

## PREFACE

### Coherent States: A contemporary panorama

Coherent states (CS) of the harmonic oscillator (also called canonical CS) were introduced in 1926 by Schrödinger in answer to a remark by Lorentz on the classical interpretation of the wave function. They were rediscovered in the early 1960s, first (somewhat implicitly) by Klauder in the context of a novel representation of quantum states, then by Glauber and Sudarshan for the description of coherence in lasers. Since then, CS have grown into an extremely rich domain that pervades almost every corner of physics and have also led to the development of several flourishing topics in mathematics. Along the way, a number of review articles have appeared in the literature, devoted to CS, notably the 1985 reprint volume of Klauder-Skagerstam, the 1990 review paper by Gilmore *et al* [2], the 1993 Oak Ridge conference [3] and the 1995 review paper by Ali *et al* [4]. Textbooks also have been published, among which one might mention the ground breaking text of Perelomov [1], focusing on the group-theoretical aspects, that of Ali-Antoine-Gazeau [6],<sup>1</sup> analyzing systematically the mathematical structure beyond the group-theoretical approach and also the relation to wavelet analysis, that of Dodonov and V.I.Man'ko [7], mostly devoted to quantum optics, that of Gazeau [8], more oriented towards the physical, probabilistic and quantization aspects, and finally the very recent one by Combescure-Robert [9].

In retrospect, one can see that the development of CS has gone through a two-phase transition. First, the (simultaneous) discovery in 1972 by Gilmore and Perelomov that CS were rooted in group theory, then the realization that CS can be defined in a purely algebraic way, as an eigenvalue problem or by a series expansion (Malkin and Man'ko, 1969; Barut and Girardello, 1971; Gazeau and Klauder, 1999; references to the original articles may be found in the textbooks quoted above). Both facts resulted in an explosive expansion of the CS literature. We thought, therefore, that the time was ripe for devoting a special issue of the Journal of Physics A to CS. However, because of limitations of space and time, it would have been impossible to get a fully representative cross-section of papers, covering all the different facets of the subject. Consequently, we have selected 37 articles, including some by a few of the originators of the field. We thank all the authors for submitting their up-to-date thoughts on this fascinating subject.

The contents of this special issue are subdivided into five categories: (1) Review papers; (2) Physics oriented CS; (3) Physics and quantum information; (4) Mathematics, general topics; (5) Mathematics, particular problems.

#### (1) *Review papers*

We start with 5 review papers. The first paper, by Klauder, surveys the many possible applications of affine variables, both in classical and in quantum physics. The second, by Sanders, proposes a grand tour of entangled CS, which are present in many fields, such as quantum optics, quantum information processing, etc. The next paper, by Rowe, surveys the field of vector CS and the attendant group representation problems (including induced representations). Then Oriti *et al* describe a particular class of CS relevant to (loop) quantum gravity. Finally, Combescure and Robert present a comprehensive review of fermionic CS, including all mathematical details.

#### (2) *Physics oriented CS*

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<sup>1</sup>A second edition of that volume is in preparation.

The six contributions in this section deal with specific physical problems. (i) Dajka-Luczka study Gazeau-Klauder cat states associated to a nonlinear Kerr oscillator, instead of the usual canonical CS leading to Schrödinger cat states. (ii) Angelova *et al* discuss squeezed CS associated to a 1D Morse potential, used in molecular physics. (iii) Bagrov *et al* study CS in a magnetic solenoid field and prove their completeness. (iv) Blasone-Jizba treat Nambu-Goldstone dynamics in spontaneously broken symmetries, using CS functional integrals. (v) Calixto *et al* describe accelerated relativistic particles in the context of spontaneous breakdown of conformal  $SU(2,2)$  symmetry, using  $SU(2,2)$  CS. (vi) Finally, Mortazavi-Tavassoly study  $f$ -deformed charge CS and their physical properties (nonclassical features, sub-Poissonian statistical behavior, etc).

### (3) *Physics and quantum information*

The second group of physically related CS contains 4 contributions with a distinct quantum information theoretic flavor. First, Thilagam describes the dynamical behavior of entanglement of a pair of qubits (excitons), using a CS basis. Next, Lavoie-de Guise study  $SU(3)$  intelligent states (i.e., minimal uncertainty states), of interest in the quantum information community. Then Muñoz constructs discrete CS for  $n$  qubits. Finally, Wagner-Kendon explore the continuous variable Deutsch-Jozsa algorithm known in quantum computing in a discrete formulation.

### (4) *Mathematics, general topics*

In this subgroup, there are 8 papers dealing with general properties of CS, independently of any particular system or application. A whole series discusses the interaction between CS and various mathematical objects: pseudodifferential operators and Weyl calculus [Unterberger]; induced representations of the affine group and intertwining operators [Elmabrok-Hutník]; measure-free CS and reproducing kernels [Horzela-Szafraniec]; extremal POV measures [Heinosaari-Pellonpää]; Hilbert  $W^*$ -modules [Bhattacharyya-Roy]; Toeplitz operators [Hutníková-Hutník]; operator localization and homogeneous structure of nilpotent Lie groups [Kisil]. In addition, Balazs *et al* consider multipliers for continuous frames, including CS or wavelet frames.

### (5) *Mathematics, particular problems*

The second group of mathematically oriented papers contains 14 contributions, devoted to CS in particular systems. We start with a paper by Gilmore, which explores the (sometimes chaotic) evolution of atomic CS under a time-periodic driving field, using sphere maps  $S^2 \rightarrow S^2$ . Next, we include a paper on CS on the 2-sphere in a magnetic field [Hall-Mitchell]; a paper on CS for a quantum particle on a Möbius strip [Cirilo-Lombardo]; a discussion of quantization on the circle [Chadzitaskos *et al*]; SUSY CS for Pöschl-Teller potentials [Bergeron *et al*]; generalized Bargmann functions and von Neumann lattices [Vourdas *et al*]; partial reconstruction for a finite CS system, using the Fock-Bargmann representation [Calixto *et al*]; phase operators for  $SU(3)$  irreps, thus for finite quantum systems [de Guise]; semiclassical CS in periodic potentials [Carles-Sparber]; complexified CS with non-Hermitian Hamiltonians [Graefe-Schubert]; minimal uncertainty states in the context of (semisimple) group representation theory [Oszmaniec]; localization operators in the time-frequency domain, i.e., in Gabor analysis [Muzhikyan-Avanesyan]; and, finally, two papers about fermionic CS [Daoud-Kibler and Trifonov].

This brief description illustrates perfectly the extreme versatility of the CS concept. As already stressed, coherent states constitute nowadays a flourishing research topic, with applications to a wide spectrum of domains. Indeed, CS are everywhere in physics: condensed matter physics, atomic physics, nuclear and particle physics, quantum optics, dynamics – both quantum and classical potentials –, quantum gravity, quantization and quantum information theory. On the other hand, CS have grown into a full-fledged domain in mathematics, incorporating many tools such as group representations, POV measures, frames, holomorphic functions, orthogonal polynomials and so on. Interestingly enough, a majority of contributions in this special issue (22 out of 37) are mathematically-minded, demonstrating the widespread interest coherent states have generated in various areas of mathematics. A third field related to CS (but almost not represented in the present collection) is signal processing. Indeed both Gabor analysis and wavelet analysis derive in the first place from CS theory, namely, CS associated to the Weyl-Heisenberg and the  $ax + b$  group, respectively. Here too, a tremendous development has taken place in the last years, another testimony to the richness of the notion of CS.

We leave it to the jury of public opinion to judge whether the call for a special issue of the journal, devoted to coherent states, has been justified.

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 Guest editors

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