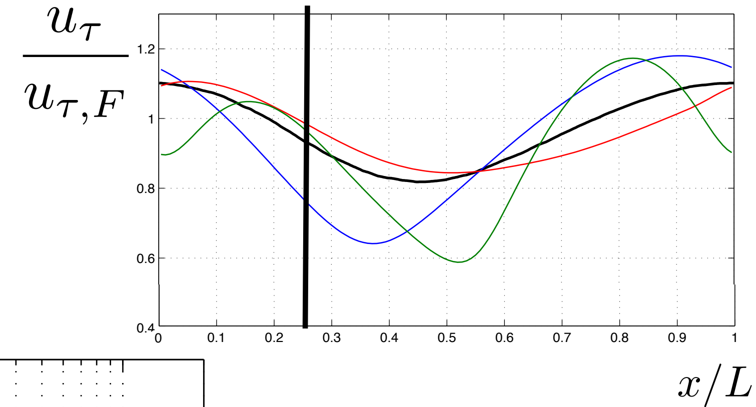


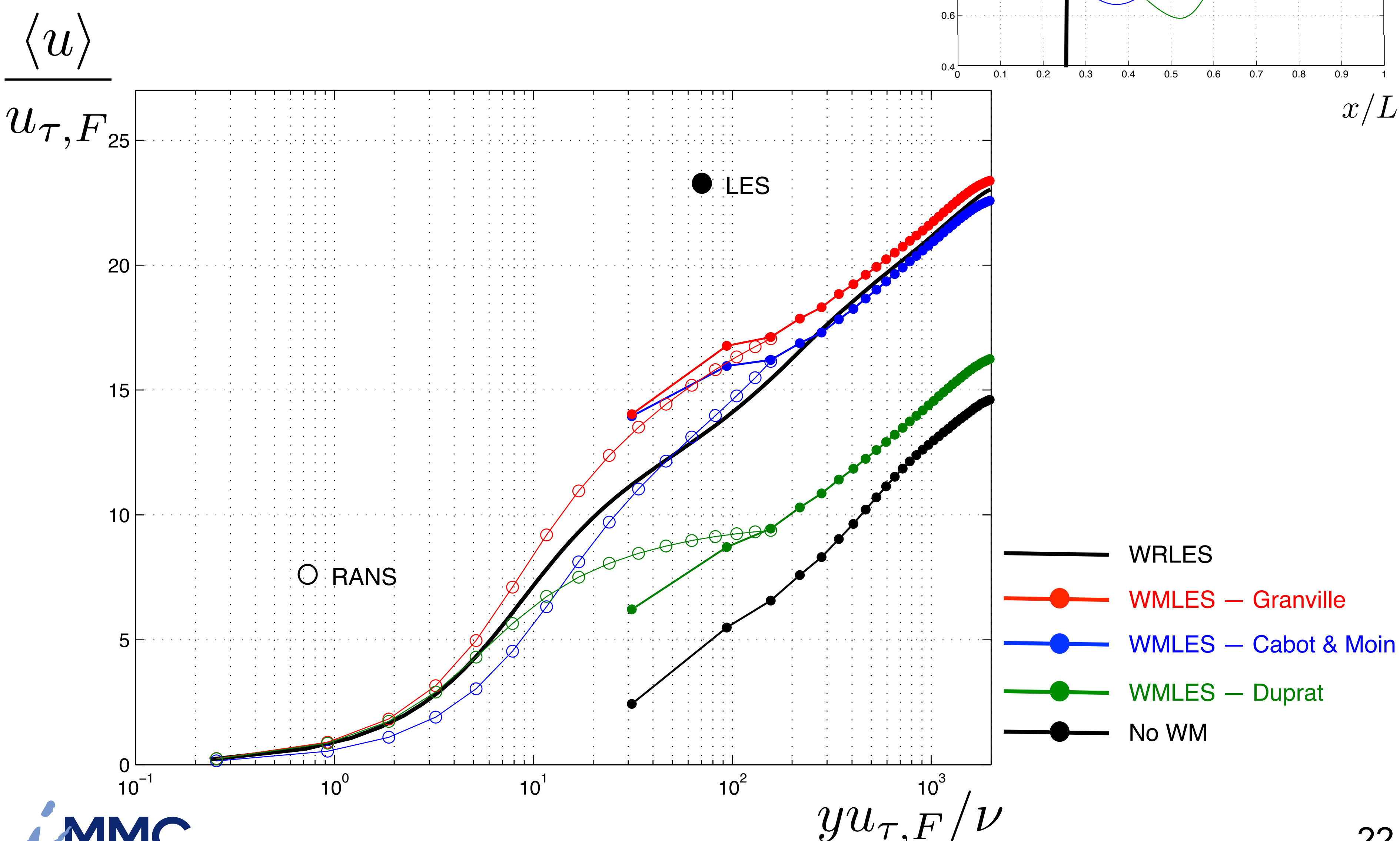


RANS sublayer models: Velocity profiles



RANS sublayer models: Velocity profiles





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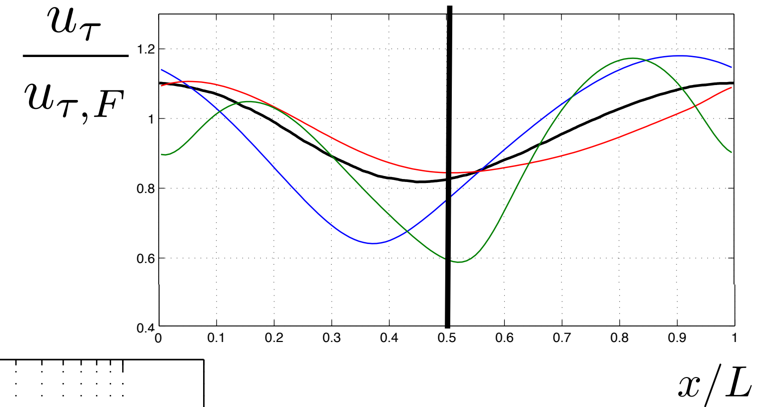
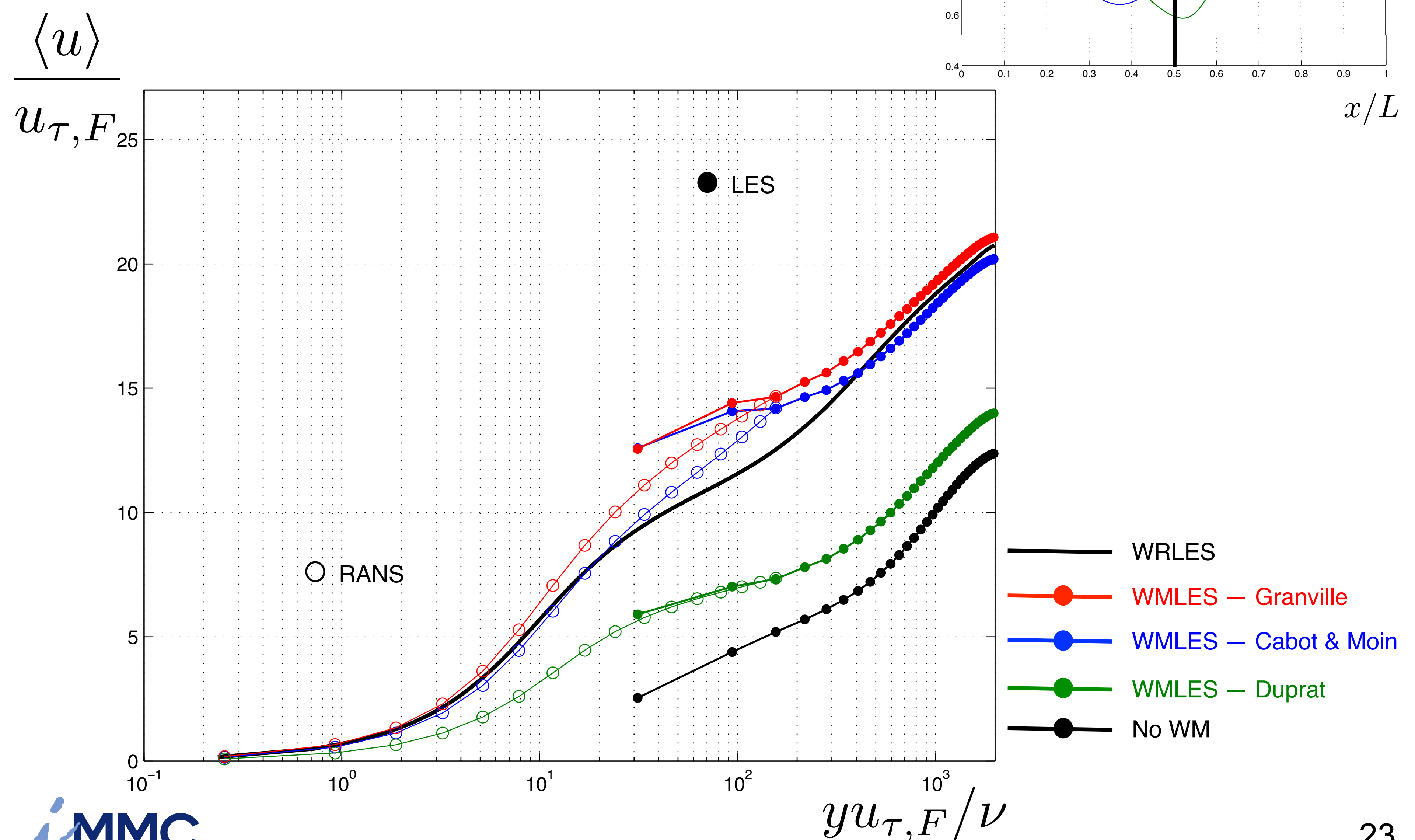
RANS sublayer models: Velocity profiles





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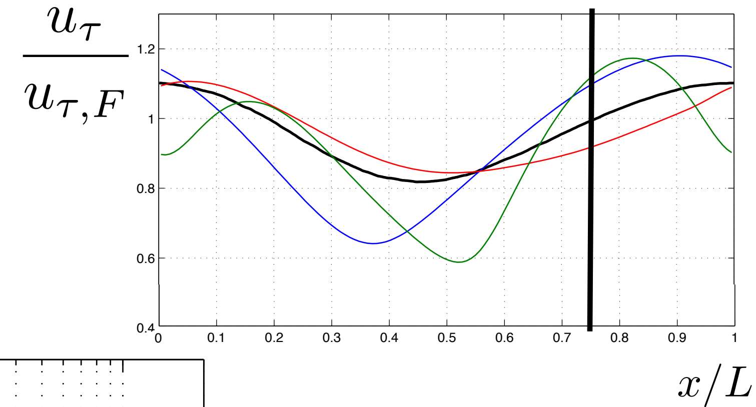
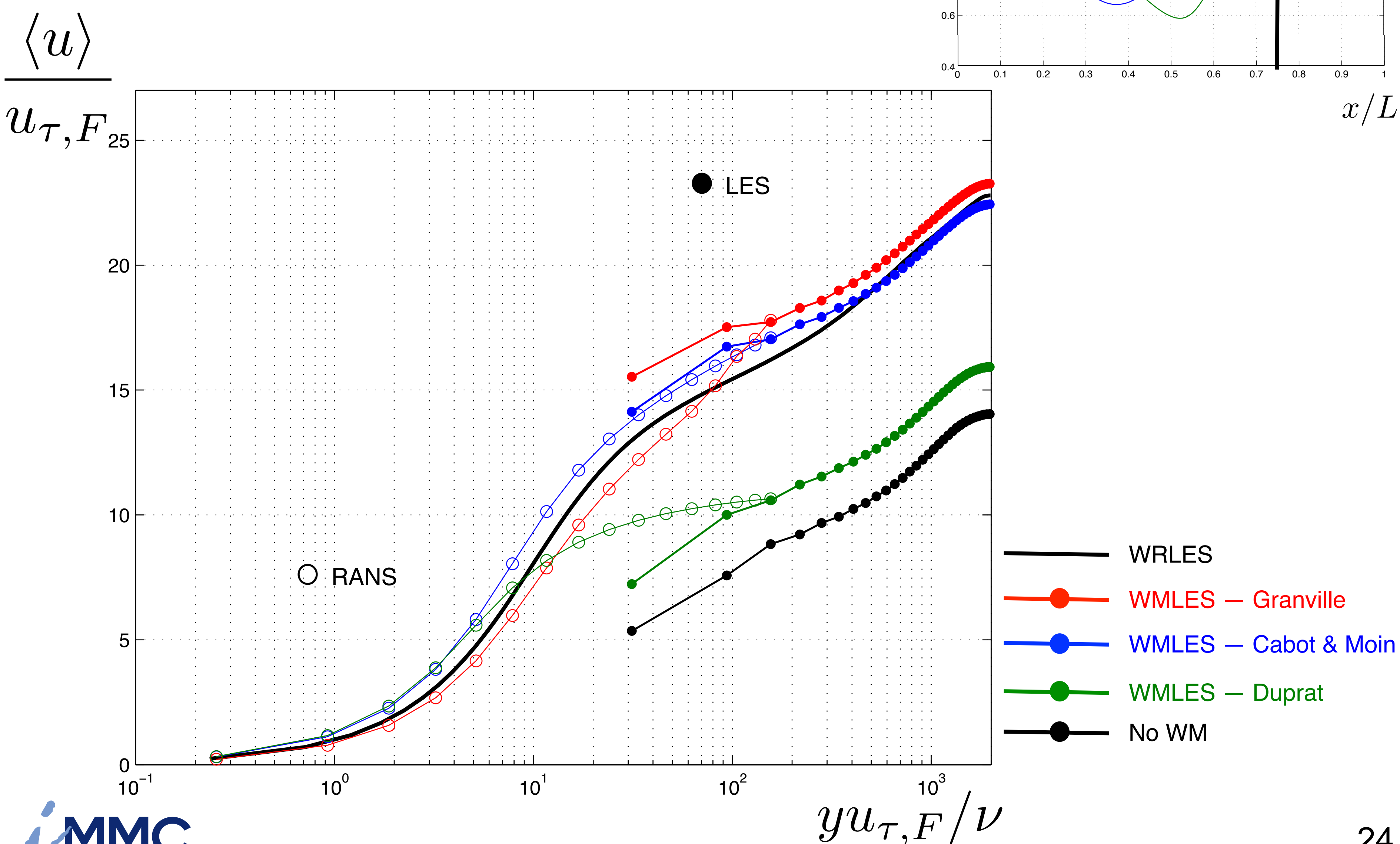
RANS sublayer models: Velocity profiles





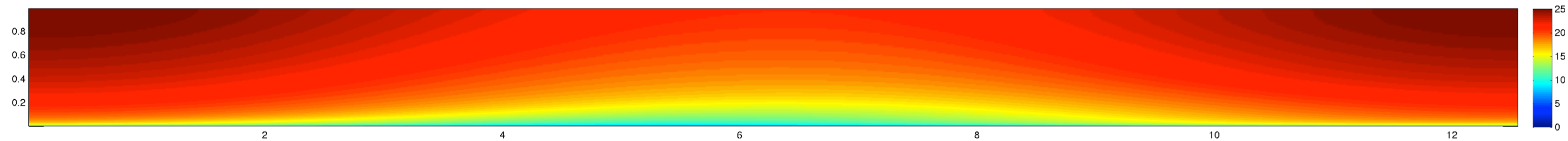
Conclusions and perspectives

- Mild pressure gradient, yet challenging test case for wall models
- Algebraic models: Reichardt law performs best for such a **mild** pressure gradient case
- RANS sublayer models: Granville ν_t model performed reasonably well
- Investigate other averaging strategies, also in time

Questions ?

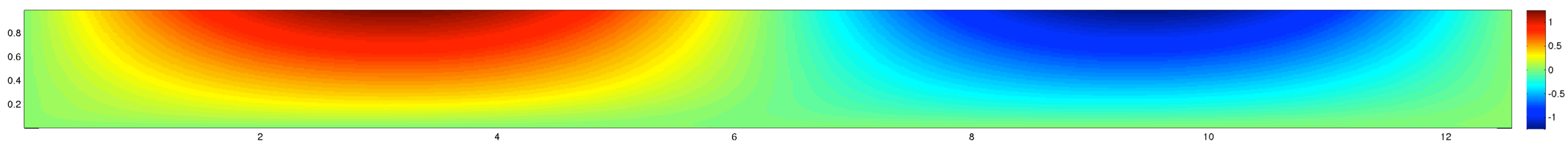
Mean fields, scale 1:1

y/h $\langle u\rangle/u_{\tau,F}$



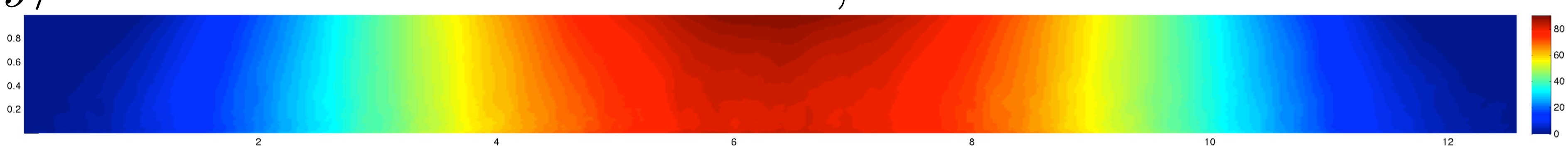
x/h

y/h $\langle v\rangle/u_{\tau,F}$



x/h

y/h $\langle P\rangle/u_{\tau,F}^2$



x/h

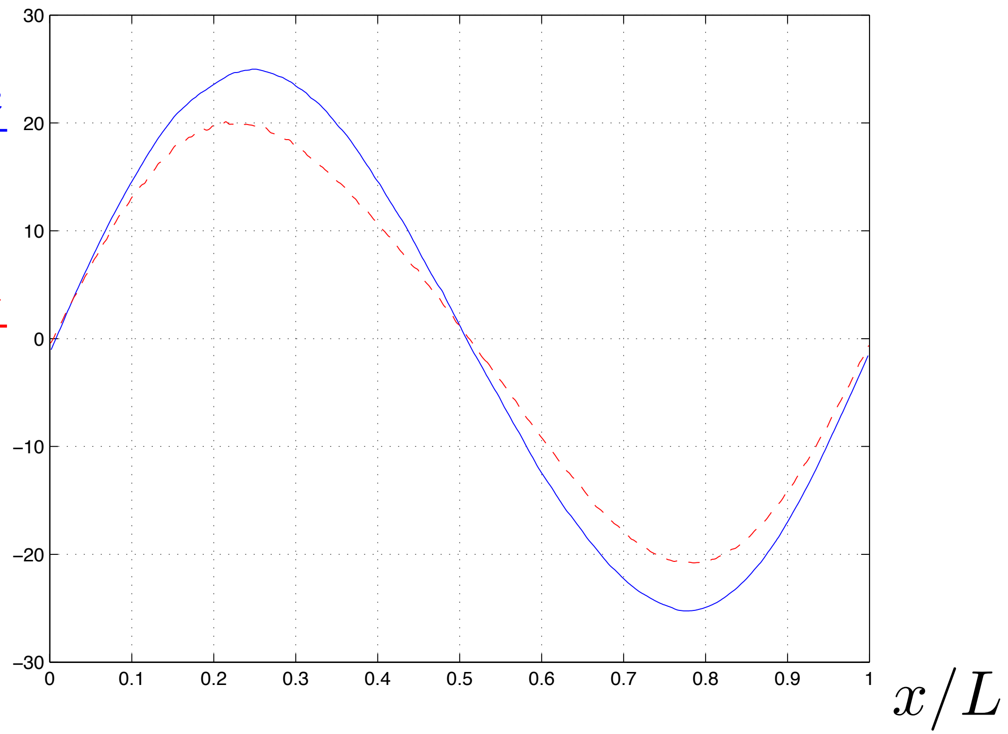
Analogy with accelerated/decelerated TBL

$$\frac{\partial u_i}{\partial x_i} = 0$$

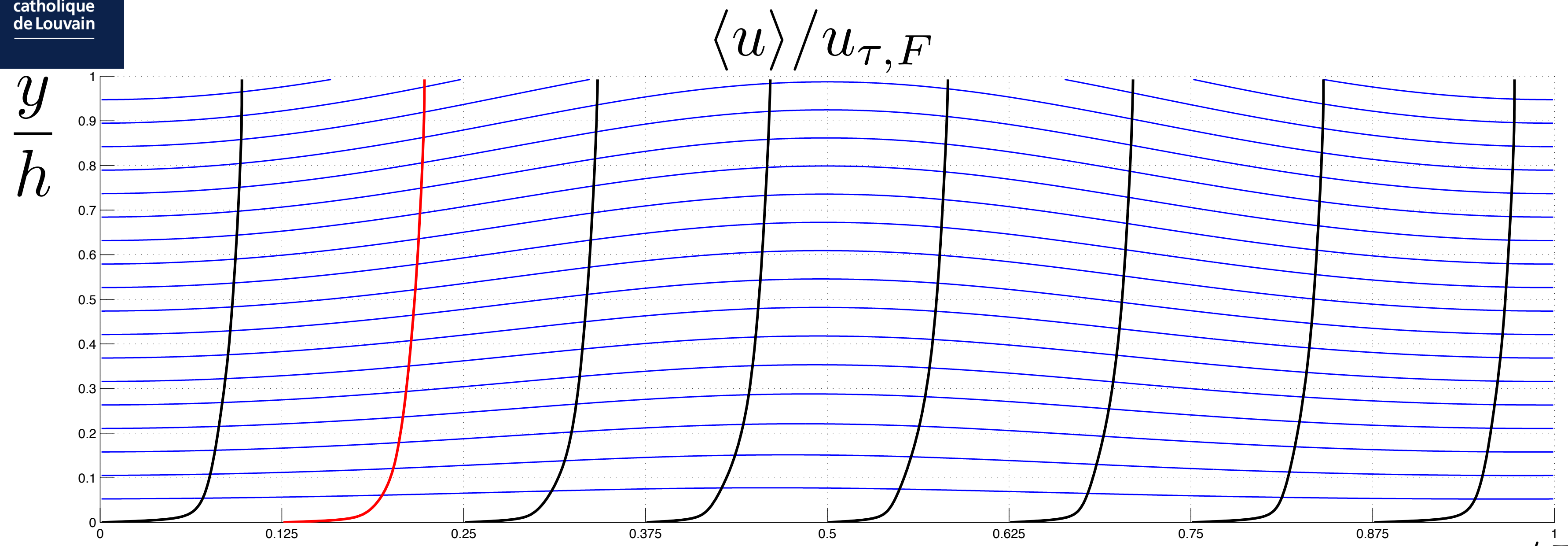
$$\frac{\partial u_i}{\partial t} + \frac{\partial u_i u_j}{\partial x_j} = u_e \frac{\partial u_e}{\partial x_i} + \frac{\partial}{\partial x_j}\Big(2\nu S_{ij}\Big)$$

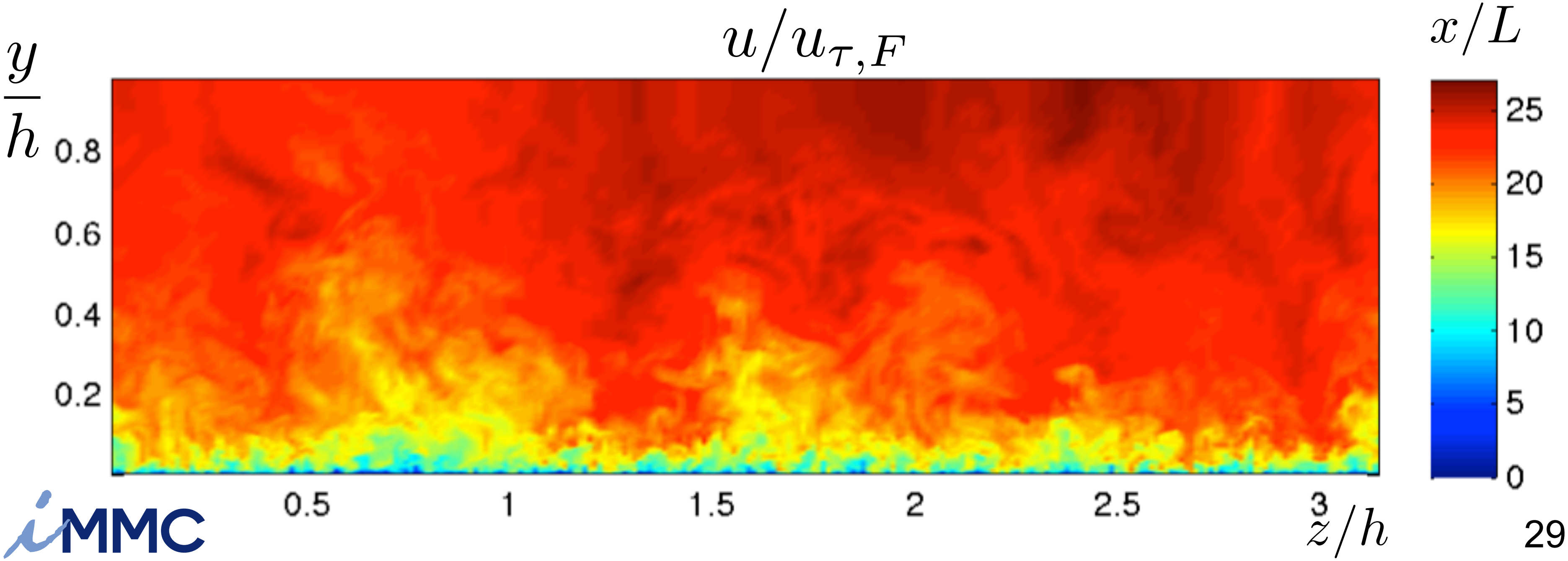
$$-\frac{h}{u_{\tau,F}^2} u_e \frac{du_e}{dx}$$

$$\frac{h}{u_{\tau,F}^2} \frac{dP_e}{dx}$$

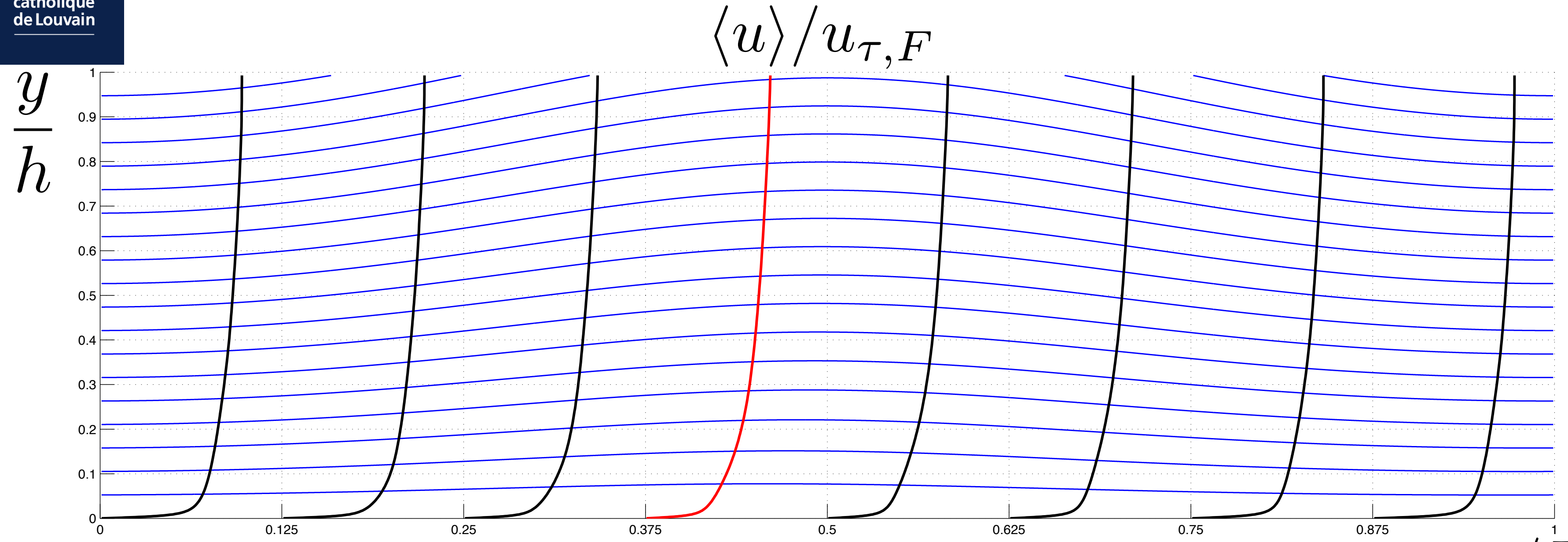


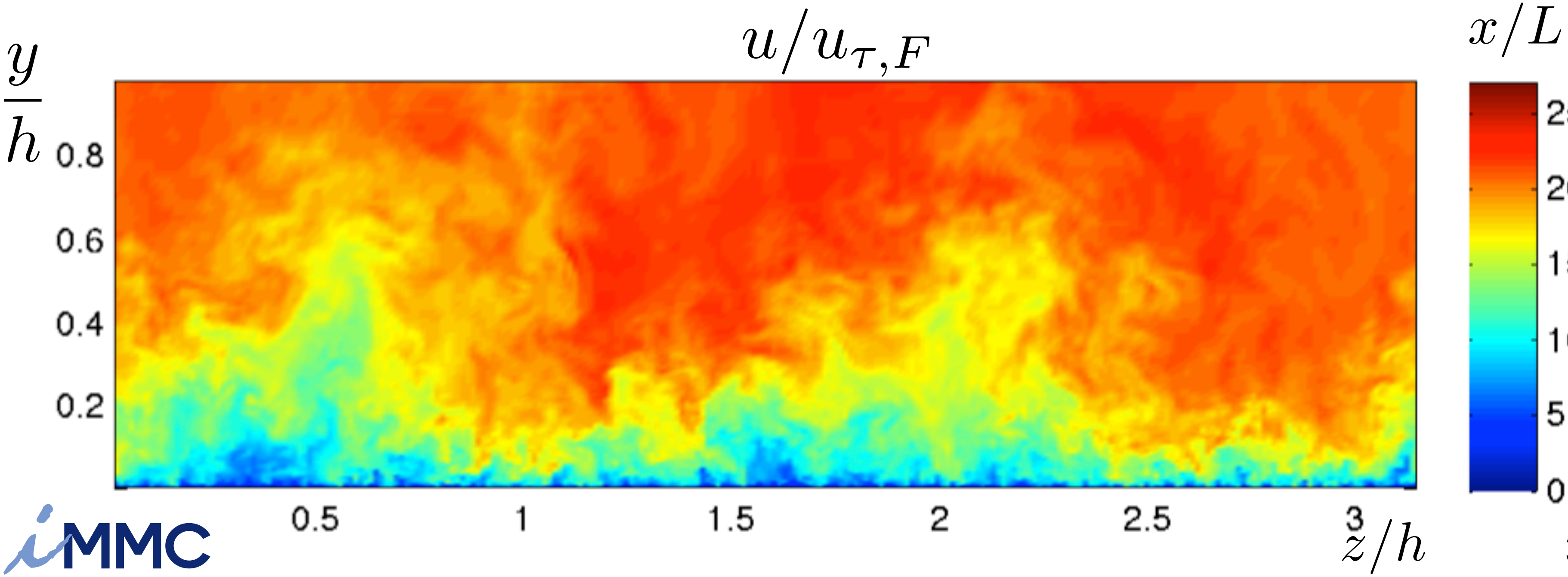
$u/u_{\tau,F}$ at x/L = 0.125



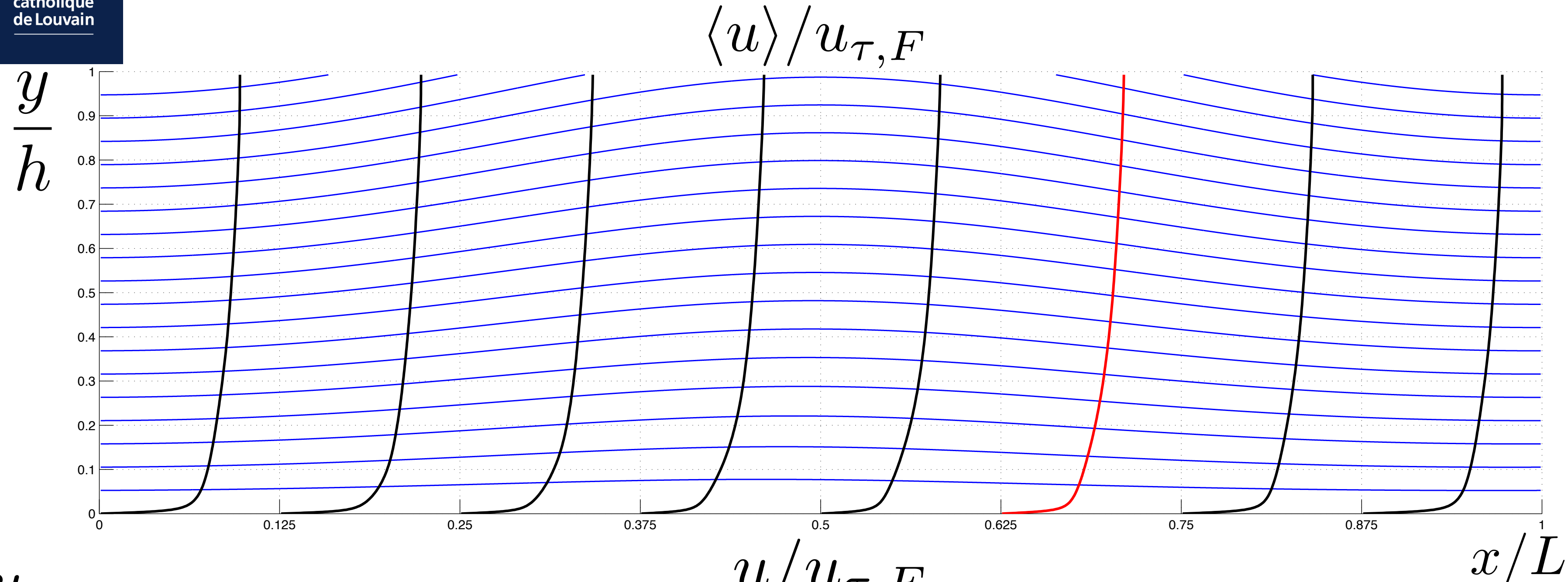


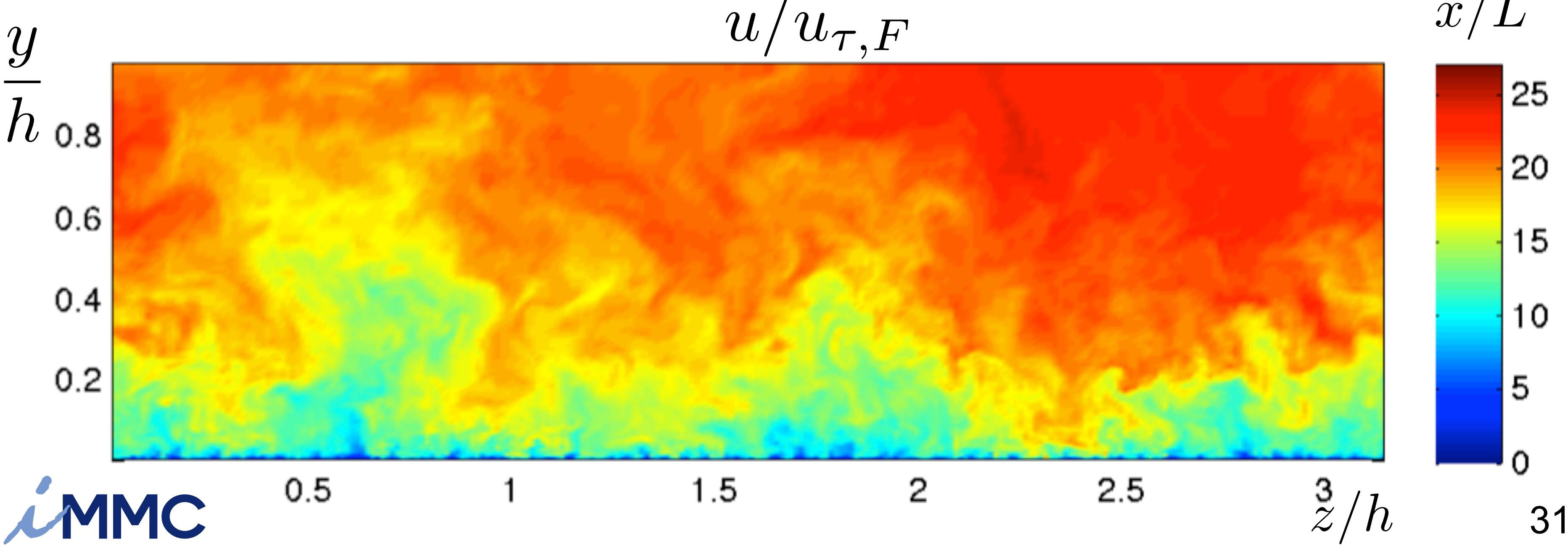
$u/u_{\tau,F}$ at x/L = 0.375



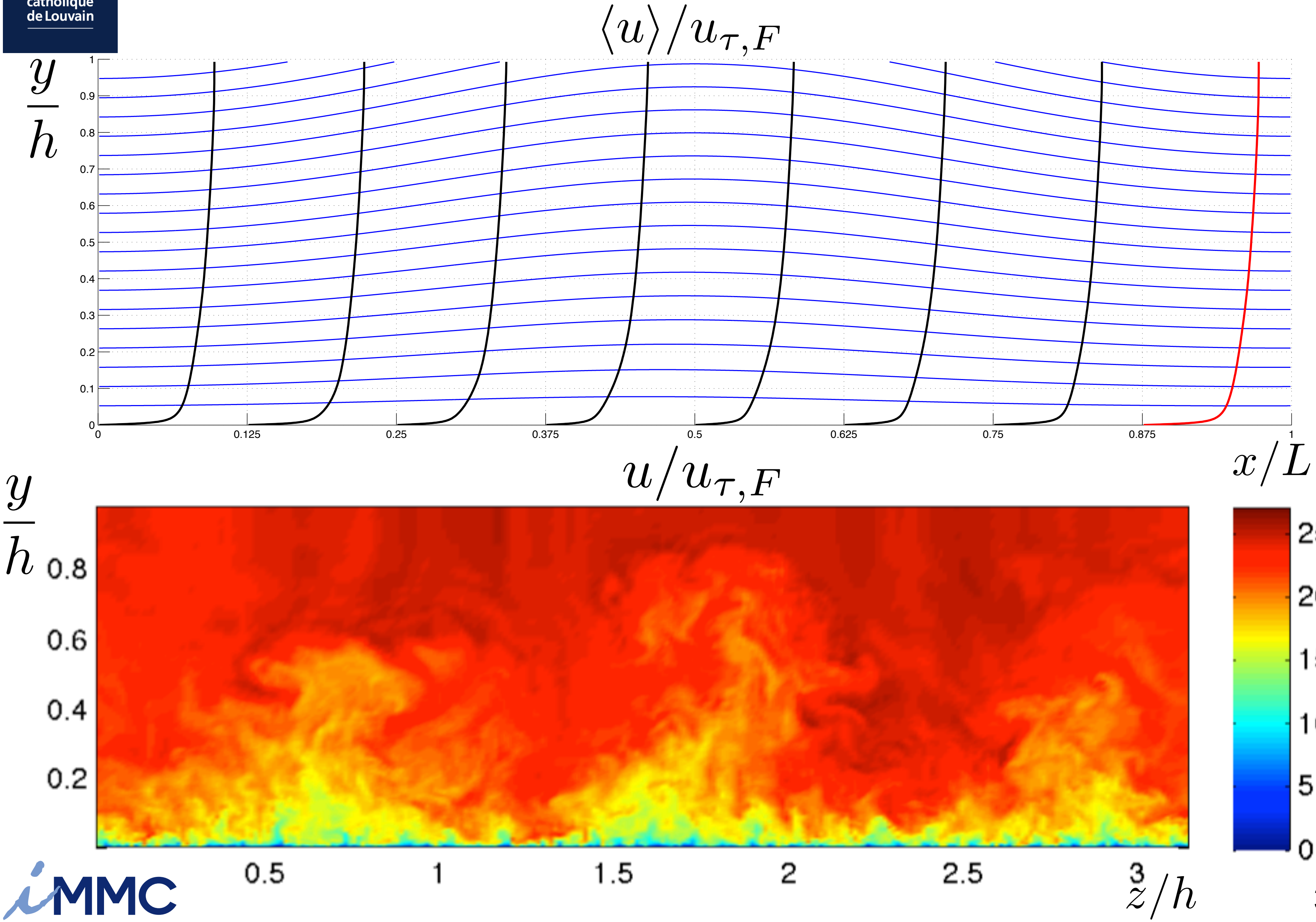


$u/u_{\tau,F}$ at $x/L = 0.625$





$u/u_{\tau,F}$ at x/L = 0.875



Shih wall function in our framework

- Compute pressure correction based on locally averaged pressure gradient :

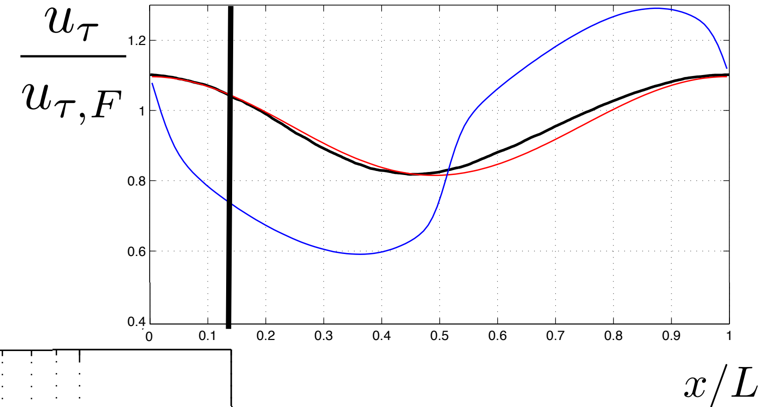
$$u_p = \left(\nu \left| \left\langle \frac{dP}{d\mathbf{x}} \right\rangle \right| \right)^{1/3}$$

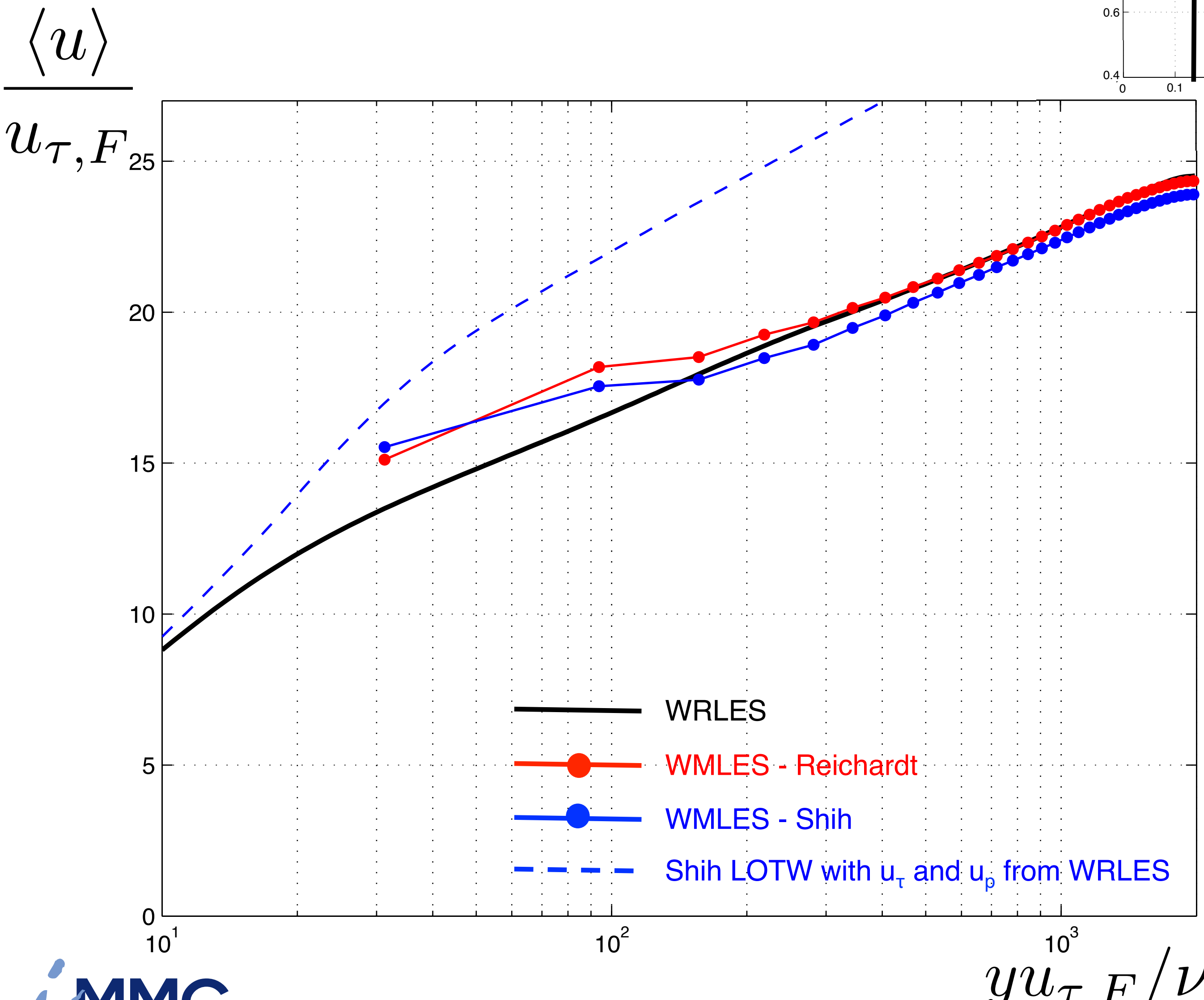
$$\mathbf{u_2} = u_p \frac{\langle dP/d\mathbf{x} \rangle}{|\langle dP/d\mathbf{x} \rangle|} f_2 \left(\frac{y u_p}{\nu} \right)$$

- Compute wall model based on corrected field:

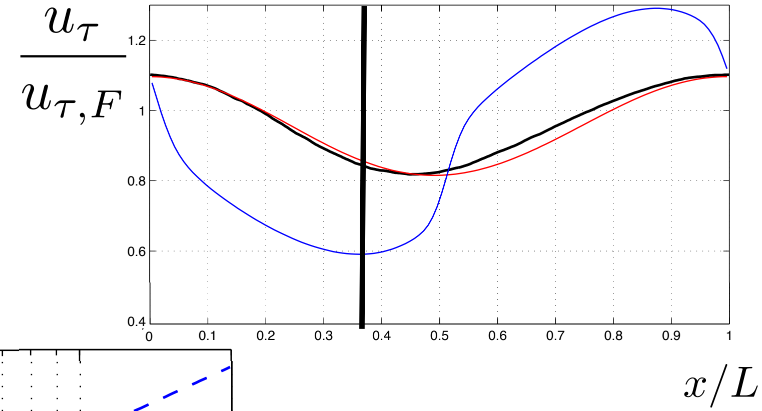
$$\mathbf{u_1} = \mathbf{u} - \mathbf{u_2}$$

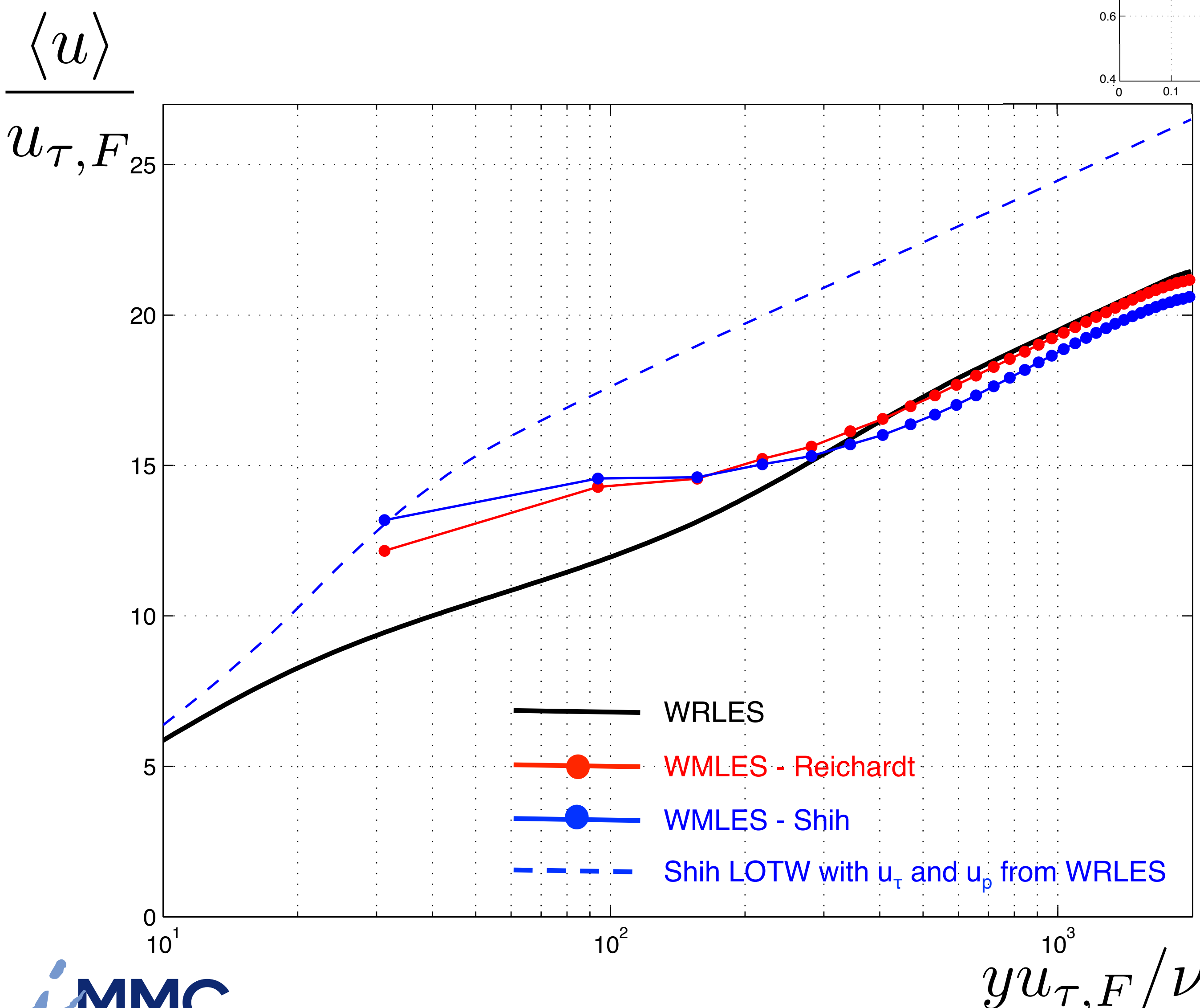
Algebraic models: Velocity profiles



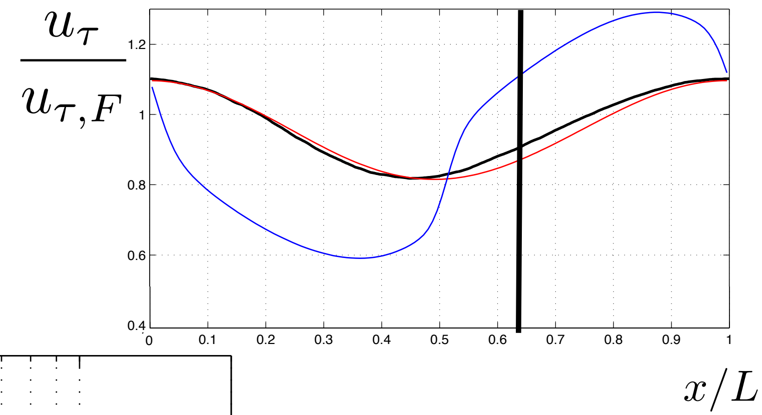


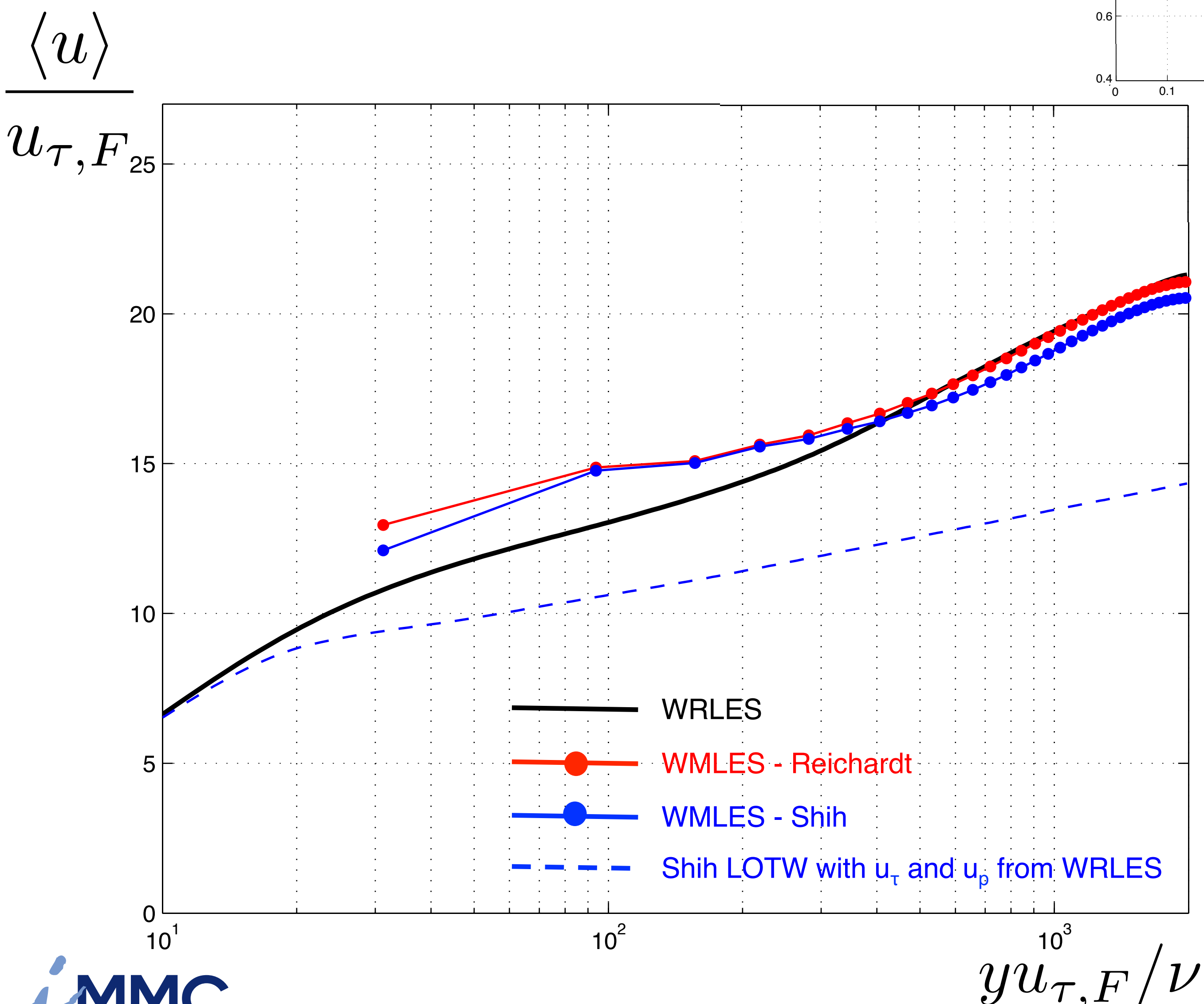
Algebraic models: Velocity profiles





Algebraic models: Velocity profiles





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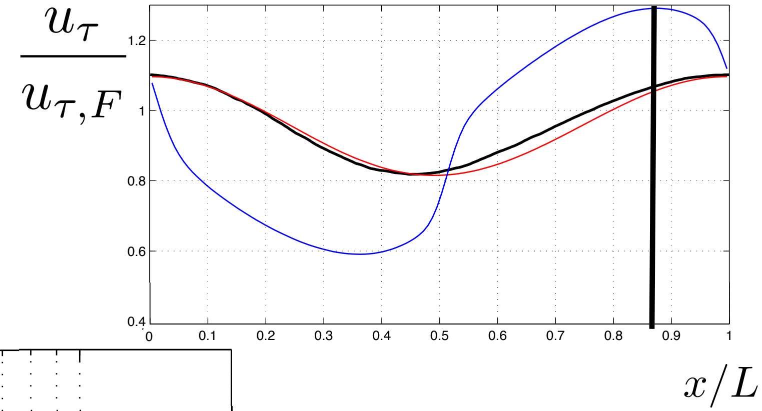
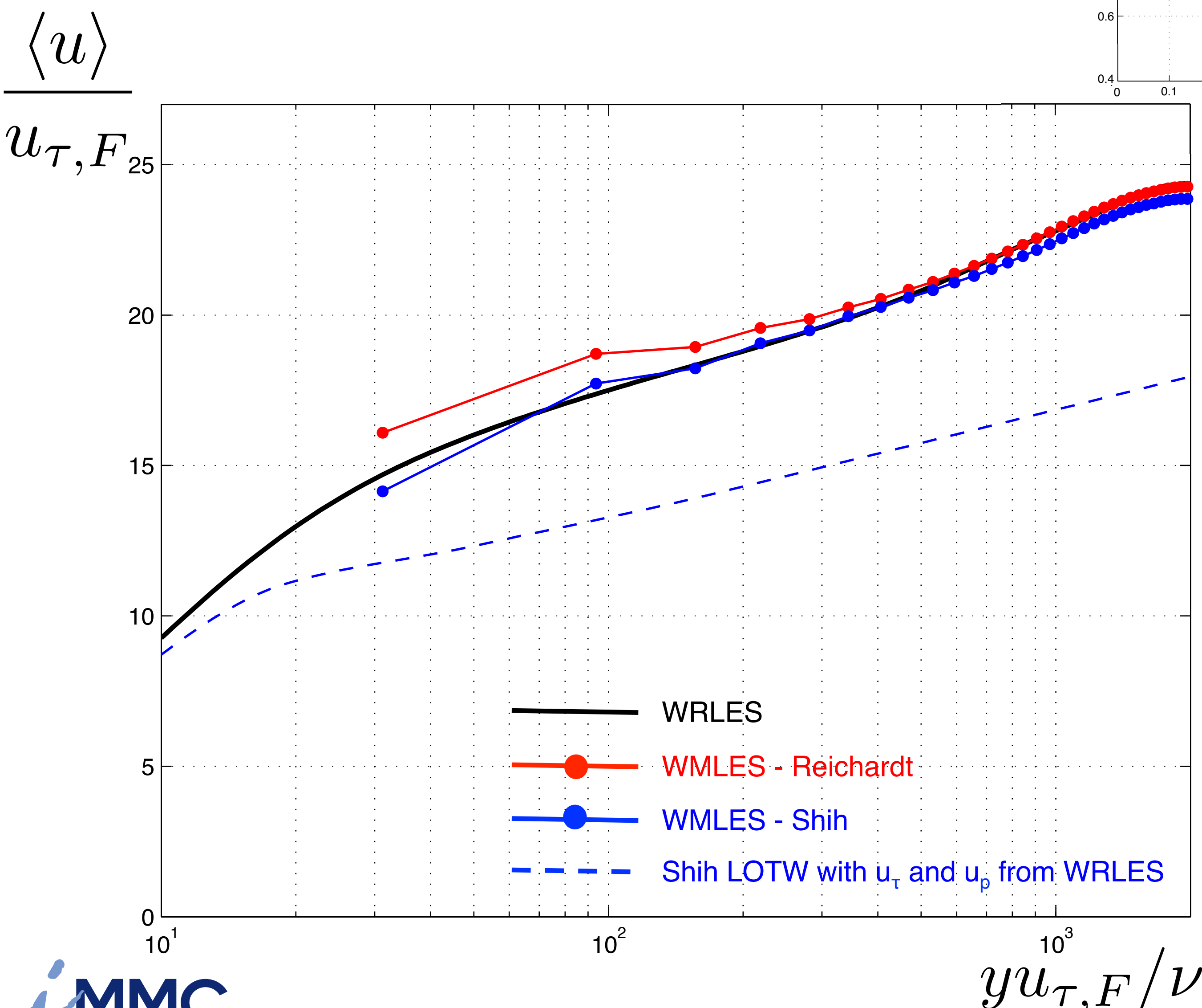
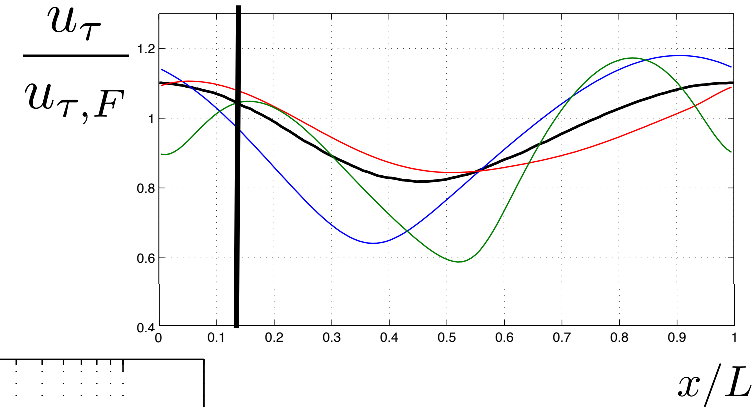
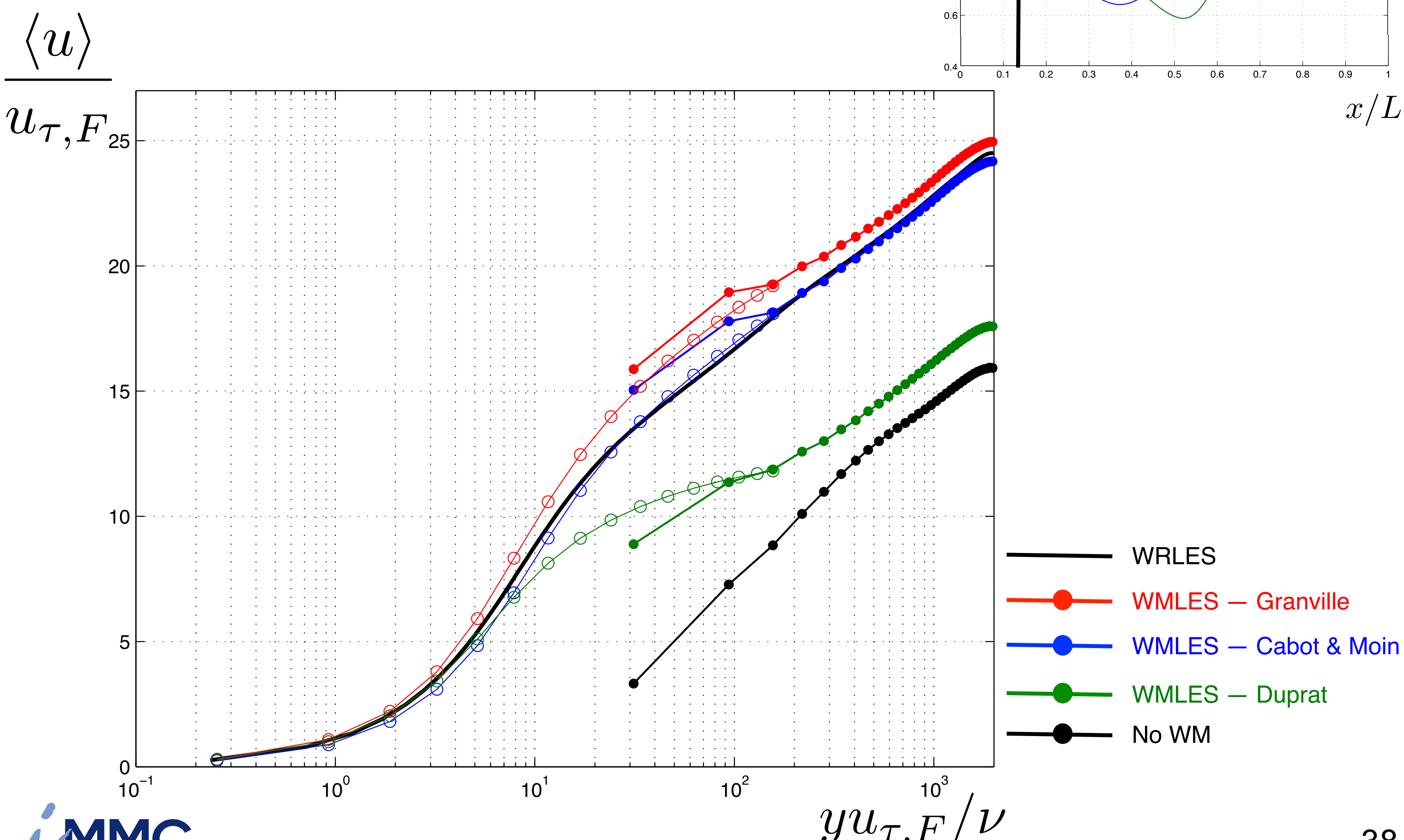


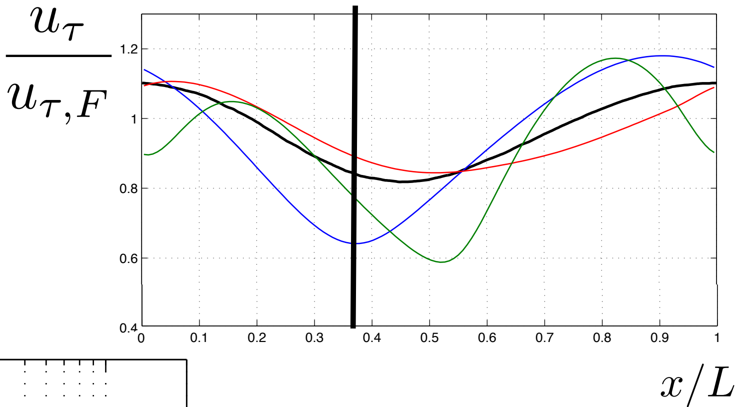


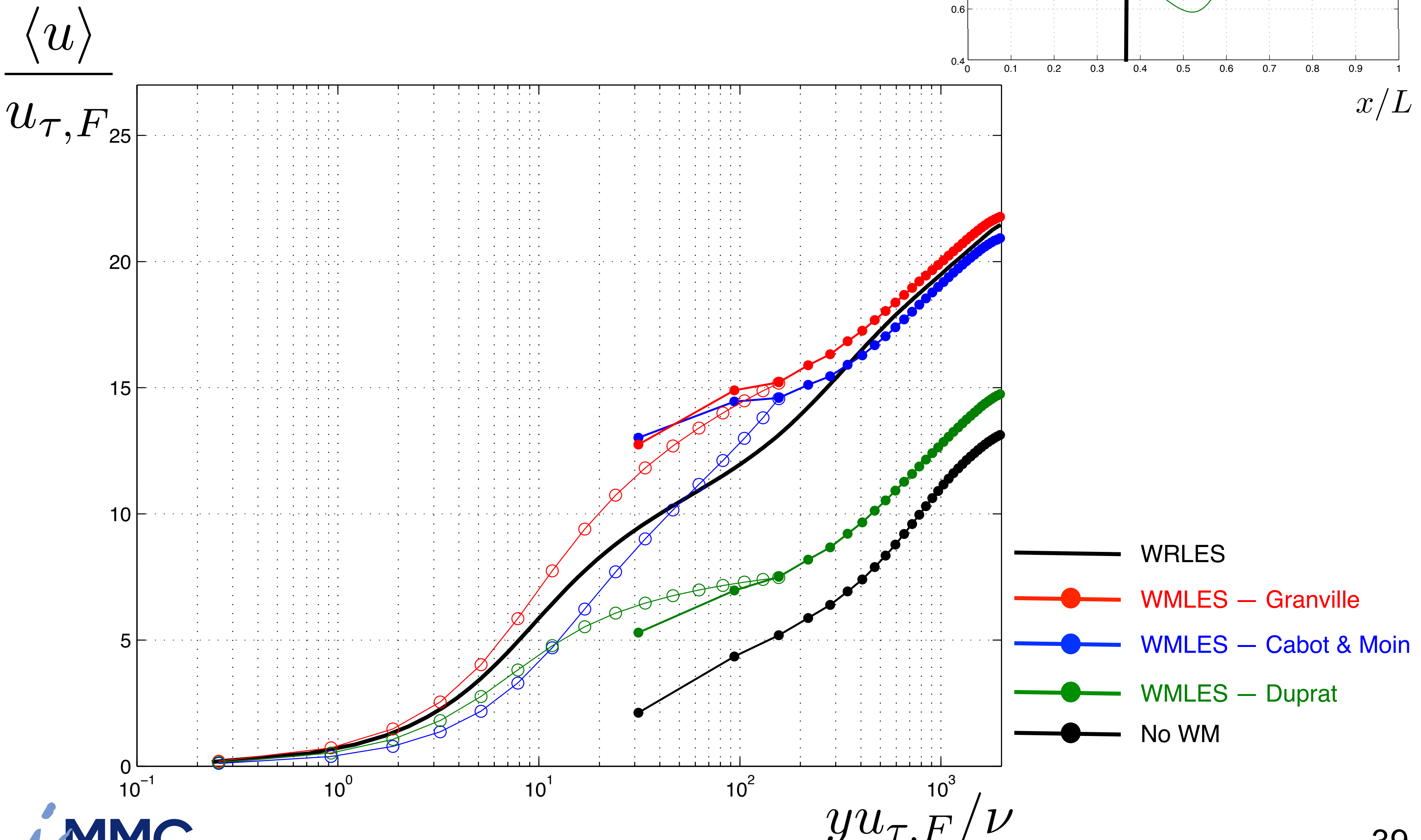
RANS sublayer models: Velocity profiles





RANS sublayer models: Velocity profiles





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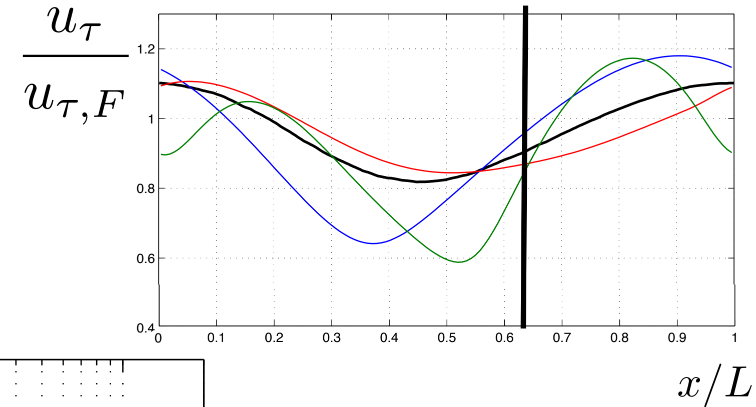

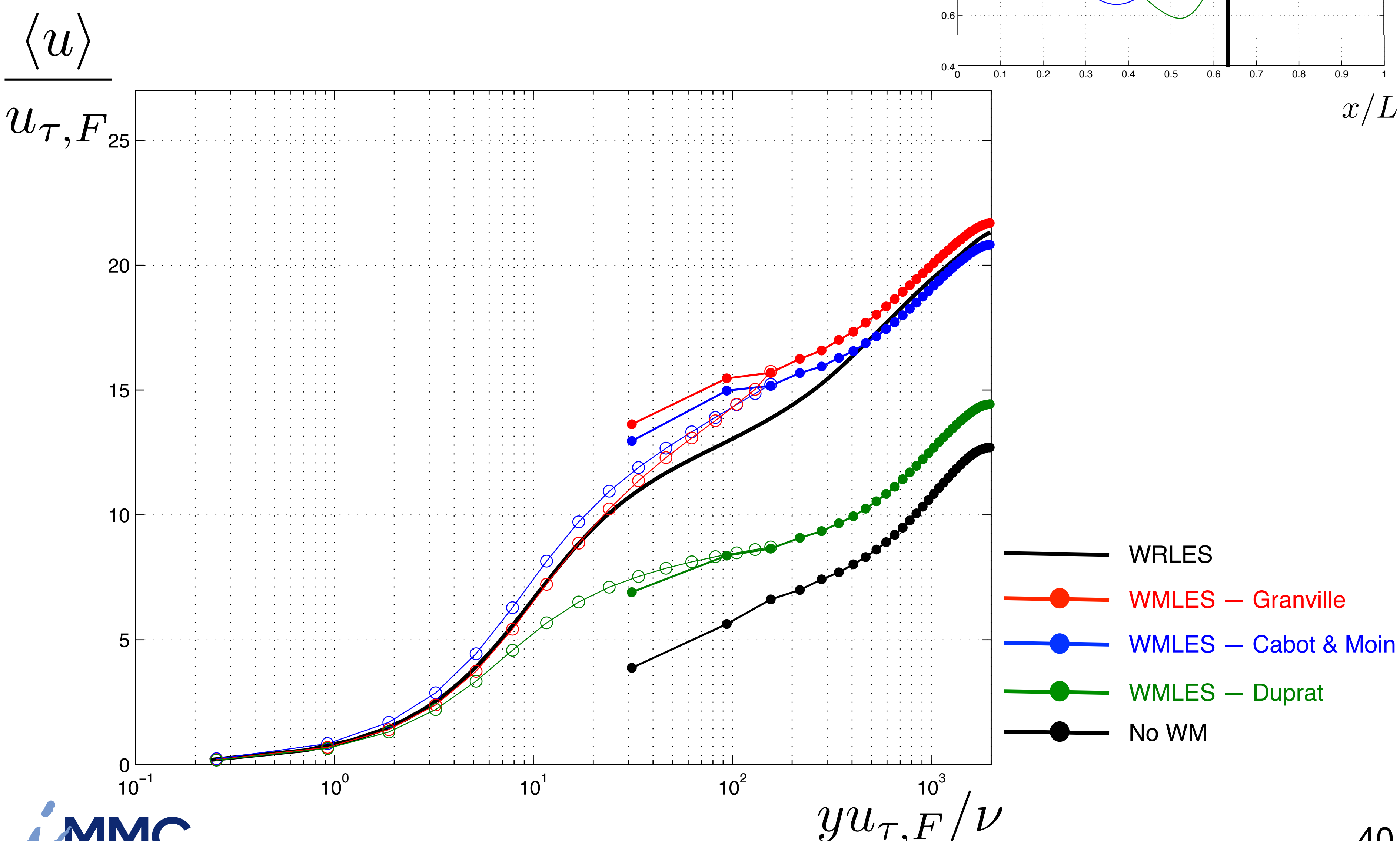

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