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A FORMAL MODEL OF KRUGMAN'S INTUITION ON THE J-CURVE

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Abstract

We use a two-good dynamic intertemporal general equilibrium model to formalize the economic intuition of Krugman about the explanation of the J-curve phenomenon in terms of habit persistence in consumption and sluggishness in capital adjustment. The results differ markedly according to the permanence or temporary nature of the shock. A short-lived terms of trade worsening may give rise to a once-for-all decrease in the marginal utility of wealth, a hump-shape response of real expense when the perturbation is at work and a definitely higher level of consumption at the new steady-state. Habitual standard of living and welfare are raised through the combination of an intertemporal speculation, habit persistence, and hysteresis effects. In accordance with recent empirical results, investment is procyclical, H-L-M effect holds, net foreign assets adjustment exhibits a J-curve, current account surplus is associated with a fall in real income. From an analytical viewpoint, a new consistent procedure to study temporary shocks in continuous time leads to formal solutions that allow to investigate accurately transitional dynamics in a complex dynamic system, comparing transitory and permanent perturbations analytically, underscoring hysteresis phenomenon, and bringing out the determinants of short and long-term reactions of macroeconomic aggregates.

Keywords: Current account; Habit Formation; Temporary shock; J-curve.

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1 Introduction

The objective of this contribution is to provide a new explanation of the J-curve phenomenon by elaborating a framework which differs from the previous two-good intertemporal optimizing continuous time models interested in terms of trade shocks in four respects: [i] the framework combines non separable additively preferences in the demand-side with adjustment costs in capital in the supply-side, [ii] an explicit and a consistent *two-step* analytical procedure for studying short-run and long-run effects of a terms of trade worsening has been applied, [iii] a systematic comparison with stylized facts is operated, [iv] the analytical solutions allow to determine rigourously and accurately the factors which influence the current account dynamics. We show that the *J*-shape response of the net foreign assets following an adverse terms of trade shock depends crucially on [i] the degree of habit persistence in consumption, [ii] the installation costs of capital, [iii] the domestic contents of consumption and investment expenditures, [iv] the length of the shock. The two last factors determine in turn the strength of the *smoothing* and *intertemporal speculation* effects which influence the real expense and investment reactions once the perturbation hits the economy.

What are the optimal responses of macroeconomic aggregates following a terms of trade shock, particularly a transitory perturbation? This question has received a lot of attention in the eighties by abstracting from capital accumulation in infinite horizon or two-period models (see e. g. Obstfeld [1982], [1983], Svensson and Razin [1983], Ostry [1988]). Sen [1990], Karayalçin [1995], Mansoorian [1993], [1998] address this issue in the nineties by introducing real money balances in the utility, heterogeneity between the individuals, or by assuming an habit-forming behavior. These contributions share the common characteristic of considering a small open economy without capital accumulation and of focusing only on an unanticipated permanent terms of trade shock. Sen and Turnovsky [1989], Murphy [1992], Servèn [1999] analyze transitory terms of trade perturbations by considering capital adjustment costs and by restricting intertemporal preferences to be additively separable. Although they explore the effects of an unanticipated transitory change in the relative price, the first paper applies a procedure which contains an inconsistency and the second is not very explicit in the solution method (see Schubert and Turnovsky [2002]). The last paper used a framework which is unable to allow for consumption dynamics and therefore to explore its joint adjustment with net foreign assets accumulation. A dynamic intertemporal general equilibrium that captures the main characteristics of a small open economy is developed to establish the determinants of the macroeconomic aggregates' reactions in the short-run and the long-run after a permanent or a temporary terms of trade shock. Domestic firms accumulate physical capital subject to installation costs and the households' consumption behavior is habit-forming. We follow a recent literature strand that explores the dynamics of consumption, saving, and investment by incorporating non additive separable preferences (Karayalçin [1994], Ikeda and Gombi [1998]).

The justification for focusing on terms of trade disturbances is threefold. First, they are supply-side and demand-side shocks because they affect both consumption and investment choices. A change in the price of an intermediary input or in total factor productivity are typically supply-side perturbations. A variation in government spending affects only the demand-side aggregates through a wealth effect. Second a relative price disturbance gives rise to income, smoothing, intratemporal, and intertemporal speculation effects which enlarge the scope of economic aggregates' responses. Third, regressions performed by Fischer [1993] and Easterly et al. [1993] show that terms of trade variations influence economic growth. According to panel data estimations of Loayza et al. [2000], terms of trade play a key role in explaining saving rates variations. Other empirical works show that a great part of real income, current account and

net exports fluctuations can be attributed to terms of trade shocks, particularly for small open economies (see e. g. Fox et al. [2002], Otto [2003]).

The framework of Ikeda and Gombi [1998] is extended by considering that consumption and investment have an import content. Our analysis departs from the study of the authors in the kind of shock considered, which in turn, results in new conclusions and expand the effects at work. More importantly, it differs in the analytical method applied to study unanticipated transitory terms of trade shocks. This new solution procedure has been recently proposed by Schubert and Turnovsky [2002] which corrects an inconsistency in the preceding solution method initiated by Sen and Turnovsky [1990]. Moreover, this framework is more suitable than its predecessors which study the current account response to terms of trade worsening. First, by assuming habit-forming consumers we allow for a slow adjustment of real expense following the relative price shock. This sluggish response of consumption in accordance with “excess smoothness in consumption” is no longer obtained by Obstfeld [1983] and Servén [1995]. Second, by considering that investment has an import content, our study highlights relevant factors that determine optimal investment response following an unanticipated transitory terms of trade perturbation. Obstfeld [1982] and Mansoorian [1993], [1998] assume no capital accumulation and consider only permanent terms of trade shocks. Sen and Turnovsky [1989] incorporates a labor-leisure arbitrage and assumes that investment has zero import content. Third, the model is sufficiently tractable to investigate the factors that determine the consumption, investment, and external position reactions to an unanticipated temporary and permanent future terms of trade deterioration. The international business cycle models like those of Mendoza [1995], Senhadji [1998], [2003], Kose [2002] provide complex frameworks with imported capital goods, multiple sectors, intersectoral costs of reallocation of production factors, recursive preferences, but cannot be solved analytically.

Number of empirical studies have shown that terms of trade changes have a large temporary component (see e. g. Reinhart and Wickham [1994] for developing countries, Cashin and Mc Dermott [2002] for developed countries) and that the duration of relative price shocks varies widely across countries (see Cashin, Mc Dermott, Patillo [2004]). Surprisingly, the analysis of transitory terms of trade perturbations in intertemporal optimizing continuous time models in an analytical way are scarce, particularly when capital accumulation is considered. When such an investigation is performed, some papers are not very explicit in their solution methods, or contain an inconsistency as emphasized by Schubert and Turnovsky [2002], or report only a numerical analysis (see for example Glenn [1997]). The study of an unanticipated temporary terms of trade worsening in the formal setup we present below allows to show a new effect at work by adopting the recent *two-step* approach recently developed by Schubert and Turnovsky [2002]. Following a transitory decline in the home goods’ relative price, real expense may rise in the short run like in the additive-utility case; more importantly real expense may be persistently higher in the long-run in the present formal setup. The combination of habit persistence in consumption, an intertemporal speculation effect, and an hysteresis phenomenon plays a key role in generating the long-run response of real expense.

In the present study, we propose new insights about the effects of various terms of trade shocks on consumption, investment, and external position dynamics. From empirical results of Leonard and Stockman [2002], a new explanation of the J-curve phenomenon is needed.¹ To this end, a framework is elaborated which allows to identify the channels through which relative price perturbations affect short and long term dynamics of macroeconomic aggregates; in particular, we formalize the economic intuition of Krugman [1989] about the interpretation of non-monotonic adjustment of the current account.²

The paper is organized as follows. In section 2, we present the framework of a two-good model of a small open economy, facing given terms of trade and world interest rate. In section 3, we analyze the equilibrium dynamics and the steady-state of the model. Section 4 explores in detail the effects of a permanent worsening of the terms of trade. In section 5, a consistent solution method for analyzing temporary shocks is applied to an adverse transitory relative price perturbation. Consumption, stock of habits, investment, and current account dynamics are studied and the results are compared to the effects of a permanent terms of trade deterioration. Conclusions and a short outlook on further research are contained in section 6. Almost all formal computations can be retrieved in the Appendix.

2 The Framework

Consider a small open economy that is populated by a constant number of identical households and firms that have perfect foresight and live forever. We normalize, without loss of generality, the number of households to one. There are four types of goods. The representative firm is completely specialized in the production of a final good that can be consumed domestically or exported. This good can also be transformed, at some cost, in capital. The domestic good is an imperfect substitute for an imported good which can be used for consumption or investment. The country is small in world good and capital markets and faces given terms of trade (price of the domestic good in terms of the foreign good), p , and world interest rate, r^* .

2.1 Structure of the Economy

Households

At each instant the representative household consumes domestic goods and foreign goods denoted by d and f . The measure of utility of consumption at t , $c(t)$, is given by the relation :

$$c(t) = c(d(t), f(t)), \quad (1)$$

where $c(\cdot)$ is a positive, increasing, concave and linearly homogeneous aggregator function. The representative household maximizes the objective function

$$U[C(0)] = \int_0^\infty u[c(d(t), f(t)), s(t)] \exp(-\delta t) dt, \quad (2)$$

where δ is the consumer's discount rate, and $s(t)$ a distributed lag on past real expenditure as (see Ryder and Heal [1973]),³

$$s(t) = \sigma \int_{-\infty}^t c(d(\tau), f(\tau)) \exp(-\sigma(t - \tau)) d\tau. \quad (3)$$

From (3), the dynamic equation of habit stocks is given by

$$\dot{s}(t) = \sigma [c(t) - s(t)]. \quad (4)$$

Following Ryder and Heal [1973], the instantaneous utility function is assumed to be : [P1] increasing in current real expenditure, $u_1 > 0$; [P2] non-increasing in past real expense, $u_2 \leq 0$; [P3] increasing in real expense at stationary state, $u_1(c, c) + u_2(c, c) > 0$; [P4] strictly concave in c and s , $u_{11}(c, s) < 0$, $u_{22} < 0$, and concave in (c, s) , $u_{11}(c, s) u_{22}(c, s) - [u_{12}(c, s)]^2 \geq 0$; [P4] $\lim_{c \rightarrow 0} u_1(c, s) = \infty$ and $\lim_{c \rightarrow 0} [u_1(c, c) + u_2(c, c)] = \infty$.

Since $c(\cdot)$ is homothetic, the household's maximization problem can be decomposed into two stages (see Frenkel and Razin [1987], chapter 6). At the first stage, the household minimizes the cost, $z_c(t) = p(t)d(t) + f(t)$, for a given level of subutility, $c(t)$, where $p(t)$ is the relative price of the domestic good. For any chosen $c(t)$, the optimal basket $(d(t), f(t))$ is a solution to

$$p_c(p(t)) c(t) = \min_{\{d(t), f(t)\}} \{p(t)d(t) + f(t) : c(d(t), f(t)) \geq c(t)\}. \quad (5)$$

The assumption that the subutility function $c(\cdot)$ is linear homogeneous implies that the total expense in consumption goods can be expressed as $z_c(t) = p_c(p(t)) c(t)$, with $p_c(p(t))$ is the unit cost function dual (or consumption-based price index) to c . Intra-temporal allocations between domestic goods and imports follows from Sheppard's Lemma (or the envelope theorem) applied to (5):

$$d(t) = p'_c(p(t)) c(t), \quad f(t) = [p_c(p(t)) - p(t)p'_c(p(t))] c(t), \quad (6)$$

with (see Deaton and Muelbauer [1980]),⁴

$$p_c(p) > 0, \quad p'_c(p) > 0, \quad p''_c(p) < 0. \quad (7)$$

In the second stage, consumers choose their real expense, c , and rates of accumulation of consumption "experience" and traded bonds to maximize (2) subject to (3) and the flow budget constraint,

$$\dot{b}(t) = r^*b(t) + [D(t) + w(t)] - p_c(p(t)) c(t), \quad (8)$$

and initial conditions $s(0) = s_0$, $b(0) = b_0$. Households' income consists of interest earnings, $r^*b(t)$, dividend payments on equity holdings, $D(t)$; moreover, households inelastically supply one unit of labor services and receive the wage, $w(t)$, per unit of time. The real stock of foreign assets held by the household, $b(t)$, is denominated in terms of the imported good since we assume that external borrowing and lending are measured in units of the foreign good.

Firms

Perfectly competitive firms produce output, Y , from labor and capital, k , by means of a constant returns to scale production function, which is assumed to have the usual neoclassical properties of positive and diminishing marginal products. Like Abel and Blanchard [1983], the installation cost function $\psi(I(t)/k(t))$, is assumed to have the following properties:

$$\psi(0) = 0, \quad \psi'(\cdot) > 0, \quad 2\psi'(\cdot) + \frac{I}{k}\psi''(\cdot) > 0. \quad (9)$$

Following Gavin [1992] and Serv  n [1995], [1999], we assume that domestic and imported goods are converted in an investment good according to a linearly homogeneous technology:

$$J(t) = J(J_D(t), J_F(t)), \quad (10)$$

where J_D and J_F denote domestic and foreign inputs combined into the investment process. Since $J(\cdot)$ is homogeneous of degree one, the investment decision can be done in two stages like consumption decision. As the solution to the cost minimizing of expenditure on investment goods it can be defined by

$$p_I(p(t)) J(t) = \min_{\{J_D(t), J_F(t)\}} \{p(t)J_D(t) + J_F(t) : J(J_D(t), J_F(t)) \geq J(t)\}, \quad (11)$$

where the exact investment price index is a function of terms of trade and has the following properties

$$p_I(p) > 0, \quad p'_I(p) > 0, \quad p''_{II}(p) < 0. \quad (12)$$

From Sheppard's lemma, we obtain investment demand for the domestic and imported goods:

$$J_D(t) = p'_I(p(t)) J(t), \quad J_F(t) = [p_I(p(t)) - p(t)p'_I(p(t))] J(t). \quad (13)$$

In the second stage, the representative firm maximizes the present value of anticipated future cash flow:

$$\max_{\{I(t), l(t)\}} V[0] = \int_0^\infty D(t) e^{-r^* t} = \int_0^\infty \left\{ pF(k, l) - wl - p_I(p) I \left[1 + \psi \left(\frac{I}{k} \right) \right] \right\} e^{-r^* t} dt, \quad (14a)$$

$$\text{subject to} \quad \dot{k}(t) = I(t), \quad (14b)$$

$$\text{and the initial condition} \quad k(0) = k_0. \quad (14c)$$

2.2 Macroeconomic Equilibrium

To obtain the macroeconomic equilibrium, we first derive the optimality conditions for households and firms and combine these with the accumulation equations. This leads to the set of equations⁵

$$u_1(c, s) + \sigma \xi = p_c(p) \lambda, \quad (15a)$$

$$pF_l(k, 1) = w, \quad (15b)$$

$$q = p_I(p) \left[1 + \psi \left(\frac{I}{k} \right) + \left(\frac{I}{k} \right) \psi' \left(\frac{I}{k} \right) \right], \quad (15c)$$

$$\dot{\lambda} = 0, \quad \text{i.e.} \quad \lambda = \bar{\lambda}, \quad (15d)$$

$$\dot{\xi} = (\delta + \sigma) \xi - u_2(c, s), \quad (15e)$$

$$\dot{q} = r^* q - \left[pF_k(k, 1) + p_I(p) \left(\frac{I}{k} \right)^2 \psi' \left(\frac{I}{k} \right) \right], \quad (15f)$$

$$\dot{b} = r^* b + pF(k, 1) - p_c(p) c - p_I(p) I \left[1 + \psi \left(\frac{I}{k} \right) \right], \quad (15g)$$

and dynamic equations (4) and (14b), and the transversality conditions

$$\lim_{t \rightarrow \infty} \bar{\lambda} b \exp(-r^* t) = \lim_{t \rightarrow \infty} \xi s \exp(-r^* t) = \lim_{t \rightarrow \infty} q k \exp(-r^* t) = 0, \quad (16)$$

where λ , ξ , q are the co-state variables associated with dynamic equations (8), (3), and (14b).

The solution of the differential equation (15e) using (16) is given by

$$\xi(t) = \int_t^\infty u_2(c(\tau), s(\tau)) e^{-(\delta+\sigma)(\tau-t)} d\tau. \quad (17)$$

The shadow price of habits' stock is equal to the present discounted value of marginal utility of consumption "experience", $u_2 \leq 0$, which depreciates at the rate σ .

Solving (15f) forward and ruling out "bubble trajectories", we obtain

$$q(t) = \int_t^\infty \left\{ pF_k[k(\tau), 1] + p_I(p) \left(\frac{I}{k} \right)^2 \psi' \left(\frac{I}{k} \right) \right\} e^{-r^*(\tau-t)} d\tau. \quad (18)$$

According to (18), the shadow price of capital is equal to the present discounted value of the sum of the marginal product of capital and the reduction of the marginal cost induced by an increase in the capital stock for a given flow of investment, both expressed in the foreign good.

The first static efficiency condition (15a) requires that along an optimal path the sum of marginal current utility of real expense and its marginal contribution to the future felicity stream derived from a higher habitual standard of living is equal to the marginal utility of wealth in the form of internationally traded bonds measured in terms of the domestic good, $p_c \lambda$. The second static efficiency condition (15b) establishes the usual equality between the marginal productivity of labor and the real wage. Equation (15c) equates the ratio of market price of installed capital to the replacement cost of capital, i.e. the Tobin q , to the marginal cost investment.

With a constant rate of time preference and an exogenous world interest rate, we require that

$$\delta = r^*, \quad (19)$$

in order to generate an interior solution. This standard assumption made in the literature implies that the marginal utility of wealth, λ , must remain constant over time (see (15d)), and gives rise to the zero-root problem (see Sen and Turnovsky [1990]).

Finally, the first transversality condition of (16) rules out the possibility of running up infinite debt or credit and ensures that the nation remains intertemporally solvent.

3 Equilibrium Dynamics and the Steady-State

Equilibrium Dynamics

The static efficiency condition (15c) implies that the rate of investment is a function of Tobin q :

$$\frac{I}{k} = \kappa \left(\frac{q}{p_I(p)} \right), \quad \kappa'(\cdot) > 0, \quad \kappa(1) = 0. \quad (20)$$

From (20), the rate of investment rises when the market price of capital is higher than investment replacement cost, that is to say the Tobin's q , denoted by ν , is greater than one.

Total differentiation of equation (15a), substitution of (4) and (15e), and elimination of ξ using (15a) lead to the following dynamic equation of real expense (see appendix A):

$$\dot{c} = \frac{1}{u_{11}} [(\delta + \sigma)(u_1 - p_c(p)\bar{\lambda}) + \sigma u_2 - u_{12}\sigma(c - s)]. \quad (21)$$

Inserting the short run static solution (20) in (14b) and (15f), linearizing these with dynamic equations (3) and (21) around the steady-state, and denoting $\bar{x} = \bar{s}, \bar{\xi}, \bar{k}, \bar{q}, \bar{p}, \bar{c}$ the long term values of $x = s, \xi, k, q, p, c$, we obtain in a matrix form

$$\begin{pmatrix} \dot{s}, \dot{c}, \dot{k}, \dot{q} \end{pmatrix}^T = A \begin{pmatrix} s(t) - \bar{s}, c(t) - \bar{c}, k(t) - \bar{k}, q(t) - \bar{q} \end{pmatrix}^T, \quad (22)$$

where A is given by

$$A \equiv \begin{pmatrix} -\sigma & \sigma & 0 & 0 \\ \frac{\delta+2\sigma}{u_{11}} \left[u_{12} + \frac{\sigma}{\delta+2\sigma} u_{22} \right] & \delta + \sigma & 0 & 0 \\ 0 & 0 & 0 & \kappa'(1) \frac{\bar{k}}{p_I(\bar{p})} \\ 0 & 0 & -\bar{p} F_{kk} & r^* \end{pmatrix}. \quad (23)$$

The Fisherian separation theorem implies that the matrix is block recursive. As we will see below, the number of predetermined variables equals the number of negative eigenvalues and the number of jump variables equals the number of positive eigenvalues, so there exists a unit convergent path towards the steady-state.

The characteristic roots obtained from A_{11} write as follows:⁶

$$\mu_i \equiv \frac{1}{2} \left\{ \delta \pm \sqrt{(\delta + 2\sigma)^2 + \frac{4\sigma(\delta + 2\sigma)}{u_{11}}\Gamma} \right\} \gtrless 0, \quad i = 1, 2, \quad (24)$$

where we let

$$\Gamma = u_{12} + \frac{\sigma}{\delta + 2\sigma} u_{22} \lesseqgtr 0. \quad (25)$$

The sign of Γ depends on u_{12} . If the marginal utility of real expense is sufficiently increasing in stock of habits, the preferences of the representative consumer display “adjacent complementarity” and Γ is positive (see Ryder and Heal [1973]). If u_{12} has a negative or a low positive value, Γ is negative and preferences are said to display “distant complementarity”.

We denote respectively $\mu_1 < 0$ and $\mu_2 > 0$ the stable and unstable real eigenvalues satisfying

$$\mu_1 < 0 < r^* < \mu_2, \quad (26)$$

under the condition (see Obstfeld [1992])⁷

$$u_{12} + \frac{\sigma}{\delta + 2\sigma} u_{22} < - \left(\frac{\delta + \sigma}{\delta + 2\sigma} \right) \frac{1}{u_{11}}, \quad (27)$$

so that the long-run equilibrium is a saddle-point in (s, c) -space. While the habits’ stock evolves always gradually, the real expense, c , can jump instantaneously in response to a new information.

The general solution of the linearized system A_{11} may be written as

$$s(t) = \bar{s} + A_1 e^{\mu_1 t} + A_2 e^{\mu_2 t}, \quad (28a)$$

$$c(t) = \bar{c} + A_1 \left(\frac{\sigma + \mu_1}{\sigma} \right) e^{\mu_1 t} + A_2 \left(\frac{\sigma + \mu_2}{\sigma} \right) e^{\mu_2 t}, \quad (28b)$$

where A_1 and A_2 are arbitrary constants. Invoking the transversality conditions requires that $A_2 = 0$. Starting from an initial “consumption experience” $s(0) = s_0$, the stable dynamic time paths followed by s and c are given by

$$s(t) = \bar{s} + (s_0 - \bar{s}) e^{\mu_1 t}, \quad (29a)$$

$$c(t) = \bar{c} + \left(\frac{\sigma + \mu_1}{\sigma} \right) (s_0 - \bar{s}) e^{\mu_1 t}, \quad (29b)$$

where $A_1 = s_0 - \bar{s}$. When the stock of habit consumption is expected to be higher, real expense and habitual standard of living co-vary in the same or in an opposite direction according to whether $(\sigma + \mu_1)$ is positive (adjacent complementarity, $\Gamma > 0$) or negative (distant complementarity, $\Gamma < 0$), i. e. according to whether the marginal current utility of real expense is sufficiently strongly increasing or decreasing (or weakly increasing) in future “consumption experience” with respect to the increase of marginal desutility of habits.

The two real characteristic roots obtained from A_{22} write as follows:

$$\chi_i \equiv \frac{1}{2} \left\{ r^* - \sqrt{(r^*)^2 - 4 \frac{\kappa'(1) \bar{k} \bar{p} F_{kk}}{p_I(\bar{p})}} \right\} \gtrless 0, \quad i = 1, 2. \quad (30)$$

The stable and unstable eigenvalues satisfy

$$\chi_1 < 0 < r^* < \chi_2. \quad (31)$$

Hence the dynamics describe a saddle-point in (k, q) -space.

The general solution of the linearized system A_{22} may be written as

$$k(t) = \bar{k} + B_1 e^{\chi_1 t} + B_2 e^{\chi_2 t}, \quad (32a)$$

$$q(t) = \bar{q} + B_1 \left(\frac{p_I \chi_1}{\kappa'(1) \bar{k}} \right) e^{\chi_1 t} + B_2 \left(\frac{p_I \chi_2}{\kappa'(1) \bar{k}} \right) e^{\chi_2 t}. \quad (32b)$$

The stable adjustment path is given by

$$k(t) = \bar{k} + (k_0 - \bar{k}) e^{\chi_1 t}, \quad (33a)$$

$$q(t) = \bar{q} + \left(\frac{p_I \chi_1}{\kappa'(1) \bar{k}} \right) (k_0 - \bar{k}) e^{\chi_1 t}, \quad (33b)$$

where $B_1 = k_0 - \bar{k}$. The solution (33b) indicates that the stable branch is downward-sloping.

Substituting the short-run solution (20), linearizing the dynamic equation of the foreign asset stock (15g) in the neighborhood of the steady-state, using the fact that $\bar{p} F_k(\bar{k}, 1) = r^* p_I(\bar{p})$ at the steady-state, and substituting the general solutions (28b), (32a), and (32b), we obtain the general (linearized) solution for b :

$$\begin{aligned} b(t) = & \bar{b} + \left[(b_0 - \bar{b}) + p_I B_1 + p_I B_2 + \frac{p_c(\sigma + \mu_1)}{\sigma(\mu_1 - r^*)} A_1 + \frac{p_c(\sigma + \mu_2)}{\sigma(\mu_2 - r^*)} A_2 \right] e^{r^* t} - \\ & - p_I B_1 e^{\chi_1 t} - p_I B_2 e^{\chi_2 t} - \frac{p_c(\sigma + \mu_1)}{\sigma(\mu_1 - r^*)} A_1 e^{\mu_1 t} - \frac{p_c(\sigma + \mu_2)}{\sigma(\mu_2 - r^*)} A_2 e^{\mu_2 t}. \end{aligned} \quad (34)$$

The intertemporal solvency condition for the small economy imply $A_2 = 0$ and the linearized version of the nation's intertemporal budget constraint

$$(b_0 - \bar{b}) = -p_I (k_0 - \bar{k}) - \frac{p_c(\sigma + \mu_1)}{\sigma(\mu_1 - r^*)} (s_0 - \bar{s}), \quad (35)$$

where the initial stocks, k_0, b_0, s_0 are given. The stable solution for $b(t)$ consistent with long-run solvency writes as follows

$$(b(t) - \bar{b}) = -p_I (k(t) - \bar{k}) - \frac{p_c(\sigma + \mu_1)}{\sigma(\mu_1 - r^*)} (s(t) - \bar{s}). \quad (36)$$

This equation describes the relationship between the stock of internationally traded bonds, the stock of physical capital, and the stock of consumption habits along a stable path. Time derivatives of solutions (33a) and (36) and elimination of $A_1 e^{\mu_1 t}$ and $B_1 e^{\chi_1 t}$ allow to express the change of capital and foreign assets stocks in term of deviations from steady state as

$$\dot{k}(t) = \chi_1 (k(t) - \bar{k}), \quad (37a)$$

$$\dot{b}(t) = \mu_1 (b(t) - \bar{b}) + p_I (\mu_1 - \chi_1) (k(t) - \bar{k}), \quad (37b)$$

where the multiplier $(\mu_1 - \chi_1)$ can be positive or negative according to whether the speed of adjustment of consumption habits, $|\mu_1|$, is lower or greater than this of capital stock, $|\chi_1|$. The

new differential equation system has two stable roots $\mu_1 < 0$ and $\chi_1 < 0$; hence the steady-state is a stable node in (k, b) -space.

Steady-State

The steady-state of the economy is obtained by setting $\dot{c}, \dot{s}, \dot{k}, \dot{q}, \dot{b} = 0$ and is defined by the following set of equations:

$$u_1(\bar{c}, \bar{s}) + \frac{\sigma}{\delta + \sigma} u_2(\bar{c}, \bar{s}) = p_c(\bar{p}) \bar{\lambda}, \quad (38a)$$

$$\bar{c} = \bar{s}, \quad (38b)$$

$$\bar{q} = p_I(\bar{p}), \quad (38c)$$

$$r^* p_I(\bar{p}) = \bar{p} F_k(\bar{k}, 1), \quad (38d)$$

$$r^* \bar{b} + \bar{p} F(\bar{k}, 1) - p_c(\bar{p}) \bar{c} = 0, \quad (38e)$$

and the intertemporal solvency condition

$$(b_0 - \bar{b}) = -p_I(k_0 - \bar{k}) - \frac{p_c(\sigma + \mu_1)}{\sigma(\mu_1 - r^*)} (s_0 - \bar{s}). \quad (38f)$$

From (38a), the marginal utility of consumption along a constant path,⁸ i. e. the sum of marginal current utility of real expense (u_1) and the capitalized value of marginal utility of “consumption experience” ($u_2/(\delta + \sigma)$) adjusted by the parameter σ is equal to the marginal utility of wealth in terms of the domestic good, $p_c \bar{\lambda}$. The equation (38b) requires that real expense is equal to habits’ stock at the steady-state. Equation (38c) asserts that long-run investment is zero when the market price of installed capital is equal to its replacement cost, i. e. when the Tobin’s q is equal to one. Equation (38d) indicates that in the long-run the marginal product of capital, $\bar{p} F_k$, is equal to its user cost, $r^* p_I$, both expressed in terms of the foreign good. Equation (38e) implies that in the steady-state equilibrium, the current account must be zero, that is, the gross national product, $r^* \bar{b} + \bar{p} \bar{Y}$, must be equal to the total expenditure in consumption goods, $p_c(\bar{p}) \bar{c}$. Finally, the linearized version of the nation’s intertemporal budget constraint (38f) implies that the steady-state depends on the initial stocks k_0 , b_0 , and s_0 . This dependency upon initial conditions comes from the assumptions of infinitely lived maximizing agents having a constant rate of discount and facing perfect capital markets and leads to hysteresis effects, that is, temporary terms of trade disturbances have permanent effects (see Sen and Turnovsky [1989], [1990]).⁹

System (38) may be solved for the steady-state values by applying the two-step solution method described by Schubert [2002] and Schubert and Turnovsky [2002]. We first solve equations (38a) to (38e) as functions of marginal utility of wealth expressed in terms of the foreign good, $\bar{\lambda}$, and of the terms of trade, p . This yields to the following functions

$$\bar{s} = \bar{c} = t(\bar{\lambda}, p), \quad t_{\bar{\lambda}} < 0, \quad t_p < 0, \quad (39a)$$

$$\bar{k} = u(p), \quad u_p > 0, \quad (39b)$$

$$\bar{q} = p_I(p), \quad p'_I > 0, \quad (39c)$$

$$\bar{b} = v(\bar{\lambda}, p), \quad v_{\bar{\lambda}} < 0, \quad v_p < 0. \quad (39d)$$

In the second step, we insert these functions into the economy’s intertemporal budget constraint (eq (38f)), which may be solved for the equilibrium value of the marginal utility of wealth:

$$\bar{\lambda} = g(s_0, k_0, b_0, p), \quad \lambda_s \geq 0, \quad \lambda_k < 0, \quad \lambda_b < 0, \quad \lambda_p < 0. \quad (40)$$

Substituting then $\bar{\lambda}$ into the other steady-state functions (39) gives the conventional steady-state values of the economy as functions of the terms of trade and the initial conditions, k_0 , b_0 and s_0 (except for supply side's variables, k and q).

4 A permanent Deterioration of Terms of Trade

We investigate the effects of a permanent decrease in the relative price of the domestic good, p , from p_0 to p_1 , which occurs at time $t = 0$, where the economy is originally in steady state. Like Obstfeld [1983] and Sen and Turnovsky [1989], we assume that the small open economy is a net exporter of the domestic good at the steady-state, that is $(\bar{Y} - \bar{d}) > 0$. All agents perfectly understand the permanence of the terms of trade deterioration, but its occurrence at time $t = 0$ is unanticipated. Because of perfect foresight assumption, the transitional dynamics are affected by the expected long-run state of the economy.

Steady-State Effects of an Unanticipated Permanent Terms of Trade Deterioration

The long-run effects after a permanent change in the relative price of the home good, consistent with long-run solvency, are obtained from the total differential of the equilibrium system (38):

$$\frac{d\bar{c}}{dp} = \frac{d\bar{s}}{dp} = \frac{\sigma(\mu_1 - r^*)}{p_c \mu_1 (\sigma + r^*)} (\bar{Y} - \bar{d}) > 0, \quad (41a)$$

$$\frac{d\bar{k}}{dp} = -\frac{F_k(1 - \alpha_I)}{pF_{kk}} > 0, \quad \frac{d\bar{q}}{dp} = p'_I(p) > 0, \quad (41b)$$

$$\frac{d\bar{\lambda}}{dp} = \frac{\sigma(\mu_1 - r^*)}{p_c \mu_1 (\sigma + r^*)} \frac{(\bar{Y} - \bar{d})}{p_c} \left[u_{11} + \frac{\delta + 2\sigma}{\delta + \sigma} \left(u_{12} + \frac{\sigma}{\delta + 2\sigma} u_{22} \right) \right] - \frac{\alpha_c \bar{\lambda}}{p} < 0, \quad (41c)$$

$$\frac{d\bar{b}}{dp} = -\frac{(\sigma + \mu_1)(\bar{Y} - \bar{d})}{\mu_1(\sigma + r^*)} + \frac{p_I F_k(1 - \alpha_I)}{pF_{kk}} \leq 0, \quad (41d)$$

$$\frac{d\bar{n}x}{dp} = -r^* \frac{d\bar{b}}{dp} = (\bar{Y} - \bar{d}) + pF_k \frac{d\bar{k}}{dp} - p_c \frac{d\bar{c}}{dp}, \quad (41e)$$

where the net exports expressed in terms of the foreign good are denoted by nx ; they are defined as the difference between the domestic output and absorption, the latter being equal to the sum of consumption and investment expenditure, i. e. $nx(t) = pF[k(t), 1] - p_c c(t) - p_I J(t)$.

A permanent decrease in p reduces the long term values of real expense, $\bar{c}$, and of consumption habits, $\bar{s}$, of the same amount (see (41a)). These changes are proportional to the loss¹⁰ in the purchasing power of exports in terms of consumption goods, $[(\bar{Y} - \bar{d})/p_c] dp < 0$. The difference with time-additive utility is that the modification of c in the long-run is greater (in absolute value) the higher is habits' persistence, i.e. the lower is $|\mu_1|$. This can be explained in an intuitive way; a drop in p implies a fall in real income which incites households to decrease their real expenditure. When preferences display adjacent complementarity, i.e. the persistence of consumption habits is high, consumers reduce initially c but less than the drop in real income which implies a decumulation of financial wealth and hence amplify the long-run effects of the negative income shock on consumption. This process can be unstable if condition (27) is removed (see Becker and Murphy [1988]).

From expression (41c), the permanent decrease in p has two influences which work in the same direction on marginal utility of wealth expressed in terms of the foreign good. First, a fall in p lowers domestic real income which induces a drop in consumption. This income effect

is proportional to $(\bar{Y} - \bar{d})$ (see the first term on the right hand side of eq (41c)). Second, a negative change in p favors the consumption of the domestically produced good, d , and incites households to reduce the level of their consumption in the foreign good. This substitution effect is proportional to α_c , the share of domestic goods in consumption expenditure (see the second term on the right hand side of eq (41c)). These two adjustments lead to an increase in marginal utility of wealth expressed in terms of the foreign good.

When the expenditure in capital goods has an import content, a decrease in p leads to a drop in the marginal product of capital expressed in the foreign good greater in absolute value than the reduction of the user cost of capital. Adjustment in the long-run calls for a lower capital stock, $\bar{k}$. At the new steady-state, the shadow price of installed capital is definitely reduced to the level $\bar{q}_1 = p_I(p_1)$ and Tobin's q is equal to unity.

The change in the foreign assets stock, $\bar{b}$, is the result of two forces. First, in response to an adverse permanent terms of trade perturbation, financial wealth increases or decreases according to households' preferences display distant $((\sigma + \mu_1)^{dist} < 0)$ or adjacent complementarity $((\sigma + \mu_1)^{adj} > 0)$, i. e.

$$\frac{d\bar{a}}{dp} = -\frac{p_c(\sigma + \mu_1)}{\sigma(\mu_1 - r^*)} \frac{d\bar{s}}{dp} \lesseqgtr 0 \quad \text{according to} \quad (\sigma + \mu_1) \lesseqgtr 0,$$

where the non-human wealth, $a = b + qk$, is the sum of the value of internationally traded bonds and the value of domestic equities, both expressed in terms of the foreign good. Second, a drop in the relative price of domestic goods discourages investment which affects positively the stock of foreign bonds. Finally, we assume that under adjacent complementarity, the savings flow dominates the investment flow, i.e the long-run value of the foreign assets' stock decreases following a permanent terms of trade deterioration. This assumption can be expressed in a more formal way :

$$\left. \frac{d\bar{b}}{dp} \right|^{adj} = -p_I \frac{d\bar{k}}{dp} - \frac{p_c(\sigma + \mu_1)}{\sigma(\mu_1 - r^*)} \frac{d\bar{s}}{dp} > 0, \quad (42)$$

where the notation *adj* means that we refer to the adjacent complementarity case. At the new steady-state, the equilibrium current account must be zero, $\bar{c}a = 0$; the trade balance must mirror the balance of net interest earnings, i. e. $\bar{n}x = -r^*\bar{b}$. Under adjacent complementarity, the economy must raise net exports to compensate the losses in interest earnings, i. e. $\bar{n}x_1 = -r^*\bar{b}_1 > \bar{n}x_0 = -r^*\bar{b}_0$.

Transitional Dynamics

We assume that the economy is initially in an “old” steady-state. Due to the zero-root property, the marginal utility of wealth jumps instantaneously at time $t = 0$ to its new steady-state value, $\bar{\lambda}$, and remains constant from thereon. Whereas s_0 is predetermined, real expense is free to jump at time $t = 0$ to situate the economy on the stable branch :

$$\frac{dc(0)}{dp} = \left[1 - \left(\frac{\sigma + \mu_1}{\sigma} \right) \right] \geq \frac{d\bar{c}}{dp} \quad \text{according to} \quad (\sigma + \mu_1) \lesseqgtr 0, \quad (43a)$$

$$= -\frac{\mu_1}{\sigma} \frac{d\bar{c}}{dp} > 0, \quad (43b)$$

where we differentiated the stable solution (29) evaluated at time $t = 0$ with respect to p . In the additive-utility case, a permanent deterioration of terms of trade leads to a decline in real expense equal to the drop in real income. The economy jumps immediately to the new steady-state and saving is not affected (see Obstfeld [1983], p. 139). When preferences of habit-forming

consumers display adjacent complementarity, a negative income shock calls for a (decreasing) gradual response of c . This sluggish consumption adjustment follows an initial jump of real expense less important than its decline in the long-run (see eq (43a)). Agents choose $c(0)$ by assigning a positive weight to the initial consumption habits' stock, s_0 , and a weight less than one to the annuity value of total wealth deflated by the consumption price index, $r^* [b_0 + W(0)] / p_c$, i.e. the permanent income (see appendix B).

When the representative household tries to maintain his consumption habits, he decumulates financial wealth after the permanent deterioration of terms of trade. The economic intuition behind this result rests on the degree of habit persistence in consumption determined by μ_1 which in turn depends on the value of u_{12} . With adjacent complementarity ($\Gamma > 0$), a consumption experience decrease in the long-run implies an initial rise in the rate of time preference, i. e. :

$$\frac{d\rho(0)}{dp} = -\frac{u_{11}(\delta + \sigma)}{p_c \bar{\lambda}} \left[\left(\frac{\sigma + \mu_1}{\sigma} \right)^{adj} + \frac{\delta + 2\sigma}{u_{11}(\delta + \sigma)} \Gamma \right] \frac{d\bar{s}}{dp} < 0. \quad (44)$$

where the expression in square brackets is negative (see appendix A). In words, the high positive value of the cross-partial derivative of the felicity function, u_{12} , implies that the marginal utility of current real expense is greater than the marginal utility of future real expense, so that consumption is always biased toward the present. Figure 5 depicts the adjustment of c and s following a permanent deterioration of terms of trade. Once initial jump of c is operated, the real expense and consumption habits' stock monotonically decreased along the stable branch $U_1 U_1$. In the same time, the rate of time preference falls toward the fixed discount rate, δ , as s converges to its new steady-state value.

With k_0 being predetermined, the initial jump in $q(0)$, following an unanticipated permanent deterioration of terms of trade is given by:

$$\frac{dq(0)}{dp} = \frac{d\bar{q}}{dp} - \left(\frac{p_I \chi_1}{\kappa'(1) \bar{k}} \right) \frac{d\bar{k}}{dp} > \frac{d\bar{q}}{dp} > 0, \quad (45)$$

where we differentiate (33b) evaluated in $t = 0$ with respect to p . The relative price of home goods' decline leads, as explained in the discussion of the steady-state, to a perfectly decrease in the capital stock and thus gives rise to an instantaneous downward jump of Tobin's q , q/p_I , on the new stable branch $X_1 X_1$ (see figure 4). The depressing effect on investment from the decrease in q is greater the larger is the gap between the short-run decrease of the market price of capital, $q(0)$, and the new investment price index, $p_I(p_1) = \bar{q}_1$, i.e the higher is the import content of capital goods.¹¹ Figure 4 depicts the decumulation of capital stock, $\dot{k}(t) < 0$, which slows down over time, $\ddot{k}(t) > 0$. The decline of k raises its marginal productivity over time, and therefore its market price at a decreasing rate.

Having discussed the dynamic transition of real expense and investment towards the new long-run equilibrium of the economy, we turn now to the current account and net exports adjustment following a worsening of the terms of trade. The transitional dynamics can be best described by the use of phase portraits for the stable manifold. Because there are two sluggish variables, the equilibrium system follows a two-dimensional stable path determined by the evolution of the capital and habits' stocks. The construction of phase portraits is described in appendix C. Because we are interested in providing a new explanation of the J-curve phenomenon, we restrict the study by assuming that [i] the representative household's preferences exhibit adjacent complementarity ($(\sigma + \mu_1)^{adj} > 0$), [ii] the adjustment speed $|\mu_1|$ of habits is higher than the adjustment speed $|\chi_1|$ of capital, and [iii] inequality (42), according to which a permanent terms of trade worsening reduces the stock of foreign assets in the long-run, holds. Transitional dynamics

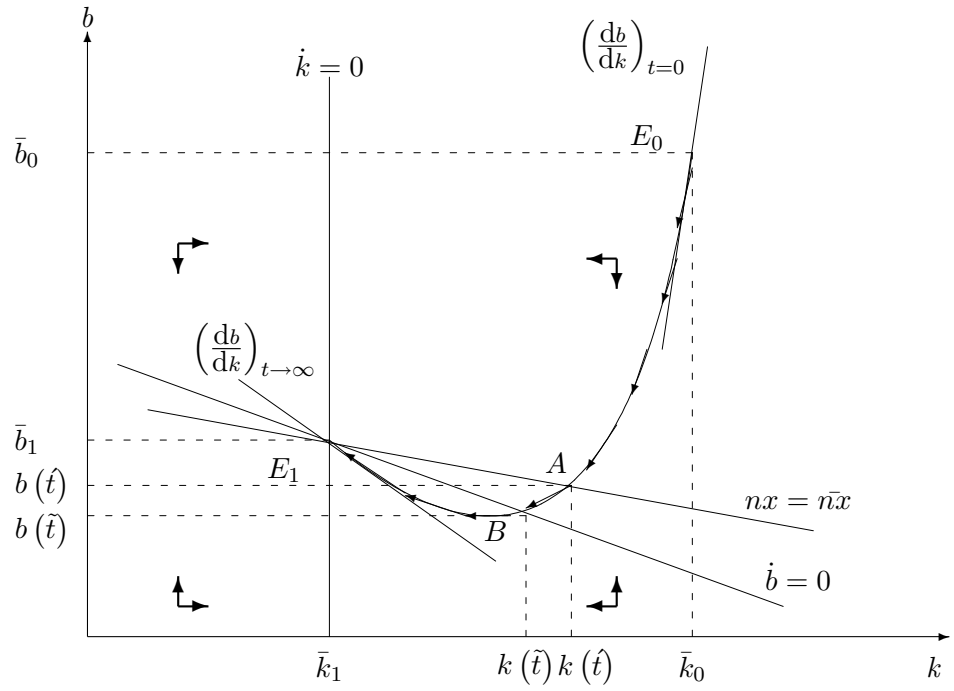


Figure 1: Permanent terms of trade worsening and net foreign assets adjustment in the (k, b) -space: the J-Curve - $\mu_1 < \chi_1$ - Adjacent complementarity.

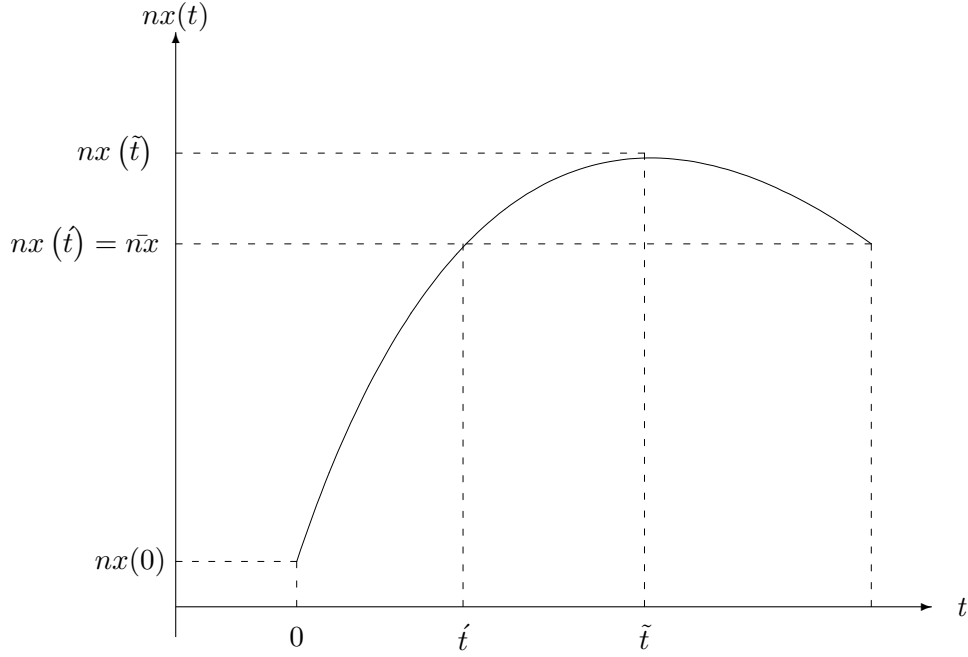


Figure 2: Permanent terms of trade worsening and real net exports adjustment: the overshooting phenomenon.

can be disentangled in two phases by noting that there exists a date $t = \tilde{t}$ (with $ca(\tilde{t}) = 0$) such that the stock of foreign assets overshoots along the stable trajectory.¹² To summarize, we find

$$ca(t) = \dot{b}(t) \leq 0, \quad \text{pour } t \leq \tilde{t}, \quad (46a)$$

$$ca(t) = \dot{b}(t) > 0, \quad \text{pour } t > \tilde{t}. \quad (46b)$$

An unanticipated permanent worsening of terms of trade leads to an immediate decumulation of foreign bonds and a deterioration in the trade balance, the initial level of b being predetermined:

$$\dot{b}(0) = ca(0) = \left[p_I \chi_1 \frac{d\bar{k}}{dp} + \mu_1 \frac{p_c(\sigma + \mu_1)}{\sigma(\mu_1 - r^*)} \frac{d\bar{s}}{dp} \right] dp < 0, \quad (47)$$

where we differentiated (36) with respect to time, then evaluated the expression at time $t = 0$, and assumed a permanent fall in p . The initial current account deficit is a consequence of the fact that the initial negative flow of saving, $\dot{a}(0) = S(0) < 0$, more than outweighs the influence of the initial negative flow of investment. The assumption of habit-forming behavior gives rise to a decline in real expense less than proportional to the reduction in permanent income, i. e. an “excess smoothness” of consumption, and therefore a fall in financial wealth. At the same time, high installation costs imply a slow adjustment speed of capital stock, and then a small initial decumulation of equipment goods.

During the first phase $0 < t \leq \tilde{t}$, consumption habits and physical capital decrease smoothly. Since the representative household reduces its financial wealth faster than the representative firm decumulates its capital stock, the non-human wealth a approaches its steady-state faster than k . The fact that habit persistence effects dominate investment discouraging effects until date $\tilde{t}$ gives rise to an overshooting of net foreign assets’ adjustment, i. e. $b(\tilde{t}) < \bar{b}_1 < \bar{b}_0$. After its initial

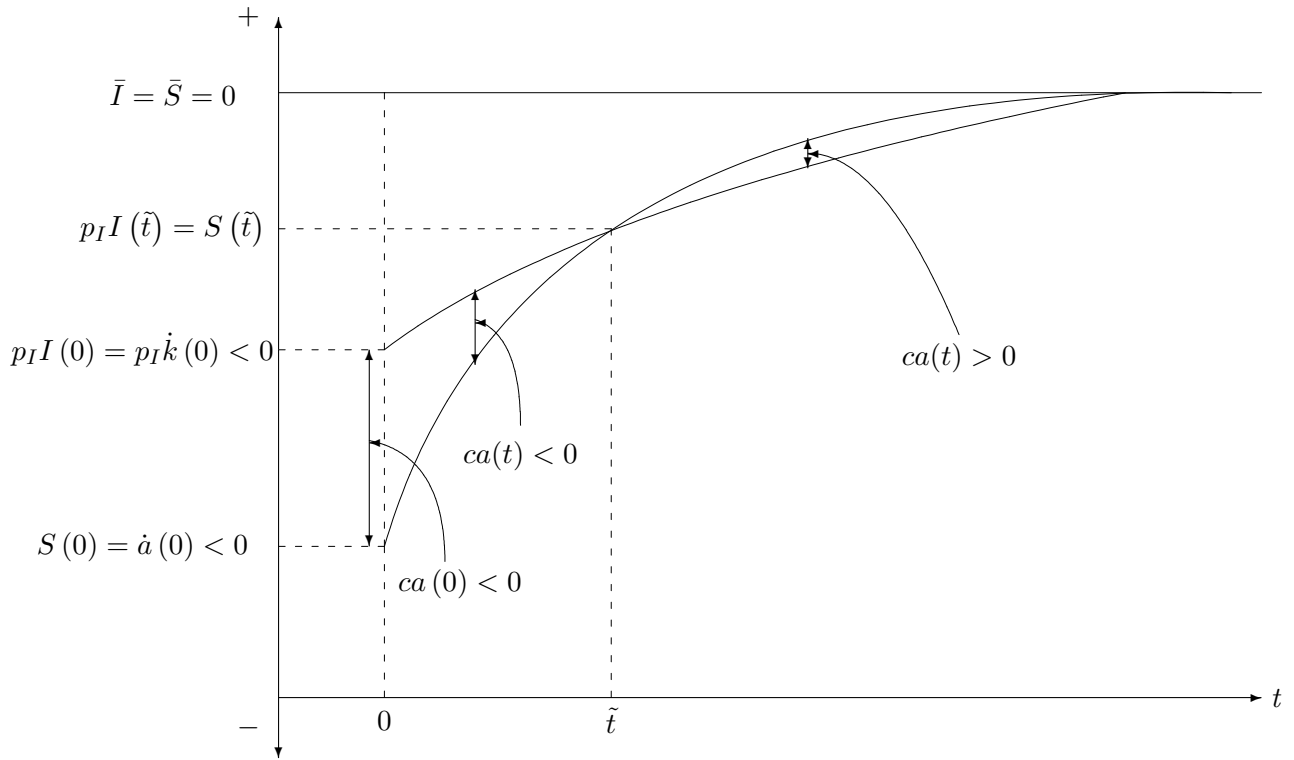


Figure 3: Permanent terms of trade worsening: saving and investment impulse functions

worsening, net exports improve faster than the current account since $nx(t) = \ddot{b}(t) - r^*\dot{b}(t) > 0$ with $ca(t) = \dot{b}(t) < 0$ and $\dot{ca}(t) = \ddot{b}(t) > 0$. We can see it graphically since the locus $nx(t) = \bar{nx}$ is flatter than the demarcation line $\dot{b}(t) = 0$ (see appendix C and figure 1).¹³ This is because at time $\tilde{t}$, the losses in interest earnings must be compensated by an improvement in net exports. By denoting $\hat{t}$ the date at which net exports reach a level at point A equal to their long-run value, the discussion above can be summarized as follows:

$$nx(t) \leq \bar{nx}, \quad \text{pour } 0 < t \leq \hat{t}, \quad (48a)$$

$$nx(t) > \bar{nx}, \quad \text{pour } \hat{t} < t < \tilde{t}. \quad (48b)$$

At some point of time, $t = \tilde{t}$, the trajectory cuts the demarcation line $\dot{b}(t) = 0$. From point B , the deterioration in the net foreign asset position is then followed by a current account surplus. Over the period $t > \tilde{t}$, investment influence becomes dominant over the habit effects. The rate at which the representative firm wishes to decumulate its physical capital stock now outweighs the rate at which representative household desires to decrease its financial wealth (see figure 3). This exercises a rise in the stock of foreign assets which converges in direction to its new steady-state level $\bar{b}_1$ at point E_1 . Turning to the trade balance adjustment, the effect of a decline in capital goods on the net exports operates through three channels: [i] the reduction of real output, [ii] the deceleration of the negative investment flow as k approaches its new steady-state value, and [iii] the rise in marginal installation costs for a given investment flow.¹⁴ Net exports decline over time but remain above their steady-state level during the second phase.

Finally, at the new steady-state, investment has ceased, capital and hence output have fallen. Levels of habits and real expense are lower than at the initial steady-state. As decumulation of foreign bonds outweighs their accumulation during the second phase, b is definitively reduced. Since current account must be balanced in the long-run, the losses in interest earnings from abroad must be compensated by an improvement of the trade balance. Short term and long term effects in the present formal setup following an unanticipated terms of trade worsening may provide some explanations of recent empirical results:

1. By using non-parametric methods, Leonard and Stockman [2002] provide some evidence of a J-curve phenomenon but their evidence does not support standard explanation. In accordance with authors' results, we find that the second phase of current account adjustment is associated with a low real income level which results from the decline in both the home goods' relative price and the stock of physical capital.
2. We provide a formalization of Krugman's economic intuition about interpretation of the J-curve phenomenon. The current account adjustment display a deficit-surplus sequence following a terms of trade deterioration when consumption habits effects are dominant first and then are more than compensated by investment effects. This new explanation depends on the interaction between habit persistence in consumption and equipment goods' installation costs which determine respectively adjustment speeds of saving and investment. Non-monotonic transitional dynamics of net foreign assets are ruled out for identical adjustment speeds (see eq (37b)).
3. Business cycle properties documented by Mendoza [1995] and Senhadji [1998] for developing countries and by Cardi [2004] for developed countries, particularly the negative contemporaneous correlation between trade balance and the relative price of imports, suggest the presence of a H-L-M effect (see figure 2). Its existence finds strong empirical support in Otto [2003] by using a structural vector autoregression model and rests on the strength of saving decline with respect to negative investment flow in our framework (see eq (42)).

4. Backus et al. [1994] provide a restatement of the J-curve phenomenon in an “international real business cycle” framework. In accordance with their empirical results, transitional dynamics show that net exports are at first negatively correlated with the relative price of imports and then positively correlated¹⁵. More precisely, net exports exhibit a hump-shaped response (see figure 2).

5 An Unanticipated Temporary Deterioration of Terms of Trade

Obstfeld [1983] was the first to study the implications of an unanticipated transitory terms of trade deterioration in an intertemporal optimizing framework. He showed that a temporary terms of trade shock generates both a smoothing effect and an intertemporal speculation effect. The first effect follows from the differential between the current and the permanent incomes. The second is induced by the transitory variation of the consumption-based real interest rate. Following a negative terms of trade perturbation which lasts a short period, the fall of real interest rate in terms of consumption provides an incentive for raising present real expense. Once the relative price comes back to its initial level, real expense is definitively reduced because of hysteresis effect. Sen and Turnovsky [1989] and Serv  n [1999] enlarge Obstfeld’s formal setup by allowing for capital accumulation in presence of installation costs. In their framework, the external asset position is essentially determined by the investment side of the model. Although a transitory terms of trade shock may generate a discontinuity in real expense adjustment, the intertemporal additive separable preferences’ assumption implies that real expense trajectory remains flat over the period $0 \leq t < T$ when the shock is at work, and over the period $t \geq T$ once the shock ends. The study of an unanticipated transitory terms of trade perturbation in the present formal setup allows to provide some new insights of the real expense and foreign asset position transitional dynamics by introducing a habit-forming behavior as we shall see below.¹⁶ In addition, the analytical techniques developed in this paper differ considerably from previous studies in that they allow to determine formal solutions for investment and consumption blocks, and then for current account, without retaining functional forms. Finally, it permits to emphasize the relevant factors for investment and consumption decisions following a temporary terms of trade worsening in an intertemporal general equilibrium model.

We now investigate the effects in the short run and in the long run of a terms of trade disturbance which lasts over the period $(0, T)$. For reason of clarity, we leave formal solutions in appendix D where we display the major steps.¹⁷ This technical appendix substantiates the discussion. The following notation is used: subscripts refer to the corresponding periods, the subscription “perm” denotes a steady-state in case of a permanent terms of trade change, and “temp” refers to an unanticipated transitory terms of trade perturbation.

Period 1 ($0 \leq t < T$)

When the transitory terms of trade disturbance is at work, the economy follows an unstable

path:

$$s(t) = \bar{s}_1 + A_1 e^{\mu_1 t} + A_2 e^{\mu_2 t}, \quad (49a)$$

$$c(t) = \bar{c}_1 + A_1 \left(\frac{\sigma + \mu_1}{\sigma} \right) e^{\mu_1 t} + A_2 \left(\frac{\sigma + \mu_2}{\sigma} \right) e^{\mu_2 t}, \quad (49b)$$

$$k(t) = \bar{k}_1 + B_1 e^{\chi_1 t} + B_2 e^{\chi_2 t}, \quad (49c)$$

$$q(t) = \bar{q}_1 + B_1 \left(\frac{p_I \chi_1}{\kappa'(1) \bar{k}} \right) e^{\chi_1 t} + B_2 \left(\frac{p_I \chi_2}{\kappa'(1) \bar{k}} \right) e^{\chi_2 t}, \quad (49d)$$

$$b(t) = \bar{b}_1 + \left[\frac{(\bar{Y} - \bar{d})}{r^*} + \frac{\mu_2 p_c (\sigma + r^*)}{r^* \sigma (\mu_1 - r^*)} t_p \right] e^{r^*(t-T)} dp - p_I (B_1 e^{\chi_1 t} + B_2 e^{\chi_2 t}) - \Upsilon_1 A_1 e^{\mu_1 t} - \Upsilon_2 A_2 e^{\mu_2 t}, \quad (49e)$$

$$nx(t) = \bar{n}x_1 + p_I (\chi_2 B_1 e^{\chi_1 t} + \chi_1 B_2 e^{\chi_2 t}) + [\mu_2 \Upsilon_1 A_1 e^{\mu_1 t} + \mu_1 \Upsilon_2 A_2 e^{\mu_2 t}], \quad (49f)$$

where we set

$$\Upsilon_1 = \frac{p_c (\sigma + \mu_1)}{\sigma (\mu_1 - r^*)} \geq 0, \quad \Upsilon_2 = \frac{p_c (\sigma + \mu_2)}{\sigma (\mu_2 - r^*)} > 0.$$

Period 2 ($t \geq T$)

Once the terms of trade shocks ends, the convergence of the economy toward the final steady-state is governed by the following equations:

$$s(t) = \bar{s}_2 + A'_1 e^{\mu_1 t}, \quad (50a)$$

$$c(t) = \bar{c}_2 + A'_1 \left(\frac{\sigma + \mu_1}{\sigma} \right) e^{\mu_1 t}, \quad (50b)$$

$$k(t) = \bar{k}_2 + B'_1 e^{\chi_1 t}, \quad (50c)$$

$$q(t) = \bar{q}_2 + B'_1 \left(\frac{p_I \chi_1}{\kappa'(1) \bar{k}} \right) e^{\chi_1 t}, \quad (50d)$$

$$b(t) = \bar{b}_2 - p_I B'_1 e^{\chi_1 t} - \frac{p_c (\sigma + \mu_1)}{\sigma (\mu_1 - r^*)} A'_1 e^{\mu_1 t}, \quad (50e)$$

$$nx(t) = \bar{n}x_2 + p_I \chi_2 B'_1 e^{\chi_1 t} + \mu_2 \frac{p_c (\sigma + \mu_1)}{\sigma (\mu_1 - r^*)} A'_1 e^{\mu_1 t}, \quad (50f)$$

5.1 Supply-Side Dynamics

Although we adopted a formal setup for the investment-side closely related to the framework specified¹⁸ by Gavin [1992] and Serv  n [1999] and our results are qualitatively the same, the solution method we apply to study temporary terms of trade disturbances effects yield to some new formal solutions for temporary shocks which allow for an analytical comparison with formal solutions obtained with permanent shocks. The extension of the *two-step* method to a two-good model with trade in capital goods leads to a richer description of investment paths.

Since steady-state values (117b) and (117c) of capital stock and its market price are only function of terms of trade and do not depend on marginal utility of wealth, a transitory home goods' relative price disturbance has not permanent effects on investment-side aggregates. Therefore, k and q revert back to their initial levels when the shock ends. The long-run effects of an unanticipated temporary terms of trade worsening can be summarized by the following inequality:

$$\bar{k}_0 = \bar{k}_2 > \bar{k}_1, \quad \bar{q}_0 = \bar{q}_2 > \bar{q}_1. \quad (51)$$

The dynamic evolutions of k and q when the terms of trade disturbance is active are governed by the couple of equations (123c) and (123d) which contain an explosive part, i. e. terms with $e^{\mu_2 t}$; that is why we say that period 1 is unstable. From the solution prevailing in the case of a temporary terms of trade shock, it can be shown that the initial impact on the market price of installed capital is dampened with respect to its initial jump following a permanent terms of trade shock. By evaluating (123d)) at time $t = 0$, using the fact that $B_1 = -B_2 - u_p dp$, and differentiating with respect to p , one obtains¹⁹

$$\left. \frac{dq(0)}{dp} \right|_{temp} = (1 - e^{-\chi_2 T}) \left[\left. \frac{d\bar{q}_1}{dp} \right|_{temp} - \frac{p_I \chi_1}{\kappa'(1) \bar{k}} \left. \frac{d\bar{k}_1}{dp} \right|_{temp} \right] = (1 - e^{-\chi_2 T}) \left. \frac{dq(0)}{dp} \right|_{perm} > 0. \quad (52)$$

After an unanticipated temporary drop of p , the market price of equipment goods jumps instantaneously in the same direction as after a definitive reduction of p ; nevertheless, its initial jump is scaled down by the term $(1 - e^{-\chi_2 T})$ which is:

1. an increasing function of the length T of the shock;
2. a decreasing function of capital installation costs.²⁰

Symmetrically to consumption response emphasized by Obstfeld [1982] in a small open economy without capital accumulation, a temporary terms of trade disturbance generates two effects on investment-side variables.²¹ Since all domestic firms are forward-looking with perfect foresight, they are aware of the whole time path of their present discounted value of future marginal products, which in turn determines their response. Moreover, they perfectly understand the temporary nature of the perturbation and perfectly expect a terms of trade improvement at time T . Whereas the shock is transitory, the discounting rate is no longer equal to r^* over the period $(0, T)$ but equal to the investment-based real rate of interest, i. e. $r^I = r^* - \dot{p}/p$, which is lower than the real world interest rate. On the one hand, a drop in the home goods' relative price discourages investment by lowering capital marginal productivity below its user cost, as long as investment expenditure has an import content. Since the negative shock is transitory, q declines less compared to its reaction to a negative permanent shock: this is the *transitory profitability* effect. On the other hand, as the relevant real rate of interest is relatively low while firms expect a rise in home goods' relative price, there is an incentive to raise investment because the price of future real investment in terms of present real investment is transitorily higher.

To emphasize the relevant factors for optimal investment reactions in the short term, we express the gap between the market price value $q(0)$ and its steady-state level at period 1 (see appendix E, eq (136)):

$$q(0) - \bar{q}_1 = \frac{p_I}{p} \left[\frac{r^*(1 - \alpha_I)}{\chi_2} (1 - e^{-\chi_2 T}) - e^{-\chi_2 T} \alpha_I \right] dp, \quad (53)$$

where $\bar{q}_1 = p_I(p_1)$ (see eq (124c)). From (53), a transitory unfavorable terms of trade movement, $dp = p_1 - p_0 < 0$, implies an initial jump of q below or above its period 1's steady-state value, i. e. $\nu(0) < 1$ or $\nu(0) > 1$, according to whether the first term or the second term in square brackets predominates. The former represents the *transitory profitability* effect which is an increasing function of [1] the share of imported capital goods in investment expenditure, $(1 - \alpha_I)$, and [2] the shock's length, T . The latter represents the *intertemporal speculation* effect which works in an opposite direction on investment. The shorter-lasting is the shock, the less the present value of future marginal product is affected and therefore the smaller is the fall in the market price of installed capital; the higher is the domestic content in investment expenditure, α_I , the greater is

the fall in the investment-based real interest rate r^I below the world interest rate r^* . This last effect encourages the domestic firms to benefit from the transitorily low level of the investment price index.

We now turn to the shape of the trajectory in (k, q) -space depicted on figure 4 during the unstable period 1. The slope of the transitional path changes over time. Under the condition that the transitory negative shock displays a high persistence which implies that the *transitory profitability* effect predominates initially, the transitional path may change its direction at time $t = \hat{t}$ such that $0 < \hat{t} < T$ (see appendix E.1). Results regarding the shape of investment trajectory over period 1 can be summarized as follows:

$$\left. \frac{dq(t)}{dk(t)} \right|_{0 < t < \hat{t}}^{T > \hat{T}} < \left. \frac{dq(t)}{dk(t)} \right|_{t=0}^{T > \hat{T}} < 0 < \left. \frac{dq(t)}{dk(t)} \right|_{\hat{t} < t < T}^{T > \hat{T}}, \quad \text{for } T > \hat{T}, \quad (54a)$$

$$0 < \left. \frac{dq(t)}{dk(t)} \right|_{0 \leq t < T}^{T < \hat{T}}, \quad \text{for } T < \hat{T}. \quad (54b)$$

When the unfavorable terms of trade shock is quite persistent, i. e. $T > \hat{T}$, the flow of investment is negative at first, declines towards zero over the period $0 < t < \hat{t}$ (the trajectory slope raises towards infinity in absolute value), and becomes positive over the period $\hat{t} < t < T$ (see eq (54a)). When the negative perturbation lasts a short time, the capital stock adjusts along a monotonic path with a positive slope in the (k, q) -space.

Case $T > \hat{T}$

The figure 4 depicts the case of a persistent negative terms of trade shock, i. e. $T > \hat{T}$. The economy jumps initially at point A such that:

$$q(0)|_{perm} < q(0)|_{temp} < \bar{q}_1 = p_I(p_1). \quad (55)$$

The impact change of the market price of installed capital is so large that the Tobin's q is less than unity which in turn encourages domestic firms to reduce their stock of equipment goods. The economy moves along a trajectory which lies above the saddle-path X_1X_1 for a permanent drop in p . While the capital stock is decreasing until the economy reaches point B , the marginal product raises which induces a rise in the market price q . As we prove in appendix E.1, there must be a time $\hat{t}$ in period 1 (with $0 < \hat{t} < T$) where the fall in the stock of equipment goods stops. At point B , Tobin's q is equal to unity,²² i. e. $q(\hat{t}) = p_I(p_1)$. Given domestic firms know that the relative price of home goods revert back to its initial level in the near future, they are incited to accumulate capital goods in order to benefit from their low cost. Beyond time $\hat{t}$, investment expenditure increases due to the rising market value of capital, q :

$$\dot{q}(t) = \frac{p_I}{\kappa'(1)\bar{k}} \left[\chi_1^2 \frac{B_1}{dp} e^{\chi_1 \hat{t}} + \chi_2^2 \frac{B_2}{dp} e^{\chi_2 \hat{t}} \right] dp > 0, \quad (56)$$

where $B_1/dp < 0$ and $B_2/dp < 0$.²³ At the same time, the growth of capital stock is operated at an increasing rate:

$$\ddot{k}(t) = \left[\chi_1^2 \frac{B_1}{dp} e^{\chi_1 t} + \chi_2^2 \frac{B_2}{dp} e^{\chi_2 t} \right] dp > 0. \quad (57)$$

This acceleration of investment expenditure and hence capital accumulation raises production even faster. The economy moves along the unstable and explosive path BC after time $\hat{t}$. This increase in the market price of installed capital boosts investment again and leads to increases of

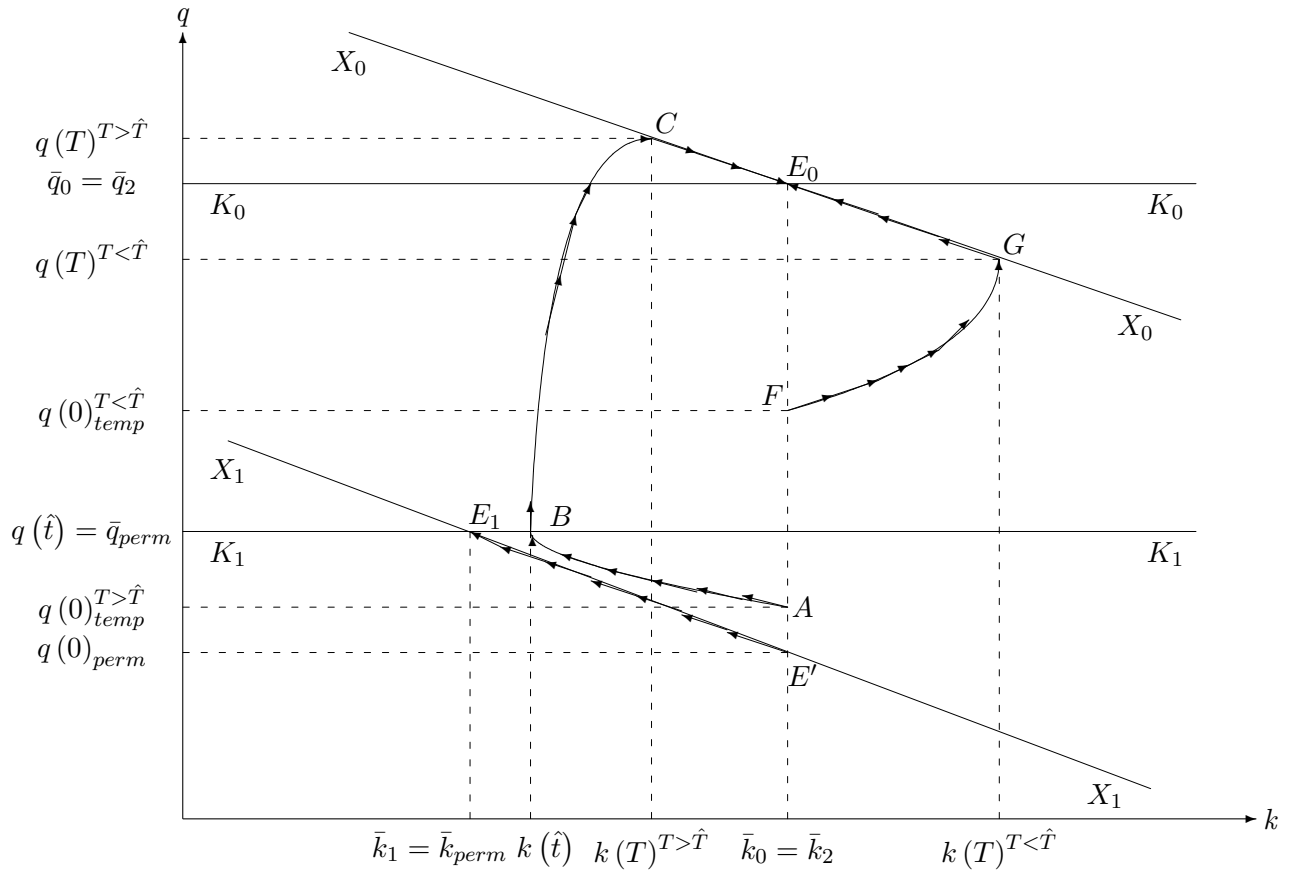


Figure 4: Permanent and temporary terms of trade worsening and investment dynamics: the *capital profitability effect* and the *intertemporal speculation effect*

the capital stock at a growing rate. Therefore, the dividend yield falls continuously. Arbitrage calls then for successively higher capital gains to guarantee the equality with the investment-based real rate of interest, $r^I(t)$. The expectation of a rising q increases the actual market price of installed capital, expectations become self-fulfilling, and this leads to successively higher expected increases of q . The only stabilizing element in the dynamic adjustment is the correctly anticipated terms of trade improvement at time T .

When the relative price of home goods turns back at its initial level (i. e. $p_T = p_2 = p_0$), there are no *news effects*, since all domestic firms anticipated the end of terms of trade disturbance. Since the capital stock and its market price cannot change discretely, $k(T^-) = k(T^+)$ and $q(T^-) = q(T^+)$, and the relative price p rises abruptly, the no-arbitrage condition between capital and traded bonds requires that the capital gain jumps at time T , that is $\dot{q}(T^-) \neq \dot{q}(T^+)$ (see eq (15f)). At instant T , the rise of p generates two effects of investment expenditure. The price of new capital rises immediately which in turn raises the investment-based real interest to the level $r^I = r^*$ and remains constant thereafter. At the same time, the growth of the profitability of capital encourages the firms to pursue the equipment goods' accumulation in order to restore k towards its initial level $\bar{k}_2 = \bar{k}_0$. The economy is now situated on trajectories which guide it towards the ultimate steady-state 2, that is situated on the stable saddle path labelled X_0X_0 . Since the market price of installed capital is still above its steady-state value, $q(T^+) > \bar{q}_2$, that is, above the price of new equipment goods, $p_I(p_2)$, as can be seen from point C on figure 4, investment flow keeps on being positive.²⁴ From this, we are able to determinate the sign of B'_1/dp by evaluating the solution (125c) at time T^+ :

$$k(T^+) - \bar{k}_2 = \frac{B'_1}{dp} e^{\chi_1 T^+} dp < 0, \quad (58)$$

Since $k(T^+) < \bar{k}_2$, we deduce that $B'_1/dp > 0$. Over period 2, the capital stock increases at a decreasing rate and therefore the economy converges towards its initial stock of equipment goods. As accumulation of capital becomes smaller as time passes, its market price decreases at a falling rate.

Case $T < \hat{T}$

When the negative terms of trade shock displays a weak persistence and/or the share of home goods in investment expenditure is important, the *intertemporal speculation* effect may outweigh the *profitability of capital* effect. This implies that the second term of the expression (53) dominates the first term. Therefore, the market price of capital jumps initially at point F such that:

$$q(0)|_{perm} < \bar{q}_1 = p_I(p_1) < q(0)|_{temp}, \quad (59)$$

which in turn leads to a Tobin's q higher than unity and a positive flow of investment, $I(0) > 0$ (see eq (138b)):

$$\dot{k}(0) = \left[\chi_1 \frac{B_1}{dp} + \chi_2 \frac{B_2}{dp} \right] dp > 0. \quad (60)$$

Over the unstable period 1, the capital stock (and its market price) increases at a growing rate:

$$\dot{k}(t) = \left[\chi_1 \frac{B_1}{dp} e^{\chi_1 t} + \chi_2 \frac{B_2}{dp} e^{\chi_2 t} \right] dp > 0, \quad (61a)$$

$$\ddot{k}(t) = \left[\chi_1^2 \frac{B_1}{dp} e^{\chi_1 t} + \chi_2^2 \frac{B_2}{dp} e^{\chi_2 t} \right] dp > 0, \quad (61b)$$

which follow from equation (60) and the fact that $e^{\chi_1 t} < e^{\chi_2 t}$. Over time, the positive value of the slope of the trajectory in the (k, q) -space becomes higher. Although the economy moves along an unstable and explosive path, all domestic firms perfectly anticipate that the negative terms of trade worsening will end in the near future and that the marginal product will rise at time T . Because the marginal productivity of capital is transitory low and the rise of the stock of capital goods intensifies it, the no-arbitrage condition calls for increases in the market price of capital ($\dot{q}(t) > 0$). Since k rises at a growing rate, the successive increases of q must be higher (i. e. $\dot{q}$ becomes bigger). At time T , the temporary terms of trade disturbances end and the economy must be situated on the stable path $X_0 X_0$. Now, the market price of installed capital is below unity since $q(T^+) < p_I(p_2) = \bar{q}_2$ as we graphed in figure 4. At point G , the capital stock $k(T^+)$ is greater than its long-run value $\bar{k}_2$ which implies that:

$$k(T^+) - \bar{k}_2 = \frac{B'_1}{dp} e^{\chi_1 T^+} dp > 0, \quad (62)$$

and $B'_1/dp < 0$. Investment expenditure jumps discretely from a positive to a negative rate. Although the terms of trade improvement at time T makes the profitability of capital greater, the price of new capital rises abruptly which in turn raises the investment-based real interest rate. Since the latter effect dominates the former, investment is discouraged. Over the stable period 2, the stock of equipment goods decreases towards its initial long-run value, $\bar{k}_2 = \bar{k}_0$. Over time, disinvestment reduces the capital stock and leads to an increase of the market price of installed capital. Since hysteresis effects are absent in the supply-side, the economy reaches its ultimate long-run steady-state 2 at point E_0 with its initial capital stock k_0 . We shall see below that temporary shocks have permanent effects on consumption aggregates, and therefore the financial wealth and the stock of foreign assets.

5.2 Demand-Side Dynamics

We describe now the short term and long term effects of an unanticipated temporary terms of trade worsening on the real expense and the stock of consumption habits. We begin by evaluating respectively the steady-state changes for period 1 and the ultimate deviation with respect to the initial steady-state:

$$\left. \frac{d\bar{c}_1}{dp} \right|_{temp} = t_{\bar{\lambda}} \left. \frac{d\bar{\lambda}}{dp} \right|_{temp} + t_p = t_{\bar{\lambda}} \lambda_p \left(1 - e^{-r^* T} \right) - t_p \frac{r^*}{\mu_1} \left(e^{-r^* T} - e^{-\mu_2 T} \right) + t_p \geq 0, \quad (63a)$$

$$\left. \frac{d\bar{c}}{dp} \right|_{temp} = \left. \frac{d\bar{\lambda}}{dp} \right|_{temp} = t_{\bar{\lambda}} \lambda_p \left(1 - e^{-r^* T} \right) - t_p \frac{r^*}{\mu_1} \left(e^{-r^* T} - e^{-\mu_2 T} \right) \geq 0, \quad (63b)$$

where $t_{\bar{\lambda}} < 0$, $t_p < 0$, and $\lambda_p < 0$. The direction of changes of the steady-state value $\bar{c}_1 = \bar{s}_1$ and of the ultimate and sustainable steady-state value $\bar{c}_2 = \bar{s}_2$ with respect to $\bar{c}_0 = \bar{s}_0$ remain undetermined. Nevertheless, we are able to emphasize the effects at work by inspecting the expression of the equilibrium change of the marginal utility of wealth following a transitory terms of trade disturbance, that is:

$$\left. \frac{d\bar{\lambda}}{dp} \right|_{temp} = \lambda_p \left(1 - e^{-r^* T} \right) - \frac{p'_c \bar{\lambda}}{p_c} \frac{r^*}{\mu_1} \left(e^{-r^* T} - e^{-\mu_2 T} \right) \leq 0 \quad (64)$$

where $p'_c/p_c \equiv \alpha_c/p$, and $(e^{-r^* T} - e^{-\mu_2 T})$ is positive because $\mu_2 > r^*$. An adverse transitory relative price's disturbance exerts on $\bar{\lambda}$ two possibly offsetting effects:

1. A *wealth effect*. The first term on the right-hand side of (64) is usually obtained by considering separable additive preferences and a temporary relative price disturbance or a habit-forming behavior and a transitory productivity shock (or a transitory intermediary input price perturbation).²⁵ The term $0 < (1 - e^{-r^*T}) < 1$ is a monotonic increasing function of T . Since $\lambda_p < 0$ denotes the steady-state change of λ for a permanent change in p , we see that the change of λ for a temporary change in p is smaller but of the same direction. The *wealth effect* comes from the fall of the real income. As the disturbance is transitory, the current income decline is greater than the drop in permanent income. This implies that the *wealth effect* following a permanent shock, λ_p , is scaled down by a *smoothing effect* reflected by the term $-e^{-r^*T}\lambda_p$. Following a negative temporary terms of trade shock, the *wealth effect* incites households to reduce their real expense through the rise of the marginal utility of wealth which is nevertheless dampened with respect to a permanent disturbance. Since agents know that the decrease in p is only temporary, the present value of the necessary reduction in real expense to satisfy the intertemporal budget constraint is less than for an equal but permanent decline in p .
2. An *intertemporal speculation effect combined with an hysteresis effect* which is no longer obtained by preceding studies.²⁶ This effect works in an opposite direction of the *wealth effect*. This phenomenon reflected by the term $-\frac{\alpha_c \bar{\lambda}}{p} \frac{r^*}{\mu_1} (e^{-r^*T} - e^{-\mu_2 T}) > 0$ may outweigh the *wealth effect* if the shock's persistence is not too high (i. e. a low value of T) and/or the share of domestic goods in consumption expenditure is important (i. e. a high value of α_c) and/or the habit persistence in consumption is strong (i. e. a low value of $|\mu_1|$). As emphasized by Obstfeld [1983], following an unanticipated fall in p which lasts only a short period of time, the transitory low value of the consumption index price, $p_c(p_1) < p_c(p_2)$, makes an incentive for the households to benefit from the temporary decline of the consumption-based real rate of interest, $r^c < r^*$.

In the time-additive utility case, the marginal utility of wealth definitively increases without ambiguity (see Obstfeld [1983], p. 140). The assumption of intertemporal separable preferences combined with the hypothesis of perfect capital international markets imply that the dynamic system possesses an eigenvalue equal to zero and a positive eigenvalue equal to r^* , that is $\mu_1 = 0$ and $\mu_2 = r^*$. The counterpart is that the second term on the right-hand side of (64) disappears. Since the eigenvector corresponding to the positive root is null, the temporal path of real expense must be flat over the periods 1 and 2. At time $t = 0$, the real expense may jump upward or downward following a temporary terms of trade worsening. At time T , the real expense is reduced without ambiguity in order to guarantee that the current account is zero in the long-run since the net foreign assets are permanently reduced. At the opposite, with a habit-forming behavior, c adjusts progressively. The dynamics of real expense over the unstable and stable periods affect the accumulation of internationally traded bonds which in turn influence the once-for-all jump of the marginal utility of wealth. By assuming a habit-forming behavior, $\bar{\lambda}$ may be permanently lower after a transitory decline in p depending on the strength of the *intertemporal speculation effect*.

From expression (63b), we see that the long-run change of *real expense* is completely determined by the direction of the initial and definitive deviation of $\bar{\lambda}$. First, we have the phenomenon of *hysteresis* which comes from the *knife-edge* condition,²⁷ that is from the imposed equality $\delta = r^*$. This result is usually obtained in small open economy models which suppose a constant time preference rate and perfect world capital markets. Second, when the households's preferences exhibit an adjacent complementarity and agents expect a rise of consumption experience in the long-run, the transitional dynamics call for an accumulation of wealth. In words, when

terms of trade revert back to their initial level at time T , the households accumulate foreign bonds over time in order to reach a higher stock of consumption habits. At the opposite, if the *wealth effect* dominates the *intertemporal speculation effect*, households reduce initially and in the long-run their real expense and their consumption experience. The difference with the time-additive utility case is that c and s adjust smoothly because of an *inertia* phenomenon. Since multiple cases may emerge and we are only interested in providing a new explanation to the *J-curve* phenomenon, we shall restrict our discussion to the adjacent complementarity case.²⁸

The initial response of $\bar{\lambda}$ exercises *news effects* on the real expense, whereas the change in the terms of trade itself constitutes a *disturbance effect* (see eq (39a)). The impact effect on real expense follows from the stable solution of c prevailing for period 1 (see appendix E.2):

$$\left. \frac{dc(0)}{dp} \right|_{temp} = -\frac{\mu_1}{\sigma} \left. \frac{d\bar{c}}{dp} \right|_{perm} \left(1 - e^{-r^*T} \right) + \frac{\mu_2}{\sigma} t_p \left(e^{-r^*T} - e^{-\mu_2 T} \right) \geq 0, \quad (65a)$$

$$= \frac{\mu_1}{\sigma} \left[\frac{A_1}{dp} + \frac{\mu_2}{\mu_2 - \mu_1} t_p e^{-\mu_2 T} \right], \quad (65b)$$

The first term on the right-hand side of (65a) captures the *wealth effect* dampened by the *smoothing effect*. The latter is greater the shorter is the shock's length reflected by the parameter T , that is the higher is the gap between the current and the permanent incomes. Following an adverse terms of trade perturbation, the fall of c is moderated with respect to the case of a permanent decrease of p . The second term reflects the *intertemporal speculation effect* which works against the *wealth effect*. Intuitively, since agents know that the decline in the relative price p and hence the advantage of a lower cost of consumption goods last only temporarily, they wish to benefit from it by consuming at higher rates. Under the conditions of a low value of T , a high share of domestic goods α_c in consumption expenditure, and a strong habit persistence, the domestic households may be encouraged to raise initially their real expense (i. e. $c(0) > \bar{c}_0$).²⁹ The economic forces at stake in the choice of the optimal initial reaction of real expense may be summarized as follows. On the one hand, the adverse perturbation reduces real wealth and induce the representative household to decrease its real expense. This *real wealth effect* is weakened by the *smoothing effect* because of the temporary nature of the shock and therefore the differential between the current income and the permanent income. On the other hand, the transitory fall of the consumption price index which makes the consumption-based real interest rate lower and therefore makes the real cost of borrowing in terms of consumption cheaper, constitutes an incentive for households to increase their real expense. This *real interest rate effect* is more likely to prevail the higher is the differential between $r^c(t)$ and r^* which raises with the domestic content of consumption, and the closer is the date such that terms of trade revert back to their initial level which strengthens the incentive to benefit of the low cost of consumption. At the opposite, for a sufficient high value of the shock's length T , the *wealth effect* predominates and the domestic households reduce initially their real expense (i. e. $c(0) < \bar{c}_0$). A striking difference with the time-additive utility case concerning the initial response of c is the presence of a new term $-\frac{\mu_2}{\sigma} t_p e^{-\mu_2 T} > 0$ which dampens the *intertemporal speculation effect*. This dampening effect arises from the households' wish to smooth their consumption change which makes the real expense less responsive (see Carrol, Overland, and Weil [2000]). Symmetrically, the stronger is the habit persistence in consumption, i. e. the lower is the speed $|\mu_1|$ of habits, the weaker is the response of c to the *wealth effect*.

Once the real expense has jumped, the initial changes of c and s are given by :

$$\left. \frac{d\dot{s}(0)}{dp} \right|_{temp} = \mu_1 \left[-\left. \frac{d\bar{c}}{dp} \right|_{perm} \left(1 - e^{-r^*T} \right) + \frac{\mu_2}{\mu_1} t_p \left(e^{-r^*T} - e^{-\mu_2 T} \right) \right] = \sigma \left. \frac{dc(0)}{dp} \right|_{temp} \geq 0, \quad (66a)$$

$$\left. \frac{d\dot{c}(0)}{dp} \right|_{temp} = -\mu_1 \left(\frac{\sigma + \mu_1}{\sigma} \right) \left[\left. \frac{d\bar{c}_1}{dp} \right|_{temp} - \frac{\sigma + r^*}{\sigma + \mu_1} t_p e^{-\mu_2 T} \right] \geq 0, \quad (66b)$$

$$= (\sigma + \mu_1) \left. \frac{dc(0)}{dp} \right|_{temp} + \frac{\mu_1 \mu_2}{\sigma} t_p e^{-\mu_2 T}, \quad (66c)$$

Expression (66a) indicates that the stock of habits go in the same direction that the initial jump of real expense. If the shock's length is not too short and preferences display adjacent complementarity, $(\sigma + \mu_1)^{adj} > 0$, the initial change of real expense is determined by its initial jump in the same way that a permanent shock. Nevertheless, if the relative price perturbation is short-living, the second term on the right-hand side may predominate. The economic intuition behind this result may be explained as follows. When the negative disturbance is not too persistent, the *smoothing* and the *intertemporal speculation effects* outweigh the *wealth effect* which imply higher consumption habits at the steady state of period 1. In the adjacent complementarity case, a higher consumption experience in the long-run implies that the marginal utility of real expense in the future is greater than in the present. The *rising habitual standard effect* leads to a growing path over time for real expense while the value of the time preference rate is transitorily below the real rate of interest, $r^c(t)$. At some date, the value of the time preference rate becomes greater than r^c and the path of real expense is decreasing until date T . When the shock is short-living, the representative household anticipates perfectly a rise in the price of consumption in the near future which in turn encourages him to reduce the real expense after its initial rise ($c(0) > \bar{c}_0$). The real expense dynamics over period 1 can be clarified from the following dynamic equation (see appendix A, eq (105)) :

$$\dot{c}(t) = -\frac{p_c \lambda}{u_{11}} \left(r^* - \alpha_c \frac{\dot{p}(t)}{p(t)} - \rho(t) \right), \quad (67)$$

where period 1's solution of the time preference rate is given by (see appendix E.2, eq (155))

$$\rho(t) = \delta + \frac{(\delta + \sigma) u_{11}}{p_c \bar{\lambda}} \left[\Lambda_1^{adj} \frac{A_1}{dp} e^{\mu_1 t} dp + \Lambda_2 \frac{A_2}{dp} e^{\mu_2 t} \right] dp \quad (68)$$

with $\delta = r^*$, $\Lambda_1^{adj} < 0$, $\Lambda_2 > 0$ and $u_{11} < 0$. The dynamic equation (67) differs from the standard one obtained in the additive-utility case by the variable nature of the time preference rate, $\rho(t)$. This implies that real expense adjusts progressively following a shock (permanent or temporary). Equation (67) differs too from the optimal dynamics of real expense obtained in the recursive-utility case by the constancy of the psychological time discount rate, δ . Once the shock hits the small country, the time preference rate may initially be greater or lower than the real rate of interest. The gap between $\rho(0)$ and r^c influences the initial variation of real expense which will be positive if the time preference rate falls sufficiently below its steady-state value.

The slope of the transitional path in the (s, c) -space is obtained by calculating the time derivatives of formal solutions of c and s prevailing over period 1 (see (123a)-(123b))

$$\frac{dc(t)}{ds(t)} = \frac{\mu_1 \left(\frac{\sigma + \mu_1}{\sigma} \right) \frac{A_1}{dp} e^{\mu_1 t} + \mu_2 \left(\frac{\sigma + \mu_2}{\sigma} \right) \frac{A_2}{dp} e^{\mu_2 t}}{\mu_1 \frac{A_1}{dp} e^{\mu_1 t} + \mu_2 \frac{A_2}{dp} e^{\mu_2 t}}. \quad (69)$$

Since the dynamics of c and s are the result of multiple forces which work in opposite ways, the changes over time of real expense and consumption habits may change of sign before time T since the economy follows unstable paths. We denote respectively $\hat{t}$ and $\acute{t}$ the dates such that $\dot{s}(\hat{t}) = 0$ and $\dot{c}(\acute{t}) = 0$. As we discussed above, a short-lived terms of trade negative disturbance gives rise to non-monotonic adjustment of real expense which increases first ($0 \leq t \leq \hat{t}$) and decreases until the relative price reverts back to its initial level ($\hat{t} < t < T$). When the shock is more persistent, the trajectory cuts the locus labelled $\dot{s} = 0$ (see figure 5); real expense reduces further and the consumption habits begins to decrease at an increasing rate. Finally, a highly persistent terms of trade worsening leads to an initial drop in real expense ($c(0) < \bar{c}_0$) since the *wealth effect* outweighs the *smoothing* and the *intertemporal speculation effects*. Nevertheless, the fall of c is moderated by the factor $0 < (1 - e^{-r^*T}) < 1$ and by the term $\frac{\mu_2}{\sigma} t_p (e^{-r^*T} - e^{-\mu_2 T}) dp > 0$ which reflects the incentive to benefit from the transitory lower level of the consumption index price.

According to the degree of terms of trade shock's persistence, three cases depicted on figure 5 may be considered :

1. A short-lived terms of trade worsening: $0 < T < \hat{T}$; the length of the shock is sufficiently short for encouraging the domestic households to rise initially their real expense.
2. A medium-lived terms of trade worsening: $\hat{T} < T < \acute{T}$. The length of the shock implies a fall in the short-run and the long-run of real expense, but less than in the permanent case.
3. A long-lived terms of trade worsening: $T > \acute{T}$. The shock is sufficiently persistent to lead to a change in c greater than in the permanent case.

Case $T < \hat{T}$

Under the assumptions of [i] a short-lived negative disturbance ($T < \hat{T}$), [ii] a high domestic content in consumption expenditure (a high value of α_c), and [iii] a strong habit persistence in consumption (a low value of $|\mu_1|$), the marginal utility of wealth is definitively reduced (see eq (64)) since the *intertemporal speculation effect* dominates the *income effect* dampened by the *smoothing effect*.³⁰ As in the additive-utility case with a temporary terms of trade shock or in the habit-forming case with a productivity shock, the equilibrium change of the marginal utility of wealth is a scaled-down factor of its reaction to a permanent change in p (see the first term on the right hand-side of (64)). With habit-forming consumers confronted with a change in the relative price which last only a short period, the transitory low value of the consumption price index exercises an incentive to increase real expense in the long run. The following inequalities follow from the once-for-all decrease of $\bar{\lambda}$ (see eq (63a) and (63b)):

$$\bar{\lambda} < \bar{\lambda}_0, \quad \bar{c}_1 > \bar{c}_2 > \bar{c}_0 > \bar{c}_{perm}, \quad (70)$$

where $d\bar{\lambda} = \bar{\lambda} - \bar{\lambda}_0$. The initial response of the real expense, c , is described by the expression :

$$\left. \frac{dc(0)}{dp} \right|_{temp}^{T < \hat{T}} = \left. \frac{dc(0)}{dp} \right|_{perm} \left(1 - e^{-r^*T} \right) + \frac{\mu_2}{\sigma} t_p \left(e^{-r^*T} - e^{-\mu_2 T} \right) < 0, \quad (71)$$

where the negative sign follows from the *intertemporal speculation* that predominates. From (71), we deduce (see eq (65b)):

$$\frac{A_1}{dp} > -\frac{\mu_2}{\mu_2 - \mu_1} t_p e^{-\mu_2 T} = \frac{A_2}{dp} - \frac{r^*}{\mu_2 - \mu_1} t_p e^{-\mu_2 T} > 0, \quad (72)$$

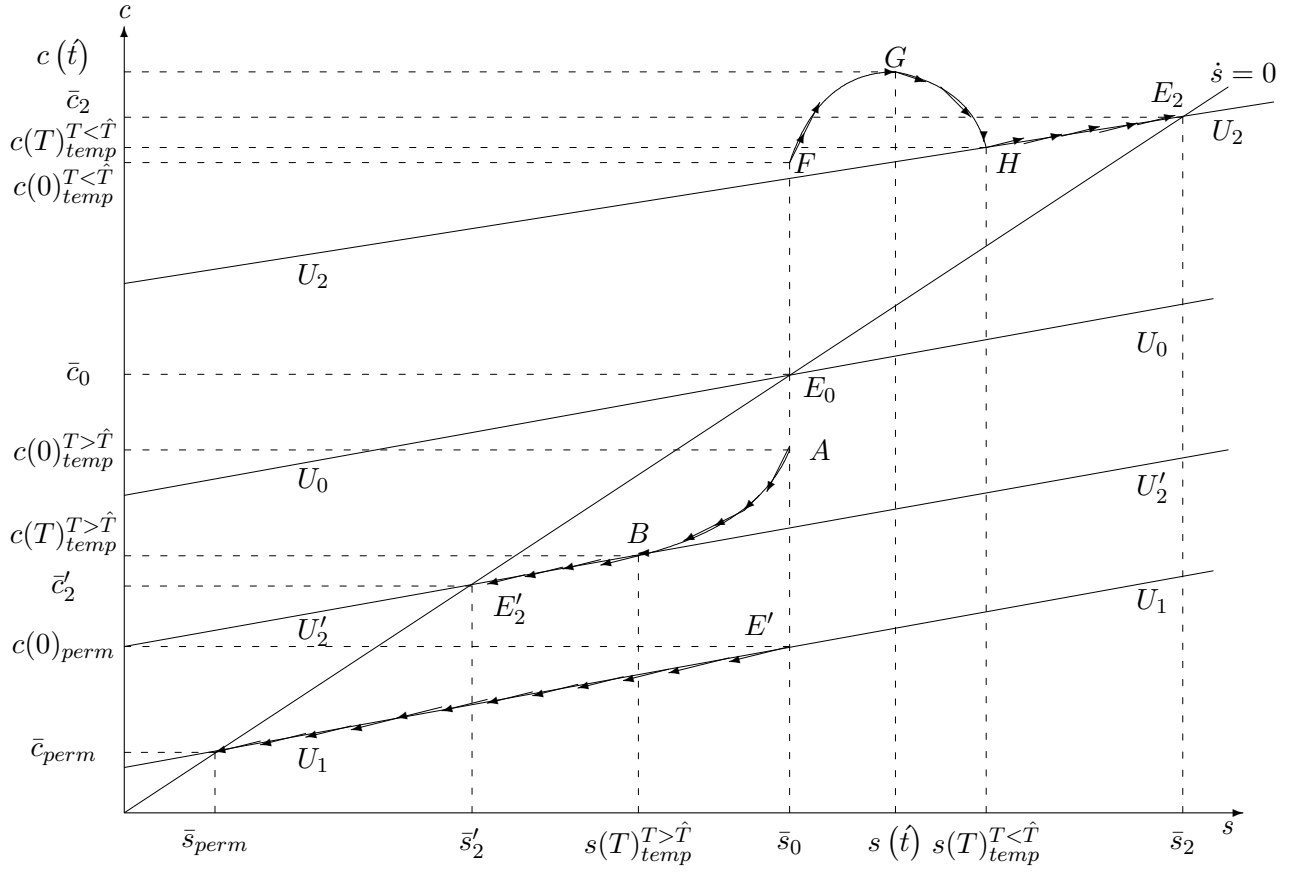


Figure 5: Permanent and temporary terms of trade worsening and real expense dynamics: the *income, smoothing, intertemporal speculation, inertia, and hysteresis* effects

where we used the fact that $-\mu_2 = \mu_1 - r^*$ and substituted the expression of A_2/dp (see eq (134b)). According to expression (66a), an upward initial jump in c gives rise to a positive change in consumption experience, $\dot{s}(0) > 0$. As the terms of trade negative disturbance is not too long, the dynamics of s over the period 1 are monotonic and the path will be situated on the stable branch labelled U_2U_2 at time T before the trajectory cuts the locus $\dot{s}(t) = 0$ (labelled $\dot{s} = 0$). The initial optimal reaction of c , $c(0)$, is driven by the expected higher habitual standard of living at the new period 1's steady-state which in turn is not a sustainable steady-state as we shall see later. Under the assumption of adjacent complementarity (adopted in this section), the positive effect of a rise of s in the long-run on the marginal utility of future real expense is strong enough to encourage households to raise c in a smaller proportion than the rise prevailing in the long-run. This optimal consumption choice leads to a fall in the time preference rate below or above the real rate of interest depending on shock's length. Since $\Lambda_1^{adj} A_1/dp < 0$, the dynamics of the time preference rate are monotonic over the period 1. The striking difference between a very short-lived and a short-lived shock comes from the extent of the initial time preference rate's fall. This can be seen formally:

$$\dot{c}(0) = -\frac{p_c \lambda}{u_{11}} \left(r^* - \alpha_c \frac{\dot{p}}{p} - \rho(0) \right), \quad (73)$$

where the initial change of real expense is positive if $r^c = r^* - \alpha_c \frac{\dot{p}}{p} > \rho(0)$. The solution (155) of the time preference rate evaluated at time $t = 0$ require some comments:

$$\rho(0) - r^* = \frac{(\delta + \sigma) u_{11}}{p_c \bar{\lambda}} (c(0) - \bar{c}_1) + \frac{(\delta + 2\sigma)}{p_c \bar{\lambda}} \Gamma^{adj} (\bar{s}_0 - \bar{s}_1), \quad (74)$$

where we used the fact that $\delta = r^*$, $s(0) = \bar{s}_0$, and $\Gamma^{adj} = u_{12} + \frac{\sigma}{\delta + 2\sigma} u_{22} > 0$. The first and second terms on the right-hand side of (74) influence respectively positively and negatively the gap $\rho(0) - r^*$. The nearest is the initial value of real expense after the disturbance from its period 1's steady-state value (with $(c(0) - \bar{c}_1)$), and the greatest is the gap $\bar{s}_0 - \bar{s}_1 < 0$ in absolute value, the most likely a decline in the time preference rate below the world interest rate. This is quite intuitive. The fall in ρ will be higher the greater is the (positive) discrepancy between the marginal utility of future real expense (that is $u(\bar{c}_1, \bar{s}_1)$) and the marginal utility of current real expense (that is $u(c(0), \bar{s}_0)$), keeping in mind that $u_{11} < 0$ and $u_{12} > 0$. According to (73), the greatest is the fall of the time preference rate, the highest is the domestic content in consumption expenditure, and the most likely a positive change in real expense following its upward initial jump. Over the unstable period 1, the time preference rate increases as the consumption habits raise :

$$\dot{\rho}(t) = \frac{(\delta + \sigma) u_{11}}{p_c \bar{\lambda}} \left[\Lambda_1^{adj} \mu_1 \frac{A_1}{dp} e^{\mu_1 t} + \Lambda_2 \mu_2 \frac{A_2}{dp} e^{\mu_2 t} \right] dp < 0, \quad (75)$$

with $\Lambda_1^{adj} < 0$, $\Lambda_2 > 0$, $A_1 dp > 0$, and $A_2/dp > 0$. At time $t = \hat{t}$, the time preference rate is equal to the real rate of interest and the real expense stops increasing, $\dot{c}(\hat{t}) = 0$ (see appendix E.2). As $\rho(t)$ keeps on increasing since the consumption experience rises over the entire period 1, the time preference rate becomes higher than r^c which in turn leads to a decreasing temporal path of c until time T , that is (see appendix A, eq (105))

$$\dot{c}(t) = -\frac{p_c \lambda}{u_{11}} \left(r^* - \alpha_c \frac{\dot{p}}{p} - \rho \right) < 0, \quad \hat{t} < t < T. \quad (76)$$

As time T approaches, the representative household lowers its real expense since he perfectly anticipates a rise in the marginal utility of wealth measured in terms of the domestic good. Since

the period 1's steady-state, $(\bar{s}_1, \bar{c}_1)$, is not viable in solvency terms, the economy cannot raise real expense over the entire period 1 and must establish the level of consumption habits at time T to a level that satisfies the overall intertemporal budget constraint. The decrease of real expense and the rise in consumption habits decelerate as time passes. The dynamics of c and s over the period 1 can be summarized as follows :

$$c(0) > \bar{c}_0, \quad \dot{c}(0) > 0, \quad \dot{c}(t) > 0, \quad 0 < t < \dot{t}, \quad \dot{c}(t) < 0, \quad \dot{t} < t < T < \dot{t}, \quad \ddot{c}(t) < 0, \quad (77a)$$

$$\dot{s}(0) > 0, \quad \dot{s}(t) > 0, \quad 0 < t \leq T < \dot{t}, \quad \ddot{s}(t) < 0. \quad (77b)$$

Over period 1, the economy moves along an unstable and explosive path. The only and merely partially stabilizing element in this dynamic adjustment is the correctly anticipated expiration of the negative terms of trade disturbance at time T .

The slope of the transitional path in (s, c) -space given by (69) changes over time as we can see from figure 5:

$$\left. \frac{dc(t)}{ds(t)} \right|_{t=0} > \left. \frac{dc(t)}{ds(t)} \right|_{0 < t < \dot{t}} > 0 = \left. \frac{dc(t)}{ds(t)} \right|_{t=\dot{t}} > \left. \frac{dc(t)}{ds(t)} \right|_{\dot{t} < t < T^-} > \left. \frac{dc(t)}{ds(t)} \right|_{t=T^-}. \quad (78)$$

When the perturbation is short-living, the time preference rate falls but less than the real interest rate which in turn implies a decreasing temporal path of real expense until time T . The dynamics of c and s are similar to those prevailing over the period 1's second phase (that is over $\dot{t} < t < T$) when the shock is short-living. The latter case is quite interesting since the real expense dynamics exhibit a hump-shape response which is no longer obtained when preferences display distant complementarity or the shock is weakly persistent. The dynamic adjustment differs greatly with the time-additive utility case or the recursive utility case since the representative household may benefit from an increase of real expense over the period $0 \leq t < \dot{t}$.

By differentiating solutions (123a) and (125a), and evaluating respectively at time T^- and time T^+ , we immediately see that the change in the consumption habits stock is continuous in the neighborhood of time T , that is $\dot{s}(T^-) = \dot{s}(T^+) > 0$ (see appendix E.2, eq (151a) and eq (152a)). This allows to determine the sign of the constant A'_1/dp :

$$\dot{s}(T^-) = \mu_1 \left[\frac{A_1}{dp} e^{\mu_1} + \frac{\mu_2}{\mu_2 - \mu_1} t_p \right] dp = \dot{s}(T^+) > 0 \quad \Rightarrow \quad \frac{A_1}{dp} + \frac{\mu_2}{\mu_2 - \mu_1} t_p e^{-\mu_1 T} = \frac{A'_1}{dp} > 0, \quad (79)$$

where we have dropped the superscripts “-”, “+” from T in the exponential functions to save notation. At time T when the negative perturbation is ended, there are no *news effects* since all agents perfectly anticipated that change in domestic relative price and the marginal utility of wealth remains constant in the neighborhood of time T , $\lambda(T^-) = \lambda(T^+) = \bar{\lambda}$. Only the improvement in the terms of trade, that is $p_1 - p_2 < 0$, constitutes a *disturbance effect* and exercises an impact on the real expense. The long-run value c falls because of the rise of the marginal utility of wealth measured in terms of the domestic good, $p_c(p_2) \bar{\lambda} > p_c(p_1) \bar{\lambda}$ and establishes to the level $\bar{c}_2 < \bar{c}_1$. The economy is now situated on a stable path labelled $U_2 U_2$ from which it converges toward the ultimate and viable steady-state 2. Since the consumption habits at time T^+ , $s(T^+)$, are below the long-term consumption experience, $\bar{s}_2$, the real expense and s increase along the stable trajectory $U_2 U_2$. The expiration of the terms of trade disturbance leads to an abruptly rise in the consumption-based real interest rate, that is $r^c(T^+) = r^*$, which remains constant during the convergence toward the new steady-state. From period 2's solution (161) of $\rho(t)$ evaluated at time T^+ , we see formally that the time preference rate is below the world interest rate:

$$\rho(T^+) - r^* = \frac{(\delta + \sigma) u_{11}}{p_c \bar{\lambda}} \Lambda_1^{adj} \frac{A'_1}{dp} e^{\mu_1 T} dp < 0, \quad (80)$$

where $\Lambda_1^{adj} < 0$ and $A'_1/dp > 0$ (see (79)). When preferences display adjacent complementarity, the marginal utility of real expense is strongly increasing with the habitual standard of living ($\Gamma > 0$), and the expectation of a higher value of consumption habits in the final steady-state implies that the time preference rate is below its long-run value, $\delta = r^*$. The explanation comes from the marginal utility of future real expense which is higher than the marginal utility of current real expense. This latter, in turn, encourages to forgo present for future real expense. The temporal path of c is increasing over period 2

$$\dot{c} = -\frac{p_c \lambda}{u_{11}} (r^* - \rho) > 0, \quad t \geq T, \quad (81)$$

The time preference rate rises and converges toward the psychological time discount rate

$$\dot{\rho}(t) = \frac{(\delta + \sigma) u_{11}}{p_c \bar{\lambda}} \Lambda_1^{adj} \dot{s}(t) > 0, \quad (82)$$

as consumption habits increase ($\dot{s}(t) > 0$) at a decreasing rate ($\ddot{s}(t) < 0$)

$$\dot{s}(t) = \mu_1 \frac{A'_1}{dp} e^{\mu_1 t} dp > 0. \quad (83)$$

The extension of Ikeda and Gombi [1998] one-good framework to a two-good model combined with new solution procedure for studying temporary shock analytically leads to interesting results which differ markedly from those of the previous authors.³¹ First, following a short-lived temporary terms of trade worsening, the real expense exhibits a hump-shaped response over the unstable period 1 which is no longer obtained by Ikeda and Gombi, or in the additive-utility case (see Obstfeld [1983], Serv  n [1999]), or with Epstein [1987a] preferences (see Cardi [2004], chapter 6). Moreover, in the distant complementarity case which is not studied in this paper for reason of space (results are however straightforward), the trajectory moves clockwise. While the real expense dynamics is monotonic, the stock of habits rises first and decreases once the trajectory cuts the locus $\dot{s} = 0$. Second, the application of the Schubert and Turnovsky [2002] two-step method permits an explicit and consistent analysis of temporary terms of trade shock. We find that the once-for-all jump in the marginal utility of wealth after a relative price change depends of two possibly offsetting effects : an *income effect* dampened by the *smoothing effect* and an *intertemporal speculation effect* moderated by a *disutility effect*. Analytical solutions allow for defining rigourously and accurately the trajectories of demand-side aggregates and determining the conditions under which economic aggregates display a monotonic or non-monotonic adjustment over the unstable period 1. Third, our analysis underscores the key role of the gap between the consumption-based real interest rate and time (variable) preference rate on the real expense dynamics.

Case $\hat{T} < T < \acute{T}$

Under the assumptions of [i] a medium-lived negative disturbance ($T > \hat{T}$), [ii] a low domestic content in consumption expenditure, the new long-run equilibrium of the marginal utility of wealth is definitely higher (see eq (64)). As the terms of trade worsening lasts a medium-period, the *smoothing* and *intertemporal speculation effects* are dominated by the *income effect*. From (64), (63a), and (63b) it then follows:³²

$$\bar{\lambda} > \bar{\lambda}_0, \quad \bar{c}_0 > \bar{c}_1 > \bar{c}_2 > \bar{c}_{perm}, \quad (84)$$

where the last inequality holds for a length of the shock $T < \acute{T}$, with $\acute{T}$ the degree of terms of trade disturbance such that the change in c following a permanent or a transitory shock are

equal. Following an unanticipated persistent terms of trade worsening, the real expense, c , falls since

$$\left. \frac{dc(0)}{dp} \right|_{temp}^{T > \hat{T}} = \left. \frac{dc(0)}{dp} \right|_{perm} \left(1 - e^{-r^*T} \right) + \frac{\mu_2}{\sigma} t_p \left(e^{-r^*T} - e^{-\mu_2 T} \right) > 0. \quad (85)$$

According to (85), the initial drop of c is moderated with respect to the initial fall following a permanent shock. Intuitively, as the length of the shock, T , raises, the reaction of real expense approaches the response in the permanent case.³³ The larger T , the longer-lasting the real income decline (or the purchasing power of real exports), and the more distant in the future the rise in the consumption price index. This encourages the representative household to decrease initially its real expense.

The gap at time $t = 0$ between the time preference rate and the world interest rate is given by

$$\rho(0) - r^* = \frac{(\delta + \sigma) u_{11}}{p_c \bar{\lambda}} \left[\Lambda_1^{adj} \frac{A_1}{dp} + \Lambda_2 \frac{A_2}{dp} \right] dp > 0. \quad (86)$$

From (86), the inequality $\rho(0) > r^* > r^c$ holds and the real expense begins to decrease after its initial drop

$$\dot{c}(0) = -\frac{p_c \lambda}{u_{11}} \left(r^* - \alpha_c \frac{\dot{p}}{p} - \rho(0) \right) < 0. \quad (87)$$

Following a persistent terms of trade worsening, the response of c is driven by the expectation of the negative long-run change of the habitual standard of living (see (84)). The real expense decreases but less than its steady-state change because of a high habit persistence in consumption.³⁴ When the marginal utility of real expense, $u_{12} > 0$, is sufficiently increasing in the consumption experience, a fall in the habitual standard of living in the long-run implies that the marginal utility of current real expense is greater than the marginal utility of future real expense. This leads to an initial rise in the time preference rate above the world interest rate as we see formally from (86).

Over the period 1, s and c co-moves along an unstable path

$$\dot{s}(t) = \left[\mu_1 \frac{A_1}{dp} e^{\mu_1 t} + \mu_2 \frac{A_2}{dp} e^{\mu_2 t} \right] dp < 0, \quad (88a)$$

$$\dot{c}(t) = \left[\mu_1 \left(\frac{\sigma + \mu_1}{\sigma} \right) \frac{A_1}{dp} e^{\mu_1 t} + \mu_2 \left(\frac{\sigma + \mu_2}{\sigma} \right) \frac{A_2}{dp} e^{\mu_2 t} \right] dp < 0, \quad (88b)$$

where

$$\frac{A_1}{dp} = -\left. \frac{d\bar{c}_1}{dp} \right|_{temp} - \frac{A_2}{dp} < 0. \quad (89)$$

since $(d\bar{c}_1/dp)^{T > \hat{T}}_{temp} > 0$ and $A_2/dp > 0$. Over period 1, the time preference rate decreases toward its steady-state value, δ . As time T approaches, the household perfectly anticipates a rise in the marginal utility of wealth measured in terms of the domestic good. The decrease in c begins to accelerate, $\ddot{c}(t) < 0$, and the stock of habits decrease at an increasing rate, $\ddot{s}(t) < 0$.

At time T , the terms of trade revert back to their initial level, $p_T = p_2 > p_1$. The relative price improvement has a negative impact on the long-run change of c (and s) due to the rise from $p_c(p_1) \bar{\lambda}$ to the level $p_c(p_2) \bar{\lambda}$. The economy is now located on a stable trajectory along which it converges to the ultimate steady-state, $(\bar{s}_2, \bar{c}_2)$. Consumption experience and real expense keeps

on decreasing further

$$\dot{s}(t) = \mu_1 \frac{A'_1}{dp} e^{\mu_1 t} dp < 0, \quad t \geq T, \quad (90a)$$

$$\dot{c}(t) = \mu_1 \frac{A'_1}{dp} \left(\frac{\sigma + \mu_1}{\sigma} \right) e^{\mu_1 t} dp < 0, \quad t \geq T, \quad (90b)$$

where $A'_1/dp < 0$ since $A_1/dp < 0$ (see eq (89) and eq (134c)). Once the original level of terms of trade is restored, the consumption-based real interest rate augments abruptly but remains below the time preference rate as we can see formally

$$\rho(T^+) - r^* = \frac{(\delta + \sigma) u_{11}}{p_c \bar{\lambda}} \Lambda_1^{adj} \frac{A'_1}{dp} e^{\mu_1 T^+} dp > 0. \quad (91)$$

The real expense dynamics over period 2 in an habit-forming model differ markedly with the dynamics prevailing in the additive-utility case from the variable nature of the time preference rate. This relationship

$$\dot{c} = -\frac{p_c \lambda}{u_{11}} (r^* - \rho(t)) < 0, \quad t \geq T, \quad (92)$$

highlights the real expense adjustment after time T . As $\rho(t)$ approaches the psychological time discount rate, the decrease of c becomes lower and lower in absolute value. At the new steady-state, the households are endowed with a lower habitual standard of living and a lower real expense. Although the rise of the marginal utility of wealth expressed in terms of the foreign good is moderated by the *smoothing* and the *intertemporal speculation* terms (see eq (64)), the real expense decline in the long-run may eventually be larger than in the permanent case as T converges to infinity.

The present framework, that is a two-good small open economy model with habit formation and capital installation costs, leads to some new results concerning the adjustment of real expense which in turn calls for new empirical studies. The real expense series should [i] follow a *Difference Stationnary* process since transitory shocks have permanent effects, [ii] be negatively correlated with the cyclical component of relative price of exports when the first-order auto-correlation of p is low, [iii] be positively correlated with the transitory component of p for a sufficiently high auto-correlation, and [iv] in the latter case, the correlation should increase with the degree of persistence of the cyclical component of p .

5.3 Current Account and Net Export Dynamics: a J-Curve?

Long-Term Effects

We investigate the effects of a transitory terms of trade worsening on the current account and net exports, both expressed in terms of the foreign good. The complexity of analytical solutions for $b(t)$ and $nx(t)$ leave undetermined the sign of the expressions. For this reason, we begin by preliminary remarks which allow to handle the adjustment of external asset position presented below. As in the additive-utility case, there is an *income effect* dampened by a *smoothing effect*: the agents reduce their stock of foreign bonds to finance the transitory fall of the current real income. As in an habit-forming small open economy model hit by an income shock, the initial decrease in real expense is larger the less is the habit persistence in consumption. In a two-good small open economy model with a habit-forming behavior and capital costs of adjustment, a transitory decline in the home goods' relative price may encourage the domestic households and firms to raise initially their real expense and their investment through an *intertemporal*

speculation effect. While in the long-run, the stock of equipment goods returns back to its initial level, the real expense may eventually reach a higher level at the new steady-state through an *hysteresis effect* combined with an *inertia effect*. This result comes from the dependence of real expense on consumption experience which in turn affects the accumulation of net foreign assets. The habit-persistence induces a departure from the time separable utility thanks to a viscosity phenomenon which leads to a gap between the short-term and long-term habitual standard of living and therefore induces a transitory change in the time preference rate. The habit-forming adjustment departs from the recursive-utility case in the permanent effects of transitory terms of trade shocks and the fixity of the psychological time discount rate.

The multiplicity of effects at work in the present framework leads to a multiple possible adjustments. Although it would be interested to investigate all possibilities, we set some assumptions for reasons of space and in order to provide a new explanation of the non-monotonic adjustment of the current account in accordance with the recent empirical results obtained by Leonard and Stockman [2002]. The three assumptions are the following:

1. The households' preferences display adjacent complementarity, that is $(\sigma + \mu_1)^{adj} > 0$ (and therefore $\Gamma > 0$, $\Upsilon_1 < 0$), which implies a low speed of adjustment in the stock of habits (a low value of $|\mu_1|$).
2. The transitory terms of trade worsening is medium-living which implies a low value of T ($T < \hat{T}$).
3. The domestic contents in consumption and investment expenditures are great (high values of α_c and α_I).

Under these three assumptions, the *intertemporal speculation effect* predominates. The stock of physical capital increases over the unstable period 1 and returns back to its initial level over the stable period 2. In the same time, the real expense displays a hump-shape response over period 1 and rises once the terms of trade disturbance has ended. The marginal utility of wealth is definitively reduced which in turn implies the following inequalities:

$$\bar{b}_1 > \bar{b}_2 > \bar{b}_0 > \bar{b}_{perm}; \quad \bar{n}x_{perm} > \bar{n}x_0 > \bar{n}x_2 > \bar{n}x_1. \quad (93)$$

Since the relative price perturbation has no permanent effects on k , and therefore on the real domestic product, a fall in $\bar{\lambda}$ implies a permanent rise in *real expense* which in turn must be exactly outweighed by higher interest revenues in order to guarantee the equilibrium of the current account at the new steady-state. Formally, in the new long-run equilibrium, the gross national product must be equal to the absorption, that is

$$p_2 F(\bar{k}_2, 1) + r^* \bar{b}_2 - p_c(p_2) \bar{c}_2 = 0, \quad (94)$$

where $p_2 = p_0$ and $\bar{k}_2 = \bar{k}_0$. When $\bar{c}_2 > \bar{c}_0$, the long-run foreign assets stock must be higher, $\bar{b}_2 > \bar{b}_0$, in order to allow the small open economy to reach the steady-state.

From equation (56), the transition over period 1 does not take place along a path converging toward a viable steady-state. This unsustainable long-run equilibrium would lead the small open economy to intertemporal insolvency. The unstable nature of period 1's transitional path can be seen from the term in brackets in expression (56) which represents the explosive (unstable) component due to the presence of the term e^{r^*t} . This term is an increasing function of time t and is composed of two terms which both exert a negative effect on the current account: the first term, $\frac{(\bar{Y}-\bar{d})}{r^*} dp < 0$ refers to the *smoothing effect*, and the second term, $\frac{\mu_2}{r^*} \frac{p_c(\sigma+r^*)}{\sigma(\mu_1-r^*)} t_p dp < 0$ refers to the *intertemporal speculation effect*.

Impact Effects

Starting off from an initial steady-state (period 0), the initial increase in real expense and investment expenditure leads to an increase in domestic absorption. In the same time, as the initial stock of physical is predetermined, the decline in the relative price of home goods induces a fall in the real income. Therefore, the trade balance deteriorates and the current account turns negative at time $t = 0^+$ (see figure 6). Formally, after tedious computations, the initial response of the current account can be calculated as

$$\begin{aligned} \left. \frac{dca(0)}{dp} \right|_{temp}^{adj} &= p_I \chi_1 \left. \frac{d\bar{k}}{dp} \right|_{perm} + \mu_1 \frac{p_c(\sigma + \mu_1)}{\sigma(\mu_1 - r^*)} \left. \frac{d\bar{c}}{dp} \right|_{perm} - p_I \left[\chi_1 \left. \frac{d\bar{k}}{dp} \right|_{perm} e^{-\chi_2 T} - \right. \\ &\quad \left. \frac{\kappa'(1)\bar{k}}{p} e^{-\chi_2 T} \alpha_I \right] - p_c \frac{\mu_1}{\sigma} \left[\left. \frac{d\bar{c}}{dp} \right|_{perm} e^{-r^* T} + t_p \frac{\mu_2}{\mu_1} (e^{-r^* T} - e^{-\mu_2 T}) \right] > 0. \quad (95) \end{aligned}$$

The above expression merits some comments:

1. The two first terms on the right-hand side of (95) represent the initial variation in the stock of internationally traded bonds after a permanent terms of trade shock (see eq (42)). We have assumed that in the adjacent complementarity case, the positive effect on $ca(0)$ of the initial drop in investment (first term) is outweighed by the negative effect induced by the fall in saving following a permanent drop in p (see (42)). Under this condition, the H-L-M effect holds in accordance with empirical facts.
2. The third term in brackets, $-p_I \chi_1 \left. \frac{d\bar{k}}{dp} \right|_{perm} e^{-\chi_2 T} dp < 0$ moderates the first term since it dampens the fall in k . It has a negative effect on $ca(0)$ which is larger the shorter-lasting is the terms of trade disturbance. The decline in the profitability in the stock of equipment goods expressed in present value is lower in absolute value the shorter is the shock's duration. The fourth term in square brackets, $p_I \frac{\kappa'(1)\bar{k}}{p} e^{-\chi_2 T} \alpha_I dp < 0$, represents the *intertemporal speculation effect* at work in the supply block. The transitory fall of the price of new equipment encourages the domestic firms to benefit from the decline in the real cost of capital goods. It exercises a negative impact on $ca(0)$ through the initial rise in investment $I(0) > 0$.
3. The fifth term in brackets, $-p_c \frac{\mu_1}{\sigma} \left. \frac{d\bar{c}}{dp} \right|_{perm} e^{-r^* T} dp < 0$, reflects the negative effect of the consumption *smoothing* behavior on the initial current account. The fall in the real income in present value is lower the shorter-lasting is the shock, and therefore the rise in the marginal utility of wealth rises with the length of the shock. In words, the negative differential between the current and the permanent incomes encourages the domestic households to maintain its real expense by reducing its foreign assets stock. The sixth term in brackets, $-p_c \frac{\mu_1}{\sigma} t_p \frac{\mu_2}{\mu_1} (e^{-r^* T} - e^{-\mu_2 T}) dp < 0$, refers to the *intertemporal speculation effect* which has a negative impact on $ca(0)$. The transitory lower level of the consumption price index constitutes an incentive for domestic households to benefit from the temporary fall in the real cost of consumption. This incentive is stronger the shorter-living is the shock and the higher is the domestic content of consumption expenditure. The habit persistence in consumption plays an ambiguous role. When the speed of adjustment of s , $|\mu_1|$, is low and the households expect a higher habitual standard of living at the period 1 steady-state, the initial rise in c is dampened since adjustment calls for rising wealth. In words, the households wants to smooth the change in c (see Fuhrer [2000]). The period 1 long run equilibrium

is clearly unsustainable in terms of solvency and will lead the economy to bankrupt if this path would be followed forever. When preferences display distant complementarity (the value of $|\mu_1|$ is high), the real expense is quite sensitive to the long-run change in s and the disutility induced by huge variations of real expense is low since the behavior of households exhibit a mean-reverting behavior (see Obstfeld [1992]). In the former case, the initial current account is less affected than in the latter case. Nevertheless, once the real expense has jumped, c may rise in the former case if the shock is not too short. In the distant complementarity case, the temporal path of real expense is decreasing without ambiguity.

Date $t = 0^+$

The results may be summarized by the following inequalities

$$c(0) > \bar{c}_0, \quad \dot{c}(0) > 0, \quad \dot{s}(0) > 0, \quad \dot{k}(0) > 0, \quad ca(0) = \dot{b}(0) < 0. \quad (96)$$

The rise in the real expense and investment through the intertemporal speculation effect deteriorates the current account.

Transitional Dynamics and the J-Curve Phenomenon

The transitional dynamics of the net foreign asset position can be best described by the use of a phase portrait in the (k, b) -space depicted in figure 7. The adjustment of the current account, $ca(t) = \dot{b}(t)$, is drawn on figure 8. It represents the impulse response of the current account following a short-lived terms of trade shock.

Once the control variables have jumped, the current account and net exports expressed in terms of the foreign good are given by

$$\begin{aligned} ca(t) = \dot{b}(t) = r^* & \left[\frac{(\bar{Y} - \bar{d})}{r^*} + \frac{\mu_2 p_c (\sigma + r^*)}{r^* \sigma (\mu_1 - r^*)} t_p \right] e^{r^*(t-T)} dp - p_I \dot{k}(t) - \\ & - \left[\Upsilon_1 \mu_1 \frac{A_1}{dp} e^{\mu_1 t} + \Upsilon_2 \mu_2 \frac{A_2}{dp} e^{\mu_2 t} \right] dp \leq 0, \end{aligned} \quad (97a)$$

$$nx(t) - \bar{n}x_1 = p_I \left[\chi_2 \frac{B_1}{dp} e^{\chi_1 t} + \chi_1 \frac{B_2}{dp} e^{\chi_2 t} \right] dp + \left[\mu_2 \Upsilon_1 \frac{A_1}{dp} e^{\mu_1 t} + \mu_1 \Upsilon_2 \frac{A_2}{dp} e^{\mu_2 t} \right] dp > 0 \quad (97b)$$

where $nx(t) = \dot{b}(t) - r^*b(t) = ca(t) - r^*b(t)$, $\left[\frac{(\bar{Y} - \bar{d})}{r^*} + \frac{\mu_2 p_c (\sigma + r^*)}{r^* \sigma (\mu_1 - r^*)} t_p \right] dp < 0$, $B_1/dp \leq 0$, $B_2/dp < 0$, $A_1/dp > 0$, $A_2/dp > 0$, $\Upsilon_1 < 0$, under the above assumptions and $\Upsilon_2 > 0$.

Period 1 ($0 < t \leq \hat{t}$)

Over the period $0 < t \leq \hat{t}$, the evolution of economic aggregates may be summarized as follows

$$\begin{aligned} \dot{c}(t) & \geq 0, \quad \ddot{c}(t) < 0, \quad \dot{s}(t) > 0, \quad \ddot{s}(t) < 0, \\ \dot{k}(t) & > 0, \quad \ddot{k}(t) > 0, \quad ca(t) = \dot{b}(t) < 0, \quad \dot{ca}(t) = \ddot{b}(t) < 0, \quad nx(t) < 0, \end{aligned} \quad (98)$$

where $\hat{t} < T$ is a date such that $\dot{c}(\hat{t}) = 0$. At the beginning of period 1, the *smoothing effect* and the *intertemporal speculation effect* which influence the consumption dynamics contribute to deteriorate the current account. Formally, the first term of (97a) in brackets which represents the explosive component of the current account's temporal path indicates that the real expense

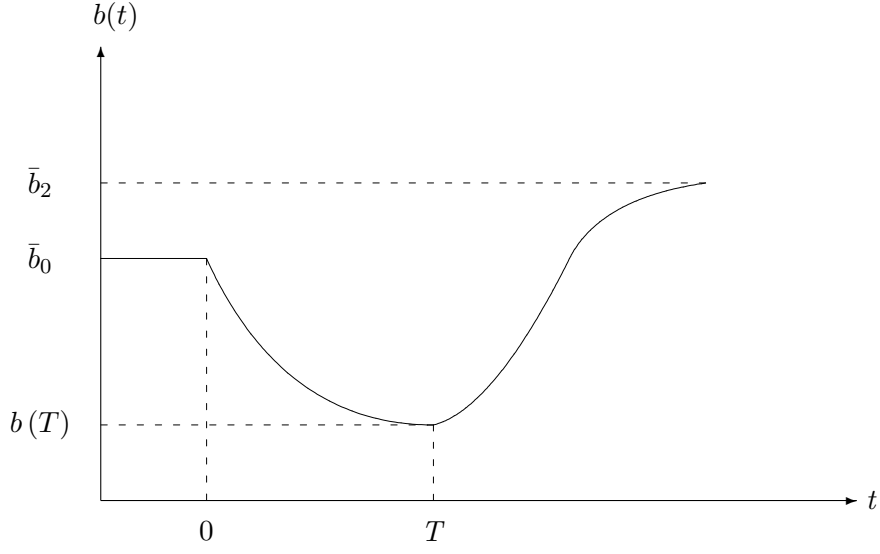


Figure 6: Temporary terms of trade worsening and the net foreign assets adjustment: the J-Curve

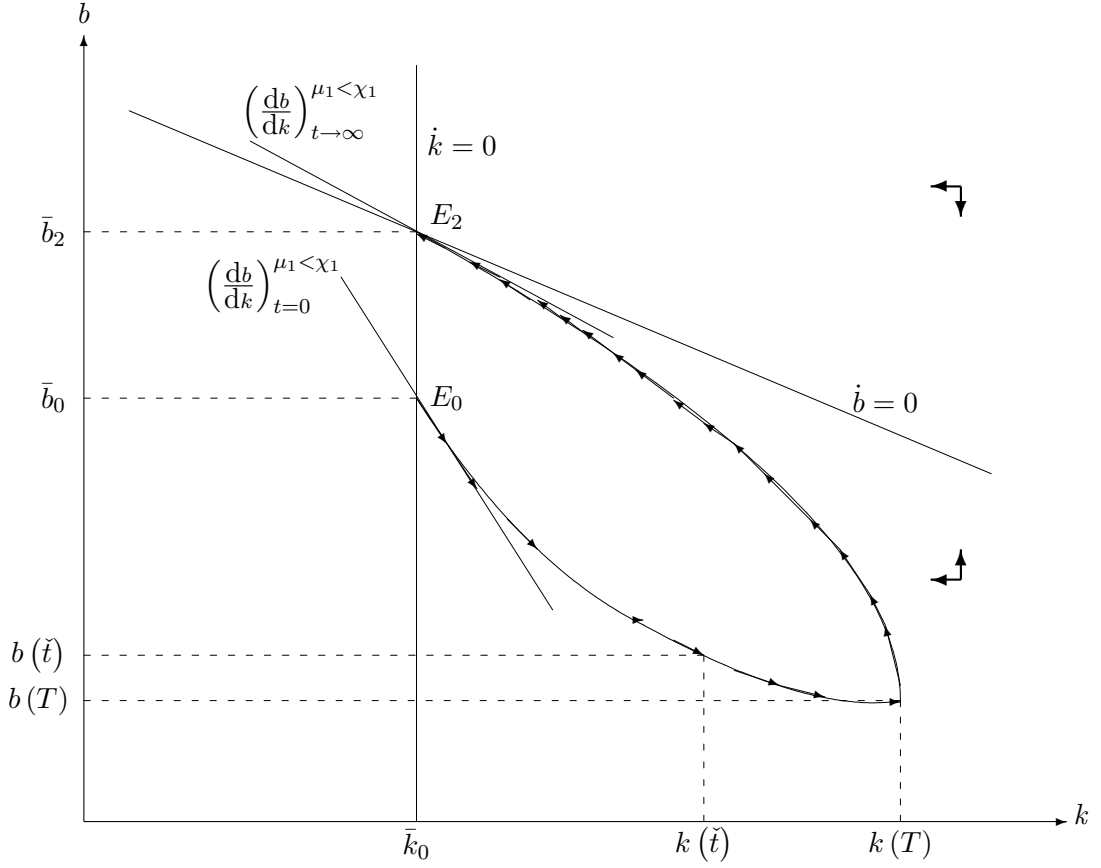


Figure 7: Temporary terms of trade worsening and the transitional path in the (k, b) -space

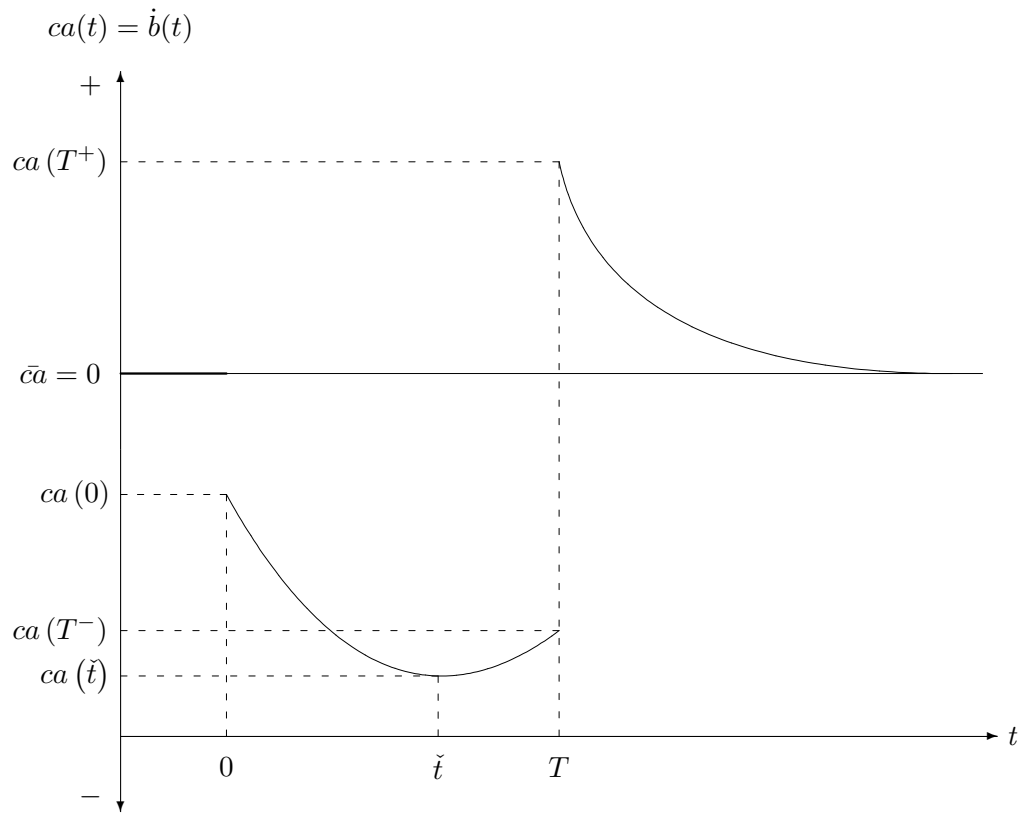


Figure 8: Temporary terms of trade worsening and the current account adjustment

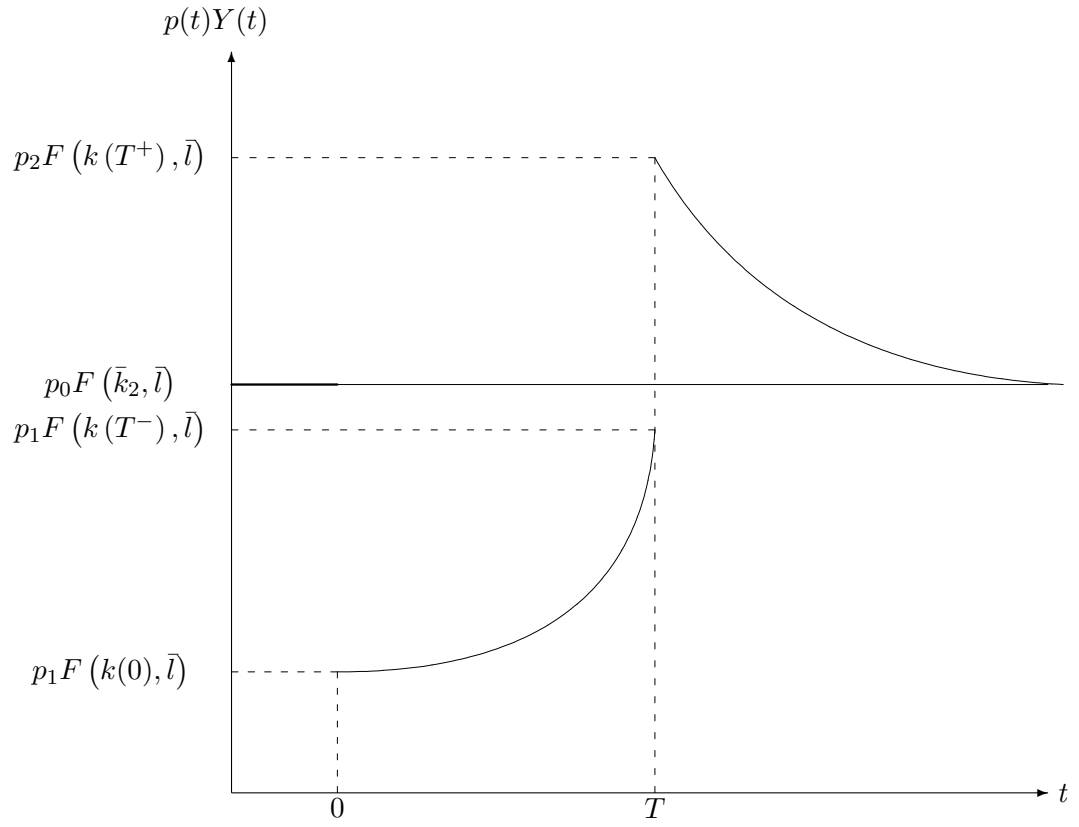


Figure 9: Temporary terms of trade worsening and the real income dynamics

adjustment and therefore the decumulation of net foreign assets will not be viable if it is pursued forever. Under the assumption that the *income effect* is dominated by the *smoothing* and *intertemporal speculation* effects, the domestic households expect a higher consumption experience at the unsustainable period 1 steady-state. When preferences are supposed to display an adjacent complementarity, the real expense adjustment exhibits an *inertia* phenomenon which in turn implies a slow and progressive rise of c . This *viscosity effect* contributes to improve the current account or to limit the decumulation of foreign bonds (see the last term of (97a) in brackets). In the same time, the investment in physical capital is positive, $I(t) > 0$, as the domestic firms wish to benefit from the transitory decline in the real cost of capital goods. The rise of $k(t)$ exercises a positive effect on the net exports by raising the real domestic product and reducing the marginal cost of investment. Nevertheless, the growth of $c(t)$ and of $I(t)$ ($\dot{I}(t) = \ddot{k}(t) > 0$) contributes to deteriorate the net exports.³⁵ By differentiating (97b), we see that the rise in k leads to a deterioration of the trade balance.

Period 1 ($\acute{t} < t < T$)

Over the period $\acute{t} < t < T$, the evolution of economic aggregates may be summarized as follows

$$\begin{aligned} \dot{c}(t) < 0, \quad \ddot{c}(t) < 0, \quad \dot{s}(t) > 0, \quad \ddot{s}(t) < 0, \\ \dot{k}(t) > 0, \quad \ddot{k}(t) > 0, \quad ca(t) = \dot{b}(t) < 0, \quad \dot{ca}(t) = \ddot{b}(t) > 0, \quad nx(t) > 0, \end{aligned} \quad (99)$$

where the current account deficit is assumed to decrease in absolute value over time due to the decrease of c at an increasing rate. This consumption adjustment is supposed to compensate the effect of investment dynamics. On the one hand, the growth of k and the decrease of c have a positive effect on real net exports, and therefore on the current account. On the other hand, the decline in interest incomes due to the fall in b and the rise of investment contribute to deteriorate the net foreign asset position.

Period 2 ($t \geq T$)

At time T , the relative price of home goods is restored to its original level. Since the switch of terms of trade was perfectly anticipated, no new information is diffused. The level of economic aggregates remain unchanged in the neighborhood of time T . The consumption and investment-based real rate of interest increase abruptly to the level of the world interest rate, that is $r^c(T^+) = r^I(T^+) = r^*$, and remain constant as the economy converges toward the final steady-state. The rise in the marginal utility of wealth measured in terms of the domestic goods induce a fall in the real expense steady-state from the level $\bar{c}_1$ to the level $\bar{c}_2$. At the same time, the absence of hysteresis effects on supply-side aggregates imply that the stock of physical capital must return to its initial level, $\bar{k}_2 = \bar{k}_0$. The real income reaches its maximum at time T^+ , that is $p_2 F[k(T^+), 1]$. The current account becomes unambiguously positive as the terms of trade improve

$$ca(T^+) = ca(T^+) = \dot{b}(T^+) = -p_I \chi_1 \frac{B'_1}{dp} e^{\chi_1 T^+} dp - \mu_1 \Upsilon_1 \frac{A'_1}{dp} e^{\mu_1 T^+} dp > 0, \quad (100)$$

where $A'_1/dp > 0$, $B'_1/dp < 0$ under the assumptions set above.

The economy switches back to a sustainable transition, consistent with intertemporal solvency, that satisfies the unique and overall intertemporal national budget constraint

$$b_T - \bar{b}_2 = -p_I (k_T - \bar{k}_2) - \frac{p_c (\sigma + \mu_1)}{\sigma (\mu_1 - r^*)} (s_T - \bar{s}_2), \quad (101)$$

where the economy's unstable period 1 transition is contained in stocks k_T , s_T , and b_T serving as new initial conditions. According to the intertemporal solvency condition (101), the stocks of consumption habits and of capital goods move respectively in the same and in an opposite direction with the stock of foreign assets.

By differentiating the formal solutions (57), we find that economic aggregates evolve as follows

$$\begin{aligned} \dot{c}(t) &> 0, & \ddot{c}(t) &< 0, & \dot{s}(t) &> 0, & \ddot{s}(t) &< 0, \\ \dot{k}(t) &< 0, & \ddot{k}(t) &> 0, & ca(t) = \dot{b}(t) &> 0, & \dot{ca}(t) = \ddot{b}(t) &< 0, & nx(t) &< 0. \end{aligned} \quad (102)$$

Since the change in the stock of habits is continuous, the consumption experience keeps on increasing. The adjustment calls for a growth of real expense as the preferences exhibit adjacent complementarity. The consumption and capital goods adjustments deteriorate the trade balance by respectively raising absorption and lowering real income. As the fall in k decreases in absolute value, that is $\dot{I}(t) > 0$, the net exports are declining without ambiguity over the stable period 2. Thanks to the *inertia effect*, the real expense rises slowly and progressively as the level of terms of trade is favorable, a stable transition calls for rising wealth through the accumulation of internationally traded bonds. Since the real income is above its long-run equilibrium value as depicted on figure 9, and the investment is negative, and the real expense is below its steady-state, the current account remains positive along the stable path.

Once the small open economy reaches the final steady-state, the stock of equipment goods is restored to its initial level and the market price of capital is equal to the original investment price index. This differs from Sen and Turnovsky [1989]'s result as we abstracted from the labor-leisure arbitrage. Henceforth, the real income is unaffected by an unanticipated temporary terms of trade shock. In empirical words, the real domestic product should follow a *Trend Stationary* process. From a theoretical viewpoint, consumption and investment decisions may be linked by assuming endogenous terms of trade or incorporating a non-traded sector at the price of an increasing analytical complexity (see for example Sen and Turnovsky [1991], Brock [1996]). Concerning demand-side aggregates, the real expense and the habitual standard of living may be definitively higher in the new long-run equilibrium. This result departs markedly from the conclusions of Obstfeld [1983] and Serv  n [1999] who assume separable additive intertemporal preferences.

6 Conclusion

Our contribution is twofold. From an economic viewpoint, we provide a new explanation of the J-curve phenomenon in terms of habit persistence in consumption and capital adjustment costs. The non-monotonic adjustment of the net foreign assets position displays a sequence deficit-surplus following a permanent or a (short-) medium-lived terms of trade worsening. In accordance with Leonard and Stockman's empirical results, the surplus phase of current account (second period) is associated with a decline in the real income. Beyond the formalization of Krugman's economic intuition, the present framework extends Obstfeld's analysis by introducing an habit-forming behavior. It has been shown that real expense adjustment may exhibit a hump-shape response when the adverse transitory disturbance is at work and may eventually reach a higher level at the new steady-state. From an analytical viewpoint, we propose a two-good extension of the Ikeda and Gombi framework and apply a new formal procedure to investigate short term and long term effects of an unanticipated temporary terms of trade worsening. The *two-step method* suggested by Schubert and Turnovsky to study the effects of transitory perturbations guarantees that arbitrage relationships are respected and allows for an analytical comparison

with permanent shocks. At the opposite of the international business cycles, we obtain some analytical solutions from which the transitional dynamics have been accurately characterized and factors of real expense, investment, net exports, and current adjustment are determined. Using Schubert and Turnovsky's procedure to analyze the effects of temporary shocks shows its capacity to deal in a consistent and explicit way with complex dynamic models in continuous time and one can expect that further research in international macroeconomic dynamics may take advantage from this mathematical tool.

The new explanation of the J-curve phenomenon suggested in this paper calls for new empirical studies which may focus on an extension of the structural model initiated by Glick and Rogoff [1995] to a two-good framework with habit-forming consumers. The introduction of habit persistence in consumption in the Glick and Rogoff discrete-time formalization has been performed by Gruber [2002] and leads to encouraging results. The work needs to be pursued by considering that the consumption and investment expenditures have an import content and by exploring the effects of relative price changes in an empirical way.

The complexity of the model elaborated in the present paper restricts the possible extensions. If consumption durable goods are introduced, the analytical tractability is no longer maintained and a numerical analysis is necessary. A future interesting study would be to incorporate some consumption durable goods by abstracting from an habit-forming behavior. We may expect that the adjustment would be close to the one obtained by assuming distant complementarity in preferences. Although after a terms of trade permanent worsening, a sequence deficit-surplus is not possible, the current account may exhibit a J-curve adjustment (under some conditions) after an unanticipated transitory terms of trade worsening. A two-country model will be also an interesting way to extend our model as it would endogenize the world interest rate as well as the relative price. Such a formalization has been recently performed by Gombi and Ikeda [2003] by abstracting from capital accumulation in a one-good framework.

A Time preference rate, real interest rate, and real expense dynamics

Define the rate of time preference, ρ , as the negative of the logarithmic rate of change of the present-value of marginal utility of real expense along a locally constant consumption path (see Epstein [1987a], Gombi and Ikeda [2003]):

$$\rho \equiv - \left. \frac{d \ln \{ [u_1(c, s) + \sigma \xi] \exp(-\delta t) \}}{dt} \right|_{\dot{c}(t)=0}. \quad (103)$$

Substitute the dynamic equations of habits' stock and its shadow price, i. e. (4) and (15e), and eliminate ξ using (15a), the time preference rate is given by:

$$\rho = \frac{1}{p_c \lambda} [(\delta + \sigma)(u_1 - p_c \lambda) + \sigma u_2 - u_{12} \sigma (c - s)] + \delta. \quad (104)$$

When the time path of terms of trade is expected not to be constant, the dynamic equation of real expense writes now as follows:

$$\dot{c} = \frac{1}{u_{11}} \left[(\delta + \sigma)(u_1 - p_c \lambda) + \sigma u_2 - u_{12} \sigma (c - s) + p_c \lambda \left(\delta - r^* + \alpha_c \frac{\dot{p}}{p} \right) \right].$$

In using the expression (104) of time preference rate, we obtain the familiar dynamic equation of consumption along an optimal path:

$$\dot{c} = - \frac{p_c \lambda}{u_{11}} \left(r^* - \alpha_c \frac{\dot{p}}{p} - \rho \right), \quad (105)$$

where α_c is the share of consumption expenditure allocated to domestic goods. According to equation (105), the real expense increases when the consumption-based real interest rate, $r^c = r^* - \alpha_c \frac{\dot{p}}{p}$, is higher than the rate of time preference, ρ . The difference from the time-additive utility function is that the rate of time preference is variable and not constant (see e. g. Obstfeld [1983], Serv  n [1999]). Moreover, it differs from recursive preferences in the constancy of psychological time discount rate. Since the long-run value of δ is constant, a disturbance implies a transitory adjustment of the time preference rate toward its steady-state value which is the psychological time discount rate.

Linearize (104) in the neighborhood of the steady-state, use the fact that $\bar{\rho} = \delta$ in the long-run, and substitute stable solutions (29) to obtain :

$$\rho(t) = \delta + \frac{u_{11}(\delta + \sigma)}{p_c \bar{\lambda}} \left[\left(\frac{\sigma + \mu_1}{\sigma} \right) + \frac{\delta + 2\sigma}{u_{11}(\delta + \sigma)} \Gamma \right] (s(t) - \bar{s}), \quad (106)$$

where $\Gamma \geq 0$ is given by (25). Being an eigenvalue of the matrix (23), μ_1 is a solution to the quadratic equation $\Gamma(\delta + 2\sigma)/u_{11}(\delta + \sigma - \mu_1) = (\sigma + \mu_1)/\sigma$. Therefore, the expression in square brackets represents the difference between the slopes of the stable branch and the slope of the locus $\dot{c}(t) = 0$, the former being always smaller than the latter. When preferences display adjacent complementarity (resp. distant complementarity), i. e. $\Gamma > 0$ (resp. $\Gamma < 0$), the expression in square brackets is negative (resp. positive).

B Real expense, habit persistence, and permanent income

Solving (15g) and invoking (16), when terms of trade are expected to be constant, i. e. $p(t) = p$ for all t , we obtain:

$$\int_0^\infty p_c c(\tau) e^{-r^* \tau} d\tau = b(0) + W(0),$$

where $W(0)$ denotes non-financial wealth, defined as the present discounted value of the future flow of output net of investment expenditure expressed in terms of the foreign good, i.e.

$$W(0) = \int_0^\infty \{pF[k(\tau), 1] - p_I J(\tau)\} e^{-r^* \tau} d\tau.$$

By substituting the stable solution (29) for $c(t)$ into the intertemporal budget constraint, we can obtain the long-run real expense level, $\bar{c}$, which satisfies the intertemporal solvency condition given a stable adjustment along the path (29)

$$\bar{c} = \frac{r^* (\sigma + \mu_1)}{\mu_1 (\sigma + r^*)} s_0 - \frac{\sigma (r^* - \mu_1) r^* [b_0 + W(0)]}{\mu_1 (\sigma + r^*) p_c}, \quad (107)$$

where $r^* [b_0 + W(0)] / p_c$ is permanent income defined as the annuity value of financial and non financial wealth deflated by the consumption price index. Evaluate (29) in $t = 0$, use the fact that $\bar{s} = \bar{c}$, and substitute into the expression above, one can determine the optimal initial level of real expense

$$c(0) = \frac{(\sigma + \mu_1)}{(\sigma + r^*)} s_0 + \frac{(r^* - \mu_1) r^* [b_0 + W(0)]}{(\sigma + r^*) p_c}. \quad (108)$$

C Graphical construction of stable manifolds

Net exports

In a first step, we determine the stable solution for net exports. Use the definition of the trade balance out and at the steady-state, i. e. respectively $nx(t) = \dot{b}(t) - r^* b(t)$ and $\bar{nx} = -r^* \bar{b}$, substitute the stable solution for the stock of foreign assets and its time derivative, one obtains :

$$nx(t) = \bar{nx} + \chi_2 p_I B_1 e^{\chi_1 t} + \mu_2 \frac{p_c (\sigma + \mu_1)}{\sigma (\mu_1 - r^*)} A_1 e^{\mu_1 t}, \quad (109a)$$

$$= \bar{nx} - \mu_2 (b(t) - \bar{b}) + p_I (\mu_1 - \chi_1) (k(t) - \bar{k}), \quad (109b)$$

where the second line is calculated from (37b) and allows for expressing the deviation ($nx(t) - \bar{nx}$) in terms of deviations of foreign assets and capital stocks from their steady-state values.

Demarcation lines

Turning to the demarcation lines, we obtain from (37a), (37a), and (109b):

$$\left(\frac{db}{dk} \right)_{\dot{k}=0} = +\infty, \quad \frac{\partial \dot{k}}{\partial k} = \chi_1 < 0, \quad (110a)$$

$$\left(\frac{db}{dk} \right)_{\dot{b}=0} = -\frac{p_I (\mu_1 - \chi_1)}{\mu_1} \geq 0 \quad \text{according to} \quad \mu_1 \geq \chi_1, \quad \frac{\partial \dot{b}}{\partial b} = \mu_1 < 0, \quad (110b)$$

$$\left(\frac{db}{dk} \right)_{nx=\bar{nx}} = \frac{p_I (\mu_1 - \chi_1)}{\mu_2}. \quad (110c)$$

The $\dot{k} = 0$ line is a vertical line. The slopes of $\dot{b} = 0$ and $nx = \bar{n}x$ lines are positive or negative according to the speed of adjustment of the consumption habits' stock, $|\mu_1|$, is smaller or greater than the speed of adjustment of the capital stock, $|\chi_1|$. As $\mu_2 > r^*$, it follows that the demarcation line $nx = \bar{n}x$ is flatter than the demarcation line $\dot{b} = 0$.

Transitional trajectory in the (k, b) -space

The slope of the transitional path in the (k, b) -space is obtained by calculating the time derivatives of (37a) and (37b):

$$\frac{db/dt}{dk/dt} = \frac{db}{dk} = -\frac{p_I \chi_1 \frac{d\bar{k}}{dp} dp + \frac{p_c(\sigma+\mu_1)}{\sigma(\mu_1-r^*)} \mu_1 \frac{d\bar{s}}{dp} e^{(\mu_1-\chi_1)t} dp}{\chi_1 \frac{d\bar{k}}{dp} dp} \gtrless 0, \quad (111)$$

where we let $A_1 = -d\bar{s}$ and $B_1 = -d\bar{k}$. If $(\mu_1 - \chi_1) < 0$, then the numerator's second term of (111) becomes smaller as time passes. There exists a date $t = \tilde{t}$ for which the slope of the transitional path becomes zero:

$$\tilde{t} = \frac{1}{\mu_1 - \chi_1} \log \left[\frac{-p_I \chi_1 d\bar{k}/dp}{\mu_1 \frac{p_c(\sigma+\mu_1)}{\sigma(\mu_1-r^*)} d\bar{s}/dp} \right] > 0,$$

where the expression in square brackets is positive and smaller than one if inequality (42) holds.

Under the assumptions of adjacent complementarity in preferences, $(\sigma + \mu_1)^{adj} > 0$, and high adjustment costs which imply $\mu_1 < \chi_1$,³⁶

$$\left(\frac{db}{dk} \right)_{t=0}^{\mu_1 < \chi_1, adj} = -\frac{p_I \chi_1 \frac{d\bar{k}}{dp} dp + \frac{p_c(\sigma+\mu_1)}{\sigma(\mu_1-r^*)} \mu_1 \frac{d\bar{s}}{dp} dp}{\chi_1 \frac{d\bar{k}}{dp} dp} > 0, \quad (112a)$$

$$\left(\frac{db}{dk} \right)_{t \rightarrow \infty}^{\mu_1 < \chi_1} = -p_I < \left(\frac{db}{dk} \right)_{b=0}^{\mu_1 < \chi_1} = -p_I \left(1 - \frac{\chi_1}{\mu_1} \right) < 0, \quad (112b)$$

where we assumed that inequality (42) holds. The slope of the transitional trajectory in (k, b) -space changes over time. Comparing the slope of the trajectory at the initial point, at time $t = \tilde{t}$, at the steady-state with the slope of the $\dot{b} = 0$ line, we find:

$$\left(\frac{db}{dk} \right)_{t=0}^{\mu_1 < \chi_1, adj} > 0 = \left(\frac{db}{dk} \right)_{t=\tilde{t}}^{\mu_1 < \chi_1, adj} > \left(\frac{db}{dk} \right)_{b=0}^{\mu_1 < \chi_1} > \left(\frac{db}{dk} \right)_{t \rightarrow \infty}^{\mu_1 < \chi_1}. \quad (113)$$

D Temporary terms of trade disturbance: a consistent solution method for the two-good model

Viable steady-state and the two-step method

Following Schubert and Turnovsky [2002], we define a viable steady-state i starting at time T_i to be one that is consistent with long-run solvency, given the stocks of capital, k_{T_i} , foreign bonds, b_{T_i} , and consumption habits, s_{T_i} . We rewrite the system of steady-state equations (38)

for an arbitrary period i :

$$u_1(\bar{c}_i, \bar{s}_i) + \frac{\sigma}{\delta + \sigma} u_2(\bar{c}_i, \bar{s}_i) = p_c(p_i) \bar{\lambda}_i, \quad (114a)$$

$$\bar{c}_i = \bar{s}_i, \quad (114b)$$

$$\bar{q}_i = p_I(p_i), \quad (114c)$$

$$r^* p_I(p_i) = p_i F_k(\bar{k}_i, 1), \quad (114d)$$

$$r^* \bar{b}_i + p_i F(\bar{k}_i, 1) - p_c(p_i) \bar{c}_i = 0, \quad (114e)$$

together with the intertemporal solvency condition

$$(b_{T_i} - \bar{b}_i) = -p_I(p_i) (k_{T_i} - \bar{k}_i) - \frac{p_c(p_i)(\sigma + \mu_1)}{\sigma(\mu_1 - r^*)} (s_{T_i} - \bar{s}_i). \quad (114f)$$

The new *consistent* procedure consists in two steps. In a **first step**, we solve the system (114a)-(114e) for $\bar{s}_i$, $\bar{c}_i$, $\bar{k}_i$, $\bar{q}_i$, and $\bar{b}_i$ as functions of the marginal utility of wealth, $\bar{\lambda}_i$, and terms of trade, p_i . Totally differentiating equations (114a)-(114e) yields in matrix form

$$\begin{pmatrix} u_{11} + \frac{\sigma}{\delta + \sigma} u_{21} & u_{12} + \frac{\sigma}{\delta + \sigma} u_{22} & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & p_i F_{kk} & 0 \\ -p_c(p_i) & 0 & p_i F_k & r^* \end{pmatrix} \begin{pmatrix} d\bar{c}_i \\ d\bar{s}_i \\ d\bar{k}_i \\ d\bar{b}_i \end{pmatrix} = \begin{pmatrix} p_c(p_i) d\bar{\lambda}_i + p'_c(p_i) \bar{\lambda} dp_i \\ 0 \\ -F_k [1 - \alpha_I(p_i)] dp_i \\ -(\bar{Y}_i - \bar{d}_i) dp_i \end{pmatrix}, \quad (115)$$

and

$$d\bar{q}_i = p'_I(p_i) dp_i. \quad (116)$$

The variables $\bar{\lambda}_i$ and p_i determine the following steady-state values

$$\bar{s}_i = \bar{c}_i = t(\bar{\lambda}_i, p_i), \quad (117a)$$

$$\bar{k}_i = u(p_i), \quad (117b)$$

$$\bar{q}_i = p_I(p_i), \quad (117c)$$

$$\bar{b}_i = v(\bar{\lambda}_i, p_i), \quad (117d)$$

with partial derivatives given by

$$t_{\bar{\lambda}} \equiv \frac{\partial \bar{s}_i}{\partial \bar{\lambda}_i} = -\frac{r^* p_i F_{kk} p_c}{G} < 0, \quad (118a)$$

$$t_p \equiv \frac{\partial \bar{s}_i}{\partial p_i} = -\frac{r^* p_i F_{kk} p'_c \bar{\lambda}_i}{G} = \frac{p'_c \bar{\lambda}_i}{p_c} t_{\bar{\lambda}} < 0, \quad (118b)$$

$$u_p \equiv \frac{\partial \bar{k}_i}{\partial p_i} = -\frac{F_k(1 - \alpha_I)}{p_i F_{kk}} > 0, \quad (118c)$$

$$v_{\bar{\lambda}} \equiv \frac{\partial \bar{b}_i}{\partial \bar{\lambda}_i} = -\frac{p_i F_{kk} p_c^2}{G} = \frac{p_c}{r^*} t_{\bar{\lambda}} < 0, \quad (118d)$$

$$\begin{aligned} v_p \equiv \frac{\partial \bar{b}_i}{\partial p_i} &= -\frac{[p_i F_k^2(1 - \alpha_I) - p_i F_{kk}(\bar{Y}_i - \bar{d}_i)] \left[u_{11} + \frac{\delta + 2\sigma}{\delta + \sigma} \left(u_{12} + \frac{\sigma}{\delta + 2\sigma} u_{22} \right) \right]}{G} - \\ &\quad - \frac{p_i F_{kk} p'_c p_c \bar{\lambda}_i}{G} = -p_I u_p + \frac{p_c}{r^*} t_p - \frac{(\bar{Y}_i - \bar{d}_i)}{r^*} < 0, \end{aligned} \quad (118e)$$

where $G \equiv -r^* p_i F_{kk} \left[u_{11} + \frac{\delta+2\sigma}{\delta+\sigma} \left(u_{12} + \frac{\sigma}{\delta+2\sigma} u_{22} \right) \right] < 0$. The sign of the expression in square brackets is negative under condition (27) of saddle-point stability. The **second step** consists to determine the equilibrium change of $\bar{\lambda}_i$ by taking the total differential of the intertemporal solvency condition (114f):

$$\begin{aligned} \left[v_{\bar{\lambda}} + t_{\bar{\lambda}} \frac{p_c(\sigma + \mu_1)}{\sigma(\mu_1 - r^*)} \right] d\bar{\lambda}_i &= db_{T_i} + p_I dk_{T_i} + \frac{p_c(\sigma + \mu_1)}{\sigma(\mu_1 - r^*)} ds_{T_i} - \\ &- \left[v_p + p_I u_p + t_p \frac{p_c(\sigma + \mu_1)}{\sigma(\mu_1 - r^*)} \right] dp_i, \end{aligned} \quad (119)$$

from which we derive

$$\lambda_p \equiv \frac{d\bar{\lambda}_i}{dp_i} = - \frac{v_p + p_I u_p + t_p \frac{p_c(\sigma + \mu_1)}{\sigma(\mu_1 - r^*)}}{v_{\bar{\lambda}} + t_{\bar{\lambda}} \frac{p_c(\sigma + \mu_1)}{\sigma(\mu_1 - r^*)}} < 0, \quad (120a)$$

$$\lambda_b \equiv \frac{d\bar{\lambda}_i}{db_{T_i}} = \frac{1}{v_{\bar{\lambda}} + t_{\bar{\lambda}} \frac{p_c(\sigma + \mu_1)}{\sigma(\mu_1 - r^*)}} = \frac{G}{D} < 0, \quad (120b)$$

$$\lambda_k \equiv \frac{d\bar{\lambda}_i}{dk_{T_i}} = \frac{p_I}{v_{\bar{\lambda}} + t_{\bar{\lambda}} \frac{p_c(\sigma + \mu_1)}{\sigma(\mu_1 - r^*)}} < 0, \quad (120c)$$

$$\lambda_s \equiv \frac{d\bar{\lambda}_i}{ds_{T_i}} = \frac{\frac{p_c(\sigma + \mu_1)}{\sigma(\mu_1 - r^*)}}{v_{\bar{\lambda}} + t_{\bar{\lambda}} \frac{p_c(\sigma + \mu_1)}{\sigma(\mu_1 - r^*)}} \geq 0 \quad \text{according to} \quad (\sigma + \mu_1) \geq 0. \quad (120d)$$

We may solve for the equilibrium value of $\bar{\lambda}_i$ as a function of initial stocks at time T_i and terms of trade:

$$\bar{\lambda} = g(s_{T_i}, k_{T_i}, b_{T_i}, p_i), \quad \lambda_s \geq 0, \quad \lambda_k < 0, \quad \lambda_b < 0, \quad \lambda_p < 0. \quad (121)$$

Formal solutions for temporary terms of trade disturbances

We assume that the small open economy is initially in steady-state equilibrium, denoted by the subscript $i = 0$:

$$s_0 = \bar{s}_0 = t(\bar{\lambda}_0, p_0) = t(g(s_0, k_0, b_0, p_0), p_0), \quad (122a)$$

$$k_0 = \bar{k}_0 = u(p_0), \quad (122b)$$

$$q_0 = \bar{q}_0 = p_I(p_0), \quad (122c)$$

$$b_0 = \bar{b}_0 = v(\bar{\lambda}_0, p_0) = v(g(s_0, k_0, b_0, p_0), p_0), \quad (122d)$$

$$\lambda_0 = \bar{\lambda}_0 = g(s_0, k_0, b_0, p_0). \quad (122e)$$

We suppose now that terms of trade change unexpectedly at time $t = 0$ from the original level p_0 to level p_1 over the period $0 \leq t < T$, and they revert back at time T permanently to their initial level, $p_T = p_2 = p_0$.

Period 1 ($0 \leq t < T$)

Whereas the terms of trade' disturbance is at work, the economy follows unstable transitional

paths:

$$s(t) = \bar{s}_1 + A_1 e^{\mu_1 t} + A_2 e^{\mu_2 t}, \quad (123a)$$

$$c(t) = \bar{c}_1 + A_1 \left(\frac{\sigma + \mu_1}{\sigma} \right) e^{\mu_1 t} + A_2 \left(\frac{\sigma + \mu_2}{\sigma} \right) e^{\mu_2 t}, \quad (123b)$$

$$k(t) = \bar{k}_1 + B_1 e^{\chi_1 t} + B_2 e^{\chi_2 t}, \quad (123c)$$

$$q(t) = \bar{q}_1 + B_1 \left(\frac{p_I \chi_1}{\kappa'(1) \bar{k}} \right) e^{\chi_1 t} + B_2 \left(\frac{p_I \chi_2}{\kappa'(1) \bar{k}} \right) e^{\chi_2 t}, \quad (123d)$$

$$b(t) = \bar{b}_1 + \left[(b_0 - \bar{b}_1) + p_I B_1 + p_I B_2 + \frac{p_c (\sigma + \mu_1)}{\sigma (\mu_1 - r^*)} A_1 + \frac{p_c (\sigma + \mu_2)}{\sigma (\mu_2 - r^*)} A_2 \right] e^{r^* t} - p_I B_1 e^{\chi_1 t} - p_I B_2 e^{\chi_2 t} - \frac{p_c (\sigma + \mu_1)}{\sigma (\mu_1 - r^*)} A_1 e^{\mu_1 t} - \frac{p_c (\sigma + \mu_2)}{\sigma (\mu_2 - r^*)} A_2 e^{\mu_2 t}, \quad (123e)$$

where the steady-state values $\bar{s}_1$, $\bar{c}_1$, $\bar{k}_1$, $\bar{q}_1$, and $\bar{b}_1$ are given by the following functions (determined from (114a)-(114e) with $i = 1$)

$$\bar{s}_1 = \bar{c}_1 = t(\bar{\lambda}, p_1), \quad (124a)$$

$$\bar{k}_1 = u(p_1), \quad (124b)$$

$$\bar{q}_1 = p_I(p_1) \quad (124c)$$

$$\bar{b}_1 = v(\bar{\lambda}, p_1), \quad (124d)$$

where the marginal utility of wealth remains constant over periods 1 and 2 at level $\bar{\lambda}_1 = \bar{\lambda}_2 = \bar{\lambda}$ after its initial jump at time $t = 0$.

Period 2 ($t \geq T$)

Once terms of trade revert back to their initial level, the economy follows stable paths

$$s(t) = \bar{s}_2 + A'_1 e^{\mu_1 t}, \quad (125a)$$

$$c(t) = \bar{c}_2 + A'_1 \left(\frac{\sigma + \mu_1}{\sigma} \right) e^{\mu_1 t}, \quad (125b)$$

$$k(t) = \bar{k}_2 + B'_1 e^{\chi_1 t}, \quad (125c)$$

$$q(t) = \bar{q}_2 + B'_1 \left(\frac{p_I \chi_1}{\kappa'(1) \bar{k}} \right) e^{\chi_1 t}, \quad (125d)$$

$$b(t) = \bar{b}_2 - p_I B'_1 e^{\chi_1 t} - \frac{p_c (\sigma + \mu_1)}{\sigma (\mu_1 - r^*)} A'_1 e^{\mu_1 t}, \quad (125e)$$

where the steady-state values $\bar{s}_2$, $\bar{c}_2$, $\bar{k}_2$, $\bar{q}_2$, and $\bar{b}_2$ are given by the following functions (determined from (114a)-(114e) with $i = 2$)

$$\bar{s}_2 = \bar{c}_2 = t(\bar{\lambda}, p_2), \quad (126a)$$

$$\bar{k}_2 = u(p_2), \quad (126b)$$

$$\bar{q}_2 = p_I(p_2) \quad (126c)$$

$$\bar{b}_2 = v(\bar{\lambda}, p_2). \quad (126d)$$

During the transition period 1, the economy accumulates capital, foreign assets, and consumption habits stocks. Since this period is by its very nature unstable, it would lead the nation

to violate its intertemporal budget constraint. By contrast, the adjustment process taking place in period 2 is stable and must satisfy the economy's intertemporal budget constraint. At the same time, the zero-root problem requires the equilibrium value of marginal utility of wealth to adjust once-and-for-all when the shock hits the economy. So λ remains constant over the periods 1 and 2. The aim of the *two-step method* is to calculate the deviation of λ such that the country satisfies one single and overall intertemporal budget constraint, given the new relevant initial conditions k_T , b_T , and s_T prevailing when the shock ends and accumulated over the unstable period. Therefore, for the country to remain intertemporally solvent, we require:

$$b_T - \bar{b}_2 = -p_I (k_T - \bar{k}_2) - \frac{p_c (\sigma + \mu_1)}{\sigma (\mu_1 - r^*)} (s_T - \bar{s}_2). \quad (127)$$

In order to determine the six constants A_1 , A_2 , B_1 , B_2 , A'_1 , et B'_1 , and the equilibrium value of marginal utility of wealth, we impose three conditions:

1. Initial conditions $k(0) = k_0$, $b(0) = b_0$, $s(0) = s_0$ must be met.
2. Economic aggregates s , c , k , q remain continuous at time T .
3. The intertemporal solvency constraint (127) must hold.³⁷

Set $t = 0$ in solutions (123a) and (123c), equate (123a) and (125a), (123b) and (125b), (123c) and (125c), (123d) and (125d) evaluated at time $t = T$, one obtains

$$\bar{s}_1 + A_1 + A_2 = s_0, \quad (128a)$$

$$\bar{s}_1 + A_1 e^{\mu_1 T} + A_2 e^{\mu_2 T} = \bar{s}_2 + A'_1 e^{\mu_1 T}, \quad (128b)$$

$$\bar{c}_1 + A_1 \left(\frac{\sigma + \mu_1}{\sigma} \right) e^{\mu_1 T} + A_2 \left(\frac{\sigma + \mu_2}{\sigma} \right) e^{\mu_2 T} = \bar{c}_2 + A'_1 \left(\frac{\sigma + \mu_1}{\sigma} \right) e^{\mu_1 T}, \quad (128c)$$

$$\bar{k}_1 + B_1 + B_2 = k_0, \quad (128d)$$

$$\bar{k}_1 + B_1 e^{\chi_1 T} + B_2 e^{\chi_2 T} = \bar{k}_2 + B'_1 e^{\chi_1 T}, \quad (128e)$$

$$\bar{q}_1 + B_1 \left(\frac{p_I \chi_1}{\kappa' (1) \bar{k}} \right) e^{\chi_1 T} + B_2 \left(\frac{p_I \chi_2}{\kappa' (1) \bar{k}} \right) e^{\chi_2 T} = \bar{q}_2 + B'_1 \left(\frac{p_I \chi_1}{\kappa' (1) \bar{k}} \right) e^{\chi_1 T}, \quad (128f)$$

where we used the continuity condition.

Evaluating k_T , s_T , and b_T from respectively (123a), (123c), and (123e), substituting into (127), and using functions of steady-state values $\bar{k}_i$, $\bar{s}_i$, and $\bar{b}_i$ given by (122) (for $i = 0$), (124) (for $i = 1$), and (126) (for $i = 2$), the intertemporal solvency condition can be rewritten as

$$\begin{aligned} & v(\bar{\lambda}, p_1) + \left[(v(\lambda_0, p_0) - v(\bar{\lambda}, p_1)) + p_I B_1 + p_I B_2 + \frac{p_c (\sigma + \mu_1)}{\sigma (\mu_1 - r^*)} A_1 + \frac{p_c (\sigma + \mu_2)}{\sigma (\mu_2 - r^*)} A_2 \right] e^{r^* T} - \\ & - p_I B_1 e^{\chi_1 T} - p_I B_2 e^{\chi_2 T} - \frac{p_c (\sigma + \mu_1)}{\sigma (\mu_1 - r^*)} A_1 e^{\mu_1 T} - \frac{p_c (\sigma + \mu_2)}{\sigma (\mu_2 - r^*)} A_2 e^{\mu_2 T} - v(\bar{\lambda}, p_2) \\ = & -p_I [u(p_1) + B_1 e^{\chi_1 T} + B_2 e^{\chi_2 T} - u(p_2)] - \frac{p_c (\sigma + \mu_1)}{\sigma (\mu_1 - r^*)} \left[t(\bar{\lambda}, p_1) + A_1 e^{\mu_1 T} + \right. \\ & \left. + A_2 e^{\mu_2 T} - t(\bar{\lambda}, p_2) \right]. \end{aligned} \quad (129)$$

Then, we approximate the steady-state changes with the differentials by using the dummy

notation

$$\bar{s}_1 - \bar{s}_0 \equiv t(\bar{\lambda}, p_1) - t(\lambda_0, p_0) = t_{\bar{\lambda}} d\bar{\lambda} + t_p dp, \quad (130a)$$

$$\bar{s}_2 - \bar{s}_1 \equiv t(\bar{\lambda}, p_2) - t(\bar{\lambda}, p_1) = -t_p dp, \quad (130b)$$

$$\bar{k}_1 - \bar{k}_0 \equiv u(p_1) - u(p_0) = u_p dp, \quad (130c)$$

$$\bar{k}_2 - \bar{k}_1 \equiv u(p_2) - u(p_1) = -u_p dp, \quad (130d)$$

$$\bar{q}_2 - \bar{q}_1 \equiv p_I(p_2) - p_I(p_1) = -p'_I dp, \quad (130e)$$

$$\bar{b}_1 - \bar{b}_0 \equiv v(\bar{\lambda}, p_1) - v(\lambda_0, p_0) = v_{\bar{\lambda}} d\bar{\lambda} + v_p dp, \quad (130f)$$

$$\bar{b}_2 - \bar{b}_1 \equiv v(\bar{\lambda}, p_2) - v(\bar{\lambda}, p_1) = -v_p dp, \quad (130g)$$

where $d\bar{\lambda} \equiv \bar{\lambda} - \lambda_0$. By substituting these expressions in (128) and (129), we obtain finally

$$A_1 + A_2 = -t_{\bar{\lambda}} d\bar{\lambda} - t_p dp, \quad (131a)$$

$$A_1 e^{\mu_1 T} + A_2 e^{\mu_2 T} - A'_1 e^{\mu_1 T} = -t_p dp, \quad (131b)$$

$$A_1 \left(\frac{\sigma + \mu_1}{\sigma} \right) e^{\mu_1 T} + A_2 \left(\frac{\sigma + \mu_2}{\sigma} \right) e^{\mu_2 T} - A'_1 \left(\frac{\sigma + \mu_1}{\sigma} \right) e^{\mu_1 T} = -t_p dp, \quad (131c)$$

$$B_1 + B_2 = -u_p dp, \quad (131d)$$

$$B_1 e^{\chi_1 T} + B_2 e^{\chi_2 T} - B'_1 e^{\chi_1 T} = -u_p dp, \quad (131e)$$

$$B_1 \left(\frac{p_I \chi_1}{\kappa'(1) \bar{k}} \right) e^{\chi_1 T} + B_2 \left(\frac{p_I \chi_2}{\kappa'(1) \bar{k}} \right) e^{\chi_2 T} - B'_1 \left(\frac{p_I \chi_1}{\kappa'(1) \bar{k}} \right) e^{\chi_1 T} = -p'_I dp, \quad (131f)$$

and

$$A_1 \Upsilon_1 + A_2 \Omega_1 + p_I B_1 + p_I B_2 - v_{\bar{\lambda}} d\bar{\lambda} = \Phi_1, \quad (132)$$

where we set

$$\Upsilon_1 \equiv \frac{p_c(\sigma + \mu_1)}{\sigma(\mu_1 - r^*)}, \quad (133a)$$

$$\Omega_1 \equiv \left[\left(\frac{p_c(\sigma + \mu_1)}{\sigma(\mu_1 - r^*)} - \frac{p_c(\sigma + \mu_2)}{\sigma(\mu_2 - r^*)} \right) e^{-\mu_1 T} + \frac{p_c(\sigma + \mu_2)}{\sigma(\mu_2 - r^*)} \right], \quad (133b)$$

$$\Phi_1 \equiv \left[v_p - \left(v_p + p_I u_p + t_p \frac{p_c(\sigma + \mu_1)}{\sigma(\mu_1 - r^*)} \right) e^{-r^* T} \right] dp. \quad (133c)$$

The solutions are given by:³⁸

$$\frac{A_1}{dp} = -t_{\bar{\lambda}}\lambda_p \left(1 - e^{-r^*T}\right) - t_p \left[\left(1 + \frac{\mu_1}{\mu_2 - \mu_1} e^{-\mu_2 T}\right) - \frac{r^*}{\mu_1} \left(e^{-r^*T} - e^{-\mu_2 T}\right) \right] \leq 0, \quad (134a)$$

$$\frac{A_2}{dp} = \frac{\mu_1}{\mu_2 - \mu_1} e^{-\mu_2 T} t_p > 0, \quad (134b)$$

$$\frac{A'_1}{dp} = \frac{A_1}{dp} + \frac{\mu_2}{\mu_2 - \mu_1} e^{-\mu_1 T} t_p \leq 0, \quad (134c)$$

$$\frac{B_1}{dp} = -u_p \left(1 + \left(\frac{\chi_1}{\chi_2 - \chi_1}\right) e^{-\chi_2 T}\right) + \frac{\kappa'(1)\bar{k}}{p_I} \frac{e^{-\chi_2 T}}{\chi_2 - \chi_1} p'_I \leq 0, \quad (134d)$$

$$\frac{B_2}{dp} = \left(\frac{\chi_1}{\chi_2 - \chi_1}\right) e^{-\chi_2 T} u_p - \frac{\kappa'(1)\bar{k}}{p_I} \frac{1}{\chi_2 - \chi_1} e^{-\chi_2 T} p'_I < 0, \quad (134e)$$

$$\frac{B'_1}{dp} = \frac{B_1}{dp} + \left(\frac{\chi_2}{\chi_2 - \chi_1}\right) e^{-\chi_1 T} u_p - \frac{\kappa'(1)\bar{k}}{p_I} \frac{1}{\chi_2 - \chi_1} e^{-\chi_1 T} p'_I \leq 0, \quad (134f)$$

$$\frac{d\bar{\lambda}}{dp} \Big|_{temp} = \lambda_p \left(1 - e^{-r^*T}\right) - \lambda_b t_p \frac{p_c(\sigma + r^*)}{\sigma(\mu_1 - r^*)} \left(e^{-r^*T} - e^{-\mu_2 T}\right) \leq 0, \quad (134g)$$

where we used the following relationships

$$\begin{aligned} \lambda_b \left[v_{\bar{\lambda}} + t_{\bar{\lambda}} \frac{p_c(\sigma + \mu_1)}{\sigma(\mu_1 - r^*)} \right] &= 1, \\ \lambda_b \left[v_p + p_I u_p + t_p \frac{p_c(\sigma + \mu_1)}{\sigma(\mu_1 - r^*)} \right] &= -\lambda_p, \\ \lambda_b t_p \frac{p_c(\sigma + r^*)}{\sigma(\mu_1 - r^*)} &= \frac{p'_c \bar{\lambda} r^*}{p_c \mu_1}, \\ v_{\bar{\lambda}} = \frac{p_c}{r^*} t_{\bar{\lambda}}, \quad t_p &= \frac{p'_c \bar{\lambda}}{p_c} t_{\bar{\lambda}}. \end{aligned}$$

E Temporary Terms of Trade Disturbances: Impact Effects and Trajectories

E.1 Supply-Side

Long-term Effects

Given the considered steady-states, the changes in the physical capital stock can be calculated as:

$$\frac{d\bar{k}_1}{dp} \Big|_{temp} = \frac{d\bar{k}}{dp} \Big|_{perm} = u_p > 0, \quad \frac{d\bar{k}_2}{dp} \Big|_{temp} = -u_p < 0, \quad \frac{d\bar{k}}{dp} \Big|_{temp} = 0. \quad (135)$$

Impact Effects

First, we determine the gap between the initial jump of the capital market price, $q(0)$, and its steady-state value prevailing at period 1, $\bar{q}_1 = p_I(p_1)$. Evaluating (123d) at time $t = 0$, and

using the fact that $B_1 = -B_2 - u_p \tau dp$, yields:

$$\begin{aligned} q(0) - \bar{q}_1 &= - \left(\frac{p_I \chi_1}{\kappa'(1) \bar{k}} \right) (1 - e^{-\chi_2 T}) u_p dp - p'_I e^{-\chi_2 T} dp, \\ &= \frac{p_I}{p} \left[\frac{r^* (1 - \alpha_I)}{\chi_2} (1 - e^{-\chi_2 T}) - \alpha_I e^{-\chi_2 T} \right] dp. \end{aligned} \quad (136)$$

The second equality (136) is obtained by substituting

$$-u_p \frac{p_I \chi_1}{\kappa'(1) \bar{k}} = \frac{r^* (1 - \alpha_I)}{\chi_2} \frac{p_I}{p}, \quad (137)$$

which follows from using these relationships: $\chi_1 \chi_2 = \kappa'(1) \frac{\bar{k}}{p_I(\bar{p})} \bar{p} F_{kk}$, $r^* p_I(\bar{p}) = \bar{p} F_k(\bar{k}, 1)$, $u_p = -\frac{F_k(1-\alpha_I)}{\bar{p} F_{kk}}$.

Second, by differentiating solution (123c) with respect to time t , evaluating at time $t = 0$, and differentiating with respect to p , one obtains an expression of the initial response of investment:

$$\left. \frac{dI(0)}{dp} \right|_{temp} = \frac{\kappa'(1) \bar{k}}{p} \left[\frac{r^* (1 - \alpha_I)}{\chi_2} (1 - e^{-\chi_2 T}) - \alpha_I e^{-\chi_2 T} \right], \quad (138a)$$

$$= (1 - e^{-\chi_2 T}) \left. \frac{dI(0)}{dp} \right|_{perm} - \frac{\kappa'(1) \bar{k}}{p} e^{-\chi_2 T} \alpha_I, \quad (138b)$$

where substitution of (137) yields to the equality (138a). From (138), we are able to determine the length of the shock, denoted by $\hat{T}$, for which the two effects compensate:

$$\hat{T} = -\frac{1}{\chi_2} \log \left[\frac{r^* (1 - \alpha_I) / \chi_2}{\alpha_I + r^* (1 - \alpha_I) / \chi_2} \right] \quad (139)$$

Once its initial jump is performed, the market price of capital evolves as follows:

$$\left. \frac{d\dot{q}(0)}{dp} \right|_{temp} = -\chi_1 \left(\frac{p_I \chi_1}{\kappa'(1) \bar{k}} \right) \left(1 - \frac{r^*}{\chi_1} e^{-\chi_2 T} \right) u_p - r^* p'_I e^{-\chi_2 T}, \quad (140a)$$

$$= (1 - e^{-\chi_2 T}) \left. \frac{d\dot{q}(0)}{dp} \right|_{perm} - \frac{r^* p_I}{p} e^{-\chi_2 T} < 0. \quad (140b)$$

Transitional Dynamics

The slope of the transitional path in (k, q) -space over the unstable period 1 is obtained by calculating the time derivatives of (123c) and (123d):

$$\frac{dq(t)}{dk(t)} = \frac{\chi_1 \frac{B_1}{dp} \left(\frac{p_I \chi_1}{\kappa'(1) \bar{k}} \right) e^{\chi_1 t} + \chi_2 \frac{B_2}{dp} \left(\frac{p_I \chi_2}{\kappa'(1) \bar{k}} \right) e^{\chi_2 t}}{\chi_1 \frac{B_1}{dp} e^{\chi_1 t} + \chi_2 \frac{B_2}{dp} e^{\chi_2 t}}, \quad (141)$$

where the sign of the denominator's expression can change over time. There exists a date $t = \hat{t}$ for which the two terms of the denominator compensate mutually:

$$\chi_1 \frac{B_1}{dp} e^{\chi_1 \hat{t}} + \chi_2 \frac{B_2}{dp} e^{\chi_2 \hat{t}} = 0 \quad \Leftrightarrow \quad \hat{t} = \frac{1}{\chi_1 - \chi_2} \log \left[-\frac{\chi_2 B_2 / dp}{\chi_1 B_1 / dp} \right], \quad (142)$$

where $\hat{t} > 0$ under the condition that the term in square brackets is positive and smaller than one. In words, the investment flow changes of sign during the unstable period 1 if and only if $\frac{dk(0)}{dp}\big|_{temp} = \chi_1 \frac{B_1}{dp} + \chi_2 \frac{B_2}{dp} > 0$, i. e. when the *transitory profitability* effect predominates initially and encourages the domestic firms to decumulate the stock of equipment goods following the temporary terms of trade worsening.

The slope of the transitional path in (k, q) -space over the stable period 2 is obtained by calculating the time derivatives of (125c) and (125d):

$$\frac{dq(t)}{dk(t)} = \frac{\chi_1 \frac{B'_1}{dp} \left(\frac{p_I \chi_1}{\kappa'(1)\bar{k}} \right) e^{\chi_1 t}}{\chi_1 \frac{B'_1}{dp} e^{\chi_1 t}} = \frac{p_I \chi_1}{\kappa'(1)\bar{k}} < 0. \quad (143)$$

The slope of the trajectory along the transitional path towards the ultimate steady state 2, that is $(\bar{k}_2, \bar{q}_2)$, is without ambiguity negative whatever the degree of the shock's persistence.

E.2 Demand-Side

Long-Term Effects

As the adjustments of *real expense* and of the stock of consumption habits are characterized by an *hysteresis* phenomenon and since the transitional dynamics are driven by the agents' expectations of the ultimate steady-state under the perfect foresight's assumption, we start our analysis with the investigation of the long-run steady-state effects of an unanticipated transitory change in p .

From (134g), the equilibrium change in $\bar{\lambda}$ is given by

$$\frac{d\bar{\lambda}}{dp}\bigg|_{temp} = \lambda_p \left(1 - e^{-r^*T} \right) - \frac{p'_c \bar{\lambda} r^*}{p_c \mu_1} \left(e^{-r^*T} - e^{-\mu_2 T} \right). \quad (144)$$

Starting off from an "old" (initial) steady-state $\bar{s}_0 = \bar{c}_0$, for the period 1 steady-state change of real expense it follows then immediately from (130a)

$$\frac{d\bar{c}_1}{dp}\bigg|_{temp} = \frac{d\bar{s}_1}{dp}\bigg|_{temp} = t_{\bar{\lambda}} \frac{d\bar{\lambda}}{dp}\bigg|_{temp} + t_p \geq 0, \quad (145a)$$

$$= t_{\bar{\lambda}} \lambda_p \left(1 - e^{-r^*T} \right) - t_p \frac{r^*}{\mu_1} \left(e^{-r^*T} - e^{-\mu_2 T} \right) + t_p, \quad (145b)$$

$$= \frac{d\bar{c}}{dp}\bigg|_{perm} \left(1 - e^{-r^*T} \right) - t_{\bar{\lambda}} \lambda_p e^{-r^*T} - t_p \frac{r^*}{\mu_1} \left(e^{-r^*T} - e^{-\mu_2 T} \right), \quad (145c)$$

where we use the fact that $t_p = p'_c \bar{\lambda} t_{\bar{\lambda}} / p_c$ to obtain (145b) and substitute the expression of the long-run change in c following a permanent shock, $(d\bar{c}/dp)_{perm} = t_{\bar{\lambda}} \lambda_p + t_p$.

Since $\bar{s}_2 = \bar{c}_2 = t(\bar{\lambda}, p_2)$, the ultimate steady-state changes compared to the initial $\bar{c}_0$ for an unanticipated transitory shock can be calculated as

$$\frac{d\bar{c}}{dp}\bigg|_{temp} = t_{\bar{\lambda}} \lambda_p \left(1 - e^{-r^*T} \right) - t_p \frac{r^*}{\mu_1} \left(e^{-r^*T} - e^{-\mu_2 T} \right) \geq 0. \quad (146)$$

Impact Effects

By evaluating the formal solution (123b) for period 1 at time $t = 0$, differentiating with respect to p , and using the fact that $A_1 = -(\bar{c}_1 - \bar{c}_0) - A_2$, the initial response of real expense following an unanticipated transitory terms of trade shock may write as follows

$$\left. \frac{dc(0)}{dp} \right|_{temp} = \left. \frac{d\bar{c}_1}{dp} \right|_{temp} - \left(\frac{\sigma + \mu_1}{\sigma} \right) \left. \frac{d\bar{c}_1}{dp} \right|_{temp} + \frac{A_2}{dp} \left(\frac{\mu_2 - \mu_1}{\sigma} \right), \quad (147a)$$

$$= -\frac{\mu_1}{\sigma} \left. \frac{d\bar{c}}{dp} \right|_{perm} \left(1 - e^{-r^*T} \right) + \frac{\mu_2}{\sigma} t_p \left(e^{-r^*T} - e^{-\mu_2 T} \right) \geq 0, \quad (147b)$$

where we use the fact that $r^* = \mu_1 + \mu_2$ and $(d\bar{c}/dp)|_{perm} = t_{\bar{\lambda}} (d\bar{\lambda}/dp)|_{perm} + t_p$.

Once real expense reacted immediately after the temporary perturbation, the initial changes of the stock of consumption habits and real expense can be obtained by differentiating solutions (123a) and (123b) with respect to time and evaluating at time $t = 0$:

$$\begin{aligned} \left. \frac{d\dot{s}(0)}{dp} \right|_{temp} &= \mu_1 \frac{A_1}{dp} + \mu_2 \frac{A_2}{dp}, \\ &= -\mu_1 \left. \frac{d\bar{s}}{dp} \right|_{perm} \left(1 - e^{-r^*T} \right) + \mu_2 t_p \left(e^{-r^*T} - e^{-\mu_2 T} \right) = \sigma \left. \frac{dc(0)}{dp} \right|_{temp} \geq 0, \end{aligned} \quad (148a)$$

$$\begin{aligned} \left. \frac{d\dot{c}(0)}{dp} \right|_{temp} &= \mu_1 \left(\frac{\sigma + \mu_1}{\sigma} \right) \frac{A_1}{dp} + \mu_2 \left(\frac{\sigma + \mu_2}{\sigma} \right) \frac{A_2}{dp}, \\ &= -\mu_1 \left(\frac{\sigma + \mu_1}{\sigma} \right) \left[\left. \frac{d\bar{c}_1}{dp} \right|_{temp} - \frac{\sigma + r^*}{\sigma + \mu_1} t_p e^{-\mu_2 T} \right] \geq 0, \end{aligned} \quad (148b)$$

where we use the fact that $\mu_2 \left(\frac{\sigma + \mu_2}{\sigma} \right) - \mu_1 \left(\frac{\sigma + \mu_1}{\sigma} \right) = (\sigma + r^*) (\mu_2 - \mu_1) / \sigma$.

Transitional Dynamics

Period 1 ($0 \leq t < T$)

We investigate the trajectories of the real expense and the consumption experience and evaluate under which condition these macroeconomic aggregates change of direction over the unstable period 1. Differentiate the solutions (123a) and (123a) with respect to time, one obtains the evolution of s and c over period 1 following a temporary terms of trade shock:

$$\dot{s}(t) = \left[\mu_1 \frac{A_1}{dp} e^{\mu_1 t} + \mu_2 \frac{A_2}{dp} e^{\mu_2 t} \right] dp \leq 0, \quad (149a)$$

$$\dot{c}(t) = \left[\mu_1 \left(\frac{\sigma + \mu_1}{\sigma} \right) \frac{A_1}{dp} e^{\mu_1 t} + \mu_2 \left(\frac{\sigma + \mu_2}{\sigma} \right) \frac{A_2}{dp} e^{\mu_2 t} \right] dp \leq 0. \quad (149b)$$

Denote respectively $\hat{t}$ and $\hat{t}$ the dates such that $\dot{s}(\hat{t}) = 0$ and $\dot{c}(\hat{t}) = 0$. Equalize (149a)-(149b) to zero, we obtain:

$$\hat{t} = \frac{1}{\mu_1 - \mu_2} \log \left[-\frac{\mu_2 \left(\frac{\sigma + \mu_2}{\sigma} \right) \frac{A_2}{dp}}{\mu_1 \left(\frac{\sigma + \mu_1}{\sigma} \right) \frac{A_1}{dp}} \right] < \hat{t} = \frac{1}{\mu_1 - \mu_2} \log \left[-\frac{\mu_2 A_2 / dp}{\mu_1 A_1 / dp} \right] < T. \quad (150)$$

The trajectory of c and s change of sign over period 1 if and only if:

1. $A_1 / dp > 0$ since $A_2 / dp > 0$ and $\mu_1 < 0$ which ensures that the terms in square brackets are positive (see (150));

2. $\mu_1 \left(\frac{\sigma + \mu_1}{\sigma} \right) \frac{A_1}{dp} + \mu_2 \left(\frac{\sigma + \mu_2}{\sigma} \right) \frac{A_2}{dp} = \frac{d\dot{c}(0)}{dp} \Big|_{temp} < 0$ which guarantees that the terms in square brackets are less than one.

This situation emerges when the expectation of a greater habitual standard of living dominates initially the expectation of rise in the marginal utility of wealth measured in terms of the domestic good induced by the perfectly anticipated terms of trade improvement, i. e. under the condition that $\dot{c}(0) > 0$ following an unanticipated temporary terms of trade worsening. In other cases, the trajectory in the (s, c) -space is monotonic.

We evaluate the solutions (123a) and (123b) from unstable period 1 at time $t = T^-$:

$$\begin{aligned} \dot{s}(T^-) &= \left[\mu_1 \frac{A_1}{dp} e^{\mu_1 T^-} + \mu_2 \frac{A_2}{dp} e^{\mu_2 T^-} \right] dp, \\ &= \left[\mu_1 \frac{A_1}{dp} e^{\mu_1 T^-} + \frac{\mu_2 \mu_1}{\mu_2 - \mu_1} t_p \right] dp, \end{aligned} \quad (151a)$$

$$\begin{aligned} \dot{c}(T^-) &= \left[\mu_1 \left(\frac{\sigma + \mu_1}{\sigma} \right) \frac{A_1}{dp} e^{\mu_1 T^-} + \mu_2 \left(\frac{\sigma + \mu_2}{\sigma} \right) \frac{A_2}{dp} e^{\mu_2 T^-} \right] dp, \\ &= \left[\mu_1 \left(\frac{\sigma + \mu_1}{\sigma} \right) \frac{A_1}{dp} e^{\mu_1 T^-} + \left(\frac{\sigma + \mu_2}{\sigma} \right) \frac{\mu_2 \mu_1}{\mu_2 - \mu_1} t_p \right] dp, \end{aligned} \quad (151b)$$

where we substitute expression (134b) of the constant A_2/dp to obtain (151a) and (151b).

Period 2 ($t \geq T$)

We evaluate solutions (125a) and (125b) from unstable period 1 at time $t = T^+$:

$$\begin{aligned} \dot{s}(T^+) &= \mu_1 \frac{A'_1}{dp} e^{\mu_1 T^+} dp, \\ &= \left[\mu_1 \frac{A_1}{dp} e^{\mu_1 T^+} + \frac{\mu_2 \mu_1}{\mu_2 - \mu_1} t_p \right] dp = \dot{s}(T^-) \lesseqgtr 0, \end{aligned} \quad (152a)$$

$$\begin{aligned} \dot{c}(T^+) &= \mu_1 \left(\frac{\sigma + \mu_1}{\sigma} \right) \frac{A'_1}{dp} e^{\mu_1 T^+} dp, \\ &= \left[\mu_1 \left(\frac{\sigma + \mu_1}{\sigma} \right) \frac{A_1}{dp} e^{\mu_1 T^+} + \left(\frac{\sigma + \mu_1}{\sigma} \right) \frac{\mu_2 \mu_1}{\mu_2 - \mu_1} t_p \right] dp \neq \dot{c}(T^-), \end{aligned} \quad (152b)$$

where we substitute the expression of the constant A'_1/dp given by (134c).

By differentiating solutions (125a) and (125b) with respect to time, we determine in which direction s and c evolve over time in direction to the final steady-state :

$$\dot{s}(t) = \mu_1 \frac{A'_1}{dp} e^{\mu_1 t} dp \lesseqgtr 0, \quad (153a)$$

$$\dot{c}(t) = \mu_1 \left(\frac{\sigma + \mu_1}{\sigma} \right) \frac{A'_1}{dp} e^{\mu_1 t} dp \lesseqgtr 0. \quad (153b)$$

First, the slope of the transitional path in the (s, c) -space is positive. Second, as the trajectory over the stable period 2 is monotonic and the change of consumption habits remains constant at time T , i. e. $\dot{s}(T^-) = \dot{s}(T^+)$, the sign of the constant A'_1/dp may be determined as follows by using (151a) and (152a) :

$$\mu_1 \left[\frac{A_1}{dp} e^{\mu_1 T} + \frac{\mu_2}{\mu_2 - \mu_1} t_p \right] dp \gtrless 0 \quad \Rightarrow \quad \frac{A_1}{dp} + \frac{\mu_2}{\mu_2 - \mu_1} t_p e^{-\mu_1 T} = \frac{A'_1}{dp} \gtrless 0, \quad (154)$$

where $\mu_1 < 0$ and $dp < 0$ since we consider a decline in p .

Time preference rate dynamics

As we noted in appendix A, the habit-forming behavior implies a variable time preference rate. Following an unanticipated terms of trade worsening, the time preference rate rises initially and then decreases over time toward its steady-state value, δ . The reaction of ρ reflects the temporary gap between the marginal utility of current real expense and the marginal utility of future real expense. We turn now to the dynamics of the time preference rate prevailing after an unanticipated temporary terms of trade shock.

Period 1 ($0 \leq t < T$)

The solution of the time preference rate over the unstable period 1 is obtained by linearizing (104) in the neighborhood of the steady-state $(\bar{s}_1, \bar{c}_1)$ and using the fact that $\bar{\rho}_1 = \delta$:

$$\rho(t) = \bar{\rho}_1 + \frac{(\delta + \sigma) u_{11}}{p_c \bar{\lambda}} (c(t) - \bar{c}_1) + \frac{(\delta + 2\sigma)}{p_c \bar{\lambda}} \left[u_{12} + \frac{\sigma}{\delta + 2\sigma} u_{22} \right] (s(t) - \bar{s}_1), \quad (155a)$$

$$= \delta + \frac{(\delta + \sigma) u_{11}}{p_c \bar{\lambda}} \left[\Lambda_1 \frac{A_1}{dp} e^{\mu_1 t} dp + \Lambda_2 \frac{A_2}{dp} e^{\mu_2 t} \right] dp, \quad (155b)$$

where

$$\Lambda_1 = \left(\frac{\sigma + \mu_1}{\sigma} \right) + \frac{(\delta + 2\sigma)}{(\delta + \sigma) u_{11}} \Gamma \leq 0, \quad \text{according to} \quad \left(\frac{\sigma + \mu_1}{\sigma} \right) \geq 0, \quad (156a)$$

$$\Lambda_2 = \left(\frac{\sigma + \mu_2}{\sigma} \right) + \frac{(\delta + 2\sigma)}{(\delta + \sigma) u_{11}} \Gamma > 0. \quad (156b)$$

with Γ given by (25).

The impact effect of a temporary terms of trade disturbance on ρ is obtained by evaluating (155) at time $t = 0$, and differentiating with respect to p :

$$\left. \frac{d\rho(0)}{dp} \right|_{temp} = \frac{(\delta + \sigma) u_{11}}{p_c \bar{\lambda}} \left[\Lambda_1 \frac{A_1}{dp} + \Lambda_2 \frac{A_2}{dp} \right] \gtrless 0, \quad (157)$$

où $u_{11} < 0$, $\Lambda_1^{adj} < 0$, $\Lambda_1^{dist} > 0$, $\Lambda_2 > 0$, et $A_2/dp > 0$.

The time preference rate evolves as follows :

$$\dot{\rho}(t) = \frac{(\delta + \sigma) u_{11}}{p_c \bar{\lambda}} \left[\Lambda_1 \mu_1 \frac{A_1}{dp} e^{\mu_1 t} + \Lambda_2 \mu_2 \frac{A_2}{dp} e^{\mu_2 t} \right] dp \gtrless 0, \quad (158)$$

where we differentiate (155) with respect to time. Denote $\bar{t}$ the date such that $\dot{\rho}(\bar{t}) = 0$, that is :

$$\bar{t} = \frac{1}{\mu_1 - \mu_2} \log \left[-\frac{\Lambda_2 \mu_2 A_2 / dp}{\Lambda_1 \mu_1 A_1 / dp} \right] < T, \quad (159)$$

where $A_2/dp > 0$, $\Lambda_2 > 0$, $\mu_1 < 0$, et $\mu_2 > 0$.

The initial variation of the time preference rate is given by :

$$\dot{\rho}(0) = \frac{(\delta + \sigma) u_{11}}{p_c \bar{\lambda}} \left[\Lambda_1 \mu_1 \frac{A_1}{dp} + \Lambda_2 \mu_2 \frac{A_2}{dp} \right] dp \gtrless 0. \quad (160)$$

The dynamics of the time preference rate over the unstable period 1 may be non monotonic if and only if :

1. $\Lambda_1 A_1 / dp > 0$ which guarantees that the term in square brackets of the expression (159) is positive;
2. $\Lambda_1 \mu_1 \frac{A_1}{dp} + \Lambda_2 \mu_2 \frac{A_2}{dp} < 0$, that is $\left. \frac{d\dot{\rho}(0)}{dp} \right|_{temp} > 0$ (see eq (160)), which implies that the term in square brackets of the expression (159) is less than unity.

Period 2 ($t \geq T$)

The solution of the time preference rate over the stable period 2 is obtained by linearizing (104) in the neighborhood of the steady-state $(\bar{s}_2, \bar{c}_2)$ and using the fact that $\bar{\rho}_2 = \delta$:

$$\begin{aligned} \rho(t) &= \bar{\rho}_2 + \frac{(\delta + \sigma) u_{11}}{p_c \bar{\lambda}} (c(t) - \bar{c}_2) + \frac{(\delta + 2\sigma)}{p_c \bar{\lambda}} \left[u_{12} + \frac{\sigma}{\delta + 2\sigma} u_{22} \right] (s(t) - \bar{s}_2), \\ &= \delta + \frac{(\delta + \sigma) u_{11}}{p_c \bar{\lambda}} \Lambda_1 \frac{A'_1}{dp} e^{\mu_1 t} dp. \end{aligned} \quad (161)$$

By differentiating (161) with respect to time, the time preference rate evolves over the stable period 2 toward the value of the psychological time discount rate as follows :

$$\dot{\rho}(t) = \frac{(\delta + \sigma) u_{11}}{p_c \bar{\lambda}} \mu_1 \Lambda_1 \frac{A'_1}{dp} e^{\mu_1 t} dp \geq 0. \quad (162)$$

E.3 External Asset Position and Net Exports

Long-term Effects

The long-run changes in the stock of foreign assets and net exports following an unanticipated transitory terms of trade shock are given by

$$\left. \frac{d\bar{b}_1}{dp} \right|_{temp} = v_{\bar{\lambda}} \left. \frac{d\bar{\lambda}}{dp} \right|_{temp} + v_p \geq 0, \quad \left. \frac{d\bar{b}_2}{dp} \right|_{temp} = -v_p > 0, \quad \left. \frac{d\bar{b}}{dp} \right|_{temp} = v_{\bar{\lambda}} \left. \frac{d\bar{\lambda}}{dp} \right|_{temp} \geq 0, \quad (163a)$$

$$\left. \frac{d\bar{n}x_1}{dp} \right|_{temp} = -r^* \left. \frac{d\bar{b}_1}{dp} \right|_{temp}, \quad \left. \frac{d\bar{n}x_2}{dp} \right|_{temp} = -r^* \left. \frac{d\bar{b}_2}{dp} \right|_{temp}. \quad (163b)$$

Impact Effects

In order to simplify the solution (123e), we rewrite the term in square brackets as follows³⁹

$$\begin{aligned}
& (b_0 - \bar{b}_1) + p_I B_1 + p_I B_2 + \frac{p_c(\sigma + \mu_1)}{\sigma(\mu_1 - r^*)} A_1 + \frac{p_c(\sigma + \mu_2)}{\sigma(\mu_2 - r^*)} A_2 \\
= & - \left[v_{\bar{\lambda}} \frac{d\bar{\lambda}}{dp} \Big|_{temp} + v_p \right] dp - p_I u_p dp - \frac{p_c(\sigma + \mu_1)}{\sigma(\mu_1 - r^*)} \left[t_{\bar{\lambda}} \frac{d\bar{\lambda}}{dp} \Big|_{temp} + t_p \right] dp - \\
& - \frac{A_2}{dp} \left[\frac{p_c(\sigma + \mu_1)}{\sigma(\mu_1 - r^*)} - \frac{p_c(\sigma + \mu_2)}{\sigma(\mu_2 - r^*)} \right] dp, \\
= & - \left[v_{\bar{\lambda}} + t_{\bar{\lambda}} \frac{p_c(\sigma + \mu_1)}{\sigma(\mu_1 - r^*)} \right] \frac{d\bar{\lambda}}{dp} \Big|_{temp} dp - \left[v_p + p_I u_p + \frac{p_c(\sigma + \mu_1)}{\sigma(\mu_1 - r^*)} t_p \right] dp + \frac{p_c(\sigma + r^*)}{\sigma(\mu_1 - r^*)} t_p e^{-\mu_2 T} dp, \\
= & - \frac{1}{\lambda_b} \frac{d\bar{\lambda}}{dp} \Big|_{temp} dp + \frac{\lambda_p}{\lambda_b} dp + \frac{p_c(\sigma + r^*)}{\sigma(\mu_1 - r^*)} t_p e^{-\mu_2 T} dp, \\
= & \left[\frac{\lambda_p}{\lambda_b} + \frac{p_c(\sigma + r^*)}{\sigma(\mu_1 - r^*)} t_p \right] e^{-r^* T} dp, \tag{164a} \\
= & \left[\frac{(\bar{Y} - \bar{d})}{r^*} + \frac{\mu_2 p_c(\sigma + r^*)}{r^* \sigma(\mu_1 - r^*)} t_p \right] e^{-r^* T} dp. \tag{164b}
\end{aligned}$$

Period 1 ($0 \leq t < T$)

The transitions of the net foreign assets and net exports during the unstable period 1 are respectively governed by the following equations

$$\begin{aligned}
b(t) = & \bar{b}_1 + \left[\frac{\lambda_p}{\lambda_b} + \frac{p_c(\sigma + r^*)}{\sigma(\mu_1 - r^*)} t_p \right] e^{r^*(t-T)} dp - p_I (B_1 e^{\chi_1 t} + B_2 e^{\chi_2 t}) - \\
& - \Upsilon_1 A_1 e^{\mu_1 t} - \Upsilon_2 A_2 e^{\mu_2 t}, \tag{165a}
\end{aligned}$$

$$nx(t) = \bar{n}x_1 + p_I (\chi_2 B_1 e^{\chi_1 t} + \chi_1 B_2 e^{\chi_2 t}) + [\mu_2 \Upsilon_1 A_1 e^{\mu_1 t} + \mu_1 \Upsilon_2 A_2 e^{\mu_2 t}], \tag{165b}$$

where we set

$$\Upsilon_1 = \frac{p_c(\sigma + \mu_1)}{\sigma(\mu_1 - r^*)} \geq 0, \quad \Upsilon_2 = \frac{p_c(\sigma + \mu_2)}{\sigma(\mu_2 - r^*)} > 0. \tag{166}$$

The solution (165b) for net exports is obtained by using the fact that $nx(t) = \dot{b}(t) - r^* b(t)$, substituting the solution (165a) and its time derivative. By remembering that at the period 1 steady-state, $\bar{n}x_1 = -r^* \bar{b}_1$ since $\bar{c}a_1 = 0$, one finally determines (165b).

Period 2 ($t \geq T$)

$$b(t) = \bar{b}_2 - p_I B'_1 e^{\chi_1 t} - \frac{p_c(\sigma + \mu_1)}{\sigma(\mu_1 - r^*)} A'_1 e^{\mu_1 t}, \tag{167a}$$

$$nx(t) = \bar{n}x_2 + p_I \chi_2 B'_1 e^{\chi_1 t} + \mu_2 \frac{p_c(\sigma + \mu_1)}{\sigma(\mu_1 - r^*)} A'_1 e^{\mu_1 t}, \tag{167b}$$

where $\bar{n}x_2 = -r^* \bar{b}_2$ (since $\bar{c}a_2 = 0$).

By differentiating the equation (165a) with respect to time, evaluating at time $t = 0$, and differentiating with respect to p , one obtains the initial response of the current account:

$$\begin{aligned} \left. \frac{dca(0)}{dp} \right|_{temp} &= \left. \frac{d\dot{b}(0)}{dp} \right|_{temp} = r^* \left[\frac{\lambda_p}{\lambda_b} + \frac{p_c(\sigma + r^*)}{\sigma(\mu_1 - r^*)} t_p \right] e^{-r^*T} - p_I \left[\chi_1 \frac{B_1}{dp} + \chi_2 \frac{B_2}{dp} \right] - \\ &\quad - \Upsilon_1 \mu_1 \frac{A_1}{dp} - \Upsilon_2 \mu_2 \frac{A_2}{dp}, \end{aligned} \quad (168)$$

By substituting $(A_1/dp) = -(\bar{d}\bar{c}/dp)|_{temp} - t_p - (A_2/dp)$, the expression (168) may be rewritten as follows

$$\begin{aligned} \left. \frac{dca(0)}{dp} \right|_{temp} &= r^* \left[\frac{\lambda_p}{\lambda_b} + \frac{p_c(\sigma + r^*)}{\sigma(\mu_1 - r^*)} t_p \right] e^{-r^*T} - p_I \left. \frac{d\dot{k}(0)}{dp} \right|_{temp} + \\ &\quad + \mu_1 \Upsilon_1 \left[t_{\bar{\lambda}} \left. \frac{d\bar{\lambda}}{dp} \right|_{temp} + t_p \right] + \frac{A_2}{dp} [\mu_1 \Upsilon_1 - \mu_2 \Upsilon_2]. \end{aligned} \quad (169)$$

By substituting the expression of $\lambda_b = \frac{r^* \sigma (\mu_1 - r^*)}{p_c \mu_1 (\sigma + r^*)} \frac{1}{t_{\bar{\lambda}}}$, by using the fact that $r^* = \mu_1 + \mu_2$ and that $(\bar{d}\bar{c}/dp)_{perm} = t_{\bar{\lambda}} \lambda_p + t_p$,

$$\begin{aligned} \left. \frac{dca(0)}{dp} \right|_{temp} &= p_I \chi_1 \left. \frac{d\bar{k}}{dp} \right|_{perm} + \mu_1 \frac{p_c(\sigma + \mu_1)}{\sigma(\mu_1 - r^*)} \left. \frac{d\bar{c}}{dp} \right|_{perm} - p_I \left[\chi_1 \left. \frac{d\bar{k}}{dp} \right|_{perm} e^{-\chi_2 T} - \right. \\ &\quad \left. \frac{\kappa'(1) \bar{k}}{p} e^{-\chi_2 T} \alpha_I \right] - p_c \frac{\mu_1}{\sigma} \left[\left. \frac{d\bar{c}}{dp} \right|_{perm} e^{-r^*T} + t_p \frac{\mu_2}{\mu_1} (e^{-r^*T} - e^{-\mu_2 T}) \right] \leq 0. \end{aligned} \quad (170)$$

Notes

¹“Our evidence supports the J-curve in the data but not its common explanation” (Leonard and Stockman [2002], p. 485).

²“As recent experience has confirmed, the response of trade flows to the exchange rate take years, both because consumers are slow to change habits and, even more important, because many changes in supply and sourcing require long-term investment decisions” (Krugman [1989], p. 33).

³The parameter $\sigma \geq 0$ determines the relative weight of real expenditure at different times. Performing the differentiation of (3) with respect to time, one obtains a motion equation of habits stock where σ is the coefficient (or speed) of adjustment. For example, taking a value of $\sigma = 0.6$ in the line of empirical results of Constantinides [1990], then the time required to close 95% of the discrepancy between $s(t)$ and $c(t)$ by changes in $s(t)$ following a change in $c(t)$ is five years (because $e^{-0.6t} = 0.05$ for $t \approx 4.99$).

⁴The unit cost dual function, $p_c(\cdot)$, is defined as the minimum total expense in consumption goods, z_c , such that $c = c(d(t), f(t)) = 1$, for a given level of terms of trade, p . The minimized unit cost function depends on the terms of trade and is expressed in terms of the foreign good.

⁵The dynamic equation of the foreign assets' stock is obtained by the substitution of the dividend flow expression, $D(t)$ (see (14a)).

⁶We refer to A_{11} and A_{22} as the submatrices composed respectively by the two first and two last lines and columns of matrix A .

⁷When intertemporal preferences display distant complementarity, inequality (27) is always satisfied. This inequality is necessary to ensure that adjacent complementarity is not too strong and that the dynamic system exhibits a saddle point stability (see Becker and Murphy [1988]).

⁸The left hand side of (38b) is the Volterra derivative applied to the functional (2) expressed in current value and obtained along a constant path (see Ryder and Heal [1973]).

⁹The dependency on initial conditions can be avoided thanks to the assumptions of an endogenous psychological rate of discount (see e. g. Obstfeld [1982], Ikeda [2001]), imperfect world capital markets (see e. g. Fisher and Terrell [2000]), finite life (see e. g. Matsuyama [1987]) or a continuum of infinitely lived agents with different birth dates (see e. g. Weil [1989]).

¹⁰Following Sen and Turnovsky [1989] and Serv  n [1999], we assume that the small open economy is a net exporter of the home good, and symmetrically a net importer of the foreign good at the steady-state. This assumption is formalized as follows:

$$p(\bar{Y} - \bar{d}) = (\bar{f} - r^*\bar{b}) > 0.$$

¹¹To see it, evaluate the stable solution (33b) at $t = 0$ letting $d\bar{k} \equiv \bar{k} - k_0$, and substitute (41b) to obtain

$$q(0) - \bar{q}_1 = \left(\frac{p_I \chi_1}{\kappa'(1) \bar{k}} \right) \frac{F_k(1 - \alpha_I)}{p F_{kk}} dp.$$

From the expression above, the gap between the short run shadow value of capital, $q(0)$, and its new steady state value, $\bar{q}_1 = p_I(p_1)$, is greater, i. e. the decline in Tobin's q is larger, the higher is the share of import goods in investment expenditure, $(1 - \alpha_I)$.

¹²Non-monotonic convergent paths of net foreign assets are also obtained by Matsuyama [1987] in a finite horizon model with capital adjustment costs, by Karayal  n [1994], [1995] respectively in an infinite horizon model with recursive preferences and installation costs and in a small open economy model allowing heterogeneity across agents, by Mansoorian [1998] in a model with durable consumption goods and habit-forming behavior, and by Ikeda and Gombi [1998] in a closely related framework with only one good.

¹³The current account is negative over the period $(0, \tilde{t})$. As the foreign assets' stock falls at a decreasing rate, the current account becomes less negative over time until date $\tilde{t}$ when the current account is balanced and b overshoots. We can see it more formally by differentiating (37b) with respect to time:

$$\ddot{b}(t) = \mu_1 \dot{b}(t) + p_I(\mu_1 - \chi_1) \dot{k}(t) \geq 0, \quad \text{for } t \leq \tilde{t}.$$

¹⁴The effect of a decrease in the domestic capital stock on the net exports can be formally highlighted by differentiating the expression of nx with respect to k :

$$\begin{aligned} \frac{\partial nx}{\partial k} &= \frac{\partial}{\partial k} \left[pF[k, 1] - p_c c - p_I \dot{k} \left[1 + \psi \left(\frac{\dot{k}}{k} \right) \right] \right], \\ &= p \frac{\partial F[k, 1]}{\partial k} - p_I \frac{\partial \dot{k}}{\partial k} [1 + \psi(\cdot)] - p_I \dot{k} \frac{\partial \psi(\dot{k}/k)}{\partial k}, \\ &= p F_k - p_I \chi_1 [1 + \psi(\cdot)] + p_I \left(\frac{\dot{k}}{k^2} \right)^2 \psi'(\cdot) > 0, \end{aligned}$$

where we used the stable solution (33a) and differentiated the installation cost function for a given flow of investment.

¹⁵More precisely, whereas the relative price of imports rise permanently above its initial level (i. e. p decreases), net exports remain at first below their steady-state value, then overshoot, and finally decline towards their long-run value. Therefore, net exports and the relative price of imports co-vary at first in opposite directions, and then in the same direction. Backus et al. [1994] obtain the same response for the trade balance (see p. 95).

¹⁶Mansoorian [1993], [1998] studies exclusively the impact of a permanent terms of trade deterioration by introducing habit formation behavior in an economy without capital accumulation. Sen and Turnovsky [1989] analyze the response of economic aggregates after a temporary terms of trade disturbance by allowing for a labor-leisure choice and capital adjustment costs. They do not consider habit-forming consumers and capital good imports. The framework of Ikeda and Gombi [1998] is relatively close to ours but we allow for two consumption and capital goods. Moreover, the authors apply the solution procedure proposed by Sen and Turnovsky [1990] which contain an inconsistency (see Schubert [2002], chapter 2, and Schubert and Turnovsky [2002]). Finally, our study differs from Serv  n's analysis in retaining dependent intertemporal preferences and in the analytical method we have chosen to study an unanticipated transitory terms of trade shock.

¹⁷For details of derivations, see Cardi ([2004], Appendix C.14).

¹⁸We retained a formalization of the supply-side analogue to these specified by Gavin [1992] and Serv  n [1999] who extend Abel and Blanchard's model to an open economy by allowing trade in capital and consumption goods. Gavin [1992] studies temporary terms of trade disturbances only graphically and Serv  n [1995] applies the solution procedure proposed by Buiter [1984] for continuous time models. This analytical method is more suited to 2×2 dynamic systems with one predetermined and one non-predetermined variable.

¹⁹We used the following equalities:

$$\left. \frac{d\bar{q}_1}{dp} \right|_{temp} = \left. \frac{d\bar{q}}{dp} \right|_{perm} = p'_I; \quad \left. \frac{d\bar{k}_1}{dp} \right|_{temp} = u_p = \left. \frac{d\bar{k}}{dp} \right|_{perm} = m_p.$$

²⁰To clarify this point, remember that the dampening term $(1 - e^{-\chi_2 T})$ is an increasing function of the unstable eigenvalue of $\chi_2 > 0$ which in turn raises with the derivative of the inverse function $\kappa'(1)$ evaluated at the steady-state; the value of the latter is greater the smaller are capital installation costs. In conclusion, the term $(1 - e^{-\chi_2 T})$ is a decreasing function of adjustment costs.

²¹Serv  n [1999] provides a discussion about the response of investment following a transitory terms of trade improvement. With the help of formal solutions, we are able to get some new information about investment transitional dynamics.

²²At time $\hat{t}$, the investment flow is null $I(\hat{t}) = 0$, which in turn implies that:

$$q(\hat{t}) - p_I(p_1) = \frac{p_I}{\kappa'(1)\bar{k}} \left[\chi_1 \frac{B_1}{dp} e^{\chi_1 \hat{t}} + \chi_2 \frac{B_2}{dp} e^{\chi_2 \hat{t}} \right] dp = \frac{p_I}{\kappa'(\cdot)\bar{k}} \dot{k}(\hat{t}) = 0,$$

where we have evaluated the solution (123d) at time $\hat{t}$.

²³In the case of a persistent negative terms of trade shock ($T > \hat{T}$), we have shown in appendix E.1 that the investment flow is initially negative; formally, we have:

$$\dot{k}(0) = \left[\chi_1 \frac{B_1}{dp} + \chi_2 \frac{B_2}{dp} \right] dp < 0.$$

Since $B_2/dp < 0$ and $dp < 0$, we deduce that $B_1/dp < 0$.

²⁴To see this more formally, note that $\frac{I}{k} = \kappa(\bar{q}_2/p_I(p_2)) = 0$ and $\kappa' > 0$. Next, $q(T^+) > \bar{q}_2$, as we can see from figure 4. It follows that $\frac{I(T^+)}{k(T^+)} = \kappa(q(T^+)/p_I(p_2)) > 0$

²⁵See Cardi [2004], Chapter 5.

²⁶See e. g. Obstfeld [1983], Sen and Turnovsky [1989], Serv  n [1999] who study the effects of an unanticipated terms of trade shock.

²⁷See Turnovsky [1997] for a discussion about the property of hysteresis.

²⁸Cardi [2004] provides a new explanation of *J-curve* phenomenon following an adverse transitory price shock by considering the distant complementarity case (see Chapter 5).

²⁹To clarify this point, remember that the partial derivative of the real expense function (see eq (39a)) with respect to the terms of trade, that is t_p may be written as follows (see eq (118b)):

$$t_p = \frac{u_1 + \frac{\sigma}{\delta+\sigma} u_2}{\left[u_{11} + \frac{\delta+2\sigma}{\delta+\sigma} \left(u_{12} + \frac{\sigma}{\delta+2\sigma} u_{22} \right) \right]} \frac{\alpha_c}{p} < 0,$$

where we use the fact that $\bar{\lambda} = \left[u_1 + \frac{\sigma}{\delta+\sigma} u_2 \right] / p_c$ (see eq (38a)) and $\alpha_c \equiv p'_c p / p_c$. The right-hand side of the above expression is greater the higher is α_c the lower is the denominator $u_{11} + \frac{\delta+2\sigma}{\delta+\sigma} \left(u_{12} + \frac{\sigma}{\delta+2\sigma} u_{22} \right) < 0$. The latter is less negative the higher is the positive value of u_{12} when the households' preferences display an adjacent complementarity.

³⁰The direction of the once-for-all change of the marginal utility of wealth is not clear cut since its jump is the result of multiple economic forces which work in opposite directions. There is an *income effect* dampened by the *smoothing effect* and an *intertemporal speculation effect* moderated by a *disutility effect* which results from the large changes of real expense. From expression (64) of the equilibrium change of $\bar{\lambda}$, there exists a length of the shock denoted by $\hat{T}$ for which the influences of the *income effect* and the *intertemporal speculation effect* on the variation of the marginal utility of wealth are equal. The shorter-lasting the worsening in the terms of trade, the more likely a decline in $\bar{\lambda}$: since the representative household expects a perfectly rise in p (an improvement in the terms of trade) and therefore a decline in the consumption-based real interest rate in the near future, the incentive to raise its real expense initially and in the long-run becomes greater. As we assume a habit-forming behavior with adjacent complementarity in preferences, the expected rise of the habitual standard of living encourages the household to accumulate foreign bonds once the terms of trade revert back to their initial level and the economy converges toward its final steady-state. The *inertia* of real expense makes possible a positive accumulation of internationally traded bonds over the stable period 2. Since the accumulation of b over period 2 is greater than the decumulation over period 1, the equilibrium change of $\bar{\lambda}$ which guarantees the respect of the national intertemporal solvency constraint is negative.

³¹The impact effects on consumption consist only of the wealth effect since Ikeda and Gombi [1998], in a framework close to ours, consider an adverse transitory productivity shock, a temporary increase in government spending, and a temporary increase in capital taxes.

³²Following a persistent terms of trade worsening, the ultimate change of c may be lower or greater in absolute value than the change in c following a permanent disturbance. On figure 5, we assume that the $\bar{c}_2 > \bar{c}_{perm}$. This inequality holds if the adverse terms of trade disturbance is not too long-lived.

³³By inserting $\lim_{T \rightarrow \infty}$ in the expression (85), one obtains the initial reaction of real expense in the permanent case.

³⁴As Obstfeld [1992] underscored in a one-good model without physical capital accumulation, the representative households makes its optimal consumption choice by putting a positive weight to the initial habitual standard of living, $\bar{s}_0$, and a weight less than one to the real permanent income, $\frac{r^*[b_0 + W(0)]}{p_c}$

³⁵Net exports expressed in terms of the foreign good are defined as follows

$$nx(t) = pF[k(t), 1] - p_c c(t) - p_I I(t) \left[1 + \psi \left(\frac{I(t)}{k(t)} \right) \right].$$

By differentiating the above expression with respect to time, one obtains

$$\dot{nx}(t) = \left[pF_k + p_I \left(\frac{I(t)}{k(t)} \right)^2 \psi'(\cdot) \right] \dot{k}(t) - p_c \dot{c}(t) - q(t)I(t),$$

where we use the fact that $q = p_I(p) \left[1 + \psi \left(\frac{I}{k} \right) + \left(\frac{I}{k} \right) \psi'(\cdot) \right]$.

³⁶By using expressions (24) and (30) of eigenvalues μ_1 and χ_1 , the fact that $\delta = r^*$, and the relationship $pF_k = r^*p_I$ prevailing at the steady-state, we deduce that $\mu_1 \geq \chi_1$ if:

$$\kappa'(1) \geq \frac{1}{r^* \zeta_{F_k, k}} \frac{\sigma(\delta + \sigma)}{u_{11}} \left[u_{11} + \frac{\delta + 2\sigma}{\delta + \sigma} \left(u_{12} + \frac{\sigma}{\delta + 2\sigma} u_{22} \right) \right] > 0,$$

where $\zeta_{F_k, \bar{k}} \equiv -F_{kk}\bar{k}/F_k > 0$ is the elasticity of marginal productivity of capital with respect to capital. The right-hand side term can be interpreted as a critical value of the inverse function's slope, $\kappa'(1)$, evaluated at the steady-state. From the properties of the installation cost function, $\psi(\cdot)$, $\kappa'(1)$ is higher (resp. smaller) the smaller are the installation costs (resp. higher).

³⁷The third condition imply that net foreign assets remain continuous at time T .

³⁸See Schubert [2002], Appendix A.7 for a similar procedure with one good and additive separable intertemporal preferences.

³⁹See Cardi [2004] (Appendix C.14) for details of derivation.

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