Université catholique de Louvain
Faculté des économiques, sociales, politiques et de communication
Ecole des sciences économiques

Essays on Economic Growth and Development

Nguyen Thang DAO

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Supervisor
Prof. Julio DÁVILA (Université catholique de Louvain)

Referees
Prof. David DE LA CROIX (Université catholique de Louvain)
Prof. Jean HINDRIKS (Université catholique de Louvain)
Prof. Cuong LE VAN (Paris School of Economics)

President of the jury
Prof. Mathieu PARENTI (Université catholique de Louvain)

To my BIG FAMILY, including a yet unborn generation.

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1 General introduction

This thesis consists of four main essays contributing to the theoretical literature of Economic Growth and Development covering issues of environmental externalities, optimal taxation, demography, gender inequality, endogenous technological progress, and comparative development, etc. in variations of overlapping generations modeling. The choice of overlapping generations modelling not only allows to study the long run growth of economies but also captures interactions of different generations of individuals. The first essay is devoted to *normative economics* considering overlapping generations economies with environmental externalities. The overlapping generations modeling in this chapter provides new implications about the role of tax and transfer policies. The remaining three essays are devoted to *positive economics* within the unified growth theory framework to explain many aspects of development process across societies. While the second essay studies an extreme case that societies can be locked in stagnation under specific geographical and environmental conditions, the two last essays study the convergences to modern sustained economic growth along with demographic transition.

In the first essay, we consider Diamond (1965) overlapping generations economies with environmental externalities in which pollution results from both consumption and production (which combines two main streams of assumptions about pollution in the literature of environmental economics). The environmental quality is a public good entering in the life-time utilities of agents. We compare the competitive equilibrium steady state with the benevolent social planner's optimal steady state. We show that the dynamical inefficiency of a competitive equilibrium steady state with capital-labor ratio exceeding the

golden rule ratio still holds. Moreover, the range of dynamically efficient steady state capital ratios increases with the effectiveness of the environmental maintenance technology, and decreases for more polluting production technologies. We propose four alternative tax and transfer policies that decentralize as a competitive outcome the transition to the social planner's steady state. These policies are: (1) *Taxes on consumption,* (2) *Taxes on consumption and capital income,* (3) *Taxes on consumption and production, and* (4) *Taxes on consumption, production and labor income.* Finally, we compare and discuss the implementation of these policies.

The second essay studies the important role of geographical and environmental conditions for the development process of a society. This essay extends the model in Galor and Weil (2000, AER), "*Technology, Population and Growth: From Malthusian Stagnation to Demographic Transition and Beyond*", by introducing geographical factors and the possibility of technology losses to make stand out how the interplay between available "land", its suitability for living and producing, and its degree of isolation, may lock a society in Malthusian stagnation and never take off. Specifically, we study the development of a society with initial conditions characterized by a small population, zero-education and low level of technology, and we characterize the set of geographical factors that prevent such a society from reaching the critical population size guaranteeing technological growth. As a consequence, the technological level will remain below the take-off threshold, locking the society at zero-education. Zero-education associated with small size of population cannot guarantee the technological progress needed to offset recurrent losses of technology, so that the society cannot expand its "productive land" to enhance fertility and reach a bigger population. This negative feedback loop prevents the society from escaping stagnation. The theory in this essay may be illustrated by the case of Tasmania island, where extreme losses of technology took place after it was separated from mainland Australia due to the rising sea level. With a stable population of 4.000, Tasmania had, at the time of arrival of Europeans, shown no sign of escaping stagnation

and had the simplest material culture and technology of any people in the modern world due to its isolation and small size of population.

The third essay studies the comparative development across societies by considering the differences in geographical advantage for agricultural production between societies. We will consider in this essay two-sector overlapping generations economies to explain the development from an agricultural regime, through industrialization, to a manufacturing regime occurring along with the demographic transition, technological progress, and the changing in labor structure over time towards an increasing fraction of skilled labor. We will show that the manufacturing sector requires a technological accumulation to be economically viable. And when the level of technology is sufficiently high, it becomes more complementary to skilled labor rather than to unskilled labor, so that households tend to raise less unskilled offspring in order to raise more skilled ones. We will see in this essay that the geographical advantage for agricultural production helps a society to be more prosperous in the agricultural regime, but delays the timing of industrialization as well as the timing of the demographic transition. Hence, a society with more geographical advantage for agriculture may be left behind by another with less geographical advantage for agriculture when technology accumulates enough. This essay may explain why Asian countries, particularly China and India, were ahead in GDP per-capita of European countries until 11th century, but the industrialization took place in Europe before Asia, making Europe overtake Asia.

The last essay sets up a unified growth model capturing issues of endogenous technological progress, gender inequality in education, fertility and the complex interaction between these issues to explain some common stylized facts of development processes across societies: (1) *the negative correlation between fertility and female-to-male education ratio;* (2) *the positive correlation between per-capita income and female-to-male education ratio;* (3) *the negative correlation between female-to-male earning ratio and fertility;* (4) *the decline over time in*

human capital gap and earning gap between male and female workers; (5) *the increase over time in labor-force participation of women,* and (6) *the demographic transition of many societies entering the regime of modern sustained economic growth.* There are three main components to the mechanism linking technology, gender inequality in education, and fertility: First, increases in the level of technology not only increase the return to human capital but also reduce, in traditional societies, women's time doing housework, leaving them with more time for child care and labor-force participation, because technological progress creates labor-saving products for housework. Second, the decrease in women's time devoted to housework in the future makes households today invest relatively less in education for their sons in order to invest more in education for their daughters because the marginal return to female education is greater than to male education, therefore improving the gender equality in education. Third, the better gender equality in education, in turn, accelerates the technological progress. This positive feedback loop generates a demographic transition accompanied by accelerated economic growth, firstly increasing in fertility and output simultaneously, followed by a decline in fertility along with sustained economic growth. The mechanism inducing a demographic transition is that, in the early stage of development when the level of technology is sufficiently low associated with low human capital and big relative gap between male and female human capital, the technological progress just helps women to save their time in doing housework, but does not help them supply their labor to the market until the level of technology is still below a given threshold because the opportunity cost for raising children is still low, which make households to increase the fertility. However, when the technology accumulates to a sufficiently high level (exceeding the threshold), improving significantly the gender equality in education and making the opportunity cost of raising children is rather high, then technological progress will make households decrease their fertility and improve the quality of their children.

So far, the first essay has been accepted for publication in the *Journal of Public Economic Theory* with title "Implementing steady state efficiency in overlapping generations economies with environmental externalities". The second essay has been published in the *Economics Letters* with the same title. The two last essays have been being revised in order to submit in refereed journals soon.

2 Optimal taxation in OLG economies with environmental externalities

coauthored with Julio Dávila

We consider in this chapter overlapping generations economies with pollution resulting from both consumption and production. The competitive equilibrium steady state is compared to the optimal steady state from the social planner's viewpoint. We show that the dynamical inefficiency of a competitive equilibrium steady state with capital-labor ratio exceeding the golden rule ratio still holds. Moreover, the range of dynamically efficient steady state capital ratios increases with the effectiveness of the environment maintenance technology, and decreases for more polluting production technologies. We characterize some tax and transfer policies that decentralize as a competitive equilibrium outcome the transition to the social planner's steady state.

Keywords: overlapping generations, environmental externality, tax and transfer policy.

2.1 Introduction

Environmental externalities have been studied in economies with overlapping generations since long. In particular, the effects of environmental externalities on dynamic inefficiency, productivity, health and longevity of agents have been addressed, as well as the policy interventions that may be needed. While in most papers pollution is assumed to come from production, and the environment is supposed to improve or degrade by itself at a constant rate (Gutiérrez 2008;

Jouvet et al. 2000; Jouvet, Pestieau and Ponthiere 2010; Marini and Scaramozzino 1995; Pautrel 2007), other papers assume that pollution comes from consumption (John and Pecchenino 1994; John et al. 1995; Ono 1996). As a consequence of the differing assumptions, the effect of environmental externalities on capital accumulation vary widely across papers. Specifically, John et al. (1995) showed that when only consumption pollutes, the economy accumulates less capital than what would be optimal. Conversely, Gutiérrez (2008) showed that when only production pollutes, the economy accumulates instead more capital than the optimal level. This is so because in John et al. (1995) agents pay taxes to maintain environment when young, so that an increased pollution reduces their savings; however, in Gutiérrez (2008) pollution increases health costs in old age, leading agents to save more to pay for them. The difference seems therefore to come from when the taxes are paid (when young or old) rather than from whether pollution comes from production or consumption. Another main difference between John et al. (1995) and Gutiérrez (2008) is their different assumptions about the ability of environment to recover from pollution. John et al. (1995) assumes that environment naturally degrades over time, while Gutiérrez (2008) assumes that environment recovers naturally.

This chapter aims at identifying the net impact of both production and consumption on environment by allowing for the two types of pollution simultaneously. Moreover, as in John et al. (1994, 1995), we assume that the environment degrades naturally over time at a constant rate and that young agents devote part of their income to maintain it.[1] In this setup, we characterize the range of dynamically inefficient capital-labor ratios. Next, we introduce taxes and transfer policies that decentralize the first-best steady state, and the transition to it, as a competitive equilibrium steady state.

The rest of the chapter is organized as follows. Section 2.2 introduces the model. Section 2.3 characterizes its competitive equilibria. Sec-

[1] In John et al. (1994, 1995), only the consumption of old agents pollutes, young agents do not consume. In Ono (1996), it is assumed that consumption of both young and old agents degrade the environment but with a period lag. Here, we assume also that consumptions of both old and young agents and production pollute without decay.

tion 2.4 presents the problem of the social planner, defines the efficient allocation with and without discounting, and characterizes the range of dynamically inefficient capital ratios (Proposition 2.1). The competitive equilibrium steady state and the planner's steady state are compared in Section 2.5, where we introduce some tax and transfer schemes that decentralize the planner's steady state and the transition to it as market outcome (from Proposition 2.2 to Proposition 2.5). Section 2.6 discusses the participation constraints of these policies. Section 2.7 discusses these policies and show conditions under which a policy is both preferred by agents (Proposition 2.6), as well as more easily implementable. Section 2.8 concludes the chapter.

2.2 The model

We consider the overlapping generations economy in Diamond (1965) with a constant population of identical agents. At each period t output can be produced out of capital and labor according to a constant returns to scale neoclassical production function $F(K_t, L_t)$. This production is assumed to satisfy the condition $F_K(k,1) + F_{KK}(k,1)k > 0$ for all $k > 0$ where $k = K/L$, which guarantees the existence of competitive equilibrium dynamics (see Appendix A2.3). Note that this property holds for all Constant Elasticity of Substitution (CES) production functions $F(K_t, L_t) = [\theta K_t^\rho + (1-\theta)L_t^\rho]^{1/\rho}$, $\theta \in (0,1)$, $\rho \in (-\infty, 1]$. Capital fully depreciates in each period. The representative firm maximizes profits solving under perfect competition

$$\max_{K_t, L_t \geq 0} F(K_t, L_t) - r_t K_t - w_t L_t$$

so that the rental rate of capital and wage rate are, in each period t, the marginal productivity of capital and labor respectively, i.e.

$$r_t = F_K(K_t, L_t) \tag{2.1}$$

$$w_t = F_L(K_t, L_t) \tag{2.2}$$

The size of each generation is normalized to one. Each agent lives two periods, say young and old. When young, an agent is endowed with one unit of labor which he supplies inelastically, so that $L_t = 1$ for all t, since population is constant. Agents born in period t divide their wage w_t between consumption when young c_0^t, investment in maintaining the environment m^t, and savings k^t lent to firms for a return rate r_{t+1} to be used in $t+1$ as capital, so that $K_{t+1} = k^t$ since population is normalized to 1. The return to savings $r_{t+1}k^t$ is used up as old age consumption. Agents born at date t have preferences over their consumptions when young and old $(c_0^t, c_1^t) \in \mathbb{R}_+^2$ and the environmental quality when old, $E_{t+1} \in \mathbb{R}$, represented by $u(c_0^t) + v(c_1^t) + \phi(E_{t+1})$ with u', v', $\phi' > 0$, u'', v'', $\phi'' < 0$, and $u'(0) = v'(0) = +\infty$, $u'(+\infty) = v'(+\infty) = 0$, $\phi'(+\infty) = 0$, $\phi'(-\infty) = +\infty$.[2]

Environmental quality evolves according to

$$E_{t+1} = (1-b)E_t - \alpha F(K_{t+1}, L_{t+1}) - \beta(c_0^{t+1} + c_1^t) + \gamma m^t \qquad (2.3)$$

for some $\alpha, \beta, \gamma > 0$ and $b \in (0, 1]$. That is to say, environmental quality converges autonomously to a natural level normalized to zero at a rate b that measures the speed of reversion to this level. Nonetheless, production and consumption degrade environmental quality by an amount $\alpha F(K_{t+1}, 1)$ and $\beta(c_0^{t+1} + c_1^t)$ respectively, while young agents can improve the environmental quality they will enjoy when old by an amount γm^t if they devote a portion m^t of their labor income to that

[2]Green preferences are necessary to capture the externality, and this can be achieved in several ways. Many papers in the literature introduce the stock of pollution as a public bad entering the utility function instead of environmental quality. Alternatively, pollution makes agents pay a health cost that reduces utility because of consuming less. In this chapter, following John et al. (1994, 1995), we introduce the environmental quality as a public good entering the utility function of agents. This approach to the impact of environment on choices of agents seems more adequate for several reasons. First, the health and utility of agents in fact depend on the quality of environment around rather than on the quantity of pollution. Second, from the dynamics of environmental quality in (2.3), environmental quality captures anyway the effect of pollution, and moreover not only the impact of current pollution and abatement activities but those in the past also. Third, without any activity the environmental quality converges (upgrades or depreciates itself) to a level normalized to zero; while for the alternative approach, environment just upgrades itself via a rate of decay of the stock of pollution, so that environment never degrades itself which prevents capturing the reversion to wildeness that may render unfit environment for human activity.

end.[3]

The life-time utility maximization problem of the representative agent is

$$\max_{\substack{c_0^t, c_1^t, k^t, m^t \geq 0 \\ E_t,\, E_{t+1}^e}} u(c_0^t) + v(c_1^t) + \phi(E_{t+1}^e) \tag{2.4}$$

subject to

$$c_0^t + k^t + m^t = w_t \tag{2.5}$$

$$c_1^t = r_{t+1} k^t \tag{2.6}$$

$$E_t = (1-b)E_{t-1} - \alpha F(k^{t-1}, 1) - \beta(c_0^t + c_1^{t-1}) + \gamma m^{t-1} \tag{2.7}$$

$$E_{t+1}^e = (1-b)E_t - \alpha F(K_{t+1}, 1) - \beta(c_0^{t+1,e} + c_1^t) + \gamma m^t \tag{2.8}$$

where E_{t+1}^e is, at t, the expected state of environment at $t+1$, given the expected consumption of the next generation young agent $c_0^{t+1,e}$ and E_{t-1}, c_1^{t-1}, k^{t-1}, m^{t-1}, w_t, r_{t+1}. Since the representative agent is assumed to be negligible within his own generation, he thinks of the impact of his savings k^t on aggregate capital K_{t+1} to be negligible as well, ignoring that actually $K_{t+1} = k^t$ at equilibrium. This assumption implies that he does not internalize the impact of the savings decision on environment via production. Notwithstanding, the agent considers the impact of his consumption and maintenance choices not to be negligible. This is meant to capture the idea that agents care actually, not on the global environment, but on the nearby environment on which their consumption and maintenance choices have a direct

[3] One can thus interpret environmental quality as including any characteristic that make the environment more apt for human life, like the cleanness of rivers and atmosphere, and the quality of soil or groundwater, etc. It also includes the state of forests, agricultural lands, parks and gardens, which left on their own naturally revert to wilderness, unless subject to regular maintenance.

One can consider the dynamics of environmental quality in a more general form, $E_{t+1} = (1-b)E_t - \left([\alpha F(K_{t+1}, L_{t+1})]^{\frac{1}{\rho}} + [\beta(c_0^{t+1} + c_1^t) - \gamma m^t]^{\frac{1}{\rho}} \right)^{\rho}$, where $\rho \in (-\infty, 1]$ is the elasticity of substitution between pollution from production and pollution from consumption after maintaining. However, the quantitative results and analysis do not change crucially.

impact. Anyway given that there is a single representative agent, at equilibrium local and global environment coincide. Since production is not in the hands of the agent (although he supplies the necessary capital through his savings), that he disregards his impact on environment through production is the natural assumption to make.

An interior optimal choice $(c_0^t, c_1^t, k^t, m^t, E_t, E_{t+1}^e)$ for agent t is therefore characterized by the first-order conditions

$$u'(c_0^t) - [\beta(1-b) + \gamma]\, \phi'(E_{t+1}^e) = 0 \tag{2.9}$$

$$v'(c_1^t) - \left[\beta + \frac{\gamma}{r_{t+1}}\right] \phi'(E_{t+1}^e) = 0 \tag{2.10}$$

$$c_0^t + k^t + m^t - w_t = 0 \tag{2.11}$$

$$c_1^t - r_{t+1} k^t = 0 \tag{2.12}$$

$$E_t - (1-b)E_{t-1} + \alpha F(k^{t-1}, 1) + \beta(c_0^t + c_1^{t-1}) - \gamma m^{t-1} = 0 \tag{2.13}$$

$$E_{t+1}^e - (1-b)E_t + \alpha F(K_{t+1}, 1) + \beta(c_0^{t+1,e} + c_1^t) - \gamma m^t = 0 \tag{2.14}$$

to be an implicit function of E_{t-1}, c_1^{t-1}, k^{t-1}, m^{t-1}, w_t, r_{t+1} and $c_0^{t+1,e}$ as long as the Jacobian matrix of the left-hand-side of the system above with respect to c_0^t, c_1^t, k^t, m^t, E_t, E_{t+1}^e is regular at the solution. The existence and regularity of the optimal solution is established in Appendix A2.1. For these FOCs to be not only necessary but also sufficient for the solution to be a maximum, the second order conditions (SOCs) are shown to hold at equilibrium in Appendix A2.2.

2.3 Competitive equilibria

The perfect foresight competitive equilibria are characterized by (i) the agent's utility maximization under the budget constraints, with correct expectations, (ii) the firms' profit maximization determining factors' prices, and (iii) the dynamics of environment. Therefore, a

competitive equilibrium allocation $\{c_0^t,\, c_1^t,\, k^t,\, m^t,\, E_{t+1}\}_t$ is solution to the system of equations

$$u'(c_0^t) - [\beta(1-b)+\gamma]\,\phi'(E_{t+1}) = 0 \tag{2.15}$$

$$v'(c_1^t) - \left[\beta + \frac{\gamma}{F_K(k^t,1)}\right]\phi'(E_{t+1}) = 0 \tag{2.16}$$

$$c_0^t + k^t + m^t - F_L(k^{t-1},1) = 0 \tag{2.17}$$

$$c_1^t - F_K(k^t,1)k^t = 0 \tag{2.18}$$

$$E_{t+1} - (1-b)E_t + \alpha F(k^t,1) + \beta(c_0^{t+1} + c_1^t) - \gamma m^t = 0 \tag{2.19}$$

Note that the feasibility of the allocation of resources is guaranteed by the agent's budget constraints (2.17) and (2.18), since at t

$$c_0^t + c_1^{t-1} + k^t + m^t = F_K(k^{t-1},1)k^{t-1} + F_L(k^{t-1},1) = F(k^{t-1},1)$$

The perfect foresight competitive equilibria of this economy follow a dynamics represented by a first-order difference equation, because of the regularity of the associated Jacobian matrix of the left hand side of the system of equations above with respect to $c_0^{t+1}, c_1^t, k^t, m^t, E_{t+1}$ (see Appendix A2.3).

A perfect foresight competitive equilibrium steady state, in particular, is a (c_0, c_1, k, m, E) solution to the system of equations

$$u'(c_0) - [\beta(1-b)+\gamma]\,\phi'(E) = 0 \tag{2.20}$$

$$v'(c_1) - \left[\beta + \frac{\gamma}{F_K(k,1)}\right]\phi'(E) = 0 \tag{2.21}$$

$$c_0 + k + m - F_L(k, 1) = 0 \tag{2.22}$$

$$c_1 - F_K(k, 1)k = 0 \tag{2.23}$$

$$bE + \alpha F(k, 1) + \beta(c_0 + c_1) - \gamma m = 0 \tag{2.24}$$

2.4 The social planner's choice with and without discounting

In this section, we consider the optimal allocation from the viewpoint of a social planner that allocates resources in order to maximize a weighted sum of the welfare of all current and future generations. The allocation selected by the social planner, which is optimal in the Pareto sense, is a solution to the problem

$$\max_{\{c_0^t, c_1^t, k^t, m^t, E_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \frac{1}{(1+R)^t} \left[u(c_0^t) + v(c_1^t) + \phi(E_{t+1})\right] \tag{2.25}$$

subject to, $\forall t = 0, 1, 2, ...,$

$$c_0^t + c_1^{t-1} + k^t + m^t = F(k^{t-1}, 1) \tag{2.26}$$

$$E_{t+1} = (1 - b)E_t - \alpha F(k^t, 1) - \beta(c_0^{t+1} + c_1^t) + \gamma m^t \tag{2.27}$$

given some initial conditions c_1^{-1}, k^{-1}, E_0, where $0 \leq R$ is the social planner's subjective discount rate.[4] The first constraint (2.26) of the problem is the resource constraint of the economy in period t requiring that the total output in that period is split into consumptions of the current young and old, savings for next period's capital, and environmental maintenance. The second constraint (2.27) is the dynamics of the environmental quality.

The social planner's choice of a steady state is a $(\bar{c}_0, \bar{c}_1, \bar{m}, \bar{k}, \bar{E})$ satisfying (see Appendix A2.4)

[4]The discount rate R is strictly positive when the social planner cares less about a generation's welfare the further away in the future that generation is, while R equals to zero when she cares about all generations equally, no matter how far in the future they may be.

$$u'(\bar{c}_0) = (1+R)\frac{\gamma+\beta(1+R)}{b+R}\phi'(\bar{E}) \tag{2.28}$$

$$v'(\bar{c}_1) = \frac{\gamma+\beta(1+R)}{b+R}\phi'(\bar{E}) \tag{2.29}$$

$$F_K(\bar{k},1) = \frac{\gamma(1+R)}{\gamma-(1+R)\alpha} \tag{2.30}$$

$$\bar{c}_0 + \bar{c}_1 + \bar{k} + \bar{m} = F(\bar{k},1) \tag{2.31}$$

$$b\bar{E} + \alpha F(\bar{k},1) + \beta(\bar{c}_0+\bar{c}_1) - \gamma\bar{m} = 0 \tag{2.32}$$

(the planner's discount rate R cannot be arbitrarily high for the optimal steady state to be characterized as above, specifically $\gamma > (1+R)\alpha$ needs to hold, which requires $\gamma > \alpha$, so that $F_K(\bar{k},1) > 0$). More specifically, in the case of the social planner caring about all generations equally, i.e. $R = 0$, the planer's steady state is the so-called golden rule steady state $\{c_0^*, c_1^*, k^*, m^*, E^*\}$ that maximizes the utility of the representative agent and is characterized by being a solution to the system

$$u'(c_0^*) = \frac{\gamma+\beta}{b}\phi'(E^*) \tag{2.33}$$

$$v'(c_1^*) = \frac{\gamma+\beta}{b}\phi'(E^*) \tag{2.34}$$

$$F_K(k^*,1) = \frac{\gamma}{\gamma-\alpha} \tag{2.35}$$

$$c_0^* + c_1^* + k^* + m^* = F(k^*,1) \tag{2.36}$$

$$bE^* + \alpha F(k^*,1) + \beta(c_0^*+c_1^*) - \gamma m^* = 0 \tag{2.37}$$

Note that, from (2.33) and (2.34), the marginal utility of consumption of the young agent must equal that of the consumption of the old agent.

Diamond (1965) shows that in the standard OLG model without pollution externalities, a competitive equilibrium steady state whose capital per worker exceeds the golden rule level is dynamically inefficient. In this chapter, we consider instead an economy with pollution externalities coming from both production and consumption, in which the environment degrade itself over time, and where the quality of the environment can be improved through maintenance. It turns out that, as in Diamond (1965), the golden rule capital ratio of this economy with pollution externalities is still the highest level of capital ratio that is dynamically efficient.[5]

> **Proposition 2.1**: *In a Diamond (1965) overlapping generations economy with consumption and production pollution, for an efficient enough cleaning technology, compared to the marginal polluting impact of production (specifically, for $\gamma > \alpha$ in the model), the golden rule capital ratio (i.e. the planner's steady state choice without discounting) is the highest dynamically efficient capital ratio.*

Proof: Since $F_{KK}(k,1) < 0$ for all k, the planner's optimal capital ratio $\bar{k}$ is implicitly defined to be a differentiable function $\bar{k}(R)$ of R by the condition

$$F_K(\bar{k},1) = \frac{\gamma(1+R)}{\gamma-(1+R)\alpha}$$

whose derivative, by the implicit function theorem, is

$$\bar{k}'(R) = \frac{1}{F_{KK}(\bar{k}(R),1)}\left(\frac{\gamma}{\gamma-(1+R)\alpha}\right)^2 < 0$$

[5] In a different framework and when pollution externalities are large enough, Gutiérrez (2008) has shown the existence of dynamically efficient competitive equilibrium steady state capital ratios that exceed the golden rule capital ratio. Specifically, when (i) pollution externalities come only from production; (ii) environment recovers itself at a constant rate; (iii) no resource is devoted to maintaining the environment; and (iv) the pollution externality decreases the utility of the agents only indirectly by requiring each agent to pay for extra health costs in the old age, Gutiérrez (2008) shows the existence of a "super golden rule" level of capital ratio (beyond the golden rule level) such that any stationary capital ratio exceeding this level is necessarily dynamically inefficient.

So, $\bar{k}$ is decreasing in R. Hence, $\bar{k}(R)$ is maximal when $R = 0$, which is corresponds to the golden rule level of capital k^*. Q.E.D.

Proposition 2.1 shows that any steady state capital ratio exceeding k^* is dynamically inefficient. From (2.35) the golden rule capital ratio k^* is decreasing in the production pollution parameter α. It is, however, increasing in the environment maintaining technology γ. Hence, the more polluting is production, the smaller the range of steady state allocations that are dynamically efficient for some discount factor R. Similarly, the more effective is the environment maintenance technology, the bigger the range of steady state allocations that are dynamically efficient for some discount factor R.

2.5 Policy implementation of the planner's optimal steady state

In this section, we provide tax and transfer policies allowing to implement the planner's optimal steady state. The participation constraints of these policies will be discussed in Section 2.6. Ono (1996) and Gutiérrez (2008) introduced also tax and transfer schemes to decentralize the golden rule steady state in the context of the pollution externalities they consider (from consumption and production only, respectively). However, their schemes uphold the golden rule once the economy is already at that steady state. In this section we provide instead policies that lead the economy towards the social planer's first best steady state and will keep it there once reached. The policies fulfill this in two stages. In the first stage (in the period $t-1$), taxes and transfers are set in order to make the agent born in period $t-1$ choose $c_0^{t-1} = c_0^*, c_1^{t-1} = c_1^*, k^{t-1} = k^*$ and $E_t^e = E^*$ (but not $m^{t-1} = m^*$ or $E_{t-1} = E^*$). For the sake of avoiding unnecessary cumbersome notation the argument is presented for the case $R = 0$, although it can be rewritten for any $R \geq 0$). Then, in the second stage, taxes and transfers are reset to uphold the planner's steady state from period t onward. The first scheme based on the taxation of consumption is presented next in detail. The subsequent schemes work analogously.

2.5.1 Taxes on consumptions

As in Ono (1996) we consider first taxes on consumption along with lump-sum taxes and transfers. Letting τ_0^t and τ_1^t be the tax rate on agent t's consumption when young and old respectively, T_0^t a lump-sum tax (if positive) levied on agent t's income when young, and T_1^t a lump-sum transfer (if positive) to the same agent when old at date $t+1$, the problem of agent t is then

$$\max_{\substack{c_0^t, c_1^t, k^t, m^t \geq 0 \\ E_t, E_{t+1}^e}} u(c_0^t) + v(c_1^t) + \phi(E_{t+1}^e) \tag{2.38}$$

subject to

$$(1+\tau_0^t)c_0^t + k^t + m^t = w_t - T_0^t \tag{2.39}$$

$$(1+\tau_1^t)c_1^t = r_{t+1}k^t + T_1^t \tag{2.40}$$

$$E_t = (1-b)E_{t-1} - \alpha F(k^{t-1}, 1) - \beta(c_0^t + c_1^{t-1}) + \gamma m^{t-1} \tag{2.41}$$

$$E_{t+1}^e = (1-b)E_t - \alpha F(K_{t+1}, 1) - \beta(c_0^{t+1,e} + c_1^t) + \gamma m^t \tag{2.42}$$

given E_{t-1}, c_1^{t-1}, k^{t-1}, m^{t-1}, w_{t-1}, $c_0^{t+1,e}$ and r_{t+1}. Note again that in equation (2.42), the agent, being negligible within his generation, ignores the fact that $K_{t+1} = k^t$ and hence is unable to internalize the effect of the savings decisions on environment through the aggregate output.

Proposition 2.2: *In a Diamond (1965) overlapping generations economy with pollution from both consumption and production, the planner's steady state can be implemented at a given period t by the following period-by-period balanced-budget policy:*

announce at $t-1$ that the following consumption tax rates and lump-sum transfers will apply,

(1) *to generation born at $t-1$*

$$\tau_0^{t-1} = \frac{\gamma + \beta - [\gamma + \beta(1-b)]\, b}{\gamma b}$$

$$\tau_1^{t-1} = \frac{\gamma + \beta(1-b)}{(\gamma - \alpha)b} - 1$$

$$T_0^{t-1} = \frac{1}{\gamma + \beta(1-b)} \begin{pmatrix} \gamma \\ 1-b \\ -1 \end{pmatrix}'$$

$$\begin{pmatrix} F_L(k^{t-2}, 1) - (1+\tau_0)c_0^* - k^* \\ (1-b)E_{t-2} - \alpha F(k^{t-2}, 1) - \beta((1+\tau_0)c_0^* + c_1^{t-2}) + \gamma m^{t-2} \\ E^* + \alpha F(k^*, 1) + \beta(c_0^* + c_1^*) \end{pmatrix}$$

$$T_1^{t-1} = (1+\tau_1)c_1^* - F_K(k^*, 1)k^*$$

(2) *to generation born from t onwards*

$$\tau_0 = \frac{\gamma + \beta - [\gamma + \beta(1-b)]\, b}{\gamma b}$$

$$\tau_1 = \frac{\gamma + \beta(1-b)}{(\gamma - \alpha)b} - 1$$

$$T_0 = F_L(k^*, 1) - (1+\tau_0)c_0^* - k^* - m^*$$

$$T_1 = (1+\tau_1)c_1^* - F_K(k^*, 1)k^*$$

Proof: See Appendix A2.5.

2.5.2 Taxes on consumptions and capital income

In the section 2.5.1, we introduced taxes on consumptions in which the tax rates differ between consumptions of the old and the young. In reality, however, this tax scheme seems to be difficult to apply because it discriminates between young and old agents. In order to avoid the discrimination, a unique rate of consumption tax τ^t should be applied. Beside that, a capital income tax τ_k^t and a system of lump-sum tax T_0^t (if positive) and lump-sum transfer T_1^t (if positive), levied on agent t's incomes, are introduced to show that the best steady state allocation can be achieved. The problem of agent t under the tax policy is then

$$\max_{\substack{c_0^t, c_1^t, k^t, m^t \geq 0 \\ E_t, E_{t+1}^e}} u(c_0^t) + v(c_1^t) + \phi(E_{t+1}^e) \tag{2.43}$$

subject to

$$(1+\tau^t)c_0^t + k^t + m^t = w_t - T_0^t \tag{2.44}$$

$$(1+\tau^t)c_1^t = (1-\tau_k^t)r_{t+1}k^t + T_1^t \tag{2.45}$$

$$E_t = (1-b)E_{t-1} - \alpha F(k^{t-1}, 1) - \beta(c_0^t + c_1^{t-1}) + \gamma m^{t-1} \tag{2.46}$$

$$E_{t+1}^e = (1-b)E_t - \alpha F(K_{t+1}, 1) - \beta(c_0^{t+1,e} + c_1^t) + \gamma m^t \tag{2.47}$$

given E_{t-1}, c_1^{t-1}, k^{t-1}, m^{t-1}, w_t, $c_0^{t+1,e}$ and r_{t+1}. Note again that in equation (2.47), the agent, being negligible within his generation, ignores the fact that $K_{t+1} = k^t$ and hence is unable to internalize the effect of the savings decisions on environment through the aggregate output.

Proposition 2.3: *In a Diamond (1965) overlapping generations economy with pollution from both consumption and production, the planner's steady state can be implemented at a given period t by the following period-by-period balanced-budget policy:*

announce at $t-1$ that the following consumption tax rate, capital income tax rate and lump-sum transfers will apply,

(1) *to generation born at $t-1$*

$$\tau^{t-1} = \frac{\gamma + \beta - [\gamma + \beta(1-b)]\, b}{\gamma b}$$

$$\tau_k^{t-1} = \frac{\alpha(\gamma+\beta) - \beta b^2(\gamma - \alpha) - \alpha\beta b}{\gamma\,[\gamma + \beta(1-b)]}$$

$$T_0^{t-1} = \frac{1}{\gamma + \beta(1-b)} \begin{pmatrix} \gamma \\ 1-b \\ -1 \end{pmatrix}'$$

$$\begin{pmatrix} F_L(k^{t-2},1) - (1+\tau)c_0^* - k^* \\ (1-b)E_{t-2} - \alpha F(k^{t-2},1) - \beta((1+\tau)c_0^* + c_1^{t-2}) + \gamma m^{t-2} \\ E^* + \alpha F(k^*,1) + \beta(c_0^* + c_1^*) \end{pmatrix}$$

$$T_1^{t-1} = (1+\tau)c_1^* - (1-\tau_k)F_K(k^*,1)k^*$$

(2) *to all generations born from period t onwards*

$$\tau = \frac{\gamma + \beta - [\gamma + \beta(1-b)]\, b}{\gamma b}$$

$$\tau_k = \frac{\alpha(\gamma+\beta) - \beta b^2(\gamma - \alpha) - \alpha\beta b}{\gamma\,[\gamma + \beta(1-b)]}$$

$$T_0 = F_L(k^*,1) - (1+\tau)c_0^* - k^* - m^*$$

$$T_1 = (1+\tau)c_1^* - (1-\tau_k)F_K(k^*,1)k^*$$

Proof: The proof for this proposition is similar to the proof for proposition 2.2. Q.E.D.

2.5.3 Taxes on consumptions and production

We still keep the non-discriminatory tax rate τ^t on consumptions and the system of lump-sum tax T_0^t (if positive) and lump-sum transfer T_1^t (if positive). However, we now introduce a Pigouvian tax on production instead of tax on capital income. In any period t, let τ_p^t be the tax paid by firms per one unit of output produced in period t. We will design taxes and transfers policy ensuring the government's budget to be balanced and achieving the planner's steady state through competitive markets.

Under the production tax, the problem that the firm must solve in period t is

$$\max_{K_t} \quad (1-\tau_p^t)F(K_t,1) - r_t K_t - w_t \tag{2.48}$$

The returns to capital and labor are at equilibrium respectively

$$r_t = (1-\tau_p^t)F_K(k^{t-1},1) \tag{2.49}$$

$$w_t = (1-\tau_p^t)F_L(k^{t-1},1) \tag{2.50}$$

Under the taxes and transfers policy, the agent t's problem is

$$\max_{\substack{c_0^t, c_1^t, k^t, m^t \geq 0 \\ E_t,\, E_{t+1}^e}} u(c_0^t) + v(c_1^t) + \phi(E_{t+1}^e) \tag{2.51}$$

subject to

$$(1+\tau^t)c_0^t + k^t + m^t = w_t - T_0^t \tag{2.52}$$

$$(1+\tau^t)c_1^t = r_{t+1}k^t + T_1^t \tag{2.53}$$

$$E_t = (1-b)E_{t-1} - \alpha F(k^{t-1},1) - \beta(c_0^t + c_1^{t-1}) + \gamma m^{t-1} \tag{2.54}$$

$$E_{t+1}^e = (1-b)E_t - \alpha F(K_{t+1},1) - \beta(c_0^{t+1,e} + c_1^t) + \gamma m^{t-1} \tag{2.55}$$

given E_{t-1}, c_1^{t-1}, k^{t-1}, m^{t-1}, w_t, $c_0^{t+1,e}$ and r_{t+1}. Note again that in equation (2.55), the agent, being negligible within his generation, ig-

nores the fact that $K_{t+1} = k^t$ and hence is unable to internalize the effect of the savings decisions on environment through the aggregate output.

Proposition 2.4: *In a Diamond (1965) overlapping generations economy with pollution from both consumption and production, the planner's steady state can be implemented at a given period t by the following period-by-period balanced-budget policy:*

announce at $t-1$ that the following consumption tax rate, production tax rates and lump-sum transfers will apply,

(1) *to generation born at $t-1$*

$$\tau^{t-1} = \frac{\gamma + \beta - [\gamma + \beta(1-b)]\, b}{\gamma b}$$

$$\tau_p^t = \frac{\alpha(\gamma+\beta) - \beta b^2(\gamma - \alpha) - \alpha\beta b}{\gamma\,[\gamma + \beta(1-b)]}$$

$$T_0^{t-1} = \frac{1}{\gamma + \beta(1-b)} \begin{pmatrix} \gamma \\ 1-b \\ -1 \end{pmatrix}'$$

$$\begin{pmatrix} F_L(k^{t-2}, 1) - (1+\tau)c_0^* - k^* \\ (1-b)E_{t-2} - \alpha F(k^{t-2}, 1) - \beta((1+\tau)c_0^* + c_1^{t-2}) + \gamma m^{t-2} \\ E^* + \alpha F(k^*, 1) + \beta(c_0^* + c_1^*) \end{pmatrix}$$

$$T_1^{t-1} = (1+\tau)c_1^* - (1-\tau_p)F_K(k^*, 1)k^*$$

(2) *to all generations born from period t onwards*

$$\tau = \frac{\gamma + \beta - [\gamma + \beta(1-b)]\, b}{\gamma b}$$

$$\tau_p = \frac{\alpha(\gamma+\beta) - \beta b^2(\gamma - \alpha) - \alpha\beta b}{\gamma\,[\gamma + \beta(1-b)]}$$

$$T_0 = (1-\tau_p)F_L(k^*,1) - (1+\tau)c_0^* - k^* - m^*$$

$$T_1 = (1+\tau)c_1^* - (1-\tau_p)F_K(k^*,1)k^*$$

Proof: The proof for this proposition is similar to the proof for proposition 2.2. Q.E.D.

2.5.4 Taxes on consumption, production and labor income

We now modify the tax and transfer policy introduced in section 2.5.3 by using the labor income tax rate τ_w^t to replace the lump-sum tax T_0^t. All other things are kept the same as in the section 2.5.3. Under this policy, the agent t's problem is

$$\max_{\substack{c_0^t, c_1^t, k^t, m^t \geq 0 \\ E_t, E_{t+1}^e}} u(c_0^t) + v(c_1^t) + \phi(E_{t+1}^e) \tag{2.56}$$

subject to

$$(1+\tau^t)c_0^t + k^t + m^t = (1-\tau_w^t)w_t \tag{2.57}$$

$$(1+\tau^t)c_1^t = r_{t+1}k^t + T_1^t \tag{2.58}$$

$$E_t = (1-b)E_t - \alpha F(k^{t-1},1) - \beta(c_0^t + c_1^{t-1}) + \gamma m^{t-1} \tag{2.59}$$

$$E_{t+1}^e = (1-b)E_t - \alpha F(K_{t+1},1) - \beta(c_0^{t+1,e} + c_1^t) + \gamma m^t \tag{2.60}$$

given E_{t-1}, c_1^{t-1}, k^{t-1}, m^{t-1}, w_t, $c_0^{t+1,e}$ and r_{t+1}. Note again that in equation (2.60), the agent, being negligible within his generation, ignores the fact that $K_{t+1} = k^t$ and hence is unable to internalize the effect of the savings decisions on environment through the aggregate output.

Proposition 2.5: *In a Diamond (1965) overlapping generations economy with pollution from both consumption and production, the planner's steady state can be implemented at a given period t by the following period-by-period balanced-budget policy:*

announce at $t-1$ that the following consumption tax rate, production tax rate, labor tax rate and lump-sum transfer will apply,

(1) *to generation born at $t-1$*

$$\tau^{t-1} = \frac{\gamma + \beta - [\gamma + \beta(1-b)]\, b}{\gamma b}$$

$$\tau_p^t = \frac{\alpha(\gamma+\beta) - \beta b^2(\gamma - \alpha) - \alpha\beta b}{\gamma\,[\gamma + \beta(1-b)]}$$

$$\tau_w^{t-1} = \frac{1}{B} \begin{pmatrix} \gamma \\ 1-b \\ -1 \end{pmatrix}' \begin{pmatrix} F_L(k^{t-2},1) - (1+\tau)c_0^* - k^* \\ (1-b)E_{t-2} - \alpha F(k^{t-2},1) - \beta((1+\tau)c_0^* + c_1^{t-2}) + \gamma m^{t-2} \\ E^* + \alpha F(k^*,1) + \beta(c_0^* + c_1^*) \end{pmatrix}$$

$$T_1^{t-1} = (1+\tau)c_1^* - (1-\tau_p)F_K(k^*,1)k^*$$

where $B = [\gamma + \beta(1-b)]\, F_L(k^{t-2},1)$.[6]

(2) *to all generations born from period t onwards*

$$\tau = \frac{\gamma + \beta - [\gamma + \beta(1-b)]\, b}{\gamma b}$$

[6]Note that this denominator differs from the corresponding denominator in Proposition 4 by a factor $F_L(k^{t-2},1)$. This is obvious since τ_w^{t-1} is the tax rate on labor income while T_0^{t-1} is a lump-sum tax on income.

$$\tau_p = \frac{\alpha(\gamma+\beta) - \beta b^2(\gamma-\alpha) - \alpha\beta b}{\gamma\left[\gamma + \beta(1-b)\right]}$$

$$\tau_w = 1 - \frac{(1+\tau)c_0^* + k^* + m^*}{(1-\tau_p)F_L(k^*, 1)}$$

$$T_1 = (1+\tau)c_1^* - (1-\tau_p)F_K(k^*, 1)k^*$$

Proof: The proof for this proposition is similar to the proof for proposition 2.2. Q.E.D.

2.6 Participation constraint - A sufficient condition

In this section, we discuss about the participation constraints of tax and transfer schemes. We will provide a sufficient condition under which there always exist a period $t-1$ onwards such that these schemes can be implemented. Let us focus on the sufficient condition for the first scheme, say taxes on consumptions. Because of the equivalence of the schemes introduced in the Section 2.5, then this sufficient condition also holds for the subsequent schemes.

Note that, in the first stage of taxation we need the young agent chooses his young consumption and savings at c_0^* and k^*, respectively. So the participation constraint for the taxes on consumptions in the first stage of taxation, say period $t-1$, is

$$T_0^{t-1} \leq F_L(k^{t-2}, 1) - (1+\tau_0)c_0^* - k^*$$

We know that without any taxation scheme, the economy converges to a steady state characterized by (c_0, c_1, k, m, E) satisfying the system from (2.20) to (2.24) as stated in the Section 2.3. If the policy

taxes on consumptions in the subsection 2.5.1 can conduct the economy from competitive steady state to the social planner's steady state, then there always exists a period t that such a policy can be implemented. Now suppose that at period $t-1$ the economy is being at the competitive characterized by (c_0, c_1, k, m, E), then the participation constraint becomes

$$T_0^{t-1} \leq F_L(k,1) - (1+\tau_0)c_0^* - k^*$$

That is to say

$$\frac{1}{\gamma + \beta(1-b)} \begin{pmatrix} \gamma \\ 1-b \\ -1 \end{pmatrix}'$$

$$\begin{pmatrix} F_L(k,1) - (1+\tau_0)c_0^* - k^* \\ (1-b)E - \alpha F(k,1) - \beta((1+\tau_0)c_0^* + c_1) + \gamma m \\ E^* + \alpha F(k^*,1) + \beta(c_0^* + c_1^*) \end{pmatrix}$$

$$\leq F_L(k,1) - (1+\tau_0)c_0^* - k^* \tag{2.61}$$

Note that $bE = \gamma m - \alpha F(k,1) - \beta(c_0 + c_1)$ and $bE^* = \gamma m^* - \alpha F(k^*,1) - \beta(c_0^* + c_1^*)$, then (2.61) is equivalent to

$$\begin{pmatrix} \gamma \\ 1-b \\ -1 \end{pmatrix}' \begin{pmatrix} F_L(k,1) - (1+\tau_0)c_0^* - k^* \\ E - \beta((1+\tau_0)c_0^* - c_0) \\ (1-b)E^* + \gamma m^* \end{pmatrix}$$

$$\leq [\gamma + \beta(1-b)] [F_L(k,1) - (1+\tau_0)c_0^* - k^*]$$

$$\Leftrightarrow \ (1-b)(E + \beta c_0) - (1-b)E^* - \gamma m^* \leq \beta(1-b) [F_L(k,1) - k^*]$$

i.e.

$$\frac{E - E^*}{\beta} \leq F_L(k,1) - k^* + c_0 + \frac{\gamma m^*}{\beta(1-b)} \tag{2.62}$$

Lemma 2.1: $E < E^*$.

Proof: We know that the competitive steady state differs from the social planner's steady state, then it holds

$$u(c_0) + v(c_1) + \phi(E) < u(c_0^*) + v(c_1^*) + \phi(E^*) \tag{2.63}$$

We prove the Lemma 2.1 by a contradiction. In effect, suppose that $E \geq E^*$, then

$$\phi'(E) \leq \phi'(E^*)$$

We know that

$$u'(c_0) = [\beta(1-b) + \gamma]\,\phi'(E) \qquad \text{and} \qquad u'(c_0^*) = \tfrac{\gamma+\beta}{b}\phi'(E^*)$$
$$v'(c_1) = \left[\beta + \tfrac{\gamma}{F_K(k,1)}\right]\phi'(E) \qquad \text{and} \qquad v'(c_1^*) = \tfrac{\gamma+\beta}{b}\phi'(E^*)$$

Note that the gross return of capital at the competitive steady state should is greater than or equal to 1, i.e. $F_K(k,1) \geq 1$, otherwise agents will not have incentives to lent their savings to the firms. Since $u(\cdot)$ and $v(\cdot)$ are increasing and concave functions in $\mathbb{R}_+$, and $\beta(1-b)+\gamma < \frac{\gamma+\beta}{b}$ and $\beta + \frac{\gamma}{F_K(k,1)} < \frac{\gamma+\beta}{b}$, then $c_0 > c_0^*$ and $c_1 > c_1^*$ which lead to

$$u(c_0) + v(c_1) + \phi(E) > u(c_0^*) + v(c_1^*) + \phi(E^*)$$

contradicting to (2.63). Therefore, $E < E^*$. Q.E.D.

From the Lemma 2.1, the left hand side of the participation constraint (2.62) is strictly negative. Now we state in the Lemma 2.2 below the sufficient condition under which the taxes on consumptions can be implemented, i.e. the participation constraint (2.62) always holds.

Lemma 2.2: *If* $\frac{(\gamma-\beta)\beta}{\alpha\gamma} \leq \frac{1}{1-b}$, *then there always exists a period* $t-1$ *onwards that such the taxes on consumptions policy stated in the Proposition 2.2 can be implemented.*

Proof: We prove that under the condition $\frac{(\gamma-\beta)\beta}{\alpha\gamma} \leq \frac{1}{1-b}$, the right hand side of the participation constraint (2.62) is always nonnegative, i.e. the participation constraint holds. The right hand side of (2.62) is nonnegative if

$$k^* \leq F_L(k,1) + c_0 + \frac{\gamma m^*}{(1-b)\beta} \tag{2.64}$$

In effect, we will prove (2.64) by a contradiction. Suppose that $k^* > F_L(k,1) + c_0 + \frac{\gamma m^*}{(1-b)\beta}$, then we have

$$\begin{aligned}
bE^* &= \gamma m^* - \alpha F(k^*,1) - \beta(c_0^* + c_1^*) \\
&= \gamma m^* - \alpha(c_0^* + c_1^* + k^* + m^*) - \beta(c_0^* + c_1^*) \\
&= (\gamma-\beta)m^* - (\alpha+\beta)(c_0^* + c_1^* + m^*) - \alpha k^* \\
&< (\gamma-\beta)m^* - (\alpha+\beta)(c_0^* + c_1^* + m^*) - \alpha\left[F_L(k,1) + c_0 + \frac{\gamma m^*}{(1-b)\beta}\right] \\
&= \left[\gamma - \beta - \frac{\alpha\gamma}{(1-b)\beta}\right] m^* - (\alpha+\beta)(c_0^* + c_1^* + m^*) - \alpha\left[F_L(k,1) + c_0\right]
\end{aligned}$$

and we know that

$$\begin{aligned}
bE &= \gamma m - \alpha F(k,1) - \beta(c_0 + c_1) \\
&= (\gamma-\beta)m - (\alpha+\beta)(c_0 + c_1 + m) - \alpha k
\end{aligned}$$

Since $k^* > F_L(k,1) + c_0 + \frac{\gamma m^*}{(1-b)\beta} \Rightarrow k^* > k$, and $F(k^*,1) - k^* > F(k,1) - k$ because $\forall \tilde{k} < k^*$ it holds $F_K(\tilde{k},1) - 1 > F_K(k^*,1) - 1 = \frac{\alpha}{\gamma-\alpha} > 0$. That is to say

$$c_0^* + c_1^* + m^* > c_0 + c_1 + m$$

From the Lemma 2.1, $E^* > E$, then it is straightforward that $m^* > m$. So if $\frac{(\gamma-\beta)\beta}{\alpha\gamma} \leq \frac{1}{1-b}$, i.e. $\gamma - \beta - \frac{\alpha\gamma}{(1-b)\beta} \leq 0$, then

$$\left[\gamma - \beta - \frac{\alpha\gamma}{(1-b)\beta}\right] m^* - (\alpha+\beta)(c_0^* + c_1^* + m^*) - \alpha\left[F_L(k,1) + c_0\right]$$
$$< (\gamma-\beta)m - (\alpha+\beta)(c_0 + c_1 + m) - \alpha k$$

i.e.

$$bE^* < bE \quad \Leftrightarrow \quad E^* < E$$

which contradicts to the Lemma 2.1.

Therefore, $k^* \leq F_L(k,1) + c_0 + \frac{\gamma m^*}{(1-b)\beta}$, implying the participation constraint (2.62) holds. Q.E.D.

2.7 Discussion of policies

The four alternative tax and transfer policies analyzed above conduct the economy to the same outcome, the social optimum, through the same Pareto-improving path. So from a welfare point of view, the agents are indifferent between the four policies. However, the agents pay lower taxes and/or receive higher transfers under some of the policies, compared to the others, which may make them likelier to be voted for and implemented. In our setting, the tax and transfer policy is announced at the beginning of period $t-1$. Therefore, we will analyze the taxes and transfers that agents born in period $t-1$ pay and receive, respectively, as well as those for agents born from period t onwards.

For convenience of notation, we shall denote by subscripts 1, 2, 3, and 4 respectively the policies "Taxes on consumptions", "Taxes on consumptions and capital income", "Taxes on consumptions and production", and "Taxes on consumptions, production, and labor income".

The following proposition summarizes how the policies compare in terms of net taxes and transfer they generate from period $t-1$ onwards.

> **Proposition 2.6:** *In a Diamond (1965) overlapping generations economy with pollution from both consumption and production, if at period* $t-1$ *one of the tax and transfer policies is announced, then*
>
> (1) *in period* $t-1$*, young agents pay the same amount of taxes under the four policies above. Hence, old agents in period* $t-1$ *receive the same amount of transfers under these policies.*
>
> (2) *from period* t *onwards, all agents are equally treated under the policies* 1 *and* 2*; and they are also equally treated under the policies* 3 *and* 4*, specifically*
>
> (2a) *If* $\alpha(\gamma+\beta)-\beta b^2(\gamma-\alpha)-\alpha\beta b > (<)0$*, so that* $\tau_p > (<)0$*, then they pay less (more) taxes when young and receive more (less) net transfers when old with policies* 3 *and* 4 *than with the two other.*
>
> (2b) *If* $\alpha(\gamma+\beta)-\beta b^2(\gamma-\alpha)-\alpha\beta b = 0$*, so that* $\tau_p = 0$ *and* $\tau_k = 0$*, then policies* 1*,* 2*, and* 3 *coincide, and agents are equally treated by all four policies.*

Proof: See Appendix A2.6.

According to Proposition 2.6, which policy would be voted for by the agents depends on the condition guaranteeing the production tax rate τ_p to be positive, or negative, or zero. When $\alpha(\gamma+\beta)-\beta b^2(\gamma-\alpha)-\alpha\beta b > 0$, the Proposition 2.6 shows that the policies 3 and 4 will likely be preferred by the agents since they pay directly less taxes when young and receive more net transfer when old with these policies than with the others. That is because under policies 3 and 4, firms pay part of the taxes while under the others they do not, and agents may not perceive taxes paid by the firm as undistributed income. In fact, when firms pay taxes, wages decrease. Actually, each agent being

negligible takes into account only how taxes paid directly affect his/her budget constraints, disregarding how taxes paid by firms affect his/her budget constraints. Therefore, they will likely vote for policies 3 and 4. Now, from the policy maker's point of view, the only difference between policies 3 and 4 is the labor income tax versus the lump-sum tax raised on the income of the young agents. For the same amount of tax, the labor income tax seems more easily implementable because of being proportional on the agents' wages while the policy maker may have to set a heterogeneous lump-sum tax applying to heterogeneous agents in reality.

In the case $\alpha(\gamma + \beta) - \beta b^2(\gamma - \alpha) - \alpha\beta b < 0$, agents may prefer policies 1 and 2, since under policies 3 and 4, agents pay a higher tax when young and receive a lower transfer when old to subsidize for production. Indeed, being negligible, agents cannot internalize that their paid tax subsidize production that will increase wages making the policies equivalent. From the policy maker's point of view, policy 2 may be more easily implementable than policy 1, since policy 1 requires discriminating between young and old consumers about tax rates on consumption, the implementation of which may be costly. For policy 2, however, the policy maker applies a single consumption tax rate to both young and the old, along with a capital income tax on the returns to savings lent to firms via the banking system, and hence easy to collect at a low cost.

For the case $\alpha(\gamma+\beta)-\beta b^2(\gamma-\alpha)-\alpha\beta b = 0$, agents are equally treated by all four policies. Moreover, under this condition the policies 1, 2, and 3 exactly coincide. In this case, as discussed above, from the policy maker's point of view the policy 4 is likely to be more easily implementable than the others.

In this chapter, we mention in the policy 2 to tax on capital income combining with tax on consumption in decentralizing the planner's steady state as a competitive equilibrium steady state. That is to say, there is a room for the capital income tax to improve the social welfare. As a matter of fact, the room for taxation of capital income are still controversial in the literature. The arguments against

the capital income tax include Diamond and Mirrlees (1971a, 1971b), Chamley (1986), Judd (1987) and others. The arguments favor in this tax can be found in Aiyagari (1995), Chamley (2001), and recently in Dávila (2012), and others. Dávila (2012) shows that the conclusions for *whether capital income tax is good or bad* depend on the way framing the models in which the question is address, say overlapping generations models or neoclassical growth model. Basically, in the neoclassical growth model the resources constraint of the social planner is exactly the market constraint faced by agents, so that the market equilibrium allocation is Pareto efficient. But in the framework of overlapping generations model there is a difference between resources constraints faced by the agents and the social planner, hence the competitive equilibrium allocation is typically not Pareto efficient without the adequate interventions. The setup in this chapter which is an overlapping generations model, indeed, differs from the setups in Diamond and Mirrlees (1971a, 1971b), Chamley (1986), and others in that aspect. Therefore, as mentioned in Dávila (2012), there is a rationale for capital income taxation to be applied to improve the social welfare in our framework.

2.8 Conclusion

We have presented a general equilibrium overlapping generations model with environmental externalities from both production and consumption. For such a model we proved that the competitive equilibrium steady state is not the planner's steady state, for any discount rate the social planner may use. The pollution externality from consumption does not affect the range of dynamically inefficient capital ratios, whereas the pollution externality from production does. The higher the production pollution parameter α, the larger the inefficient range. The environment maintaining technology γ also plays a role in determining the best steady state capital ratio k^*. The cleaner the environment maintaining technology, the smaller the range of the dynamically inefficient allocations. By comparing the competitive steady state and

the best steady state, we designed balanced budget taxes and transfer policies that decentralize the planner's steady state. We also discussed the desirability and implementability of each policy from the viewpoints of both the agents and the policy maker.

This chapter makes many simplifying assumptions, such as the technology being exogenous, the population growth rate being zero and there being only one production sector. In addition, this chapter just mentions a sufficient condition to implement the tax and transfer policy in decentralizing the planner's steady state. A necessary condition should be studied. Further developments including endogenous technology and fertility, the impact of human capital accumulation, as well as the necessary condition for applying such tax and transfer policies are left for future research.

2.9 Appendix

A2.1. Existence of the agent's optimal solution

By substituting (2.11), (2.12), (2.13) and (2.14) into (2.9) and (2.10) the existence of solution to the system of the first order conditions (2.9)-(2.14) is equivalent to the existence of solution to the system of two following equations

$$u'(c_0^t) - [\beta(1-b)+\gamma]\,\phi'(E_{t+1}^e) = 0 \tag{2.65}$$

$$v'(c_1^t) - \frac{\beta + \frac{\gamma}{r_{t+1}}}{\beta(1-b)+\gamma} u'(c_0^t) = 0 \tag{2.66}$$

where

$$E_{t+1}^e = (1-b)\left[(1-b)E_{t-1} - \alpha F(k_{t-1},1) - \beta(c_0^t + c_1^{t-1}) + \gamma m^{t-1}\right]$$

$$-\alpha F(K_{t+1},1) - \beta(c_0^{t+1,e} + c_1^t) + \gamma(w_t - c_0^t - \frac{c_1^t}{r_{t+1}})$$

From (2.66), by implicit function theorem we can treat c_1^t as a function of c_0^t, $c_1^t = \varphi(c_0^t)$ where $\varphi'(\cdot) > 0$, $\varphi(0) = 0$ and $\varphi(+\infty) = +\infty$. We rewrite

$$E_{t+1}^{e} = Q - [\beta(1-b)+\gamma]\, c_0^t - (\beta + \frac{\gamma}{r_{t+1}})\varphi(c_0^t)$$

where

$$Q = (1-b)\left[(1-b)E_{t-1} - \alpha F(k_{t-1},1) - \beta c_1^{t-1} + \gamma e^{t-1}\right]$$
$$-\alpha F(K_{t+1},1) - \beta c_0^{t+1,e} + \gamma w_t$$

Now the system of equations (2.65) and (2.66) leads to the following equation

$$u'(c_0^t) - [\beta(1-b)+\gamma]\, \phi'(Q - [\beta(1-b)+\gamma]\, c_0^t - (\beta + \frac{\gamma}{r_{t+1}})\varphi(c_0^t)) = 0 \tag{2.67}$$

The existence of the agent's optimal solution is equivalent to the existence of solution to equation (2.67). In effect, set

$$\triangle = u'(c_0^t) - [\beta(1-b)+\gamma]\, \phi'(Q - [\beta(1-b)+\gamma]\, c_0^t - (\beta + \frac{\gamma}{r_{t+1}})\varphi(c_0^t))$$

is a continuous function of c_0^t. We have,

$$\lim_{c_0^t \to +\infty} \triangle = -\infty < 0 \qquad \text{and} \qquad \lim_{c_0^t \to 0^+} \triangle = +\infty > 0$$

We also find that $\triangle$ is a monotone function of c_0^t since

$$\frac{\partial \triangle}{\partial c_0^t} = u''(c_0^t)$$
$$+ [\beta(1-b)+\gamma]\left[\beta(1-b)+\gamma + (\beta + \frac{\gamma}{r_{t+1}})\varphi'(c_0^t)\right]\phi''(E_{t+1}^{e}) < 0$$

So there exists a unique solution $c_0^t > 0$ to (2.67), meaning that there exists a unique optimal solution of the agent.

A2.2. Checking the SOCs for the maximization problem of the agent

For the FOCs to be sufficient conditions to characterize a (local) max-

imum to the optimization problem, we have to check the sufficient SOCs. The Lagrangian of the maximization problem is

$$\begin{aligned} Z_t = u(c_0^t) + v(c_1^t) + \phi(E_{t+1}^e) + \lambda_1^t(c_0^t + k^t + m^t - w_t) + \lambda_2^t(c_1^t - r_{t+1}k^t) \\ + \lambda_3^t\left(E_t - (1-b)E_{t-1} + \alpha F(k^{t-1}, 1) + \beta(c_0^t + c_1^{t-1}) - \gamma m^{t-1}\right) \\ + \lambda_4^t\left(E_{t+1}^e - (1-b)E_t + \alpha F(k^t, 1) + \beta(c_0^{t+1,e} + c_1^t) - \gamma m^t\right) \end{aligned}$$

whose bordered Hessian will appear as

$$\bar{H} = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -r_{t+1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \beta & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \beta & 0 & -\gamma & b-1 & 1 \\ 1 & 0 & \beta & 0 & u''(c_0^t) & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & \beta & 0 & v''(c_1^t) & 0 & 0 & 0 & 0 \\ 1 & -r_{t+1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & -\gamma & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & b-1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & \phi''(E_{t+1}^e) \end{pmatrix}$$

The sufficient SOCs for a maximum are

$$(-1)^5\left|\bar{H}_5\right| = -\begin{vmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -r_{t+1} & 0 & 0 \\ 0 & 0 & 0 & 0 & \beta & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & \beta & 0 & -\gamma & b-1 \\ 1 & 0 & \beta & 0 & u''(c_0^t) & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & \beta & 0 & v''(c_1^t) & 0 & 0 & 0 \\ 1 & -r_{t+1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & -\gamma & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & b-1 & 0 & 0 & 0 & 0 & 0 \end{vmatrix}$$

$$= -\left(\beta(1-b)+\gamma\right)^2 r_{t+1}^2 v''(c_1^t) - \left(\beta r_{t+1} + \gamma\right)^2 u''(c_0^t) > 0$$

$$(-1)^6 \left|\bar{H}_6\right| = \begin{vmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -r_{t+1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \beta & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \beta & 0 & -\gamma & b-1 & 1 \\ 1 & 0 & \beta & 0 & u''(c_0^t) & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & \beta & 0 & v''(c_1^t) & 0 & 0 & 0 & 0 \\ 1 & -r_{t+1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & -\gamma & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & b-1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & \phi''(E_{t+1}^e) \end{vmatrix}$$

$$= r_{t+1}^2 v''(c_1^t) u''(c_0^t) + \phi''(E_{t+1}^e) \left|\bar{H}_5\right| > 0$$

which guarantees that the solution to the agent's problem is a maximum indeed.

A2.3. Competitive equilibrium dynamics

The competitive equilibrium conditions impose on $(c_0^{t+1}, c_1^t, k^t, m^t, E_{t+1})$ a dynamics described by a first-order difference equation, since

$$u'(c_0^t) - [\beta(1-b) + \gamma]\, \phi'(E_{t+1}) = 0$$

$$v'(c_1^t) - \left[\beta + \frac{\gamma}{F_K(k^t, 1)}\right] \phi'(E_{t+1}) = 0$$

$$c_0^t + k^t + m^t - F_L(k^{t-1}, 1) = 0$$

$$c_1^t - F_K(k^t, 1) k^t = 0$$

$$E_{t+1} - (1-b)E_t + \alpha F(k^t, 1) + \beta(c_0^{t+1} + c_1^t) - \gamma m^t = 0$$

implicitly define it to be a function of its lagged value $(c_0^t, c_1^{t-1},\ k^{t-1},\ m^{t-1},\ E_t)$. In effect, the associated Jacobian matrix with respect to $(c_0^{t+1}, c_1^t, k^t, m^t,\ E_{t+1})$

$$J = \begin{pmatrix} 0 & 0 & 0 & 0 & G \\ 0 & v''(c_1^t) & D & 0 & H \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & -C & 0 & 0 \\ \beta & \beta & \alpha F_K(k^t, 1) & -\gamma & 1 \end{pmatrix}$$

where

$$C = F_K(k^t, 1) + F_{KK}(k^t, 1)k^t > 0$$

$$D = \frac{\gamma F_{KK}(k^t, 1)}{F_K(k^t, 1)^2} \phi'(E_{t+1}) < 0$$

$$G = -\left[\beta(1-b) + \gamma\right] \phi''(E_{t+1}) > 0$$

$$H = -\left[\beta + \frac{\gamma}{F_K(k^t, 1)}\right] \phi''(E_{t+1}) > 0$$

is regular, since

$$\det(J) = G \begin{vmatrix} 0 & v''(c_1^t) & D & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & -C & 0 \\ \beta & \beta & \alpha F_K(k^t, 1) & -\gamma \end{vmatrix} = -G\beta \left[D + Cv''(c_1^t)\right] > 0$$

Since, the Jacobian matrix is regular for all $(c_0^{t+1}, c_1^t, k^t, m^t, E_{t+1})$ then it is evidently regular at the solution. This implies that for all competitive equilibrium $(c_0^{t+1}, c_1^t, k^t, m^t, E_{t+1})_t$ there exists, for all t, a function $\psi : \mathbb{R}^5 \to \mathbb{R}^5$ such that

$$\begin{pmatrix} c_0^{t+1} \\ c_1^t \\ k^t \\ m^t \\ E_{t+1} \end{pmatrix} = \psi \begin{pmatrix} c_0^t \\ c_1^{t-1} \\ k^{t-1} \\ m^{t-1} \\ E_t \end{pmatrix}$$

A2.4. Solving the problem of the social planner

The Lagrange function for this problem is

$$\mathcal{L} = \sum_{t=0}^{+\infty} \frac{1}{(1+R)^t} \left[u(c_0^t) + u(c_1^t) + \phi(E_{t+1})\right]$$

$$+ \sum_{t=0}^{+\infty} \frac{\mu_t}{(1+R)^t} \left[F(k^{t-1}, 1) - c_0^t - c_1^{t-1} - k^t - m^t\right]$$

$$+ \sum_{t=0}^{+\infty} \frac{\eta_t}{(1+R)^t} \left[E_{t+1} - (1-b)E_t + \alpha F(k^t, 1) + \beta(c_0^{t+1} + c_1^t) - \gamma m^t\right]$$

The FOCs of the maximization problem are

$$\frac{\partial \mathcal{L}}{\partial c_0^t} = \frac{u'(c_0^t)}{(1+R)^t} - \frac{\mu_t}{(1+R)^t} + \frac{\beta\eta_{t-1}}{(1+R)^{t-1}} = 0$$

$$\frac{\partial \mathcal{L}}{\partial c_1^t} = \frac{v'(c_1^t)}{(1+R)^t} - \frac{\mu_{t+1}}{(1+R)^{t+1}} + \frac{\beta\eta_t}{(1+R)^t} = 0$$

$$\frac{\partial \mathcal{L}}{\partial E_{t+1}} = \frac{\phi'(E_{t+1})}{(1+R)^t} + \frac{\eta_t}{(1+R)^t} - \frac{\eta_{t+1}(1-b)}{(1+R)^{t+1}} = 0$$

$$\frac{\partial \mathcal{L}}{\partial k^t} = -\frac{\mu_t}{(1+R)^t} + \frac{\mu_{t+1}F_K(k^t, 1)}{(1+R)^{t+1}} + \frac{\eta_t \alpha F_K(k^t, 1)}{(1+R)^t} = 0$$

$$\frac{\partial \mathcal{L}}{\partial m^t} = -\frac{\mu_t}{(1+R)^t} - \frac{\eta_t \gamma}{(1+R)^t} = 0$$

that is to say

$$u'(c_0^t) - \mu_t + \beta\eta_{t-1}(1+R) = 0$$

$$v'(c_1^t) - \frac{\mu_{t+1}}{1+R} + \beta\eta_t = 0$$

$$\phi'(E_{t+1}) + \eta_t - \frac{\eta_{t+1}(1-b)}{1+R} = 0$$

$$-\mu_t + \frac{\mu_{t+1} F_K(k^t, 1)}{1+R} + \eta_t \alpha F_K(k^t, 1) = 0$$

$$-\mu_t - \eta_t \gamma = 0$$

At the steady state,

$$u'(\bar{c}_0) = \mu - \beta\eta(1+R)$$

$$v'(\bar{c}_1) = \frac{\mu}{1+R} - \beta\eta$$

$$\phi'(\bar{E}) = -\eta + \frac{(1-b)\eta}{1+R}$$

$$F_K(\bar{k}, 1) = \frac{\mu(1+R)}{\mu + \alpha\eta(1+R)}$$

$$\mu = -\eta\gamma$$

Therefore,

$$u'(\bar{c}_0) = (1+R)\frac{\gamma + \beta(1+R)}{b+R}\phi'(\bar{E})$$

$$v'(\bar{c}_1) = \frac{\gamma + \beta(1+R)}{b+R}\phi'(\bar{E})$$

$$F_K(\bar{k}, 1) = \frac{\gamma(1+R)}{\gamma - (1+R)\alpha}$$

A2.5. Proof of Proposition 2.2

Note first that, since a competitive equilibrium steady state under a period-by-period balanced-budget stationary policy of consumption tax rates τ_0, τ_1 and lump-sum transfers T_0, T_1 is characterized by

$$u'(c_0) = [\beta(1-b) + \gamma(1+\tau_0)]\,\phi'(E)$$

$$v'(c_1) = \left[\beta + \frac{\gamma}{F_K(k,1)}(1+\tau_1)\right]\phi'(E)$$

$$(1+\tau_0)c_0 + k + m = F_L(k,1) - T_0$$

$$(1+\tau_1)c_1 = F_K(k,1)k + T_1$$

$$E = (1-b)E - \alpha F(k,1) - \beta(c_0 + c_1) + \gamma m$$

and

$$\tau_0 c_0 + \tau_1 c_1 + T_0 = T_1$$

and since the planner's steady state is characterized by

$$u'(c_0^*) = \frac{\gamma+\beta}{b}\phi'(E^*)$$

$$v'(c_1^*) = \frac{\gamma+\beta}{b}\phi'(E^*)$$

$$F_K(k^*,1) = \frac{\gamma}{\gamma-\alpha}$$

$$c_0^* + c_1^* + k^* + m^* = F(k^*,1)$$

$$E^* = (1-b)E^* - \alpha F(k^*,1) - \beta(c_0^* + c_1^*) + \gamma m^*$$

then for the stationary competitive equilibrium under τ_0, τ_1, T_0, T_1 to be the planner's steady state it is necessary and sufficient that

$$\beta(1-b) + \gamma(1+\tau_0) = \frac{\gamma+\beta}{b}$$

$$\beta + \frac{\gamma}{F_K(k^*,1)}(1+\tau_1) = \frac{\gamma+\beta}{b}$$

$$(1+\tau_0)c_0^* + k^* + m^* = F_L(k^*,1) - T_0$$

$$(1+\tau_1)c_1^* = F_K(k^*,1)k^* + T_1$$

i.e. that

$$\tau_0 = \frac{\gamma+\beta-[\gamma+\beta(1-b)]\,b}{\gamma b}$$

$$\tau_1 = \frac{\gamma+\beta(1-b)}{(\gamma-\alpha)b} - 1$$

$$T_0 = F_L(k^*, 1) - (1 + \tau_0)c_0^* - k^* - m^*$$

$$T_1 = (1 + \tau_1)c_1^* - F_K(k^*, 1)k^*$$

In other words, such a policy supports the planner's steady state *once the economy is there*. There is then the additional problem of *moving the economy to the planner's steady state*.

To address this additional problem, it should be noticed that the choice by agent t of $c_0^t, c_1^t, k^t, m^t, E_t, E_{t+1}^e$ depends on past decisions, in particular on k^{t-1} through $w_t = F_L(k^{t-1}, 1)$, and on $c_1^{t-1}, m^{t-1}, E_{t-1}$ through the environment dynamics. We shall show next that there exists a balanced-budget policy towards generation $t-1$ that makes it choose c_0^*, c_1^*, k^* and $E_t^e = E^*$ (but not m^* or $E_{t-1} = E^*$), and another policy towards generation t, namely the policy above, that makes it choose c_0^*, c_1^*, k^* and $E_{t+1}^e = E^*$ as well as m^* and $E_t = E^*$. Once there the same policy keeps the economy at the planner's steady state.

In effect, agent $t-1$'s choice at a perfect foresight equilibrium is characterized by the conditions

$$u'(c_0^{t-1}) = [\beta(1-b) + \gamma(1+\tau_0)]\phi'(E_t)$$

$$v'(c_1^{t-1}) = [\beta + \frac{\gamma}{F_K(k^{t-1}, 1)}(1+\tau_1)]\phi'(E_t)$$

$$(1+\tau_0)c_0^{t-1} + k^{t-1} + m^{t-1} = F_L(k^{t-2}, 1) - T_0^{t-1}$$

$$(1+\tau_1)c_1^{t-1} = F_K(k^{t-1}, 1)k^{t-1} + T_1^{t-1}$$

$$E_{t-1} = (1-b)E_{t-2} - \alpha F(k^{t-2}, 1) - \beta(c_0^{t-1} + c_1^{t-2} + T_1^{t-2}) + \gamma m^{t-2}$$

$$E_t = (1-b)E_{t-1} - \alpha F(k^{t-1}, 1) - \beta(c_0^t + c_1^{t-1}) + \gamma m^{t-1}$$

given past decisions $c_1^{t-2}, k^{t-2}, m^{t-2}, E_{t-2}$ and c_0^t. Note that, for the government's budget to be balanced at $t-1$, generation $t-2$ receives as a transfer T_1^{t-2} all the taxes raised at $t-1$, i.e.

$$T_1^{t-2} = \tau_0 c_0^{t-1} + T_0^{t-1}$$

Keeping this in mind, it turns out that there exist transfers T_0^{t-1}, T_1^{t-1} such that under τ_0, τ_1 above generation $t-1$ chooses c_0^*, c_1^*, k^* and $E_t^e = E^*$. In effect, there is a solution in $c_0^{t-1}, T_0^{t-1}, T_1^{t-1}, m^{t-1}, E_{t-1}, E_t$ to the system above with $c^{t-1} = c_1^*$ and $k^{t-1} = k^*$, i.e. a solution to

$$u'(c_0^{t-1}) = [\beta(1-b) + \gamma(1+\tau_0)]\phi'(E_t)$$

$$v'(c_1^*) = [\beta + \frac{\gamma}{F_K(k^*,1)}(1+\tau_1)]\phi'(E_t)$$

$$(1+\tau_0)c_0^{t-1} + k^* + m^{t-1} = F_L(k^{t-2},1) - T_0^{t-1}$$

$$(1+\tau_1)c_1^* = F_K(k^*,1)k^* + T_1^{t-1}$$

$$E_{t-1} = (1-b)E_{t-2} - \alpha F(k^{t-2},1) - \beta((1+\tau_0)c_0^{t-1} + c_1^{t-2} + T_0^{t-1}) + \gamma m^{t-2}$$

$$E_t = (1-b)E_{t-1} - \alpha F(k^*,1) - \beta(c_0^t + c_1^*) + \gamma m^{t-1}$$

(where c_1^{t-1} and k^{t-1} have been fixed at the levels c_1^* and k^* respectively) since, given the conditions characterizing the planner's steady state, from the second equation necessarily $E_t = E^*$, which in turn implies, from the first equation, that $c_0^{t-1} = c_0^*$. The fourth equation directly determines

$$T_1^{t-1} = (1+\tau_1)c_1^* - F_K(k^*,1)k^*$$

and the three other equations constitute the following regular linear system in m^{t-1}, T_0^{t-1}, and E_{t-1}

$$T_0^{t-1} + m^{t-1} = F_L(k^{t-2},1) - (1+\tau_0)c_0^* - k^*$$

$$E_{t-1} + \beta T_0^{t-1} = (1-b)E_{t-2} - \alpha F(k^{t-2},1) - \beta((1+\tau_0)c_0^* + c_1^{t-2}) + \gamma m^{t-2}$$

$$(1-b)E_{t-1} + \gamma m^{t-1} = E_t + \alpha F(k^*,1) + \beta(c_0^t + c_1^*)$$

with solution

$$\begin{pmatrix} m^{t-1} \\ T_0^{t-1} \\ E_{t-1} \end{pmatrix} = \frac{1}{A} \begin{pmatrix} (1-b)\beta & b-1 & 1 \\ \gamma & 1-b & -1 \\ -\gamma\beta & \gamma & \beta \end{pmatrix}$$

$$\cdot \begin{pmatrix} F_L(k^{t-2},1) - (1+\tau_0)c_0^* - k^* \\ (1-b)E_{t-2} - \alpha F(k^{t-2},1) - \beta((1+\tau_0)c_0^* + c_1^{t-2}) + \gamma m^{t-2} \\ E_t + \alpha F(k^*,1) + \beta(c_0^t + c_1^*) \end{pmatrix}$$

where $A = \gamma + \beta(1-b)$ is the determinant of the matrix of coefficients.

Thus, under the following policy of consumption tax rates and lump-sum transfers

$$\tau_0^{t-1} = \frac{\gamma + \beta - [\gamma + \beta(1-b)]b}{\gamma b}$$

$$\tau_1^{t-1} = \frac{\gamma + \beta(1-b)}{(\gamma - \alpha)b} - 1$$

$$T_0^{t-1} = \frac{1}{A} \begin{pmatrix} \gamma \\ 1-b \\ -1 \end{pmatrix}'$$

$$\cdot \begin{pmatrix} F_L(k^{t-2},1) - (1+\tau_0)c_0^* - k^* \\ (1-b)E_{t-2} - \alpha F(k^{t-2},1) - \beta((1+\tau_0)c_0^* + c_1^{t-2}) + \gamma m^{t-2} \\ E_t + \alpha F(k^*,1) + \beta(c_0^t + c_1^*) \end{pmatrix}$$

$$T_1^{t-1} = (1+\tau_1)c_1^* - F_K(k^*,1)k^*$$

generation $t-1$ makes at equilibrium the choices

$$c_0^{t-1} = c_0^*$$
$$c_1^{t-1} = c_1^*$$
$$k^{t-1} = k^*$$

$$m^{t-1} = \frac{1}{A} \begin{pmatrix} (1-b)\beta \\ b-1 \\ 1 \end{pmatrix}'$$

$$
. \begin{pmatrix} F_L(k^{t-2},1) - (1+\tau_0)c_0^* - k^* \\ (1-b)E_{t-2} - \alpha F(k^{t-2},1) - \beta((1+\tau_0)c_0^* + c_1^{t-2}) + \gamma m^{t-2} \\ E_t + \alpha F(k^*,1) + \beta(c_0^t + c_1^*) \end{pmatrix}
$$

$$
E_{t-1} = \frac{1}{A} \begin{pmatrix} -\gamma\beta \\ \gamma \\ \beta \end{pmatrix}'
$$

$$
. \begin{pmatrix} F_L(k^{t-2},1) - (1+\tau_0)c_0^* - k^* \\ (1-b)E_{t-2} - \alpha F(k^{t-2},1) - \beta((1+\tau_0)c_0^* + c_1^{t-2}) + \gamma m^{t-2} \\ E_t + \alpha F(k^*,1) + \beta(c_0^t + c_1^*) \end{pmatrix}
$$

$$
E_t = E^*
$$

Note that the policy above depends on elements known at the time $t-1$ of announcing it, except for the transfer T_0^{t-1}, which depends as well on the expected (and, at a perfect foresight equilibrium, the actual) first-period consumption of generation t, c_0^t. The choice of m^{t-1} and E_{t-1} by generation $t-1$ depends on it accordingly. Nevertheless, under the announcement at $t-1$ that the same consumption tax rates will be applied to generation t as well, along with the stationary balanced-budget transfers implementing the planner's steady state, i.e. under the policy

$$
\tau_0^t = \frac{\gamma + \beta - [\gamma + \beta(1-b)]b}{\gamma b}
$$

$$
\tau_1^t = \frac{\gamma + \beta(1-b)}{(\gamma - \alpha)b} - 1
$$

$$
T_0 = F_L(k^*,1) - (1+\tau_0)c_0^* - k^* - m^*
$$

$$
T_1 = (1+\tau_1)c_1^* - F_K(k^*,1)k^*
$$

it is perfectly foreseen at $t-1$ that at equilibrium $c_0^t = c_0^*$. More specifically, under this policy generation t's choice is $c_0^*, c_1^*, k^*, m^*, E_t = E^*$ and $E_{t+1}^e = E^*$, since this choice solves

$$
u'(c_0^t) = [\beta(1-b) + \gamma(1+\tau_0)]\phi'(E_{t+1}^e)
$$

$$v'(c_1^t) = [\beta + \frac{\gamma}{F_K(k^t, 1)}(1 + \tau_1)]\phi'(E_{t+1}^e)$$

$$(1 + \tau_0)c_0^t + k^t + m^t = F_L(k^*, 1) - T_0^t$$

$$(1 + \tau_1)c_1^t = F_K(k^t, 1)k^t + T_1^t$$

$$E_t = (1 - b)E_{t-1} - \alpha F(k^*, 1) - \beta(c_0^t + c_1^*) + \gamma m^{t-1}$$

$$E_{t+1}^e = (1 - b)E_t - \alpha F(k^t, 1) - \beta(c_0^{t+1,e} + c_1^t) + \gamma m^t$$

given that $E_{t+1}^e, c_0^{t+1,e}$ are perfectly foreseen to be E^*, c_0^* when the policy is left unchanged for all generations from generation t onwards, and the next-to-last equation is guaranteed to be satisfied from the choices of E_{t-1} and m^{t-1} by generation $t-1$ at $t-1$ (it is the last equation in generation $t-1$'s system).

Finally, note that at period t the policy is balanced as well (at $t-1$ and from $t+1$ onwards it is so by construction), i.e.

$$T_1^{t-1} = \tau_0 c_0^t + \tau_1 c_1^{t-1} + T_0^t$$

that is to say

$$(1+\tau_1)c_1^* - F_K(k^*, 1)k^* = \tau_0 c_0^* + \tau_1 c_1^* + F_L(k^*, 1) - (1+\tau_0)c_0^* - k^* - m^*$$

which boils down to

$$c_0^* + c_1^* + k^* + m^* = F_K(k^*, 1)k^* + F_L(k^*, 1)$$

which is guaranteed by the feasibility of the planner's steady state. Q.E.D.

A2.6. Proof of Proposition 2.6

(1) It should be noticed that, under each policy above, each agent born in period $t-1$ chooses $c_0^{t-1} = c_0^*$, $c_1^{t-1} = c_1^*$, $k^{t-1} = k^*$, $E_t^e = E^*$ (but not $m^{t-1} = m^*$ and not $E_{t-1} = E^*$). So, young agents pay the following amounts Γ_i^{t-1} of taxes under each policy i[7]

[7]Since the tax rates are unchanged over time, except for the tax rate on labor income, we have removed the time superscripts for all tax rates, but not for the tax rate on labor income in period $t-1$.

$$\text{Policy 1}: \quad \Gamma_1^{t-1} = \tau_0 c_0^* + T_0^{t-1}$$

$$\text{Policy 2}: \quad \Gamma_2^{t-1} = \tau c_0^* + T_0^{t-1}$$

$$\text{Policy 3}: \quad \Gamma_3^{t-1} = \tau c_0^* + T_0^{t-1}$$

$$\text{Policy 4}: \quad \Gamma_4^{t-1} = \tau c_0^* + \tau_w^{t-1} F_L(k^{t-2}, 1)$$

From the quations determining tax rates and lump-sum taxes and transfers in Propositions 2.2 to 2.5, we know that $\tau_0 = \tau$, and the lump-sum taxes imposed on the incomes of agents born in period $t-1$ in policy 1, 2, and 3 are exactly identical. And it is straightforward from equation defining τ_w^{t-1} (in Proposition 2.5) to find that $\tau_w^{t-1} F_L(k^{t-2}, 1) = T_0^{t-1}$. Therefore, we have

$$\Gamma_1^{t-1} = \Gamma_2^{t-1} = \Gamma_3^{t-1} = \Gamma_4^{t-1}$$

i.e., the agents born in period $t-1$ pay the same amount of taxes when young under each of the policies above.

(2) First of all, we prove that from period t onwards, agents are treated equally under policies 1 and 2, and they are also treated equally under policies 3 and 4. It should be noted that from period t onwards, under each policy above, agents choose $c_0^{t+i} = c_0^*$, $c_1^{t+i} = c_1^*$, $k^{t+i} = k^*$, $m^{t+i} = m^*$, $E_{t+i} = E^*$, and $E_{t+i+1}^e = E^*$ $\forall i \geq 0$. So, from period t onwards, each young agent has to pay an amount of taxes corresponding to each alternative policy as follows[8]

$$\text{Policy 1}: \quad \Gamma_1 = \tau_0 c_0^* + T_{0,1} = F_L(k^*, 1) - c_0^* - k^* - m^*$$

$$\text{Policy 2}: \quad \Gamma_2 = \tau c_0^* + T_{0,2} = F_L(k^*, 1) - c_0^* - k^* - m^*$$

$$\text{Policy 3}: \quad \Gamma_3 = \tau c_0^* + T_{0,3} = (1-\tau_p) F_L(k^*, 1) - c_0^* - k^* - m^*$$

$$\text{Policy 4}: \quad \Gamma_4 = \tau c_0^* + \tau_w F_L(k^*, 1) = (1-\tau_p) F_L(k^*, 1) - c_0^* - k^* - m^*$$

Hence, we have

$$\Gamma_1 = \Gamma_2 \qquad \text{and} \qquad \Gamma_3 = \Gamma_4 \tag{2.68}$$

[8] In this proof, we add subscripts 1, 2, 3 and 4 into corresponding lump-sum taxes and lump-sum transfers to refer that they belong to corresponding policy 1, 2, 3, or 4.

Each old agent from period t onwards receives an amount of net transfers (lump-sum transfer after subtracting taxes paid) corresponding to each alternative policy as follows

$$\text{Policy 1}: \quad \Theta_1 = T_{1,1} - \tau_1 c_1^* = c_1^* - F_K(k^*, 1)k^*$$

$$\text{Policy 2}: \quad \Theta_2 = T_{1,2} - \tau c_1^* - \tau_k F_K(k^*, 1)k^* = c_1^* - F_K(k^*, 1)k^*$$

$$\text{Policy 3}: \quad \Theta_3 = T_{1,3} - \tau c_1^* = c_1^* - (1 - \tau_p)F_K(k^*, 1)k^*$$

$$\text{Policy 4}: \quad \Theta_4 = T_{1,4} - \tau c_1^* = c_1^* - (1 - \tau_p)F_K(k^*, 1)k^*$$

Hence, we have

$$\Theta_1 = \Theta_2 \qquad \text{and} \qquad \Theta_3 = \Theta_4 \tag{2.69}$$

(2.68) and (2.69) imply that agents are treated equally under policies 1 and 2, and they are also treated equally under policies 3 and 4.

(2a) If $\alpha(\gamma + \beta) - \beta b^2(\gamma - \alpha) - \alpha\beta b > (<)0$, then $\tau_k, \tau_p > (<)0$, therefore

$$\Gamma_1 = \Gamma_2 > (<)\Gamma_3 = \Gamma_4 \qquad \text{and} \qquad \Theta_1 = \Theta_2 < (>)\Theta_3 = \Theta_4$$

which imply (2a) of Proposition 2.6.

(2b) If $\alpha(\gamma + \beta) - \beta b^2(\gamma - \alpha) - \alpha\beta b = 0$, then $\tau_k, \tau_p = 0$, therefore

$$\Gamma_1 = \Gamma_2 = \Gamma_3 = \Gamma_4 \qquad \text{and} \qquad \Theta_1 = \Theta_2 = \Theta_3 = \Theta_4$$

i.e., in this case agents will be equally treated under all policies.

We prove next that, in this case the policies 1, 2, and 3 coincide. In effect, from equations determining the taxes and transfers in the Proposition 2.3 and Proposition 2.4, it is straightforward that policies 2 and 3 coincide. By comparing equations in Proposition 2.3 (or Proposition 2.4) with equations in Proposition 2.2, we now complete the proof by showing that under the condition $\alpha(\gamma + \beta) - \beta b^2(\gamma - \alpha) - \alpha\beta b = 0$ the tax rates τ_0 and τ_1, which are determined in Proposition 2.2, are equal.

We have

$$\alpha(\gamma + \beta) - \beta b^2(\gamma - \alpha) - \alpha\beta b = 0 \quad \Leftrightarrow \quad \gamma + \beta = \frac{\beta b^2 \gamma}{\alpha} - \beta b^2 + \beta b$$

We transform τ_0 and τ_1 as follows

$$\tau_0 = \frac{\gamma + \beta - [\gamma + \beta(1-b)]\, b}{\gamma b}$$

$$= \frac{\frac{\beta b^2 \gamma}{\alpha} - \beta b^2 + \beta b - [\gamma + \beta(1-b)]\, b}{\gamma b} = \frac{\beta b}{\alpha} - 1$$

and

$$\tau_1 = \frac{\gamma + \beta(1-b)}{(\gamma - \alpha)b} - 1 = \frac{\frac{\beta b^2 \gamma}{\alpha} - \beta b^2 + \beta b - \beta b}{(\gamma - \alpha)b} - 1 = \frac{\beta b}{\alpha} - 1$$

Therefore, $\tau_0 = \tau_1 = \tau$. Hence, policy 1 also coincides with policies 2 and 3. Q.E.D.

3 Can geography lock a society in stagnation?

coauthored with Julio Dávila

We extend Galor and Weil (2000) by including geographical factors in order to show that under some initial conditions, an economy may be locked in Malthusian stagnation and never take off. Specifically, we characterize the set of geographical factors for which this happens, and this way we show how the interplay of the available "land", its suitability for living, and its degree of isolation, determines whether an economy can escape stagnation.

Keywords: Geographical factors, loss of technology, human capital.

3.1 Introduction

Galor and Weil (2000) advanced a unified growth model to explain the transition to modern growth as the result of the interaction between population, technology, and output. In their model, the authors show that the transition from stagnation to sustained growth is an *inevitable* outcome when the driving forces for technological progress are the education and size of the population. Specifically, in Galor and Weil (2000), technological progress is assumed to appear even for zero education investments and arbitrarily small populations so that, eventually, Malthusian stagnation vanishes endogenously, leaving the arena to modern growth forces and letting thus the economy take off and converge to a modern steady state growth. In this chapter we study conditions under which take-off is not inevitable, but rather stagnation is.

In order to understand how a society can be locked in stagnation it is useful to identify what exactly drives a take-off in Galor and Weil (2000). A key ingredient to the mechanism proposed there and that takes the economy out of stagnation is the positive dependence of technological progress on population. Still it is worth noting that this dependence is, nevertheless, not indispensable for an explanation. In effect, for instance Galor and Moav (2002) show that a society can still take off without having to assume a positive effect of population on technological progress. In this case, it is the composition of the population (in terms of the households' preferences about quality vs. quantity of their offspring) rather than the size of the population that matters in order to spur technological progress. In effect, the appearance of a fraction (even a tiny one) of "quality-loving mutant" households suffices for a society to take off in the long-run, regardless of population size, by triggering a change in the composition of the population. Still, if the population size does not matter in Galor and Moav (2002) it is because of their explicit assumption according to which the costs (not related to education) of rearing a child do not depend on the population size. Nevertheless, population density is known to have an impact on the childrearing costs that are unrelated to education. Specifically, evidence shows that when households have small dwellings, child production is more costly and households have fewer children (see De la Croix and Gosseries (2012), citing evidence from Goodsell (1937) and Thompson (1938)). It is precisely this kind of interplay between a population and its environment —and its impact on growth— what our model aims at capturing.[1]

In this chapter, we build on Galor and Weil (2000), introducing geographical factors instead, in order to show that, under some initial conditions, an economy may be locked in stagnation, with a small population, a basic technology, and no education, *even if population size has, per se, a positive effect on technological progress* (so that the economy should eventually take off instead according to Galor and

[1]Whether the Galor and Moav (2002) population-composition mechanism allows too for a stagnation trap when population composition itself affects the childrearing costs remains an open question.

Weil (2000)). In order to show this, we take into account too the often overlooked role of technology losses in the determination (along with education investments and population size) of the technological level of the society.[2] The key mechanism is, in this case, that recurrent technology losses allow for technological progress only if the population size is large enough to offset them. When this is the case, the level of technology will increase until it reaches a threshold beyond which the returns to education are high enough to trigger investment in human capital, the tipping point where education kicks in and from which sustained growth obtains. Nonetheless, societies whose geographical factors cannot support a sufficiently large population never escape stagnation. This paper therefore makes stand out clearly the role of geographical factors —such as the amount of available land or, more generally, environmental resources, its suitability for living and production, and its degree of isolation— in the creation of a stagnation trap.

It is interesting to note an alternative mechanism that De la Croix and Dottori (2008) propose to explain the road to stagnation followed in Easter island in particular. In that paper the authors argue that the population collapse in Easter island was the result of a population race —that played the role of an arms race given the labor-intensive warring technology— triggered by the non-cooperative bargaining between clans about the allocation of the society's total output (in case of disagreement, a war would break out whose outcome would be de-

[2]Diamond (1997) provides evidence that some societies show no sign of escaping stagnation on their own due to losses of technology and culture, in particular small and isolated societies. An extreme case, but by no means the only documented one (see Aiyar *et al.* (2008) for technological losses driven by population shocks and, more generally, footnote 5 below), took place on the Tasmania island. Aborigines in Tasmania were separated from mainland Australians due to rising sea level around 10.000 years ago. With a stable population of 4.000, Tasmanians had, at the time of arrival of Europeans, the simplest material culture and technology of any people in the modern world. Like mainland Aborigines, they were hunter-gatherers but they lacked many technologies and artifacts widespread on the mainland. Some technologies were brought to Tasmania when it was still a part of the Australian mainland, and were subsequently lost in Tasmania's cultural isolation. For example, the disappearance of fishing, and of awls, needles, and other bone tools, around 1500 BC (Diamond 1997, pp. 312–13). Diamond argues that a small population of 4.000 was able to survive for 10.000 years, but was not enough to prevent significant losses of technology and culture, as well as the failure to invent new technology, leaving it with a uniquely simplified material culture.

termined by relative population sizes of the belligerent clans, so that in order to improve their bargaining power, each clan would increase its size to the point of jointly depleting natural resources and leading eventually the society to collapse). Therefore, in De la Croix and Dottori (2008) the conflict-driven population race is the prime cause of stagnation in an environment whose resources are bounded but not necessarily insufficient for sustaining take-off in the absence of conflict. On the contrary, in this chapter, the cause of stagnation is a geography unable to support a population large enough to offset technology losses.[3]

The rest of the chapter is organized as follows. Section 3.2 introduces the model. Section 3.3 characterizes its equilibria. Geographical factors under which an economy is unable to escape stagnation are studied in section 3.4. Specifically, we show that a society for which the population guaranteeing technological progress, and the technology guaranteeing education investment, implies a high enough *effective population density*, never escapes stagnation, under some initial conditions. Section 3.5 makes a summary and concludes the chapter.

3.2 The model

3.2.1 Geographical factors

We refer by "land" to the set of geographical and environmental conditions supporting the life and economic activity of a society (obviously, living and production conditions depend on how suitable for that the ecosystem around us is). How much of this land can be put to productive use depends on the interplay of its intrinsic suitability for that purpose and the level of technology. The suitability of land captures its adequacy for people to live and work in the ecosystem as a whole,

[3]Mariani et al. (2010) address a related topic noting the possibility of an environmental poverty trap in a set-up in which environmental quality and life expectancy are jointly determined. In such a setup multiple equilibria are possible according to which agents either invest in both environmental quality and longevity, or do not, which may lock an economy in an environmental poverty trap in the latter case. However, the mechanism there is basically a coordination problem where population growth and its interaction with the environment are abstracted.

such as temperature, humidity, orography, river density, bio-diversity, etc. Typically, suitability and technological constraints prevent people to make the most of their environment, i.e. the available land. For instance, people may just occupy the part of their geographical territory that is most suitable for their lives, or may be unable to tap certain resources with the current technology. We refer to the fraction X_t of the available land X that is put to productive use at period t as "productive land" and its size depends positively on its suitability θ and (moreover concavely) on the technological level $A_t \geq 0$, i.e.

$$X_t = \chi(\theta, A_t)X \tag{3.1}$$

with $\chi(\theta, A_t) \in (0,1)$, $\chi_\theta(\theta, A_t) > 0$, $\chi_A(\theta, A_t) > 0$, $\chi_{AA}(\theta, A_t) < 0$.

3.2.2 Production and technology

The productivity of each household in period t is determined by its human capital $h_t \geq 0$ and the technological level A_t, so that the output per household in period t is

$$y_t = f(A_t)h_t$$

where $f(A_t) > 0$, $f'(A_t) > 0$.[4]
The technological level in period $t+1$ is

$$A_{t+1} = [1 - \lambda(\omega)][1 + g_t]A_t \tag{3.2}$$

where g_t is the rate of technological progress in t, and $\lambda(\omega) \in (0,1)$ is a rate of loss of technology that depends *positively* on the degree of isolation ω of the society, i.e. $\lambda'(\omega) > 0$ (a higher ω corresponds to a higher isolation),[5] so that $[1-\lambda(\omega)][1+g_t]$ is technological growth rate

[4] In order to make stand out clearly the importance of the interplay between population, education, and environment, we abstract from land as an input of the production function. Introducing land in the production function does not change the qualitative analysis.

[5] Diamond (1997) addresses the role of geography in losses of technology. He argues that technologies must not only be acquired but maintained too, which may depend on unpredictable factors, among them cultural prejudices and fads that see economically useful technologies become devalued. A famous example (see Diamond (1997), pp. 257–258) is the loss of gun production technologies in isolated Japan under the Shogunate, when the Samurai class worked against the acceptance of firearms because a cultural preference of swords as class symbols as well as works

between periods t and $t+1$. As in Galor and Weil (2000), we assume that $g_t \geq 0$ depends positively on the average education $e_t \geq 0$ and the size $L_t \geq 0$ of the working generation in period t, i.e.

$$g_t = g(e_t, L_t) \tag{3.3}$$

with $g_e(e_t, L_t),\ g_L(e_t, L_t) > 0$, as well as $g(0, L_t) > 0$ and $\lim_{L_t \to 0^+} g(0, L_t) = 0$, so that technological progress (before losses) is positive as long as population is too.

From (3.2) and (3.3) we know that if the education of the working generation t is zero, then the economy has positive technological growth if, and only if, the population size is large enough, i.e.

$$[1 - \lambda(\omega)][1 + g(0, L_t)] > 1 \quad \Leftrightarrow \quad g(0, L_t) > \frac{\lambda(\omega)}{1 - \lambda(\omega)}$$

which implies that for positive technological growth to exist it must hold $L_t > \underline{L}$, where $\underline{L}$ is the smallest population able to sustain technological progress given ω, i.e. satisfying

of art. Such a phenomenon could not happen lastingly (for obvious reasons) in less isolated European countries, even though similar attitudes towards guns occasionally happened. Another instance of technology losses due to isolationist attitudes is the abandonment of oceangoing navigation techniques by the Chinese under the Ming after the explorations by Zheng He. A non isolated society that temporarily turns against useful technology can reacquire it easily by diffusion from neighboring societies, while its chances to reacquire it are increasingly hampered by higher degrees of isolation.

In this paper we make abstraction of the causes of technology losses and, for the sake of simplicity, assume a rate (not necessarily large) of recurrent technology losses $\lambda(\omega)$ that depends positively on the degree ω of isolation of the society (alternatively these losses can be made, more realistically, random, but this would not change qualitatively the results). This rate may also depend on the society's population size, education level, technological level, etc. Indeed, a larger and better educated population may be better at maintaining technological knowledge due to dissemination scale and interaction of people. These effects can however be captured in technological progress factor $g(e, L)$ in (3.3). A high technological level itself may help a society from losing technologies in two ways: (i) through better storage devices in which to save technologies, and (ii) better communications and transportation to offset isolation. This paper, however, focuses on societies in very early stages of development without widespread literacy and modern communications.

Aiyar et al. (2008) focus on a different phenomenon of technology regress based on external shocks reducing the population in societies in which the transmission of technology is embodied in the human capital instead of recorded. They argue that, when the population shrinks, aggregate demand falls, leading to some technologies to become unprofitable at the margin. As a consequence, those out-of-use technologies are not transmitted to the next generation, and hence lost until rediscovered by chance.

$$g(0, \underline{L}) = \frac{\lambda(\omega)}{1 - \lambda(\omega)} \tag{3.4}$$

Applying the implicit function theorem to (3.4), $\underline{L}$ is implicitly defined to be a function $\mathcal{L}$ of ω, i.e.

$$\underline{L} = \mathcal{L}(\omega) \qquad \text{such that} \quad \mathcal{L}'(\omega) = \frac{\lambda'(\omega)}{[1 - \lambda(\omega)]^2 g_L(0, \underline{L})} > 0.$$

In other words, the more isolated a society is, the bigger the population needed to offset technology losses and generate technological growth. For less isolated societies a smaller population suffices to offset technology losses because the latter are partially offset by technology diffusion from neighboring societies too.

An illustration of the effect of population size in offsetting technology losses is provided by the divergence in technological level between Easter island and Hawaii at the time of the arrival of Europeans in 1770s. Both islands are extremely isolated and had the same cultural background of Polynesian colonizers, but Hawaii had a bigger population (due to more resources available for living and production), which had resulted in a more advanced technology than Easter island by the time Europeans arrived.[6]

3.2.3 Households

In each period t there is a generation of L_t identical working households. Each household lives for two periods. In the first period (say childhood) $t-1$ it uses up a fraction of its parent's time. In the second period (say parental) t it is endowed with one unit of time which it allocates between child-rearing and production. The preferences of the household born in period $t-1$ are defined over the number and quality (i.e. human capital) of its (household-)children, n_t and h_{t+1} respectively, as well as from its consumption c_t in period t as follows

[6]We thank David de la Croix for pointing us this fact.

$$u_t = \gamma \ln(n_t h_{t+1}) + (1-\gamma) \ln c_t \tag{3.5}$$

Each household chooses a number and quality of children under the constraint of the unit of time available for child-rearing and production. The only input required to produce both child quantity and quality is time. We assume that the time to raise children physically, regardless of education investment, is decreasing in per household resources X_t/L_t.[7] For simplicity, we assume that the cost in time for raising n_t children physically is $(\frac{L_t}{X_t})^\beta n_t$, where $\beta \in (0,1)$. We define $\frac{L_t}{X_t}$ as the *effective population density*, i.e. the density of the population with respect to *productive* land only. So the opportunity cost of raising n_t children with education e_{t+1} is $y_t n_t[(\frac{L_t}{X_t})^\beta + e_{t+1}]$. Hence, the agent born at date $t-1$ maximizes at date t its utility (3.5) under the following budget constraint

$$y_t n_t[(\frac{L_t}{X_t})^\beta + e_{t+1}] + c_t \leq y_t \tag{3.6}$$

Galor and Weil (2000) assume that human capital formation of children born at date t, h_{t+1}, depends positively on education investment e_{t+1} they receive, and negatively on the growth rate of technological progress g_t from period t to period $t+1$, their rationale being that education lessens the obsolescence of human capital due to a changing technology. As a consequence, households have incentives to invest in education whenever technological progress is high enough, regardless the level of technology. It can be argued, however, that the incentives to educate their offspring depend not on technological progress, i.e. the rate of change g_t of the technological level, but on the level of technology A_{t+1} itself. In effect, in an economy with a high enough level of technology agents have incentives to educate their offspring in order to able them to make use of the technology, even in the absence of technological progress. Hence, we assume that human capital at $t+1$ is eroded by an increasing technology A_{t+1}, but it increases

[7] This idea is introduced in Goodsell (1937) and Thompson (1938), recently cited by de la Croix and Gosseries (2012) to take into account that when households have small dwellings, child production is more costly and households have fewer children.

with education e_{t+1} with a return that is higher the higher the level of technology, i.e.

$$h_{t+1} = h(e_{t+1}, A_{t+1}) \tag{3.7}$$

with $h_e(e, A) > 0$, $h_A(e, A) < 0$, and $h_{eA}(e, A) > 0$ at interior points. It is moreover assumed that households are endowed with some human capital even in the absence of education and any technological sophistication, i.e. $h(0, 0) > 0$, so that production can take place. Also it is assumed (i) $h_e(0, 0) = 0$, i.e. education does not increase human capital in the absence of technology, (ii) $\lim_{A \to +\infty} h(0, A) = 0$ so that an unbounded increase in the level of technology wipes out human capital in the limit, and (iii) the return to education remains bounded away from zero as technology grows unboundedly, i.e. $\lim_{A \to +\infty} h_e(0, A) > 0$.

Household's optimization

Each household t chooses the quantity n_t and quality h_{t+1} of its offspring, as well as consumption c_t, so as to maximize its utility. From (3.5), (3.6), and (3.7), the optimization problem is

$$\max_{n_t, e_{t+1}} \gamma \ln \left[n_t h(e_{t+1}, A_{t+1}) \right] + (1 - \gamma) \ln \left[(1 - n_t [(\frac{L_t}{X_t})^\beta + e_{t+1}]) y_t \right]$$

The household's optimization problem is solved in the Appendix A3.6. The first-order condition (FOC) with respect to n_t gives us

$$n_t = \frac{\gamma}{(\frac{L_t}{X_t})^\beta + e_{t+1}} \tag{3.8}$$

And the FOC with respect to e_{t+1} requires the following relationship between e_{t+1} and A_{t+1}, $\frac{L_t}{X_t}$ to hold:

$$G\left(e_{t+1}, A_{t+1}, \frac{L_t}{X_t}\right) \equiv h_e(e_{t+1}, A_{t+1})[(\frac{L_t}{X_t})^\beta + e_{t+1}] - h(e_{t+1}, A_{t+1})$$

$$\begin{cases} = 0 \text{ if } e_{t+1} > 0 \\ \leq 0 \text{ if } e_{t+1} = 0 \end{cases} \tag{3.9}$$

Proposition 3.1: *In the economy set up above, there exists, for each effective population density, a technological level* $\hat{A}_{t+1} = \hat{A}\left(\frac{L_t}{X_t}\right) > 0$*, such that households educate their offspring if, and only if, technology exceeds that level, i.e.*

$$e_{t+1} = e\left(A_{t+1}, \frac{L_t}{X_t}\right) \begin{cases} = 0 \text{ if } A_{t+1} \leq \hat{A}\left(\frac{L_t}{X_t}\right) \\ \\ > 0 \text{ if } A_{t+1} > \hat{A}\left(\frac{L_t}{X_t}\right) \end{cases}$$

Moreover, $\hat{A}'\left(\frac{L_t}{X_t}\right) < 0$.

***Proof*:** We prove that, for each $\frac{L_t}{X_t}$ there exists a unique $\hat{A}_{t+1}$ such that $G(0, \hat{A}_{t+1}, \frac{L_t}{X_t}) = 0$. From the assumptions on $h(e_{t+1}, A_{t+1})$ and the equation (3.9), we find that $G(0, A_{t+1}, \frac{L_t}{X_t})$ is monotonically increasing in A_{t+1},

$$\frac{\partial G(0, A_{t+1}, \frac{L_t}{X_t})}{\partial A_{t+1}} = h_{eA}(0, A_{t+1})(\frac{L_t}{X_t})^{\beta} - h_A(0, A_{t+1}) > 0$$

Furthermore, from the assumptions made on h it follows that $G(0, 0, \frac{L_t}{X_t}) < 0$ while $\lim_{A_{t+1} \to +\infty} G(0, A_{t+1}, \frac{L_t}{X_t}) > 0$. So, there exists a unique $\hat{A}_{t+1} > 0$, given $\frac{L_t}{X_t}$, such that $G(0, \hat{A}_{t+1}, \frac{L_t}{X_t}) = 0$, and therefore, as it follows from (3.9), $e_{t+1} = 0$ for $A_{t+1} \leq \hat{A}_{t+1}$.
Applying the implicit function theorem to $G(0, \hat{A}_{t+1}, \frac{L_t}{X_t}) = 0$, we get $\hat{A}_{t+1} = \hat{A}(\frac{L_t}{X_t})$, and

$$\hat{A}'(\frac{L_t}{X_t}) = \frac{-\beta h_e(0, \hat{A}_{t+1})(\frac{L_t}{X_t})^{\beta-1}}{h_{eA}(0, \hat{A}_{t+1})(\frac{L_t}{X_t})^{\beta} - h_A(0, \hat{A}_{t+1})} < 0.$$

Q.E.D.

3.3 Equilibria

We look for the initial conditions and geographical factors preventing a given economy to escape stagnation, that is to say, such that the technological level stays always below the threshold triggering education investment. The corresponding equilibria are therefore characterized by $e_t = 0$, for all t, on top of (i) the households' utility maximization under constraints, (ii) the determination of output, (iii) the population dynamics, (iv) the technological progress dynamics, and (v) the determination of productive land. Therefore, a competitive equilibrium is fully determined by the following system of equations (3.10)-(3.15), given β, γ, θ, ω, X, L_0, and A_0:

$$\text{FOCs :} \qquad n_t = \gamma \left(\frac{X_t}{L_t} \right)^{\beta} \tag{3.10}$$

$$e_{t+1} = 0 \tag{3.11}$$

$$\text{Production :} \qquad y_t = f(A_t) h_t \tag{3.12}$$

$$\text{Population :} \qquad L_{t+1} = n_t L_t \tag{3.13}$$

$$\text{Technology :} \qquad A_{t+1} = [1 - \lambda(\omega)][1 + g(0, L_t)] A_t \tag{3.14}$$

$$\text{Land :} \qquad X_t = \chi(\theta, A_t) X \tag{3.15}$$

The competitive equilibrium system above are characterized by the reduced equilibrium dynamics of the population L_t and technology A_t:

$$L_{t+1} = \gamma \left(\chi(\theta, A_t) X \right)^{\beta} L_t^{1-\beta} \tag{3.16}$$

$$A_{t+1} = [1 - \lambda(\omega)][1 + g(0, L_t)] A_t \tag{3.17}$$

for a given initial conditions L_0, A_0 (and $e_0 = 0$).

3.4 Stagnation trap

This section studies the conditions on geographical factors (X, θ, ω), i.e. the amount of "land" available, its suitability, and its degree of isolation, under which an economy starting from specific initial conditions never escapes stagnation. Specifically we characterize the set of geographical factors that do not allow an economy to reach the critical population size $\mathcal{L}(\omega)$ guaranteeing technological growth. As a consequence, the technological level will remain below the take-off threshold, locking the economy at zero-education. Zero-education associated with small population cannot guarantee a technological progress able to offset the losses of technology, so that the economy cannot expand its productive land to enhance fertility and reach a bigger population. This negative feedback loop prevents the economy from escaping stagnation.

Proposition 3.2: *An economy with land X, a given suitability θ for it, and a degree of isolation ω such that*

$$\frac{\mathcal{L}(\omega)}{\chi(\theta, \hat{A}(\mathcal{L}(\omega)/X))X} \geq \gamma^{1/\beta},$$

i.e. with too high effective population density (specifically above $\gamma^{1/\beta}$) —at the population level $\mathcal{L}(\omega)$ guaranteeing technological progress, and the level of technology triggering education investment $\hat{A}(\mathcal{L}(\omega)/X)$— will be locked in a stable steady state with $\tilde{e} = 0$, i.e. no education investment, the lowest level of technology $\tilde{A} = 0$, and a small population $\tilde{L}$, with

$$\tilde{L} = \gamma^{1/\beta}\chi(\theta, 0)X < \mathcal{L}(\omega)$$

for initial conditions $L_0 < \mathcal{L}(\omega)$, $e_0 = 0$, and $A_0 \leq \hat{A}\left(\frac{\mathcal{L}(\omega)}{\chi(\theta,0)X}\right)$, i.e. for an initial population not big enough to guarantee technological progress, no initial education investment, and a level of technology smaller than the one triggering education investment under zero technology.

***Proof*:** The claim follows from the fact that, starting from $e_0 = 0$ and $L_0 < \mathcal{L}(\omega)$, if $L_t < \mathcal{L}(\omega)$, then $L_{t+1} < \mathcal{L}(\omega)$, for all t. In effect, assume $L_t < \mathcal{L}(\omega)$. Given that χ is increasing in A, $L_t < \mathcal{L}(\omega)$, and $\hat{A}' < 0$,

$$A_t \leq \hat{A}\left(\frac{\mathcal{L}(\omega)}{\chi(\theta, 0)X}\right)$$

implies

$$A_t < \min\left\{\hat{A}\left(\frac{L_t}{\chi(\theta, A_t)X}\right), \hat{A}\left(\frac{\mathcal{L}(\omega)}{X}\right)\right\}$$

Moreover $L_t < \mathcal{L}(\omega)$ implies

$$A_{t+1} = [1 - \lambda(\omega)][1 + g(0, L_t)]A_t < A_t$$

and hence

$$A_{t+1} < \hat{A}\left(\frac{L_t}{\chi(\theta, A_t)X}\right)$$

so that $e_{t+1} = 0$ and

$$L_{t+1} = \gamma\left(\frac{\chi(\theta, A_t)X}{L_t}\right)^{\beta} L_t < \gamma\left(\frac{\chi(\theta, \hat{A}(\mathcal{L}(\omega)/X))X}{\mathcal{L}(\omega)}\right)^{\beta} \mathcal{L}(\omega) \leq \mathcal{L}(\omega)$$

(where the last inequality comes from the assumed excessive effective population density). As a consequence, $L_{t+1} < \mathcal{L}(\omega)$ and $e_t = 0$, for all $t = 0, 1, 2, \ldots$. Moreover

$$A_{t+1} = [1 - \lambda(\omega)]^{t+1} \prod_{i=0}^{t} [1 + g(0, L_i)]A_0$$

with $[1 - \lambda(\omega)][1 + g(0, L_t)] < 1$, for all t, so that the technological level converges monotonically to $\tilde{A} = 0$, and the population converges,

according to (3.16), to the level $\tilde{L}$ solution to

$$\gamma \left(\frac{\chi(\theta, 0) X}{L} \right)^{\beta} = 1$$

that is to say

$$\tilde{L} = \gamma^{1/\beta} \chi(\theta, 0) X$$

The economy will, therefore, be locked in the stable steady state characterized by

$$(\tilde{L}, \tilde{A}, \tilde{e}) = \left(\gamma^{1/\beta} \chi(\theta, 0) X,\, 0,\, 0 \right).$$

Q.E.D.

3.5 Conclusion

In early stages of development, i.e. with a small population and a low technological level giving households no incentive to educate their children (as in the first case in Proposition 3.1), if the geographical factors (X, θ, ω) —i.e. land, its suitability, and its degree of isolation— do not allow for a sufficiently large population (i.e. $L < \mathcal{L}(\omega)$), there will be no technological growth in the long run, as well as no education investment to enhance technological progress. As a consequence, the economy will be locked in stagnation (as stated in Proposition 3.2).

If, on the contrary, the geographical factors allow for a sufficiently large population able to offset losses of technology, then the mechanism for the economy to take off is similar to the one in Galor and Weil (2000): for a large enough population, technological growth appears, the increase in the level of technology over time increases the returns to education, households educate their children, and this triggers sustained technological progress and growth, forcing the economy to take off and leave Malthusian stagnation.

By showing how in a set up close to Galor and Weil (2000) — complemented with geographical factors and the possibility of technology

losses— an economy can get trapped in Malthusian stagnation, this chapter makes stand out the role of some geographical and environmental conditions for the development process.

3.6 Appendix

A3.1. Solving optimization problem of household

$$\max_{n_t>0, e_{t+1}\geq 0} \gamma \ln\left[n_t h(e_{t+1}, A_{t+1})\right] + (1-\gamma)\ln\left[(1-n_t[(\frac{L_t}{X_t})^{\beta}+e_{t+1}])y_t\right]$$

The Kuhn-Tucker conditions are

$$\begin{pmatrix} \frac{\gamma}{n_t} - \frac{(1-\gamma)[(\frac{L_t}{X_t})^{\beta}+e_{t+1}]}{1-n_t[(\frac{L_t}{X_t})^{\beta}+e_{t+1}]} \\ \frac{\gamma h_e(e_{t+1},A_{t+1})}{h(e_{t+1},A_{t+1})} - \frac{(1-\gamma)n_t}{1-n_t[(\frac{L_t}{X_t})^{\beta}+e_{t+1}]} \end{pmatrix} = \eta_t \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$

$$\eta_t e_{t+1} = 0$$

$$\eta_t \geq 0$$

i.e.

$$n_t = \frac{\gamma}{(\frac{L_t}{X_t})^{\beta}+e_{t+1}}$$

$$\frac{h_e(e_{t+1}, A_{t+1})}{h(e_{t+1}, A_{t+1})} - \frac{1}{(\frac{L_t}{X_t})^{\beta}+e_{t+1}} = -\eta_t \leq 0$$

$$\eta_t e_{t+1} = 0$$

$$\eta_t \geq 0$$

i.e.

$$n_t = \frac{\gamma}{(\frac{L_t}{X_t})^{\beta}+e_{t+1}}$$

$$h_e(e_{t+1}, A_{t+1})\left[(\frac{L_t}{X_t})^{\beta}+e_{t+1}\right] - h(e_{t+1}, A_{t+1}) \begin{cases} = 0 \text{ if } e_{t+1} > 0 \\ \leq 0 \text{ if } e_{t+1} = 0 \end{cases}$$

Since the optimization problem is not convex, then we have to check the second order conditions. We examine two cases: (i) the positivity constraint of education is binding, and (ii) it is not binding.

- If the positivity constraint of education is binding, i.e. $e_{t+1} = 0$, the bordered Hessian matrix is

$$\bar{H} = \begin{pmatrix} 0 & 0 & -1 \\ 0 & -\frac{\gamma}{n_t^2} - \frac{(1-\gamma)[(\frac{L_t}{X_t})^\beta + e_{t+1}]^2}{\left(1-n_t[(\frac{L_t}{X_t})^\beta + e_{t+1}]\right)^2} & -\frac{1-\gamma}{\left(1-n_t[(\frac{L_t}{X_t})^\beta + e_{t+1}]\right)^2} \\ -1 & -\frac{1-\gamma}{\left(1-n_t[(\frac{L_t}{X_t})^\beta + e_{t+1}]\right)^2} & \gamma\frac{h_{ee}h - h_e^2}{h^2} - \frac{(1-\gamma)n_t^2}{\left(1-n_t[(\frac{L_t}{X_t})^\beta + e_{t+1}]\right)^2} \end{pmatrix}$$

where for lightening notation, we denote $h = h(e_{t+1}, A_{t+1})$, $h_e = h_e(e_{t+1}, A_{t+1})$, and $h_{ee} = h_{ee}(e_{t+1}, A_{t+1})$. We have

$$\left|\bar{H}_1\right| = 0$$

$$\left|\bar{H}_2\right| = \frac{\gamma}{n_t^2} + \frac{(1-\gamma)[(\frac{L_t}{X_t})^\beta + e_{t+1}]^2}{\left(1-n_t[(\frac{L_t}{X_t})^\beta + e_{t+1}]\right)^2} > 0$$

So, the solution to the household's problem in this case is a maximum indeed.

- If the positivity constraint of education is not binding, i.e. $e_{t+1} > 0$, the Hessian matrix is

$$H = \begin{pmatrix} -\frac{\gamma}{n_t^2} - \frac{(1-\gamma)[(\frac{L_t}{X_t})^\beta + e_{t+1}]^2}{\left(1-n_t[(\frac{L_t}{X_t})^\beta + e_{t+1}]\right)^2} & -\frac{1-\gamma}{\left(1-n_t[(\frac{L_t}{X_t})^\beta + e_{t+1}]\right)^2} \\ -\frac{1-\gamma}{\left(1-n_t[(\frac{L_t}{X_t})^\beta + e_{t+1}]\right)^2} & \gamma\frac{h_{ee}h - h_e^2}{h^2} - \frac{(1-\gamma)n_t^2}{\left(1-n_t[(\frac{L_t}{X_t})^\beta + e_{t+1}]\right)^2} \end{pmatrix}$$

We have

$$|H_1| = -\frac{\gamma}{n_t^2} - \frac{(1-\gamma)[(\frac{L_t}{X_t})^\beta + e_{t+1}]^2}{\left(1-n_t[(\frac{L_t}{X_t})^\beta + e_{t+1}]\right)^2} < 0$$

$$|H_2| = |H| = \begin{vmatrix} -\frac{\gamma}{n_t^2} - \frac{(1-\gamma)[(\frac{L_t}{X_t})^\beta + e_{t+1}]^2}{\left(1-n_t[(\frac{L_t}{X_t})^\beta + e_{t+1}]\right)^2} & -\frac{1-\gamma}{\left(1-n_t[(\frac{L_t}{X_t})^\beta + e_{t+1}]\right)^2} \\ -\frac{1-\gamma}{\left(1-n_t[(\frac{L_t}{X_t})^\beta + e_{t+1}]\right)^2} & \gamma\frac{h_{ee}}{h} - \gamma\frac{h_e^2}{h^2} - \frac{(1-\gamma)n_t^2}{\left(1-n_t[(\frac{L_t}{X_t})^\beta + e_{t+1}]\right)^2} \end{vmatrix}$$

We know that, at the solution we have

$$n_t[(\frac{L_t}{X_t})^\beta + e_{t+1}] = \gamma$$

and

$$\frac{h_e}{h} = \frac{1}{(\frac{L_t}{X_t})^\beta + e_{t+1}} = \frac{n_t}{\gamma}$$

So we get

$$|H_2| = \begin{vmatrix} \frac{-\gamma}{n_t^2(1-\gamma)} & \frac{-1}{1-\gamma} \\ \frac{-1}{1-\gamma} & \gamma\frac{h_{ee}}{h} - \frac{n_t^2}{(1-\gamma)\gamma} \end{vmatrix}$$

$$= \frac{-\gamma^2}{n_t^2(1-\gamma)}\frac{h_{ee}}{h} + \frac{1}{(1-\gamma)^2} - \frac{1}{(1-\gamma)^2} = \frac{-\gamma^2}{n_t^2(1-\gamma)}\frac{h_{ee}}{h} > 0$$

So the solution to the household's problem in this case is also a maximum indeed.

4 From agriculture to manufacture: How does Geography matter?

This chapter extends Galor and Mountford (2006) by introducing geographical advantage for agricultural production to explain some historical facts about development process across the globe. We show that the development from an agricultural regime through industrialization to a manufacturing regime occurs simultaneously to the demographic transition and the change in labor structure towards an increasing fraction of skilled labor due to technological progress. The manufacturing sector is economically viable when the technological level is sufficiently high. During the industrialization, the technological progress makes technology become more complementary to skilled labor than to unskilled labor, so that individuals tend to decrease the number of unskilled offspring in order to increase the number of skilled ones. This chapter also shows that a geographical advantage for agriculture helps an economy to be more prosperous in the agricultural regime, but delays the timing of industrialization and the timing of demographic transition. Hence, an economy with more geographical advantage for agriculture may be overtaken in the development process by another with less geographical advantage for agriculture when the level of technology is high enough.

***Keywords*:** Agricultural sector, manufacturing sector, technological level, technological progress, geographical advantage for agriculture.

4.1 Introduction

The development process from agriculture to manufacture along with the demographic transition and the different timings of industrialization, which has lead to the divergence across countries, are interesting topics in development economics that have been subject of intensive research in recent years. This chapter aims at contributing a mechanism helping to explain the development from the agricultural regime to the manufacturing regime occurring along with the demographic

transition and the change in labor structure towards an increasing fraction of skilled labor. Moreover, we highlight the role of geographical advantages for agricultural production in shaping the patterns of economic development, contributing to the divergence and reversals in economic performance across societies. We show that when the technological level is low enough, only the agricultural production is operative. Over time, when the technological level is sufficiently high, the manufacturing production becomes economically viable, and the fraction of skilled labor increases along with the accumulation of technology. This chapter sheds light on the fact that a geographical advantage for agricultural production helps an economy to be more prosperous in the agricultural regime but delays its industrialization and demographic transition. Moreover, the economic performances may be reversed in the development process between countries with more geographical advantage for agriculture and those with less geographical advantage.

The closest papers in the literature to this chapter may be Ashraf and Galor (2012), "*Cultural Diversity, Geographical Isolation, and the Origin of the Wealth of Nations*", and Galor and Mountford (2006), "*Trade and the Great Divergence: The Family Connection*". This chapter, however, differs significantly from Ashraf and Galor (2012) at least in two fundamental aspects: (i) Ashraf and Galor (2012) considers the exogenous effects of geographical isolation on cultural diversity affecting the creation of knowledge to explain the asymmetric evolution across societies, while this chapter explains the asymmetric evolution across societies by considering the entire geographical environment and resources which are advantageous for agricultural production; (ii) Ashraf and Galor (2012) does not take into account the demographic transition in explaining the development process across societies, while this chapter does. This chapter also differs significantly from Galor and Mountford (2006) at least in two aspects: (i) By not taking into account the role of geography, Galor and Mountford (2006) shows international trade to be a prime cause of the Great Divergence in per capita income across countries in the last two centuries, while here we argue that geography is a deeper fundamental cause; (ii)

While this chapter explains the switching from agricultural production to manufacturing production, along with demographic transition, due to technological progress during the development process, Galor and Mountford (2006) does not.

The two last centuries are characterized by the significant technological progress associated with the industrial revolution, the demographic transition, and the generalization of basic education. As a consequence, most societies got out of Malthusian stagnation and experienced a considerable increase in the income per capita and human capital, as well as a decline in the growth rate of population as depicted in the Figure 4.1 for the case of US. The different timings of the transition from agricultural production to manufacturing production across societies have shaped considerably the contemporary world economy (as depicted in the Figure 4.2) and need to be explained.

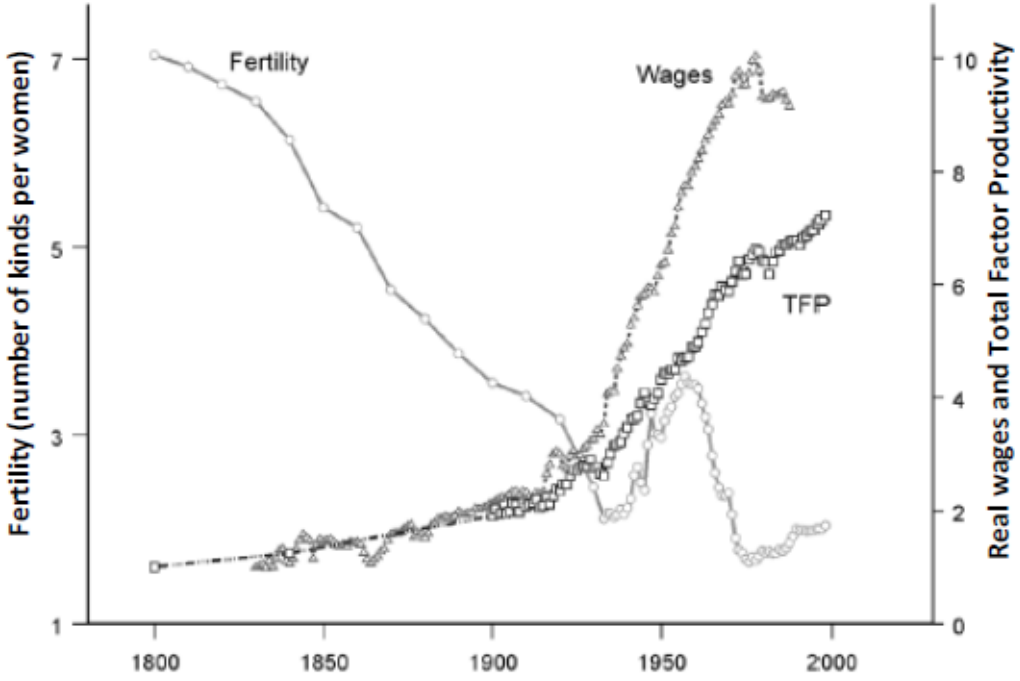

Figure 4.1. Technological progress, fertility, and return to labor; US 1800 - 2000. Quoted in Greenwood and Seshadri (2005)

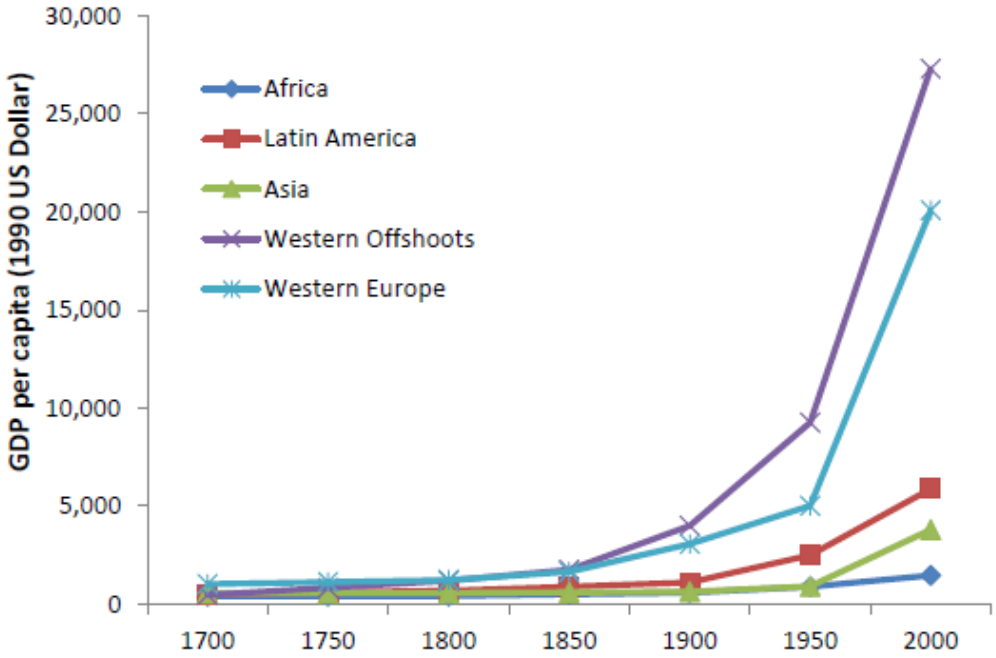

Figure 4.2. The differential timings of the take-off across regions. Source: Maddison (2003)

The divergence across societies in the two last centuries is also marked by the divergence in the per capita level of industrialization (measuring per capita volume of industrial production) which can also be explained in this chapter.

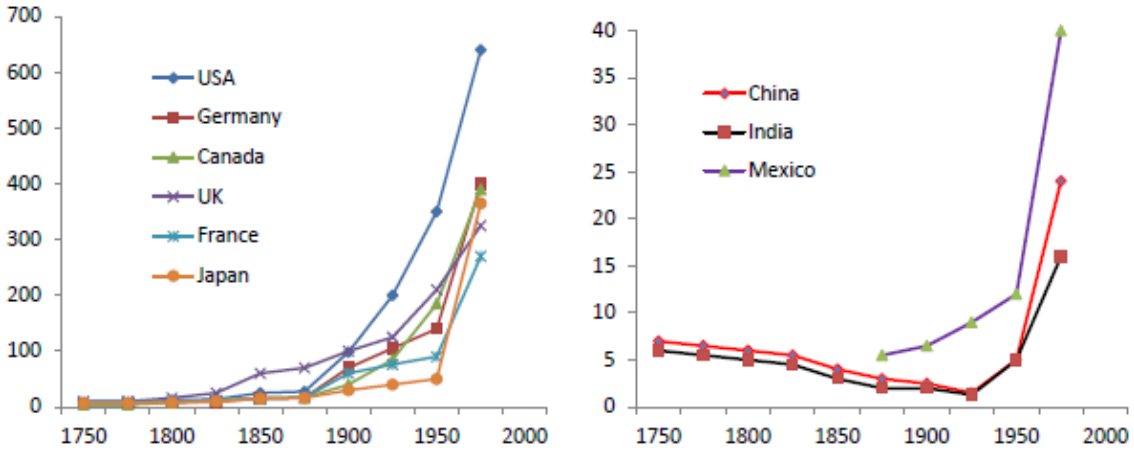

Figure 4.3. Per capita levels of industrialization (UK in 1900 = 100). Source: Bairoch (1982)

The industrial revolution in Western Europe is reflected by the sharp acceleration in process of urbanization, as depicted in the Figure 4.4, which proxies the change in the structure of labor towards an increasing proportion of manufacturing workers.

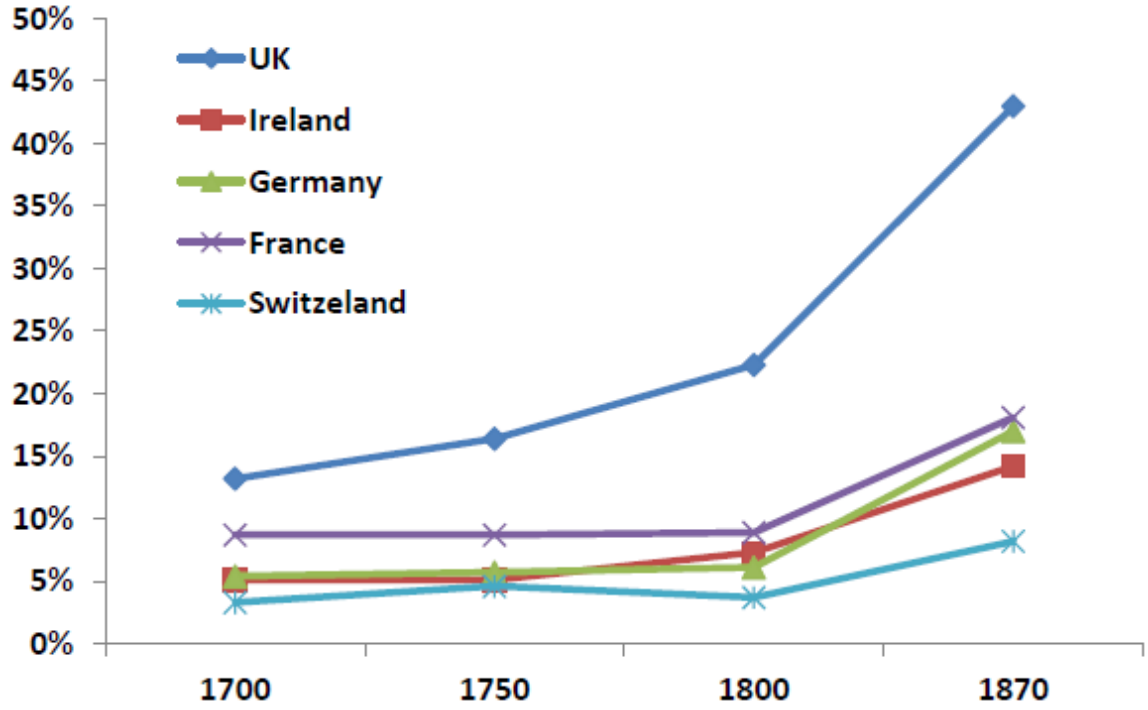

Figure 4.4. Urbanization rate (percentage of city with population larger than 10,000) in Western Europe: 1700–1870. Sources: Bairoch (1988) and De Vries (1984).

Historical evidence shows that the wealth of Asia, particularly China and India, was ahead of Europe until 11th century. Nonetheless, Asia had been already overtaken by the time of the Industrial Revolution in the 18th century and beyond. This story is interesting and can be explained in this chapter.

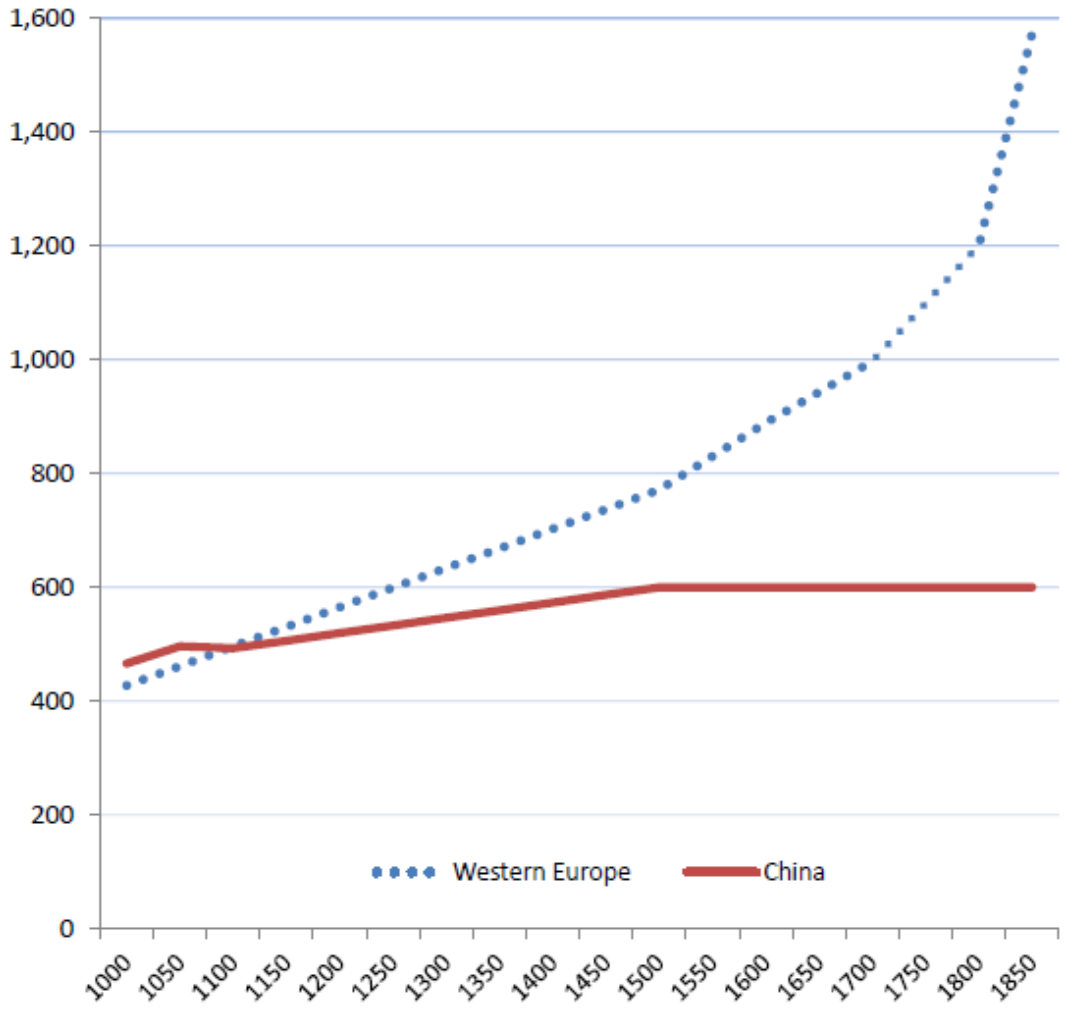

Figure 4.5. Comparative levels of GDP per capita (1990 US Dollar): China and Western Europe, 1000 - 1850 A.D. Source: Maddison (2003)

This chapter extends Galor and Mountford (2006) by introducing a geographical advantage for agricultural production to explain the historical facts depicted in Figures 4.1 to 4.5 above. The rest of the chapter is organized as follows. Section 4.2 reviews the literature. Section 4.3 introduces the benchmark model. Section 4.4 analyzes the equilibrium in the labor market. The effects of technology on fertility and labor structure are analyzed in section 4.5. The competitive equilibrium and dynamical system are presented in section 4.6. Section 4.7 discusses the role of geography and a tale of two countries. Section 4.8 concludes the chapter.

4.2 Literature Review

There is a huge literature in the research fields of this chapter. In this section, we try to review two strands of economic growth and development literature which relate to this chapter: (i) the emergence of human capital and the demographic transition in unified growth models, and (ii) the causes of economic growth and the great divergence.

4.2.1 Human capital and demographic transition in unified growth models

The role of human capital formation on the demographic transition leading to the great divergence across societies has been emphasized in the unified growth theories advanced by Galor and Weil (2000) and Galor and Moav (2002), and confirmed empirically by Glaeser et al. (2004), and recently by Becker and Woessman (2009) and Becker et al. (2010, 2011). More recently, Cervellati and Sunde (2013) also develop a unified growth model to study the mechanism generating economic growth and the demographic transition as well as development. These papers, in general, show theoretically and quantitatively that the increase in technologically-driven demand for human capital and its effect on educational investment play a central role in the transition from Malthusian stagnation, through the demographic transition, to modern sustained economic growth. Doepke (2004) advances a unified growth model to examine whether government policies on education and child labor can account for cross-country variations in fertility decline. He shows that education subsidies have only minor effects, while accounting for child labor regulation is important. Dao (2013) (see chapter 5 below) also proposes a mechanism linking technology, gender inequality, and fertility in a unified growth model to explain the demographic transition accompanied with accelerated growth. Chapter 5 below highlights the role of technological progress not only in freeing women from housework, but also in improving the equality in human capital between males and females, hence contributing significantly to the demographic transition during development process.

Hansen and Prescott (2002) advance a two-sector and one-good overlapping generations model, close to our model, to explain the transition from Malthusian technology to Solow technology along with the change in labor structure (the fraction of labor in the Solow sector) and the demographic transition. The model in this chapter, however, differs from the one in Hansen and Prescott (2002) in some fundamental aspects: (i) our model considers a fertility choice in the utility maximization of the individual, while in Hansen and Prescott (2002), fertility is a consequence of consumption by young households; (ii) Hansen and Prescott (2002) does not take into account the heterogeneity of children, while our model does; and (iii) in Hansen and Prescott (2002), technological progress in both sectors is exogenous, while our model endogenizes technological progress.

4.2.2 The causes of economic growth and the great divergence

What triggered economic growth, and why some countries are so rich and some others so poor are controversial topics in the literature of economic growth. According to Acemoglu (2009, p.109), any explanation for the differences in income per capita based on the differences across countries in technology, physical capital, as well as human capital is incomplete. He argues that if technology, physical capital, and human capital are so important in understanding differences in the wealth of nations, then *why do not some countries improve their technology, accumulate physical capital, and invest in human capital as much as others do*? So there must be other and deeper reasons that are the fundamental causes of economic growth.

So what could these fundamental causes be? Innumerable causes for economic growth have been proposed in the literature by economists, historians, and social scientists. Acemoglu (2009) classifies the main hypotheses into four categories: (i) the luck hypothesis, (ii) the culture hypothesis, (iii) the institutions hypothesis, and (iv) the geography hypothesis. He refers by the "luck" hypothesis to the set of causes leading to divergence in economic performances between countries that are otherwise identical. They can be just different selections

among multiple equilibria. He mentions that "luck" explanations are often theoretically grounded. The empirical plausibility of this hypothesis is another matter. The influences of geographical, cultural, and institutional factors have been at the center of the debate on the explanation of the remarkable transformation of the world income distribution over last two centuries.

The significant influence of cultural factors on economic development of Europe and Asia is originally pioneered by Weber (1905, 1922), and promoted later by Hall (1986), Lal (1998), and Landes (1998, 2006), among others.[1] According to Weber, the origins of the industrialization in Western Europe can be traced to the Protestant reformation and particularly the rise of Calvinism. Ashraf and Galor (2012) state that: "the Weberian viewpoint places the proclivity of European culture towards rationalism and the objective "*disenchantment of the world*" at the forefront in explaining the rise of industry in the Western world". Indeed, his ideas have been proposed as explanations for why Latin American countries are relatively poor (because of their Iberian culture), while North American countries are more prosperous (because of their Anglo-Saxon culture), (see Acemoglu 2009, p.122). However, according to Acemoglu, there are two challenges to the theories of economic growth based on culture: (i) the difficulty of measuring culture; and (ii) the accounting for miracles, such as those of Asian Tigers[2]. If Asian cultural values are crucial for the miracles of these Asian countries, then why these values did not lead to growth before? If these cultural values are crucial for the miracle of South Korea then why they do not spur growth in North Korea?, etc. Acemoglu (2009) argues that these challenges are solvable if we take into account the role of institutions. In this light, hence, culture may be viewed as a complement to institutional factors.

The influence of institutions was first emphasized by North and Thomas (1973), and then advanced by North (1981), Landes (1998), and oth-

[1] As mentioned in Ashraf and Galor (2011), the term "culture" refers to the set of society's norms, beliefs, customs, traditions, taboos, codes of conduct, etc., and it is therefore distinct from the notion of "institutions", which has been regarded in the literature as embodying the sociopolitical environment determined by constitutions, laws, and property rights.

[2] Asian Tigers refer to South Korea, Singapore, Taiwan, and Hong Kong

ers, arguing that institutional factors, which facilitate the property rights and enhance technological progress as well as the diffusion of knowledge, were fundamental factors in the early European transition to sustained economic growth and the great divergence across the globe. Recently, Acemoglu et al. (2005) show that economic outcomes are shaped by economic institutions through the incentives and constraints they impose on economic agents. There is generally a conflict over social choices because different groups and individuals typically benefit differently from different economic institutions. Such a conflict is ultimately solved in favor of groups with greater political power. The political power of each group is determined by political institutions and the distribution of resources. They argue that: "Economic institutions encouraging economic growth emerge when political institutions allocate power to groups with interests in broad-based property rights enforcement, when they create effective constraints on power-holders, and when there are relatively few rents to be captured by power-holders". So, differences in political institutions lead to differences in economic performance, and hence lead to divergence across societies.

The effects of geography on economic growth and divergence between societies have been emphasized recently by Jones (1981), Diamond (1997), Gallup, Sachs and Mellinger (1998), and Ashraf and Galor (2012), and others. The geography hypothesis, first and foremost, is the fact that not all regions of the world are equally apt for living and production. Nature, that is, the ecological and geographical environment of nations may play a major role in their economic experiences. There are at least three main branches of geography hypothesis, each emphasizing different mechanism for how geography affects prosperity. The first one, and also the earliest one, is proposed by Montesquieu in 1748. He believed that climate, in particular heat, shaped human attitudes and effort, and through this channel, it affects both economic and social outcomes. The second one, which is developed by Gunnar Myrdal, emphasizes the impact of geography on the technologies available to a society, especially in agriculture. Myrdal (1968, p.2121) wrote: "*Serious study of the problems of underdevelopment should take*

into account the climate and its impacts on soil, vegetation, animals, humans and physical assets - in short, on living conditions in economic development". The third variant of the geography hypothesis, which is proposed by Jefferey Sachs, links poverty in many areas of the world to their disease burden, emphasizing that "the burden of infectious disease is higher in the tropics than in the temperate zones" (Sachs, 2001). In this chapter, the geographical factor is likely closer to the second version of geography hypothesis.

In the widely popular book "*Guns, Germs, and Steel: The Fates of Human Societies*", Jared Diamond (1997) provides a historical account along with research results from other sciences such as biology, geography, archeology, epidemiology, etc., to explain why the world becomes so unequal across communities. And why some regions, peoples, and cultures developed more quickly than others. Diamond pushes the series of causes and consequences back to 13,000 years ago to reach the conclusion that the origin of the great divergence is due to initial differences in geographical and biological conditions. Like espousing the second view of geography hypothesis, he argues that geographical differences between Americas and Eurasia determined the timing and nature of settled agriculture and, by means of this channel, shaped whether societies were able to develop complex organizations and advanced civilian and military technologies. Although whether the study of human societies can be pursued as scientifically is still a controversy, the work of Diamond depicts the most general picture of human history during the last 13,000 years and shows a way for other sciences to develop theories of development.

Ashraf and Galor (2012) provide a mechanism linking geographical isolation to cultural diversity, affecting the creation and accumulation of knowledge, to explain the asymmetric evolutions across societies. They show that societies characterized by less geographical vulnerability to cultural diffusion benefited from enhanced assimilation, lower cultural diversity, and more intense accumulation of society-specific human capital. So, these societies were more efficient with respect to their production-possibility frontier and flourished in the agricultural stage of development. However, the lack of cultural diffusion

diminished the ability of these societies to adapt to new technological paradigms, which delayed their industrialization and, hence, their take-off to a sustained growth regime. Their empirical analysis shows that (i) geographical isolation prevalent in pre-industrial period has had a persistent negative impact on the extent of contemporary cultural diversity; (ii) pre-industrial geographical isolation had a positive impact on economic development in the agricultural stage but has had a negative impact on income per capita in the course of industrialization; and (iii) cultural diversity has had a positive impact on economic development in the industrialization process.

Other approaches to explain the asymmetric evolutions across the globe are based on the roles of ethnic, linguistic, religious fractionalization, and human genetic diversity. The effects of ethnolinguistic fractionalization on comparative development across societies are examined empirically by Easterly and Levine (1997) and Alesina et al. (2003). These papers have demonstrated that geopolitical factors, which brought a high degree of ethnic fractionalization in some regions of the world, led to the implementation of poor institutions and, hence, to a divergence in the development process across societies. Ashraf and Galor (2013) advance and empirically establish a hypothesis that, in the prehistoric course of *Homo-sapiens* out of Africa, the variation in migratory distance to various destinations across the globe has affected the genetic diversity and has had a persistent hump-shaped effect on comparative economic development. Ashraf and Galor (2013) show that the low diversity of Native American and the high diversity of African have been detrimental for the development process of these regions, while the intermediate levels of diversity of European and Asian have been conducive for their development process.

4.3 The benchmark model

In this chapter, we focus on the geography hypothesis proposed by Myrdal (1968) as mentioned in the literature review section, i.e. we consider the impact of geography on the technology available to a society, especially in agriculture. Of course, geography is not everything

in explaining the asymmetric development across countries, but from previous research in the literature, geography seems to play a crucial role, at least, in the early stages of development.

We consider competitive overlapping generations economies in the process of development with two sectors of production, agriculture and manufacture, and one final output. We assume that the factors of production in the agricultural sector are land and unskilled labor, and the only factor of production in the manufacturing sector is skilled labor. In any period, each individual in the economy chooses a number of skilled children and unskilled children as well as consumption, under the budget constraint, so as to maximize his/her utility.

4.3.1 Environmental resources for agriculture

We refer by "land", which is absent in Galor and Mountford (2006), to the entire geographical environment and natural resources of the economy supporting agricultural production. Due to technological constraint, an economy may not make the most of the available "land" (i.e. entire natural resources and geographical environment) for agricultural production, e.g. people may just occupy the part of their territory and resources that are the most suitable for agriculture. This "part of land" is called "productive land" whose size in period t, X_t, is determined by

$$X_t = \chi(A_t)X \tag{4.1}$$

where $\chi(A_t) \in (0,1)$ is the level of accessibility to the "land" depending on the level of technology in period t, A_t; $\chi'(A_t) > 0$, $\chi''(A_t) < 0$; X is the entire natural resources and geographical environment for agricultural production. Without lost of generality, we normalize $X = 1$ for simplicity. Hereafter, therefore, the natural resources and geographical environment available for agricultural production in period t is $X_t = \chi(A_t)$.

4.3.2 Production

In every period t, output can be produced in the agricultural sector and/or in the manufacturing sector. The agricultural sector uses unskilled labor and "land" as factors of production to produce output according to Cobb-Douglas production technology

$$Y_t^a = \chi(A_t)^\alpha (L_t^u)^{1-\alpha}; \qquad \alpha \in (0,1) \tag{4.2}$$

where L_t^u is the aggregate amount of unskilled labor of the economy in period t.

Indeed, the way the use of "land" for agricultural production is modeled here differs from that in Ashraf and Galor (2012) by making it depend on technology. In Ashraf and Galor (2012), land for agricultural production is always fixed at $X = 1$ regardless of the level of technology. That paper, however, introduces a level of technology specific for agriculture whose dynamics can be traced to depend on the geographical isolation of the economy. That is to say Ashraf and Galor (2012) consider the effect of a geographical factor differing from that in this chapter for agricultural production.

We assume that, in the context of an early stage of development, there is no property rights to "land" (i.e. geographical environment and resources), so that the return to land is zero[3] and the return to labor, or *the inverse demand for labor*, in the agricultural sector in period t is

$$w_t^u = \left(\frac{\chi(A_t)}{L_t^u}\right)^\alpha \tag{4.3}$$

[3]We can justify the assumption of "no property rights to land" as: (i) "land" in our model stands for the entire geographical environment and resources then basically "land" is collectively owned with the proceeds distributed as a lump sum to the population; (ii) we consider a completely aggregate economy, so the model disregards the heterogeneity across households, say labourers and "landlords"; in addition, (iii) although there is no property right to these resources, the economy cannot engage in the over-exploit action of common resources known as the "tragedy of commons" because the accessibility to these resources is constrained by the technological level through the function $\chi(A)$.

Similar to Ashraf and Galor (2012), the output of the manufacturing sector in period t is determined by a linear, constant returns to scale production technology such that

$$Y_t^m = A_t L_t^s \tag{4.4}$$

where L_t^s is the aggregate amount of skilled labor of the economy in period t. The return to labor, or *the inverse demand for labor*, in the manufacturing sector in period t is

$$w_t^s = A_t \tag{4.5}$$

The total labor force of the economy in period t is

$$L_t = L_t^u + L_t^s \tag{4.6}$$

4.3.3 Technology

The dynamics of technological level is defined by

$$A_{t+1} = (1 + g_t)A_t \tag{4.7}$$

where g_t is the rate of technological progress between periods t and $t + 1$. We assume that g_t depends on the average skill of labor in the economy, i.e. it is determined by the fraction of skilled labor in the economy. In particular,

$$g_t = g(h_t)$$

where $h_t = L_t^s / L_t$ is the fraction of skilled labor over the total labor force of the economy in period t; $g(h) > 0$, $g'(h) > 0$, $\forall h \geq 0$.

4.3.4 Individuals

Basically, the individual's problem in this chapter follows from Galor and Mountford (2006). We use superscripts "u" and "s" to denote

for unskilled and skilled labor respectively. In every period $t \in \mathbb{N}$, a generation consists of L_t^u identical unskilled individuals working in the agricultural sector and L_t^s identical skilled individuals working in the manufacturing sector. Individuals live for two periods. In the first period, say childhood, they consume a fraction of their parent's time. In the second period, say adulthood, they allocate their time endowment between child-rearing and labor force participation.

As in Galor and Mountford (2006), the utility of an adult $i \in \{s, u\}$ in period t comes from consumption and the total potential income of his/her children. In particular,

$$u_t^i = \gamma \ln(w_{t+1}^s n_t^{s,i} + w_{t+1}^u n_t^{u,i}) + (1-\gamma) \ln c_t^i \tag{4.8}$$

where $n_t^{s,i}$ and $n_t^{u,i}$ are the number of children to be skilled and unskilled workers respectively; w_{t+1}^s and w_{t+1}^u are their wages in period $t+1$ respectively; c_t^i is total amount of consuming agricultural goods and manufacturing goods by individual i. We assume here these two kinds of good are perfectly substitutable.

The budged constraint of an adult i in period t is

$$c_t^i + w_t^i(\phi^s n_t^{s,i} + \phi^u n_t^{u,i}) \leq w_t^i \tag{4.9}$$

where ϕ^s and ϕ^u are costs in time to raise one offspring to be a skilled worker and a unskilled worker, respectively. We assume that $\phi^s > \phi^u$, implying raising a skilled child is more costly than raising an unskilled one.

An adult individual $i \in \{s, u\}$ in period t chooses the number of skilled and unskilled children and consumption under the budget constraint (4.9) so as to maximize his/her utility (4.8), i.e.

$$\max_{c_t^i > 0;\, n_t^{s,i}, n_t^{u,i} \geq 0} \gamma \ln\left(w_{t+1}^s n_t^{s,i} + w_{t+1}^u n_t^{u,i}\right) + (1-\gamma) \ln c_t^i$$

subject to

$$c_t^i + w_t^i\left(n_t^{s,i}\phi^s + n_t^{u,i}\phi^u\right) \leq w_t^i$$

Solving the individual's problem above (see Appendix A4.1), we have

$$c_t^i = (1-\gamma)w_t^i \tag{4.10}$$

$$n_t^{s,i}\phi^s + n_t^{u,i}\phi^u = \gamma \tag{4.11}$$

where

$$n_t^{s,i} = 0,\ n_t^{u,i} = \frac{\gamma}{\phi^u} \quad \text{if} \quad \frac{w_{t+1}^s}{\phi^s} < \frac{w_{t+1}^u}{\phi^u}$$

$$n_t^{s,i} = \frac{\gamma}{\phi^s},\ n_t^{u,i} = 0 \quad \text{if} \quad \frac{w_{t+1}^s}{\phi^s} > \frac{w_{t+1}^u}{\phi^u}$$

$$n_t^{s,i} > 0 \quad \text{and} \quad n_t^{u,i} > 0 \quad \text{only if} \quad \frac{w_{t+1}^s}{\phi^s} = \frac{w_{t+1}^u}{\phi^u}.$$

It is obvious from the homotheticity of preferences that the fertility and the composition between skilled and unskilled children do not depend on the income of their parent. Hence, hereafter in every period t, we remove the superscript i from variables $n_t^{s,i}$ and $n_t^{u,i}$ to write as n_t^s and n_t^u respectively.

To guarantee the population never collapses, we assume that

$$\frac{\gamma}{\phi^s} \geq 1 \qquad (A4.1).$$

The assumption $(A4.1)$ guarantees the growth rate of population is always non-negative. This assumption is rather consistent with the reality which is observed widely that population grows in most countries.

We know from (4.3) that the return to unskilled labor in period $t+1$ depends on the size of unskilled population, L_{t+1}^u. And it will be apparent in section 4.4 (in Corollary 4.1) that the viability of the manufacturing sector in period $t+1$ depends on the size of population, L_{t+1}. When the manufacturing sector is viable in period $t+1$, the individuals will be indifferent between raising skilled and unskilled

children when it holds $\frac{w^s_{t+1}}{\phi^s} = \frac{w^u_{t+1}}{\phi^u}$. That is to say the choices of numbers of skilled children and unskilled children of individuals in period t depend on the expected structure of labor in period $t+1$. We assume that individuals in period t has a perfect foresight on the structure of labor in period $t+1$, h^e_{t+1}. Hence, in the perspective of individuals, the choices of skilled and unskilled children are determined by

$$n^s_t \phi^s + n^u_t \phi^u = \gamma$$

and

$$\frac{n^s_t}{n^s_t + n^u_t} = h^e_{t+1}$$

i.e.

$$n^s_t = \frac{\gamma h^e_{t+1}}{(\phi^s - \phi^u)h^e_{t+1} + \phi^u} \equiv n^s(h^e_{t+1})$$

$$n^u_t = \frac{\gamma(1 - h^e_{t+1})}{(\phi^s - \phi^u)h^e_{t+1} + \phi^u} \equiv n^u(h^e_{t+1})$$

4.4 Equilibrium in the labor market

It is a fact that, as it will be apparent later, in the early stages of development, when the technological level is sufficiently low so that the labor productivity of the manufacturing sector is low relative to that of the agricultural sector, then output is produced only by the agricultural sector. However, in later stages of development, when the technological level is high enough, increasing the labor productivity of the manufacturing sector, then the manufacturing sector is economically viable.

The inverse demand for labor in the agricultural sector, as in (4.3), increases without bound when employment in that sector decreases. This implies that in a closed economy the agricultural sector is operative in every period. In contrast to the agricultural sector, the

manufacturing sector is operative if, and only if the productivity in this sector is high enough. Proposition 4.1 and Corollary 4.1 below state the condition under which the manufacturing sector is economically viable.

> **Proposition 4.1**: *In any period t, there exists a unique threshold of technological level, $\hat{A}_t = \hat{A}(L_t)$, such that the manufacturing sector is operative if, and only if:*
>
> $$A_t \geq \hat{A}(L_t) \qquad \text{where} \quad \hat{A}'(L_t) < 0$$

***Proof*:** It follows from (4.3), (4.5), and the individual's optimization problem in period $t-1$ that:

(i) If $\frac{A_t}{\phi^s} > \left(\frac{\chi(A_t)}{L_t^u}\right)^{\alpha} / \phi^u$ then $n_{t-1}^s > 0$, and

(ii) $n_{t-1}^s > 0$ only if $\frac{A_t}{\phi^s} \geq \left(\frac{\chi(A_t)}{L_t^u}\right)^{\alpha} / \phi^u$.

Now we prove that

$$n_{t-1}^s \begin{cases} > 0 \quad \text{if} \quad \frac{A_t}{\phi^s} > \left(\frac{\chi(A_t)}{L_t}\right)^{\alpha} / \phi^u & (1) \\ = 0 \quad \text{if} \quad \frac{A_t}{\phi^s} \leq \left(\frac{\chi(A_t)}{L_t}\right)^{\alpha} / \phi^u & (2) \end{cases}$$

(Note that in the denominators there is L_t instead of L_t^u).

The statement (1) can be rewritten as

(1) If $\frac{A_t}{\phi^s} > \left(\frac{\chi(A_t)}{L_t}\right)^{\alpha} / \phi^u$ then $L_t^u < L_t$ (i.e. $L_t^s > 0$ or $n_{t-1}^s > 0$).

We prove (1) by a contradiction. In effect, if $L_t^u = L_t$ (i.e. $L_t^s = 0$ or $n_{t-1}^s = 0$) then

$$\frac{A_t}{\phi^s} > \left(\frac{\chi(A_t)}{L_t}\right)^{\alpha} / \phi^u$$

is equivalent to

$$\frac{A_t}{\phi^s} > \left(\frac{\chi(A_t)}{L_t^u}\right)^{\alpha} / \phi^u$$

which implies (from (i) above) that $n_{t-1}^s > 0$, leading to a contradiction with $n_{t-1}^s = 0$.

The statement (2) can be rewritten as
(2) If $\frac{A_t}{\phi^s} \leq \left(\frac{\chi(A_t)}{L_t}\right)^{\alpha} / \phi^u$ then $L_t^u = L_t$ (i.e. $L_t^s = 0$ or $n_{t-1}^s = 0$).

We prove (2) by establishing its negation. In effect, if $L_t^u < L_t$ (i.e. $L_t^s > 0$ or $n_{t-1}^s > 0$) then, from (ii), it holds

$$\frac{A_t}{\phi^s} \geq \left(\frac{\chi(A_t)}{L_t^u}\right)^{\alpha} / \phi^u.$$

Moreover, since $L_t^u < L_t$ then $\left(\frac{\chi(A_t)}{L_t^u}\right)^{\alpha} / \phi^u > \left(\frac{\chi(A_t)}{L_t}\right)^{\alpha} / \phi^u$, from which we have

$$\frac{A_t}{\phi^s} > \left(\frac{\chi(A_t)}{L_t}\right)^{\alpha} / \phi^u$$

which is the negation of the condition $\frac{A_t}{\phi^s} \leq \left(\frac{\chi(A_t)}{L_t}\right)^{\alpha} / \phi^u$.

Hence, the manufacturing sector is economically viable in period t (i.e. $n_{t-1}^s > 0$) if, and only if, the marginal return to labor in that sector per unit of time raising skilled labor, A_t/ϕ^s, is higher than in the agricultural sector per unit of time raising unskilled labor, $\left(\frac{\chi(A_t)}{L_t}\right)^{\alpha} / \phi^u$, when the entire labor force is employed in the agricultural sector, i.e.

$$\frac{A_t}{\phi^s} > \frac{\left(\frac{\chi(A_t)}{L_t}\right)^{\alpha}}{\phi^u} \qquad \Leftrightarrow \qquad \frac{A_t}{\chi(A_t)^{\alpha}} > \frac{\phi^s}{\phi^u} L_t^{-\alpha}$$

Let $\psi(A_t) = \frac{A_t}{\chi(A_t)^{\alpha}}$, which is monotonically increasing in A_t. In effect,

$$\psi'(A_t) = \frac{\chi(A_t) - \alpha A_t \chi'(A_t)}{\chi(A_t)^{\alpha+1}} > 0$$

since $\chi(A_t) \in (0,1)$ is strictly increasing concave and $\alpha \in (0,1)$, it holds $\chi(A_t) - \alpha A_t \chi'(A_t) > 0$. Moreover,

$$\psi(0) = 0 \qquad \text{and} \qquad \lim_{A_t \to +\infty} \psi(A_t) = +\infty$$

Therefore, given L_t, there exists a unique $\hat{A}_t$ such that

$$\frac{\hat{A}_t}{\chi(\hat{A}_t)^{\alpha}} = \frac{\phi^s}{\phi^u} L_t^{-\alpha} \tag{4.12}$$

By applying the implicit function theorem, we have

$$\hat{A}_t = \hat{A}(L_t) \qquad \text{where} \quad \hat{A}'(L_t) = \frac{-\alpha\phi^s}{L_t^{1+\alpha}\phi^u} \frac{\chi(\hat{A}_t)^{1+\alpha}}{\chi(\hat{A}_t) - \alpha \hat{A}_t \chi'(\hat{A}_t)} < 0$$

Moreover, it is straightforward that

$$\lim_{L_t \to 0^+} \hat{A}(L_t) = +\infty \qquad \text{and} \qquad \lim_{L_t \to +\infty} \hat{A}(L_t) = 0$$

Q.E.D.

Corollary 4.1: *Given $A_t > 0$, there always exists a unique threshold size of population, $\hat{L}_t$, such that the manufacturing sector is operative in period t if, and only if*

$$L_t \geq \hat{L}_t = \left(\frac{\phi^s}{\phi^u}\right)^{1/\alpha} \frac{\chi(A_t)}{A_t^{1/\alpha}}.$$

***Proof*:** It is straightforward from the necessary and sufficient conditions for the manufacturing sector to be operative, i.e., $A_t/\phi^s \geq \left(\frac{\chi(A_t)}{L_t}\right)^{\alpha} / \phi^u$. Q.E.D.

The Figure 4.6 below presents the equilibrium in the labor market and shows intuitively when the manufacturing sector is viable.

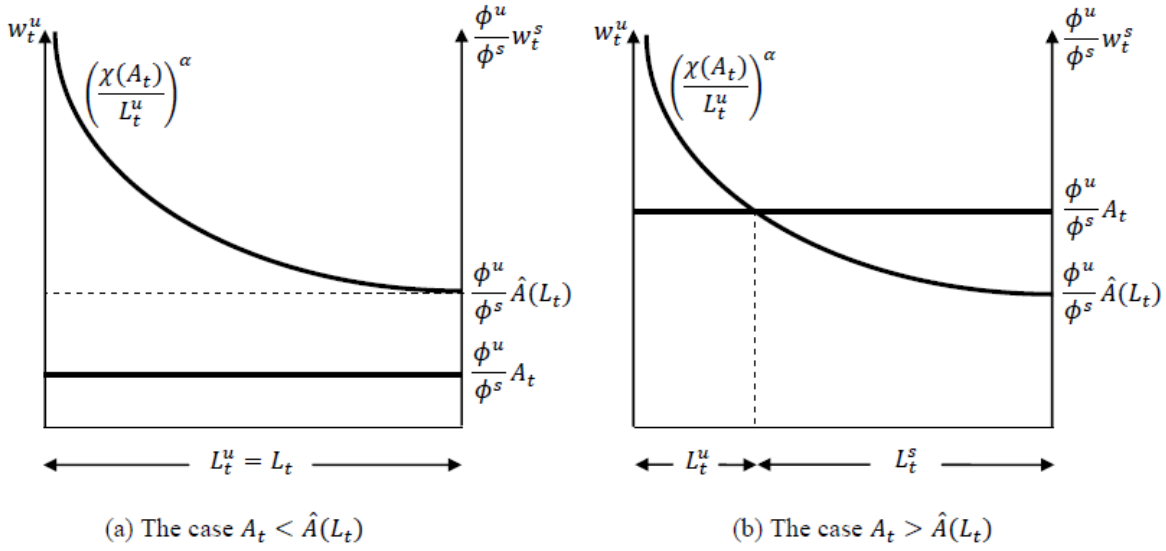

Figure 4.6. The equilibrium in the labor market

4.5 Impact of technology on fertility and the composition of labor

Before examining the dynamical system of the economy in the next section, it is interesting to analyze in this section the impact of technological progress on fertility and the structure of labor. These analyses will help us to understand better the simultaneous evolution of fertility and the structure of labor along with technology.

When the manufacturing is viable, then at the equilibrium it holds

$$\frac{w_t^s}{\phi^s} = \frac{w_t^u}{\phi^u}$$

$$\text{i.e.} \qquad \frac{A_t}{\phi^s} = \frac{\left(\frac{\chi(A_t)}{L_t^u}\right)^\alpha}{\phi^u} \quad \Leftrightarrow \quad L_t^u = \left(\frac{\phi^s}{\phi^u}\right)^{1/\alpha} \frac{\chi(A_t)}{A_t^{1/\alpha}} \tag{4.13}$$

So the fraction of skilled labor is determined by

$$h_t = 1 - \frac{L_t^u}{L_t} = \begin{cases} 0 & \text{if} \quad A_t \leq \hat{A}(L_t) \\ 1 - \left(\frac{\phi^s}{\phi^u}\right)^{1/\alpha} \frac{\chi(A_t)}{A_t^{1/\alpha} L_t} & \text{if} \quad A_t \geq \hat{A}(L_t) \end{cases} \equiv h(A_t, L_t) \tag{4.14}$$

We have

$$\frac{\partial h_t}{\partial A_t} = \begin{cases} 0 & \text{if} \quad A_t < \hat{A}(L_t) \\ \left(\frac{\phi^s}{\phi^u}\right)^{1/\alpha} \frac{\frac{1}{\alpha}\chi(A_t) - \chi'(A_t)A_t}{A_t^{\frac{1+\alpha}{\alpha}} L_t} > 0 & \text{if} \quad A_t > \hat{A}(L_t) \end{cases}$$

and

$$\lim_{A_t \to +\infty} h_t = \lim_{A_t \to +\infty} \left[1 - \left(\frac{\phi^s}{\phi^u}\right)^{1/\alpha} \frac{\chi(A_t)}{A_t^{1/\alpha} L_t}\right] = 1$$

We know from (4.11), lagged one period, that

$$n_{t-1}^s \phi^s + n_{t-1}^u \phi^u = \gamma \tag{4.15}$$

The fertility rate of the economy in period $t-1$ is

$$n_{t-1} = n_{t-1}^s + n_{t-1}^u$$

At the perfect foresight competitive equilibrium, we have $h_t^e = h_t$. It is straightforward that, the fraction of skilled labor in period t can be rewritten as

$$h_t = \frac{n_{t-1}^s}{n_{t-1}^s + n_{t-1}^u} \tag{4.16}$$

From (4.14), (4.15), and (4.16) we have

$$n^s_{t-1} = \frac{\gamma h(A_t, L_t)}{(\phi^s - \phi^u)h(A_t, L_t) + \phi^u} \equiv n^s(h(A_t, L_t))$$

$$n^u_{t-1} = \frac{\gamma(1 - h(A_t, L_t))}{(\phi^s - \phi^u)h(A_t, L_t) + \phi^u} \equiv n^u(h(A_t, L_t))$$

$$n_{t-1} = n^s_{t-1} + n^u_{t-1} = \frac{\gamma}{(\phi^s - \phi^u)h(A_t, L_t) + \phi^u} \equiv n(h(A_t, L_t))$$

As in (4.14), when $A_t \leq \hat{A}(L_t)$ then $h(A_t, L_t) = 0$, therefore, in this case

$$n^s_{t-1} = 0$$

$$n_{t-1} = n^u_{t-1} = \frac{\gamma}{\phi^u}$$

and when $A_t > \hat{A}(L_t)$ then

$$\frac{\partial n^s_{t-1}}{\partial A_t} = \frac{\gamma\phi^u}{\left[(\phi^s - \phi^u)h(A_t, L_t) + \phi^u\right]^2}\frac{\partial h(A_t, L_t)}{\partial A_t} > 0$$

$$\frac{\partial n^u_{t-1}}{\partial A_t} = \frac{-\gamma\phi^s}{\left[(\phi^s - \phi^u)h(A_t, L_t) + \phi^u\right]^2}\frac{\partial h(A_t, L_t)}{\partial A_t} < 0$$

$$\frac{\partial n_{t-1}}{\partial A_t} = \frac{\gamma(\phi^u - \phi^s)}{\left[(\phi^s - \phi^u)h(A_t, L_t) + \phi^u\right]^2}\frac{\partial h(A_t, L_t)}{\partial A_t} < 0$$

And

$$\lim_{A_t \to +\infty} n_{t-1} = \lim_{A_t \to +\infty} n^s_{t-1} = \frac{\gamma}{\phi^s} \qquad \text{and} \qquad \lim_{A_t \to +\infty} n^u_{t-1} = 0$$

Figure 4.7 below presents intuitively the impact of technology on fertility and the structure of labor.

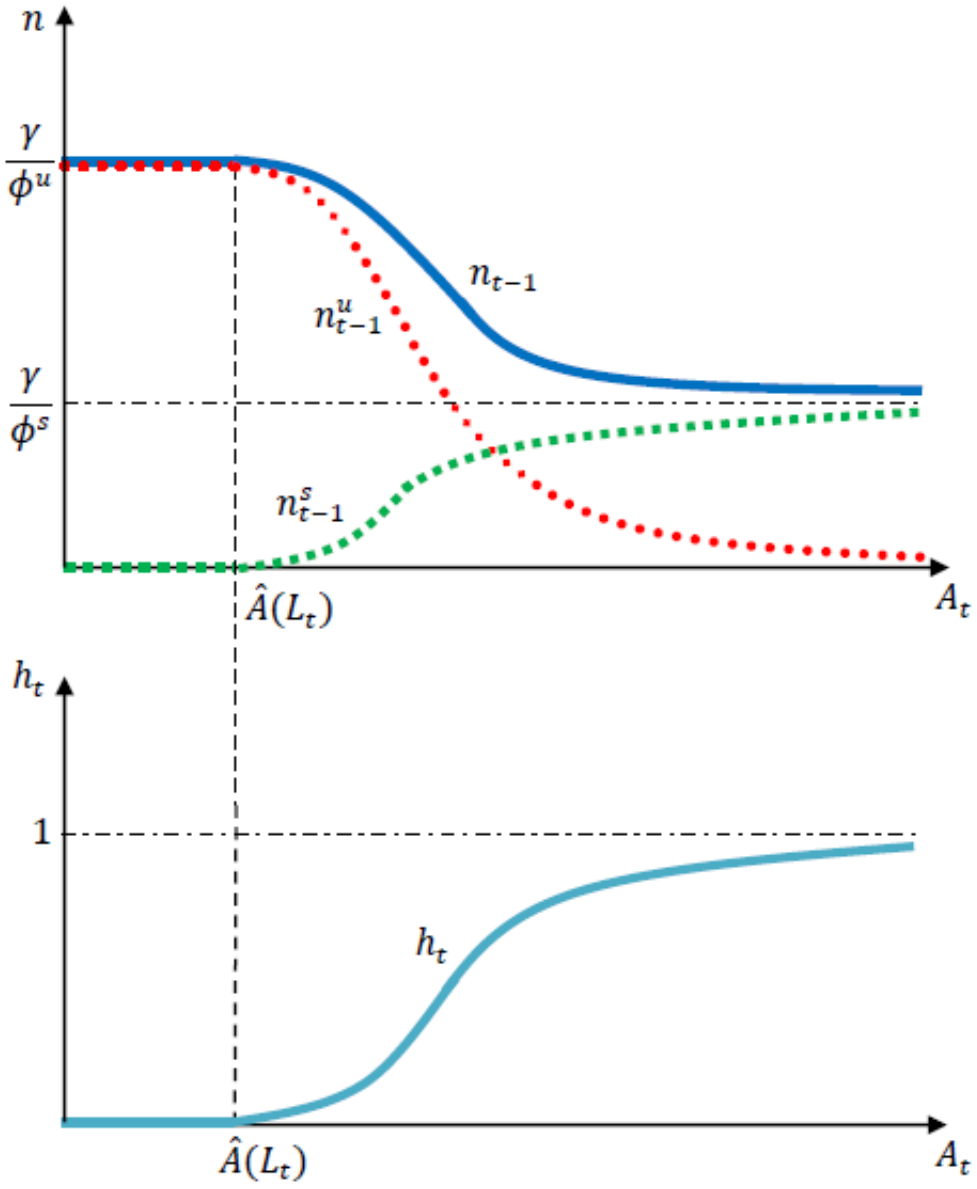

Figure 4.7. Fertility and structure of labor against technology

4.6 Competitive equilibrium and Dynamical system

The perfect foresight competitive equilibrium of the economy is characterized by (i) the household utility maximization under the constraints, (ii) the equilibrium in the labor markets, (iii) the dynamics of the technological level, and (iv) the dynamics of population and

the structure of labor. Therefore, the perfect foresight competitive equilibrium is a sequence of $\{c_t^i,\, n_t^s,\, n_t^u,\, w_t^s,\, w_t^u,\, A_{t+1},\, L_{t+1}, h_{t+1}\}_{t\geq 0}$ is determined by the following system of equations

$$c_t^i = (1-\gamma)w_t^i \qquad i \in \{s,u\}$$

$$n_t^s = \frac{\gamma h_{t+1}^e}{(\phi^s - \phi^u)h_{t+1}^e + \phi^u}$$

$$n_t^u = \frac{\gamma(1-h_{t+1}^e)}{(\phi^s - \phi^u)h_{t+1}^e + \phi^u}$$

$$w_t^s = A_t$$

$$w_t^u = \left(\frac{\chi(A_t)}{L_t(1-h_t)}\right)^{\alpha}$$

$$A_{t+1} = \left[1 + g(h_t)\right] A_t$$

$$L_{t+1} = \frac{\gamma L_t}{(\phi^s - \phi^u)h_{t+1} + \phi^u}$$

$$h_{t+1}^e = h_{t+1}$$

$$h_{t+1} = \begin{cases} 0 & \text{if} \quad A_{t+1} \leq \hat{A}(L_{t+1}) \\ \\ 1 - \left(\frac{\phi^s}{\phi^u}\right)^{1/\alpha} \frac{\chi(A_{t+1})}{A_t^{1/\alpha} L_{t+1}} & \text{if} \quad A_{t+1} \geq \hat{A}(L_{t+1}) \end{cases}$$

for given A_0, L_0, and h_0; where $\hat{A}(L_{t+1})$ is the only solution to (4.12) given L_{t+1}.

The perfect foresight competitive equilibrium can be fully characterized by the following reduced system describing the equilibrium dynamics of technological level, population size, and structure of labor.

$$A_{t+1} = [1 + g(h_t)]\, A_t \tag{4.17}$$

$$L_{t+1} = \frac{\gamma L_t}{(\phi^s - \phi^u)h_{t+1} + \phi^u} \tag{4.18}$$

$$h_{t+1} = \begin{cases} 0 & \text{if} \quad A_{t+1} \leq \hat{A}(L_{t+1}) \\ 1 - \left(\frac{\phi^s}{\phi^u}\right)^{1/\alpha} \frac{\chi(A_{t+1})}{A_{t+1}^{1/\alpha} L_{t+1}} & \text{if} \quad A_{t+1} \geq \hat{A}(L_{t+1}) \end{cases} \tag{4.19}$$

for given initial conditions A_0, L_0, and h_0.

This system defines $\{A_{t+1},\, L_{t+1},\, h_{t+1}\}$ to be a function of its lagged value $\{A_t,\, L_t,\, h_t\}$. In effect, when $A_{t+1} \leq \hat{A}(L_{t+1})$ then

$$A_{t+1} = [1 + g(h_t)]\, A_t$$

$$L_{t+1} = \frac{\gamma L_t}{\phi^u}$$

$$h_{t+1} = 0$$

while when $A_{t+1} \geq \hat{A}(L_{t+1})$, we have

$$A_{t+1} = [1 + g(h_t)]\, A_t$$

$$L_{t+1}\left[(\phi^s - \phi^u)h_{t+1} + \phi^u\right] = \gamma L_t$$

$$L_{t+1}(1 - h_{t+1}) = \left(\frac{\phi^s}{\phi^u}\right)^{1/\alpha} \frac{\chi\left([1 + g(h_t)]\, A_t\right)}{\left([1 + g(h_t)]\, A_t\right)^{\alpha}}$$

implying that there always exists an equilibrium.

We now study the development process of a country from the agricultural regime transiting to the manufacturing regime (or so-called the industrial regime). Consider an economy in its early stages of development characterized by a sufficiently low level of technology, i.e. $A_0 < \hat{A}_0 = \hat{A}(L_0)$ such that only the agricultural sector is operative. In this regime, fertility is very high and constant at a rate γ/ϕ^u, and the entire labor force consists of unskilled labor, because individuals have no economic incentives to raise skilled offspring, leading to technology grow very slowly at a rate $g(0)$.

Over time, from date t onwards, when technology has accumulated enough, i.e. $A_t \geq \hat{A}_t$, guaranteeing the manufacturing sector to be viable, then individuals raise less unskilled offspring in order to raise more skilled ones, because of their utility maximization behavior, and the increase in the technological level. This makes the structure of labor change towards an increasing fraction of skilled workers in the labor force. Because the cost in time of raising a skilled offspring is higher than that of raising an unskilled one, then the increase in the technological level leads to a decrease in fertility accompanied with an increase in the fraction of skilled worker. The higher fraction of skilled worker, in turn, speeds up the technological progress. This feedback loop between technology and the fraction of skilled worker accelerates the industrialization of the economy characterized by the expansion of the manufacturing sector along with the simultaneous reduction in agricultural sector. Over time, the fertility converges to a constant low rate γ/ϕ^s, the fraction of skilled labor converges to 1.

4.7 Geography and a Tale of two countries

We define the timing of industrialization to be the time when the manufacturing sector starts to be economically viable, i.e. when it holds

$$A_t/\phi^s = \left(\frac{\chi(A_t)}{L_t}\right)^{\alpha}/\phi^u$$

This section studies how geography affects the timing and speed of industrialization in the development process of an economy, and it also analyzes a tale of two countries, which differ only in the geography, about the possibility of reversal of fortunes between them. We know from the Proposition 4.1 that the manufacturing sector is economically viable if, and only if

$$\frac{A_t}{\phi^s} \geq \frac{\left(\frac{\chi(A_t)}{L_t}\right)^\alpha}{\phi^u} \qquad \text{i.e} \qquad \frac{A_t}{\chi(A_t)^\alpha} \geq \frac{\phi^s}{\phi^u L_t^\alpha}$$

which implies $A_t \geq \hat{A}(L_t)$.

We say that, country C has more geographical advantage for agriculture production than country B if the following property holds

$$\chi_C(A) > \chi_B(A) \qquad \forall A.$$

Proposition 4.2 below states the effect of geographical advantage for agricultural production on the timing of industrialization.

> **Proposition 4.2:** *In the overlapping generations economy above, the more geographical advantage for agriculture, the later the timings of industrialization and demographic transition.*

***Proof*:** In effect, let us consider two countries B and C which have the same initial conditions $A_0 > 0$, $L_0 > 0$, and $h_0 = 0$ and are identical except that country C has a geographical advantage for agricultural production over country B, i.e.

$$\chi_C(A_t) > \chi_B(A_t) \qquad \forall A_t$$

which implies

$$\frac{A_t}{\chi_B(A_t)^\alpha} > \frac{A_t}{\chi_C(A_t)^\alpha} \qquad \forall A_t. \tag{4.20}$$

Assume that both countries start from the agricultural production regime. As shown in section 4.4, before the manufacturing sector is operative, i.e. when the level of technology is sufficiently low, the population grows at a constant rate γ/ϕ^u. So the populations of countries B and C are the same during the agricultural development process until at least one of these countries starts having manufacturing production, and hence during this process the growth rates of technological levels in the two countries are the same at a constant $g(0)$. So (4.20) implies that the manufacturing sector is operative in country B before in country C, i.e. the timing of industrialization in country B precedes that of country C.

The industrialization in country B triggers the decline in fertility along with the change in the labor structure towards an increase in the fraction of skilled labor. Hence, the demographic transition in country B also precedes that of country C.

Q.E.D.

The Proposition 4.2 shows that the timing of industrialization in a country with less geographical advantage for agricultural production, country B, precedes that of a country with a geographical advantage for agricultural production, country C. This is because during the agricultural development process, the return to labor in agricultural production in country B is always less than that in country C, while the returns to labor in the manufacturing sector, if it were operative, in both countries would be the same due to the same level of technology. So, although in the agricultural development process country C is more prosperous than country B, country C needs a higher level of technology (than country B) to start industrializing, i.e. country C needs more time for technological accumulation to incentivize individuals to raise skilled offspring. As a result, the timing of industrialization in country B precedes that of country C. Country B enters the industrialization process along with a reduction of the fertility rate, an increase of the fraction of skilled labor, and an accelerated technological progress, while country C continues its agricultural development with a high fertility rate and slow technological progress.

4.8 Conclusion

This chapter develops a unified endogenous growth model to explain the development from agricultural production to manufacturing production along with the demographic transition and an increasing fraction of skilled workers in the labor force. The theoretical results are consistent with the phase of industrial revolution and beyond in Western Europe and Western Offshoots. By taking into account the geographical factor, this chapter sheds some light on the fact that a geographical advantage for agricultural production makes a country more prosperous in agricultural regime but delays its industrialization and demographic transition, as well as delays the labor structure change towards an increasing fraction of skilled labor, hence inhibiting technological progress. This may help to explain the reversal in fortunes during the development process between Europe and Asia. This chapter points to geography as a fundamental cause of growth and divergence across societies.

This chapter builds on Galor and Mountford (2006) and Ashraf and Galor (2012) introducing geographical advantage for agricultural production in a single set-up. This single set-up, however, provides some implications, still absent in both Galor and Mountford (2006) and Ashraf and Galor (2012), for empirical investigations. They are: (i) a high total factor productivity in agriculture delays the demographic transition and industrialization; (ii) fertility, the structure of labor, and growth are all shaped by a geographical advantage for agricultural production.

Of course, geography, particularly geographical advantage for agricultural production, is not everything in explaining the development process and divergence across societies. However, this chapter contributes a geographical viewpoint to this literature. Other viewpoints based too on geography, culture, institutions, ethnic, linguistic, religious fractionalization, etc., and also geography to explain the asymmetric development across globe are left for future research, both in theory and empirics.

4.9 Appendix

A4.1. Solving the individual's problem

The utility maximization of individual $i \in (s,u)$ in period t is

$$\max_{c_t>0;\, n_t^{s,i}, n_t^{u,i} \geq 0} \gamma \ln\left(w_{t+1}^s n_t^{s,i} + w_{t+1}^u n_t^{u,i}\right) + (1-\gamma)\ln c_t^i$$

$$\text{subject to} \quad c_t^i + w_t^i\left(n_t^{s,i}\phi^s + n_t^{u,i}\phi^u\right) \leq w_t^i$$

Because the maximization problem is convex, then the Kuhn-Tucker conditions are necessary and sufficient for a maximum. The Kuhn-Tucker conditions of this problem are

$$\begin{pmatrix} \frac{1-\gamma}{c_t^i} \\ \frac{\gamma w_{t+1}^s}{w_{t+1}^s n_t^{s,i} + w_{t+1}^u n_t^{u,i}} \\ \frac{\gamma w_{t+1}^u}{w_{t+1}^s n_t^{s,i} + w_{t+1}^u n_t^{u,i}} \end{pmatrix} = \lambda_1 \begin{pmatrix} 1 \\ w_t^i \phi^s \\ w_t^i \phi^u \end{pmatrix} + \lambda_2 \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix} + \lambda_3 \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}$$

$$c_t^i + w_t^i\left(n_t^{s,i}\phi^s + n_t^{u,i}\phi^u\right) - w_t^i \leq 0$$

$$n_t^{s,i} \geq 0$$

$$n_t^{u,i} \geq 0$$

$$\lambda_1\left[c_t^i + w_t^i\left(n_t^{s,i}\phi^s + n_t^{u,i}\phi^u\right) - w_t^i\right] = 0$$

$$\lambda_2 n_t^{s,i} = 0$$

$$\lambda_3 n_t^{u,i} = 0$$

$$\lambda_1, \lambda_2, \lambda_3 \geq 0$$

i.e.

$$\frac{1-\gamma}{c_t^i} = \lambda_1 > 0$$

$$c_t^i + w_t^i \left(n_t^{s,i} \phi^s + n_t^{u,i} \phi^u \right) - w_t^i = 0$$

$$n_t^{s,i} \geq 0$$

$$n_t^{u,i} \geq 0$$

$$\frac{\gamma w_{t+1}^s}{w_{t+1}^s n_t^{s,i} + w_{t+1}^u n_t^{u,i}} = \frac{1-\gamma}{c_t^i} w_t^i \phi^s - \lambda_2$$

$$\frac{\gamma w_{t+1}^u}{w_{t+1}^s n_t^{s,i} + w_{t+1}^u n_t^{u,i}} = \frac{1-\gamma}{c_t^i} w_t^i \phi^u - \lambda_3$$

$$\lambda_2 n_t^{s,i} = 0$$

$$\lambda_3 n_t^{u,i} = 0$$

$$\lambda_2, \lambda_3 \geq 0$$

- (i) If at the solution the positivity constraint on skilled children is binding and the one to unskilled children is not binding, i.e. $n_t^{s,i} = 0$, $n_t^{u,i} > 0$, then $\lambda_2 \geq 0$, $\lambda_3 = 0$, and hence

$$\frac{\gamma}{n_t^{u,i}} = \frac{1-\gamma}{c_t^i} w_t^i \phi^u$$

$$c_t^i + w_t^i n_t^{u,i} \phi^u = w_t^i$$

Therefore, we have

$$c_t^i = (1-\gamma)w_t^i$$

$$n_t^{u,i} = \frac{\gamma}{\phi^u}$$

- (ii) If at the solution the positivity constraint on skilled children is not binding and the one on unskilled children is binding, i.e. $n_t^{s,i} > 0$, $n_t^{u,i} = 0$, then $\lambda_2 = 0$, $\lambda_3 \geq 0$, and

$$\frac{\gamma}{n_t^{u,i}} = \frac{1-\gamma}{c_t^i} w_t^i \phi^s$$

$$c_t^i + w_t^i n_t^{s,i} \phi^s = w_t^i$$

Therefore, we have

$$c_t^i = (1-\gamma)w_t^i$$

$$n_t^{s,i} = \frac{\gamma}{\phi^s}$$

- (iii) If solution is interior, i.e. $n_t^{s,i} > 0$, $n_t^{u,i} > 0$, then $\lambda_2 = 0$, $\lambda_3 = 0$, and hence

$$\frac{\gamma w_{t+1}^s}{w_{t+1}^s n_t^{s,i} + w_{t+1}^u n_t^{u,i}} = \frac{1-\gamma}{c_t^i} w_t^i \phi^s$$

$$\frac{\gamma w_{t+1}^u}{w_{t+1}^s n_t^{s,i} + w_{t+1}^u n_t^{u,i}} = \frac{1-\gamma}{c_t^i} w_t^i \phi^u$$

which imply that it must hold

$$\frac{w_{t+1}^s}{w_{t+1}^u} = \frac{\phi^s}{\phi^u}$$

Therefore,

$$\frac{\gamma w_{t+1}^u}{w_{t+1}^s n_t^{s,i} + w_{t+1}^u n_t^{u,i}} = \frac{1-\gamma}{c_t^i} w_t^i \phi^u \quad \Leftrightarrow \quad \frac{\gamma}{\frac{w_{t+1}^s}{w_{t+1}^u} n_t^{s,i} + n_t^{u,i}} = \frac{1-\gamma}{c_t^i} w_t^i \phi^u$$

$$\Leftrightarrow \quad \frac{\gamma}{n_t^{s,i}\phi^s + n_t^{u,i}\phi^u} = \frac{1-\gamma}{c_t^i} w_t^i \tag{4.21}$$

We also have

$$c_t^i + w_t^i \left(n_t^{s,i}\phi^s + n_t^{u,i}\phi^u \right) = w_t^i \tag{4.22}$$

then from (4.21) and (4.22) we have

$$c_t^i = (1-\gamma) w_t^i$$

$$n_t^{s,i}\phi^s + n_t^{u,i}\phi^u = \gamma$$

So, in all cases (i), (ii), and (iii), we have

$$c_t^i = (1-\gamma) w_t^i$$

$$n_t^{s,i}\phi^s + n_t^{u,i}\phi^u = \gamma$$

Now, we will clarify under what conditions each corresponding case occurs. We know from the two last equations that the allocation of resources between raising children and consumption is always constant. The last equation shows that the total time for raising children is also always constant. So the optimization problem of individuals boils

down to determine the optimal choice between skilled and unskilled children so as to maximize their total income, $w^s_{t+1}n^{s,i}_t + w^u_{t+1}n^{u,i}_t$, under the constraint $n^{s,i}_t\phi^s + n^{u,i}_t\phi^u = \gamma$.

From the last equation we have

$$0 \leq n^{s,i}_t \leq \frac{\gamma}{\phi^s}$$

$$0 \leq n^{u,i}_t \leq \frac{\gamma}{\phi^u}$$

We have

$$w^s_{t+1}n^{s,i}_t + w^u_{t+1}n^{u,i}_t = w^s_{t+1}n^{s,i}_t + w^u_{t+1}\frac{\gamma - n^{s,i}_t\phi^s}{\phi^u}$$

$$= \left(w^s_{t+1} - w^u_{t+1}\frac{\phi^s}{\phi^u}\right)n^{s,i}_t + w^u_{t+1}\frac{\gamma}{\phi^u}$$

- If $\frac{w^s_{t+1}}{\phi^s} < \frac{w^u_{t+1}}{\phi^u}$, then $\left(w^s_{t+1}n^{s,i}_t + w^u_{t+1}n^{u,i}_t\right)_{\max} = w^u_{t+1}\frac{\gamma}{\phi^u}$ when $n^{s,i}_t = 0$ and $n^{u,i}_t = \frac{\gamma}{\phi^u}$.

- If $\frac{w^s_{t+1}}{\phi^s} > \frac{w^u_{t+1}}{\phi^u}$, then $\left(w^s_{t+1}n^{s,i}_t + w^u_{t+1}n^{u,i}_t\right)_{\max} = \left(w^s_{t+1} - w^u_{t+1}\frac{\phi^s}{\phi^u}\right)\frac{\gamma}{\phi^s}$ when $n^{s,i}_t = \frac{\gamma}{\phi^s}$ and $n^{u,i}_t = 0$.

- If $\frac{w^s_{t+1}}{\phi^s} = \frac{w^u_{t+1}}{\phi^u}$, then any composition satisfying $n^{s,i}_t\phi^s + n^{u,i}_t\phi^u = \gamma$ is an optimal solution.

In summary, we have

$$n^{s,i}_t = 0,\ n^{u,i}_t = \frac{\gamma}{\phi^u} \quad \text{if} \quad \frac{w^s_{t+1}}{\phi^s} < \frac{w^u_{t+1}}{\phi^u}$$

$$n^{s,i}_t = \frac{\gamma}{\phi^s},\ n^{u,i}_t = 0 \quad \text{if} \quad \frac{w^s_{t+1}}{\phi^s} > \frac{w^u_{t+1}}{\phi^u}$$

$$n_t^{s,i} > 0,\ n_t^{u,i} > 0 \quad \text{only if} \quad \frac{w_{t+1}^s}{\phi^s} = \frac{w_{t+1}^u}{\phi^u}.$$

5 Technology, Gender inequality, and Fertility

This chapter proposes a new mechanism linking technology, gender inequality in education, and fertility in a unified growth model. There are three main components to the mechanism: First, increases in the level of technology not only increase the return to human capital but also reduce women's time in doing housework, leaving women with more time for child care and labor-force participation, since technological progress creates labour-saving products for doing housework. Second, the decrease in women's time devoted to housework in the future make households today invest relatively less in education for their sons in order to invest more in education for their daughters because the marginal return to female education is higher than that to male education, therefore, improving the gender equality in education. Third, the better gender equality in education, in turn, accelerates the technological progress. This positive feedback loop generates a demographic transition accompanied with accelerated economic growth, firstly increases in fertility and output simultaneously, and then a decline in fertility along with sustained output growth.

Keywords: Technological level, technological progress, gender inequality in education, fertility, human capital.

5.1 Introduction

Recently, there has been a renewed interest, both from theoretical and empirical viewpoints, in the relationships between gender inequality, fertility and growth to explain some stylized facts during development processes of societies, such as Galor and Weil (1996), Klasen (2002), Lagerlof (2003), Klasen and Lamanna (2009), Doepke and Tertilt (2009), De la Croix and Vander Donckt (2010), Diebolt and Perrin (2013a, b), and others. The stylized facts observed widely across societies are: (i) a negative correlation between fertility and female-

to-male education ratio; (ii) a positive correlation between per-capita income and female-to-male education ratio; (iii) a negative correlation between female-to-male earning ratio and fertility; (iv) a decline over time in the human capital gender gap and earning gap between male and female workers; (v) an increase over time in the labor-force participation of women, and (vi) a demographic transition of societies entering the regime of modern sustained economic growth.

Gender gaps versus fertility and per-capita income growth

The Figure 5.1 below provides the cross-79-country picture of per-capita income (in logarithms, US dollar), and the fertility rate, against gender inequality in education in years 1970 and 2000 which are measured by the ratio of the number of schooling years of women over that of men. The gender inequality in education looks strongly positively correlated with per-capita income, and negatively correlated with fertility.

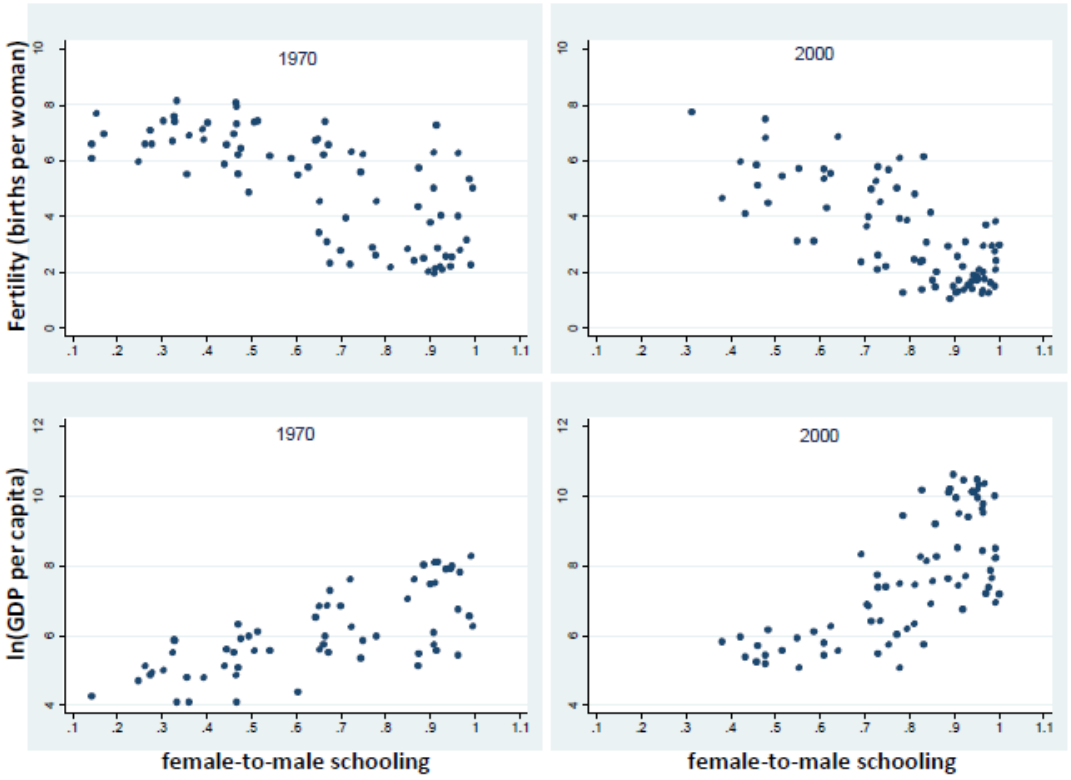

Figure 5.1. Cross-country plots of fertility and per-capita income against gender equality in education. Source: World Bank (2013).

The strong negative correlation between gender inequality in earning and fertility is showed by the dynamics of the gender gap in earnings and fertility in Sweden from 1870 to 1924. One can argue that the gap in earnings between men and women is due to the gap in education between them. So the Figure 5.2 below may reflect the upper graphs in Figure 5.1 above.

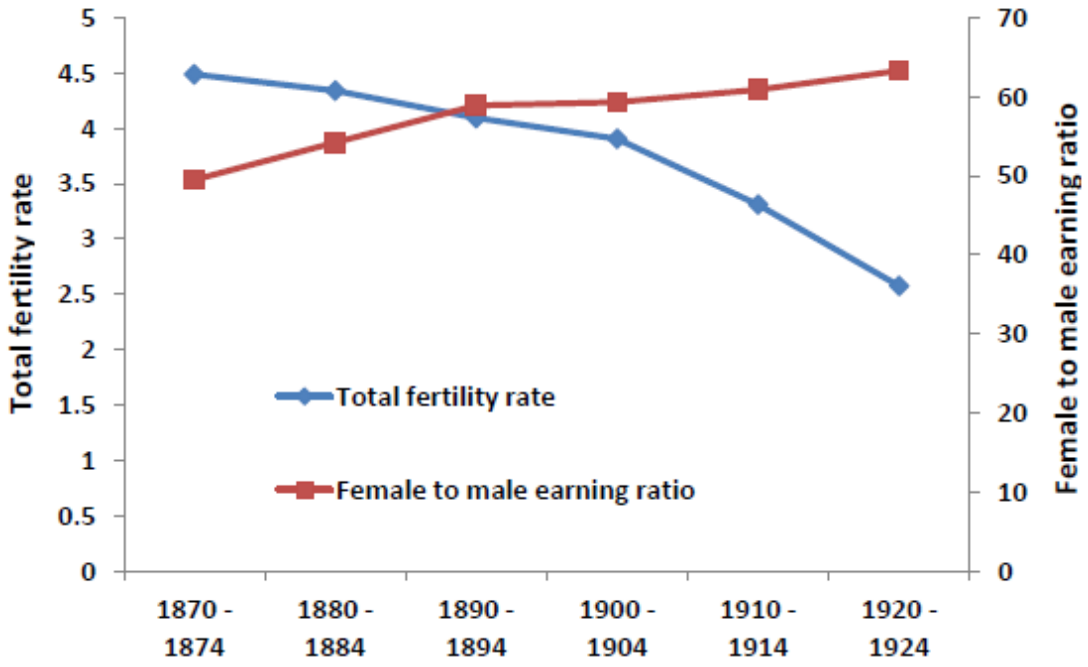

Figure 5.2. Female relative wages and fertility rates in Sweden 1870–1924. Source: Schultz (1985)

The negative correlation between gender inequality in income and fertility has been well explained in the literature, such as in Galor and Weil (1996), Lagerlof (2003), and others. Like these previous research, we argue in this chapter that an increase in female relative wage increases the opportunity cost of raising children more than household income, which makes fertility decline (when the level of technology is sufficiently high). However, this chapter re-explains this stylized fact in a different unified growth framework with a different mechanism leading to a decline in gender inequality in income. Galor and Weil (1996) argue the decline in the gender wage gap over time to be due to the accumulation of physical capital, because physical capital is more

complementary to women's labor than men's. Lagerlof (2003) treats the relative human capital gap between women and men, and hence relative wage gap between them, as exogenous (by fixing this relative gap) and considers impacts of the different relative gender wage gaps on the divergence in fertility and the long run growth across societies. We show in this chapter that the technological progress makes the gender relative gap in human capital decline, and hence improves the gender equality in relative income.

Gender gap in human capital

The Figure 5.3 below uses literacy rates to stand for human capital. The literacy rate is not a unique measurement for human capital. However, the evolution of these rates for men and women can be a proxy for the evolution of human capital, in particular it gives an intuition for the decline in the gender gap in education as well as human capital.

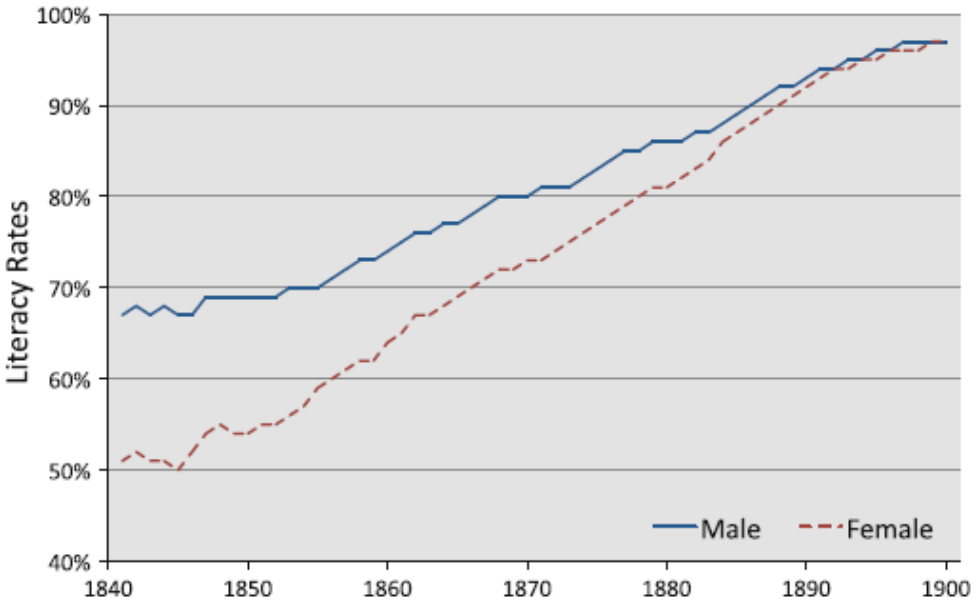

Figure 5.3. The decline in human capital gap between male and female: England 1840 - 1900. Source:[1] Cipolla (1969)

[1]Quoted in Galor (2012)

Most of papers in the related literature study the effects of gender inequality in education or human capital on economic growth, as well as the effect of economic growth on gender inequality in education. A list of selected papers includes Barro and Lee (1994), Dollar and Gatti (1999), Klasen (2002), Lagerlof (2003), and Klasen and Lamanna (2009), etc. However, a question addressed here is why does gender inequality in human capital decrease during the development process? And how does this dynamics of human capital relate to other stylized facts?

Labor-force participation of women

The Figure 5.4 below shows the increase over time in female labor-force participation along with the decrease in fertility in OECD countries.

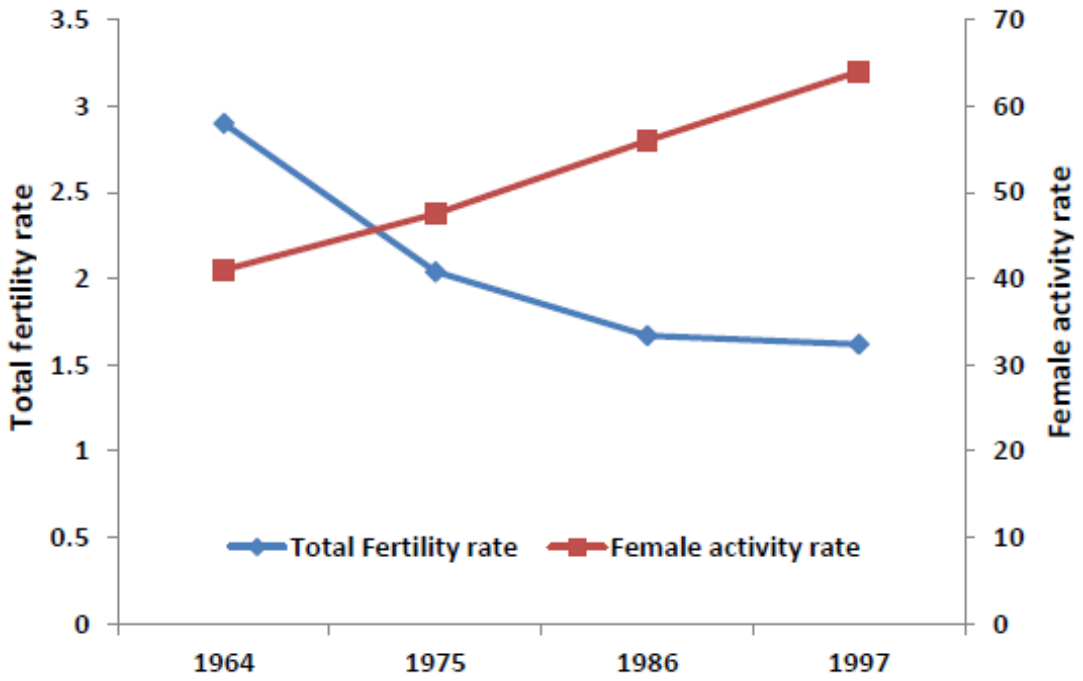

Figure 5.4. Total fertility rate and female activity rate in OECD countries 1960 - 1997. Source: Adserà (2004).

The increase over time in female labor-force participation along with the decrease in fertility due to decreases in the gender wage gap between female and male is examined theoretically in Galor and Weil (1996), Lagerlof (2003), and Bloom et al. (2009). Greenwood et al. (2005) explain the increase in female labor-force participation by technological progress creating labor-saving products for doing housework in a model with constant fertility. This chapter is based on the impact of technological progress on women's time devoted to housework as proposed in Greenwood et al. (2005). However, this chapter differs from Greenwood et al. (2005) is that it endogenizes technological progress and analyzes the improvement in gender inequality in education due to the impact of technological progress on women's time devoted to housework, while explaining the demographic transition during the development process.

Evolution of income growth and population growth in western Europe

The population and output of western Europe experienced three distinct regimes during the last couple of millenia: Malthusian stagnation, the demographic transition, and modern sustained growth. After thousands of years in Malthusian stagnation characterized by very low growth rates in both per-capita income and population, the economy entered the demographic transition in which both growth rates of per-capita income and population increased simultaneously. Later on, the growth rate of per-capita income still increased while the growth rate of population fell, the economy entered then the regime of modern sustained growth (see Figure 5.5). This stylized fact, indeed, is explained by Galor and Weil (2000) and Galor and Moav (2002) in unified growth models, and recently confirmed empirically by Becker et al. (2010, 2011). Like these papers, this chapter also highlights the role of human capital on the technological progress and the demographic transition and the interactions between these issues during the development process. These papers, however, ignore the gender issue and its interactions with technology and fertility, while this chapter does.

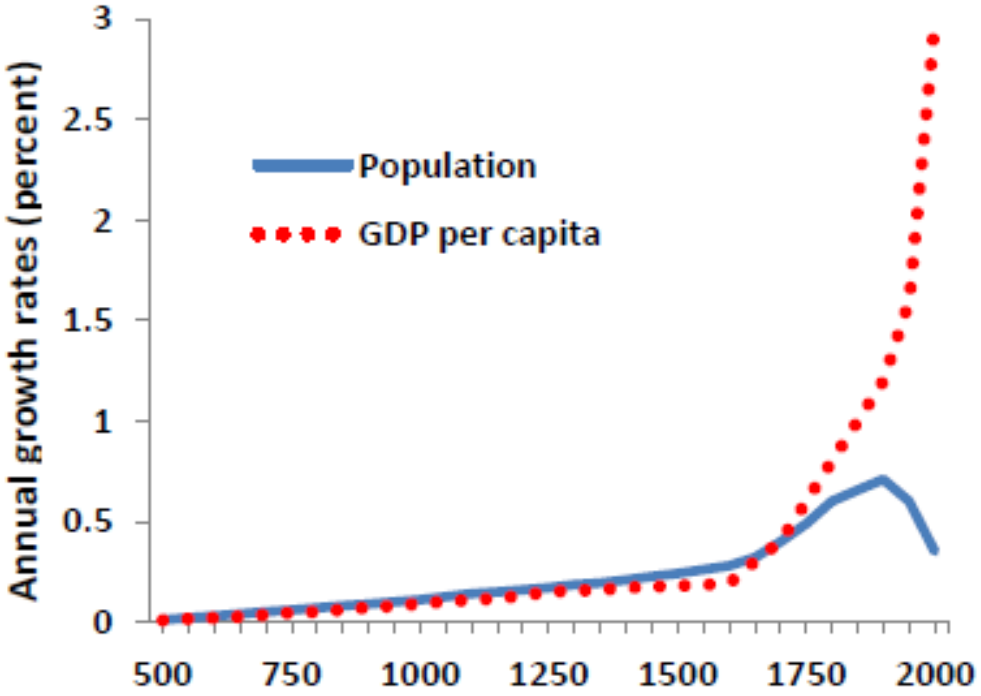

Figure 5.5. The growth rates of per-capita income and population of western Europe in the three regimes. Source: Madison (2001)

The closest literature to this chapter may be the paper of Diebolt and Perrin (2013b), which also proposes a unified growth model to explain the development process considering gender inequality. Basically, Diebolt and Perrin (2013b) also stress the importance of human capital accumulation for economic growth and the positive effect of technological progress on skilled human capital through an increase in the return to education. The most important differences between Diebolt and Perrin (2013b) and this chapter, which leads to different explanations for development process from stagnation to modern sustained growth, are: (i) Diebolt and Perrin (2013b) assume that individuals invest education for themselves when they are adults, while in this chapter we assume that individuals receive educational investment from their parents when they were children; (ii) Diebolt and Perrin (2013b) consider the *bargaining power* of the wife, which depends on both incomes of the wife and the husband, determining the equality in human capital between the wife and the husband, while this chapter considers the *positive effect of technological progress on the possible*

female labor supply which makes the households increase the share in educational investment for their daughters. Therefore, the mechanism for the transition from stagnation to modern sustained economic growth in Diebolt and Perrin (2013b) differs from the one proposed in this chapter. In their model, over time, technological progress triggers the female empowerment which induces women to invest more in human capital for themselves, contributing to human capital accumulation, and hence fostering economic growth. In parallel, the higher female human capital increases the opportunity cost of raising children, making the fertility decline. The mechanism in this chapter, however, is that the technological progress increases the possible female labor supply which makes the households increase the share in educational investment for their daughters. The better equality in human capital between women and men, in turn, accelerates technological progress. These feedback interactions also generate both demographic and economic transitions. However, in Diebolt and Perrin (2013b), the growth rate of population, proxied by fertility, always declines over time along with technological progress and bargaining power of the wife, while it is a fact, which is *captured* in this chapter, that the growth rate of population increases during the early stages of development (before the demographic transition). And because Diebolt and Perrin (2013b) assume that adult individuals spend time for educating themselves to increase their human capital, then although their model generates the decline over time in fertility with the increase in human capital, the explanation for the increase over time in female labor-force participation (corresponding to the stylized fact depicted in the Figure 5.4) is still absent. In addition, Diebolt and Perrin (2013b) assume the human capital formation is linear in education investment (for given other factors, the marginal return to education investment is always constant when education investment exceeds a fixed cost), while we assume in this chapter that the human capital formation is an increasing and concave function of education investment. Therefore, the positive effect of gender equality in education on technological progress comes from a positive externality of women's human capital on their children's human capital formation, while in our model this positive

effect comes from the higher marginal return to female education in human capital formation.

Although there has been a huge empirical literature considering the relationship between gender inequality and economic growth, the theoretical literature on this issue seems rather limited. In addition, to the best of my knowledge, no published paper so far has explained all stylized facts above in one single theoretical model. Most papers in the related literature only explain the combinations of some of the stylized facts above. This chapter aims at contributing a simple unified growth model capturing technological progress, gender inequality in education, fertility and the complex interaction between these issues to explain all stylized facts listed above. This chapter shows that the demographic transition to modern sustained growth, the decline over time in the human capital and earning gender gaps, and the increase over time of the labor-force participation of women are inevitable outcomes of the development process when the driving force for technological progress is the average human capital.

The rest of the chapter is organized as follows. Section 5.2 reviews the related literature on gender inequality and economic growth. Section 5.3 introduces the model. Section 5.4 analyzes the effects of technological progress on gender inequality in education, fertility, and female labor-force participation. The competitive equilibrium and the dynamical system are identified in section 5.5. Section 5.6 analyzes the development process to explain the stylized facts. Section 5.7 concludes the chapter.

5.2 Related literature

Cubers and Reignite (2012) provide an excellent review on gender inequality and economic growth both in theory and empirics. They consider gender inequality in its many aspects. From their review we find that, most papers in the literature conclude that gender inequality is harmful for economic growth, and economic growth help to improve gender equality.

Among previous theoretical works considering gender inequality in growth models, the most cited may be Galor and Weil (1996). The authors advance a theory to interpret the loop relationship among gender gap, fertility and growth in which higher wages of women reduce fertility by raising the opportunity cost of children. The lower fertility, in turn, raises the level of per-capita physical capital. In Galor and Weil (1996), the only difference between men and women, resulting in the gender wage gap, is that men have more physical strength than women. Physical capital is more complementary to mental labor than to physical labor, so women's wage is then increased relatively to that of men due to capital accumulation. The model exhibits multiple steady state equilibria, one in which fertility is high while output and capital per worker are low, and hence women's relative wage is low. The other is characterized by low fertility, high output and capital per worker, hence high women's relative wage. They conclude that countries with a high initial level of capital per worker will converge to a high income level equilibrium with low fertility and high relative wages for women. The opposite would be true for countries with a low initial level of capital per worker. Cavalcanti and Tavares (2007) use Galor and Weil (1996) model, extended to consider for the size of government, to argue that the increase in income per capita and decline in fertility are accompanied by two changes in structure: (i) an increase in the share of government expenditure in total output; and (ii) an increase in women's labor force participation. These two changes are causally related. The approach in Galor and Weil (1996), however, assume that women and men have identical human capital implying male and female receive the same educational investment, whereas differences in education are observed widely in many countries with females typically receiving less education than males.

Lagerlof (2003) examines the links between gender inequality in human capital and long run economic growth. The author points out a threshold of relative equality in human capital between women and men beyond which (i.e. for relatively high equality) the economy can exhibit sustained growth, otherwise the economy converges to a non-growing steady state, a Malthusian trap. The paper of Lagerlof (2003)

implies that inequality in human capital can result in high fertility, low economic growth, and continued gender inequality in providing human capital for males and females, thus generating a poverty trap that calls for public intervention. The paper of Lagerlof, however, assumes that (for whatever reason) men have more human capital than women and the relative inequality in human capital between them is fixed, i.e. it treats this relative inequality as an exogenous variable. In fact, gender inequality may derive from culture, and its size may change endogenously along with technological progress.

Greenwood et al. (2005) provide a channel through which economic growth affects positively gender equality in employment. They argue that technological progress in the household sector is embodied in the form of new labor-saving consumer durables which free up women's time devoted to housework, making them to increase their labor force participation. In this approach, however, the authors explicitly assume that the technological progress and gender gap are exogenous, and ignore the fertility factor as well as education investment.

Doepke and Tertilt (2009) propose an interesting mechanism of a positive effect of growth on gender equality. The authors investigate men's incentive to share power with their wives. They argue that, from a man's perspective, he wants his wife to have no rights. But he cares about his daughters's marital power bargaining vis-à-vis his son-in-law because he is altruistic to his children. In Doepke and Tertilt (2009), an expansion of a wife's legal rights increases human capital investment for her children, helping them to find quality spouses which are also in the preference of their father. Therefore, the father gains from the increasing power of his children's future mothers-in-law because his children will have quality spouses. That is to say men face a trade-off between the rights they want for their own wives and the rights of other women in society. This trade-off shifts over time because of the changing role of human capital driven by technological progress. The authors show that when the returns to education are low, men will vote for the regime in which all family decisions are made solely by the husband. When technological progress changes the importance of human capital men may vote for the regime under which decisions

are made jointly by husband and wife. De la Croix and Vander Donckt (2010) propose a model, which is also based on the intrahousehold bargaining between man and woman, capturing several aspects of gender inequality (such as survival gap, wage gap, social and institutional gap, and educational gap) to analyze their impacts on demographic and economic outcomes for least developed countries. The authors point out the key measures to ease these countries out of poverty trap are to promote survival probabilities of female and infant, which make women more likely active in the market, leading to female education to be more important. A better female education increases the bargaining power of women in the households' decision process, hence decreasing fertility and improving the quality of children, as well as fosters economic growth. One can find more analytical works relating to woman's rights and marital power bargaining in Basu (2006), Fernandez (2009), Doepke and Tertilt (2011), Doepke et al. (2012), and more recently Diebolt and Perrin (2013a, b).

In parallel to theoretical studies, a huge empirical literature has also examined the complex relationship between gender inequality and economic growth. The availability of comprehensive international datasets has allowed the emergence of a large number of time series, cross-section, and panel data empirical studies of this topic. An early study by Barro and Lee (1994) reported a "puzzling" finding that gender inequality in education might increase economic growth. The authors find that when they include male and female primary and secondary schooling in regression, the coefficient associated with female schooling is negative. However, more recent papers have shown the opposite appears, i.e. gender inequality in education reduces economic growth (Dollar and Gatti 1999, Forbes 2000, Knowles et al. 2002, Klasen 2002, Abu-Ghaida and Klasen 2004, Klasen and Lamanna 2009). These papers also explain why Barro and Lee (1994) found the opposite effect and their use of more careful econometric techniques yield the finding that gender inequality in education inhibits economic growth.[2]

[2]Many authors show that Barro and Lee (1994) identified the absence of regional dummy variables, particularly for Latin America and East Asia, making their estimation biased. Their biased

Dollar and Gatti (1999) study the effects of gender gaps in education, health and life expectancy, the legal and economic equality in society and marriage, and the degree of women's empowerment on economic growth. In contrast to Barro and Lee (1994), they find that the coefficient associated with female education is positive, whereas that associated with male education is negative but statistically insignificant. However, the positive effect of female education on growth is nonlinear: increase in female education has no effect on economic growth for countries with very low female education. But, in countries with relatively high female education, increasing it spurs economic growth. Dollar and Gatti (1999) also provide a strong evidence that increases in per capital income lead to improvements in gender equality in education and health care. And, they conclude that societies that have a preference for not investing in girls pay a price for it in terms of slower growth and lower income.

By using cross-country data 1960 - 2000 and panel regression, Klasen (2002) and Klasen and Lamanna (2009) show that gender inequality in education directly affects economic growth by lowering the average level of human capital and indirectly affects economic growth through its impact on population and investment. In contrast to Barro and Lee (1994), they find that the negative coefficient associated with female education disappears when the multicolinearity problems are taken into account and dummy variables of regions are added. Interestingly, these two papers estimate lower and upper bounds of the effect of gender inequality on economic growth. The findings of these papers differ significantly from Dollar and Gatti (1999) in the point that they find that the negative effect of gender inequality in education not only appears among countries with relative high female education, but appears among countries with low female education also. They justify the causes for this difference by the fact that they use a larger

finding may also be related to multicolinearity. In most countries, female and male education are closely correlated, making it difficult to estimate their individual effects. Large standard errors for male and female education in Barro and Lee (1994) and the sudden reversal of this finding in other specifications is a strong evidence of this problem. For more discussion of these issue, see Dollar and Gatti (1999), Forbes (2000), Klasen (2002), and Klasen and Lamanna (2009).

time-series dataset (1960 - 2000 rather than 1975 - 1990). They use a different measure of human capital (the total year of schooling rather than the share of the adult population with secondary education). And they claim that the multicolinearity problem were ignored in Dollar and Gatti (1999).

Along with these empirical studies above, many papers also find out a negative effect of gender inequality in education on economic growth. An incomplete list of papers includes Hill and King (1995), Tzannatos (1999), Lorgelly and Owen (1999), Forbes (2000), Knowles et al. (2002), and Abu-Ghaida and Klasen (2004), etc.

5.3 The model

We consider a two-sex overlapping generations economy in which people live for two periods. In the first period, they are children and they are raised as well as provided educational investment by their parents. In the second period, they are adults, getting married to be households. The representative agent hereafter is the household (or couple). In the adulthood period, agents are endowed with one unit of time to raise children, to do housework, and to supply labor to market to earn income for their consumption and educational investment for their children.

5.3.1 Preferences and constraints

In any period $t \in \mathbb{N}$, the economy consists of L_t identical households. Each household is composed by one man and one woman. Each member in household is endowed one unit of time. Households allocate their time resources for labor supplied to the market to earn income, for child-rearing, and for housework. As in Becker (1985, p52), we assume that in a household, the woman is fully responsible for child-rearing and doing other housework.[3] The time devoted to raising chil-

[3]One may think that working in the market requires not only human capital but physical strength of workers also. So men have more advantage to supply their labor to the market than women have. One may also justify that for some psychological reasons as well as conventions of societies,

dren and doing housework cannot be used to work in the market. We use indexes m and f to denote for sexes male and female respectively. Households born at date $t-1$ have preferences over their consumptions at date t, the number of their children, and the income that their children can earn when they are adults as follows

$$u_t = \gamma \ln \left(n_t w_{t+1} \left[h^m_{t+1} + [1 - \varphi(A_{t+1})] h^f_{t+1} \right] \right) + (1-\gamma) \ln c_t \quad (5.1)$$

where c_t is consumption of household in period t; n_t is the number of children (since the basic unit of analysis in this model is a couple then n_t is in fact the number of couples of children that each couple t has)[4]; w_{t+1} is the return to human capital in period $t+1$; h^m_{t+1} and h^f_{t+1} are human capital of each male and female children respectively; A_{t+1} is the level of technology in period $t+1$; and $\varphi(A_{t+1})$ is the time that women will spend for housework in period $t+1$.

We assume here that the time for doing housework is *inevitable* and households in period t perfectly foresee that their daughters will spend that time doing housework in the period $t+1$, while households do not have perfect foresight on the fertility of their offspring. Therefore, households consider $w_{t+1} \left[h^m_{t+1} + [1 - \varphi(A_{t+1})] h^f_{t+1} \right]$ as the potential income that each couple of their offspring can earn when they are adults.

Greenwood et al. (2005) provide a theoretical framework to argue that technological advancements in the household sector play crucial role in liberating women from housework. It is obvious that, along with technological progress, it is the appearance of household sector products such as washing machines, vacuum cleaners, refrigerators, etc., which helps women save time in doing housework. The appearance of some other products, such as frozen foods and ready made clothes, due to technological progress also liberate women from housework. So,

women are supposed to be responsible for child care and other housework. In fact, introducing time for men to do housework does not change the qualitative analysis as long as men take care of the housework less than women do.

[4]Here we also assume that the gender birth ratio (male over female) is $1:1$ which is closed to the natural gender ratio $1.05:1$.

it is rather plausible to assume that the time devoted to housework is decreasing in the level of technology. Hence, we assume that[5]

$$\varphi'(A) < 0,\ \varphi''(A) > 0 \quad \text{and} \quad 0 = \lim_{A \to +\infty} \varphi(A) < \bar{\varphi} = \varphi(0) < 1 \qquad (A5.1)$$

Human capital formation for a child with sex $i \in \{m, f\}$ is

$$h_{t+1}^i = (e_{t+1}^i)^\theta \qquad (5.2)$$

where $\theta \in [\frac{1}{2}, 1)$ for a reason to be apparent in section 5.4.2; e_{t+1}^i is educational investment for a child with sex $i \in \{m, f\}$.

The budget constraint of the household born at date $t-1$ is

$$c_t + n_t(\rho w_t h_t^f + e_{t+1}^m + e_{t+1}^f) \leq w_t(h_t^m + [1 - \varphi(A_t)]h_t^f) \qquad (5.3)$$

where ρ is the cost in time required to raise one couple of children physically. Since only the wife takes care children in the household and the time raising children cannot be used to work in the market, then the opportunity cost for raising one couple of children is $\rho w_t h_t^f$.

Since each person in period t is endowed one unit of time, then the time constraint of the woman in period t is

$$\rho n_t + \varphi(A_t) \leq 1 \qquad (5.4)$$

5.3.2 Production and technology

In each period t, output can be produced out of human capital according to the following production function

$$Y_t = f(A_t)H_t \qquad (5.5)$$

[5]The assumption $\lim_{A \to +\infty} \varphi(A) = 0$ is a simplification. The analysis does not change qualitatively if we set $\lim_{A \to +\infty} \varphi(A) = \underline{\varphi} \in (0, \bar{\varphi})$.

where Y_t is aggregate output produced in period t; H_t is aggregate human capital supplied to production in period t; $f(A_t) > 0$, $f'(A_t) > 0\ \forall A_t > 0$;[6] and $\lim_{A_t \to +\infty} f(A_t) = \bar{f}$.

The output per household in period t is

$$y_t = \frac{Y_t}{L_t} = f(A_t)\left(h_t^m + [1 - \rho n_t - \varphi(A_t)]h_t^f\right) \tag{5.6}$$

The return to human capital in period t is

$$w_t = f(A_t) \tag{5.7}$$

The dynamics of technology is

$$A_{t+1} = (1 + g_t)A_t \tag{5.8}$$

where g_t is the rate of technological progress between periods t and $t+1$. We assume that g_t depends on the average human capital of the working generation t, i.e.

$$g_t = g\left(\frac{h_t^m + h_t^f}{2}\right) \tag{5.9}$$

where $g(h) > 0$, $g'(h) > 0$, $\forall h \geq 0$.

5.3.3 Household's optimization

Households of generation t choose the optimal mixture of quantity and quality of their children and supply their remaining time, after finishing housework, in the labor market to consume their wages so as to maximize their inter-temporal utility function. Substituting (5.2) into (5.1), the optimization of the representative household is

[6] Here we simplify the production function by ignoring the role of physical capital to focus on the role of human capital.

$$\max_{\substack{c_t, n_t > 0 \\ e^m_{t+1}, e^f_{t+1} \geq 0}} \gamma \ln \left(n_t w_{t+1} \left[(e^m_{t+1})^\theta + [1 - \varphi(A_{t+1})](e^f_{t+1})^\theta \right] \right) + (1-\gamma) \ln c_t$$

subject to

$$c_t + n_t(\rho w_t h^f_t + e^m_{t+1} + e^f_{t+1}) \leq w_t(h^m_t + [1 - \varphi(A_t)]h^f_t) \tag{5.10}$$

$$\rho n_t + \varphi(A_t) \leq 1 \tag{5.11}$$

for given w_t, h^m_t, h^f_t, A_t, perfectly foreseen w_{t+1}, A_{t+1}, and parameters γ, θ, ρ.

Solving this problem (see Appendix A5.1 and A5.2), we get the optimal choice of a household

$$n_t = \begin{cases} \frac{\gamma(1-\theta)}{\rho} \left(\frac{h^m_t}{h^f_t} + 1 - \varphi(A_t) \right) & \text{if} \quad \gamma(1-\theta) \left(\frac{h^m_t}{h^f_t} + 1 - \varphi(A_t) \right) + \varphi(A_t) < 1 \\ \frac{1-\varphi(A_t)}{\rho} & \text{if} \quad \gamma(1-\theta) \left(\frac{h^m_t}{h^f_t} + 1 - \varphi(A_t) \right) + \varphi(A_t) \geq 1 \end{cases} \tag{5.12}$$

$$c_t = \begin{cases} (1-\gamma) w_t (h^m_t + [1 - \varphi(A_t)]h^f_t) & \text{if} \quad \rho n_t + \varphi(A_t) < 1 \\ \frac{(1-\gamma) w_t h^m_t}{1-\gamma+\gamma\theta} & \text{if} \quad \rho n_t + \varphi(A_t) = 1 \end{cases} \tag{5.13}$$

$$e^m_{t+1} = \begin{cases} \frac{\theta \rho w_t h^f_t}{(1-\theta)(1+[1-\varphi(A_{t+1})]^{\frac{1}{1-\theta}})} & \text{if} \quad \rho n_t + \varphi(A_t) < 1 \\ \frac{\gamma\theta\rho w_t h^m_t}{(1-\gamma+\gamma\theta)[1-\varphi(A_t)](1+[1-\varphi(A_{t+1})]^{\frac{1}{1-\theta}})} & \text{if} \quad \rho n_t + \varphi(A_t) = 1 \end{cases} \tag{5.14}$$

$$e^f_{t+1} = \begin{cases} \frac{\theta \rho w_t h^f_t}{(1-\theta)(1+[1-\varphi(A_{t+1})]^{\frac{1}{\theta-1}})} & \text{if} \quad \rho n_t + \varphi(A_t) < 1 \\ \frac{\gamma\theta\rho w_t h^m_t}{(1-\gamma+\gamma\theta)[1-\varphi(A_t)](1+[1-\varphi(A_{t+1})]^{\frac{1}{\theta-1}})} & \text{if} \quad \rho n_t + \varphi(A_t) = 1 \end{cases} \tag{5.15}$$

From (5.12) we find that when the time constraint of the wife is not binding, then the fertility is increasing in the human capital gap between the husband and the wife. In the case the time constraint of

the wife is not binding, consumption of the household is increasing linearly in the potential income of household (abstracting the opportunity costs for doing housework) and educational investment for each child are increasing linearly in the potential income of the wife. While when the time constraint of the wife is binding, i.e. the wife spends her full time child-rearing and doing housework, then the consumption of the household, and educational investment for male and female children are increasing linearly in the real income of the household, i.e. the income of the husband.

5.4 Gender inequality and fertility with technology

Before examining the dynamical system, it is interesting to analyze the impact of technological progress on gender inequality in education and fertility. These analyses will help us to understand better the simultaneous evolution of gender inequality in education and fertility along with technology.

5.4.1 Gender inequality in education

We define the following measure of gender inequality (female over male) in education in period t

$$\mu_t = \frac{e_t^f}{e_t^m} \tag{5.16}$$

where (5.16) implies that a complete equality in education between genders in period t appears when $\mu_t = 1$, and an education bias toward male (female) when $\mu_t < (>)1$.

From (5.14) and (5.15), we have

$$\mu_{t+1} = \frac{1 + [1 - \varphi(A_{t+1})]^{\frac{1}{1-\theta}}}{1 + [1 - \varphi(A_{t+1})]^{\frac{1}{\theta-1}}} = [1-\varphi(A_{t+1})]^{\frac{1}{1-\theta}} \equiv \mu(A_{t+1}) < 1 \tag{5.17}$$

$$\mu'(A_{t+1}) = \frac{1}{\theta - 1}[1 - \varphi(A_{t+1})]^{\frac{\theta}{1-\theta}}\varphi'(A_{t+1}) > 0$$

$$\mu''(A_{t+1}) = \frac{1}{\theta - 1}\times$$

$$\left[\frac{\theta}{1-\theta}[1 - \varphi(A_{t+1})]^{\frac{2\theta-1}{1-\theta}}\varphi'(A_{t+1})^2 + [1 - \varphi(A_{t+1})]^{\frac{\theta}{1-\theta}}\varphi''(A_{t+1})\right] < 0$$

So the gender inequality in education is biased towards male, strictly increasing, and concave in the level of technology. Under the assumption $(A5.1)$ we have

$$\lim_{A_{t+1}\to+\infty}\mu(A_{t+1}) = 1 \quad \text{and} \quad \lim_{A_{t+1}\to 0^+}\mu(A_{t+1}) = (1-\bar{\varphi})^{\frac{1}{1-\theta}}$$

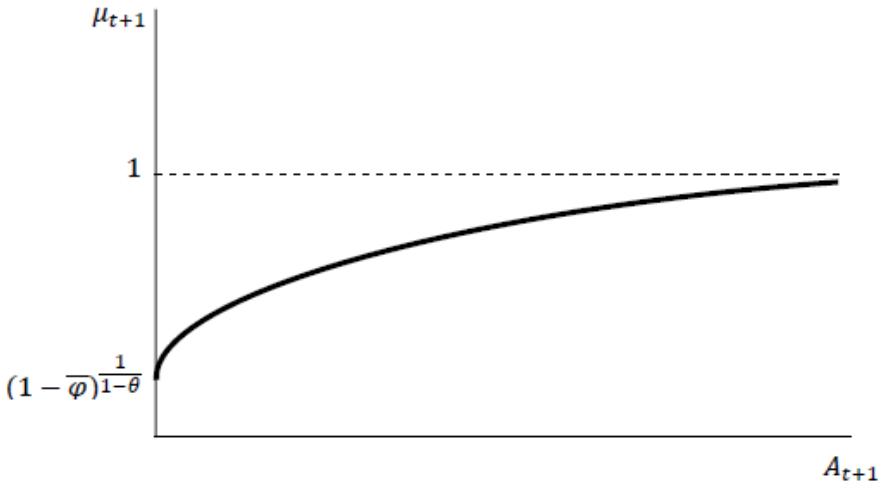

Figure 5.5. Gender inequality in education against technology

From (5.14) and (5.15), we also have

$$e^m_{t+1} + e^f_{t+1} = \begin{cases} \frac{\theta\rho w_t h^f_t}{1-\theta} & \text{if } \rho n_t + \varphi(A_t) < 1 \\ \\ \frac{\gamma\theta\rho w_t h^m_t}{(1-\gamma+\gamma\theta)[1-\varphi(A_t)]} & \text{if } \rho n_t + \varphi(A_t) = 1 \end{cases} \tag{5.18}$$

From (5.17) we find that the gender inequality in education is always biased towards males, i.e. male children receive more educational investment than female children do. This is because when children become adults in the period $t+1$, the women have to spend a fraction of time to do housework while men do not. The time devoted to housework cannot earn income. So economically, for a given amount for education investment per one couple of children as in (5.18), a rational household in period t invests less education on their daughter and invest more on their son. Interestingly, the inequality in education is decreasing in the level of technology in period $t+1$. The reason is rather intuitive, when a higher level of technology in period $t+1$ reduces the time that women devote to housework, then they have a chance to increase their participation to the labor force, leading their parents in period t, by utility optimization behavior, to invest more in education for their daughters to increase their labor productivity. As a result, educational investments for male children decrease. Consequently, the inequality in education decreases with an increase in technology.

Now it is interesting to examine the impact of gender inequality in education on the growth rate of technological progress.

> **Proposition 5.1:** *In the overlapping generations economy above, in any period t, the better the gender equality in education, the higher the growth rate of technological progress. This also implies that the growth rate of technological progress, g_t, is maximum when complete gender equality in education prevails.*

***Proof*:** In effect, from (5.18) with one period lagged, let us denote

$$\Sigma_t = e_t^m + e_t^f$$

which is determined in period $t-1$ and independent of the gender inequality in period t.

The growth rate of technology between period t and $t+1$ is defined in (5.9), i.e.

$$g_t = g\left(\frac{(e_t^m)^\theta + (e_t^f)^\theta}{2}\right) = g\left(\frac{(\Sigma_t - e_t^f)^\theta + (e_t^f)^\theta}{2}\right)$$

so that

$$\frac{\partial g_t}{\partial e_t^f} = g'\left(\frac{(\Sigma_t - e_t^f)^\theta + (e_t^f)^\theta}{2}\right)\frac{\theta}{2}\left[(e_t^f)^{\theta-1} - (\Sigma_t - e_t^f)^{\theta-1}\right]$$

Since $g'(\cdot) > 0$ and $\frac{1}{2} \leq \theta < 1$ then

$$\frac{\partial g_t}{\partial e_t^f}\begin{cases} > 0 & \Leftrightarrow \quad e_t^f < \Sigma_t/2 \\ = 0 & \Leftrightarrow \quad e_t^f = \Sigma_t/2 \\ < 0 & \Leftrightarrow \quad e_t^f > \Sigma_t/2 \end{cases} \tag{5.19}$$

Since from (5.16) and (5.17), it holds

$$\mu_t = \frac{e_t^f}{\Sigma_t - e_t^f} < 1 \quad \forall t \quad \text{then} \quad e_t^f < \Sigma_t/2 \quad \forall t$$

so that an increase in education of women e_t^f (when $e_t^f < \Sigma_t/2$), i.e. a better gender equality in education, leads to a higher growth rate of technological progress. And (5.19) also implies that in the period t, the growth rate of technological progress g_t is maximum when a complete gender equality in education prevails, i.e. $\mu_t = 1$.

Q.E.D.

Proposition 5.1 is consistent with empirical results in Klasen (2002) showing the positive effect of gender equality in education on economic growth. Indeed, Proposition 5.1 shows a positive effect of gender equality in education on the growth rate of technology which, in turn, affects positively the income growth. This is because in any period t the marginal return to education is higher for the female children

than for male children. Hence, for a given amount of educational investment for children, investing more in female children until complete gender equality appears would increase the average human capital for the economy. As a result, the higher the average human capital, the higher the growth rate of technological progress. The empirical evidence for the statement in Proposition 5.1 and its mechanism can be found in Klasen (2002).[7]

5.4.2 Fertility and labor-force participation of women

The equation (5.17) still holds when we step back one period, i.e.

$$\mu_t = \frac{e_t^f}{e_t^m} = [1 - \varphi(A_t)]^{\frac{1}{1-\theta}} \tag{5.20}$$

so that

$$\frac{h_t^m}{h_t^f} = \frac{(e_t^m)^\theta}{(e_t^f)^\theta} = [1 - \varphi(A_t)]^{\frac{\theta}{\theta-1}} \tag{5.21}$$

Substituting (5.21) into (5.12), we have

$$n_t = \begin{cases} \frac{\gamma(1-\theta)}{\rho}\left([1-\varphi(A_t)]^{\frac{\theta}{\theta-1}} + 1 - \varphi(A_t)\right) \equiv n_a(A_t) \text{ if } n_a(A_t) \leq \frac{1-\varphi(A_t)}{\rho} \\ \frac{1-\varphi(A_t)}{\rho} \equiv n_b(A_t) \qquad \text{if } n_a(A_t) \geq \frac{1-\varphi(A_t)}{\rho} \end{cases} \tag{5.22}$$

For the existence of a solution to $n_a(A_t) = n_b(A_t)$, i.e. the existence of a threshold for the technological level below which women will spend full time doing housework and raising children, we assume that

$$\bar{\varphi} > 1 - \left[\frac{\gamma(1-\theta)}{1-\gamma(1-\theta)}\right]^{1-\theta} \qquad (A5.2)$$

[7] Klasen (2002) shows that gender inequality in education directly affects economic growth by lowering the average level of human capital and indirectly affects economic growth through its impact on population and investment.

The assumption $A5.2$ requires that at some low level of technology, the time devoted to housework of women is sufficiently high to make them not to supply their labor to the market. If this assumption did not hold, the time constraint of women would be never binding for all $A_t > 0$, as derived from (5.22). That is to say, women would always supply their labor to the market regardless how low the strictly positive contemporary level of technology is, and the fertility would be always decreasing in the level of technology. Nevertheless, in the early stages of development, fertility is typically observed to be increasing with the level of technology, and women supply their labor to the market when the return to their labor was sufficiently high.

Lemma 5.1: *Under assumptions* $A5.1$ *and* $A5.2$*, there exists a unique* $A^* > 0$ *such that*

$$n_a(A^*) = n_b(A^*).$$

***Proof*:** In effect we consider the equation

$$n_a(A_t) = n_b(A_t)$$

That is to say

$$\gamma(1-\theta)\left([1-\varphi(A_t)]^{\frac{\theta}{\theta-1}} + 1 - \varphi(A_t)\right) = 1 - \varphi(A_t)$$

or equivalently

$$\varphi(A_t) = 1 - \left[\frac{\gamma(1-\theta)}{1-\gamma(1-\theta)}\right]^{1-\theta}$$

Under assumption $A5.1$, $\varphi(A_t)$ is an invertible function. Thus, under assumption $A5.2$, there exists a unique $A^* > 0$ solving $n_a(A_t) = n_b(A_t)$, where

$$A^* = \varphi^{-1}\left(1 - \left[\frac{\gamma(1-\theta)}{1-\gamma(1-\theta)}\right]^{1-\theta}\right)$$

Q.E.D.

So, under Lemma 5.1, we can rewrite (5.22) as follows

$$n_t = \begin{cases} \frac{\gamma(1-\theta)}{\rho}\left([1-\varphi(A_t)]^{\frac{\theta}{\theta-1}} + 1 - \varphi(A_t)\right) \equiv n_a(A_t) \text{ if } A_t \geq A^* \\ \\ \frac{1-\varphi(A_t)}{\rho} \equiv n_b(A_t) \qquad\qquad \text{if } A_t \leq A^* \end{cases} \tag{5.23}$$

Equation (5.23) implies that in any period t women participate in the labor market if, and only if, the contemporary level of technology A_t is sufficiently high (i.e. $A_t > A^*$), otherwise women will spend their full time doing housework and raising children. So, A^* is thus the highest level of technology for which the women do not work in the market.

To see the impact of technology on female participation in the labor market, note first that $\forall A_t$,

$$n_a'(A_t) = \frac{\gamma(1-\theta)}{\rho}\left(\frac{\theta}{1-\theta}[1-\varphi(A_t)]^{\frac{1}{\theta-1}} - 1\right)\varphi'(A_t) < 0$$

since $\theta \in [\frac{1}{2}, 1)$ as mentioned in section 5.3.1.

$$n_a''(A_t) = \frac{\gamma(1-\theta)}{\rho} \times$$

$$\left[\frac{\theta\varphi'(A_t)^2}{(1-\theta)^2}[1-\varphi(A_t)]^{\frac{2-\theta}{\theta-1}} + \left(\frac{\theta}{1-\theta}[1-\varphi(A_t)]^{\frac{1}{\theta-1}} - 1\right)\varphi''(A_t)\right] > 0$$

$$\lim_{A_t \to +\infty} n_a(A_t) = \frac{2\gamma(1-\theta)}{\rho} \text{ and}$$

$$n_a(0) = \frac{\gamma(1-\theta)}{\rho}\left([1-\bar{\varphi}]^{\frac{\theta}{\theta-1}} + 1 - \bar{\varphi}\right)$$

while

$$n_b'(A_t) = \frac{-\varphi'(A_t)}{\rho} > 0$$

$$n_b''(A_t) = \frac{-\varphi''(A_t)}{\rho} < 0$$

$$\lim_{A_t \to +\infty} n_b(A_t) = \frac{1}{\rho} > \frac{2\gamma(1-\theta)}{\rho} \quad \text{and}$$

$$n_b(0) = \frac{1-\bar{\varphi}}{\rho} < n_a(0)$$

Moreover, the time devoted to the labor market of women is

$$\mathcal{L}(A_t) = \begin{cases} 1 - \rho n_a(A_t) - \varphi(A_t) & \text{if} \quad A_t \geq A^* \\ \\ 0 & \text{if} \quad A_t \leq A^* \end{cases} \tag{5.24}$$

with

$$\mathcal{L}'(A_t) = -\rho n_a'(A_t) - \varphi'(A_t) > 0 \quad \forall A_t \geq A^*$$

$$\mathcal{L}''(A_t) = -\rho n_a''(A_t) - \varphi''(A_t) < 0 \quad \forall A_t \geq A^*$$

and

$$\lim_{A_t \to +\infty} \mathcal{L}(A_t) = 1 - 2\gamma(1-\theta)$$

So Figure 5.6 below describes the impact of technology on fertility and female participation in the labor market.

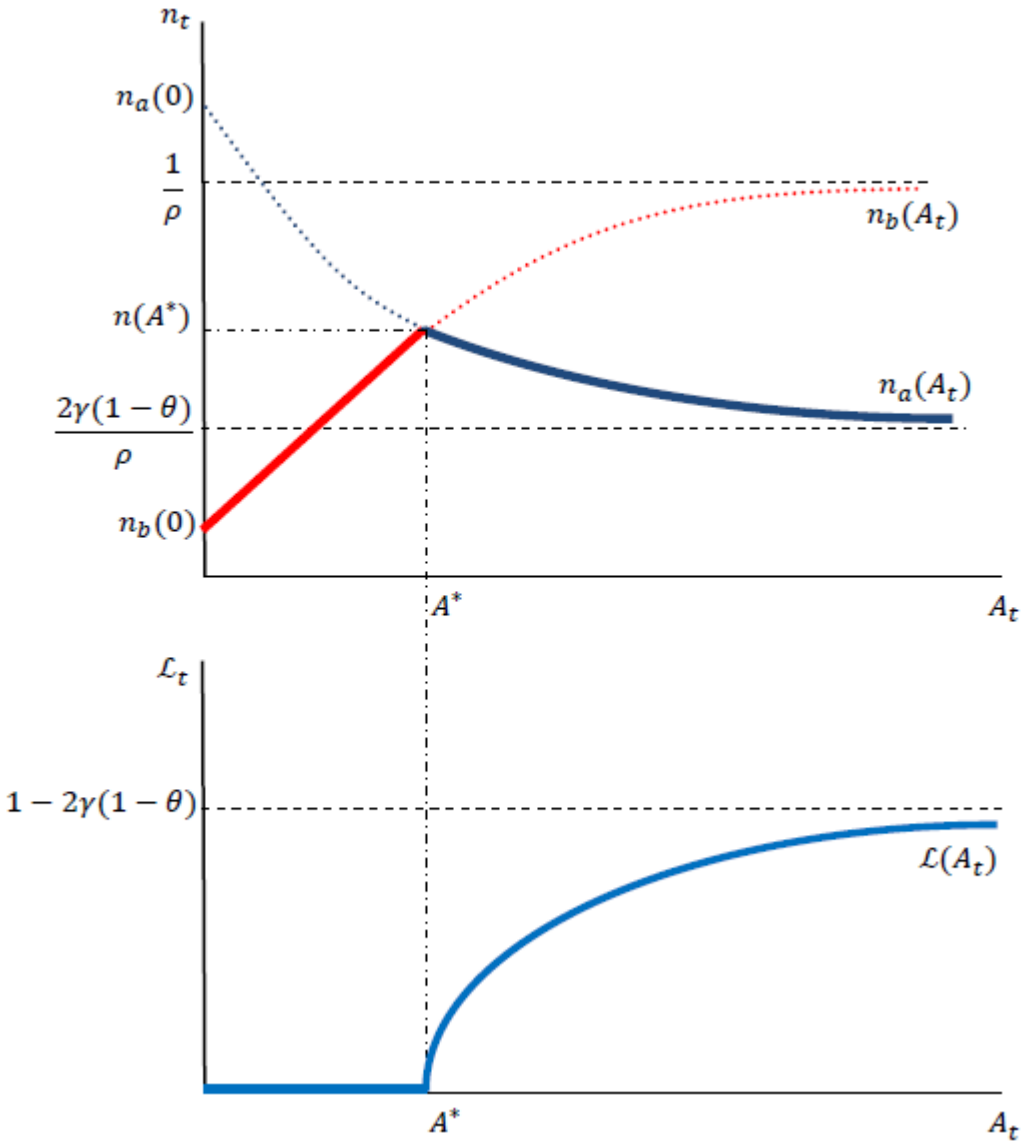

Figure 5.6. Fertility and female labor-force participation against technology

So, when the technological level is low enough (i.e. $A_t < A^*$), women spend full time doing housework and raising children. When the technological level increases but is still low, the time for doing housework decreases and the time for raising children increases, then in this case an increase in the technological level leads to an increase in fertility. When technological level exceeds the threshold A^*, women do housework and raise children part time, and participate the labor market in their remaining time. Technological progress makes the human capital gap between men and women decrease, as expressed in (5.21),

and thus reduces the relative earning gap between men and women. This reduction in the relative earning gap implies an increase in the earnings of women. This increase in the earnings of women leads to an increase in the opportunity cost of raising children. From (5.18) we find that the educational investment for one couple of children increases in the potential earnings of women when the technological level is large enough (i.e. $A_t > A^*$). So when the cost of raising children physically increases, households will trade less quantity for higher quality children. Therefore, along with the decline in fertility and increase in educational investment due to the increase in technology, the labor-force participation of women increases.

5.5 Competitive equilibrium and dynamics

The competitive equilibrium of the economy in period t is characterized by (i) the household's utility maximization under the constraints, (ii) the aggregate output equating the total return to human capital, (iii) the dynamics of the technological level, and (iv) the dynamics of population. Therefore, a competitive equilibrium $\{c_t, n_t, e^m_{t+1}, e^f_{t+1}, Y_t, w_t, A_{t+1}, L_{t+1}\}$ is determined by the following system of equations

$$c_t = \begin{cases} (1-\gamma)w_t[(e^m_t)^\theta + [1-\varphi(A_t)](e^f_t)^\theta] & \text{if} \quad A_t \geq A^* \\ \\ \frac{1-\gamma}{1-\gamma+\gamma\theta} w_t (e^m_t)^\theta & \text{if} \quad A_t \leq A^* \end{cases}$$

$$n_t = \begin{cases} \frac{\gamma(1-\theta)}{\rho}\left[\frac{(e^m_t)^\theta}{(e^f_t)^\theta} + 1 - \varphi(A_t)\right] & \text{if} \quad A_t \geq A^* \\ \\ \frac{1-\varphi(A_t)}{\rho} & \text{if} \quad A_t \leq A^* \end{cases}$$

$$e^m_{t+1} = \begin{cases} \frac{\theta\rho w_t (e^f_t)^\theta}{(1-\theta)(1+[1-\varphi(A_{t+1})]^{\frac{1}{1-\theta}})} & \text{if} \quad A_t \geq A^* \\ \\ \frac{\gamma\theta\rho w_t (e^m_t)^\theta}{(1-\gamma+\gamma\theta)[1-\varphi(A_t)](1+[1-\varphi(A_{t+1})]^{\frac{1}{1-\theta}})} & \text{if} \quad A_t \leq A^* \end{cases}$$

$$e_{t+1}^f = \begin{cases} \frac{\theta\rho w_t (e_t^f)^\theta}{(1-\theta)(1+[1-\varphi(A_{t+1})]^{\frac{1}{\theta-1}})} & \text{if} \quad A_t \geq A^* \\ \\ \frac{\gamma\theta\rho w_t (e_t^m)^\theta}{(1-\gamma+\gamma\theta)[1-\varphi(A_t)](1+[1-\varphi(A_{t+1})]^{\frac{1}{\theta-1}})} & \text{if} \quad A_t \leq A^* \end{cases}$$

$$Y_t = f(A_t) L_t \left((e_t^m)^\theta + [1 - \rho n_t - \varphi(A_t)](e_t^f)^\theta \right)$$

$$w_t = f(A_t)$$

$$A_{t+1} = \left[1 + g\left(\frac{(e_t^m)^\theta + (e_t^f)^\theta}{2} \right) \right] A_t$$

$$L_{t+1} = n_t L_t$$

for given A_t, L_t, e_t^m, e_t^f.

The competitive equilibrium can be fully characterized by the following reduced system describing the equilibrium dynamics of the level of technology A_{t+1} and educational investments for male and female children e_{t+1}^m, e_{t+1}^f.

$$A_{t+1} = \left[1 + g\left(\frac{(e_t^m)^\theta + (e_t^f)^\theta}{2} \right) \right] A_t \tag{5.25}$$

$$e_{t+1}^m = \begin{cases} \frac{\theta\rho f(A_t)(e_t^f)^\theta}{(1-\theta)(1+[1-\varphi(A_{t+1})]^{\frac{1}{1-\theta}})} & \text{if} \quad A_t \geq A^* \\ \\ \frac{\gamma\theta\rho f(A_t)(e_t^m)^\theta}{(1-\gamma+\gamma\theta)[1-\varphi(A_t)](1+[1-\varphi(A_{t+1})]^{\frac{1}{1-\theta}})} & \text{if} \quad A_t \leq A^* \end{cases} \tag{5.26}$$

$$e_{t+1}^f = \begin{cases} \frac{\theta\rho f(A_t)(e_t^f)^\theta}{(1-\theta)(1+[1-\varphi(A_{t+1})]^{\frac{1}{\theta-1}})} & \text{if} \quad A_t \geq A^* \\ \\ \frac{\gamma\theta\rho f(A_t)(e_t^m)^\theta}{(1-\gamma+\gamma\theta)[1-\varphi(A_t)](1+[1-\varphi(A_{t+1})]^{\frac{1}{\theta-1}})} & \text{if} \quad A_t \leq A^* \end{cases} \tag{5.27}$$

for a given initial condition A_0, e_0^m, and e_0^f.

To prove the convergence, which is stated in Proposition 5.2 below, of the dynamic system (5.25)-(5.27) , we have to prove the following Lemma 5.2.

Lemma 5.2: *For a dynamic equation* $x_{t+1} = a_t x_t^{\alpha}$ *with* $\alpha \in (0,1)$, $x_0 > 0$, *and* $\lim_{t\to+\infty} a_t = a > 0$, *then*

$$\lim_{t\to+\infty} x_t = a^{\frac{1}{1-\alpha}}.$$

***Proof*:** See Appendix A5.3.

Proposition 5.2: *The overlapping generations economy above, with any initial conditions* $A_0 > 0$, e_0^m, *and* e_0^f, *will converge to a regime of sustained growth characterized by a constant growth rate of technology, a constant fertility rate, constant education investments, and complete gender equality in education.*

***Proof*:** In effect, for this economy, the level of technology increases unboundedly over time because the driving force for technological progress is positive investment in human capital. The technological progress appears even when the average human capital is very small. So it is straightforward from (5.26), (5.27) and the Lemma 5.2 that the educational investments will converge to a constant and equal level between male and female children. In particularly,

$$\bar{e} = \lim_{t\to+\infty} e_t^m = \lim_{t\to+\infty} e_t^f = \left[\frac{\theta\rho\bar{f}}{2(1-\theta)}\right]^{\frac{1}{1-\theta}}$$

From (5.23), the fertility rate will converge to a constant rate,

$$\bar{n} = \lim_{t\to+\infty} n_a(A_t) = \frac{2\gamma(1-\theta)}{\rho}$$

And the technology will grow at a constant rate

$$\bar{g} = g\left(\left[\frac{\theta\rho\bar{f}}{2(1-\theta)}\right]^{\frac{\theta}{1-\theta}}\right)$$

Q.E.D.

The statement in the Proposition 5.2 is rather consistent with stylized facts of the developed world where a modern sustained growth regime prevails characterized by unbounded economic growth, low and decreasing fertility rate, and high human capital.

5.6 Analysis

It would be interesting to link the theoretical results of this chapter to the development process of western Europe characterized by three distinct regimes, (i) the Malthusian regime where both per-capita income and growth rate of population are very low; (ii) the demographic transition where both growth rate of population and per-capita income increase simultaneously; and later (iii) the modern sustained growth regime where the growth rate of population falls while per-capita income grows. During the development process of modern sustained growth, the labor-force participation of women increases along with a decline in gender inequalities in education and income.

Consider an economy in the early stage of development with very low initial level of technology and low human capital of both men and women. The technological level is low enough (i.e. $A_0 < A^*$) for women to have to spend their full time raising children and doing housework. The low technological level prevents women from participating in the labor market in two ways. First, it directly requires a large fraction of women's time to do housework. Second, it indirectly creates a gender inequality in education that makes women receive less educational investment from their parents, hence the opportunity cost of raising children physically becomes cheap, so that households prefer to increase their number of children rather than supplying the

woman's labor to the market. Since, in this period, the income of a household is very low due to low human capital and low technological level then educational investments for their children are very small. Therefore, human capital are very low for both male and female children. Moreover, the growth rate of technology is very small as well because of low average human capital, which is the driving force for technological progress. Consequently, households invest very little in education for their daughters since they anticipate the large fraction of time that their daughters have to devote to housework due to the low level of technology in the next period. Since men always supply their labor inelastically to the market then, while women have to spend a large fraction of their time for housework, households allocate a large fraction of education investment to their sons, making the education of their sons and daughters very unequal. In addition, in this stage of development, housework requires a very large fraction of women's time then the remaining time for raising children is very small, making the population growth rate low. Because the driving force of technological progress is the average human capital and technology grows even when the average human capital is small, over time, technological level increases, while still being low (i.e. $A_t < A^*$), making the time for housework decrease, and making women have more time to take care of their children. In this stage, although the technological progress can increase the earnings of women, i.e. increase the opportunity cost of raising children physically, households increase the number of their children. Consequently, the fertility rate increases.

Over time, fertility increases along with technological progress and reaches the maximum when the technological level reaches A^*, generating a demographic transition. When the technological level is high enough (i.e. $A_t > A^*$), women will participate the labor market and their participation increases due to two effects of technology. First, the technological progress helps women save time in doing housework, leaving them more time for childcare and labor-force participation. Second, the technological progress improves the return to human capital, thus increases the earnings of women, meaning that the opportunity cost of raising children physically increases. The model shows

that the fertility decreases due to the increase in the technological level. We also know the educational investment for one couple of children from (5.26) and (5.27) when $A_t > A^*$ that

$$e_{t+1}^m + e_{t+1}^f = \frac{\theta\rho f(A_t)(e_t^f)^\theta}{1-\theta}$$

i.e. this educational investment increases with the level of technology. So, in this period, households trade low quantity for high quality children. The increases in the level of technology make households invest relatively less in education for male children in order to invest more education for female children because the return to female education is higher than that to male education. Consequently, the gender equality in education improves over time, accelerating the technological progress. This feedback loop puts the economy to enter the regime of sustained economic growth.

5.7 Conclusion

This chapter develops a unified growth model capturing technological progress, gender inequality in education and fertility and the complex interaction between them to explain some stylized facts characterizing development process. Particularly, it proposes a new mechanism linking technology, gender inequality and fertility to shed light on the transition from stagnation through demographic transition to modern sustained growth, along with the improvement in gender equality in education and income, as well as the increase in female labor-force participation. These processes are the *inevitable* outcomes of development when the driving force for technological progress is average human capital. This chapter highlights the positive effect of gender equality in education on economic growth, widely recognized in the literature, through accelerating technological progress. This chapter also shows that technological progress may increase female labor-force participation not only by freeing them from doing housework due to the appearance of time-saving household-sector products but also by leading households to reduce fertility due to the increase in the return

of human capital, i.e the increase in the cost of raising children. In addition, technological progress also makes households trade quantity for higher quality children.

5.8 Appendix

A5.1. Solving the household's optimization problem

$$\max_{\substack{c_t,n_t>0 \\ e^m_{t+1},e^f_{t+1}\geq 0}} \gamma \ln\left(n_t w_{t+1}\left[(e^m_{t+1})^\theta + [1-\varphi(A_{t+1})](e^f_{t+1})^\theta\right]\right) + (1-\gamma)\ln c_t$$

subject to

$$c_t + n_t(\rho w_t h^f_t + e^m_{t+1} + e^f_{t+1}) \leq w_t(h^m_t + [1-\varphi(A_t)]h^f_t)$$

$$\rho n_t + \varphi(A_t) \leq 1$$

The Kuhn Tucker conditions are

$$\begin{pmatrix} \frac{1-\gamma}{c_t} \\ \frac{\gamma}{n_t} \\ \frac{\gamma\theta(e^m_{t+1})^{\theta-1}}{(e^m_{t+1})^\theta+[1-\varphi(A_{t+1})](e^f_{t+1})^\theta} \\ \frac{\gamma[1-\varphi(A_{t+1})]\theta(e^f_{t+1})^{\theta-1}}{(e^m_{t+1})^\theta+[1-\varphi(A_{t+1})](e^f_{t+1})^\theta} \end{pmatrix} = \lambda_1 \begin{pmatrix} 1 \\ \rho w_t h^f_t + e^m_{t+1} + e^f_{t+1} \\ n_t \\ n_t \end{pmatrix} + \lambda_2 \begin{pmatrix} 0 \\ 0 \\ -1 \\ 0 \end{pmatrix}$$

$$+\lambda_3 \begin{pmatrix} 0 \\ 0 \\ 0 \\ -1 \end{pmatrix} + \lambda_4 \begin{pmatrix} 0 \\ \rho \\ 0 \\ 0 \end{pmatrix}$$

$$c_t + n_t(\rho w_t h^f_t + e^m_{t+1} + e^f_{t+1}) - w_t(h^m_t + [1-\varphi(A_t)]h^f_t) \leq 0$$

$$-e^m_{t+1} \leq 0$$

$$-e^f_{t+1} \leq 0$$

$$\rho n_t + \varphi(A_t) - 1 \leq 0$$

$$\lambda_1 \Big[c_t + n_t(\rho w_t h_t^f + e_{t+1}^m + e_{t+1}^f) - w_t(h_t^m + [1 - \varphi(A_t)]h_t^f) \Big] = 0$$

$$\lambda_2 e_{t+1}^m = 0$$

$$\lambda_3 e_{t+1}^f = 0$$

$$\lambda_4[\rho n_t + \varphi(A_t) - 1] = 0$$

$$\lambda_1, \lambda_2, \lambda_3, \lambda_4 \geq 0$$

For these conditions, it is straightforward to show that $\lambda_1 > 0$, because

$$\lambda_1 = \frac{1 - \gamma}{c_t} \neq 0$$

since $\gamma \in (0, 1)$ and $c_t > 0$. That is to say the budget constraint is binding.

Since $\theta - 1 < 0$, then it is also straightforward from conditions

$$\frac{\gamma\theta(e_{t+1}^m)^{\theta-1}}{(e_{t+1}^m)^\theta + [1 - \varphi(A_{t+1})](e_{t+1}^f)^\theta} = \lambda_1 n_t - \lambda_2$$

$$\frac{\gamma[1 - \varphi(A_{t+1})]\theta(e_{t+1}^f)^{\theta-1}}{(e_{t+1}^m)^\theta + [1 - \varphi(A_{t+1})](e_{t+1}^f)^\theta} = \lambda_1 n_t - \lambda_3$$

that $e_{t+1}^m > 0$ and $e_{t+1}^f > 0$ to guarantee the left-hand-sides (both numerators and denominators) to be determined. Therefore, $\lambda_2 = \lambda_3 =$

0. That is to say, the positivity constraints of education investments for male and female children are never binding at the optimal solution.

Now we consider two cases: (i) the time constraint of the woman is not binding; and (ii) it is binding.

- (i) If $\rho n_t + \varphi(A_t) < 1$, then $\lambda_4 = 0$, so that

$$\frac{1-\gamma}{c_t} = \lambda_1 > 0$$

$$\frac{\gamma}{n_t} = \frac{1-\gamma}{c_t}(\rho w_t h_t^f + e_{t+1}^m + e_{t+1}^f) \tag{5.28}$$

$$\frac{\gamma\theta(e_{t+1}^m)^{\theta-1}}{(e_{t+1}^m)^\theta + [1-\varphi(A_{t+1})](e_{t+1}^f)^\theta} = \frac{1-\gamma}{c_t} n_t \tag{5.29}$$

$$\frac{\gamma[1-\varphi(A_{t+1})]\theta(e_{t+1}^f)^{\theta-1}}{(e_{t+1}^m)^\theta + [1-\varphi(A_{t+1})](e_{t+1}^f)^\theta} = \frac{1-\gamma}{c_t} n_t \tag{5.30}$$

$$c_t + n_t(\rho w_t h_t^f + e_{t+1}^m + e_{t+1}^f) - w_t(h_t^m + [1-\varphi(A_t)]h_t^f) = 0 \tag{5.31}$$

From (5.29) and (5.30) we have

$$\frac{(e_{t+1}^m)^{\theta-1}}{[1-\varphi(A_{t+1})](e_{t+1}^f)^{\theta-1}} = 1$$

$$\Rightarrow \quad e_{t+1}^f = [1-\varphi(A_{t+1})]^{\frac{1}{1-\theta}} e_{t+1}^m \tag{5.32}$$

From (5.28) we have

$$\frac{1-\gamma}{c_t}n_t = \frac{\gamma}{\rho w_t h_t^f + e_{t+1}^m + e_{t+1}^f} \tag{5.33}$$

Substitute (5.32) and (5.33) into (5.29) we have

$$\frac{\theta}{(1+[1-\varphi(A_{t+1})]^{\frac{1}{1-\theta}})e_{t+1}^m} = \frac{1}{\rho w_t h_t^f + (1+[1-\varphi(A_{t+1})]^{\frac{1}{1-\theta}})e_{t+1}^m}$$

So, we obtain

$$e_{t+1}^m = \frac{\theta\rho w_t h_t^f}{(1-\theta)(1+[1-\varphi(A_{t+1})]^{\frac{1}{1-\theta}})} \tag{5.34}$$

Hence,

$$e_{t+1}^f = \frac{\theta\rho w_t h_t^f}{(1-\theta)(1+[1-\varphi(A_{t+1})]^{\frac{1}{\theta-1}})} \tag{5.35}$$

From (5.28), (5.31), (5.34) and (5.35) we have

$$\frac{1}{\gamma}\left(\rho w_t h_t^f + \frac{\theta\rho w_t h_t^f}{1-\theta}\right) n_t = w_t(h_t^m + [1-\varphi(A_t)]h_t^f)$$

Hence,

$$n_t = \frac{\gamma(1-\theta)}{\rho}\left(\frac{h_t^m}{h_t^f} + 1 - \varphi(A_t)\right) \tag{5.36}$$

and

$$c_t = (1-\gamma)w_t(h_t^m + [1-\varphi(A_t)]h_t^f) \tag{5.37}$$

So, the solution is characterized by four equations (5.34)-(5.37).

- (ii) If $\rho n_t + \varphi(A_t) = 1$, so that

$$\frac{1-\gamma}{c_t} = \lambda_1 > 0$$

$$n_t = \frac{1-\varphi(A_t)}{\rho} \tag{5.38}$$

$$\frac{\gamma\rho}{1-\varphi(A_t)} = \frac{1-\gamma}{c_t}(\rho w_t h_t^f + e_{t+1}^m + e_{t+1}^f) + \lambda_4\rho \tag{5.39}$$

$$\frac{\gamma\theta(e_{t+1}^m)^{\theta-1}}{(e_{t+1}^m)^\theta + [1-\varphi(A_{t+1})](e_{t+1}^f)^\theta} = \frac{1-\gamma}{c_t}\frac{1-\varphi(A_t)}{\rho} \tag{5.40}$$

$$\frac{\gamma[1-\varphi(A_{t+1})]\theta(e_f^{t+1})^{\theta-1}}{(e_{t+1}^m)^\theta + [1-\varphi(A_{t+1})](e_{t+1}^f)^\theta} = \frac{1-\gamma}{c_t}\frac{1-\varphi(A_t)}{\rho} \tag{5.41}$$

$$c_t + \frac{1-\varphi(A_t)}{\rho}(\rho w_t h_t^f + e_{t+1}^m + e_{t+1}^f) - w_t(h_t^m + [1-\varphi(A_t)]h_t^f) = 0 \tag{5.42}$$

From (5.40) and (5.41) we have

$$\frac{(e_{t+1}^m)^{\theta-1}}{[1-\varphi(A_{t+1})](e_{t+1}^f)^{\theta-1}} = 1$$

$$\Rightarrow \quad e_{t+1}^{f} = [1-\varphi(A_{t+1})]^{\frac{1}{1-\theta}} e_{t+1}^{m} \tag{5.43}$$

Substitute (5.43) into (5.40) we have

$$\frac{\gamma\theta}{(1+[1-\varphi(A_{t+1})]^{\frac{1}{1-\theta}})e_{t+1}^{m}} = \frac{1-\gamma}{c_t}\frac{1-\varphi(A_t)}{\rho}$$

$$\Rightarrow \quad c_t = \frac{1-\gamma}{\gamma\theta}\frac{1-\varphi(A_t)}{\rho}(1+[1-\varphi(A_{t+1})]^{\frac{1}{1-\theta}})e_{t+1}^{m} \tag{5.44}$$

Substitute (5.43) and (5.44) into (5.42) we have

$$\frac{1-\varphi(A_t)}{\rho}\times$$

$$\left\{\frac{1-\gamma}{\gamma\theta}(1+[1-\varphi(A_{t+1})]^{\frac{1}{1-\theta}})e_{t+1}^{m} + \rho w_t h_t^{f} + (1+[1-\varphi(A_{t+1})]^{\frac{1}{1-\theta}})e_{t+1}^{m}\right\}$$

$$= w_t(h_t^{m} + [1-\varphi(A_t)]h_t^{f})$$

$$\Leftrightarrow \frac{1-\gamma+\gamma\theta}{\gamma\theta}(1+[1-\varphi(A_{t+1})]^{\frac{1}{1-\theta}})e_{t+1}^{m} = \frac{\rho w_t h_t^{m}}{1-\varphi(A_t)}$$

So, we obtain

$$e_{t+1}^{m} = \frac{\gamma\theta\rho w_t h_t^{m}}{(1-\gamma+\gamma\theta)[1-\varphi(A_t)](1+[1-\varphi(A_{t+1})]^{\frac{1}{1-\theta}})} \tag{5.45}$$

Hence,

$$e_{t+1}^{f} = \frac{\gamma\theta\rho w_t h_t^m}{(1-\gamma+\gamma\theta)[1-\varphi(A_t)](1+[1-\varphi(A_{t+1})]^{\frac{1}{\theta-1}})} \tag{5.46}$$

Substitute (5.45) into (5.44) we have

$$c_t = \frac{(1-\gamma)w_t h_t^m}{1-\gamma+\gamma\theta} \tag{5.47}$$

Finally, substitute (5.45), (5.46), and (5.47) into (5.39) we have

$$\lambda_4 = \frac{\gamma(1-\theta)}{1-\varphi(A_t)} - \frac{h_t^f}{h_t^m}(1-\gamma+\gamma\theta) \tag{5.48}$$

Hence, in this case, the optimal solution is

$$c_t = \frac{(1-\gamma)w_t h_t^m}{1-\gamma+\gamma\theta}$$

$$n_t = \frac{1-\varphi(A_t)}{\rho}$$

$$e_{t+1}^{m} = \frac{\gamma\theta\rho w_t h_t^m}{(1-\gamma+\gamma\theta)[1-\varphi(A_t)](1+[1-\varphi(A_{t+1})]^{\frac{1}{1-\theta}})}$$

$$e_{t+1}^{f} = \frac{\gamma\theta\rho w_t h_t^m}{(1-\gamma+\gamma\theta)[1-\varphi(A_t)](1+[1-\varphi(A_{t+1})]^{\frac{1}{\theta-1}})}$$

A5.2. Checking the SOCs for the maximization problem of household

Since the opitimization problem is not convex, then for the FOCs to be sufficient conditions to characterize a (local) maximum to the optimization problem, we have to check the sufficient SOCs. We know

from Appedix A5.1 that the positivity constraints of education investments for male and female children are never binding at the solution, while the budget constrant is always binding at the solution, and the time constraint of women can be binding or nonbinding. So we have to check the bordered Hessian matrix in two cases: (i) the time constraint of women is not binding; and (ii) it is binding.

- (i) If $\rho n_t + \varphi(A_t) < 1$, the bordered Hessian matrix of the problem appears as

$$\bar{H}^i = \begin{pmatrix} 0 & 1 & E & n_t & n_t \\ 1 & \frac{\gamma-1}{c_t^2} & 0 & 0 & 0 \\ E & 0 & -\frac{\gamma}{n_t^2} & 0 & 0 \\ n_t & 0 & 0 & B & C \\ n_t & 0 & 0 & C & D \end{pmatrix}$$

where B, C, and D are defined below (note that for lightening notations, we denote $e_{t+1}^m = e_m$, $e_{t+1}^f = e_f$, and $\varphi(A_{t+1}) = \varphi$).

$$B = \frac{\partial^2 u_t}{\partial e_m^2} = \gamma\theta \frac{(\theta-1)e_m^{\theta-2}\left[e_m^\theta + (1-\varphi)e_f^\theta\right] - \theta e_m^{2\theta-2}}{\left[e_m^\theta + (1-\varphi)e_f^\theta\right]^2}$$

$$= \frac{-\gamma\theta e_m^{\theta-2}\left[e_m^\theta + (1-\theta)(1-\varphi)e_f^\theta\right]}{\left[e_m^\theta + (1-\varphi)e_f^\theta\right]^2} < 0$$

$$D = \frac{\partial^2 u_t}{\partial e_f^2} = \gamma\theta(1-\varphi) \frac{(\theta-1)e_f^{\theta-2}\left[e_m^\theta + (1-\varphi)e_f^\theta\right] - (1-\varphi)\theta e_f^{2\theta-2}}{\left[e_m^\theta + (1-\varphi)e_f^\theta\right]^2}$$

$$= \frac{-\gamma\theta(1-\varphi)e_f^{\theta-2}\left[(1-\theta)e_m^\theta + (1-\varphi)e_f^\theta\right]}{\left[e_m^\theta + (1-\varphi)e_f^\theta\right]^2} < 0$$

$$C = \frac{\partial^2 u_t}{\partial e_m \partial e_f} = \frac{-\gamma\theta^2(1-\varphi)e_m^{\theta-1}e_f^{\theta-1}}{\left[e_m^\theta + (1-\varphi)e_f^\theta\right]^2} < 0$$

$$E = \rho w_t h_t^f + e_m + e_f > 0$$

Now we prove two following properties:

Property 1: $BD > C^2$.

In effect, $BD > 0$, $C^2 > 0$, and

$$\frac{BD}{C^2} = \frac{\left[e_m^\theta + (1-\theta)(1-\varphi)e_f^\theta\right]\left[(1-\theta)e_m^\theta + (1-\varphi)e_f^\theta\right]}{\theta^2(1-\varphi)e_m^\theta e_f^\theta}$$

$$= \frac{(1-\theta)e_m^{2\theta} + [(1-\theta)^2+1](1-\varphi)e_m^\theta e_f^\theta + (1-\theta)(1-\varphi)^2 e_f^{2\theta}}{\theta^2(1-\varphi)e_m^\theta e_f^\theta}$$

$$= \frac{(1-\theta)e_m^{2\theta} + (1-\theta)(1-\varphi)^2 e_f^{2\theta}}{\theta^2(1-\varphi)e_m^\theta e_f^\theta} + \frac{2(1-\theta)}{\theta^2} + 1 > 1$$

i.e. $BD > C^2$ ■

Property 2: $2C > B + D$.

We know that $C < 0$, $B < 0$, and $D < 0$, then the property 2 is equivalent to

$$\frac{B+D}{2C} > 1$$

In effect,

$$\frac{B+D}{2C} =$$

$$\frac{e_m^{\theta-2}\left[e_m^{\theta}+(1-\theta)(1-\varphi)e_f^{\theta}\right]+(1-\varphi)e_f^{\theta-2}\left[(1-\theta)e_m^{\theta}+(1-\varphi)e_f^{\theta}\right]}{2\theta(1-\varphi)e_m^{\theta-1}e_f^{\theta-1}}$$

$$= \frac{e_m^{2\theta-2}+(1-\varphi)^2 e_f^{2\theta-2}}{2\theta(1-\varphi)e_m^{\theta-1}e_f^{\theta-1}} + V$$

where $V = \frac{(1-\theta)[e_f^{\theta}e_m^{\theta-2}+e_f^{\theta-2}e_m^{\theta}]}{2\theta e_m^{\theta-1}e_f^{\theta-1}} > 0$.

Applying the trivial inequality, $e_m^{2\theta-2}+(1-\varphi)^2 e_f^{2\theta-2} \geq 2(1-\varphi)e_m^{\theta-1}e_f^{\theta-1}$, we have

$$\frac{B+D}{2C} \geq \frac{1}{\theta} + V > 1$$

i.e. $2C > B + D$ ■

The sufficient SOCs for a maximum in this case are

$$\left|\bar{H}_2^i\right| = \begin{vmatrix} 0 & 1 & E \\ 1 & \frac{\gamma-1}{c_t^2} & 0 \\ E & 0 & -\frac{\gamma}{n_t^2} \end{vmatrix} = \frac{1-\gamma}{c_t^2}E^2 + \frac{\gamma}{n_t^2} > 0$$

$$\left|\bar{H}_3^i\right| = \begin{vmatrix} 0 & 1 & E & n_t \\ 1 & \frac{\gamma-1}{c_t^2} & 0 & 0 \\ E & 0 & -\frac{\gamma}{n_t^2} & 0 \\ n_t & 0 & 0 & B \end{vmatrix} = B\left|\bar{H}_2^i\right| + \frac{(\gamma-1)\gamma}{c_t^2} < 0$$

$$\left|\bar{H}_4^i\right| = \left|\bar{H}^i\right| = \begin{vmatrix} 0 & 1 & E & n_t & n_t \\ 1 & \frac{\gamma-1}{c_t^2} & 0 & 0 & 0 \\ E & 0 & -\frac{\gamma}{n_t^2} & 0 & 0 \\ n_t & 0 & 0 & B & C \\ n_t & 0 & 0 & C & D \end{vmatrix}$$

$$= -\begin{vmatrix} 1 & E & n_t & n_t \\ 0 & -\frac{\gamma}{n_t^2} & 0 & 0 \\ 0 & 0 & B & C \\ 0 & 0 & C & D \end{vmatrix} + \frac{\gamma-1}{c_t^2}\begin{vmatrix} 0 & E & n_t & n_t \\ E & -\frac{\gamma}{n_t^2} & 0 & 0 \\ n_t & 0 & B & C \\ n_t & 0 & C & D \end{vmatrix}$$

$$= \left[\frac{\gamma}{n_t^2} + \frac{1-\gamma}{c_t^2}E^2\right](BD - C^2) + \frac{(1-\gamma)\gamma}{c_t^2}(2C - B - D) > 0$$

under properties 1 and 2 above.

So the solution to the agent's problem in this case is a maximum indeed.

- (ii) If $\rho n_t + \varphi(A_t) = 1$, the bordered Hessian matrix of the problem appears as

$$\bar{H}^{ii} = \begin{pmatrix} 0 & 0 & 1 & E & n_t & n_t \\ 0 & 0 & 0 & \rho & 0 & 0 \\ 1 & 0 & \frac{\gamma-1}{c_t^2} & 0 & 0 & 0 \\ E & \rho & 0 & -\frac{\gamma}{n_t^2} & 0 & 0 \\ n_t & 0 & 0 & 0 & B & C \\ n_t & 0 & 0 & 0 & C & D \end{pmatrix}$$

The sufficient SOCs for a maximum in this case are

$$\left|\bar{H}_3^{ii}\right| = \begin{vmatrix} 0 & 0 & 1 & E & n_t \\ 0 & 0 & 0 & \rho & 0 \\ 1 & 0 & \frac{\gamma-1}{c_t^2} & 0 & 0 \\ E & \rho & 0 & -\frac{\gamma}{n_t^2} & 0 \\ n_t & 0 & 0 & 0 & B \end{vmatrix}$$

$$= B\rho^2 - n_t^2 \begin{vmatrix} 0 & 0 & \rho \\ 0 & \frac{\gamma-1}{c_t^2} & 0 \\ \rho & 0 & -\frac{\gamma}{n_t^2} \end{vmatrix} = \left(B - n_t^2 \frac{1-\gamma}{c_t^2}\right)\rho^2 < 0$$

$$\left|\bar{H}_4^{ii}\right| = \left|\bar{H}^{ii}\right| = \begin{vmatrix} 0 & 0 & 1 & E & n_t & n_t \\ 0 & 0 & 0 & \rho & 0 & 0 \\ 1 & 0 & \frac{\gamma-1}{c_t^2} & 0 & 0 & 0 \\ E & \rho & 0 & -\frac{\gamma}{n_t^2} & 0 & 0 \\ n_t & 0 & 0 & 0 & B & C \\ n_t & 0 & 0 & 0 & C & D \end{vmatrix}$$

$$= -\rho^2 \begin{vmatrix} 0 & 1 & n_t & n_t \\ 1 & \frac{\gamma-1}{c_t^2} & 0 & 0 \\ n_t & 0 & B & C \\ n_t & 0 & C & D \end{vmatrix} = \rho^2 \begin{vmatrix} 1 & n_t & n_t \\ 0 & B & C \\ 0 & C & D \end{vmatrix} + \rho^2 \frac{1-\gamma}{c_t^2} \begin{vmatrix} 0 & n_t & n_t \\ n_t & B & C \\ n_t & C & D \end{vmatrix}$$

$$= \rho^2 \left[BD - C^2 + \frac{1-\gamma}{c_t^2}(2C - B - D)\right] > 0$$

under properties 1 and 2 above.

So the solution to the agent's problem in this case is also a maximum indeed.

A5.3. Proof of Lemma 5.2.

In effect, since $\lim_{t\to+\infty} a_t = a > 0$ then $\forall \varepsilon \in (0, a)$, $\exists T_0$ such that $\forall t \geq T_0$, we have

$$a - \varepsilon \leq a_t \leq a + \varepsilon$$

Define

$$X_0 = Y_0 = Z_0 = x_{T_0}$$

and

$$X_{t+1} = a_{T_0+t} X_t^\alpha, \quad Y_{t+1} = (a+\varepsilon) Y_t^\alpha, \quad Z_{t+1} = (a-\varepsilon) Z_t^\alpha$$

We know that

$$\lim_{t\to+\infty} Y_t = (a+\varepsilon)^{\frac{1}{1-\alpha}} \qquad \text{and} \qquad \lim_{t\to+\infty} Z_t = (a-\varepsilon)^{\frac{1}{1-\alpha}}$$

We also have

$$Z_1 = (a-\varepsilon) X_0^\alpha \leq X_1 = a_{T_0} X_0^\alpha \leq (a+\varepsilon) X_0^\alpha = Y_1$$

and

$$\begin{aligned} & Z_2 = (a-\varepsilon) Z_1^\alpha \\ & \leq (a-\varepsilon) X_1^\alpha \leq X_2 = a_{T_0+1} X_1^\alpha \leq (a+\varepsilon) X_1^\alpha \\ & \leq (a+\varepsilon) Y_1^\alpha = Y_2 \\ & \vdots \end{aligned}$$

and so on, by induction we have

$$Z_t \leq X_t \leq Y_t, \, \forall t.$$

Hence,

$$\begin{aligned} & (a-\varepsilon)^{\frac{1}{1-\alpha}} = \lim_{t\to+\infty} Z_t \leq \lim_{T\to+\infty} \left(\inf_{t\geq T} X_t \right) \\ & \leq \lim_{T\to+\infty} \left(\sup_{t\geq T} X_t \right) \leq \lim_{t\to+\infty} Y_t = (a+\varepsilon)^{\frac{1}{1-\alpha}} \end{aligned}$$

That is to say

$$(a-\varepsilon)^{\frac{1}{1-\alpha}} \leq \lim_{T\to+\infty}\left(\inf_{t\geq T} X_t\right) \leq \lim_{T\to+\infty}\left(\sup_{t\geq T} X_t\right) \leq (a+\varepsilon)^{\frac{1}{1-\alpha}},\ \forall \varepsilon \in (0,a)$$

Hence,

$$\lim_{\varepsilon\to 0^+}(a-\varepsilon)^{\frac{1}{1-\alpha}} \leq \lim_{T\to+\infty}\left(\inf_{t\geq T} X_t\right) \leq \lim_{T\to+\infty}\left(\sup_{t\geq T} X_t\right) \leq \lim_{\varepsilon\to 0^+}(a+\varepsilon)^{\frac{1}{1-\alpha}}$$

i.e.

$$a^{\frac{1}{1-\alpha}} \leq \lim_{T\to+\infty}\left(\inf_{t\geq T} X_t\right) \leq \lim_{T\to+\infty}\left(\sup_{t\geq T} X_t\right) \leq a^{\frac{1}{1-\alpha}}$$

which implies

$$\lim_{t\to+\infty} X_t = a^{\frac{1}{1-\alpha}}$$

i.e.

$$\lim_{t\to+\infty} x_t = a^{\frac{1}{1-\alpha}}.$$

Q.E.D.

6 General Conclusion

This thesis investigates some important issues, from a theoretical perspective, in Economic Growth and Development covering environmental externalities, optimal taxation, demography, gender inequality, endogenous technological progress, and comparative development, etc. Each chapter leaves some interesting ideas as well as open questions for not only theoretical research but also particularly for empirical studies. This section discusses about such questions and possibilities for further research.

The environmental externalities in a green growth model (an extension of Diamond (1965)) is explored in the chapter 2. The main contribution of this chapter is the introduction of tax and transfer policies decentralizing as a competitive outcome the transition to the first best steady state. However, this chapter just show a sufficient condition under which such policies can be implemented. A necessary condition and an expansion of the sufficient condition should be studied in future research. Indeed, in this chapter, we study such policies in environment without uncertainty. So, this analytical framework in this chapter should be extended to a dynamic stochastic general equilibrium setup. Some questions that should be addressed in future research are: (i) which policy should be applied for a specific economy?; and (ii) how could such a policy be implemented in the reality?

Chapter 3 makes stand out the role of geographical and environmental conditions for the development process. This chapter shows the interplay of population and technological losses with geographical factors like suitability and the degree of isolation in explaining how a society can be locked in stagnation. So this chapter may suggest for some empirical research on the development of isolated societies which

show no sign of escaping stagnation in the history. For example, the positive relationship between the total environmental resource of a society, its suitability and the size of population should be tested. Does population size really play an important role for triggering technological progress in the early stage of development? And what solutions should be applied for small and isolated societies with such geographical disadvantage nowaday to help them avoid a stagnation trap?

Chapter 4 extends Galor and Mountford (2006) by introducing geographical advantage for agricultural production to explain the transition from agricultural regime to manufacturing regime of developed countries and the asymmetric development across globe. This chapter departs significantly from Galor and Mountford (2006) in mentioning that geography should be the fundamental cause, rather than international trade, of divergence across globe. It also departs significantly from Ashraf and Galor (2012) explaining the demographic transition during the development process. The theoretical predictions of this chapter suggests some testable hypotheses for empirical research which may still be absent in the literature: (i) a high total factor productivity in agriculture should delay the demographic transition and industrialization; (ii) fertility, structure of labor, and growth all should depend on geographical advantage for agricultural production. Moreover, from this chapter, we should also address the issues of measuring the geographical advantage for agriculture production.

Chapter 5 provides a new mechanism linking technology, gender inequality in education, and fertility to re-explain some well-known stylized facts of development process from Malthusian stagnation to modern sustained growth across societies like the negative (positive) correlation between gender equality and fertility (per capita income), the demographic transition, as well as the improvements in gender equality in education and female labor-force participation over time. We can find thoughout this chapter that the interactions between gender inequality in education and technology play crucial role for economic growth. These theoretical findings may suggest policy implications for less developed countries, such as advancing technology as well as reforming education system towards equalizing education opportunities

for male and female children, in order to speed up their development process toward sustained economic growth. The cause of gender inequality in education we address in the chapter 5 is that women are responsible for housework while men is not (or women are more responsible for housework than men). Indeed, there exists others approach to the cause of gender inequality in education such as culture, religion, etc. These other approaches should be taken into account in such a unified growth modeling in order to draw efficiency-improving policy implications to promote the develpment process of less developed countries.

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