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Supply Chain Modelling with Uncertainty

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Abstract

This thesis focuses on the modelling of supply chains under uncertainty. Supply chains consist of a network of companies for purchasing raw materials, transforming these materials into intermediate and finished products, and distributing the finished products to customers. Efficient and effective supply chain management involves addressing three fundamental questions: how to buy, how to make, and how to sell? These problems are highly influenced by various uncertain factors. This thesis looks into a specific issues for each of above problems. It is structured as follows.

Chapter 1 is an introductory part that presents the background of the study, the research methodology and an outline of the contributions.

Chapter 2 is dedicated to the discussion of “how to buy". Being increasingly pressed to cut procurement costs, more and more manufacturers collaborate to jointly procure components from common supplier(s) through a joint purchasing agreement. Such agreement brings benefits to some participants, but in the meanwhile it might hurt the interest of other stakeholders including consumers. Although the input cost reduction aspect of joint purchasing has been widely documented in the extant literature, why a manufacturer would not engage in joint purchasing is less explored. This chapter employs a game theory model to analyse the manufacturers' incentives towards a joint purchasing agreement.

Chapter 3 is devoted to the discussion of “how to make". To combat climate change, many countries have enacted legislations to regulate carbon footprint. Companies that failed to comply with these regulations will face heavy penalties. In this context, incorporating sustainability principles into operations in an optimal way is a pressing problem that need to be resolved. Motivated by an applied research project, this chapter develops a math programming based production planning tool that enables manufacturers to make optimal tactical decisions under complex emission reduction policies.

Chapter 4 is given to the discussion of “how to sell". Forecast sharing is

one of the major activities in supply chain collaboration. Ideally, it should lead to a win-win situation: using forecast information, the supplier deploys enough production capacity to address potential demand, and in turn, the buyer secures a dependable supply source. However, in practice, this benefit is often compromised due to forecast imprecision. When facing multiple buyers with different levels of forecast errors, a supplier has to make decision on whether to serve a big buyer (placing large orders) first or serving a well-behaved buyer (communicating accurate forecast) first. Based on a single machine scheduling model, this chapter provides insight into the optimal order fulfilment sequence.

Finally, Chapter 5 concludes the thesis and outlines avenues for future research.

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Contents

Three passions, simple but overwhelmingly strong, have governed my life: the longing for love, the search for knowledge, and unbearable pity for the suffering of mankind.

Bertrand Russell, *What I Have Lived For*, 25 July 1956.

1

Introduction

The purpose of this chapter is to contextualize the scope of the research. It includes three parts. Part 1 introduces the supply chain topics that are covered in this thesis. Part 2 summarizes the different methodologies that have been applied. Part 3 lists the main contributions of each work.

1.1 Supply chain issues related to the thesis

1.1.1 Supply chain coopetition

Bengtsson and Kock (2000) define coopetition as the dyadic and paradoxical relationship that emerges when two firms cooperate in some activities, such as in a strategic alliance, and at the same time compete with each other in other activities. Today's global supply chains are becoming increasingly dynamic and innovative, coopetition is in fashion. For example, In the consumer electronics industry, Sony and Samsung established a joint venture to produce liquid-crystal display panels together. Samsung has been a key component supplier of Apple for many years. In the automobile industry, General Motors and Peugeot shared car design and created a global procurement alliance. BMW and Daimler have also engaged in joint procurement for many years. Coopetition benefits rival firms in various aspects, such as promoting the synergy of complementary resources and capabilities, hedging against supply uncertainty, and reducing procurement costs. Despite all these advantages, many companies still choose to purchase individually. One of the reasons is that placing a joint order would implicitly involve disclosure of sensitive market information, which they might prefer not to reveal to each other. Actually, whether information disclosure is always detrimental is far from being trivial. Chapter 2 of this thesis studies the impact of sharing different type of information on firms' profitability and consumer welfare, and identify the situations under which the competing OEMs might not want to enter into coopetition.

1.1.2 Sustainability in operations management

Driven by climate change and consumers' increasing environmental concern, more and more companies integrate sustainability principles into their operations management, balancing the traditional goal of cost reduction, operational efficiency and profitability. For instance, in 2005, the CEO of Walmart set three aspirational goals: "to be supplied 100% by renewable energy; to create zero waste; and to sell products that sustain people and the environment." Since roughly 90% of Walmart's greenhouse gas emissions and other environmental impacts occur in its extended supply chain, the firm must work with suppliers and customers to achieve these goals (Plambeck, 2012). BMW aims to reduce resource consumption per vehicle by 45 percent by 2020 compared to 2006. Its long term vision is to achieve completely CO_2 free vehicle production (BMW, 2018). In reality, there are numerous sustainable practices as the ones mentioned above. A business survey conducted by Mckinsey shows that 60% of the respondents take sustainability as seriously as economic performance. The drivers range from the need for responsible business practices, pressure from customers, public goodwill and reputation to regulatory compliance (Bonini and Gorner, 2011). This thesis especially look at the legislative driver. To combat global warming, many countries have enacted legislations or mechanisms to regulate business entities' carbon footprint. One market-based mechanism is the emission trading system (ETS), or the "cap and trade" system. The principle is that the regulators set a cap on the total volume of greenhouse gases that can be emitted by all the participants in the system. The cap is then reduced progressively such that the total emissions fall over time.Chapter 2 of this thesis studies the impact of a dynamic emission allocation rule on manufacturers' optimal production planning.

1.1.3 Buyer-supplier relationship

Managing the relationships between suppliers and buyers is the essence of supply chain management. A good supplier-buyer relationship is a win-win situation. A buyer who treat his suppliers with courtesy, honesty and fairness will receive high quality goods and services at best prices at right time. The quality of supplier-buyer relationship decides the efficiency of forecast sharing. We can find many successful cases in reality. For example, Wal-Mart shared his point-of-sale data with Procter & Gamble, which result in simultaneously reduction in inventory cost by 70% and improved service level from 96% to 99% (Poirier and Reiter, 1996). However, not all the practices lead to win-win situations. For instance, Cisco, a major networking equipment supplier, had to write off 2.1 billion in excess inventory in 2001 due to the inflated demand forecast (Files, 2001). Motivated by the forecast sharing practice in the consumer electronic industry, Chapter 4 of this thesis studies a supplier's order sequencing decisions when buyers have different levels of variability with respect to forecasts. It specifically discusses about whether a more reliable buyer (communicating less variable forecast) should be prioritized in terms of maximizing total discounted revenue.

1.1.4 Supply chain uncertainty

Supply chain uncertainty is a challenge that every manager wrestles. It can arise from a variety of sources. Exogenously, firms often confront volatile demand, which may come from irregular orders (Miller, 1992), poor demand forecast due to the long lead-time gap Van Der Vorst and Beulens (2002) and the amplification of demand due to the bullwhip effect Lee *et al.* (1997) etc. Suppliers' delivery performance or product quality can be unreliable (Yang *et al.*, 2004). The changing government and macroeconomic policies, natural disasters are all sources of external uncertainty (Christopher and Peck, 2004). Endogenously, firms can encounter uncertainty from manufacturing processes such as machine break downs, productivity of labour (Christopher and Peck, 2004), and multiple dimensions in decision making process (Xu and Beamon, 2006). The direct consequence of supply chain uncertainty is increased cost, most commonly in the form of excess inventory, excess capacity in production, or the use of faster and more expensive transportation means. Simangunsong *et al.* (2012) classifies uncertainty management strategies into two broad categories, i.e., reducing uncertainty strategies and coping with uncertainty strategies. Reducing uncertainty strategies aim to bring down uncertainty at it source, for example, a manufacturer can adopt lean production strategies to reduce inherent uncertainty and work closer with suppliers and customers to control supply or demand uncertainties. Coping with uncertainty strategies do not attempt to change the source of uncertainty but try to find ways to adapt and minimize the impact of uncertainty, for example, a manufacturer can adopt advanced analytical techniques to develop more accurate forecast, or exchange information with supply chain members to have better knowledge of end-customer demand.

This thesis focuses on coping with uncertainty by different means. Specifically, Chapter 2 models the uncertain market demand by a linear information structure. Chapter 3 develops a math programming based tool that enables managers study the impact of various sources of uncertainty through scenario analysis. Chapter 4 captures the forecast uncertainty through probability distribution.

1.2 Methodology

1.2.1 Game theory models

Game theory (hereafter GT) is a powerful tool for analyzing situations in which the decisions of multiple agents affect each agent's payoff. It has found its wide application in supply chain management to develop analytical insights and help managers develop operations strategies. Li and Whang (2002) introduces five classical application of GT tools in operations management. We summarize them as follow.

1. Time-based competition. In the time sensitive market, consumers are concerned not only about the price to pay but also about the possible delays in delivery. When firms compete to supply time-sensitive

customers, the operational decisions such as inventory, scheduling, and capacity become strategic. In this context, GT addresses the incentives and behavior of consumers and firms.

2. Priority pricing for a queueing system. In a queueing system, jobs impose negative externalities in the form of queueing delays even if the average demand is strictly less than the capacity. The queueing system offers a natural setting for game theory for two reasons. First, queueing phenomena arise due to interactions among multiple decision makers. Next, negative externalities create discrepancies between individual optimization and social optimization, since an individual decision maker will not consider the negative effect she imposes on other users. The extant literature, e.g., Mendelson and Whang (1990) a game theoretic model showing how pricing can induce privately-informed selfish users to make decisions consistent with the social objective.

3. Manufacturing and marketing incentives. Coordinating manufacturing and marketing incentives has always been challenging. Production managers are incentivized to minimize inventory costs while marketing managers focus on satisfying customers therefore they want to secure a sufficient amount of inventory. The extant work such as Ackoff (1967) and Eliashberg and Steinberg (1987) have applied GT to solve this conflict of interests.

4. Information sharing in oligopoly. This topic concerns about whether competitors in oligopolistic competition would sincerely disclose their demand and/or cost information to competitors. The tradeoff facing a player is between the efficiency gain by having more information and the strategic gain or loss caused by the reaction of other players to the changed information allocation. Vives (1984), Gal-Or (1985), Shapiro (1986) are among the earlier works that address this issue.

The chapter 2 of this thesis applies GT in the information sharing aspect, specifically it comes up with a Cournot competition model with private information on both demand and technology uncertainty.

1.2.2 Combinatorial optimization

Combinatorial optimization is a well known optimization methods in operation research. It has been widely adopted in various optimization problems such as airline crew pairing, task allocation, production and transportation planning and network design. For more details, interested readers are referred to Paschos (2013). Combinatorial problem requires enumeration and search, dynamic programming and branch-and-bound are two common techniques to deal with it. Both approaches attempt to minimize the computational effort via a divide and conquer method. Dynamic programming is applicable when problems exhibit the properties of overlapping subproblems (the solution to a given subproblem can be stored and reused in a bottom-up fashion) and optimal substructure (the link between optimal solution and optimal solutions to subproblems). On the other hand, the branch-and-bound tree search uses bounding to reduce the search space. These two techniques often need to be adopted to solve particular problem. Chapter 4 combines both techniques to build a specialized algorithm for a scheduling problem.

1.2.3 Mathematical programming

Mathematical programs represent problem choices as decision variables and seek values that maximize or minimize objective functions of the decision variables subject to constraints on variable values expressing the limits on possible decision choices. Mathematical programming is the cornerstone of operation research. In this thesis, we adopted a mixed integer programming (MIP), an important branch of general mathematical programming. A MIP is an optimization model with a mix of continuous and discrete decision variables. A classical MIP can be written as:

$$\text{Min} \quad ax + by \tag{1.1}$$

$$\text{s.t.} \quad Cx + Dy \geq e \tag{1.2}$$

$$x \in \mathbb{Z}^n_+, y \in \mathbb{R}^m_+ \tag{1.3}$$

where x represents the integer variables and y represents the continuous variables. $a \in \mathbb{R}^n$ and $b \in \mathbb{R}^n$ are the objective coefficients for x and y respectively. $C \in \mathbb{R}^{k \times n}$ is a $k \times n$ coefficient matrix for integer variable x and $D \in \mathbb{R}^{k \times m}$ is a $k \times m$ coefficient matrix for continuous variable y. $e \in \mathbb{R}^k$ is the right hand side vector of the constraints.

MIP finds its wide application in operations management such as in production planning, job shop modeling, and vehicle scheduling etc. A review on MIP can be found in Nemhauser and Wolsey (1988) and Pochet and Wolsey (2006). The two most widespread algorithmic approaches to solve MIP problems are branch-and-bound and cutting plane approaches. Several solvers have already been developed to solve MIP problems, such as open source solver Coin-or branch and cut (CBC), commercial optimization solver Gurobi and CPLEX. Formulating the problem correctly is of paramount importance which allows these solvers to find optimal solution efficiently. Chapter 3 models a production planning problem as a MIP, with an objective to minimize the total costs while satisfying resources requirements. We solve the model with both CBC and Gurobi, and find out that the performance of CBC is comparable to Gurobi for moderate size problems.

1.3 Outline of the contributions

This thesis focuses on three specific issues on the topics of how to buy, how to make, and how to sell. It contains three standalone articles which can be read independently from each other. The main contributions are summarized as follows.

Chapter 2 discusses the topic of “how to buy". It is a joint work with Professor Aadhaar Chaturvedi and his former PhD student Gilles Merckx. In this work we examine whether rival firms should engage in a joint purchasing (JP) agreements. JP agreements amongst competing OEMs are negotiated to enable them to obtain rebates from their supplier(s). The iterative interactions in reaching such JP agreements result in disclosure of private information which the OEMs might prefer to keep private by procuring individually rather than jointly. This work investigates how the information sharing dimension affects (1) OEMs' motivations towards joint purchasing agreements and (2) consumer surplus, specifically in industries characterized by market demand and technology level uncertainties. Past literature has looked at competitor's incentives on sharing information, however, there is a fine line between sharing information and collusion/anti-competitive practices. Thus it is important to understand when would information sharing, which is inevitable in joint purchasing agreements, be beneficial to both the OEMs and consumers (and hence likely to be acceptable to the anti-competitive regulators). To answer these questions we investigate a multi-period duopoly model with Cournot competition in which OEMs decide on purchasing jointly (or not) in the first stage and decide on their production quantity in the second stage, under uncertainty and information asymmetry on demand and product technology. We find that a joint purchasing agreement tends to be preferred by OEMs and are also beneficial to the consumers when either the substitutability between the two OEMs products is low enough; or when the technology level uncertainty is in the intermediate range for high level of product substitutability; or when procurement cost saving through JP agreement is high enough. Otherwise, a JP agreement might harm either the OEMs or the consumers. The Managerial insight is that for OEMs that bring periodical updates to the market this chapter provides a framework to analyze how their product level technology uncertainty (i.e., the potential for the product be a “breakthough") can effect their strategic decision on purchasing jointly with a competitor.

After exploring the buying problem, chapter 3 moves on to the issue of “how to make". To combat climate change, the European Union introduced a dynamic emission allowance allocation mechanism which will take effect starting from 2021. Under this system, a manufacturer's production decision in the current period will have a significant impact on its future emission costs. This creates a challenge for manufacturers with several production units to adjust their current production planning to meet the new rule. In this chapter we develop a decision making tool for manufacturers to tackle this problem. Specifically, we formulate a mixed-integer programming problem that helps the manufacturer to make optimal decisions taking into account emission trading, production and technology selection over multiple periods. To help the manufacturer better understand the profitability of each plant, we evaluate the marginal cost of each plant. We demonstrate that the dynamic emission allocation rule has a strong impact on manufacturers' technology choices and production strategies, and incorporating this new rule into production planning can significant cut down the total operational costs in the long run and hence provide the manufacturer an edge in the global market.

Finally, chapter 4 is devoted to the theme of "how to sell". This chapter studies the impact of forecast (in)accuracy from a supplier's perspective. To ensure a high service level, a supplier must include more safety time in the quoted lead time in response to less reliable orders, which implies a delay in revenue realization. In this chapter, we explore the relationship between supply chain information flows and cash flows. Our first contribution is to provide some analytical support to the managerial intuition that forecast errors are detrimental to the supply chain performance and that a more reliable buyer should be served earlier. We also develop an efficient algorithm to find the optimal sequence that maximize the sum of expected discounted revenue given the buyer's unreliability.

2

Purchasing together or alone? Tradeoffs for Information Sharing.

2.1 Introduction

In today's complex business environment, it has become increasingly frequent that competing Original Equipment Manufacturers (OEMs) collaborate to jointly procure components from common supplier(s) through a *joint purchasing agreement*. For examples, General motors and Peugeot have agreed to share car designs and form a global procurement alliance in an attempt to tackle increasing cost pressures, while they continue to compete with each other in many markets (WSJ, 2012). BMW and Daimler have jointly purchased tyres and seat frames since 2008, and plan to broaden their cooperation in procuring more components such as software and batteries (Reuters, 2017). Nine Chinese TV makers have been together spending around 5 billion dollars each year since 2009 in jointly ordering flat screens from Taiwanese producers. European retailers Carrefour and Tesco engaged in a three-year joint purchasing agreement to get lower prices from suppliers for own-brand products, as well as from multinationals such as *Nestle* or *Unilever* (Reuters, 2018).

Entering in a joint procurement agreement can generate significant cost savings. Procurement cost is driven down due to higher bargaining power over supplier(s). Logistics cost is cut thanks to joint transport. Despite all these advantages, manufacturers often choose to purchase individually in practice. Although the input cost reduction aspect of joint purchasing (hereafter referred to as JP) has been widely documented in the extant literature, whether or when should rival OEMs prefer JP agreements over the traditional individual purchase order (hereafter referred to as IP) from their respective supplier(s) remains debatable. For instance OEMs would have to coordinate on a joint order quantity that they would place to the supplier(s). Such coordination between OEMs would implicitly involve disclosure of sensitive market information, which they might prefer not to reveal to each other.[1] Thus even though JP would give a lower procurement cost to the competing OEMs it might also involve unavoidable disclosure of private information, which makes it unclear why (or when) competing OEMs might not want to enter into JP agreements? It is this question that we try to answer in this paper by addressing the trade-off between lower procurement cost and information disclosure when entering into JP agreement.

Actually, whether information disclosure is always detrimental to competing OEMs' profitability is far from being trivial. Economics literature has shown that competing firms' incentives to disclose private information critically depends on the type of information exchanged and on the type of

[1] Alternatively OEMs might turn to group purchasing organizations (GPOs) where they can get a discounted price without exchanging private information. However, some OEMs choose to work with each other directly on a long-term basis, as above examples show.

competition. For instance, Clarke (1983), Vives (1984) and Gal-Or (1985) have shown that firms sharing private information about their common demand estimates correlates their strategies (i.e., the quantities that they bring in the market) which in turn hurts their expected profits when they are competing à la Cournot but not necessarily under Bertrand competition. However, Fried (1984) and Shapiro (1986) have shown that firms sharing private information on production costs does not necessarily correlate their strategies and hence could in fact increase their expected profits.

Given that the type of information disclosed matters, we focus in this study on one particular type of market, namely technology products where joint purchasing agreements are particularly frequent. In such markets, the speed of evolution of products expected by consumers pushes OEMs to focus their development efforts on some key components and source many other components from outside. Another striking aspect of these markets is that the norm has become to have regular (often yearly) evolutions of the product, these updates representing sometimes small improvement and sometimes breakthroughs improvements. Our model tries to capture these specific elements of markets for high-tech products.

Besides being privately informed on common demand, OEMs might also be better informed about the technology level of their own product as compared to the rival's product before these products are brought out into the market. Information on the technology level of a product includes knowledge on the extent to which a new product has been upgraded or if the new product belongs to a new generation. Upgrades to existing products or whether the product belongs to a new generation is dependent on R&D programs whose eventual success or failure is known to the undertaking firm and not to the rival firm. Hence the technology level of a product, prior to its launch, is private information of the firm. Evidently, the product's technology level would be an important determinant of its eventual market demand. In markets that see frequent introductions of new products, e.g. electronics, the competing OEMs might simultaneously (or within relatively short interval) bring out their respective products thus resulting in the demand for their products not only being influenced by their own product's technology level but also by the competing product's technology level. If these OEMs enter into a JP agreement prior to product launch for pooling their order quantities on a common components, then these firms would not only be exchanging (advertently or inadvertently) their information about *common market demand* but also their private information on the *impact* that their proprietary product technology would have on the market demand. Given long procurement and production lead time these firms typically decide on a JP agreement and consequentially their order quantities before introducing their product to the market, therefore it is natural that the decision to enter into a JP agreement should be assessed through the lens of Cournot competition. As discussed above disclosing common market demand information would usually hurt OEMs' expected profits when they are competing à la Cournot. However, the impact on OEMs profitability from disclosing information about product technology has never been studied.

Built on these observations, our objective is threefold:

1. Independently of procurement cost advantage, we first investigate the economics of information sharing (on both market demand and technology level) and characterize the conditions under which OEMs would benefit or be penalized by the information disclosure inherent to JP.

2. For situations in which OEMs are better off by concealing information, we study whether the purchasing cost advantage derived from JP can offset their cost of disclosing information.

3. We investigate the impact of JP agreements on consumer surplus to characterize situations under which JP agreements would benefit both the OEMs and the consumers. This last point is important in determining when JP agreements would be acceptable to both competing OEMs and to anti-competitive regulators.

To address the above issues we investigate a stylized four-period duopoly model with Cournot competition in which the OEMs make decisions in two periods.

- Given the long procurement and production lead times we consider JP agreement as a strategic decision that is made before the market information of each new generation of product is observed. In the first period both OEMs decide whether or not to enter into a JP agreement in which they will jointly procure some components used in their respective final products.

- In the second period both OEMs observe their private signals on the common market demand of their respective products and observe their realized technology level (that in turn influences their product demand).

- In the third period the OEMs decide upon their production quantities. If they have not agreed on JP then they decide the quantity of products that they will bring to the market without having information of their competitor and procure their components individually. On the other hand, if they have entered into a JP agreement in period 1 then through the joint ordering process the firms learn about each others information (to simplify the model, we assume that through the order quantities, firms can infer each others full information about the market and the respective products technology level).

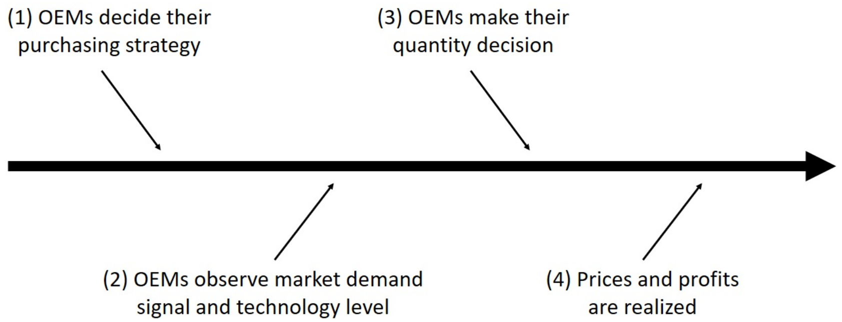


Figure 2.1. Timeline with the sequence of events.

- In the fourth (and last) period market demand is revealed to both OEMs, and the OEMs adapt their prices to clear their production quantities in a Cournot fashion.

Figure 2.1 depicts the model timeline.

We find that when the technology level is the only source of private information then the OEMs always prefer to share their information with each other. However, they prefer to conceal their private information on market demand uncertainty when it is the only source of private information and as long as product substitutability is sufficiently high. When competing OEMs' information on both technology level and market demand are private then getting into JP agreements would require them to reveal their information on both technology and market to each other. We find that sharing or not sharing technology level and market demand information can be preferred, depending on product substitutability, market demand variability, signal accuracy, as well as market impact of product technology level. This latter factor is crucial in determining OEMs' purchasing strategy when purchase cost reduction in JP is taken into consideration. If the potential impact of a "breakthrough" product is high, JP is always beneficial for the OEMs, whereas for a small impact of technology, JP is favored over IP only if product substitutability is low or the reduction in purchase cost is sufficiently high. We further find that, if JP does not give any cost reduction compared to IP then OEMs and consumers never benefit simultaneously from the information revelation inherent in JP agreements, when either market demand or technology level is the only source of private information. However, when both are sources of private information then there always exist situations in which JP would be preferred by both OEMs and consumers. These situations become more frequent as the cost reduction obtained through JP increases, suggesting that OEMs pass on a part of this cost reduction from their supplier to the consumers.

The remainder of the paper is organized as follows. In Section 2.2, we present some of the most relevant articles in the extant literature dealing with information sharing and JP. We then explain how the different aspects of the problem are modeled in section 2.3, and in the next section we give the main results for either of the two alternatives (individual or joint purchasing). In Section 2.5 we present the main results of this article comparing both sourcing options. The last section presents our conclusions and discusses the limits of our work. To keep the text flowing, we give all the proofs in the Appendix.

2.2 Related literature

This paper is principally related to two broad streams of literature dealing with 1) information sharing and 2) joint purchasing. The economics literature has investigated the benefits and disadvantages of information sharing between competing firms. In the case of Cournot competition, Clarke (1983), Vives (1984), Gal-Or (1985), and Li (1985) show that not sharing information is a dominant strategy if firms are symmetrically informed about common uncertain demand **intercept** and if goods are close substitutes. In contrast, Li (1985), Fried (1984) and Shapiro (1986) find that sharing information is a dominant strategy if firms possess private information about their own cost. The rationale is that releasing firm-specific information makes firms' decisions less correlated, which has a positive impact on firms' profits, unlike releasing common market information, which makes firms' decisions more correlated. These conclusions are reversed under Bertrand competition (Vives 1984; Gal-Or 1985). Our paper investigates information sharing under Cournot competition about both common market, i.e. the maximum price that the market is willing to accept, and about a firm-specific technology, i.e. OEM's information on the technology level of the new generation of its product.

Regarding to JP agreements among competing retailers, Keskinocak and Savaşaneril (2008) find that, under Cournot competition, whether buyers would engage in joint purchasing agreements would depend on their size and production capacity. Chen and Roma (2011) use a Bertrand model to show that, because of a reduced acquisition cost, JP is always beneficial for symmetric retailers, whereas it can be detrimental for one retailer if it has a larger market base (or if it is more cost efficient). For this retailer, letting its rival benefit from its better negotiating position (and hence lower purchasing cost) would erode its competitive advantage. In our paper, we examine the incentives for competing retailers to jointly procure from an information exchange perspective. Specifically, we study the mixed impact of uncertainty about both market demand and firm specific product technology level. To the best of our knowledge, no extant work has studied this problem. Similar to our context, Yan *et al.* (2017) analyze the impact that asymmetric private information about demand has on retailers' attitude towards joint purchasing. They find that the most informed retailer is never favorable to JP, suggesting that under information asymmetry, the cost benefits of JP do not compensate the loss of information advantage about the demand. In our work, we do not consider information asymmetry. However, we focus on the combined effect of two different types of uncertainties in motivating JP, rather than a single one as in Yan *et al.* (2017).

Chen and Roma (2011) argue that one reason why competing OEMs might prefer IP over JP agreements could be due to order size asymmetry across OEMs. Since per-unit procurement costs are typically decreasing in the quantity ordered, an OEM ordering more units would actually subsidize its competitor's purchase price through JP, and hence erode its own competitive advantage. Moreover, under JP the OEMs have to coordinate upfront in order to negotiate as a single buyer with the supplier(s).

Finally, Group purchasing organizations (GPO) have also received attention from academicians. GPO first emerged in public entities, like hospitals or schools, between which there is no competition. In that case, joining a GPO mostly lowers purchasing expenses with limited disadvantages, as notably discussed in Burns and Lee (2008) or in McKone-Sweet *et al.* (2005). However, some practical threats also exist, like the difficulty to gather a sufficient number of buyers (Liang *et al.*, 2014), or the reduced incentives for OEMs to innovate when they sell to GPO (Hu and Schwarz, 2011). Hu *et al.* (2012) and Saha *et al.* (2010) further study how those GPO affect healthcare supply chains. Yang *et al.* (2017) and Zhou *et al.* (2017) investigate the incentives for buying firms to contract with a GPO or to share information with a GPO. Different from above work, we study the strategic collaboration between buying firms without the intervention of GPO.

2.3 Model

Our model is inspired by many situations (e.g. producers of electronic devices such as TVs, smartphones, ...) where OEMs will bring periodical updates of their products to the market. In such cases the timing of release of new versions of the products is fixed well in advance by each OEM and often happens to be quite close to each other's release dates (to take advantage of the end of year holiday season for example). Moreover, selling periods are typically short relative to the long procurement lead times. We therefore chose to model the market in the Cournot framework in which the OEMs commit to the quantity of the product that they will bring into the market before releasing that new product.

We consider a duopoly with two risk neutral OEMs represented by subscripts i and j. Risk neutrality is a common assumption in the economics literature on information sharing. This assumption is also necessary to obtain analytical expressions for the equilibrium price and the profit of the OEMs. Both the OEMs simultaneously bring out a new product (which could be an upgrade to an existing product or a new generation of product) to the market. We denote by p_i and q_i the price and quantity respectively of OEM i's product on the market. The inverse demand for OEM i's product is modeled as:

$$p_i(q_i, q_j) = (P + \theta)M_i - (q_i + Kq_j), \tag{2.1}$$

where :

θ is a random variable with 0 mean and variance σ^2 representing the uncertainty about future demand.

$K \in [0, 1]$ is the substitution coefficient across products, with $K = 0$ corresponding to independent markets and $K = 1$ to perfectly substitutable products.

M_i is an indicator of the technology level of product i (more details in the next paragraph).

Note that we have normalized the slope coefficient of q_i (in OEM i's inverse demand function) to 1.

We denote OEM i's technology level by M_i. Given that the timing of the successive releases is fixed the OEMs will try to incorporate as many new features as they can for each new release and the resulting technology level would depend on the uncertain outcome of the R&D program undertaken by the respective firms. A value of 1 for OEM i's product would correspond to the normal evolution of the yearly release expected by the market. On the other hand, a high technology level of OEM i's product would correspond to some kind of "breakthrough" with a jump in the level of some important performance attribute of the product for this release. Our model corresponds to assuming the utility derived from one unit for a consumer of product i is multiplied by M_i. For simplicity, we assume that the technology level can take two states, i.e., it can turn out to be high or low with a likelihood of γ and $1 - \gamma$. The technological improvement of each new release will be known only to the firm, i.e., the OEMs are privately informed about their respective technology levels prior to committing quantities and launching (selling) their respective products. To simplify the analysis, we assume that $\gamma = 0.5$, even though the main insights of our model would carry with any other value of γ. A low technology level of OEM i's product results in $M_i = 1$ and a high technology level results in $M_i = M$, with $M > 1$. Thus M_i could either be 1 or M with a likelihood of 50%. Note that both γ and the value of M are exogenous, common knowledge and symmetric across OEMs. P and K are also exogenous and common knowledge.

The technology level is not the only source of private information that the OEMs would possess at the time of making their quantity decision. OEMs might also gain (private) information about other attributes that influence overall market demand as they move closer to introducing their product into the market. Specifically, before making the quantity decision each OEM receives a private signal Y_i, which is an unbiased estimator of θ, i.e., $\mathbb{E}[Y_i|\theta] = \theta$. This signal could, for example, arise from market research conducted independently by the OEMs to estimate the market demand. Similar to Li (1985) we assume a linear-expectation information structure, i.e. $\mathbb{E}[\theta|Y_i]$ is linear in Y_i. This information structure includes well-known conjugate pairs like normal-normal, beta-binomial and gamma-Poisson. For example, with the gamma-Poisson distribution, an OEM i would receive a signal Y_i derived from a Poisson distribution, which would then enable the OEM to formulate the gamma distribution of θ, since the distribution of θ is conditional upon the signal received by the OEM. We further define signal accuracy as $t = \dfrac{\mathbb{E}_{Y_i}[Var[Y_i|\theta]]}{\sigma^2}$. Note that we omit the subscript from t since we assume OEMs are symmetric. We assume $t \in [0, 1]$, with 0 corresponding to a perfect signal, and 1 corresponding to a signal that is no better than common information. From Li (1985), we directly obtain that $\mathbb{E}[\theta|Y_i] = \mathbb{E}[Y_j|Y_i] = \frac{Y_i}{1+t}$.

As indicated in Figure 2.1, the OEMs have two decision stages. Starting from the last stage we first determine each OEM's product quantity (q) decision after they have been privately informed about their respective technology levels and their respective market demand signal. In the individual purchasing (IP) case each OEM determines its product quantity based only on its own private information (on product technology level and market) and without knowing its competitor's private information. Moreover, each OEM individually procures the components that will be used in their respective products. We will denote by c^I the total purchase cost of all the components required for each unit of product in the individual purchasing case and assume each OEM incurs the same cost c^I.

In the joint purchasing (JP) case each OEM will have access to the competitor's private information when making its own quantity decision. As mentioned in the Introduction, in a JP agreement the two OEMs jointly buy some (or several) components that are required for the production of their respective products. Reaching an agreement on joint order quantities (of these components) will typically involve iterative discussions between the OEMs and potential suppliers which might involve sharing order forecasts. Moreover, it is very likely that some type of negotiation will arise with potential suppliers in terms of quantity versus price and also in terms of delivery lead time. Such negotiations will inevitably result in some exchange (or leakage) of private information that each OEM possesses about the common market and/or its product technology. By information exchange we do not necessarily mean that OEMs would voluntarily reveal their private information but rather that the OEMs would glean each other's information from the repeated interactions occurring in order to reach an agreement on a joint order size. Without getting into the details of such a process, we rather use a parsimonious model in which we assume that when deciding on joint order quantities OEMs become fully aware of each other's private information on both common market demand and product technology level. Thus the act of purchasing jointly allows the OEMs to make their product quantity decisions after fully knowing their competitor's private information. We will denote by c^J the total purchase cost of all the components required for each unit of product in the joint purchasing case. The purchase cost per-unit of product in case of joint purchasing will be lesser than in the individual purchasing case, i.e., $c^J < c^I$. The main argument for lowering the purchasing price is the larger joint quantity that OEMs will order from the common supplier(s). We denote by $r = (c^I - c^J)/c^I$ the percentage reduction of the purchase cost per-unit of product obtained in joint purchasing relative to individual purchasing.

After making the quantity decisions, both the OEMs bring their products to the market and the market clears as per the inverse demand function in Equation (2.1). Finally, in the first stage both the OEMs decide whether they want to enter in to a Joint Purchasing agreement or procure individually (Individual Purchasing). At the first stage (when making procurement decision) both the OEMs are not informed about their exact technology levels nor are they informed about their market demand signals. The motivation for this sequence of events is that purchasing agreements between the OEMs are strategic and are formed well in advance (sometimes spanning multiple product generations). Therefore as per our model timeline the OEMs first determine their purchasing strategy after which they are privately informed about their product technology level and receive private signals on market demand.

2.4 Analysis

To set the stage for the main focus of our research — the comparison between individual purchasing and joint purchasing agreement — we start by analyzing the individual purchasing case.

2.4.1 Individual purchasing case

Starting from the second stage, we denote OEM i's quantity decision in the IP case by $q_i^I(Y_i, M_i)$. From Equation (2.1) we characterize OEM i's conditional expected profit as

$$\mathbb{E}[\pi_i^I|Y_i, M_i] = q_i^I(Y_i, M_i)\Big((P+\mathbb{E}[\theta|Y_i])M_i - \Big(q_i^I(Y_i, M_i) + K\mathbb{E}[q_j^I(Y_j, M_j)|Y_i]\Big) - c^I\Big). \tag{2.2}$$

Moreover, we denote total consumer surplus as $CS(q_i, q_j)$ for given quantities q_i and q_j bought into the market by both the OEMs. Using Spence (1976) one can characterize the total consumer surplus from Equation (2.1) as:

$$\begin{aligned} CS(q_i, q_j) &= \int_0^{q_i} p_i(x, 0)dx + \int_0^{q_j} p_j(q_i, x)dx - p_i q_i - p_j q_j \\ &= (P+\theta)(M_i q_i + M_j q_j) - \frac{1}{2}(q_i^2 + 2K q_i q_j + q_j^2) - p_i q_i - p_j q_j \\ &= \frac{q_i(q_i + K q_j)}{2M_i} + \frac{q_j(q_j + K q_i)}{2M_j}. \end{aligned} \tag{2.3}$$

Maximizing the conditional expected profit (Equation (2.2)) provides the equilibrium quantities under the IP case, which are presented in Proposition 1.[2] We then compute the ex-ante (i.e., before OEMs observe their private information) expected profit and expected consumer surplus.

Proposition 1. *In the IP case, the unique equilibrium quantities for OEM i are given by*

$$q_i^I(Y_i, M_i = 1) = \frac{(K(1-M)+4)P - 4c^I}{4(K+2)} + \frac{4(t+1) + (1-M)K}{4(t+1)(K+2(t+1))}Y_i,$$

$$q_i^I(Y_i, M_i = M) = \frac{(M(4+K)-K)P - 4c^I}{4(K+2)} + \frac{(K+4(t+1))M - K}{4(t+1)(K+2(t+1))}Y_i.$$

OEM i's expected profit, i.e. before it observes its technology level and market demand signal, is given by

$$\begin{aligned} \mathbb{E}\pi_i^I = &\frac{((K(1-M)+4)P - 4c^I)^2 + ((M(4+K)-K)P - 4c^I)^2}{32(K+2)^2} \\ &+ \frac{\sigma^2((4(t+1)+(1-M)K)^2 + ((K+4(t+1))M - K)^2)}{32(t+1)(K+2(t+1))^2}, \end{aligned} \tag{2.4}$$

[2]Under both IP and JP, the quantity produced by an OEM corresponds to the quantity that it finally sells on the market. Moreover, the OEMs purchase components in volumes that are exactly required to produce the desired quantities of their products.

and the expected consumer surplus is given by

$$\begin{aligned}\mathbb{E}CS^I = \frac{K+2}{4}\frac{\Big((K(1-M)+4)P-4c^I\Big)^2+\Big((M(4+K)-K)P-4c^I\Big)^2}{16(K+2)^2}\\ +\frac{\sigma^2\Big(2(1+t)+K\Big)}{4}\frac{\Big(4(t+1)+(1-M)K\Big)^2+\Big((K+4(t+1))M-K\Big)^2}{16(t+1)^2\Big(K+2(t+1)\Big)^2}\\ +\frac{K\Big((K(1-M)+4)P-4c^I\Big)\Big((M(4+K)-K)P-4c^I\Big)}{32(K+2)^2}\\ +\frac{K\sigma^2\Big(4(t+1)+(1-M)K\Big)\Big((K+4(t+1))M-K\Big)}{32(t+1)^2\Big(K+2(t+1)\Big)^2}\end{aligned} \tag{2.5}$$

Proof. Using Equation (2.2), we characterize the FOCs of $\mathbb{E}_\theta\mathbb{E}_{M_j}\mathbb{E}_{Y_j}\Big[\pi_i^I|Y_i,M_i\Big]$ and $\mathbb{E}_\theta\mathbb{E}_{M_i}\mathbb{E}_{Y_i}\Big[\pi_j^I|Y_j,M_j\Big]$ for each technology level as:

$$2q_i^I(Y_i,M_i=1)=P-c^I+\mathbb{E}_\theta[\theta|Y_i]-\frac{K}{2}\mathbb{E}_{Y_j}\mathbb{E}_\theta[q_j^I(Y_j,M_j=1)+q_j^I(Y_j,M_j=M)|Y_i] \tag{2.6}$$

$$2q_i^I(Y_i,M_i=M)=PM+\mathbb{E}_\theta[\theta|Y_i]M-c^I-\frac{K}{2}\mathbb{E}_{Y_j}\mathbb{E}_\theta[q_j^I(Y_j,M_j=1)+q_j^I(Y_j,M_j=M)|Y_i] \tag{2.7}$$

$$2q_j^I(Y_j,M_j=1)=P-c^I+\mathbb{E}_\theta[\theta|Y_j]-\frac{K}{2}\mathbb{E}_{Y_i}\mathbb{E}_\theta[q_i^I(Y_i,M_i=1)+q_i^I(Y_i,M_i=M)|Y_j] \tag{2.8}$$

$$2q_j^I(Y_j,M_j=M)=PM+\mathbb{E}_\theta[\theta|Y_j]M-c^I-\frac{K}{2}\mathbb{E}_{Y_i}\mathbb{E}_\theta[q_i^I(Y_i,M_i=1)+q_i^I(Y_i,M_i=M)|Y_j] \tag{2.9}$$

In order to solve the above system of equations we first assume a specific form of equilibrium quantities (that are linear in market demand signal); we then characterize these quantities. Finally we show that these are indeed a unique solution to the above equations and hence we formulate an equilibrium. We define the linear candidate strategies as $q_i^I(Y_i,M_i=M)=H_i^0+H_i^1Y_i$, $q_i^I(Y_i,M_i=1)=L_i^0+L_i^1Y_i$, $q_j^I(Y_j,M_j=M)=H_j^0+H_j^1Y_j$ and $q_j^I(Y_j,M_j=1)=L_j^0+L_j^1Y_j$. Inserting these expressions into Equation (2.6) to (2.9) gives us 4 equations that are linear in Y_i (since q_i and q_j are linear in Y_i and Y_j respectively and moreover $\mathbb{E}_{Y_j}[Y_j|Y_i]$ too is linear in Y_i). By separating out terms containing Y_i and those not containing Y_i we obtain the following 8 equations :

$$
\begin{aligned}
2L_i^0 &= (P - c^I) - \frac{K}{2}(L_j^0 + H_j^0) &(2.10)\\
2L_i^1 &= \frac{1}{t+1} - \frac{K}{2(t+1)}(L_j^1 + H_j^1) &(2.11)\\
2H_i^0 &= (PM - c^I) - \frac{K}{2}(L_j^0 + H_j^0) &(2.12)\\
2H_i^1 &= \frac{M}{t+1} - \frac{K}{2(t+1)}(L_j^1 + H_j^1) &(2.13)\\
2L_j^0 &= (P - c^I) - \frac{K}{2}(L_i^0 + H_i^0)\\
2L_j^1 &= \frac{1}{t+1} - \frac{K}{2(t+1)}(L_i^1 + H_i^1)\\
2H_j^0 &= (PM - c^I) - \frac{K}{2}(L_i^0 + H_i^0)\\
2H_j^1 &= \frac{M}{t+1} - \frac{K}{2(t+1)}(L_i^1 + H_i^1)
\end{aligned}
$$

Solving this system of equations gives

$$
\begin{aligned}
L^0 &= L_j^0 = L_i^0 = \frac{(K(1-M)+4)P - 4c^I}{4(K+2)}\\
L^1 &= L_j^1 = L_i^1 = \frac{4(t+1) + (1-M)K}{4(t+1)(K+2(t+1))}\\
H^0 &= H_j^0 = H_i^0 = \frac{(M(4+K)-K)P - 4c^I}{4(K+2)}\\
H^1 &= H_j^1 = H_i^1 = \frac{(K+4(t+1))M - K}{4(t+1)(K+2(t+1))}.
\end{aligned}
$$

Inserting these values into q_i gives

$$
\begin{aligned}
q_i^I(Y_i, M_i = 1) &= \frac{(K(1-M)+4)P - 4c^I}{4(K+2)} + \frac{4(t+1)+(1-M)K}{4(t+1)(K+2(t+1))}Y_i.\\
q_i^I(Y_i, M_i = M) &= \frac{(M(4+K)-K)P - 4c^I}{4(K+2)} + \frac{(K+4(t+1))M - K}{4(t+1)(K+2(t+1))}Y_i.
\end{aligned}
\tag{2.14}
$$

Similarly, expressions for q_j are derived by substituting Y_i with Y_j in the above equations.

Next, we show the uniqueness of above linear equilibrium quantities. For this, we suppose $q_i^{I^*}(Y_i, M_i = M)$, $q_i^{I^*}(Y_i, M_i = 1)$, $q_j^{I^*}(Y_j, M_j = M)$,

$q_j^{I^*}(Y_j, M_j = 1)$ is another equilibrium, i.e., they solve Equation (2.6) Equation (2.7). Reformulating Equation (2.6) with $q_i^{I^*}, q_j^{I^*}$ and then subtracting $2q_i^I(Y_i, M_i = 1)$ from both sides gives

$$\begin{aligned}&2(q_i^{I^*}(Y_i, M_i = 1) - q_i^I(Y_i, M_i = 1))\\&= P - c^I - 2L_i^0 + Y_i(\frac{1}{1+t} - 2L_i^1) - \frac{K}{2}\mathbb{E}_{Y_j}\mathbb{E}_\theta[q_j^{I^*}(Y_j, M_j = 1) + q_j^{I^*}(Y_j, M_j = M)|Y_i].\end{aligned}$$

From Equation (2.10) we get $(P - c^I) - 2L_i^0 = \frac{K}{2}(L_j^0 + H_j^0)$ and from Equation (2.11) we get $\frac{1}{t+1} - 2L_i^1 = \frac{K}{2(t+1)}(L_j^1 + H_j^1)$. Therefore

$$\begin{aligned}&2(q_i^{I^*}(Y_i, M_i = 1) - q_i^I(Y_i, M_i = 1))\\&= \tfrac{K}{2}(L_j^0 + H_j^0) + \tfrac{K}{2(1+t)}(L_j^1 + H_j^1)Y_i - \tfrac{K}{2}\mathbb{E}_{Y_j}\mathbb{E}_\theta[q_j^{I^*}(Y_j, M_j = 1) + q_j^{I^*}(Y_j, M_j = M)|Y_i].\end{aligned}$$

We know that

$$\begin{aligned}&\tfrac{K}{2}(L_j^0 + H_j^0) + \tfrac{K}{2(1+t)}(L_j^1 + H_j^1)Y_i\\&= \tfrac{K}{2}(L_j^0 + L_j^1\tfrac{Y_i}{(1+t)}) + \tfrac{K}{2}(H_j^0 + H_j^1\tfrac{Y_i}{(1+t)})\\&= \tfrac{K}{2}(L_j^0 + L_j^1\mathbb{E}_{Y_j}[Y_j|Y_i]) + \tfrac{K}{2}(H_j^0 + H_j^1\mathbb{E}_{Y_j}[Y_j|Y_i])\\&= \tfrac{K}{2}\mathbb{E}_{Y_j}\mathbb{E}_\theta[q_j^I(Y_j, M_j = 1) + q_j^I(Y_j, M_j = M)|Y_i].\end{aligned}$$

Therefore,

$$\begin{aligned}&2(q_i^{I^*}(Y_i, M_i = 1) - q_i^I(Y_i, M_i = 1))\\&= \tfrac{K}{2}\mathbb{E}_{Y_j}\mathbb{E}_\theta[q_j^I(Y_j, M_j = 1) + q_j^I(Y_j, M_j = M)|Y_i]\\&-\tfrac{K}{2}\mathbb{E}_{Y_j}\mathbb{E}_\theta[q_j^{I^*}(Y_j, M_j = 1) + q_j^{I^*}(Y_j, M_j = M)|Y_i]\\&= -\tfrac{K}{2}(\mathbb{E}_{Y_j}\mathbb{E}_\theta[q_j^{I^*}(Y_j, M_j = 1) - q_j^I(Y_j, M_j = 1)|Y_i]\\&+\mathbb{E}_{Y_j}\mathbb{E}_\theta[q_j^{I^*}(Y_j, M_j = M) - q_j^I(Y_j, M_j = M)|Y_i]).\end{aligned} \tag{2.15}$$

Similarly, reformulating Equation (2.7) with $q_i^{I^*}, q_j^{I^*}$ and then subtracting $2q_i^I(Y_i, M_i = M)$ from both sides gives

$$\begin{aligned}&2(q_i^{I^*}(Y_i, M_i = M) - q_i^I(Y_i, M_i = M))\\&= PM - c^I - 2H_i^0 + Y_i(\frac{M}{1+t} - 2H_i^1)\\&-\tfrac{K}{2}\mathbb{E}_{Y_j}\mathbb{E}_\theta[q_j^{I^*}(Y_j, M_j = 1) + q_j^{I^*}(Y_j, M_j = M)|Y_i].\end{aligned}$$

From Equation (2.12) we get $(PM - c^I) - 2H_i^0 = \frac{K}{2}(L_j^0 + H_j^0)$ and from Equation (2.13) we get $\frac{M}{t+1} - 2H_i^1 = \frac{K}{2(t+1)}(L_j^1 + H_j^1)$. Following the same steps as above we obtain

$$\begin{aligned}&2(q_i^{I^*}(Y_i, M_i = M) - q_i^I(Y_i, M_i = M))\\&= -\frac{K}{2}(\mathbb{E}_{Y_j}\mathbb{E}_\theta[q_j^{I^*}(Y_j, M_j = 1) - q_j^I(Y_j, M_j = 1)|Y_i]\\&+ \mathbb{E}_{Y_j}\mathbb{E}_\theta[q_j^{I^*}(Y_j, M_j = M) - q_j^I(Y_j, M_j = M)|Y_i]).\end{aligned} \tag{2.16}$$

From Equations (2.8), (2.9) we get:

$$q_j^{I^*}(Y_j, M_j = M) = q_j^{I^*}(Y_j, M_j = 1) + \frac{(M-1)(P + \mathbb{E}_\theta[\theta|Y_j])}{2} \tag{2.17}$$

and

$$q_j^{I}(Y_j, M_j = M) = q_j^{I}(Y_j, M_j = 1) + \frac{(M-1)(P + \mathbb{E}_\theta[\theta|Y_j])}{2}. \tag{2.18}$$

Subtracting Equation (2.17) from Equation (2.18) gives

$$q_j^{I^*}(Y_j, M_j = M) - q_j^{I}(Y_j, M_j = M) = q_j^{I^*}(Y_j, M_j = 1) - q_j^{I}(Y_j, M_j = 1). \tag{2.19}$$

Taking the expectation on both sides of Equation (2.19) conditional on Y_i, we have $E_{Y_j}\mathbb{E}_\theta[q_j^{I^*}(Y_j, M_j = M) - q_j^{I}(Y_j, M_j = M)|Y_i] = E_{Y_j}\mathbb{E}_\theta[q_j^{I^*}(Y_j, M_j = 1) - q_j^{I}(Y_j, M_j = 1)|Y_i]$. Therefore Equation (2.15) and Equation (2.16) can be rewritten respectively as

$$-\frac{2}{K}(q_i^{I^*}(Y_i, M_i = 1) - q_i^{I}(Y_i, M_i = 1)) = \mathbb{E}_{Y_j}\mathbb{E}_\theta[q_j^{I^*}(Y_j, M_j = 1) - q_j^{I}(Y_j, M_j = 1)|Y_i] \tag{2.20}$$

$$-\frac{2}{K}(q_i^{I^*}(Y_i, M_i = M) - q_i^{I}(Y_i, M_i = M)) = \mathbb{E}_{Y_j}\mathbb{E}_\theta[q_j^{I^*}(Y_j, M_j = M) - q_j^{I}(Y_j, M_j = M)|Y_i]. \tag{2.21}$$

Let $g_i(Y_i) = q_i^{I^*}(Y_i, M_i = 1) - q_i^{I}(Y_i, M_i = 1)$, and $g_i'(Y_i) = q_i^{I^*}(Y_i, M_i = M) - q_i^{I}(Y_i, M_i = M)$, then Equation (2.20) and (2.21) are equivalent to

$$-\frac{2}{K}g_i(Y_i) = \mathbb{E}_{Y_j}\Big[g_j(Y_j)|Y_i\Big]$$

$$-\frac{2}{K}g_i'(Y_i) = \mathbb{E}_{Y_j}\Big[g_j'(Y_j)|Y_i\Big].$$

Since $| -\frac{2}{K} |> 1$, according to Claim 1 in the Appendix of Ha *et al.* (2011), $g_i(Y_i) = 0$ and $g_i'(Y_i) = 0$ almost surely. Therefore $q_i^{I^*}(Y_i, M_i = 1) = q_i^{I}(Y_i, M_i = 1)$ and $q_i^{I^*}(Y_i, M_i = M) = q_i^{I}(Y_i, M_i = M)$. This finishes to prove that Equations (2.14) give the unique equilibrium quantity decisions, which we present in Proposition 1.

We substitute Equations (2.6) to (2.7) into Equation (2.2) to further obtain OEMs' ex-post expected profit. For low and high technology levels respectively, these are given by

$$\mathbb{E}_{M_j}\mathbb{E}_{Y_j}\mathbb{E}_\theta[\pi_i^I|Y_i, M_i = 1] = \Big(q_i^I(Y_i, M_i = 1)\Big)^2 \tag{2.22}$$

$$\mathbb{E}_{M_j}\mathbb{E}_{Y_j}\mathbb{E}_\theta[\pi_i^I|Y_i, M_i = M] = \Big(q_i^I(Y_i, M_i = M)\Big)^2. \tag{2.23}$$

The expected profit, as presented in Proposition 1, is then obtained by taking an average of Equations (2.22) and (2.23) since the likelihood that each technology level is reached (i.e. $\gamma = 0.5$):

$$\begin{aligned}\mathbb{E}\pi_i^I &= \mathbb{E}_{Y_i}\Big[\frac{1}{2}\Big(L^0 + L^1 Y_i\Big)^2 + \frac{1}{2}\Big(H^0 + H^1 Y_i\Big)^2\Big] \\ &= \frac{1}{2}(L^0)^2 + \frac{1}{2}(H^0)^2 + \mathbb{E}_{Y_i}[Y_i](L^0L^1 + H^0H^1) + \mathbb{E}_{Y_i}[Y_i^2]\Big(\frac{1}{2}(L^1)^2 + \frac{1}{2}(H^1)^2\Big).\end{aligned} \tag{2.24}$$

Since Y_i is an unbiased estimator of θ, then $\mathbb{E}_{Y_i}[Y_i] = 0$, and $\mathbb{E}_{Y_i}[Y_i^2] = Var[Y_i] = \mathbb{E}_\theta\big[Var[Y_i|\theta]\big] + Var\big[\mathbb{E}_{Y_i}[Y_i|\theta]\big] = t\sigma^2 + \sigma^2 = (1+t)\sigma^2$. Substituting $\mathbb{E}_{Y_i}[Y_i]$, $\mathbb{E}_{Y_i}[Y_i^2]$, L^0, L^1, H^0, H^1 into Equation (2.24), we obtain

$$\begin{aligned}\mathbb{E}\pi_i^I = &\frac{((K(1-M)+4)P - 4c^I)^2 + ((M(4+K)-K)P - 4c^I)^2}{32(K+2)^2} \\ &+ \frac{\sigma^2((4(t+1) + (1-M)K)^2 + ((K+4(t+1))M - K)^2)}{32(t+1)(K+2(t+1))^2}\end{aligned}$$

We now characterize expected consumer surplus. From (2.3) we can characterize the expected consumer surplus in the IP case as:

$$\begin{aligned}\mathbb{E}CS^I &= \mathbb{E}\left[\frac{1}{2M_i}\Big(q_i^I(Y_i,M_i) + Kq_j^I(Y_j,M_j)\Big)q_i^I(Y_i,M_i)\right] \\ &+ \mathbb{E}\left[\frac{1}{2M_j}\Big(q_j^I(Y_j,M_j) + Kq_i^I(Y_i,M_i)\Big)q_j^I(Y_j,M_j)\right]\end{aligned} \tag{2.25}$$

Substituting the equilibrium quantities of IP from Equation (2.14) into Equation (2.25), we obtain:

$$\begin{aligned}\mathbb{E}CS^I = &\frac{K+2}{4}\frac{\Big((K(1-M)+4)P - 4c^I\Big)^2 + \Big((M(4+K)-K)P - 4c^I\Big)^2}{16(K+2)^2} \\ &+ \frac{\sigma^2\Big(2(1+t)+K\Big)}{4}\frac{\Big(4(t+1) + (1-M)K\Big)^2 + \Big((K+4(t+1))M - K\Big)^2}{16(t+1)^2\Big(K+2(t+1)\Big)^2} \\ &+ \frac{K\Big((K(1-M)+4)P - 4c^I\Big)\Big((M(4+K)-K)P - 4c^I\Big)}{32(K+2)^2} \\ &+ \frac{K\sigma^2\Big(4(t+1) + (1-M)K\Big)\Big((K+4(t+1))M - K\Big)}{32(t+1)^2\Big(K+2(t+1)\Big)^2}\end{aligned} \tag{2.26}$$

□

2.4.2 Joint purchasing case

As mentioned in §2.3, under JP agreement we assume that the two OEMs end up having full knowledge of their competitor's private information on technology level and market signal. Consequently, each OEM would formulate its expectation of θ conditional on both signals (Y_i and Y_j) and moreover would be informed about both M_i and M_j. Following Li (1985), we obtain that $\mathbb{E}[\theta|Y_i, Y_j] = \dfrac{Y_i + Y_j}{2+t}$. Thus the joint quantity decision would involve each OEM deciding its own order quantity, $q_i^J(Y_i, Y_j, M_i, M_j)$, that maximizes the OEM's expected profit taking into account the information gathered about the other OEM's information set during the negotiation process. Under the JP strategy, OEM i's expected profit can be expressed as:

$$\mathbb{E}[\pi_i^J|Y_i, Y_j, M_i, M_j] = q_i^J(Y_i, Y_j, M_i, M_j) \\ \Big((P + \mathbb{E}[\theta|Y_i, Y_j])M_i - \Big(q_i^J(Y_i, Y_j, M_i, M_j) + Kq_j^J(Y_j, Y_i, M_j, M_i)\Big) - c^J\Big). \tag{2.27}$$

Maximizing this conditional expected profit for each OEM gives the equilibrium quantities under JP.

Proposition 2. *In the JP case, OEM i's unique equilibrium quantity is given by*

$$q_i^J(Y_i, Y_j, M_i, M_j) = \frac{(2M_i - KM_j)}{4 - K^2}\Big(P + \frac{Y_i + Y_j}{2+t}\Big) - \frac{c^J}{K+2}. \tag{2.28}$$

OEM i's expected profit, i.e. before it observes the technology level and market demand signals, is given by

$$\mathbb{E}\pi_i^J = \frac{2(2-K)^2(c^J)(c^J - P(M+1)) + ((K^2+4)(M^2+1) - 8KM)(P^2 + \frac{2\sigma^2}{2+t})}{4(K^2-4)^2} \\ + \frac{(P - c^J)^2 + \frac{2\sigma^2}{2+t}(M^2+1) + MP(MP - 2c^J) + (c^J)^2}{4(K+2)^2} \tag{2.29}$$

and the expected consumer surplus is given by

$$\mathbb{E}CS^J = \frac{(1+K)\Big((P-c^J)^2 + \frac{2\sigma^2}{2+t}(M^2+1) + (MP-c^J)^2\Big)}{4(K+2)^2} + \frac{Kc^J\Big(c^J - P(M+1)\Big)}{2(K+2)^2} + \frac{K(KM-2)(K-2M)(P^2+\frac{2\sigma^2}{2+t})}{2(K^2-4)^2} + \frac{2(2-K)^2c^J\Big(c^J-2P(M+1)\Big) + (P^2+\frac{2\sigma^2}{2+t})\Big((KM-2)^2+(K-2M)^2\Big)}{4(K^2-4)^2} \tag{2.30}$$

Proof. From Equation (2.27) and for a given (Y_i, Y_j, M_i, M_j) we obtain the FOCs for OEM i and OEM j respectively as

$$\begin{aligned}(P+\mathbb{E}_\theta[\theta|Y_i,Y_j])M_i - Kq_j^J(Y_i,Y_j,M_i,M_j) - 2q_i^J(Y_i,Y_j,M_i,M_j) - c^J = 0\\ (P+\mathbb{E}_\theta[\theta|Y_i,Y_j])M_j - Kq_i^J(Y_i,Y_j,M_i,M_j) - 2q_j^J(Y_i,Y_j,M_i,M_j) - c^J = 0.\end{aligned}$$

Solving the above two equations and substituting $\mathbb{E}_\theta\big[\theta|Y_i,Y_j\big] = \dfrac{Y_i+Y_j}{2+t}$, we obtain

$$q_i^J(Y_i,Y_j,M_i,M_j) = \frac{(2M_i-KM_j)}{4-K^2}\Big(P+\frac{Y_i+Y_j}{2+t}\Big) - \frac{c^J}{K+2}.$$

Inserting the FOCs back into Equation (2.27) we obtain OEM i's equilibrium conditional expected profit:

$$\mathbb{E}_\theta\Big[\pi_i^J|Y_i,Y_j,M_i,M_j\Big] = \Big(q_i^J(Y_i,Y_j,M_i,M_j)\Big)^2. \tag{2.31}$$

Then, we find the expected profit through weighting the different combinations of technology levels by their respective probabilities (since $\gamma = \frac{1}{2}$, any pair of M_i and M_j together would occur with a likelihood of $\frac{1}{4}$):

$$\begin{aligned}\mathbb{E}\pi_i^J = \tfrac{1}{4}\Big\langle &\mathbb{E}_\theta\mathbb{E}_{Y_i}\mathbb{E}_{Y_j}\Big[\Big(q_i^J(Y_i,Y_j,M_i=1,M_j=1)\Big)^2\Big]\\ &+\mathbb{E}_\theta\mathbb{E}_{Y_i}\mathbb{E}_{Y_j}\Big[\Big(q_i^J(Y_i,Y_j,M_i=1,M_j=M)\Big)^2\Big]\\ &+\mathbb{E}_\theta\mathbb{E}_{Y_i}\mathbb{E}_{Y_j}\Big[\Big(q_i^J(Y_i,Y_j,M_i=M,M_j=1)\Big)^2\Big]\\ &+\mathbb{E}_\theta\mathbb{E}_{Y_i}\mathbb{E}_{Y_j}\Big[\Big(q_i^J(Y_i,Y_j,M_i=M,M_j=M)\Big)^2\Big]\Big\rangle\end{aligned} \tag{2.32}$$

Since $\mathbb{E}_{Y_i}[Y_i] = \mathbb{E}_{Y_j}[Y_j] = 0$ and since Y_i and Y_j are conditionally independent, we have

$\mathbb{E}[Y_iY_j] = \mathbb{E}_\theta[Y_iY_j|\theta] = \mathbb{E}_\theta\mathbb{E}[Y_i|\theta]\mathbb{E}[Y_j|\theta] = \mathbb{E}_\theta[\theta^2] = \sigma^2$. Using these and $\mathbb{E}_{Y_i}[Y_i^2] = \mathbb{E}_{Y_j}[Y_j^2] = (1+t)\sigma^2$, we can write

$$\mathbb{E}\pi_i^J = \frac{2(2-K)^2(c^J)(c^J - P(M+1)) + ((K^2+4)(M^2+1) - 8KM)(P^2 + \frac{2\sigma^2}{2+t})}{4(K^2-4)^2} + \frac{(P-c^J)^2 + \frac{2\sigma^2}{2+t}(M^2+1) + MP(MP-2c^J) + (c^J)^2}{4(K+2)^2}$$

Then, the expected consumer surplus in JP is characterized as:

$$\mathbb{E}CS^J = \mathbb{E}\left[\frac{1}{2M_i}\Big(q_i^J(Y_i,Y_j,M_i,M_j) + Kq_j^J(Y_i,Y_j,M_i,M_j)\Big)q_i^J(Y_i,Y_j,M_i,M_j)\right] + \mathbb{E}\left[\frac{1}{2M_j}\Big(q_j^J(Y_i,Y_j,M_i,M_j) + Kq_i^J(Y_i,Y_j,M_i,M_j)\Big)q_j^J(Y_i,Y_j,M_i,M_j)\right] \tag{2.33}$$

Substituting equilibrium quantities of JP into (2.33), we obtain:

$$\begin{aligned}\mathbb{E}CS^J = {} & \frac{(1+K)\Big((P-c^J)^2 + \frac{2\sigma^2}{2+t}(M^2+1) + (MP-c^J)^2\Big)}{4(K+2)^2} \\ & + \frac{Kc^J\Big(c^J - P(M+1)\Big)}{2(K+2)^2} + \frac{K(KM-2)(K-2M)(P^2+\frac{2\sigma^2}{2+t})}{2(K^2-4)^2} \\ & + \frac{2(2-K)^2c^J\Big(c^J - 2P(M+1)\Big) + (P^2+\frac{2\sigma^2}{2+t})\Big((KM-2)^2 + (K-2M)^2\Big)}{4(K^2-4)^2}\end{aligned} \tag{2.34}$$

□

We further consider a linear cost function where the purchase cost is decreasing in the quantities purchased, which is defined as $c = A - wQ$, with A being the purchase price without any rebate, w being the steepness of the rebate and Q being the total quantity ordered, such that $Q = q_i$ under IP and $Q = q_i + q_j$ under JP.Following the same analysis in the proof of Proposition 1 and the proof of Proposition 2, we obtain

$$\begin{aligned} q_i^I(Y_i, M_i = 1) = {} & \frac{4(1-w)(A-P) + KP(M-1)}{4(w-1)(K+2(1-w))} \\ & + \frac{-K(M-1) - 4(t+1)(w-1)}{4(t+1)(1-w)(K+2(t+1)(1-w))}Y_i. \\ q_i^I(Y_i, M_i = M) = {} & \frac{4(1-w)(A-MP) + KP(1-M)}{4(w-1)(K+2(1-w))} \\ & + \frac{-K(M-1) - 4M(t+1)(w-1)}{4(t+1)(1-w)(K+2(t+1)(1-w))}Y_i \end{aligned} \tag{2.35}$$

and

$$q_i^J(Y_i, Y_j, M_i, M_j) = \frac{A(K + w - 2) - (P + \frac{Y_i + Y_j}{2+t})(2M_i(1 - w) + M_j(w - K))}{-K^2 + 2Kw + 3w^2 - 8w + 4}. \tag{2.36}$$

It's worth noting that when $w = 0$ and $r = 0$, Equation (2.35) and (2.14) are equivalent, same for Equation (2.36) and (2.4.2).

2.5 Comparison between individual and joint purchasing

Having analyzed the IP and JP cases in the previous sections, we are now in a position to answer the questions about the preference of an OEM for either option and about what the impact of OEMs' purchase decisions is for consumer welfare? As explained earlier, the decision to get into a JP agreement will have to be made before observing its market demand signal or its technological level. To formulate this decision we compare the ex-ante, expected, profit of an OEM in the IP case (Equation (2.4)) with its expected profit for the JP case (Equation (2.29)).

We denote the difference in expected profits for the OEMs between JP and IP as $\Delta_{OEM} = \mathbb{E}\pi_i^J - \mathbb{E}\pi_i^I$ and the difference in expected consumer surplus between JP and IP as $\Delta_{CS} = \mathbb{E}CS^J - \mathbb{E}CS^I$. The full expression for Δ_{OEM} and Δ_{CS} are presented in the Equation (2.40) and (2.50) respectively of the appendix.

Proposition 3. *Δ_{OEM} is non-negative while Δ_{CS} is non-positive when $\sigma = 0$ and $r = 0$.*

Proposition 3 implies that OEMs are always better off with a JP agreement (in which they share private information on their individual technology level) when there is no market uncertainty, despite the JP agreement providing no procurement cost benefit relative to IP. This can be explained by the interactions that our model allows between competitors. Under JP, the OEMs would make their quantity decision while knowing both their own and their rival's technology levels. If an OEM has a technology advantage over the other, it would make a bigger order, to put its competitor under pressure, since OEMs affect each other's price through the quantity of products that they put on the market. In response, the rival would reduce its order to avoid flooding the market. Thus, technology level information allows OEMs' to make more informed quantity decisions which are beneficial for OEMs. We further find that the benefit of making these more informed decisions are pocketed by the OEMs at the cost of the consumers, i.e., the consumer surplus goes down when competing OEMs get into a JP agreement if there is no market uncertainty and no supply cost reduction.

Proof. We find

$$\Delta_{OEM}(\sigma^2 = 0, r = 0) = \frac{K^2P^2(12 - K^2)(M - 1)^2}{16(K^2 - 4)^2}, \tag{2.37}$$

and

$$\Delta_{CS}(\sigma^2 = 0, r = 0) = \frac{-K^2P^2(K^2 + 4)(M - 1)^2}{16(K^2 - 4)^2}, \tag{2.38}$$

with $\Delta_{OEM}(\sigma^2, r = 0) \geq 0$ and $\Delta_{CS}(\sigma^2, r = 0) \leq 0$ for $K \in [0, 1]$. □

Theorem 4. *Without procurement cost advantage (i.e., $r = 0$),*

1. *OEMs prefer JP over IP if $K < K^*$, with K^* being defined as*

$$K^* = \frac{2\left(\sqrt{2(t+1)(t+2)} - (t+1)\right)}{t+3} \geq \frac{2}{3}. \tag{2.39}$$

   Otherwise, there exists a threshold M^* such that OEMs prefer JP over IP if $M > M^*$, whereas OEMs prefer IP over JP if $M < M^*$.

2. *Consumer surplus is larger with JP if $K < \tilde{K}^*$. Otherwise, there exists a threshold $\tilde{M}^*$ such that consumer surplus is larger with JP if $M < \tilde{M}^*$, whereas consumer surplus is larger with IP if $M > \tilde{M}^*$.*

3. *When both market demand and technology level are uncertain, there always exists a non empty interval in the technology level M over which both consumers and OEMs simultaneously are better off with JP rather than IP.*

Proof. First, we show how we obtain K^* and M^*. We characterize Δ_{OEM} as a quadratic function of M:

$$\Delta_{OEM} = \frac{\alpha M^2 + \beta M + \gamma}{\xi} \tag{2.40}$$

with

$$\begin{aligned}
\alpha &= \Big\langle 4K^2\Big(K(12-K^2-5K)+60\Big)P^2+8\sigma^2\Big(K^2(16-K^2)+16(1-K)\Big)\Big\rangle t^3 \\
&\Big\langle\Big(432+K(192-24K-16K^2-K^3)\Big)P^2K^2+4\sigma^2\Big(16(4-3K) \\
&+K^2(24K-K^3-8K^2+80)\Big)\Big\rangle t^2 \\
&(K+2)\Big\langle\Big(168+K(36-14K-3K^2)\Big)P^2K^2+\sigma^2\Big(K^2(184+4K-10K^2-K^3) \\
&+64(1-K)\Big)\Big\rangle t+2K^2(12-K^2)\Big(2P^2t^4+(P^2+\sigma^2)(K+2)^2\Big) \qquad (2.41)
\end{aligned}$$

$$\begin{aligned}
\beta &= 2K^2(t+2)(t+1)(K^2-12)(K+2t+2)^2P^2+16c^Ir(t+2)(t+1)(K-2)^2 \\
&(K+2t+2)^2P+2K\sigma^2\Big(4(K^4-8K^2-32K-80)t^2-128t^3 \\
&+(K+2)(K^4+10K^3-28K^2-72K-96)t+2K(K^2-12)(K+2)^2\Big)
\end{aligned} \qquad (2.42)$$

$$\gamma = \alpha+16c^Ir(K-2)^2(t+2)(t+1)(K+2t+2)^2\Big(P-c^I(2-r)\Big) \qquad (2.43)$$

$$\xi = 16(K^2-4)^2(t^2+3t+2)(K+2t+2)^2 \qquad (2.44)$$

We let $\Omega(M)=\alpha M^2+\beta M+\gamma$, as $\xi>0$, the sign of Δ_{OEM} is determined by $\Omega(M)$. Given that $K\leq 1$ implies $\alpha\geq 0$, $\Omega(M)$ is convex in M. At $r=0$, we have

$$\Omega(M=1,r=0)=\frac{d\Omega(r=0)}{dM}=2\alpha+\beta, \qquad (2.45)$$

which implies that if $\Omega(M=1,r=0)>0$, then $\Omega(r=0)>0$ holds for any $M\geq 1$ as it increases in M. Next we define the condition for $\Omega(M=1,r=0)>0$. Using Equation (2.40), (2.41), (2.42), (2.43), (2.44) we find that

$$\Omega(M=1,r=0)=16t\sigma^2(K-2)^2(t+1)\Big(-(t+3)K^2-4(t+1)K+4(t+1)\Big). \qquad (2.46)$$

We let $l(K)=\Big(-(t+3)K^2-4(t+1)K+4(t+1)\Big)$. $\frac{d^2l(K)}{dK^2}=-(t+3)$, therefore $l(K)$ is concave in K. Since $16t\sigma^2(K-2)^2(t+1)>0$ for $\sigma^2>0$ and $K\in[0,1]$, $\Omega(M=1,r=0)>0$ if $l(K)>0$.

The two roots of $l(K)$ in K can be characterized as

$$K_{1,2}=\frac{-2\big(\pm\sqrt{2(t+1)(t+2)}+(t+1)\big)}{t+3}, \qquad (2.47)$$

with one root to be negative and one to be positive. The concavity of $l(K)$ implies that $\Omega(r=0)>0$ if $K<K^*$ with K^* be characterized as

$$K^* = \frac{2\big(\sqrt{2(t+1)(t+2)} - (t+1)\big)}{t+3}, \tag{2.48}$$

We further show that for any $K<\frac{2}{3}$, $\Omega(r=0)>0$ such that $K^*\geq\frac{2}{3}$. For this, we rewrite the inequality $l(K)>0$ as

$$\Big(t(-K^2-4K+4) - 3K^2 - 4K + 4\Big) > 0, \tag{2.49}$$

which is true if $t(-K^2-4K+4) > 3K^2+4K-4$. Since on the interval $K\in[0,\frac{2}{3}]$, $-K^2-4K+4\geq 0$ and $3K^2+4K-4\leq 0$, Inequality (2.49) is true for any $K<\frac{2}{3}$. Therefore $K^*\geq\frac{2}{3}$.

For $\Omega(M=1, r=0)<0$, i.e., $K>K^*$, the convexity of Ω implies that $\Omega(r=0)$ would be greater than 0 beyond a threshold M. This threshold is the highest root in M of $\Omega(r=0)$, namely, $M^*=\frac{\sqrt{\beta^2-4\alpha^2}-\beta}{2\alpha}>1$.

Next, we prove that there exists a unique $\tilde{K}^*$ and $\tilde{M}^*$. We characterize Δ_{CS} as a quadratic function of M:

$$\Delta_{CS} = \frac{\tilde{\alpha}M^2+\tilde{\beta}M+\tilde{\gamma}}{\tilde{\xi}} \tag{2.50}$$

with

$$\begin{aligned}
\tilde{\alpha} &= -(t+2)\Big(\sigma^2+P^2(t+1)\Big)K^6-4(t+1)^2\Big(2\sigma^2+P^2(t+2)\Big)K^5 \\
&\quad -\Big(4(t+2)(t+1)(t^2+2t+2)P^2+8\sigma^2(t^3+2t+2)\Big)K^4 \\
&\quad -16(t+1)\Big(P^2(t+2)(t+1)+2\sigma^2(-t^2+2t+1)\Big)K^3 \\
&\quad -\Big(16(t+2)(t+1)^3P^2+16\sigma^2(8t^3+20t^2+13t+2)\Big)K^2 \\
&\quad +128t\sigma^2(t+1)K+128t\sigma^2(t+1)^2
\end{aligned} \tag{2.51}$$

$$\begin{aligned}
\tilde{\beta} &= 2K^2(t+2)(t+1)(K^2+4)\Big(K+2(t+1)\Big)^2P^2+16c^Ir(K+1)(K-2)^2 \\
&\quad (t+2)(t+1)\Big(K+2(t+1)\Big)^2P+2K^2\sigma^2\Big(32Kt^3+32K(K+3)t^2 \\
&\quad +(K+2)(K^3+6K^2+44K+8)t+2(K^2+4)(K+2)^2\Big)
\end{aligned} \tag{2.52}$$

$$\tilde{\gamma} = \tilde{\alpha}+16c^Ir(K+1)(K-2)^2(t+2)(t+1)(K+2t+2)^2\Big(P-c^I(2-r)\Big) \tag{2.53}$$

$$\tilde{\xi} = 16(K^2-4)^2(t^2+3t+2)(K+2t+2)^2$$

We let $\tilde{\Omega}(M)=\tilde{\alpha}M^2+\tilde{\beta}M+\tilde{\gamma}$, as $\tilde{\xi}>0$, the sign of Δ_{CS} is determined by $\tilde{\Omega}(M)$. $\tilde{\Omega}(M)$ can be convex or concave in M depending on the value of K. Taking the second derivative of $\tilde{\alpha}$ with respect to K gives:

$$\begin{aligned}
\frac{d^2\tilde{\alpha}}{dK^2} &= -(t+2)\Big(\sigma^2+P^2(t+1)\Big)K^6-4(t+1)^2\Big(2\sigma^2+P^2(t+2)\Big)K^5 \\
&\quad -\Big(4(t+2)(t+1)(t^2+2t+2)P^2+8\sigma^2(t^3+2t+2)\Big)K^4 \\
&\quad -16(t+1)\Big(P^2(t+2)(t+1)+2\sigma^2(-t^2+2t+1)\Big)K^3 \\
&\quad -\Big(16(t+2)(t+1)^3P^2+16\sigma^2(8t^3+20t^2+13t+2)\Big)K^2,
\end{aligned} \tag{2.54}$$

which is non-positive for $K\in[0,1]$. Therefore $\tilde{\alpha}$ is concave in K. Moreover, for $t\in[0,1]$, we have $\tilde{\alpha}(K=0)=128t\sigma^2(t+1)^2\geq 0$, and $\tilde{\alpha}(K=1)=-5(t+2)(t+1)(2t+3)P^2-3\sigma^2(27t+30-8\Big(t^3+8t^2(t+1)\Big)<0$. This implies that there exists a unique $\tilde{K}^*$ such that if $K<\tilde{K}^*$ then $\tilde{\alpha}>0$ and $\tilde{\Omega}(M)$ is convex in M, and if $K>\tilde{K}^*$ then $\tilde{\alpha}<0$ and $\tilde{\Omega}(M)$ is concave in M.

At $r = 0$, we have

$$\tilde{\Omega}(M = 1, r = 0) = \frac{d\tilde{\Omega}(r = 0)}{dM} = 2\tilde{\alpha} + \tilde{\beta}. \tag{2.55}$$

Using Equation (2.50), (2.51), (2.52), (2.53) we find that

$$\tilde{\Omega}(M = 1, r = 0) = 16t\sigma^2(K-2)^2(t+1)\Big((-K^2+4K+4)t-K^3+K^2+8K+4\Big),$$

which is positive for $K \in [0,1]$, $t \in [0,1]$ and $\sigma^2 > 0$. Hence for $K \in [0,1]$, $t \in [0,1]$ and $\sigma^2 > 0$, if $K < \tilde{K}^*$, $\tilde{\Omega}(r = 0) > 0$ hold for any $M \geq 1$ (due to the convexity of $\tilde{\Omega}$). And if $K > \tilde{K}^*$, $\tilde{\Omega}(r = 0)$ would be smaller than 0 beyond a threshold $\tilde{M}^*$ (due to the concavity of $\tilde{\Omega}$). This threshold is the highest root in M of $\tilde{\Omega}(r = 0)$, namely, $\tilde{M}^* = \frac{\sqrt{\tilde{\beta}^2-4\tilde{\alpha}^2}-\tilde{\beta}}{2\tilde{\alpha}} > 1$.

Last, we show that when there is no procurement cost advantage, and both market demand and technology level are uncertain, there always exists a non empty interval on which both consumers and OEMs are better off with JP, relatively to IP. Following above analysis, there can be four situations. First, $K < min[K^*, \tilde{K}^*]$, both consumers and OEMs prefer JP over IP for $M \in [1, +\infty]$. Second, $K^* < K < \tilde{K}^*$, consumers prefer JP from $M = 1$ (and hence for any value of M) and OEMs prefer JP if $M > M^*$. In this case, OEMs and consumers prefer JP if $M \in [M^*, +\infty]$. Third, $\tilde{K}^* < K < K^*$, consumers prefer JP if $M < \tilde{M}^*$ and OEMs prefer JP from $M = 1$ (and hence for any value of M). In this case, OEMs and consumers prefer JP if $M \in [1, \tilde{M}^*]$. Fourth, $K > max[K^*, \tilde{K}^*]$, JP can be preferred by both OEMs and consumers only for $M \in [M^*, \tilde{M}^*]$. We show that in this case $\tilde{M}^* > M^*$ by contradiction. Let us assume that $\tilde{M}^* < M^*$. Then, since consumers prefer JP (IP) up to (beyond) $\tilde{M}^*$ and OEMs prefer JP (IP) beyond (up to) M^*, therefore for $\tilde{M}^* < M < M^*$ we have $\Delta_{OEM} < 0$ and $\Delta_{CS} < 0$ which implies that $\Delta_{OEM} + \Delta_{CS} < 0$.

From Equations (2.40) and (2.50) we characterize $\Delta_{OEM}(r = 0) + \Delta_{CS}(r = 0)$ as a quadratic function of M:

$$\Delta_{OEM}(r = 0) + \Delta_{CS}(r = 0) = \frac{aM^2 + b + a}{8(4 - K^2)(t^2 + 3t + 2)(K + 2t + 2)}$$

with

$$\begin{aligned}
a &= K^2(t+2)(t+1)(K+2t+2)P^2 \\
&\quad +\sigma^2\Big(4(K^2-2K+4)t^2 + (K^3+8K^2-4K+16)t + 2K^2(K+2)\Big) \\
b &= -2K^2(t+2)(t+1)(K+2t+2)P^2 - 2\sigma^2K\Big(8t^2 + (K^2+4K+12)t + 2K(K+2)\Big)
\end{aligned}$$

We let $\Lambda(M) = aM^2 + bM + a$, as $8(4-K^2)(t^2+3t+2)(K+2t+2) > 0$ for $K \in [0,1]$, the sign of $\Delta_{OEM}(r=0) + \Delta_{CS}(r=0)$ is determined by $\Lambda(M)$. Given that $K \leq 1$ implies $a \geq 0$, $\Lambda(M)$ is convex in M. At $r = 0$, we have

$$\Lambda(M=1, r=0) = 8t\sigma^2(K-2)^2(t+1), \tag{2.56}$$

which is positive for $K \in [0,1]$ and $\sigma^2 > 0$. This implies that $\Lambda(r=0)$ is positive for any $M \geq 1$, and therefore $\Delta_{OEM}(r=0) + \Delta_{CS}(r=0) > 0$, which contradicts our assertion that $\tilde{M}^* < M^*$. As a consequence, $\tilde{M}^* > M^*$ and hence both OEMs and consumers are better off over $M \in [M^*, \tilde{M}^*]$. □

In Theorem 4, we identify conditions under which OEMs and/or consumers prefer JP over IP when $r = 0$. Specifically, whether IP or JP would be preferable critically depends on the value of the parameters K and M.

Regarding the role of K, Theorem 4 first provides the condition ($K < K^*$) under which JP dominates IP for any value of M the market impact of the technology level. This condition (i.e. $K < K^*$) implies that competing OEMs would be willing to share information as long as substitutability between their respective products is below a certain threshold (K^*). To better understand this observation, one should keep in mind that JP has a two-sided information sharing impact: 1) on the one hand, it hurts the OEMs' expected profits by correlating their quantity decisions in using the same market information; but 2) on the other hand, it increases the OEMs' expected profits since they can make more informed decisions on their production quantities. Indeed, for low values of K both the benefits of sharing information and the cost of sharing information are low because the two OEMs are not really competing (a low K implies that the two OEMs are selling in independent markets). Note that this result is coherent with the findings of Vives (1984) although in their work only the case without impact of technology is considered. Moreover, we find that at these low values of K both the OEMs as well as the consumers are better off when OEMs get into a JP agreement.

However, IP tends to be favored more often as the substitutability between the products increases (i.e., as K increases). It is at higher values of K that the role of M becomes important in determining when OEMs and consumers prefer JP over IP. We show that even for $K > K^*$ OEMs would prefer JP over IP if $M > M^*$.[3] Interestingly, this result complements the earlier results in the economics literature (e.g. Vives (1984)) which state that OEMs should not share information when product substitutability is high enough. The reason being that models in the economics literature focused on information uncertainty on market attributes, like common market demand (which in our case is P). Part of our contribution is that we also bring in

[3]We provide the expression for M^* in the proof of Theorem 4 in appendix.

information uncertainty on attributes that are associated with a specific OEM, like the impact that an OEM's product technology can have on the market. In the typical case where the difference in technological specs of competing products influences their respective demands and a better product – by better we mean a product that has more advanced technological specs – has greater impact on demand (i.e., a high M), we find that OEMs might be interested in sharing their information with each other even when their respective products have high substitutability. On the contrary, for consumers we find that under high product substitutability their surplus is higher under IP when M is high, i.e., when $M > \tilde{M}^*$. Related to our previous discussion, under high product substitutability sharing information would increase the correlation effect between OEMs quantities which would hurt the OEMs and benefit the consumers. However, when M is high, the benefit (from information on the individual values M_i) of sharing information outweighs the cost for OEMs. For similar reason for low M the benefit to consumers due to correlated strategies of OEMs is high, but this benefit gets outweighed by the higher fraction of surplus that the OEMs appropriate (at the loss of consumers) as the value of M increases.

Finally, we analyze the situations in which JP agreements are beneficial to both the OEMs and the consumers. In the proof of Theorem 4 we show that, $\tilde{M}^* > M^*$ and therefore when $K > max[K^*, \tilde{K}^*]$ then the interval in which OEMs and consumers are simultaneously better off with JP is defined by $[M^*, \tilde{M}^*]$. Otherwise, the interval is $M \in [1, +\infty]$(for $K < min[K^*, \tilde{K}^*]$), or $M \in [M^*, +\infty]$(for $K^* < K < \tilde{K}^*$), or $M \in [1, \tilde{M}^*]$(for $\tilde{K}^* < K < K^*$). It follows that there always exists an interval in M over which everyone benefits from JP. However, for high values of M, the advantage that OEMs can obtain from engaging in JP agreements would be partially at the detriment of the consumers (i.e. when $M > \tilde{M}^*$), whereas for low values of M, it can be that consumers could benefit from JP even though OEMs do not find it advantageous (i.e. when $1 \leq M < M^*$).

In Figure 2.2 we numerically represent our findings of Theorem 4. In the plot (a) and plot (b) we have kept $r = 0$, so the situation corresponds exactly to Theorem 4. In the top plot we see clearly that at low K both the OEMs and the consumers are better off with JP agreement as compared to IP. However, for higher values of K the region in which JP is preferred by both OEMs and consumers narrows down, because typically the threshold value of $\tilde{M}^*$ below which JP is preferred by consumers is decreasing in K and the threshold value M^* above which JP is preferred by OEMs is increasing in K. In plot (c) we find that the region where both the OEMs and the consumers benefit with JP agreement increases as market demand variability (σ) increases. This is because with greater variability in demand the consumers are increasingly better off if OEMs share their market demand information and therefore the

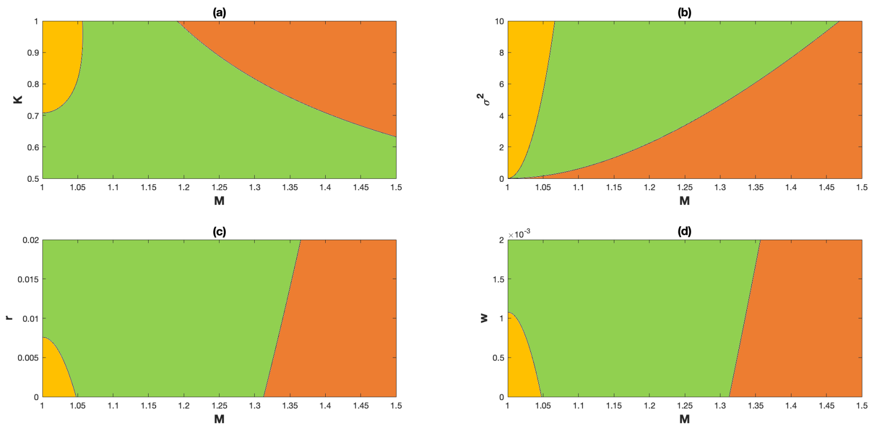


Figure 2.2. The yellow region represents the case where consumers benefit from JP but OEMs are hurt. The orange region represents the case where OEMs benefit from JP but consumers are hurt.The green region represents the case where both consumers and OEMs are better off with JP. The parameters for (a) are $r = 0$, $P = 10$, $t = 0.5$, $c^I = 0.5$, and $\sigma^2 = 5$; for (b) are $r = 0$, $P = 10$, $t = 0.5$, $c^I = 0.5$, and $K = 0.8$; for (c) are $\sigma^2 = 5$, $P = 10$, $t = 0.5$, $c^I = 0.5$, and $K = 0.8$; for (d) are $\sigma^2 = 5$, $P = 10$, $t = 0.5$, $A = 0.5$, and $K = 0.8$.

threshold value of $\tilde{M}^*$ increases in σ^2.

Proposition 5. *Δ_{OEM} and Δ_{CS} are both increasing in r.*

Proof. From Equation (2.31) and (2.32), $\mathbb{E}\pi_i^J$ increases in equilibrium quantities q_i^J. From Equation (2.33), $\mathbb{E}CS^J$ increases in equilibrium quantities q_i^J and q_j^J. Since from Proposition 2.28 the equilibrium quantities are increasing in r (as c^J decreases in r), it follows that $\mathbb{E}\pi_i^J$ and $\mathbb{E}CS^J$ are increasing in r. Moreover, $\mathbb{E}\pi_i^I$ and $\mathbb{E}CS^I$ are independent of r. Therefore, Δ_{OEM} and Δ_{CS} are increasing in r. $\square$

Finally, we obtain that the region where both OEMs and consumers are better off increases in r, as illustrated on Figure 2.2 (c). In other words, we prove through this proposition that it will always be optimal for the OEM to pass on part of the savings obtained from the supplier to the consumers. We show in Figure 2.2 (d) that this managerial insight also holds numerically with quantity dependent cost functions.
Figure 2.2 enables us to gain some insights about the examples mentioned in the §4.1 of this article. For example, in the car industry breakthroughs are

rare and not really expected between successive versions of a model, this would correspond to a relatively low value of M, but this industry is also known for aggressively pursuing economies of scale that would lead to significant cost reductions with higher volumes. Consequently we would situate companies such as BMW and Daimler in the upper left side of 2.2 (c). In the TV manufacturing industry, the competition are rather intense and TV makers upgrade their products relatively fast. The new versions of products are often equipped with breakthrough technologies such as video chat, augmented reality, multi-screen communication, AIoT and etc, which are frequently presented in the Consumer Electronics Show in Las Vegas (ChinaDaily, 2020). These technological improvements would correspond to a medium value of M, and therefore we would situate TV makers in the upper side of the green region of 2.2 (a). Regarding to the case of Carrefour and Tesco, since the two retailers operate in different countries, they would be situated in the lower side of the green region of 2.2 (a). Apart from above cases, we also observe that smartphone giants such as Huawei, Samsung and Apple rarely engage in a JP contract. We believe that these high-tech firms who compete fiercely with each other would be very willing to reveal their new product information. However the regulators would hardly approve such a JP agreement as the exchange of information on the cutting edge technologies (corresponding to a high value of M) would hurt consumers' benefits. Hence we would situate these smartphone giants in the orange region of 2.2(a).

2.6 Conclusion

Although cost advantages for competing OEMs to get into JP agreements are well understood, other strategic aspects, specially information sharing that is inherent to JP agreements, remain imperfectly understood. In this paper, we have investigated how the exchange of information about both market demand and technology level uncertainties that takes place when firms get into JP agreements, could affect their incentives to get into such agreements. While the economics literature already established that competing OEMs typically do not gain by exchanging information about market demand, little was known about the impact of sharing information on technology level, despite the fact that agreeing on common order size in JP agreements would involve information sharing on not only market demand estimates that each OEM possesses but also on each OEM's knowledge about the impact that its product specifications and technology will have on the market demand. This is specially relevant in markets where new versions of products are released periodically, i.e., OEMs do not know their rival's product technology level for the next period in advance, and hence cannot anticipate the impact of their rival's product on their own demand for the next period. In such markets, we show that either when the substitution between OEMs products is low or when the uncertainty on technology is high, then OEMs are willing to share their technology level and market demand information and hence such situation might favor JP agreements. However, competing OEMs sharing information might come at a detriment to consumers. We further find that both consumers and OEMs are better off with JP agreement (even when there exist no procurement cost advantage) when product substitutability is low. For higher levels of product substitutability we find that both the consumers and OEMs could still be better off if technology uncertainty is neither too high nor too low or when the procurement cost advantage of JP-agreement over IP is high.

In order to focus on the impact that information sharing has on competing OEMs' profitability, we assumed in this paper that JP agreements would involve OEMs fully sharing their information about their respective estimates of market demand and their technology level. In reality such an exchange of information might be more nuanced, since evidently the information is not directly communicated but is rather interpreted by OEMs from their mutual order size. These order sizes will also be influenced by the quantity discount negotiation process with the supplier. We chose not to model the negotiation process between the firms but rather assume that the outcome of such an iterative negotiation process would result in each OEM having full information about the other OEM's private information. This choice enables us to reach relatively general conclusions while limiting the number of assumptions to make. Future research on this topic could actually model such an iterative negotiation as a dynamic game to investigate the amount of information that OEMs would reveal in equilibrium through their order sizes. Such models could be further enriched if OEMs can split their orders across multiple suppliers while sharing just one supplier, as part of a JP agreement. Although these models would be more detailed, the underlying tension in exchange of information would remain the same as in our paper, i.e., OEMs would be reluctant to part with their information on common market estimates but would be more willing to share information on their respective product technology.

There is a fine line between sharing information and collusion and/or anti-competitive practices, hence it is also necessary to measure the effect that information sharing would have on consumers' surplus, in order to determine whether JP could be conflicting with antitrust regulations. Our analysis shows that there always exists an interval for the market impact of technology over which the consumers and OEMs would simultaneously benefit from JP agreements. Moreover, we obtain that this interval becomes larger with the size of the rebate offered by the suppliers to OEMs making a joint purchase. It follows that under such conditions JP can be beneficial for both OEMs and consumers. The policy implication from this is that JP agreements should be scrutinized by authorities, but should not be opposed outright.

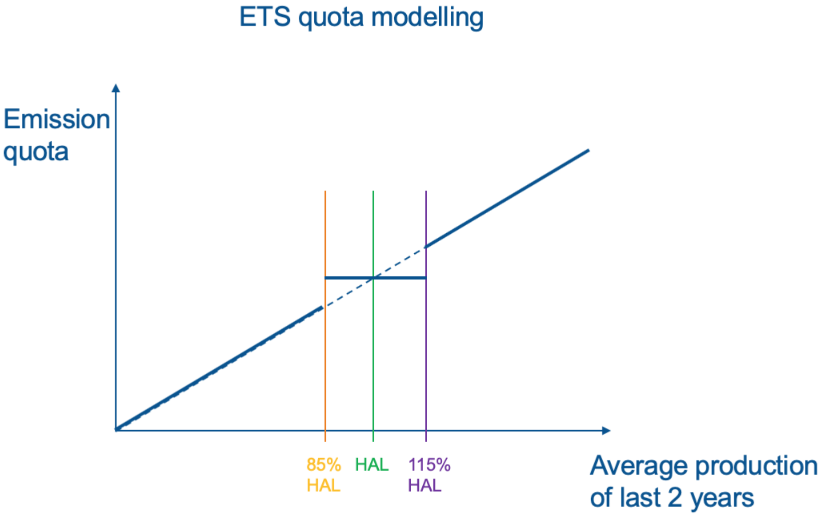
Our model could be extended in various ways, for example by considering a market with more than two OEMs or a multi-period model. We believe, however, that those two specific extensions would not result in different findings than those presented in this paper. Allowing for more sophisticated purchasing strategies than IP or JP could also be investigated. For example, OEMs could be given the option to make parallel orders to JP, as a means to try to avoid revealing their private information.

3

Multi-Period Multi-Plant Production Planning with Dynamic Emission Allowance

Figure 3.1. The Evolution of the Auction Price of Emission Allowances

Figure 3.2. The ETS rule



3.1 Introduction

Global warming is already taking a heavy toll on human well-being. Many countries have enacted legislations or mechanisms to combat climate change. The European Union Emissions Trading System (EU ETS) is the key pillar of EU's climate policy to fight global warming and curb greenhouse gas emissions. The principle is that the regulators set a "cap" on the total volume of greenhouse gases that can be emitted by all the participants in the system. The cap is then reduced progressively such that the total emissions fall over time. In practice, not all the companies are able to implement decarbonization strategies in a cost efficient way. For example, Abating CO_2 emissions is considered more difficult in ammonia, cement, ethylene, and steel sectors than in other sectors such as power, buildings and transport (de Pee *et al.*, 2018). As a consequence, to remain cost competitive, manufacturers with higher emissions would shift the productions to the regions where the regulations are weaker. To mitigate this emission "leakage" effect and address the concerns of loss of competitiveness, the regulators allocate emission allowances to the companies in the energy intensive sectors based on their historical production levels (HPL). The companies who are short of allowances can subsequently buy from those who reduced emissions through auctioning. The auction price varies according to the availability of the total number of allowances. For example, Figure 3.1 shows that the carbon price fluctuates between 18.62 and 29.46 Euro per ton of CO_2 emission in year 2019 [1]. The robust carbon price presses companies to invest in clean, low-carbon technologies. The EU ETS has gone through three phases since it was launched in 2005. In the upcoming phase four (2021-2030), the EU ETS will implement a dynamic emission allowance allocation mechanism (EU, 2019), under which the allocation of emission allowances free of charge are reviewed on the basis of a rolling average of two years. Specifically, if in the past two years a company increases or decreases its production by more than 15% compared to the HPL, the emission quota in the coming year would be adjusted proportionally, otherwise it stays at the HPL. We represent this rule graphically in Figure 4.1. From which we see that the emission quota is a piecewise linear function of the rolling average production quantities. The manufacturer gains a "bonus" when the average production of last two years is slightly above 85% of the HPL, and this "bonus" diminishes steadily as the production increases up to 115% of HPL. Under such regime, a manufacturer's production decision in the current period will have a significant impact on its future emission quota. This creates new challenges of production planning for manufacturers, such as how should they adjust the current planning to the new emission regulation? How to maximize the future quota at minimum

[1] The data is collected from the auction platform European Energy Exchange.

cost? Should they increase the production volume to keep the HPL quota? To what extent should they invest in low carbon technologies? In this paper, motivated by a research project that we undertook with an international manufacturer under the EU ETS system[2], we develop a decision making tool for manufacturers to answer the above questions. Specifically, we formulate a mixed-integer programming (MIP) problem that helps the manufacturer to make optimal decisions taking into account emission trading, production and technology selection over multiple periods. To help the manufacturer better understand the profitability of each plant, we evaluate the marginal cost of each plant under the new emission rule. We demonstrate that incorporating the dynamic emission allocation rule into production planning can significantly cut down the total operational costs in the long run and hence provide the manufacturer an edge in the global market.

The remainder of the paper is organized as follows. In section 2, we review the related literature. In section 3, we present the mathematical formulation in details. In section 4, we introduce the dataset, demonstrate the impact of the dynamic ETS on the optimal costs numerically and conduct sensitivity analysis. In section 5, we point out future research and conclude the paper.

3.2 Literature Review

Since the EU introduced the world's first international ETS system in 2005, a lot of work has been conducted to study the impact of ETS policy on supply chain performance. In the following we divided our review of the relevant work into two streams, we start by the literature that focuses on production and inventory, and we follow with articles tackling broader supply chain aspects that are in relation with ETS.

Firstly, our work is closely related to the production and inventory management literature that takes into account emission trading. In the single period context, Zhang and Xu (2013) consider a multi-item production problem where a firm uses a common capacity and carbon emission quota to produce multiple products for satisfying stochastic demand. The impact of carbon price, carbon cap on the firm's marginal costs and production decisions are discussed. Song and Leng (2012) investigate a classical news vendor problem and derive closed-form conditions under which a firm can increase its expected profit and reduce carbon emissions. Hua *et al.* (2011) examine the impact of ETS on inventory management and derive interesting findings. Chang *et al.* (2015), Yenipazarli (2016)Yenipazarli (2016), Chai *et al.* (2018) study the remanufacturing decisions under ETS. Xu *et al.* (2016)

[2]Confidentiality agreements do not allow us to reveal the name of the company and the industry involved.

study the joint production and pricing decisions of a manufacturer firm with multiple products and price sensitive demand. The effects of ETS and carbon tax regulations on the firm's profit and social welfare are compared. Wang *et al.* (2018) study the production planning and the emission reduction strategy of under-emitter firms and upper emitter firms. They derive the optimal allowance selling price for under-emitter firms, and for upper-emitter firms they identify the condition under which buying allowances is more advantageous than investing in low carbon processing. In the multi-period context, Rong and Lahdelma (2007) developed a production and emission trading optimization model for a combined heat and power producer. Li and Gu (2012) come up with an dynamic production-inventory model, and compare the optimal production-inventory strategies with or without emission permit. Gong and Zhou (2013) develop and analyze a dynamic production model to provide firms with optimal emissions trading, technology selection, and production strategies. García-Alvarado *et al.* (2015) studied the effect of ETS on the replenishment decision. Garcia-Alvarado *et al.* (2016) explore how environmental investments, capacity expansion and production and inventory planning interact to comply with a cap-and-trade mechanism. Different from the above works, our work develops a production planning model capturing the dynamic evolution of emission quotas according to the new regulation of the EU.

Apart from production planning, emission trading is also widely studied in other supply chain aspects. We summarize those works as follows. Sheu and Li (2014) investigate the effect of carbon permits under the cap-and-trade scheme on competition and green transportation strategies of global airlines using a behavioral economics approach. Pan *et al.* (2019) show that emission generating companies could collude with third-party verifiers and conceal real carbon emission data under the current carbon emission verification policy of a local government in China. They propose a new verification policy to mitigate this issue. Hu *et al.* (2020) evaluate the efficiency of ETS in achieving energy conservation and emission reduction with evidence from industrial sector in China. Zhao *et al.* (2010) investigate the efficiency of alternative systems for allocating emissions allowances and derive the long-run equilibrium solution for each. Drake *et al.* (2016) study the impact of emissions tax and emissions cap-and-trade regulation on a firm's technology choice and capacity decisions. Contradictory to common sense that the greater uncertainty jeopardizes firms' profit, they show that emissions price uncertainty under cap-and-trade results in greater expected profit than a constant emissions price under an emissions tax. Finally, Xu *et al.* (2017), Wang and Choi (2020) study the coordination contracts that improve the greenness of the supply chain.

3.3 Mathematical Model

3.3.1 Preliminary

We study a multi-period production problem where a manufacturer produces industrial commodity products and serves thousands of customers worldwide. Based on the real world case, we consider a planning horizon of five years, with two extra years to absorb the end of period effect. HPL is the five-year average production and is adjusted every 5 years, therefore it is a constant during the planning period. The manufacturer operates dozens of capacitated plants in countries in/and outside of EU ETS zone. Each plant consists of multiple production units. It incurs an annual fixed cost (e.g., cost of machine maintenance) at production unit level and a variable cost (e.g., cost of raw materials) at plant level. The manufacturer processes one raw product into two different sub-products, i.e., high quality product and regular quality product. The fraction of these two products depends on the production unit type. For example, the production unit with more advanced equipment can yield a higher fraction of high quality product. The high quality product can be downgraded to the regular quality one. The excessive inventory of regular products will be disposed of at a cost. After the production is finished, the products are transported from each plant to each customer. The products can be manufactured with two production technologies — a green technology and a cost efficient technology. The manufacturer can adopt either or both of them. The green technology produces each unit of the product at a higher production cost but generates fewer emissions, while the cost efficient technology produces each unit of the product at a lower production cost but generates more emissions. For example in Pulp and paper mills industry, steam is often generated by coal boilers and natural gas boilers. Despite that the coal is much cheaper than the natural gas, the CO_2 emissions generated by a coal boiler is twice as much as natural a gas boiler (Gong and Zhou, 2013). In each year, each plant is endowed with some emission allowances, which are determined by HPL, the production quantities and the yearly emission factor. In the first year, the production quantities and the emission quota are initialised. Starting from the second year, the basis for emission quotas are updated yearly based on the dynamic ETS rule, i.e., if the average production volume in the last two consecutive years is within the $[85\%, 115\%]$ interval of HPL, the basis for emission quota will be the same HPL, otherwise it will be the average production volume. Then the final emission allowances are equivalent to the basis multiplied by the yearly emission factor. The yearly emission factor decreases progressively, which means that the emission allowances will become more and more stringent as time goes by. At the end of each period, if one plant generates more emissions than the volume permitted by its allowances, it has to purchase allowances from other plants or from the carbon market to avoid heavy fines. Conversely, if one plant creates less emissions, it can sell the extra allowances. The emission buying price and the selling price are the same each year, they are fixed parameters. In the following we introduce the mathematical notations.

Index:
k: production unit. $k = 1, ..., K$.
p: plant ($p(k)$: plant where production unit k is located). $p = 1, ..., P$.
i: subproduct $= \{\mathcal{H}$: high quality, $\mathcal{R}$: regular quality$\}$
c: customer. $c = 1, ..., C$.
f: technology. technology $=$ $\{\mathcal{G}$: green technology, $\mathcal{C}$: cost efficient technology$\}$
t: year. $k = 1, ..., T$

Parameters:
cf_k: fixed cost for production unit k.
$cv_{p,t}$: variable cost for plant p in year t.
ci: cost of wasted inventory.
$ct_{p,c}$: transportation cost from plant p to customer c.
$p_{f,p,t}$: price of using technology f at plant p in year t.
$d_{i,c,t}$: demand for product i for customer c in year t.
iq_p: initial CO_2 quota for plant p at $t = 1$.
HPL_p: historic production level (HPL) for plant p used for the emission quota calculation.
ea_t: yearly emission factor.
cbc_t: CO_2 buying cost (or selling price) in year t.
$fe_{f,k,t}$: CO_2 emission factor for technology f at production unit k in year t.

$$ets_p = \begin{cases} 1, & \text{if plant } p \text{ is part of ETS,} \\ 0, & \text{otherwise.} \end{cases}$$

$cmin_k$: minimum annual production of production unit k if used.
$cmax_k$: maximum annual production capacity of production unit k if used.
r_k: the quantity of the raw product at production unit k at $t = 0$.
ϕ_k: fraction of regular product at production unit k.

Decision Variables:
$R_{f,k,t}$: quantity of the raw product processed by production unit k in year t by technology f.
$S_{f,k,t}$:binary variable to indicate whether technology f is used in production unit k in year t.
$X_{i,k,c,t}$: quantity of subproduct i produced by production unit k sent to customer c in year t.
$W_{k,t}$: excess quantity of regular product produced (wasted) by production

unit k in year t.
$Z_{k,t}$: binary variable to indicate whether production unit k is used in year t.
$C_{k,t}$: quantity of high quality product to be downgraded for production unit k in year t.
$E_{t,k}$: emission of CO_2 for production unit k in year t.
$LQ_{t,p}$: CO_2 emission quota for plant p in year t exclusively based on previous years average without 15% rule.
$EQ_{t,p}$: CO_2 emission quota for plant p in year t taking into account 15% rule.
$U_{t,p}$: auxiliary binary variable to compute emission quotas,

$$U_{t,p} = \begin{cases} 1, & \text{if the average emissions of past two years is greater than 115\% of } HPL_p, \\ 0, & \text{otherwise.} \end{cases}$$

$V_{t,p}$: auxiliary binary variable to compute emission quotas,

$$V_{t,p} = \begin{cases} 1, & \text{if the average emissions of past two years is greater than 85\% of } HPL_p, \\ 0, & \text{otherwise.} \end{cases}$$

$B_{t,p}$: CO_2 emission bought in year t for plant p (negative means selling).

3.3.2 Mathematical Model

The optimal production planning in our case involves several strategic decisions to make for each period such as: which production unit to open, which production technology to choose and how many product to produce in each plant, which customers to be assigned to which plants, and how many emissions to buy and sell, weighting the emission costs against the fixed costs, production costs, inventory costs, transportation costs. The mathematical model can be formulated as follows.

Objective function

$$\text{Minimize } F = \sum_k \sum_t cf_k Z_{k,t} + \sum_f \sum_t \sum_k (cv_{p(k),t} + p_{f,p(k),t}) R_{f,k,t} + \sum_k \sum_t ciW_{k,t} + \sum_t \sum_k \sum_i \sum_c ct_{p(k),c} X_{i,k,c,t} + \sum_t \sum_p B_{t,p} cbc_t \tag{3.1}$$

such that

$$\sum_k X_{i,k,c,t} = d_{i,c,t} \qquad \forall i,c,t \qquad (3.2)$$

$$\sum_f R_{f,k,t}(1-\phi_k) - C_{k,t} = \sum_c X_{\mathcal{H},k,c,t} \qquad \forall k,t \qquad (3.3)$$

$$\sum_f R_{f,k,t}\phi_k + C_{k,t} - W_{k,t} = \sum_c X_{\mathcal{R},k,c,t} \qquad \forall k,t \qquad (3.4)$$

$$\sum_f R_{f,k,t} \geq cmin_k Z_{k,t} \qquad \forall k,t \qquad (3.5)$$

$$\sum_f R_{f,k,t} \leq cmax_k Z_{k,t} \qquad \forall k,t \qquad (3.6)$$

$$R_{f,k,t} \leq cmax_k S_{f,k,t} \qquad \forall k,t \qquad (3.7)$$

$$E_{t,k} = \sum_f fe_{f,k,t} R_{f,k,t} \qquad \forall k,t \qquad (3.8)$$

$$LQ_{t,p} = \Big(\sum_{k|p(k)=p} \sum_f R_{f,k,t-1} + \sum_{k|p(k)=p} r_k \Big)/2 \qquad t=2\ \forall p|ets_p=1 \qquad (3.9)$$

$$LQ_{t,p} = \sum_{k|p(k)=p} \sum_f (R_{f,k,t-1} + R_{f,k,t-2})/2 \qquad \forall t \geq 3\ \forall p|ets_p=1 \qquad (3.10)$$

$$LQ_{t,p} \geq 1.15 \cdot HPL_p U_{t,p} \qquad \forall t \geq 2\ \forall p|ets_p=1 \qquad (3.11)$$

$$LQ_{t,p} \geq 0.85 \cdot HPL_p V_{t,p} \qquad \forall t \geq 2\ \forall p|ets_p=1 \qquad (3.12)$$

$$EQ_{t,p} \leq HPL_p + U_{t,p} \cdot 100000 \qquad \forall t \geq 3\ \forall p|ets_p=1 \qquad (3.13)$$

$$EQ_{t,p} \leq LQ_{t,p} + (V_{t,p} - U_{t,p})HPL_p \cdot 0.15 \qquad \forall t \geq 3\ \forall p|ets_p=1 \qquad (3.14)$$

$$B_{t,p} = ets_p \left(\sum_{k|p((k)=p} E_{t,k} - iq_p \right) \qquad t=1\ \forall p \qquad (3.15)$$

$$B_{t,p} = ets_p \left(\sum_{k|p((k)=p} E_{t,k} - EQ_{t,p} ea_t \right) \qquad \forall t \geq 2\ \forall p \qquad (3.16)$$

$$X_{i,k,c,t}, C_{k,t}, R_{f,k,t} \geq 0 \qquad \forall i,k,c,t \qquad (3.17)$$

$$Z_{k,t}, U_{t,p}, V_{t,p} \in \{0,1\} \qquad \forall k,t,p \qquad (3.18)$$

Constraint (3.2) is for satisfying the demand of each customer.

Constraints (3.3), (3.4)impose the material balance. (3.3) shows that the quantity of the good product minus the quantity downgraded equals to the total quantity of the good product sent to customers. (3.4) shows that the quantity of the regular product plus the quantity downgraded and minus the waste equals to the total quantity of the regular product to be shipped.

Constraint (3.5) and (3.6) impose the minimum and maximum capacity of each production unit.

Constraint (3.7) indicates which technology is adopted in each production unit.

Constraint (3.8) computes the total CO_2 emission per production unit.

Equation (3.9) and (3.10) compute the average production level of the previous two years. Equation (3.11) and Equation (3.12) determine the status variable $U_{t,p}$ and $V_{t,p}$ respectively. Equation (3.13) ensures that unless the average production of the last two years exceeded 115% of HPL, the basis for the quota cannot exceed the HPL level. Equation (3.14) establishes that outside the 85% - 115% interval, the basis for the quota allocation will be the average production level of the last two years, inside this interval the previous equation fixes the quota basis to HPL.

Constraint (3.15) and (3.16) compute the CO_2 emission bought (sold) for plants part of ETS, it is equal to the total emission minus the CO_2 quota.

Constraint (3.17) imposes the non-negativity of quantities.

Constraint (3.18) imposes binary values for the different production unit status variables.

Above problem is a MIP problem. MIP problems are NP-hard. The nature of NP-hard problems is that they could take a very long time to solve. Amongst other factors, the way of modelling a problem plays a vital role in determining whether this problem can be solved in a reasonable amount of time. In the practical case that motivated this paper, we have more complex constraints such as the quality requirements and the availability of different technologies. As a result, we have a large size MIP problem with more than 187 thousands variables and nearly 10 thousands constraints. We solved the problem using the commercial solver Gurobi. Its advanced presolve algorithm transformed the above problem into a reduced model of 48343 variables (including 393 integer variables) and 2536 constraints. MIP problems are generally solved by a branch-and-bound algorithm, which means that the problem solving time increases with the number of integer variables. Thanks to the efficient modelling, it takes us around 500 seconds to obtain the optimal solutions. The short solving time enables us to further develop the tool for marginal analysis and scenario analysis.

3.3.3 Model validation

In the practical case, the manufacturer's production planning starts from deciding which plants serve which customers, taking into account the common factors such as the distance to customers, the capacity of plants, etc. Then, each plant decides the production quantities of its production units based on some local cost optimization methods. To convince the manufacturer that our model is valid, we plug the current customer assignment to the optimization model, where we obtain the same production plans as that of made locally.

3.3.4 Marginal costs

Marginal analysis is a useful tool for managerial decision making. First of all, it evaluates the profitability of a plant or a customer. When there is a small demand change, it gives management a clear idea about how much they would lose as a result of demand shrinkage, and how much they should charge for extra demand. It also helps management to develop a robust understanding on the optimal solutions that an optimization model gives. For a standard linear programming problem, we can obtain the marginal costs by solving the dual model. Nevertheless, to develop the dual formulation for a MIP problem is rather complicated due to the integer variables. One potential approach is to look at the various variable costs for satisfying one extra unit of demand. But the marginal emission cost cannot be determined straightforwardly due to the fact that the emission quota is a nonlinear function of production quantities (see Figure 4.1), adding one unit of demand can change the current quota situation. In this paper, we present two ways to capture the real marginal costs for each product at each production unit in each year. In the first approach, we fix the decisions on the technology choice, and the plant that will serve each customer. We also fix the production quantities of each production unit except the one that we calculate the marginal cost. We keep free the auxiliary binary variables that determine the emission quotas. Then, we reoptimize the model by adding one more unit of demand. If the capacity has been saturated at the current solution, we rerun the model by deducting one unit of demand. In the second approach, we fix only the technology choices and reoptimize the model with one more unit of demand. We name the marginal costs obtained from the first approach as the "operational marginal cost", and from the second approach as the "tactical marginal cost". The two approaches serve for different decision making needs. For example, for short-term demand changes, the manufacturer has to respond it quickly, the "operational marginal cost" is the true marginal cost. Whereas for long-term changes, the manufacturer has time to plan it globally and reallocate the demand if necessary, the "tactical marginal cost" is more relevant. An interesting observation is that the "tactical marginal cost" of a production unit can be lower than its variable cost, because the demand of this production unit is partially transferred to a less expensive region. As most of the variables are fixed, calculating operational marginal cost is much faster compared to tactical marginal cost. For example, in the real world project, we have three different products, thirty-four production units and a planning horizon of seven years, resulting in 438 problems to solve. It takes 3212 seconds to calculate all the operational marginal costs but 11577 seconds to calculate all the tactical marginal costs. For a difficult problem, it can take up to 12 hours to obtain the tactical marginal costs. This shows the importance of efficient mathematical formulation.

3.3.5 Coping with uncertainty: Scenario analysis

Since it was pioneered by Charnes and Cooper (1959) and Dantzig (1955), the study of decision making under uncertainty has always been a hot topic. Although different problems require different models and solution techniques, the assumption that the probability distributions of the random variables are known exactly is prevalent in the extant work. However, in many real life applications practitioners have observed that the future demand would come from a different distribution that governing past history in an unpredictable ways (Clark and Scarf, 1960). When there is major shifts in the business environment, companies current strategies can become completely obsolete. To cope with the highly level of uncertain in the future, many companies have incorporate scenario analysis in the decision-making processes. Scenario analysis examines the alternative models of how the environment might evolve, encourage decision makers to consider extreme events and stretch their minds. A notable example is that Shell used scenario planning to navigate the oil crisis in 1970s, making it the second largest oil company in the world. The scenario based analysis typically involves a considerable number of scenarios. In order to be able to compute the optimal decisions in many scenarios, a math programming based optimization tool such as the one developed in this paper is very well adapted to such approach. In the real world project, our model examines different sources of uncertainty, such as demand, CO_2 price, transportation and production capacity, availability of alternative production technologies, and so on, which enable managers to create strategies that is robust to various situations.

3.4 Numerical Study

For confidentiality reasons, we cannot present the results based on the real data. For this reason we created an artificial data set that retains the essential characteristics of the real case, in terms of size, diversity of the plants and customer demands, trade-off between technologies. Specifically, we consider 15 plants serving 200 customers. The plants are denoted as P1, ..., P15. Each plant has 2 production units. P13, P14, P15 are outside of EU ETS zone. P1 and P2 are only equipped with the green technology, while P4 and P5 are only equipped with the cost efficient technology. The rest of the plants can employ both technologies to produce products.We consider a planning horizon of five years, with two extra years to absorb the end of period effect.

The purpose of this numerical study is twofold. First, we demonstrate the performance of our model. Second, we examine the magnitude of the cost saving when the emission costs are optimized. To do this, we compare the total costs of two different scenarios.

We program and solve the optimization problem with Pulp, a Python library for linear optimizations. It integrates with a range of open source and commercial LP solvers. It takes less than 1 minute with the open source solver *CBC* and a few seconds with the commercial solver Gurobi to solve the problem on a Macintosh with 8GB memory and a processor of 2.6 GHz Intel Core i5. We summarize the results as follow.

Figure 3.3 presents the total costs of the two scenarios. We can see that taking the ETS rule into account in the optimization model can cut down emission costs by 15%. This illustrates the importance of our study.

Figure 3.4 shows the production quantities and marginal costs of each plant in scenario 1 where the mission costs are optimised. We see that most of the plants in the ETS zone adopts the green technology. We also observe that plant $P7$ is closed. This can be explained by the marginal analysis. We see that the marginal cost of $P7$ is exceptionally high. After analyzing the data, we find out that the main reason comes from a high minimum capacity requirement of $P7$. Finally, the marginal costs of plants under the ETS are higher than the ones outside of the ETS. This is easy to understand, all other things being equal, it is always cheaper to produce in free emission cost regions.

Figure 3.5 shows the production quantities of each plant in scenario 2 where the mission costs are not taken into count in the optimization model. We see that apart from $P1$ and $P2$, in which the green technology is the only option, all other plants choose the cost efficient production technology.

From Figure 3.6 we can further see that the production of several plants are actually driven by the ETS rule. Indeed, the production of $P1$, $P2$, $P3$, $P4$, $P6$, $P9$ for some years try to match the 85% threshold. Looking back Figure 4.1 in the introduction, we observe that there is a “bonus effect" in the emission quota. If the ratio of the average production of last two years to the HPL is smaller that 85% or bigger than 115%, the emission quota for the current year is equal to the average production. If the ratio is in between, the emission quota will stays at the HPL. Therefore when the productions are at the the 85% threshold, the plants capture the most of the “bonus effect".

3.5 Conclusion

In this paper, we investigated the impact of a new emission allowance allocation mechanism on the optimal production planning. Under this new mechanism, the emission quota that a manufacturer receives each period is jointly decided by the historical production level and the production volume of the past two years. Therefore manufacturers have to plan their production

Figure 3.3. Comparison of total costs

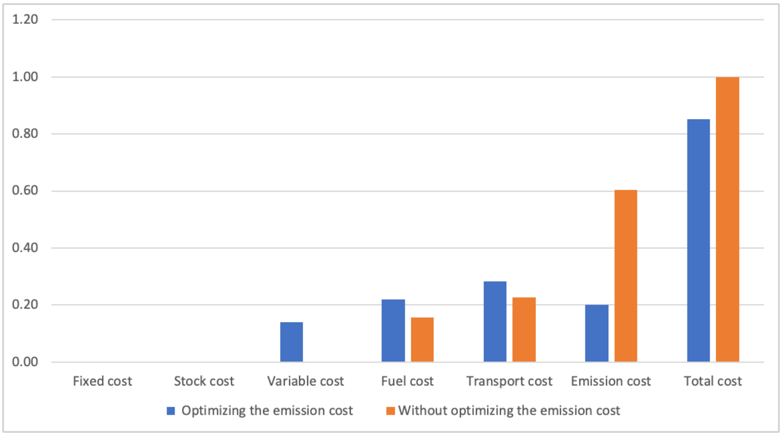


Figure 3.4. Production quantities and marginal costs under scenario 1

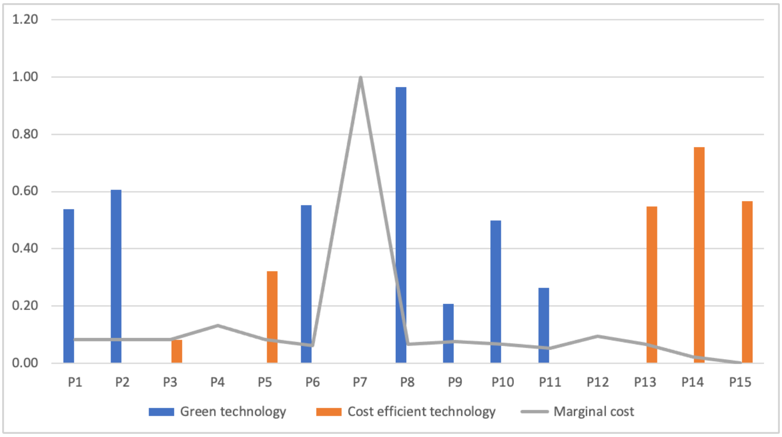


Figure 3.5. Production quantities under scenario 2

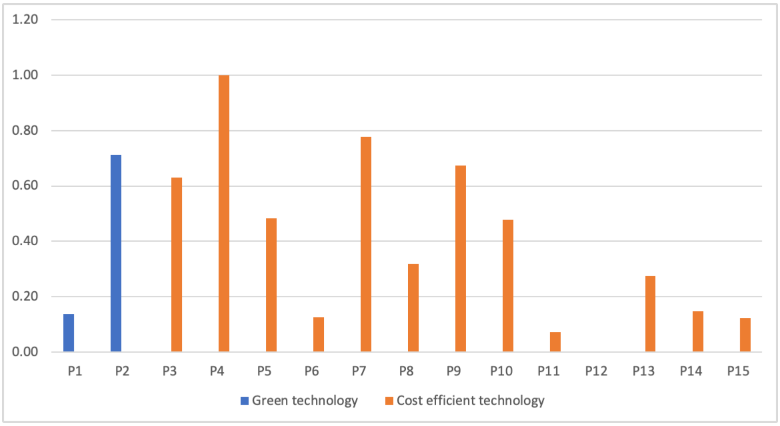
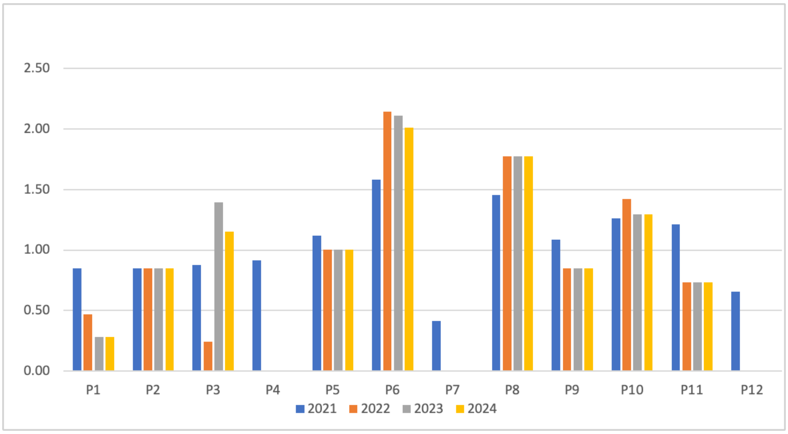


Figure 3.6. Production compared to HPL for ETS plants



strategically not only to minimize the operational costs at the current period but also to maximize the emission quota is the future. The nonlinearity of the new emission rule makes the optimal production planning nontrivial to manufacturers. The main contributions of this paper are to propose an innovative method to model the dynamic allocation rule, and to develop a math programming based optimization tool to assist decision making. The optimal solutions show that the new emission rule presses manufacturers to widely adopt low carbon technologies, and to increase the production volume to keep the future quota. To show the profitability of each plant, we propose two different methodologies to obtain the marginal costs. To help manufacturers make robust decision under uncertainty, we conduct a series of scenario analysis. Conducting more effective scenario planning and making the decision tool user-friendly will be the next steps of this research.

4

Order Sequencing with Unreliable Forecasts to Maximize Discounted Revenue

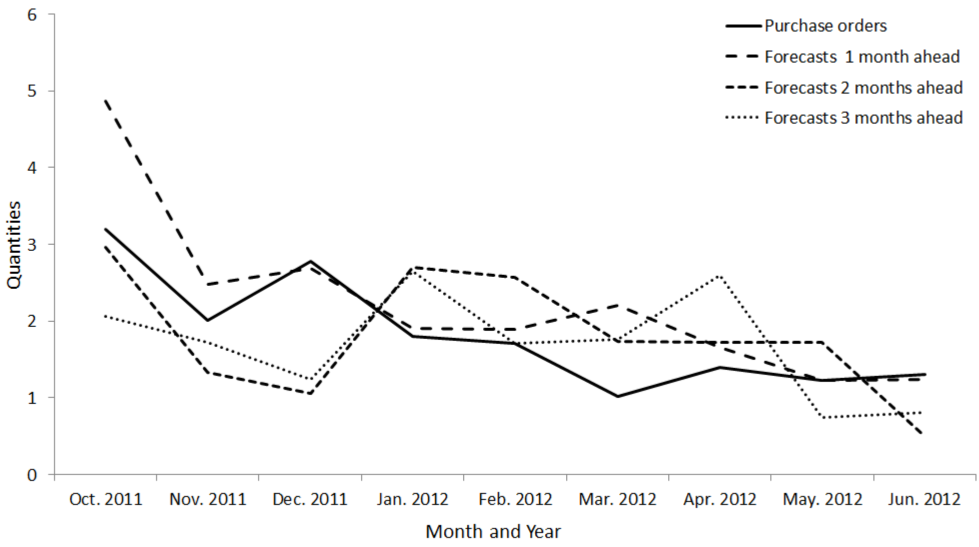
4.1 Introduction

In real-world supply chains, non-binding forecast communication is prevalent (Özer *et al.*, 2011). This allows buyers to issue some forecasted orders, or "soft orders", before committing to firm orders. For example, in an make to order supply chain[1] that we have investigated, OEMs communicate initial soft orders to the contract manufacturer on a rolling horizon basis, at least 3 months in advance; however, the actual order is not confirmed until one month before delivery. Figure 4.1 presents a sample of aggregate soft orders received by the contract manufacturer from an OEM for each month. We can observe that forecasts for the same period tend to fluctuate substantially and that their accuracy – even one month ahead – is highly variable. Terwiesch *et al.* (2005) also reports similar evidence in the semiconductor industry, and they show that suppliers penalize buyers for unreliable forecasts by providing lower service levels. Motivated by above stories, we attempt to address the following questions: should a supplier serve a reliable buyer (communicating less variable forecast) earlier? In this context, what is the impact of order sequencing on total discounted revenue?

Specifically, we consider a schedule-to-forecast, make-to-order supply chain with a supplier that sequentially serves multiple heterogeneous buyers. The buyers communicate their forecasts through soft orders on a regular and repeating basis. From past experience, the supplier observes that buyers have different levels of variability with respect to forecasts. In some situations, reliable buyers attempt to limit the discrepancies between soft orders and subsequent firm orders; while unreliable buyers only invest minimal effort in controlling forecast variability. In other cases, it is the business environment of some buyers that is intrinsically harder to predict. In any case, the supplier determines the sequence of order fulfillment and quotes due dates in response to the buyers' forecasts. To ensure a high service level, the supplier has to include more safety time in the quoted lead time for a less reliable order. The invoices will be sent on the due dates if the orders have been completed; otherwise, they will be sent only when the orders are completed. Therefore, the supplier's cash situation is not only affected by the due date quotations but also by the order completion times. The timing of cash flow is critical for suppliers to survive in today's business. Using cash conversion cycle as a metric, Tangsucheeva and Prabhu (2013) model the bullwhip effect in supply chain cash flow, and conclude that upstream members are more likely to be challenged by liquidity problems. In this paper, we investigate how to

[1]Confidentiality agreements do not allow us to reveal the names of the companies involved, but the OEM (the buyer) is a U.S. company selling consumer electronics worldwide, and the supplier is a large electronics contract manufacturer with manufacturing plants located in China.

Figure 4.1. A sample of monthly forecasts



determine the sequence of order fulfillment and quoted due dates to help suppliers improve their cash flow. The difficulty comes from the need to combine two aspects: on the one hand, starting with orders that bring larger amounts of cash; on the other hand, starting with orders that have a small variability to reduce delays. Therefore, the best production sequence in terms of optimizing revenue is not easy to determine. The objective of this paper is to provide a decision making tool for this problem.

The remainder of this paper is organized as follows. In the next section, we review related research. Then, we present the analytical model in section 4.3. Some scheduling insights are also provided in that section. In section 4.4, we propose an exact algorithm to solve our problem, and the computational experiments based on the proposed solution approaches are presented in section 4.5. Finally, we present our conclusions in the last section.

4.2 Literature review

Our work is related to several distinct topics in operations management. We summarize some of the related literature as follow.

Forecast sharing. Advanced demand information (ADI) sharing is an important coordination activity in supply chain management. A number of studies have considered different ADI scenarios (Hariharan and Zipkin, 1995; Zhu and Thonemann, 2004; Wang and Toktay, 2008; Gao *et al.*, 2012). The reliability of buyers determines the value of ADI. Lee *et al.* (1997) find that overoptimistic forecasting is one cause of the bullwhip effect, which results in excessive costs to the supplier. Based on the empirical data obtained from semiconductor equipment supply chain, Cohen *et al.* (2003) demonstrate that given the potential for unreliable forecasts, the supplier prefers to "wait and be late rather than rush and be wrong". Özer *et al.* (2011) discuss how trust in forecast information sharing affects the supplier's investment decisions. Tsay and Lovejoy (1999) , Özer and Wei (2006) , and Durango-Cohen and Yano (2006) study contracts that induce credible forecasts. We provide some analytical support to the managerial intuition that a more reliable buyer should be served earlier.

Supply chain finance. Supply chain finance is a hot topic in the current literature. In particular, a lot of research is focusing on the impact of trade credit and bank loans on the supply chain performance. For examples, Babich (2010) analyses the attractiveness of financial subsidies from buyers to suppliers. Kouvelis and Zhao (2018) work on the joint determination of procurement contracts and trade credit terms. Lee *et al.* (2018) study the impact of buyer and bank finance on the optimal ordering quantities. Tang *et al.* (2018) investigate the efficiency of buyers-provided financing and bank-provided financing under asymmetric information. Different from above approach, we aim to enrich the existing literature by investigating the impact of forecast accuracy on the supplier's cash inflow and show how the right schedule can mitigate this effect.

Stochastic scheduling. Scheduling addresses the optimal sequencing of tasks over a given period. In this paper, we formulate our model as a stochastic single machine problem with due dates (that are decision variables). The single machine problem is the basic problem in scheduling theory. The book of Pinedo (2012) provides an extensive introduction to deterministic and stochastic single machine models. Baker and Trietsch (2009) propose the concept of safe scheduling, which explicitly includes safety time when determining the quoted due date to satisfy a target service level. Due to the complexity of the stochastic model, Baker and Trietsch (2009) focus on identifying heuristic solutions. An important heuristic approach is the Weighted Smallest Variance First (WSV) rule first proposed by Soroush and Fredendall (1994). Portougal and Trietsch (2006) prove its (almost sure) asymptotic optimality when minimizing expected total weighted earliness and tardiness costs. Baker (2014) provides numerical evidence for these arguments. de Kemp *et al.* (2021) studies the performance of smallest variance first rule in appoint scheduling. Regarding optimization approaches, Baker (2014) presents a branch-and-bound type algorithm to solve a stochastic problem. Building on his work, we incorporate the idea of dynamic programming into the branch-and-bound search, and develop a hybrid algorithm that enhances the computational performance significantly.

Revenue scheduling. Chen and Hall (2008) model the revenue implications of scheduling decisions and show that efficient scheduling could stimulate customer demand and thus increase revenues. In this paper, we study the impact of forecast accuracy on the optimal scheduling. Yang (2009) considers a single machine scheduling problem to maximize total revenue, in which the revenue generated by each job is decreasing exponentially in its completion time. In our paper the revenue is discounted linearly.

4.3 The model

4.3.1 Preliminary

Motivated by the real world problem that has been introduced in the §4.1, we study a schedule-to-forecast, make-to-order supply chain with a single supplier sequentially serving multiple heterogeneous buyers repeatedly. In particular, we are considering industries such as consumer electronics contract manufacturing, or printed circuit board manufacturing or plastic extrusion. In consumer electronics contract manufacturing industry, it is often the buyer who ships components directly to the production lines, and the supplier does not need to purchase raw materials. While in printed circuit board manufacturing industry or plastic extrusion industry, the products are standard in the sense that they use the same raw materials and have identical production costs, but the products are highly customised so that the finished products are distinct and it does not make sense to have inventories of finished goods. The buyers communicate their forecasts through soft orders on a regular and repeating basis, typically every month in above industries. Based on these forecasts and the knowledge of about the reliability of each forecast, the supplier must make two decisions in each period: how to sequence the orders and what due date to promise to each buyer. The revenue lead time of an order is the elapsed time from the beginning of production until the invoice for this order can be sent. The subsequent lead time for the payment of the invoice is considered to be exogenously given and independent of operations. As the supplier use common components to process different orders, the payment of raw material is independent of the production sequence. Our research objective is to develop insights into the impact of buyers' forecast accuracy on the revenue. Therefore, we propose a model that focuses exclusively on that specific issue and keeps the rest of the model as simple as possible. Specifically, we assume that apart from their forecasting behavior, all buyers have identical characteristics such as equal bargaining power and equal value for their orders per time unit of capacity consumed.

We summarize the sequence of events as follows:

- Before production begins, the buyers place soft orders, and the supplier responds with due dates that have to meet the agreed service level.
- The supplier produces each order according to the earliest due date first rule. There is no idle time between the production of two consecutive orders.
- Until the start of its order, each buyer revises its soft orders.
- If an order is ready on time (or before), its invoice will be sent on the agreed due date.
- If an order is late, its invoice can only be sent once the order is ready.

Özer *et al.* (2011) observes in behavioral experiments that *repeated interactions induce a significant reduction in forecast inflation, leading to more effective forecast information sharing and cooperation in a supply chain.* Based on this finding, we assume that buyers do not inflate their forecasts and their initial forecasts are unbiased estimates of final demand. Regarding the distribution of forecast errors, we make the following assumptions:

- The forecast errors for the different orders are independent and are not influenced by the production sequence.
- The forecast errors for the different orders are modelled by standard normal random variables multiplied by different factors reflecting the quality of the forecasts for each order. This implies that the cumulative forecast error also follows a normal distribution, and this unique property allows us to obtain a closed-form result for the safety time reserved in the optimal due date. Because of its tractability and the Central Limit Theorem, the normal distribution plays a significant role in stochastic scheduling (see (Portougal and Trietsch, 2006) and (Baker, 2014)).

4.3.2 Model formulation

Before formulating the problem, we introduce the following notations:

$\{1, 2, ...n\}$	the set of orders; each order corresponds to one buyer,
c	the price per product unit,
w	the processing time per product unit, we set w=1,
p	the customer service level; we assume $0.5 \leq p < 1$,
β	the discount rate,
μ_i	the forecast quantity of order i,
$\{\varepsilon_1, \varepsilon_2, ..., \varepsilon_n\}$	a set of independent and normally distributed random variables, with $\mathbb{E}[\varepsilon_i] = 0$ and $Var[\varepsilon_i] = 1$,
$\alpha_i\varepsilon_i$	the forecast error of order i, $\alpha_i \geq 0$,
X_i	the purchase quantity of order i; $X_i = \mu_i + \alpha_i\varepsilon_i$,
R_i	the revenue generated by order i, $R_i = cX_i$,
γ	an order sequence,
$\gamma(j)$	the order that is in the jth position in γ,
$\gamma^{-1}(i)$	the position of order i in γ,
$D(\gamma, i)$	the due date for order i ,
$C(\gamma, i)$	the completion time for order i; $C(\gamma, i) = w \sum_{j=1}^{\gamma^{-1}(i)} X_{\gamma(j)}$,
$\xi(\gamma, i)$	the cumulative forecast error until order i in γ; $\xi(\gamma, i)$=$\sum_{j=1}^{\gamma^{-1}(i)} \alpha_{\gamma(j)}\varepsilon_{\gamma(j)}$,
$s(\gamma, i)$	the standard deviation of $\xi(\gamma, i)$; $s(\gamma, i) = \sqrt{\sum_{j=1}^{\gamma^{-1}(i)} \alpha^2_{\gamma(j)}}$,
$\Phi^{-1}(p)$	is the inverse Standard Normal cumulative distribution function,
$F^{-1}_{C(\gamma,i)}(p)$	is the inverse cumulative distribution function of the completion time of order i with the sequence γ.

To comply with the promised service level of each order, slack safety time will be included in the due dates of the orders. As a due date is only meaningful in practice if it is reliable to some extent, we will only consider positive safety times which corresponds to a service level higher than 0.5 under the Normal distribution.

Lemma 6. $D(\gamma, i) = \sum_{j=1}^{\gamma^{-1}(i)} \mu_j + s(\gamma, i)\Phi^{-1}(p)$ *is the shortest due date that will satisfy the service level.*

Proof. To satisfy the pre-determined service level p for order i, $D(\gamma, i)$ must be quoted such that $D(\gamma, i) \geq F^{-1}_{C(\gamma,i)}(p)$. The normal assumption enables us to expand $F^{-1}_{C(\gamma,i)}(p)$ as follows:

$$F^{-1}_{C(\gamma,i)}(p) = \sum_{j=1}^{\gamma^{-1}(i)} \mu_j + s(\gamma, i)\Phi^{-1}(p) \tag{4.1}$$

□

Analogous to the safety stock in inventory theory, $s(\gamma, i)\Phi^{-1}(p)$ is the safety time reserved in the optimal due date. It is influenced by the forecast errors of all the orders up to i in the sequence γ. As a result, once the sequence is decided the due-dates can be immediately be derived.

The objective of the supplier is to maximize the total discounted revenue. Obviously, the higher the value of an order, the more important it is to have it paid earlier. In effect, the supplier would intuitively prefer to begin with an order that requires only a short safety time but generates a large amount of cash. We can thus formulate the objective function as follows:

$$Maximize \qquad Z(\gamma) = \sum_{i=1}^{n} \mathbb{E}[R_i(1 - \beta max\{D(\gamma, i), C(\gamma, i)\})] \tag{4.2}$$

$\max\{D(\gamma, i), C(\gamma, i)\}$ is the revenue lead time of order i. $R_i \beta max\{D(\gamma, i), C(\gamma, i)\})$ is therefore the opportunity cost associated with the revenue R_i relative to time zero. The objective function (4.2) maximizes the sum of the expected discounted revenue. For the sake of tractability, we use linear discounting rather than exponential discounting. Linear discounting is applicable when the manufacturing lead times are short. For example, in the industry of consumer electronics or printed circuit board manufacturing, the revenue lead time of an order typically lasts a few weeks, in this case the interest is not compounded over such short horizon. Similarly, Chen and Lee (2017) also employs linear discounting to model the financial cost occurred in project supply chains. The Normal distribution assumption enables us to derive the closed form expression for (4.2). It explicitly shows that forecast uncertainty has a negative impact on the total revenue.

Theorem 7. *Under the normal assumption, maximizing* (4.2) *is equivalent to:*

$$Minimize \qquad V(\gamma) = \sum_{i=1}^{n} \mu_i s(\gamma, i) \tag{4.3}$$

Proof. Expanding (4.2), we obtain:

$$
\begin{aligned}
Z(\gamma) &= \sum_{i=1}^{n} cE[X_i] - \sum_{i=1}^{n} \left(\beta cE[X_i]E[max\{D(\gamma,i),C(\gamma,i)\}] + Cov\left[\beta cX_i, max\{D(\gamma,i),C(\gamma,i)\}\right]\right) \\
&= \sum_{i=1}^{n} c\mu_i - \sum_{i=1}^{n} \left(\beta cE[X_i]E[max\{D(\gamma,i),C(\gamma,i)\}] + Cov[\beta cX_i, C(\gamma,i)]E\left[\frac{\partial max\{D(\gamma,i),C(\gamma,i)\}}{\partial C(\gamma,i)}\right]\right) \\
&= \sum_{i=1}^{n} c\mu_i - \sum_{i=1}^{n} \beta c\mu_i \left(D_{(\gamma,i)} \int_0^{D_{(\gamma,i)}} f(C(\gamma,i))\,\mathrm{d}C(\gamma,i) + \int_{D_{(\gamma,i)}}^{\infty} C(\gamma,i) f(C(\gamma,i))\,\mathrm{d}C(\gamma,i)\right) - (1-p)\sum_{i=1}^{n} \alpha_i^2 \beta c \\
&= \sum_{i=1}^{n} c\mu_i - \sum_{i=1}^{n} \beta c\mu_i \left(pD_{(\gamma,i)} + (1-p)E[C(\gamma,i)|C(\gamma,i) > D(\gamma,i)]\right) - (1-p)\sum_{i=1}^{n} \alpha_i^2 \beta c \\
&= \sum_{i=1}^{n} c\mu_i - \sum_{i=1}^{n} \beta c\mu_i \left(p\left(\sum_{j=1}^{\gamma^{-1}(i)} \mu_j + s(\gamma,i)\Phi^{-1}(p)\right) + (1-p)\left(\sum_{j=1}^{\gamma^{-1}(i)} \mu_j + s(\gamma,i)\frac{\phi(\Phi^{-1}(p))}{1-p}\right)\right) - (1-p)\sum_{i=1}^{n} \alpha_i^2 \beta c \\
&= -\beta c\left(p\Phi^{-1}(p) + \phi(\Phi^{-1}(p))\right)\sum_{i=1}^{n} \mu_i s(\gamma,i) - \beta cp\sum_{i=1}^{n}\sum_{j=1}^{i} \mu_i\mu_j - (1-p)\sum_{i=1}^{n} \alpha_i^2\beta c + \sum_{i=1}^{n} c\mu_i
\end{aligned}
\tag{4.4}
$$

This equation results from Stein's Lemma (see (Stein, 1981; Landsman, 2006)); i.e., if X and Y are jointly normally distributed, then:

$$Cov[X, h(Y)] = Cov(X,Y)E[h'(Y)] \tag{4.5}$$

where $h(Y)$ is a function having finite $E[h'(Y)]$.

When $p \geq 0.5$, $\beta c(p\Phi^{-1}(p) + \phi(\Phi^{-1}(p)))$ is strictly positive, and the last three terms of (4.4) are not influenced by the sequence. Therefore, if (4.3) is minimized, then (4.2) is maximized. □

The reduced problem (4.3) minimizes the weighted sum of the standard deviations of the cumulative forecast errors. It is a special case of a more general single machine deterministic scheduling problem:

$$Minimize \qquad L = \sum_i w_i f(C_i) \tag{4.6}$$

where $w_i = \mu_i$, $v_j = \alpha_j^2$, $C_i = \sum_{j=1}^{i} v_j$, and f is the square root function. Objective function (4.6) minimizes the weighted sum of completion costs where the cost of each individual job is a function of its completion time. When the cost function f is linear, Smith (1956) states that it is optimal to order jobs in non-increasing order of w_n/v_n, which is known as Smith's rule or the weighted smallest processing time first rule. Höhn and Jacobs (2015) show that for piecewise linear cost functions it is strongly NP-hard. The computational complexity for square root cost functions remains open (Baker, 2014; Vásquez, 2014). The main contribution of this paper is to present an efficient algorithm to find the exact solution to problem (4.6) with a square root cost function.

4.3.3 Properties

In this section, we derive some properties of an optimal schedule, based on which we develop our algorithm.

Proposition 8. *In the absence of forecast errors, the total discounted revenue is independent of the production scheduling decisions.*

In the following proof we show that Proposition 8 is a general result (not depending on any specific distribution).

Proof. If the forecasts have no error, i.e., $\alpha_i = 0\ \forall i$, then for any order i, $D(\gamma, i) = C(\gamma, i)$. As a consequence, $Z(\gamma) = \frac{c}{2}\sum_{i=0}^{N}\sum_{j=0}^{N} X_i X_j \quad \forall\gamma$. Note that in this case, the objective function (4.3) is equal to 0 for any permutations. □

Proposition 9. *If buyer j has a higher estimate of future demand (i.e., $\mu_j \geq \mu_i$) and is more reliable (i.e., $\alpha_j^2 \leq \alpha_i^2$) than buyer i, then buyer j's order precedes buyer i's order in an optimal sequence.*

Proposition 9 suggests that a more reliable buyer with higher buying volume should be served earlier. However, in many situations managers find that not all the “big" buyers are “well behaved", the optimal schedule is thus not always straightforward. Our paper aims to provide a decision making tool for managers to manage different scenarios.

Proof. We will use a pairwise interchange argument. We consider a sequence γ in which order i precedes order j. For simplicity, we assume that there is only one order k between orders i and j; the proof can be straightforwardly extended to any number of intermediate orders. Let γ' be the new sequence, in which order i and order j are interchanged. We will show that γ' improves the objective function (4.3). The contributions of orders before i and after j to the objective are unimportant because they are not influenced by the interchange.

$$\begin{aligned} V(\gamma) &= \mu_i\sqrt{\alpha_i^2} + \mu_j\sqrt{\alpha_i^2+\alpha_j^2+\alpha_k^2} + \mu_k\sqrt{\alpha_i^2+\alpha_k^2} \\ &\geq \mu_j\sqrt{\alpha_i^2} + \mu_i\sqrt{\alpha_i^2+\alpha_j^2+\alpha_k^2} + \mu_k\sqrt{\alpha_i^2+\alpha_k^2} \\ &\geq \mu_j\sqrt{\alpha_j^2} + \mu_i\sqrt{\alpha_i^2+\alpha_j^2+\alpha_k^2} + \mu_k\sqrt{\alpha_k^2+\alpha_j^2} = V(\gamma') \end{aligned} \tag{4.7}$$

Therefore, γ' improves the objective. □

The following two results are direct consequences of Proposition 9.

Corollary 10. *If the forecasts have the same level, it is optimal to sequence the orders according to the non-decreasing variance of forecast error.*

Corollary 11. *If the forecasts have the same error distribution, it is optimal to sequence the orders according to non-increasing forecast size.*

Corollary 10 and Corollary 11 show that if buyers are symmetric in terms of buying volumes, the more reliable buyers should be prioritized, whereas if buyers are symmetric in terms of forecast error, then the buyers with larger volumes should be prioritized. In the next two Propositions we continue to show that it is optimal to prioritize the buyer who communicates an accurate forecast and assign the longest due-date to the buyer who communicates the least reliable forecast.

Proposition 12. *If order j is such that $\alpha_j = 0$ then there is an optimal sequence γ^* starting with order j, in other words $\gamma^{*-1}(j) = 1$.*

Proof. Let $\hat{\gamma}$ be a sequence off all orders except j and let $\hat{\gamma}_k$ be the same sequence with order j inserted in the kth position. From the objective function (4.3) we have

$$V(\hat{\gamma}_k) - V(\hat{\gamma}) = \mu_j s(\hat{\gamma}, k-1) \geq 0,$$

this implies that for any subsequence $\hat{\gamma}$ we have $V(\hat{\gamma}) = V(\hat{\gamma}_1)$. If $\hat{\gamma}^*$ is an optimal sequence for all orders except j then $\gamma^* = \hat{\gamma}_1^*$ is an optimal sequence for the complete set of orders. $\square$

Proposition 13. *If order j is such that $\alpha_j = M$ (where M is sufficiently large) then there is an optimal sequence γ^* ending with order j, in other words $\gamma^{*-1}(j) = n$.*

Proof. For any sequence γ where order j is not in the last position, let us call $\bar{\gamma}$ the sequence γ where we moved order j to the last position. We will show that this new sequence improves the objective value, indeed

$$V(\gamma) - V(\bar{\gamma}) = \sum_{k=\gamma^{-1}(j)+1}^{n} \mu_k \left(\sqrt{s^2(\gamma,k)} - \sqrt{s^2(\gamma,k) - M}\right) + \mu_j \left(s(\gamma, \gamma^{-1}(j)) - s(\gamma, n)\right) \tag{4.8}$$

All but the last term of (4.8) are positive and increasing in M, while the last term is negative but decreasing in M (by the concavity of the square root function). As a result for M large enough $V(\gamma) - V(\bar{\gamma}) > 0$ and it is optimal to move order j to the last position of the sequence. This also implies that for a large enough value of M, any optimal sequence must end with order j. $\square$

4.3.4 Extension

For orders that are finished in advance, the supplier often has to carry them before they can be sent. The most common reason could be that the shipping date is fixed with the transportation company, or the buyer does not accept an earlier shipment. The holding cost varies depending on the products. For example, electronic products are very sensitive to the temperature and humidity of the warehouse, and plastic extrusion products can take large space. In this section, we examine if taking into account the cost of holding finished good inventory before delivery would change the optimal order sequencing decision. To do this, we let h be the per unit holding cost, the total expected hold costs can be written as follow:

$$H(\gamma) = \sum_{i=1}^{n} \mathbb{E}[hX_i max\{D(\gamma,i) - C(\gamma,i), 0\}], \tag{4.9}$$

where $max\{D(\gamma,i) - C(\gamma,i), 0\}$ is the waiting time order i. Expanding (4.9) gives:

$$\begin{aligned}
H(\gamma) =& h\sum_{i=1}^{n} \left(E[X_i]E[max\{D(\gamma,i) - C(\gamma,i), 0\}] + Cov\left[X_i, max\{D(\gamma,i) - C(\gamma,i), 0\}\right]\right) \\
&= h\sum_{i=1}^{n} \left(E[X_i]E[max\{D(\gamma,i) - C(\gamma,i), 0\}] + Cov[X_i, C(\gamma,i)]E\left[\frac{\partial max\{D(\gamma,i) - C(\gamma,i), 0\}}{\partial C(\gamma,i)}\right]\right) \\
&= h\sum_{i=1}^{n} \mu_i \left(D_{(\gamma,i)} \int_0^{D_{(\gamma,i)}} f(C(\gamma,i))\, \mathrm{d}C(\gamma,i) - \int_0^{D_{(\gamma,i)}} C(\gamma,i) f(C(\gamma,i))\, \mathrm{d}C(\gamma,i)\right) - p\sum_{i=1}^{n} \alpha_i^2 h \\
&= h\sum_{i=1}^{n} \mu_i \left(pD_{(\gamma,i)} - pE[C(\gamma,i)|C(\gamma,i) < D(\gamma,i)]\right) - p\sum_{i=1}^{n} \alpha_i^2 h \\
&= h\sum_{i=1}^{n} \mu_i \left(p\left(\sum_{j=1}^{\gamma^{-1}(i)} \mu_j + s(\gamma,i)\Phi^{-1}(p)\right) - p\left(\sum_{j=1}^{\gamma^{-1}(i)} \mu_j - s(\gamma,i)\frac{\phi(\Phi^{-1}(p))}{p}\right)\right) - p\sum_{i=1}^{n} \alpha_i^2 h \\
&= h\left(p\Phi^{-1}(p) + \phi(\Phi^{-1}(p))\right)\sum_{i=1}^{n} \mu_i s(\gamma,i) - p\sum_{i=1}^{n} \alpha_i^2 h.
\end{aligned} \tag{4.10}$$

We then deduct the holding costs from the total revenue, which gives:

$$\begin{aligned}
Z(\gamma) - H(\gamma) =& -(\beta c + h)\left(p\Phi^{-1}(p) + \phi(\Phi^{-1}(p))\right)\sum_{i=1}^{n} \mu_i s(\gamma,i) - \beta cp\sum_{i=1}^{n}\sum_{j=1}^{i} \mu_i\mu_j \\
&- (1-p)\sum_{i=1}^{n} \alpha_i^2 \beta c + \sum_{i=1}^{n} c\mu_i + p\sum_{i=1}^{n} \alpha_i^2 h
\end{aligned} \tag{4.11}$$

It is obvious that only the first term of (4.11) is influenced by the sequence, and when $p \geq 0.5$, $(\beta c + h)(p\Phi^{-1}(p) + \phi(\Phi^{-1}(p)))$ is strictly positive. Therefore, if (4.11) is maximized, then (4.3) is also minimized. We conclude that the optimal sequence will stay the same when the holding costs are taken into account.

4.4 Exact Algorithm

In this section, we propose a hybrid algorithm that combines dynamic programming and branch-and-bound search to solve our problem. Dynamic programming and branch-and-bound are two complete enumeration techniques to deal with combinatorial optimization problems. Both approaches attempt to minimize the computational effort via a divide and conquer method. Dynamic programming is applicable when problems exhibit the properties of overlapping subproblems (the solution to a given subproblem can be stored and reused in a bottom-up fashion) and optimal substructure (the link between optimal solution and optimal solutions to subproblems). On the other hand, the branch-and-bound tree search uses bounding to reduce the search space. The idea to try to combine the advantages of dynamic programming and branch-and-bound is not new. Morin and Marsten (1976) give a detailed description of the algorithmic principle. Earlier applications of this type of hybrid algorithm for various optimization problems include: Marsten and Morin (1978), Martello and Toth (1984), Dyer *et al.* (1995), Pan (2003). In what follows we present a hybrid algorithm for solving problem (4.3).

4.4.1 Branching

The algorithm employs the breadth first search (BFS) strategy, which uses a FIFO queue to explore the nodes level by level in the search graph. A node π represents a state, or a subset of n jobs that is processed first. The dynamic programming recursion for problem (4.3) is formulated as follow:

$$V(\pi) = min_{j\in\pi}(V(\pi - \{j\}) + \mu_j \sqrt{\sum_{i\in\pi} \alpha_i^2}) \tag{4.12}$$

with initial conditions

$$V(\{j\}) = \mu_j \alpha_j, \;\; j = 1, ..., n \tag{4.13}$$

and optimal value function:

$$V(\{1, ..., n\}) \tag{4.14}$$

Clearly, the cost of an optimal sequence is determined by the recursive solution of the functional equation (4.12), which is a mathematical representation of Bellman's principle of optimality (Bellman, 1957). At each iteration with problem size $l \leq n$, there are $\frac{n!}{l!(n-l)!}$ states that have to be evaluated, and the total number of evaluations would be $\sum_{l=1}^{n} \frac{n!}{l!(n-l)!}$. Therefore if we employ pure dynamic programming to solve our problem, we will be limited to small problem sizes as the computer storage requirement grows exponentially. To overcome this limitation, we will use bounding tests to fathom certain states.

4.4.2 Bounding

The upper bound. Based on the weighted smallest variance first (WSV) rule, we propose as a heuristic to sequence the orders as follows:

$$\frac{\mu_1}{\alpha_1^2} \geq \frac{\mu_2}{\alpha_2^2} \geq ... \geq \frac{\mu_n}{\alpha_n^2}, \tag{4.15}$$

The WSV rule is an important heuristic approach for stochastic scheduling problems. The Interested reader is referred to Soroush and Fredendall (1994), Portougal and Trietsch (2006), Baker and Trietsch (2009), Baker (2014). Let γ^h denote the heuristic solution of (4.15), the upper bound (UB) on the value of (4.3) is initially set as $V(\gamma^h)$ and then updated whenever a solution has been found, i.e., $UB = min(V(\gamma^h), V(\gamma))$. If $LB(\pi) > UB$, then state π is fathomed. In order to explain the three bounding procedures tested, we introduce the notation $\bar{\pi}$ as the complement of π (i.e. the set of orders not yet scheduled in state π).

The first lower bound. It is calculated as:

$$LB_1(\pi) = V(\pi) + \sum_{l \in \bar{\pi}} \mu_l \sqrt{\sum_{i \in \pi} \alpha_i^2} \tag{4.16}$$

LB_1 ignores the variability of the orders in $\bar{\pi}$, in which case the sequence of the orders in $\bar{\pi}$ is unimportant. This is justified by Proposition 8, which holds that any sequence yields the optimal solution when there is no forecast error. LB_1 is rather loose. However, it allows us to apply a recurrence relation and calculate $LB_1(\pi)$ from $LB_1(\pi - \{j\})$ in constant time (not depending on the problem size) because

$$LB_1(\pi) = LB_1(\pi - \{j\}) + \sum_{l \in \overline{(\pi - \{j\})}} \mu_l \left(\sqrt{\sum_{i \in (\pi - \{j\})} \alpha_i^2 + \alpha_j^2} - \sqrt{\sum_{i \in (\pi - \{j\})} \alpha_i^2} \right) \tag{4.17}$$

This significantly accelerates the computation.

The second lower bound. The second lower bound is developed by Baker (2014). In the development of LB_2, orders in $\bar{\pi}$ are modified such that the kth largest order has the kth smallest variance. In this case, the optimal sequence of $\bar{\pi}$ becomes trivial. LB_2 is given by:

$$LB_2(\pi) = V(\pi) + \sum_{l \in \bar{\pi}} \mu_l \sqrt{\sum_{i \in \pi} \alpha_i^2 + \sum_{k \leq_{\bar{\pi}} l} \alpha_k^2} \tag{4.18}$$

where $i \leq_{\bar{\pi}} l$ means that order i is before or equal to order l in the sequence of modified orders in $\bar{\pi}$.

The third lower bound. The third lower bound is developed by Portougal and Trietsch (2006). In the development of LB_3, jobs in $\bar{\pi}$ are reordered according to the non-increasing ratios of μ_l/α_l^2. The contribution of each order in $\bar{\pi}$ is calculated as:

$$g_l = \frac{2\mu_l}{3\alpha_l^2}(o_l^3 - o_{l-1}^3) \tag{4.19}$$

where

$$o_l = \sqrt{\sum_{i\in\pi} \alpha_i^2 + \sum_{i\leq_{\bar{\pi}} l} \alpha_i^2} \tag{4.20}$$

Therefore, $LB_3(\pi)$ is calculated as:

$$LB_3(\pi) = \sum_{l\in\pi} \mu_l o_l + \sum_{l\in\bar{\pi}} g_l \tag{4.21}$$

Baker (2014) is the first paper to implement this lower bound. The idea behind LB_3 is to consider a relaxed problem where each job is partitioned to infinitesimal fractions with the same variance everywhere, then it is optimal to sequence each fraction according to non-increasing order of μ_l, this can be done without preemption since every fraction of job l has the same ratio of μ_l/α_l^2. The relaxed problem serves as a lower bound for the original problem.

A dominance rule

The algorithm also implements a dominance rule whereby the state space can be reduced without bounding test. For each job j, we define a set of jobs, denoted as $\mathrm{Pred}(j)$, that precede j in any optimal sequence as result of Proposition 9. If $\exists j \in \pi$ and $\bar{\pi} \cap \mathrm{Pred}(j) \neq \emptyset$, then state π is fathomed.

4.5 Computational results

Our hybrid algorithm is coded in C and implemented on a laptop computer with Intel Core i7-4960HQ Processor (6M Cache, 2.60 GHz base frequency, up to 3.80 GHz turbo frequency) and 16 GB of RAM. For each job j, the forecast size μ_j was randomly chosen from a uniform distribution between 10 and 200. Then, the forecast quality α_j was calculated as the product of μ_j and c_j. The coefficient c_j is sampled from a uniform distribution between 0.01 and 0.25, in this way the probability of a negative order size is negligible. For each size n, k random instances are created. The performance of each lower bound is compared in terms of the average running time of the hybrid algorithm for k instances and the maximum time required to run one instance.

From Table 4.1, we can see that LB_1 slightly outperforms LB_2, and LB_3 turns out to be the most efficient lower bound. The computation time is $O(1)$

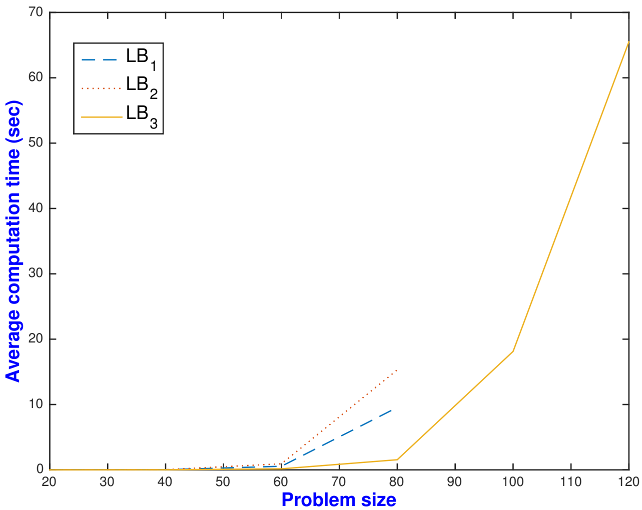


Figure 4.2. Average computation time versus problem size

for running LB_1 and $O(n)$ for running LB_2 and LB_3. Figure 4.2 depicts the increase of the average computation time with respect to the problem size. With the improved branch and bound algorithm, we are able to solve instances with up to 100 orders within a reasonable time.

We then test the the efficiency of the WSV rule. We measure its performance according to three criteria: (1) the percentage of instances in which the WSV rule yields the optimal solution; (2) the average sub-optimality gap; and (3) the maximum sub-optimality gap (the worst-case scenario). The sub-optimality gap is calculated as:

$$\frac{\text{cost of the schedule resulting from the WSV rule - cost of the optimal schedule}}{\text{cost of the optimal schedule}} \tag{4.22}$$

Table 4.2 shows that the accuracy of the WSV rule improves steadily as the problem size grows. Despite of the asymptotical optimality of the the WSV rule, it can be very inaccurate when the order size is small. We illustrate this with an example presented in table 4.3. Therefore, for the case where the number of buyers is small, it is necessary for a supplier to implement an exact algorithm to make scheduling decisions, while for the case where the number of buyers is sufficiently high, a supplier can simply follows the WSV rule.

Table 4.1. The performance of lower bounds

Problem size	Lower bound	No. of samples	Avg. time (sec)	Max. time (sec)
Size 10	LB_1	200	0,00002	
	LB_2	200	0,00004	
	LB_3	200	0,00003	
Size 20	LB_1	200	0,00079	0,006
	LB_2	200	0,00157	0,013
	LB_3	200	0,00059	0,005
Size 30	LB_1	200	0,01868	0,222
	LB_2	200	0,03391	0,326
	LB_3	200	0,00689	0,026
Size 40	LB_1	200	0,31787	7,414
	LB_2	200	0,52336	10,978
	LB_3	200	0,06426	0,442
Size 50	LB_1	200	3,95125	54,952
	LB_2	200	5,90850	72,458
	LB_3	200	0,29046	2,442
Size 60	LB_1	200	27,83542	217,031
	LB_2	200	40,72263	288,090
	LB_3	200	1,54684	13,354
Size 65	LB_1	200	75,20318	737,512
	LB_2	200	127,47912	1223,463
	LB_3	200	3,51867	15,599
Size 70	LB_3	200	7,58281	184,385
Size 80	LB_3	200	26,16763	417,898
Size 90	LB_3	200	94,24524	477,722
Size 100	LB_3	200	313,30775	1419,052

4.6 Conclusion

In this paper, we quantify the impact of forecast (in)accuracy on the supplier's revenue. We assume that to ensure a high service level, the supplier will have to include more safety time in the quoted lead time for a less reliable order which may imply a delay in revenue realization. We address our problem with a single machine stochastic scheduling model, with the objective of maximizing the sum of expected discounted revenue. The main contribution of this paper is threefold. First, we provide some analytical support for the managerial intuition that forecast errors are detrimental to the supply chain performance and that a more reliable buyer should be served earlier. Second, we show that when the number of orders is large, the supplier can simply follow WSV rule to schedule the orders. Third, we demonstrate that when the number of orders is mall, WSV can lead to very wrong solution. In this case we develop an efficient algorithm to find the optimal sequence. In closing the paper, we would like to

Table 4.2. The performance of the WSV rule

Problem size(n)	Percentage optimal	Avg. gap	Max. gap
Size 5	85,0%	0,0306%	1.19%
Size 10	71,0%	0,0184%	0,250%
Size 20	27,5%	0,0155%	0,195%
Size 30	10,5%	0,0075%	0,039%
Size 40	3,5%	0,0049%	0,027%
Size 50	1,0%	0,0034%	0,011%
Size 60	2,0%	0,0027%	0,011%
Size 70	0,0%	0,0021%	0,008%
Size 80	0,0%	0,0016%	0,005%
Size 90	0,0%	0,0015%	0,004%
Size 100	0,0%	0,0012%	0,003%

Table 4.3. An sequencing example of 5 orders

Order i	1	2	3	4	5
u_i	19	57	92	10	13
α_i^2	5.15	0.67	24.91	4.94	4.12
WSV rule	2	3	1	5	4
optimal solution	2	1	5	3	4

discuss the limitations of our study and the potential avenue of future research. Firstly, in our paper we assume that the manufacturer has entire freedom to quote due dates based on the probabilistic service level constraint. In practice it is likely that buyers would impose some constraints on due dates such as a maximum limit and a minimum limit, as each buyer might not want to have his order to be scheduled too late or too soon. These types of constraints should not be very difficult to add in our algorithm. Such constraints would reduce the running time by enabling the pruning of the search tree. Secondly, our model assumes that each order is processed sequentially. In case the manufacturer could produce some jobs in parallel, a multiple machine version of our model would have to be developed. The complexity of combining Branch&Bound and Dynamic Programming is much higher and is certainly an interesting research avenue to solve such problems.

5

Conclusion and future research direction

Buy, make, sell are three core functions in supply chains. This thesis has explored three specific topics in the context of these functions in the presence of supply chain uncertainty. In particular, chapter 2 provided insight into whether or when competing manufacturers would prefer individual purchasing over joint purchasing. This purchasing decision is greatly influenced by uncertainty on market demand and rival's product technology level. We used linear information structure and two point distribution to capture these two types of uncertainty respectively. Chapter 3 developed an optimization based production planning tool that helps manufacturers minimize total costs while meeting sustainability goals. This tool enables manufacturers conduct extensive scenario analysis to cope with various sources of uncertainty, such as uncertainty on production capacity, emission price, demand etc. Chapter 4 shed light on the supplier's optimal order fulfilment sequence given that buyers have different level of reliability in terms of forecast communication. In this work buyers' future reliability is estimated based on the past forecast sharing performance which is assumed to follow the normal distribution.

Uncertainty is the nature of supply chains that every practicing managers can not ignore. Chapter 2 and chapter 4 have taken probabilistic view to model the impact of uncertainty on decision making. The limitation of this approach is that in real life the distributions of uncertain parameters are often unavailable. For example, for a new model of self driving car, there is no historical sales data, therefore the distribution of future demand is hard to obtain. This calls for new ways to capture the stochastic character of parameters without making any assumptions on their distributions. Chapter 3 has introduced scenario planning as an alternative way to cope with uncertainty. Here I would like to continue to discuss two other distribution-free techniques, i.e., robust optimization and machine learning-based forecasting.

Robust optimization (hereafter RO) assumes that uncertain parameters take values in uncertainty sets. The goal of RO is to guarantee the feasibility and optimality of the solution for the worst case within those sets. For the earlier work on RO readers are referred to Ben-Tal and Nemirovski (1998), El Ghaoui *et al.* (1998), and Ben-Tal and Nemirovski (1999). To overcome the issue of overconservatism of the classical approach, Bertsimas and Sim (2004) proposed a new RO approach based on polyhedral uncertainty sets which allows the decision maker to adjust the level of conservatism, making trade off between performance and risk. This new technique has been applied to address the demand uncertainty in supply chain management and revenue management (Bertsimas and Thiele, 2006). I find that applying RO in a boarder sense within operations management would be an interesting avenue of future research.

Machine learning (hereafter ML) is a field of study that lies at the intersection of computer science and statistics. It has developed algorithms that allow us to build high-performance predictive models for high-dimensional data, such as Lasso regression, decision tree, random forest, Gradient Boosting algorithms, etc. These algorithms make predictions of an uncertain parameter from a set of potential predictors. These predictors, also being called features, include a variety of internal and external data, such as past sales, inventory, price, customer review, clickstream, competitors' information, weather, household panel etc. The task of ML is to learn a target function that best map the parameter to be predicted to its feature, and then use this function to generate forecasts for a new set of features. The distribution of this target function is not known in advance, it is constructed by the ML algorithm from historical observations. Feature selection and cross validation are two important steps to build high quality ML models. With the advancement of information technology, the amount of data is growing at unprecedented speed. This allows e-commerce and logistics companies to widely adopt ML to tackle stochastic issues, such as uncertainty in demand, consumer purchasing behaviour, traffic condition, etc. RO and ML can work together to protect the system from randomness. For example, we can employ ML technique to construct the uncertainty set of RO models. For any optimization problem, there are two ways to integrate the ML technique. The first one is a two-step approach, where we first use ML algorithms to estimate values of uncertain parameters, and then plug these values to a mathematical model and find the optimal solution. The second one is a one-step approach, where we construct an empirical risk minimization model to map the features of uncertain parameters to the optimal solution of the problem. Regarding to the latter approach, interested readers are referred to Ban and Rudin (2019). I see that combing optimization with machine learning for decision making would be another promising path of research to pursue.

In conclusion, the study of supply chain with uncertainty is a vast ocean. With the increasing abundance of data and methodological advances in RO and ML, we can contribute to develop more methodologies to help managers make better decisions for various supply chain problems in a stochastic environment.

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