Image completion via nonnegative matrix factorization using B-splines

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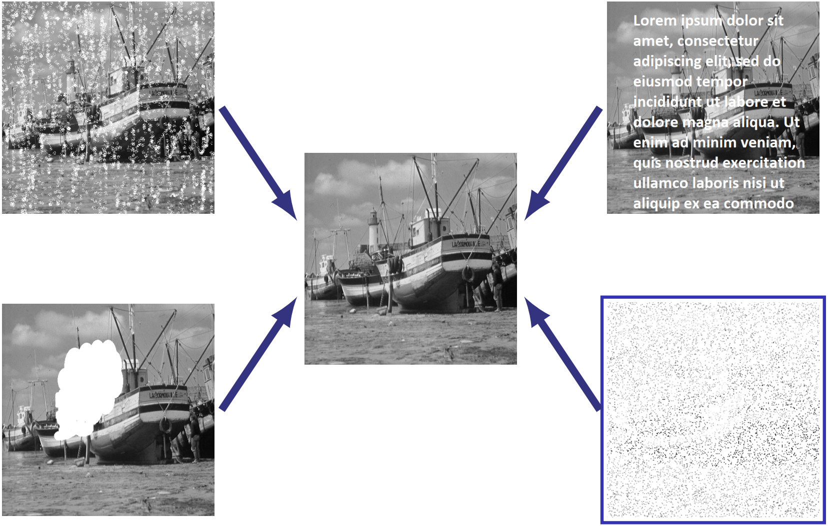
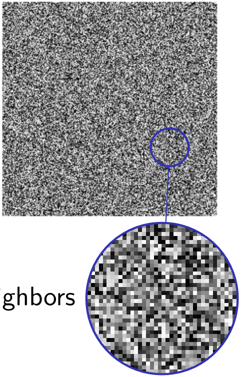
The image completion problem



Natural images are structured

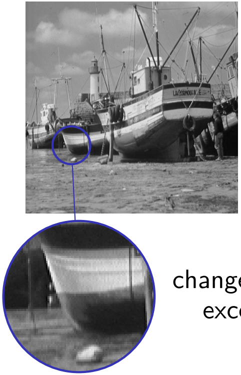
Random image

independent pixels (**high rank**)



No link with neighbors

Natural image



can be described with few elements (**low rank**)

changes are **smooth** except at edges

Take home message

- ⋄ B-Splines with nonnegative coefficients can be efficiently used for image completion.
- ⋄ Increasing progressively the number of interior knots of splines improve the accuracy of the models.

Some ideas used to complete images

- ◇ Images are smooth linear combinations of a few factors (Ji & al. 2015, Liu & al, 2018)
- ◇ **Images are linear combinations of a few smooth factors** (Yokota & al. 2016, Sadowski & Zdunek, 2018)
- ◇ Images contain structure (edges) and textures (Hong & al. 2019)

Image completion using smooth NMF

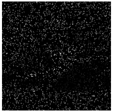
Sadowski & Zduneck:

$M \in \mathbb{R}^{m \times n}$ is an incomplete matrix with support Ω.

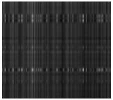
Y is a first guess for M

`Smooth-NMF(Y,A0,X0)`: returns A,X nonnegative so that $Y \simeq AX$ with columns of A splines (smooth by nature)

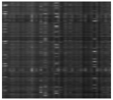
Fun SmoothNMF-IC(M, Ω, Y, A, X)
  while $||M_\Omega - (AX)_\Omega||$ too high
    $(A, X) = \texttt{Smooth-NMF}(Y, A, X)$
    $Y_2 = AX,\ Y_{2\ \Omega} = M_\Omega$
    $(X^\top, A^\top) = \texttt{Smooth-NMF}(Y_2^\top, X^\top, A^\top)$
    $Y = AX,\ Y_\Omega = M_\Omega$
  end while
  return Y
end Fun



Y



smooth cols



smooth rows

Smooth NMF

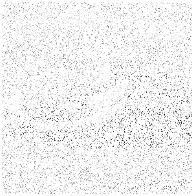
Given $Y \in \mathbb{R}^{m \times n}$, $r > 0$ and a number of splines d,
find $B \in \mathbb{R}^{(d) \times n}$, $X \in \mathbb{R}^{r \times n}$ such that:

$$\min ||Y - SBX||_F^2 \qquad A = SB \text{ and } X \text{ nonnegative}$$

where S contains discretization on m points of the B-Spline basis with $d-2$ interior knots. $A = SB$, B is a coefficient matrix.

Iteratively:

- ⋄ Update A (and B) using ADMM (Alternating Direction Method of Multipliers)
- ⋄ Update rows of X using HALS (Hierarchical Alternating Least Squares)



1324 sec
SIR: 18.19

How can we improve this method?

This method using ADMM is accurate and easy to implement. However, it is really slow. To accelerate it we should improve the Smooth-NMF function.

Update A using S-HALS instead of ADMM

S-HALS stands for splines-HALS. It also decomposes $A = SB$ and optimize B.

Main differences:

- ◇ B is updated through its columns instead of being updated as a whole
- ◇ B is imposed to be nonnegative (instead of SB)
- ◇ Updating a column of B can be done exactly with linear algebra
- ◇ the problem can be reduced: $\min ||Y - SBX||_F^2$ becomes $||L^{-1}S^\top Y - LBX||_F^2$ where $LL^\top = S^\top S$ (Cholesky)

Using S-HALS instead of ADMM speeds up significantly

ADMM

SIR: 17.8- 2217 sec.

S-HALS

SIR: 17.9 - 286 sec.

What if rows and columns are splines?

If A and X contains splines:
Given a matrix M whose entries are known on support Ω, find B and C such that

$$\min_{B,C} ||M_{\Omega} - [SB(SC)^{\top}]_{\Omega}|| + \boldsymbol{\lambda}||\boldsymbol{Y}_{\bar{\Omega}} - [\boldsymbol{SB}(\boldsymbol{SC})^{\top}]_{\bar{\Omega}}||$$

where Y is a guess of the complete image, B and C are nonnegative and S contains discretization of B-Spline basis. (Direct Image Completion, DIC)

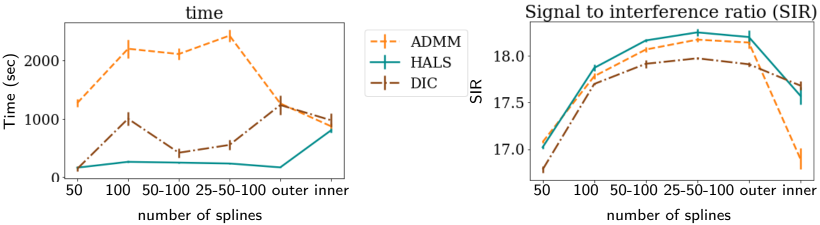
$\lambda = 0$ at first iteration and $\frac{\#\Omega}{\#\bar{\Omega}}$ after.

DIC is unable to outperform S-HALS

ADMM	S-HALS	**DIC**
		
SIR: 17.8 - 2217 sec.	SIR: 17.9 - 286 sec.	SIR: 17.79 - 1228 sec.

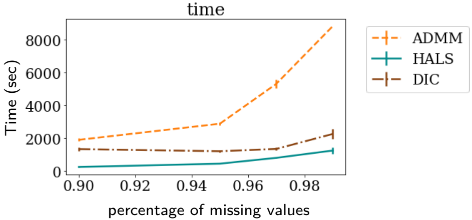
Increasing progressively the number of interior knots of splines

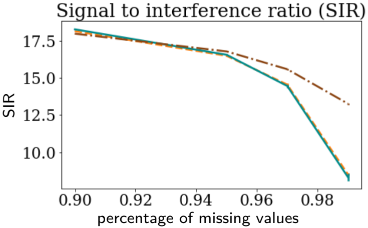
- ◇ **No increase:** d is fixed.
- ◇ **Manual increasing:** the 2-3 values that d can take are fixed. The output of d_1 is the input of d_2
- ◇ **Outer increase:** d is increased progressively at each iteration
- ◇ **Inner increase:** the number of splines is increased progressively during update of B (suggested by Sadowski & Zduneck).



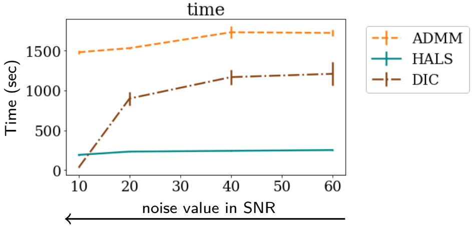
Results on difficult data

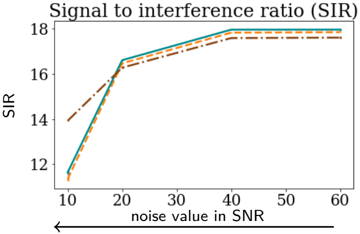
High percentage of missing values





Noisy image

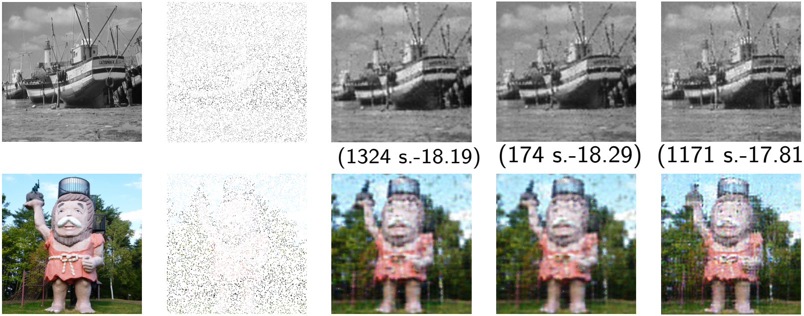




→ DIC is the most robust method

Image completion: results

Original image | Input image | ADMM | **S-HALS** | **DIC**



ADMM: (1324 s.-18.19) | S-HALS: (174 s.-18.29) | DIC: (1171 s.-17.81)

ADMM: (931 s.-14.82) | S-HALS: (188 s.-14.9) | DIC: (850 s.-14.06)

Recovered images for 90% of missing pixels. (CPU time - SIR).

Some ideas of future work

- ◇ Consider multivariate splines for colored images
- ◇ Determine the rank of the image automatically

Take home message

- B-Splines with nonnegative coefficients can be efficiently used for image completion.
- Increasing progressively the number of interior knots of splines improve the accuracy of the models.

Thank you for your attention. Do you have any question?

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