Power Allocation for Energy Efficiency Optimization in Distributed Antenna Systems

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***Abstract*—An energy efficiency (EE) maximization problem with multiple distributed antennas is considered in this paper. Besides the constraint on the total transmit power and per-antenna transmit power constraints, an additional constraint dealing with total weighted power is introduced. Moreover, a Quality-of-Service (QoS) constraint is introduced. Because the original problem is a fractional convex one, Dinkelbach's algorithm is implemented to reformulate the problem. By investigating the Karush-Kuhn-Tucker (KKT) conditions, the mathematical properties of the globally optimal solution are studied and proved, which provides a deeper understanding of the structure of power allocation among antennas. Numerical results demonstrate the effect of QoS on EE and also the structure of power allocation.**

I. Introduction

Nowadays, energy efficiency (EE) for wireless communications is becoming a main economical and societal challenge [1]. Hence, instead of maximizing the information rate under a certain power constraint, engineers wish to maximize the transmission rate per consumed energy. This is also referred to as the EE maximization problem.

Per-antenna power constraints are of practical relevance because the transmit power at each antenna is constrained by its local amplifier. However, only few contributions dealing with EE have considered per-antenna power constraints or per base station power constraints [2]–[5]. References [2] and [3] studied the EE maximization problem in distributed antenna systems (DAS). Reference [4] considered multi-cell downlink MISO systems with per base station power constraints. Reference [5] studied precoder design in multi-user MIMO systems using zero-forcing (ZF) precoding with joint per-antenna and total power constraint.

In this paper, we study the EE maximization problem for multiple transmit antenna systems, with joint total and per-antenna transmit (i.e. radio frequency (RF)) power constraints and a total power expense constraint. To the best of our knowledge, this EE maximization problem has not yet been studied. A total transmit power constraint is justified by interference and/or regulatory concerns. The per-antenna power constraint finds its origin in implementation. As far as the total expense of the consumed power constraint is concerned, it is motivated by the fact that distributed antennas might be supplied by different types of energies like solar power, wind power, or even wireless power transfer, etc. Moreover, their maintenance might imply different cost structures. As will appear later, this type of constraint leads to introducing weighting factors, making thereby this constraint non-redundant with the total power constraint. As in paper [6], where closed form solutions have been obtained for the rate optimum power allocation with joint total and per antenna power constraints for rate maximization, the mathematical properties of EE optimum power allocation are studied in the current paper. These properties provide insights into power allocation, and also help reducing the complexity of the EE maximization algorithms.

II. System Model and Problem Formulation

Consider a system with N distributed antennas serving multiple users where users are separated by various time slots like time-division multiple access (TDMA). All antennas are assumed to operate with coherent coordination, i.e. all antennas send the same signal to a certain user in each time slot [2], [5].

In a certain time slot, the channel vector between the N transmit antennas and the receiver is denoted by $\mathbf{h} = [h_1, ..., h_N]^T$, which is normalized by noise at receiver. The beamforming vector is denoted by $\mathbf{w} = [a_1, ..., a_N]^T$. It is well known that all the elements of $\mathbf{h}$ and $\mathbf{w}$ should be aligned to reach the maximum rate when each $|a_n|$ is assumed to be fixed [6]–[8]. Hence, $h_n a_n$ is a real number for every n. By denoting $q_n = |a_n|^2$, the rate function becomes

$$R = \log\left(1 + \left|\sum_{n=1}^{N} h_n a_n\right|^2\right) = \log\left(1 + \left(\sum_{n=1}^{N} \sqrt{q_n}|h_n|\right)^2\right). \tag{1}$$

Assume that each antenna n has its own amplifier efficiency $1/\kappa_n$ and a price per power consumption unit w_n (because the power at some antennas or distributed transmitters may be more precious than at others), where both $1/\kappa_n$ and w_n can be antenna-specific. The EE is defined as

$$\mathrm{EE} = \frac{\log\left(1 + \left(\sum_{n=1}^{N} \sqrt{q_n}|h_n|\right)^2\right)}{\sum_{n=1}^{N} \phi_n q_n + P_c}, \tag{2}$$

where $\phi_n = w_n \kappa_n$ is the effective weight for RF power consumption at the n-th antenna and P_c is the constant circuit power. Because of the definition of ϕ_n, (2) is a more general

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model of EE: transmitted rate information per power expense. The EE maximization problem can be formulated as

$$\max_{\{q_n\}} \quad \texttt{EE} \tag{3}$$

$$s.t. \quad \sum_{n=1}^{N} q_n \leq Q_1 \tag{4}$$

$$\sum_{n=1}^{N} \phi_n q_n \leq Q_2 \tag{5}$$

$$0 \leq q_n \leq P_n, \tag{6}$$

$$\log\left(1+\left(\sum_{n=1}^{N}\sqrt{q_n}|h_n|\right)^2\right) \geq R_{\texttt{min}}. \tag{7}$$

Hence, Q_1 is the total transmit power constraint, Q_2 is the total expense limitation, P_n is the transmit power constraint for the n-th antenna, and $R_{\texttt{min}}$ is the minimum rate. (4) and (5) are not redundant because the ϕ_n are not necessarily the same for all n. $\sum_n P_n > Q_1$ and $\sum_n \phi_n P_n > Q_2$ should be satisfied to make constraints (4) and (5) simultaneously achievable.

III. Power allocation for EE maximization

A. Concavity of rate function

The rate function in (1) is concave [8]. One can check the concavity of $\left(\sum_{n=1}^{N}\sqrt{q_n}|h_n|\right)^2$, which can be proven by the fact that the function $b(x,y)=\sqrt{xy}$ is concave and

$$\left(\sum_{n=1}^{N}\sqrt{q_n}|h_n|\right)^2 = \sum_{n=1}^{N}|h_n|^2 q_n + \sum_{m\neq n}\sqrt{q_m q_n}|h_m||h_n|. \tag{8}$$

Due to the fact that $c(d(x))$ is concave if both $c(x)$ and $d(x)$ are concave and $d(x)$ in non-decreasing, we have $\log\left(1+\left(\sum_{n=1}^{N}\sqrt{q_n}|h_n|\right)^2\right)$ is concave.

B. The KKT conditions

Because the numerator is concave and the denominator is linear in (3), we can refer to the Dinkelbach algorithm to reformulate the problem as [9]

$$\max_{\{q_n\}} \quad F_\lambda(\mathbf{q}) \triangleq \log_2\left(1+\left(\sum_{n=1}^{N}\sqrt{q_n}|h_n|\right)^2\right) - \lambda\left(\sum_{n=1}^{N}\phi_n q_n + P_c\right) \tag{9}$$

$$s.t. \quad (4)(5)(6)$$

$$\sum_{n=1}^{N}\sqrt{q_n}|h_n| \geq \sqrt{2^{R_{\texttt{min}}}-1}, \tag{10}$$

where $\mathbf{q}=[q_1,...,q_N]^T$. The EE is gradually maximized when λ is updated in each iteration until $\max_{\mathbf{q}\in(4)(5)(6)(10)} F_\lambda(\mathbf{q})=0$. Each function $F_\lambda(\mathbf{q})$ is concave. Hence, the global optimum can be found by means of the Karush-Kuhn-Tucker (KKT) conditions. The Lagrangian of $F_\lambda(\mathbf{q})$ is:

$$L(\lambda;\mathbf{q},\alpha,\beta,\{\gamma_n\},\{\rho_n\},\chi) = F_\lambda(\mathbf{q}) -\alpha\left(\sum_n q_n - Q_1\right) - \beta\left(\sum_n \phi_n q_n - Q_2\right) - \sum_n \gamma_n(q_n - P_n) +\rho_n q_n + \chi\left(\sum_{n=1}^{N}\sqrt{q_n}|h_n| - \sqrt{2^{R_{\texttt{min}}}-1}\right). \tag{11}$$

The KKT conditions are

$$\left(\frac{\chi}{2} + \frac{1}{\ln 2}\cdot\frac{\sum_{m=1}^{N}\sqrt{q_m}|h_m|}{1+\left(\sum_{m=1}^{N}\sqrt{q_m}|h_m|\right)^2}\right)\cdot\frac{|h_n|}{\sqrt{q_n}} = \lambda\phi_n + \alpha + \beta\phi_n + \gamma_n - \rho_n. \tag{12}$$

To simplify (12), we propose another way to solve (9) under $\{(4)(5)(6)(10)\}$, where we ignore the QoS constraint (10) at first. If the obtained solution (with $\max_{\mathbf{q}\in(4)(5)(6)} F_{\bar{\lambda}}(\mathbf{q})=0$, where $\bar{\lambda}$ is the optimal EE) fulfills (10), then it is the same solution as $\max_{\mathbf{q}\in(4)(5)(6)(10)} F_{\bar{\lambda}}(\mathbf{q})=0$. If the obtained solution does not fulfill (10), from the property of the Dinkelbach algorithm [9], there exist $\hat{\lambda}$ with $0\leq\hat{\lambda}<\bar{\lambda}$ such that (10) is fulfilled by the solution of $\max_{\mathbf{q}\in(4)(5)(6)} F_{\hat{\lambda}}(\mathbf{q})$ if (10) can be fulfilled by the solution of $\max_{\mathbf{q}\in(4)(5)(6)} F_0(\mathbf{q})$ which corresponds to rate maximization. The following Theorem shows that the obtained solution of $\max_{\mathbf{q}\in(4)(5)(6)} F_{\hat{\lambda}}(\mathbf{q})$ with a certain $\hat{\lambda}$ is the same as that corresponding to directly solving (3) under $\{(4)(5)(6)(7)\}$.

Theorem 1: If $\mathbf{q} = \arg\max_{\mathbf{q}\in(4)(5)(6)} F_{\bar{\lambda}}(\mathbf{q}) = 0$ does not fulfill the QoS constraint, meaning $\sum_{n=1}^{N}\sqrt{q_n}|h_n| < \sqrt{2^{R_{\texttt{min}}}-1}$, then, if one can find a $\hat{\lambda}$ in the interval $0\leq\hat{\lambda}<\bar{\lambda}$, such that $\mathbf{q}^\star = \arg\max_{\mathbf{q}\in(4)(5)(6)} F_{\hat{\lambda}}(\mathbf{q})$ satisfies $\sum_{n=1}^{N}\sqrt{(\mathbf{q}^\star)_n}|h_n| = \sqrt{2^{R_{\texttt{min}}}-1}$, then $\mathbf{q}^\star$ is the solution for $\max_{\lambda,\mathbf{q}\in(4)(5)(6)(10)} F_\lambda(\mathbf{q})=0$.

Due to the limited space, the proof is omitted.

Therefore, in the proposed algorithm, we will first ignore (10), that is, make $\chi=0$ in (12). In the case the solution does not fulfill (10), the algorithm will update λ so that (10) be satisfied with equality. From Theorem 1, the obtained solution is the one of (3) under $\{(4)(5)(6)(7)\}$ and is globally optimum because this problem is convex.

Given values of $\alpha,\beta,\{\gamma_n\},\{\rho_n\}$, the fixed point of $\mathbf{q}$ satisfying the KKT condition is the power allocation vector $\mathbf{q}$. A standard method to find the optimal $\mathbf{q}$ is to update values of $\alpha,\beta,\gamma_n,\rho_n$ by means of a sub-gradient method. However, this method converges slowly [5] and does not provide a deep insight of the power allocation structure. In the following, we exploit the mathematical structure of power allocation $\mathbf{q}$ to solve the problem, which describes the structure of power allocation at each antenna similarly as [8] and [6] have done for rate maximizations.

C. Optimal solution: four cases

To study the structure of power allocation, let us first introduce the following theorem.

Theorem 2: All ρ_n equal zero.
Due to the limited space, the proof is omitted.

Theorem 2 implies that $q_n > 0$ for all n.

Depending on whether or not the total power constraints (4) and (5) are active, we divide the solution into four cases.

1) case 1: $\alpha = \beta = 0$: In this case, both (4) and (5) are not active, i.e., $\sum_{n=1}^{N} q_n < Q_1$ and $\sum_{n=1}^{N} \phi_n q_n < Q_2$. From (12), we have

$$\sqrt{q_n} = \frac{1}{\ln 2} \cdot \frac{\sum_{m=1}^{N} \sqrt{q_m}|h_m|}{1 + \left(\sum_{m=1}^{N} \sqrt{q_m}|h_m|\right)^2} \cdot \frac{|h_n|}{\lambda \phi_n + \gamma_n}, \quad (13)$$

where for those n such that $q_n = P_n$, we have $\gamma_n > 0$, and for those n such that $q_n < P_n$, we have $\gamma_n = 0$. After some manipulations to (13), we have, if $\lambda > 0$,

$$\frac{\sqrt{P_n}\phi_n}{|h_n|} < \frac{(\lambda \ln 2)^{-1} \sum_{m=1}^{N} \sqrt{q_m}|h_m|}{1 + \left(\sum_{m=1}^{N} \sqrt{q_m}|h_m|\right)^2} \quad \texttt{if} \quad q_n = P_n \quad (14)$$

$$\frac{\sqrt{P_n}\phi_n}{|h_n|} > \frac{(\lambda \ln 2)^{-1} \sum_{m=1}^{N} \sqrt{q_m}|h_m|}{1 + \left(\sum_{m=1}^{N} \sqrt{q_m}|h_m|\right)^2} \quad \texttt{if} \quad q_n < P_n. \quad (15)$$

Note that if $\lambda = 0$, all γ_n should be positive, meaning $q_n = P_n$ for all n, which can never happen as mentioned above. Therefore, if $\lambda = 0$, the solution can not be case 1, meaning either (4) or (5) is active, or both of them are active, which is true for rate maximization ($\lambda = 0$).

From (14) and (15), all channels can be sorted in ascending order of the threshold value $\frac{P_n \phi_n^2}{|h_n|^2}$:

$$\frac{P_{\tau_1}\phi_{\tau_1}^2}{|h_{\tau_1}|^2} < \cdots < \frac{P_{\tau_N}\phi_{\tau_N}^2}{|h_{\tau_N}|^2}. \quad (16)$$

The first K channels $\tau_1, ..., \tau_K$ ($0 \le K \le N$) transmit with full power $P_{\tau_1}, ..., P_{\tau_K}$ and the remaining channels transmit with power values proportional to each other:

$$\sqrt{q_{\tau_{K+1}}} : \frac{|h_{\tau_{K+1}}|}{\phi_{\tau_{K+1}}} = \cdots = \sqrt{q_{\tau_N}} : \frac{|h_{\tau_N}|}{\phi_{\tau_N}}. \quad (17)$$

K satisfies the following relationship

$$\frac{\sqrt{P_{\tau_K}}\phi_{\tau_K}}{|h_{\tau_K}|} < \frac{(\lambda \ln 2)^{-1} \sum_{m=1}^{N} \sqrt{q_m}|h_m|}{1 + \left(\sum_{m=1}^{N} \sqrt{q_m}|h_m|\right)^2} < \frac{\sqrt{P_{\tau_{K+1}}}\phi_{\tau_{K+1}}}{|h_{\tau_{K+1}}|}. \quad (18)$$

To conclude, if neither (4) nor (5) is active, the antennas with small maximum power P_n, low effective weight ϕ_n, and strong channel h_n will first reach their maximum power. Other antennas transmit with power according to a given ratio.

2) case 2: $\alpha = 0, \beta > 0$: In this case, (4) is inactive and (5) is active, i.e., $\sum_{n=1}^{N} q_n < Q_1$ and $\sum_{n=1}^{N} \phi_n q_n = Q_2$. From (12), we have

$$\sqrt{q_n} = \frac{1}{\ln 2} \cdot \frac{\sum_{m=1}^{N} \sqrt{q_m}|h_m|}{1 + \left(\sum_{m=1}^{N} \sqrt{q_m}|h_m|\right)^2} \cdot \frac{|h_n|}{(\lambda + \beta)\phi_n + \gamma_n}. \quad (19)$$

One can observe that the only difference between (19) and (13) is that λ is replaced by $\lambda + \beta$. Similarly with case 1, the first K channels $\tau_1, ..., \tau_K$ ($0 \le K \le N$) transmit with full power $P_{\tau_1}, ..., P_{\tau_K}$ and the remaining channels transmit with power fulfilling the same proportionality as (17), where K should also satisfy (18). The only PA difference between case 2 and case 1 is that (5) is satisfied with equality. Since the power values of all channels have a fixed proportion (except the ones with full transmit power), only a unique power allocation vector $\mathbf{q}$ satisfies case 2. Let us summarize the proportionality conditions that $\mathbf{q}$ must fulfill both in case 1 and case 2:

$$\sqrt{q_{\tau_{K+1}}} : \frac{|h_{\tau_{K+1}}|}{\phi_{\tau_{K+1}}} = \cdots = \sqrt{q_{\tau_N}} : \frac{|h_{\tau_N}|}{\phi_{\tau_N}} \quad (20)$$

$$q_{\tau_n} = P_{\tau_n}, \qquad n = 1, ..., K \quad (21)$$

$$q_{\tau_N} \frac{\phi_{\tau_N}^2}{|h_{\tau_N}|^2} \frac{|h_{\tau_n}|^2}{\phi_{\tau_n}^2} > P_{\tau_n}, \qquad n = 1, ..., K \quad (22)$$

Inequality (22) is because antenna τ_N never uses its full transmit power.

To conclude, as the power values q_n (all n) increase from zero which has to happen in accordance with the proportionality condition, the number of antennas that reach their power constraints P_n increases. τ_N locates in the last position of the queue $\tau_1, ..., \tau_N$, therefore it never reaches P_{τ_N}. When (5) is satisfied with equality, the solution corresponds to case 2.

Cases 1 and 2 can be viewed as 'macro cases' because they share the same proportionality condition. Therefore they can be solved with the same algorithm.

3) case 3: $\alpha > 0, \beta = 0$: In this case, (4) is active and (5) is inactive, i.e., $\sum_{n=1}^{N} q_n = Q_1$ and $\sum_{n=1}^{N} \phi_n q_n < Q_2$. From (12), we have

$$\sqrt{q_n} = \frac{1}{\ln 2} \cdot \frac{\sum_{m=1}^{N} \sqrt{q_m}|h_m|}{1 + \left(\sum_{m=1}^{N} \sqrt{q_m}|h_m|\right)^2} \cdot \frac{|h_n|}{\lambda \phi_n + \alpha + \gamma_n}. \quad (23)$$

Different from case 1 and case 2, there is no static proportionality condition similar to (17) due to the non-zero α. The solution structure shows, however, similarity with (20) (21) (22) and corresponds to:

$$\sqrt{q_{\tau_{K+1}}} : \frac{|h_{\tau_{K+1}}|}{\lambda \phi_{\tau_{K+1}} + \alpha} = \cdots = \sqrt{q_{\tau_N}} : \frac{|h_{\tau_N}|}{\lambda \phi_{\tau_N} + \alpha} \quad (24)$$

$$q_{\tau_n} = P_{\tau_n}, \qquad n = 1, ..., K \quad (25)$$

$$q_{\tau_N} \frac{(\lambda \phi_{\tau_N} + \alpha)^2}{|h_{\tau_N}|^2} \frac{|h_{\tau_n}|^2}{(\lambda \phi_{\tau_N} + \alpha)^2} > P_{\tau_n}, \qquad n = 1, ..., K \quad (26)$$

We need to find $\alpha^\star$ such that (23) is satisfied. To this end, we introduce the following theorem:

Theorem 3: If a certain $\mathbf{q}$ satisfies (24) (25) (26) with $\alpha = 0$ and $\sum q_n = Q_1$, then we have

$$\frac{\lambda \phi_{\tau_N} \sqrt{q_{\tau_N}}}{|h_{\tau_N}|} < \frac{1}{\ln 2} \frac{\sum_{m=1}^{N} \sqrt{q_m}|h_m|}{1 + \left(\sum_{m=1}^{N} \sqrt{q_m}|h_m|\right)^2}. \quad (27)$$

Due to the limited space, the proof is omitted.

On the other hand, it is obvious that, when $\alpha = \infty$, we have

$$\frac{(\lambda\phi_{\tau_N} + \alpha)\sqrt{q_{\tau_N}}}{|h_{\tau_N}|} > \frac{1}{\ln 2}\frac{\sum_{m=1}^{N}\sqrt{q_m}|h_m|}{1+\left(\sum_{m=1}^{N}\sqrt{q_m}|h_m|\right)^2}. \quad (28)$$

Thus, a bisection method can be used to find $\alpha^\star$, such that the fixed-point solution in (23) with $\alpha = \alpha^\star$ satisfies $\sum_{n=1}^{N} q_n = Q_1$.

4) case 4: $\alpha > 0, \beta > 0$: In this case, both (4) and (5) are active, i.e., $\sum_{n=1}^{N} q_n = Q_1$ and $\sum_{n=1}^{N} \phi_n q_n = Q_2$. From (12), we have

$$\sqrt{q_n} = \frac{1}{\ln 2}\cdot\frac{\sum_{m=1}^{N}\sqrt{q_m}|h_m|}{1+\left(\sum_{m=1}^{N}\sqrt{q_m}|h_m|\right)^2}\cdot\frac{|h_n|}{(\lambda+\beta)\phi_n + \alpha + \gamma_n}. \quad (29)$$

Assume $\bar{\alpha}$ and $\bar{\beta}$ to be the optimal values for case 4. Denote the solution of (23) as $\alpha^\star$ and the solution of (19) as $\beta^\star$. An algorithm similar to the bisection method can find the value of $\bar{\alpha}$ and $\bar{\beta}$, which exploits the property of the following theorem:

Theorem 4: It is true that $0 < \bar{\alpha} < \alpha^\star$ and $0 < \bar{\beta} < \beta^\star$. Due to the limited space, the proof is omitted.

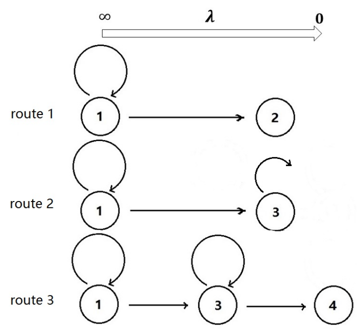

Fig. 1: State transfer of EE maximum. Number x denotes case x.

D. Behavior of power allocation as λ is updated

For rate maximization, the solution is never in case 1, i.e., the transmitter will use full total transmit power or full total consumed power, depending on either (4) or (5) is the first one being satisfied with equality when each q_n increases according to the required proportionality. In fact, rate maximization is equivalent to optimizing $F_0(\mathbf{q})$. For EE maximization, however, λ is updated so it is non-zero after the first updating. The variable λ creates a term of power penalty in $F_\lambda(\mathbf{q})$, making the case 1 solution possible. This is similar with the phenomenon in traditional EE maximization (with only a total power constraint) where the optimum transmit power may not be the maximum available transmit power. To better understand the relation between the solution and the value of λ, the following theorem is introduced.

Theorem 5: As λ decreases from infinity, the EE optimum power allocation necessarily follows one of three possible routes depicted in Fig. 1.
Due to the limited space, the proof is omitted.

As shown in Fig. 1, there are three possible routes for the optimum solution as λ decreases. All three routes begin with case 1, where the power is allocated according to (17). As λ decreases, total power and weighted total power increase.

If the constraint $\sum_n \phi_n q_n = Q_2$ is satisfied while $\sum_n q_n < Q_1$, then the solution stops at case 2 which has a unique solution and can be viewed as a special case of case 1 as in the analysis. This is referred to as route 1.

If the constraint $\sum_n q_n = Q_1$ is satisfied while $\sum_n \phi_n q_n < Q_2$, then the solution changes from case 1 to case 3. By keeping $\sum_n q_n = Q_1$ and decreasing λ, $\sum_n \phi_n q_n$ increases. On one hand, if $\sum_n \phi_n q_n < Q_2$ when $\sum_n \phi_n q_n$ reaches its maximum value when $\lambda = 0$ (as indicated by the 'open' arrow in route 2), then the solution stops at case 3, which is route 2.

On the other hand, if $\sum_n \phi_n q_n = Q_2$ before $\lambda = 0$ (as indicated by the 'closed' arrow in route 3), then the solution stops at case 4, which has a unique solution. This is referred to as route 3.

Imposing a QoS constraint is equivalent to setting a maximum value of λ for each of the three routes.

IV. Numerical results

In this section we illustrate the performance of the algorithms by means of numerical results. The parameters are set as follows: $B = 10$KHz is the bandwidth and $N = 4$. The channel is assumed to be Rayleigh distributed and has an average channel gain $G_0 d^{-n}$ where the path loss exponent $n = 4$, $G_0 = -(G_1 M_l) = -70$dB in which $G_1 = 30$dB is the gain factor at $d = 1$m. $M_l = 40$dB is the link margin compensating the hardware process variations and other noise and interference [10], the noise power $\sigma^2 = BN_0N_f$ where $N_0 = -170$dBm/Hz is the noise power spectral density and $N_f = 10$dB is the noise figure.

In Fig. 2, Subfigures (a) and (b) report EE performance versus total power constraints when EE or rate is maximized. The parameter ρ introduced enables to balance individual and total power constraints. Specifically, we set $Q_1 = \rho(\sum_n P_n)$, $Q_2 = \rho(\sum_n \phi_n P_n)$. Therefore the smaller ρ, the more impact the total power constraints (Q_1 or Q_2) will have on EE maximization. In Subfigure (a), no QoS constraint is considered. Obviously, rate maximization always tends to use the full total power available. It is observed that for small ρ, EE maximization and rate maximization produce the same EE, while for large ρ, EE maximization outperforms rate maximization. This is consistent with the analysis above. In addition, the EE gap becomes smaller for lower channel SNRs (larger d); this is because more transmit power is required to counteract the channel SNR loss for EE maximization. In Subfigure (b), a QoS constraint $R_{\min} = 1(bits/s, Hz)$ is added. In the calculation of the achieved average EE, the channel realizations for which the QoS constraint cannot be

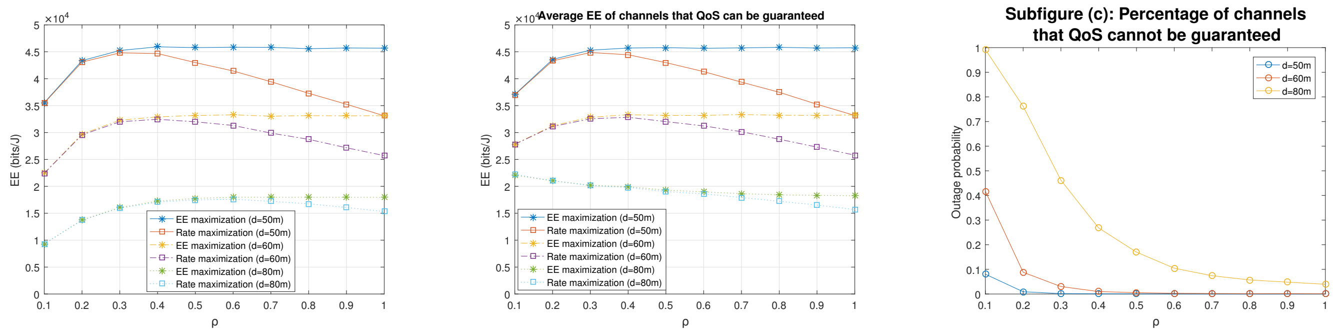

Fig. 2: Performance comparison between EE maximization and rate maximization for different total power constraints (linear HPA). Parameters: $P_c = 400\text{mW}$, $\phi = [1.5, 2, 2.5, 3]$, $P_1 = P_2 = P_3 = P_4 = 100\text{mW}$. $Q_1 = \rho\left(\sum_n P_n\right)$, $Q_2 = \rho\left(\sum_n \phi_n P_n\right)$. Subfigure (a): EE without QoS constraint; Subfigure (b): EE with QoS constraint $R_{\min} = 1(bits/s, Hz)$; Subfigure (c): outage probability with QoS constraint $R_{\min} = 1(bits/s, Hz)$.

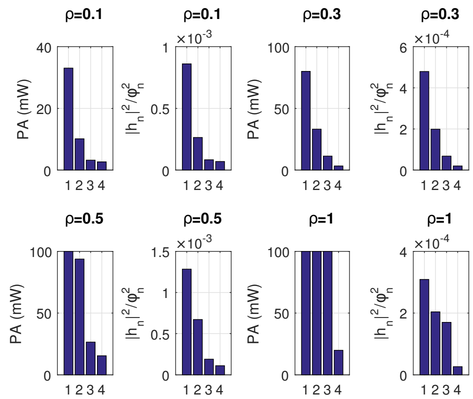

Fig. 3: Proportionality condition (linear HPA). Parameters: $\phi = [1.5, 2, 2.5, 3]$, $P_1 = P_2 = P_3 = P_4 = 100\text{mW}$, $Q_1 = \infty$, $Q_2 = \rho\left(\sum_n \phi_n P_n\right)$, $d = 100\text{m}$. $P_c = 500\text{mW}$.

fulfilled are removed and are declared to be in outage. It is observed that the EE for $d = 80\text{m}$ even decreases when ρ increases, because more 'bad' channels are likely to meet the QoS constraint for larger ρ and hence be involved, which is confirmed by the decreasing outage probability reported in Subfigure (c).

In Fig. 3, we give an insight into the proportionality condition of q_n by providing some examples of power allocation. We set Q_1 to infinity, therefore power allocation can only be constrained by Q_2, which corresponds to the 'macro cases' case 1 and case 2. It is observed that power allocation satisfies the proportionality condition for small ρ, where power allocation is not constrained by P_n. The antennas with the smallest $\frac{P_n \phi_n}{|h_n|^2}$ will first have their own power constraints being activated.

V. Conclusion

The EE maximization problem with multiple transmit antennas has been considered subject to constraints on total transmit power, per-antenna power and weighted power consumption at RF chains. The mathematical properties of the globally optimal solution were studied, which provides a deeper understanding of the structure of power allocation among antennas. Numerical results have been reported to demonstrate the effect of QoS on EE and also the structure of power allocation.

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