

Implied volatility indices as leading indicators of stock index returns?

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ABSTRACT

This paper shows that, when the VIX or VXN indices of implied volatility increase, the S&P100 and NASDAQ100 stock indices exhibit on average negative returns, hence the ‘fear factor’ associated with high levels of implied volatility in financial markets. However, attractive (from a mean-variance perspective) positive returns should then be expected on average in the immediate short-term. In this framework, very high levels of implied volatility can on a statistical basis be viewed as signalling an imminent increase in stock indices, at least on a short term basis. Our analysis also shows that average to moderately high levels of implied volatility lead to unfavorable (from a mean-variance perspective) returns. Thus traders willing to enter ‘oversold’ markets should wait until extremely high levels of implied volatility are witnessed, and their strategy should be strictly on a short-term basis.

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In an option pricing framework, volatility is the only input that cannot be directly observed by trader. Indeed, call/put feature, time-to-maturity and strike price are the basic characteristics of the option contract, while the risk-free interest rate and dividend payment are fairly easy to agree upon. Thus the unknown input when computing the price of the option is the expected volatility over the life of the option.¹ In a market economy with actively traded option contracts which express the market's view of the relevant prices for those contracts, one can solve for the volatility that equates the observed market price of the option contract with the price given by the chosen option pricing formula. This yields the implied volatility. Because of the growing importance of modelling and predicting asset volatility in modern finance, the relevance of implied volatility as a rational forecast of future realized volatility and the information content of implied volatility w.r.t. historical volatility are two important (related) research topics in the academic literature. Early studies (Day and Lewis, 1992; Canina and Figlewski, 1993; Jorion, 1995; see also a summary in Figlewski, 1997) provide rather mixed results as to which measure (implied volatility or volatility computed from historical prices) has the highest information content regarding future realized volatility. Most of these studies show that implied volatility has a reasonably large information content, but also that it is not a rational forecast of future realized volatility and that historical or ARCH-type volatility can enhance volatility forecasts based solely on implied volatility. More recently, for the S&P100 index and VIX implied volatility index, Blair, Poon, and Taylor (2001) show that historical prices (even intraday prices) do not provide much incremental information compared to the information given by the VIX index of implied volatility; moreover, the VIX index provides the best out-of-sample forecasts of realized volatility (their forecast horizon ranges from 1 to 20 trading days).

All these studies focus on the link between implied volatility and future realized volatility. Quite surprisingly, few studies deal with the possible relationship between implied volatility and future stock *returns*. This probably stems from the belief that financial markets are efficient and, as such, implied volatility cannot provide relevant

information as to whether stock prices are going up or down. This is in contrast with the opinion of non-academic market participants for whom very large implied volatility levels are usually seen as signalling attractive entry levels for long traders. Their rationale is that very high implied volatility levels indicate that one is witnessing periods of financial turmoil where investors are believed to be over-reacting and hence selling indiscriminately their financial assets to raise cash or limit losses.² For example, the VIX implied volatility index of the CBOE is now routinely discussed in financial newspapers such as Barron's or the Wall Street Journal. Quite interestingly, there is a clear reference to a possible relationship between extremely high levels of implied volatility and a 'market bottom'. For example, in an article whose title is "Fixated on the VIX: soaring volatility means fear - and opportunity", K. Tan was writing in the July 29, 2002 issue of Barron's that "A big VIX spike indicates the kind of extreme fear contrarians associate with market bottoms".³

This academic paper focuses from an empirical point of view on the possible relationship between implied volatility and future stock market returns. Our empirical application deals with the S&P100 and NASDAQ100 indices for which implied volatility indices VIX and VXN are readily available. These two implied volatility indices are also widely discussed in the academic and non-academic literature. Because the VIX and VXN indices are freely available at the CBOE internet website, they can truly be viewed as public information available to all investors and hence can reasonably be considered as possible trading signals. The S&P100 and NASDAQ100 indices are also representatives of two different class of stocks. Constituents of the S&P100 index are widely-held stocks representatives of the U.S. economy. These include financial, industrial and technological firms. As such, it is a proxy for the U.S. market 'as a whole', although not as general as the broader S&P500 index but for which no implied volatility exists. On the other hand, the NASDAQ100 index includes almost exclusively technological and bio-technological firms, or 'new economy' stocks. Our study thus deals with two types of stock market measures for which the relationship between implied volatility and returns

can differ. Regarding the NASDAQ100 dataset, we allow for possible structural change in our analysis by splitting the whole dataset (which ranges from 3/1/1995 - 2/8/2002) into a bull market period (3/1/1995 - 10/3/2000) and a bear market period (13/3/2000 - 2/8/2002).⁴ All in all, we can thus assess the validity of our results in three different settings: broad market measure (S&P100), bull market for tech or biotech firms (NASDAQ100, first period) and bear market for tech or biotech firms (NASDAQ100, second period).⁵

In a preliminary step, we first consider short term to middle term contemporaneous changes in the S&P100 and NASDAQ100 stock indices and corresponding implied volatility indices. Not surprisingly, there is a negative and statistically significant relationship between the levels of the two stock and implied volatility indices: positive stock index returns lead to decreased implied volatility levels, while negative returns lead to higher implied volatility levels. This relationship is also asymmetric in the sense that negative stock index returns yield bigger proportional changes in implied volatility measures than do positive returns. In a second step, we focus on the relationship between implied volatility and future (i.e. forward-looking) stock index returns. Our aim is to ascertain if, as thought by some market practitioners, large or very large implied volatility levels do indeed indicate over-sold markets where irrational fear predominates and hence could be viewed as short term to middle term ‘buy signals’. Our empirical methodology is close to that of Campbell and Shiller (2001) where they study the link between observed price-earnings ratios (at a time t for example) and future stock index returns (over a time period ranging from $t + 1$ to $t + n$ where n is the time horizon). Next to the study of the mean return achieved over a 1-, 5-, 10- and 60-day time horizon (long position in the stock index) subsequent to a ‘signal’ given by the implied volatility index, we also assess the trading risk (i.e. standard deviation of those returns) incurred by taking those positions. We then assess our results in the usual mean-variance framework. Our analysis shows that very high levels of implied volatility can, on a statistical basis, be viewed as signalling an imminent increase in stock indices, at least on a short

term basis. Moreover, these returns do appear to be attractive from a risk-return point of view. Quite interestingly, average to moderately high levels of implied volatility lead to unfavorable (from a mean-variance perspective) returns. Thus traders willing to enter ‘oversold’ markets should wait until extremely high levels of implied volatility are witnessed, and their strategy should be strictly on a short-term basis.

The rest of the paper is structured as follows. After this introduction, we detail the VIX and VXN implied volatility indices in Section I. We then immediately present the empirical application and results in Section II. Finally, Section III concludes.

I. The VIX and VXN indices

The VIX and VXN implied volatility indices are computed on an intradaily basis by the CBOE.⁶ By construction, the VIX (VXN) index is a weighted average of the implied volatilities computed from a total of eight call and put near-the-money, nearby and second nearby American option contracts on the underlying S&P100 (NASDAQ100) index. The weighting method ensures that VIX and VXN give the implied volatility of a hypothetical at-the-money option with a constant maturity of 22 trading days to expiry. In an efficient market where option prices reflect all available information (and because of the forward-looking nature of implied volatility), the level of VIX (VXN) “is the market’s best assessment of the expected volatility of the underlying stock index over the remaining life of the option”, a life of 22 trading days and the underlying S&P100 (NASDAQ100) index in this case (quote is from Whaley, 2000, p. 1). Details regarding the construction of the VIX index are available in Whaley (1993) or Whaley (2000).

Note that, by construction, these implied volatility indices take into account early exercise and dividend payments features, and they do not use as inputs market prices from not actively traded options (thus avoiding the recurrent problem of stale quotes for deep out-of-the money or in-the-money options).⁷ As discussed in the introduction,

these two implied volatility indices are freely available on the internet and have attracted a lot of attention in the academic and non-academic literature recently. Hence we do not deal with obscure volatility-related variables, but with two (implied) volatility levels widely accepted by the practitioner and scientific communities.

II. Empirical study

Our empirical application deals with the S&P100 index and corresponding VIX implied volatility index, and the NASDAQ100 index and corresponding VXN implied volatility index. Both implied volatility indices were detailed in Section I. At the time of this writing, it is now well known that the NASDAQ stock index has experienced a continued and formidable bear market, resulting in a slide of about 75% of its value since its all-time high reached on March 10, 2000. Thus for the NASDAQ sample, we split our dataset into 2 parts to isolate the possible distinct relationships in bull and bear markets. The first period starts on January 3, 1995 and ends on March 27, 2000 (bull market), yielding a total return of 1,082% for the NASDAQ100 index.⁸ The second period starts on March 13, 2000 and ends on August 2, 2002 (bear market), which gives a total return of -81% for the NASDAQ100 index! Splitting our dataset into 2 sub-periods known (ex-post) to exhibit pronounced bull and bear markets will allow us to test the stability of our relationships. For the S&P100 index, we do not split our dataset (which ranges from January 2, 1986 to August 2, 2002) into sub-samples as we want to test our hypotheses on the whole time period spanning 17 years. Note that this corresponds to a globally strong bull market, as the total return on this 17-year long period is equal to about 325%.

A. Definition of variables

Let us define $y1d_t$, $y5d_t$, $y20d_t$ and $y60d_t$ as the 1-, 5-, 20- and 60-day returns on the stock indices. For example, $y1d_t$ is computed as $\ln(P_t) - \ln(P_{t-1})$ and $y60d_t$ is computed as $\ln(P_t) - \ln(P_{t-60})$ where P_t is the price of the S&P100 or NASDAQ100 index at time t . Correspondingly, we define $VIX1d_t$, $VIX5d_t$, $VIX20d_t$ and $VIX60d_t$ as the 1-, 5-, 20- and 60-day relative change in the level of the VIX index. For example, $VIX5d_t$ is computed as $\ln(VIX_t) - \ln(VIX_{t-5})$.⁹

B. Contemporaneous changes in the stock index and implied volatility index

In the first part of our empirical analysis, we deal with the 1-, 5-, 20- and 60-day relative changes in the levels of the implied volatility and underlying stock indices on which the option contracts are written. More specifically we want to compare the 1-, 5-, 20- and 60-day relative changes of VIX_t and P_t , i.e. $VIX1d_t$ vs $y1d_t$, $VIX5d_t$ vs $y5d_t$, $VIX20d_t$ vs $y20d_t$ and $VIX60d_t$ vs $y60d_t$.

Figure 1 gives scatter-plots of these 4 relationships for the S&P100 index (January 2, 1986 to August 2, 2002). This figure clearly shows that rising stock index levels (i.e. positive returns) yield decreasing implied volatility levels. On the flip side, negative returns are associated with increases in implied volatility. Similar plots for the NASDAQ100 index are given in Figure 2 (bull market) and Figure 3 (bear market).

Performing an econometric analysis for one-day returns and one-day relative changes in implied volatility (S&P100 index), i.e. regressing $VIX1d_t$ on a constant and $y1d_t$ using:

$$VIX1d_t = \beta_0 + \beta_1 y1d_t + \epsilon_t, \quad (1)$$

where ϵ_t has the usual properties of the error term in an OLS regression, gives a slope ($\hat{\beta}_1$) of -3.92, with a t-statistic (computed using White's heteroskedastic-consistent standard errors) of -16.8 ($R^2 = 0.53$). For the NASDAQ100 index, regression results for $VXN1d_t$ vs $y1d_t$ give a slope of -1.69 with a t-statistic (computed using White's heteroskedastic-consistent standard errors) of -20.7 and $R^2 = 0.5$ (bull market), and -0.89 with a t-statistic of -17.5 and $R^2 = 0.47$ (bear market). Thus there is a strongly significantly negative relationship between contemporaneous changes in the price index and implied volatility index. An extended regression analysis which allows for an asymmetrical relationship between contemporaneous changes in the price index and implied volatility index can be modelled with:

$$VIX1d_t = \beta_0 D_t^- + \beta_1 D_t^+ + \beta_2 (y1d_t D_t^-) + \beta_3 (y1d_t D_t^+) + \epsilon_t, \quad (2)$$

where D_t^- is a dummy variable that is equal to 1 (0) when $y1d_t$ is negative (positive) and $D_t^+ = 1 - D_t^-$, gives estimation results $\hat{\beta}_2 = -4.72$ and $\hat{\beta}_3 = -2.87$. Both coefficients are significantly different from 0, and the null hypothesis that $H0 : \beta_2 = \beta_3$ is easily rejected. Thus negative returns lead to proportionally larger changes in implied volatility (increase) than do positive returns (decrease).

For the NASDAQ100 index (bull market) and regressing $VXN1d_t$ on these four dummy variables gives $\hat{\beta}_2 = -1.69$ and $\hat{\beta}_3 = -1.4$. Both coefficients are statistically different from 0 and the null hypothesis that $H0 : \beta_2 = \beta_3$ is also rejected. For the NASDAQ100 index (bear market), $\hat{\beta}_2 = -0.89$ and $\hat{\beta}_3 = -0.74$. Both coefficients are statistically different from 0 but the null hypothesis that $H0 : \beta_2 = \beta_3$ cannot be rejected in this case. Thus, for the NASDAQ100 index, negative returns also lead to proportionally larger changes in implied volatility than do positive returns, although the slope of the relationship is smaller than for the S&P100 index. Note that the implied volatility index takes on average much larger values for the NASDAQ100 index than for

the S&P100 index. Indeed, the average value of VXN is equal to 33.6 (bull market) and 58.3 (bear market), while the average value of VIX is equal to 21.2.

In the financial econometrics framework of GARCH models, the negative relationship between returns and (one-day-ahead conditional in this case) volatility has been successfully modelled using for example the GJR-GARCH (Glosten, Jagannathan, and Runkle, 1993) or APARCH (Ding, Granger, and Engle, 1993) models. This so-called leverage effect was already discussed by Black (1976). These models yield asymmetric news response curves, as negative shocks in asset returns lead to larger levels of conditional volatility (compared to the outcome due to positive shocks).¹⁰ This can be illustrated with the available data for the S&P100 and NASDAQ100 indices. Note that we now consider the relationship between returns and one-day-ahead conditional volatility. For daily returns, the AR(p)-GJR-GARCH model of Glosten, Jagannathan, and Runkle (1993) is formally defined by the following equations:

$$y1d_t = \delta_0 + \delta_1 y1d_{t-1} + \dots + \delta_p y1d_{t-p} + e_t, \quad (3)$$

$$e_t = \sqrt{h_t} \epsilon_t \quad (4)$$

with ϵ_t IID $t(0, 1, \nu)$, and

$$h_t = \omega + \alpha e_{t-1}^2 + \gamma e_{t-1}^2 I_{e_{t-1} < 0} + \beta h_{t-1} \quad (5)$$

and $I_{e_{t-1} < 0}$ is an indicator function which is positive if the return at time $t - 1$ is negative. When γ is significantly positive, there is an asymmetrical relationship between the conditional variance and the lagged squared error term: past negative shocks have a deeper impact on current conditional volatility than past positive shocks. For the S&P100 index over the complete 16-year period, $\gamma = 0.120$ with a t-statistic of 7.59, while

the estimated values are equal to 0.116 (t-statistic equal to 2.53) and 0.164 (t-statistic equal to 5.66) for the bear and bull markets of the NASDAQ100 index respectively. Although we do not, strictly speaking, consider contemporaneous changes in the price index and volatility index in this case, these estimation results again show that negative returns yield larger increases in the level of (conditional) volatility than do positive returns.

C. VIX and VXN indices as forward-looking indicators of future stock indices returns?

While it is clear that negative returns are associated with increased implied (and conditional if one works with ARCH-type models) volatility, there is a growing debate as to how implied volatility indices can indicate over-bought or over-sold market conditions. In this section, we tackle this issue by looking at the relationship between the level of implied volatility indices VIX (and VXN) at time t and the forward-looking (or n-day-ahead) 1-, 5-, 20- and 60-day relative changes in the underlying stock index, i.e. the S&P100 index (and the NASDAQ100 index). More specifically, we focus on the relationship between VIX_t and $r1d_t$, $r5d_t$, $r20d_t$ and $r60d_t$, where $r1d_t$, $r5d_t$, $r20d_t$ and $r60d_t$ are forward-looking 1-, 5-, 20- and 60-day relative changes in the level of the S&P100 index.¹¹ We then repeat the exercise with VXN and forward-looking relative changes in the level of the NASDAQ100 index (for both bull and bear market periods).

Figure 4 gives scatter-plots of these 4 relationships for the S&P100 index. Results are quite different from what was given in Figure 1 for the relationship between contemporaneous changes in the price index and implied volatility index. In this case, there appears to be a somewhat positive relationship between VIX_t and the n-day-ahead returns on the S&P100, but the evidence is hardly conclusive. Running a regression of $r1d_t$ against VIX_t :

$$r1d_t = \beta_0 + \beta_1 VIX_t + \epsilon_t, \quad (6)$$

where ϵ_t is the usual error term of the OLS regression, gives a slope ($\hat{\beta}_1$) of 0.008, with a t-statistic (computed using White's heteroskedastic-consistent standard errors) of 1.95 and $R^2 = 0.03$. While the null hypothesis that $H0 : \beta_1 = 0$ is almost rejected, the t-statistic is much lower than when regressing contemporaneous changes in the implied volatility index against index returns and the R^2 is almost equal to zero.¹²

Figures 5 and 6 plot the 4 relationships for the NASDAQ100 index, bull and bear market periods. Regression results for $r1d_t$ vs $VXN1d_t$ give a slope of 0.012 and $R^2 = 0.004$ (bull market), and 0.027 and $R^2 = 0.008$ (bear market). Both coefficients are statistically different from 0 as the t-statistics are equal to 2.07 and 2.33 respectively and the R^2 are very small. Hence the econometric analysis does not show a strong relationship between VIX or VXN levels and 1-day ahead returns on the S&P100 or NASDAQ100 indices.

In a seconde step, we now evaluate the possible short term to middle term positive relationship between the level of the implied volatility index and the n-day-ahead returns on the index from a practitioner's point of view. If large VIX or VXN levels are indeed 'buy signals', then one expects (on average) positive forward-looking returns when filtering our datasets to extract the data selected by those entry points. However a key issue is to assess the trading risk (or standard deviation of returns) of taking these long positions. In this framework, we filter our original datasets to extract the data that meets the following criteria: (1) 'calm' or low-volatility markets, with 3 sub-categories featuring n-day forward-looking stock index returns subsequent to observed VIX or VXN levels smaller than their 1%, 5% and 10% percentiles; (2) periods of turbulence when markets exhibit a very large volatility, with 3 sub-categories featuring n-day forward-looking stock index returns subsequent to observed VIX or VXN levels larger than their 90%, 95% and 99% percentiles.¹³

Table I gives the results for the S&P100 index for which the 1, 5, 10, 90, 95 and 99% percentiles are respectively equal to 10.76, 11.85, 12.76, 28.98, 32.71 and 45.54. For n equal to 1, 5, 20 and 60 days, we report the mean of the n -day-ahead return, the standard deviation of those returns, and the minimum and maximum observed returns. Once the time horizon is fixed (i.e. n), results in Table I show that the mean of the forward-looking returns is (in almost all cases) an increasing function of the VIX level which determined the filtering of the data. But unfortunately the risk of taking those long positions or standard deviation of the selected returns is also (in almost all cases) an increasing function of the VIX level. Furthermore in all instances the standard deviation is larger than the mean and the minimum return is always negative. Thus these n -day-ahead returns display a large variability and are not always positive. For example, buying the S&P100 index whenever VIX is larger than its 90% percentile and closing the long position after 20 days yield an average return of 2.52% with a standard deviation of 5.43%. Taking similar long positions when VIX is larger than its 95% percentile gives a mean return of 3.84% with a standard deviation of 5.62%. Results given in the top of this table also show that low-volatility markets (i.e. when the VIX level is smaller than its 1, 5 and 10% percentile) are characterized by rather small average forward-looking returns (but with a low risk). Note that the average of the 60-day-ahead returns is negative when selecting data for which VIX is smaller than its 1% percentile. Finally, the last two panels of Table I indicate that the average of the 20-day-ahead and 60-day-ahead returns is smaller for VIX levels of 99% than for VIX levels of 95%, while the standard deviation is larger.

Complete results for the bull and bear markets of the NASDAQ100 index are given in Tables II and III respectively. Upon closer examination of those results, it appears that there are key differences between the bull and bear periods, and with the results for the S&P100 index. It is no longer true that the average n -day-ahead return (and its standard deviation) is an increasing function of the implied volatility level that initiated the trade. It appears that one must separate short holding period returns (i.e. 1- and

5-day-ahead returns) and middle term holding period returns (i.e. 20- and 60-day ahead returns).

- For 1- and 5-day-ahead returns, their average and standard deviation are still increasing functions of the VXN level: traders making short-term buy trades in the NASDAQ100 index when the VXN index is high do achieve (on average) large positive returns, with a risk that generally increases along with the VXN index. Quite remarkably, this is true both for the bull and bear market periods. Even in the profound bear market, going for 1-day or 5-day holding period trades when the VXN level is large delivers on average a positive return (and for the bear market, this is also true for 20-day-ahead returns). More particularly in this bear market, taking long positions in the NASDAQ100 index when VXN is larger than its 99% percentile gives excellent results for 5- and 20-day-ahead returns, see the bottom panel of Table III.
- For 20- and 60-day-ahead returns, the bull and bear markets do not deliver the same results. During the bull market, 20- and 60-day-ahead returns present very nice risk-return properties (i.e. a large average return, a low standard deviation) when the VXN index is low. However results are poor when the VXN level is high as evidenced by the low average return and the large standard deviation. In the bear market period, 20-day-ahead returns present similar patterns as the 1- and 5-day-ahead returns, while 60-day-ahead returns are harder to characterize.

Thus, for 1- and 5-day-ahead returns (and 20-day-ahead returns in the bear market), we have approximately the same results as for the S&P100 index: large levels of implied volatility do indeed signal (on average) profitable short-term trades, but with a risk that increases with the VIX or VXN level. In this case, one must emphasize the short-term aspect of the trade as the risk-return picture darkens considerably if the trader waits too long before closing his position.

In the next three tables, i.e. Table IV for the S&P100 index, and Tables V and VI for the bull and bear market periods of the NASDAQ100 index, we report the average (and the standard deviation) of the n -day-ahead return achieved when the corresponding implied volatility index belongs to the range reported in the first column of those tables. The upper and lower bounds of those bins are chosen as the 0%, 5%, 10%, ..., 85%, 90% and 95% percentiles of the distribution of VIX and VXN (hence a total of 20 bins). Thus this analysis is quite similar to what has been discussed above, but we now characterize the risk-return relationship of the forward-looking returns over the complete (i.e. not only small or large values of VIX and VXN) distribution of the implied volatility indices. In a second step, we plot, for each stock index and n , the possible 20 combinations of μ and σ . This gives us the classic mean-variance picture of the selected returns.¹⁴ See Figures 7, 8 and 9.

A joint analysis of Figures 7, 8 and 9 and Tables IV, V and VI indicates that many n -day-ahead returns are not interesting from a mean-variance point of view. Indeed (with n fixed), for a given mean level, there are a number of possible points characterized by increasing standard deviations. Correspondingly, for a given level of risk, there are several points characterized by differing mean levels. Because of this and reframing our analysis in the usual mean-variance framework, one can split the data in Tables IV, V and VI into 2 parts or datasets.¹⁵ The first part is made up of the points deemed attractive from the mean-variance point of view, while we put the non-efficient points in the second part.¹⁶ In our analysis, we relate this selection to the corresponding bin of the implied volatility index, i.e. the trading signal. For the S&P100 index, most attractive (according to the mean-variance criteria) points are characterized by either very low or very high VIX levels. As in the previous analysis, points for which the VIX level is very low (high) are points characterized by a small (large) mean and small (large) standard deviation. For intermediate VIX levels (i.e. middle to upward range of Table IV), the mean-variance characteristics of the n -day-ahead returns are not favorable. This is particularly striking for 5-day-ahead returns (top right part of Figure 7). Although there

are some exceptions (for example the last (μ, σ) pair in Table V for $n = 60$), we conclude similarly for the NASDAQ100 index (bull market). As far as the NASDAQ100 index (bear market) is concerned, we do have the same results when $n = 1$, $n = 5$ and $n = 20$. When $n = 60$, there is no clear pattern and high VXN levels do not stand out. In both tables, as for the S&P100 index, (μ, σ) combinations for intermediate to moderately high levels of VXN are particularly unfavorable. To summarize and focusing on high levels of implied volatility only, the bottom two lines of Tables IV, V and VI (with the exception of $n = 60$ for the NASDAQ100 index), which correspond to the 90%-95% and 95%-100% percentiles bins respectively, are clearly attractive in a mean-variance framework. Therefore, there is an incentive for a trader who is keen on aggressive trading strategies to enter long (short-term) positions when the implied volatility index reaches very high levels (above the 90% percentile). It should however be emphasized that the trader must stick to short-term trades and wait for very high levels of the implied volatility index before buying the underlying index.

III. A concise summary of our results, concluding remarks and possible extensions

The empirical analysis of Section II has shown that there is a strong negative relationship between contemporaneous changes in implied volatility indices and underlying stock indices, both for the S&P100 and NASDAQ100 indices. This relationship seems to be stable over time, as the 16-year period (S&P100) and bull and bear market periods (NASDAQ100) gave the same results. It should also be stressed that periods of negative stock index returns are also periods when implied volatility increases strongly. As expressed by Equation 2, when the S&P100 index drops 1%, the level of the VIX index increases by 4.72%, i.e. a leverage of almost 5. For the NASDAQ100 index, the slope of this relationship is equal to 1.69 (bull market) and 0.89 (bear market). In the

second part of our empirical study, we show that, when implied volatility indices reach very high levels, short term forward-looking (or n-day-ahead) returns on the underlying stock indices are on average largely positive. Thus it can reasonably be argued that very high levels of implied volatility do indeed signal ‘buy’ entry points for traders who want to take long positions in the underlying index. Unfortunately, these (on average) positive returns do exhibit a large standard deviation, which generally increases along with the average return and implied volatility level which signalled the ‘buy’ entry point. Although these returns are characterized by a large standard deviation, they seem nevertheless interesting in a mean-variance framework.

A joint assessment of the two parts of Section II would thus seem to indicate that: (1) periods of very large implied volatility are, on average, periods of negative performance for stock indices (hence the ‘fear factor’ associated with high levels of implied volatility because of the recurrent negative returns that led to those implied volatility levels); (2) because of the ‘return to the mean’ mechanism, short-term positive returns should be expected on average in the subsequent periods. In this framework, very high levels of implied volatility can, on a statistical basis, be viewed as signalling an imminent return to the mean, at least on a short term basis. However, there is no free lunch as the standard deviation of the forward-looking returns does increase with the mean of those returns. Our analysis also shows that average to moderately high levels of implied volatility lead to unfavorable (from a mean-variance perspective) returns. Thus traders willing to enter ‘oversold’ markets should wait until extremely high levels of implied volatility are witnessed, and their strategy should be strictly on a short-term basis.

A possible extension of our study would be to focus on individual stocks. In this case however, computing meaningful implied volatility measures can be quite a challenge as large cross-sectional option prices datasets are needed and the stale quotes problem can be tricky to overcome. Hence the attractiveness of the VIX and VXN implied volatility indices. It would however be possible to stick with VIX and VXN implied

volatility indices and deal with forward-looking returns on individual stocks, for example by making a distinction between low-beta and high-beta stock. Indeed, for an aggressive trader who is keen on taking risky positions, it could be argued that he should buy high-beta stocks (and not the index) when implied volatility indices reach very large levels.

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Table I
Forward-looking returns (S&P100 index)

	1-day returns	5-day returns	20-day returns	60-day returns
VIX smaller than its 1% percentile (=10.76, N=41)				
Mean.	0.01	-0.09	0.64	-0.93
Std.	0.61	1.16	2.04	5.36
Min.	-2.36	-3.17	-3.76	-7.99
Max.	1.24	1.94	4.93	11.32
VIX smaller than its 5% percentile (=11.85, N=206)				
Mean.	0.07	0.30	1.05	2.84
Std.	0.52	1.13	2.06	5.17
Min.	-2.36	-3.17	-4.31	-7.99
Max.	1.55	2.56	4.98	11.32
VIX smaller than its 10% percentile (=12.76, N=414)				
Mean.	0.05	0.26	1.06	3.40
Std.	0.52	1.18	2.18	4.62
Min.	-2.36	-5.01	-6.83	-7.99
Max.	1.56	3.03	5.30	11.72
VIX larger than its 90% percentile (=28.98, N=413)				
Mean	0.19	0.96	2.52	5.28
Std.	2.31	3.48	5.43	8.24
Min.	-23.78	-13.26	-13.58	-19.24
Max.	8.53	11.96	17.05	26.58
VIX larger than its 95% percentile (=32.71, N=207)				
Mean	0.28	1.10	3.84	7.35
Std.	2.86	3.94	5.62	7.82
Min.	-23.78	-12.75	-13.58	-18.84
Max.	8.53	11.96	17.05	26.58
VIX larger than its 99% percentile (=45.54, N=42)				
Mean	0.84	1.51	1.58	6.76
Std.	2.97	4.34	7.82	8.99
Min.	-8.62	-9.04	-13.58	-4.76
Max.	8.53	11.96	16.23	26.58

Mean, standard deviation, minimum value and maximum value of the 1-, 5-, 20-, and 60-day forward-looking returns on the S&P100 index when the VIX index is lower than its 1%, 5% and 10% percentile (top of table), and when the VIX index is larger than its 90%, 95% and 99% percentile (bottom of table). The percentiles and number of observations selected are given in parenthesis. The time period is January 2, 1986 to May 8, 2002.

Table II
Forward-looking returns (NASDAQ100 index, bull market)

	1-day returns	5-day returns	20-day returns	60-day returns
VXN smaller than its 1% percentile (=18.31, N=13)				
Mean.	0.01	0.07	3.30	14.30
Std.	0.93	2.34	1.79	4.01
Min.	-1.90	-4.14	0.43	9.97
Max.	1.20	3.46	6.24	22.72
VXN smaller than its 5% percentile (=19.53, N=66)				
Mean.	0.18	0.55	3.92	13.73
Std.	0.84	1.92	2.02	4.05
Min.	-1.90	-4.14	-0.02	7.75
Max.	2.03	4.47	9.16	22.75
VXN smaller than its 10% percentile (=22.80, N=132)				
Mean.	0.28	1.44	5.01	14.19
Std.	0.98	2.23	3.29	4.90
Min.	-2.86	-4.14	-1.74	-0.89
Max.	2.75	6.55	14.26	28.47
VXN larger than its 90% percentile (=45.83, N=133)				
Mean.	0.51	2.16	4.34	8.63
Std.	2.95	5.47	10.98	21.34
Min.	-10.38	-15.60	-33.58	-32.95
Max.	6.51	14.09	24.48	52.22
VXN larger than its 95% percentile (=53.95, N=67)				
Mean.	0.56	2.41	4.27	-0.45
Std.	3.16	6.03	14.13	24.15
Min.	-5.54	-14.32	-33.58	-32.95
Max.	6.35	14.09	24.48	52.22
VXN larger than its 99% percentile (=60.20, N=14)				
Mean.	1.60	3.93	-1.42	9.33
Std.	2.96	8.76	22.23	29.16
Min.	-2.61	-14.32	-33.58	-19.37
Max.	5.93	14.09	24.48	52.22

Mean, standard deviation, minimum value and maximum value of the 1-, 5-, 20-, and 60-day forward-looking returns on the NASDAQ100 index when the VXN index is lower than its 1%, 5% and 10% percentile (top of table), and when the VXN index is larger than its 90%, 95% and 99% percentile (bottom of table). The percentiles and number of observations selected are given in parenthesis. The time period is January 3, 1995 to March 27, 2000.

Table III
Forward-looking returns (NASDAQ100 index, bear market)

	1-day returns	5-day returns	20-day returns	60-day returns
VXN smaller than its 1% percentile (=37.59, N=5)				
Mean.	-1.21	-3.20	-11.52	-30.54
Std.	2.72	2.61	4.26	4.82
Min.	-4.72	-5.96	-17.05	-36.86
Max.	1.75	0.57	-7.24	-25.56
VXN smaller than its 5% percentile (=40.79, N=27)				
Mean.	-0.96	-4.16	-8.83	-31.54
Std.	1.84	3.26	4.44	5.05
Min.	-4.72	-10.19	-18.20	-44.72
Max.	2.58	3.88	-1.20	-20.42
VXN smaller than its 10% percentile (=43.89, N=52)				
Mean.	-0.46	-2.72	-8.37	-29.21
Std.	2.04	4.20	4.54	5.96
Min.	-4.72	-10.19	-18.20	-44.72
Max.	4.25	8.01	2.96	-14.25
VXN larger than its 90% percentile (=73.60, N=53)				
Mean.	0.66	3.90	4.55	-9.90
Std.	5.54	7.16	13.46	27.05
Min.	-10.24	-13.49	-23.03	-53.31
Max.	17.20	18.38	33.84	37.54
VXN larger than its 95% percentile (=77.28, N=27)				
Mean.	1.21	3.96	7.23	-15.03
Std.	6.13	6.34	10.44	25.98
Min.	-10.24	-13.49	-17.88	-50.95
Max.	17.20	15.17	26.91	37.54
VXN larger than its 99% percentile (=83.48, N=6)				
Mean.	4.71	5.67	14.82	-27.38
Std.	9.27	4.68	5.39	24.20
Min.	-9.53	-2.60	8.07	-50.95
Max.	17.20	10.89	20.46	19.56

Mean, standard deviation, minimum value and maximum value of the 1-, 5-, 20-, and 60-day forward-looking returns on the NASDAQ100 index when the VXN index is lower than its 1%, 5% and 10% percentile (top of table), and when the VXN index is larger than its 90%, 95% and 99% percentile (bottom of table). The percentiles and number of observations selected are given in parenthesis. The time period is March 28, 2000 to May 8, 2002.

Table IV
Forward-looking returns (S&P100 index)

VIX	1-day r		5-day r		20-day r		60-day r	
	μ	σ	μ	σ	μ	σ	μ	σ
0.00 - 11.85	0.07	0.52	0.29	1.13	1.04	2.06	2.82	5.17
11.85 - 12.76	0.03	0.53	0.23	1.24	1.07	2.32	3.94	3.94
12.76 - 13.74	0.01	0.61	0.17	1.43	1.00	2.44	2.74	3.71
13.74 - 14.94	0.04	0.67	0.11	1.32	0.67	2.51	2.42	3.28
14.94 - 16.12	0.05	0.72	-0.04	1.66	0.53	2.79	2.13	4.30
16.12 - 16.96	0.01	0.88	0.32	1.90	1.03	3.02	2.66	6.38
16.96 - 17.72	0.07	0.82	0.40	1.88	1.37	4.02	2.09	8.20
17.72 - 18.37	0.11	0.78	0.45	1.77	1.01	3.56	2.29	7.26
18.37 - 19.21	0.04	0.84	0.51	1.66	0.51	4.50	1.86	8.64
19.21 - 20.17	0.08	0.88	0.30	2.22	1.32	4.66	2.48	7.52
20.17 - 21.14	0.01	0.93	0.09	2.12	1.07	4.50	2.11	9.37
21.14 - 22.06	-0.02	1.00	0.00	2.23	0.75	5.28	1.11	10.56
22.06 - 22.83	0.00	1.08	0.03	2.41	-0.40	6.54	-0.79	12.41
22.83 - 23.58	0.02	1.07	-0.14	2.49	-0.42	6.27	-0.64	10.19
23.58 - 24.47	-0.02	1.05	-0.19	3.20	-0.45	5.74	0.64	8.76
24.47 - 25.59	0.02	1.24	0.18	3.29	0.70	5.23	2.07	7.65
25.59 - 27.08	-0.16	1.32	-0.58	2.97	0.19	5.31	1.77	7.68
27.08 - 28.98	0.05	1.36	-0.06	3.25	0.10	5.10	2.34	7.17
28.98 - 32.71	0.09	1.56	0.84	2.94	1.23	4.88	3.24	8.12
32.71 -	0.29	2.87	1.09	3.95	3.84	5.63	7.40	7.81

Mean and standard deviation of the 1-, 5-, 20-, and 60-day forward-looking returns on the S&P100 index when the VIX index belongs to the interval given in the first column of the table. The time period is January 2, 1986 to May 8, 2002.

Table V
Forward-looking returns (NASDAQ100 index, bull market)

VXN	1-day r		5-day r		20-day r		60-day r	
	μ	σ	μ	σ	μ	σ	μ	σ
0.00 - 19.53	0.18	0.84	0.55	1.92	3.92	2.02	13.73	4.05
19.53 - 22.80	0.39	1.10	2.34	2.17	6.10	3.91	14.66	5.63
22.80 - 25.83	0.17	1.06	-0.02	2.88	1.14	5.05	1.78	8.86
25.83 - 27.05	0.21	1.28	0.69	3.29	1.44	5.98	5.44	10.05
27.05 - 28.00	-0.07	1.39	-0.17	3.02	0.43	4.91	6.37	6.70
28.00 - 28.58	0.07	1.47	0.37	3.15	2.54	4.98	6.75	6.72
28.58 - 29.59	0.00	1.59	0.79	3.30	1.97	5.77	7.41	8.42
29.59 - 30.38	-0.06	1.65	0.66	3.85	3.77	7.49	6.20	10.39
30.38 - 31.18	0.20	1.39	0.50	3.61	3.17	7.03	4.77	10.88
31.18 - 32.14	-0.11	1.82	0.24	4.04	0.05	6.66	3.92	11.15
32.14 - 32.98	0.26	1.75	0.34	3.34	2.45	6.93	4.79	10.93
32.98 - 33.65	-0.31	1.66	0.44	2.59	1.36	6.41	6.86	8.90
33.65 - 34.85	0.12	1.64	0.51	3.64	3.92	7.64	10.78	8.38
34.85 - 36.25	0.21	1.93	0.81	3.71	3.83	8.07	14.59	10.40
36.25 - 37.94	0.60	1.82	1.45	4.50	6.60	7.26	18.76	12.28
37.94 - 40.45	0.30	1.90	1.98	3.97	6.32	7.77	19.54	13.02
40.45 - 42.91	0.59	2.14	1.57	4.11	6.49	7.26	17.29	13.60
42.91 - 45.83	-0.10	2.51	0.66	5.33	3.79	8.17	14.26	10.59
45.83 - 53.95	0.47	2.74	1.89	4.88	4.41	6.49	17.85	12.69
53.95 -	0.56	3.16	2.41	6.03	4.27	14.13	-0.45	24.15

Mean and standard deviation of the 1-, 5-, 20-, and 60-day forward-looking returns on the NASDAQ100 index when the VXN index belongs to the interval given in the first column of the table. The time period is January 3, 1995 to March 27, 2000.

Table VI
Forward-looking returns (NASDAQ100 index, bear market)

VXN	1-day r		5-day r		20-day r		60-day r	
	μ	σ	μ	σ	μ	σ	μ	σ
0.00- 40.79	-1.06	1.87	-4.53	2.94	-8.90	4.57	-31.68	4.46
40.79- 43.90	0.09	2.07	-1.05	4.54	-7.88	4.55	-26.92	6.32
43.90- 46.78	-0.89	2.31	-1.38	5.88	-2.95	6.18	-22.37	12.19
46.78- 48.35	-0.30	2.24	-2.17	3.94	-9.92	10.59	-13.36	9.46
48.35- 49.36	-0.54	2.33	-2.12	5.08	-10.58	10.62	-13.78	11.48
49.36- 51.65	-0.23	3.55	-0.77	4.31	-6.56	10.25	-17.21	13.99
51.65- 52.80	0.22	2.16	-0.86	5.41	-8.59	8.33	-21.25	15.49
52.80- 54.15	-0.79	2.56	-2.98	5.60	-7.68	6.13	-21.32	9.98
54.15- 55.70	0.71	2.41	-0.64	5.18	-3.34	6.16	-18.63	11.36
55.70- 57.26	-0.51	3.07	-1.31	5.49	-0.32	6.42	-11.37	14.77
57.26- 59.24	-0.26	2.47	-0.04	6.06	-0.58	11.72	-11.33	13.92
59.24- 61.49	-0.74	3.04	-0.15	6.45	-5.48	14.49	-13.15	17.20
61.49- 63.70	-0.18	3.77	-1.01	7.72	-8.77	16.13	-10.52	21.97
63.70- 65.63	-1.21	3.56	-4.54	8.03	-4.03	14.93	-4.09	20.84
65.63- 67.31	-0.01	4.05	-0.56	10.50	-2.16	16.76	-2.40	20.72
67.31- 69.31	-0.21	3.78	-3.25	7.67	-6.37	16.23	-3.17	17.99
69.31- 71.53	-0.39	4.41	-3.13	9.90	-2.90	14.80	-10.79	24.30
71.53- 73.60	0.01	4.48	-0.91	8.10	1.83	13.65	-2.83	18.85
73.60- 77.28	0.09	4.91	3.83	8.05	1.77	15.74	-4.57	27.61
77.28-	1.21	6.13	3.96	6.34	7.23	10.44	-15.03	25.98

Mean and standard deviation of the 1-, 5-, 20-, and 60-day forward-looking returns on the NASDAQ100 index when the VXN index belongs to the interval given in the first column of the table. The time period is March 28, 2000 to May 8, 2002.

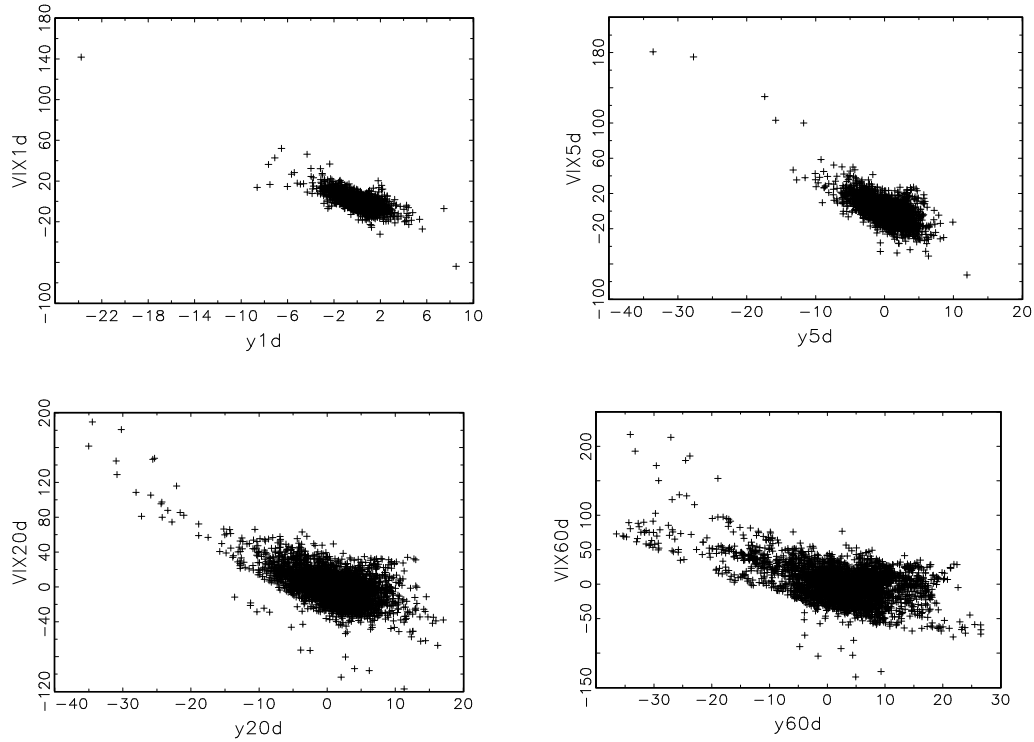


Figure 1. Contemporaneous changes in the stock index and implied volatility index (S&P100 index). This figure shows the relationship between the 1-, 5-, 20- and 60-day contemporaneous relative changes in the level of the stock index and implied volatility index. The time period is January 2, 1986 to May 8, 2002.

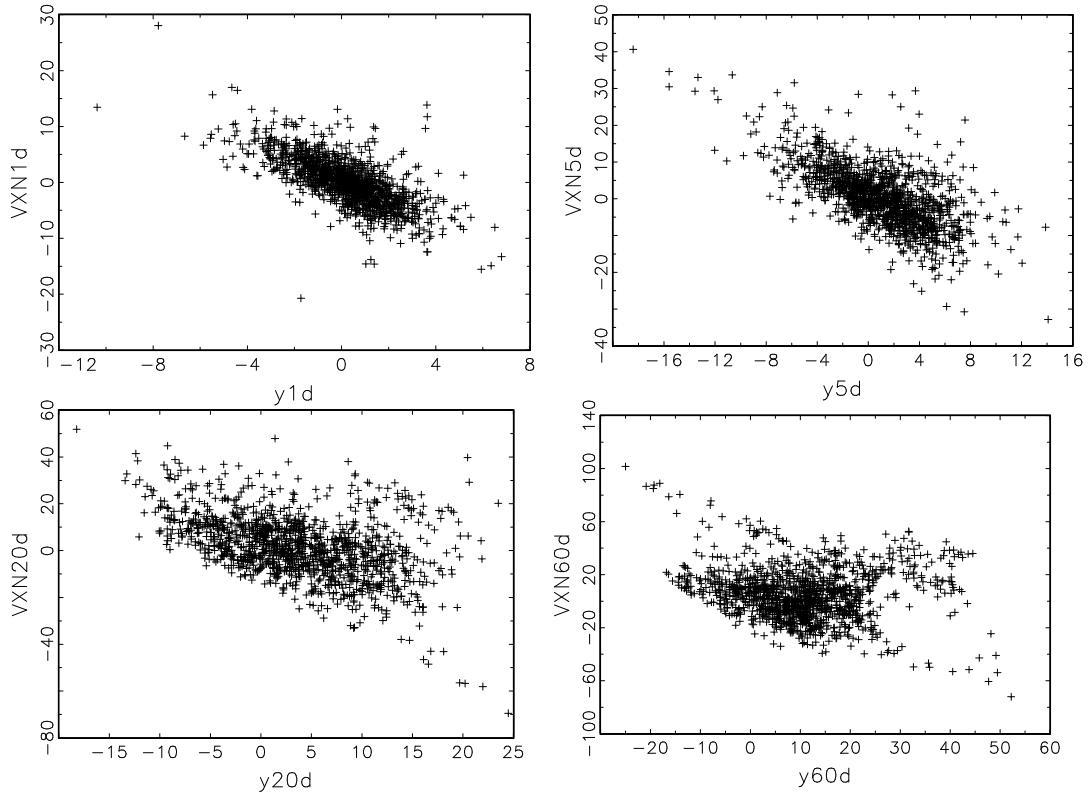


Figure 2. Contemporaneous changes in the stock index and implied volatility index (NASDAQ100 index bull market). This figure shows the relationship between the 1-, 5-, 20- and 60-day contemporaneous relative changes in the level of the stock index and implied volatility index. The time period is January 3, 1995 to March 27, 2000.

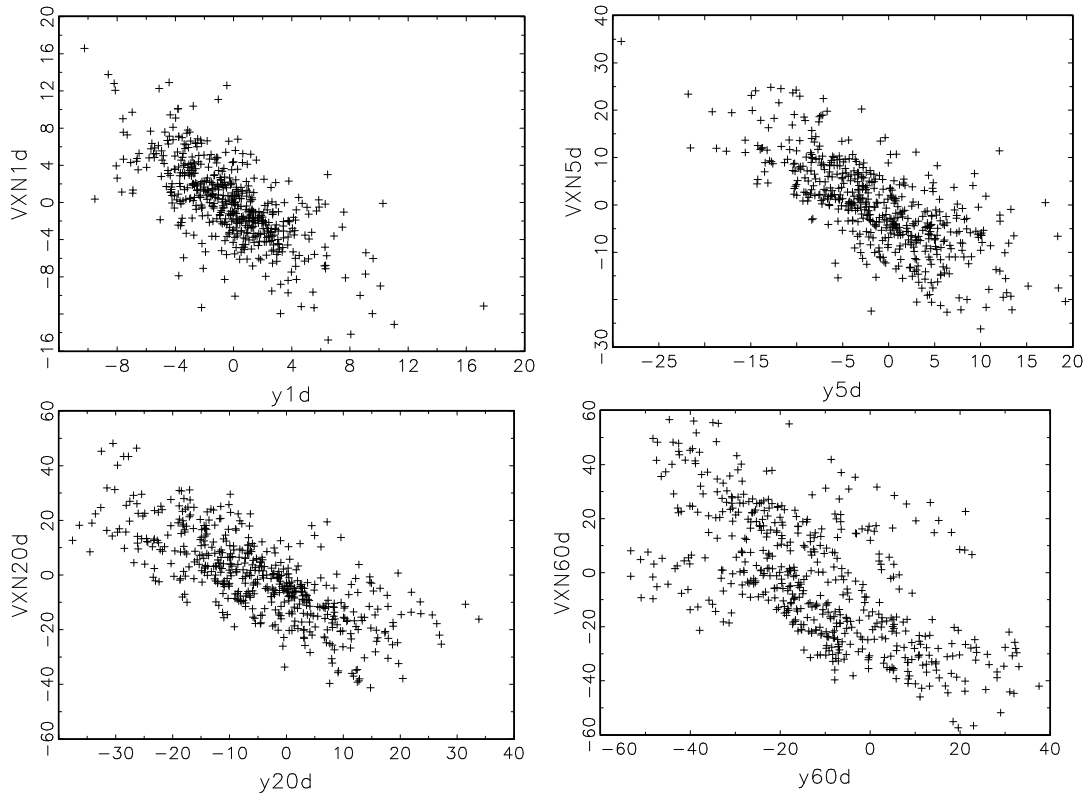


Figure 3. Contemporaneous changes in the stock index and implied volatility index (NASDAQ100 index bear market). This figure shows the relationship between the 1-, 5-, 20- and 60-day contemporaneous relative changes in the level of the stock index and implied volatility index. The time period is March 28, 2000 to May 8, 2002.

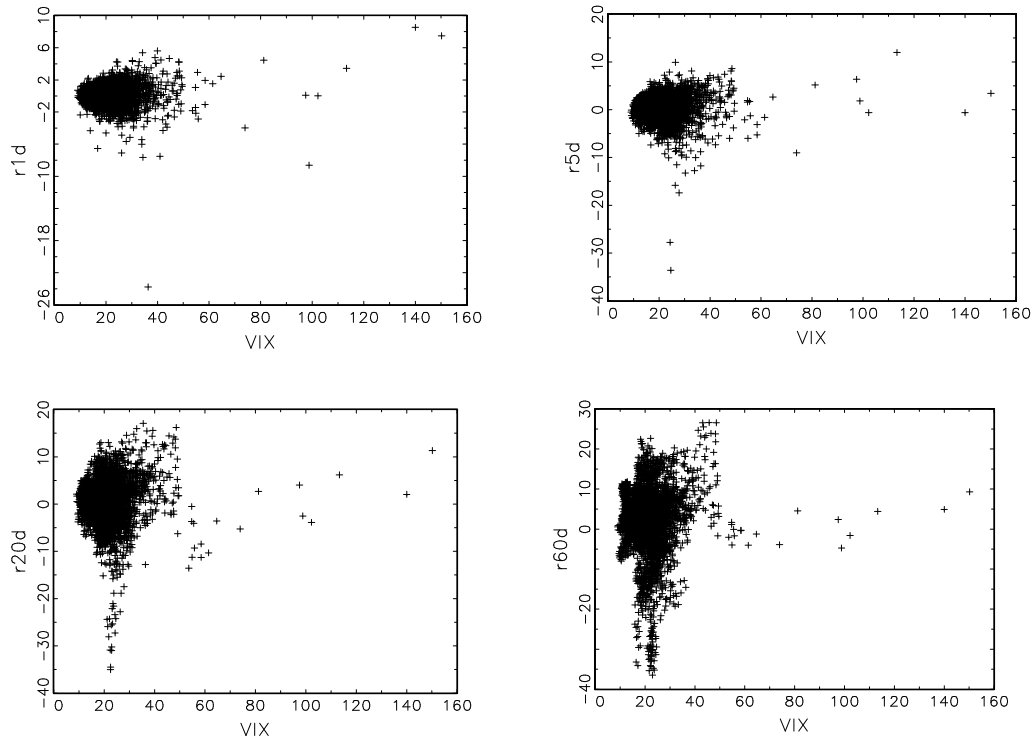


Figure 4. Scatter plots of the forward-looking returns vs implied volatility index that triggered the trade (S&P100 index). This figure shows the relationship between the 1-, 5-, 20- and 60-day forward-looking returns for the stock index and the level of implied volatility that triggered the trade. The time period is January 2, 1986 to May 8, 2002.

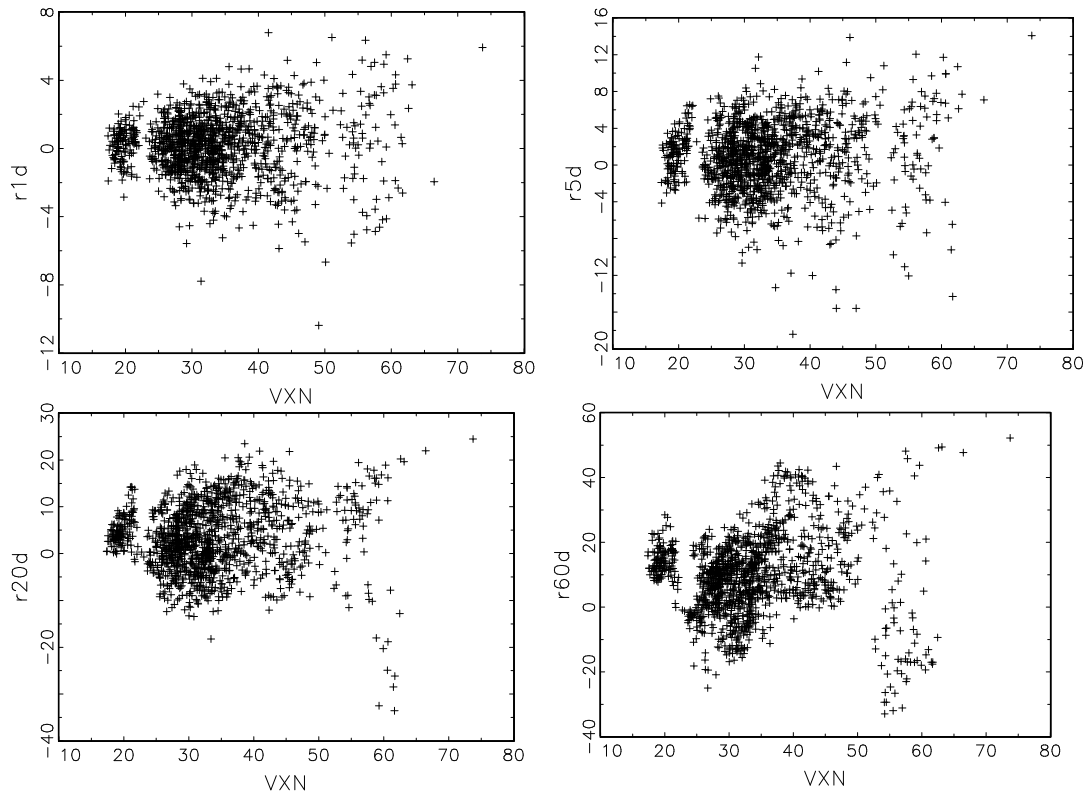


Figure 5. Scatter plots of the forward-looking returns vs implied volatility index that triggered the trade (NASDAQ100 index bull market). This figure shows the relationship between the 1-, 5-, 20- and 60-day forward-looking returns for the stock index and the level of implied volatility that triggered the trade. The time period is January 3, 1995 to March 27, 2000.

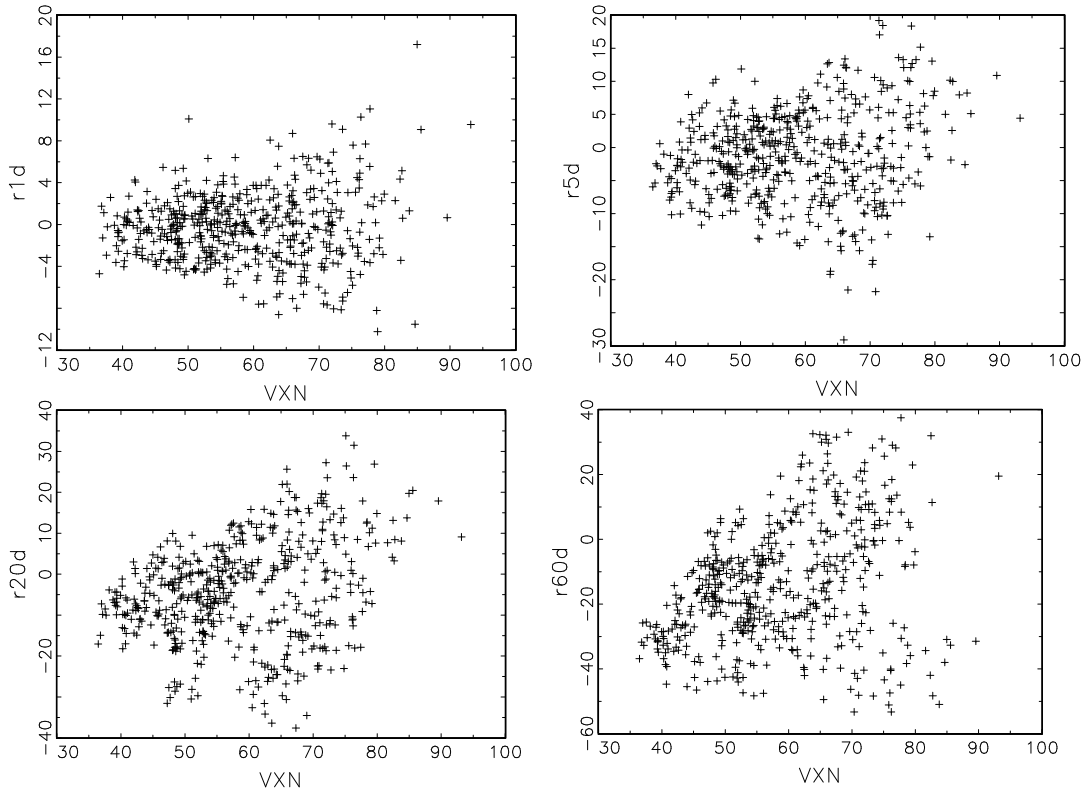


Figure 6. Scatter plots of the forward-looking returns vs implied volatility index that triggered the trade (NASDAQ100 index bear market). This figure shows the relationship between the 1-, 5-, 20- and 60-day forward-looking returns for the stock index and the level of implied volatility that triggered the trade. The time period is March 28, 2000 to May 8, 2002.

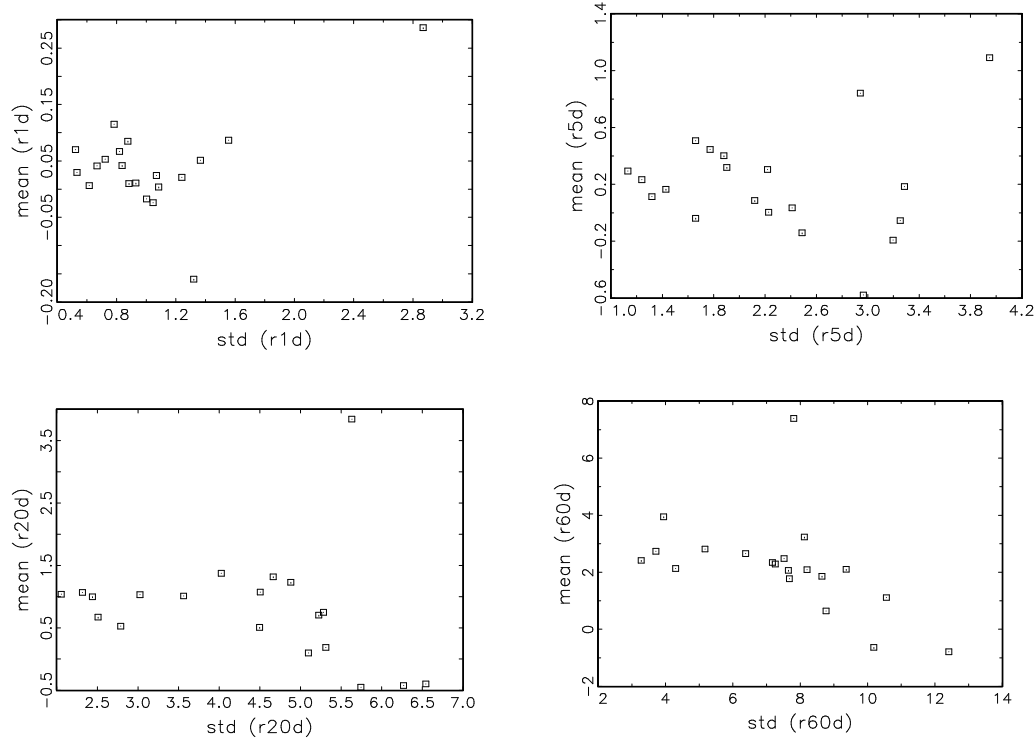


Figure 7. Mean-variance plot of the forward-looking returns (S&P100 index). This figure shows the mean-variance tradeoff for the n-day-ahead returns on the S&P100 index, i.e. the mean and standard deviation of the 1-, 5-, 20-, and 60-day forward-looking returns on the S&P100 index when the VIX index belongs to the interval given in the first column of Table IV. The time period is January 2, 1986 to May 8, 2002.

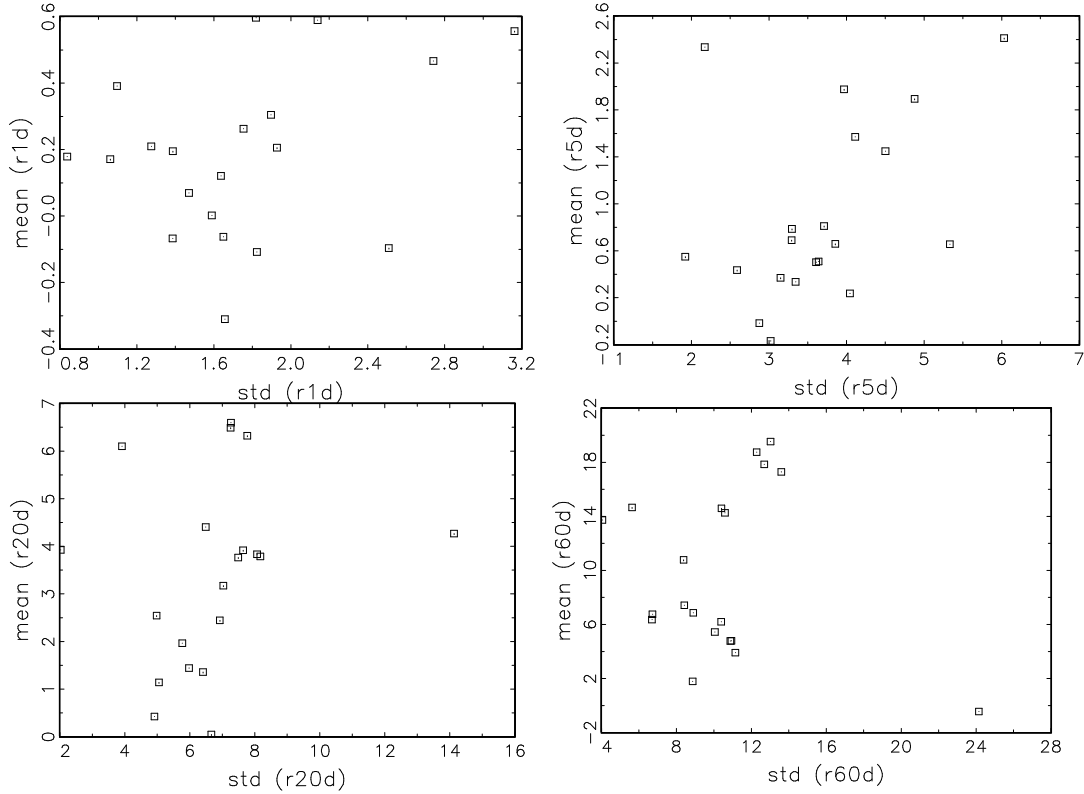


Figure 8. Mean-variance plot of the forward-looking returns (NASDAQ100 index bull market). This figure shows the mean-variance tradeoff for the n-day-ahead returns on the NASDAQ100 index (bull market), i.e. the mean and standard deviation of the 1-, 5-, 20-, and 60-day forward-looking returns on the NASDAQ100 index when the VIX index belongs to the interval given in the first column of Table V. The time period is January 3, 1995 to March 27, 2000.

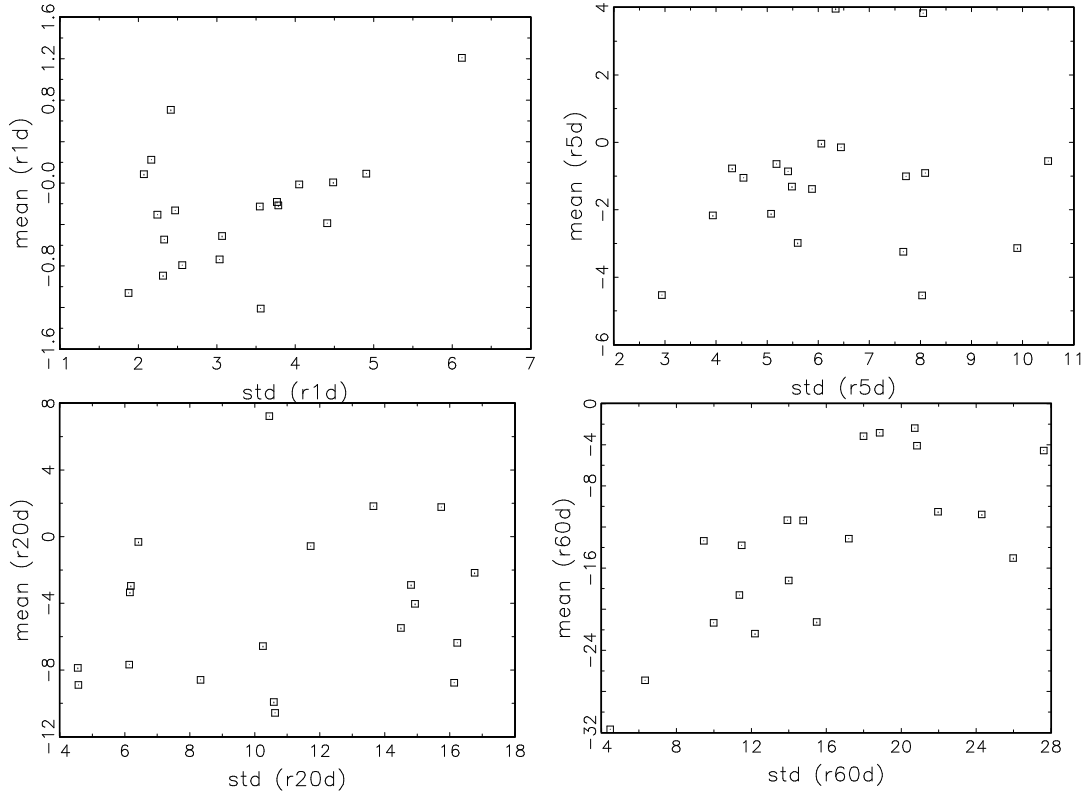


Figure 9. Mean-variance plot of the forward-looking returns (NASDAQ100 index bear market). This figure shows the mean-variance tradeoff for the n -day-ahead returns on the NASDAQ100 index (bear market), i.e. the mean and standard deviation of the 1-, 5-, 20-, and 60-day forward-looking returns on the NASDAQ100 index when the VIX index belongs to the interval given in the first column of Table VI. The time period is March 28, 2000 to May 8, 2002.

Notes

¹We assume in this paper that market participants agree on the option pricing formula, for example the usual Black and Scholes (1973) framework, as this is not a paper on the details of option pricing methods.

²They thus suppose that there are short-lived periods of market inefficiency where market participants do not act rationally but engage in ‘herding’ behaviors. See for example Shefrin (2000) for a discussion of behavioral finance.

³Even in the academic literature, the VIX implied volatility index of the CBOE for S&P100 index option contracts is dubbed the ‘fear indicator’ (Whaley, 2000).

⁴Note that the VXN implied volatility index is only available starting January 3, 1995.

⁵We stress that this study is not meant to be an assessment of market efficiency in the sense of Fama (1970), Fama (1991) or Campbell, Lo, and MacKinlay (1997), as we do not deal with the usual framework of market (for example weak-form) efficiency. Our goal is, akin to the methodology used in event studies, to focus on the mean-variance tradeoff of returns for buy-side trades triggered by very high levels of implied volatility indices.

⁶See the CBOE website at <http://www.cboe.com>.

⁷Therefore they are free from so-called mis-specification problems which affected a large number of early studies on the information content of implied volatility. See Blair, Poon, and Taylor (2001) for a discussion of potential problems and biases when computing implied volatility measures.

⁸The more general NASDAQ composite index reached its all-time high on March 10, 2000, while the NASDAQ100 index peaked at 4704.73 on March 27, 2000.

⁹ $VXN1d_t$, $VXN5d_t$, $VXN20d_t$ and $VXN60d_t$ are defined similarly.

¹⁰See Engle and Ng (1993) for a discussion of news response curves.

¹¹For example, $r5d_t$ is computed as $\ln(P_{t+5}) - \ln(P_t)$ and is the 5-day forward-looking return relative to the VIX level observed at time t .

¹²Prior to the regression analysis, a Dickey-Fuller unit root test was run on the time series of VIX_t and the null hypothesis that VIX is integrated was rejected.

¹³A similar methodology is used by Campbell and Shiller (2001) to highlight the dependence between observed price-earnings ratio and subsequent n-year-ahead market returns on stocks indices.

¹⁴Note however that all returns cannot be achieved at the same time. Indeed, as these returns are ‘triggered’ by given (and mutually exclusive) levels of VIX or VXN, they are not simultaneously available to an investor.

¹⁵Note that we do not include a risk-free asset in our analysis. Our mean-variance framework is thus made up of risky assets only.

¹⁶Of course, choosing a particular point (i.e. mean and variance levels) in the first dataset will depend on the risk-aversion of the trader.