Chapter 4

Multiple Access and Modulation Techniques

4.1 Introduction

Current powerline communication systems are characterized by a random multiple access method handled by the higher layer protocols. These techniques are more or less related to TDMA (Time Domain Multiple Access), in the sense that different users transmit their signals in separate time intervals. This method is quite simple to implement, but the obtained bit rates are suboptimal, as pointed out in Chapter 3. A better performance can be expected if all modems are allowed to transmit simultaneously.

In this chapter, a general transmission scheme is proposed for the multiuser uplink. Mutually orthogonal signatures are used by the subscriber modems to map the original binary data into spectrally shaped continuous waveforms and to allow the multiuser signal separation at the receiver. The effect of the frequency-selective channels and that of the receiver sampling unit are incorporated into the model. This provides a discrete equivalent model relating the initial data symbol streams produced by the different NTs to the received samples obtained at the LT.

The extension to downstream transmission is immediate : for a given downstream receiver, all signal components transmitted by the LT are just filtered by the same physical channel.

4.2 The multiple access model

The proposed multiple access model is depicted in Figure 4.1.

4.2.1 Bits, symbols and signatures

Binary data $d_k(n)$ are produced by the channel coder of each subscriber modem at a rate of R_k bits per second with $k \in \{1, \cdots, K\}$. The effect of error correcting codes is not analyzed in this work. The channel coder is assumed to include an ideal data scrambler, producing a sequence of mutually independent bits.

We assume that a set of N mutually orthogonal signature waveforms are available to ensure multiple access. Each user k is provided with the ability to modulate any subset $\mathcal{C}_k$ of these waveforms. A signature allocation algorithm, located in the central office, selects the appropriate subsets $\mathcal{C}_k$ with

$$\mathcal{C}_k \subseteq \{0, \cdots, N-1\} \text{ and } \mathcal{C}_k \cap \mathcal{C}_{k'} = \emptyset, \;\; k \neq k' \tag{4.1}$$

and transmits that information to the remote users through a dedicated downstream control channel. The number of signatures allocated to user k is denoted by N_k, with $\sum_k N_k = N$.

In each subscriber modem, the data sequence $d_k(n)$ is transformed into a number of parallel symbol streams $I_p(n)$ produced at a baud rate $1/T$. A given user produces as many parallel symbol streams as there are signatures in the subset $\mathcal{C}_k$. The symbols are obtained from a given PAM constellation according to a given bit coding scheme. The size 2^{b_p} of each constellation is adapted to the channel conditions and the required probability of error. The number of bits b_p associated with the different symbol sequences determines the total bit rate for each user :

$$R_k = \frac{1}{T} \sum_{p \in \mathcal{C}_k} b_p \tag{4.2}$$

Successive symbols of a given sequence are assumed to be mutually independent. The same assumption holds for symbols coming from distinct sequences. The symbol variance is denoted by σ_I^2.

4.2.2 Filter bank modulation

A filter bank (FB) including a set of N synthesis filters $s_p(n)$ defined at a chip rate $1/T_c = N/T$ is used to implement the modulation of multiple

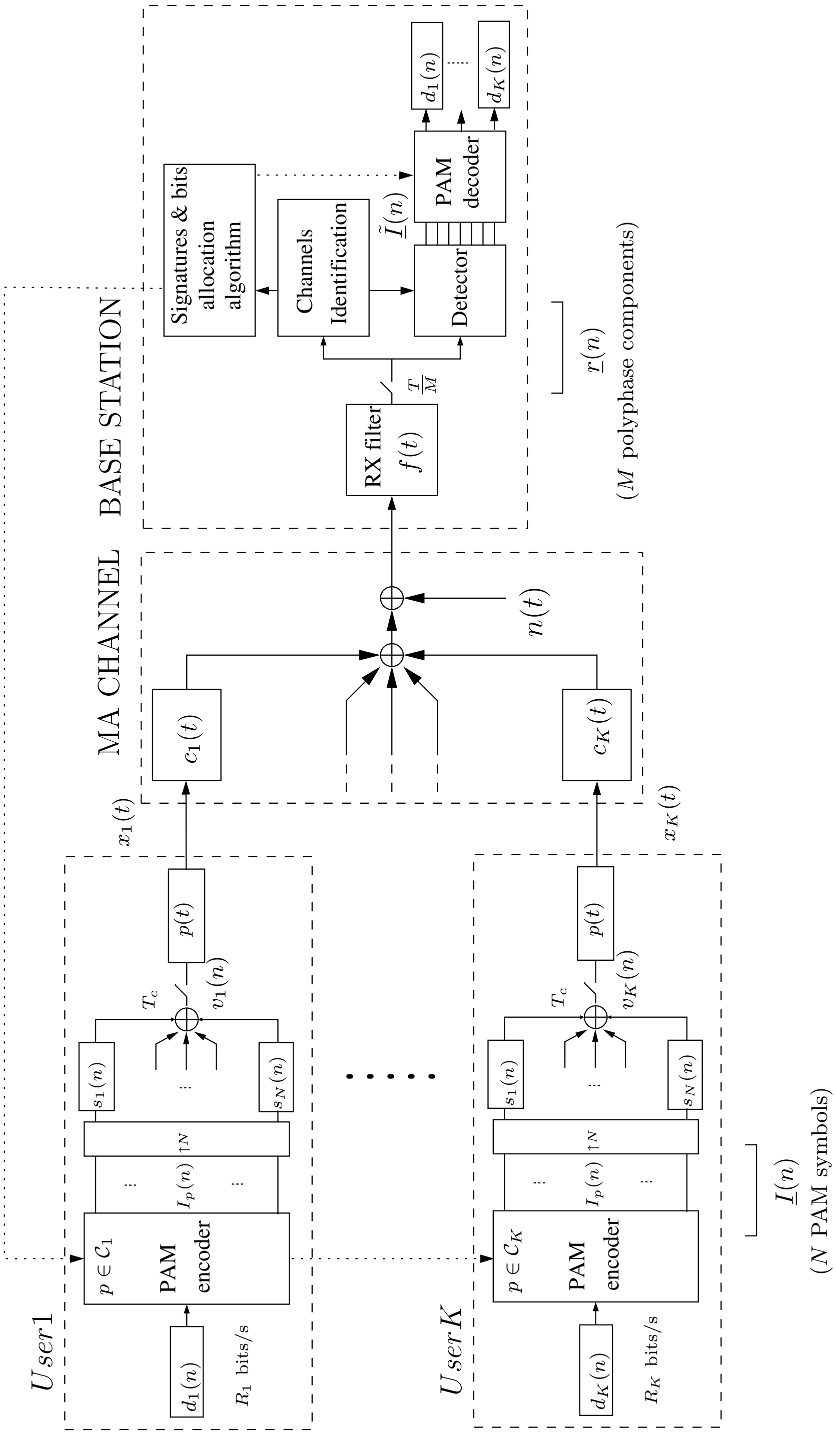

Figure 4.1: *Baseband FB multiple-access transmission scheme*

signatures efficiently in the digital domain. The synthesis filters $s_p(n)$ are considered here as signature codes. The samples produced by the synthesis operation are given by

$$v_k(n) = \sum_{p \in \mathcal{C}_k} \sum_{m=-\infty}^{+\infty} I_p(m)\, s_p(n - mN). \tag{4.3}$$

Splitting this sequence into its N type 1 polyphase components, we get

$$v_{\rho k}(n) = v_k(nN + \rho) = \sum_{p \in \mathcal{C}_k} \sum_{m=-\infty}^{+\infty} I_p(m)\, s_{\rho p}(n - m) \tag{4.4}$$

where $s_{\rho p}(m) = s_p(mN + \rho)$ and $\rho \in [0, N-1]$. Using z-transforms of the above quantities, we write :

$$V_{\rho k}(z) = \sum_{p \in \mathcal{C}_k} S_{\rho p}(z) I_p(z) \tag{4.5}$$

where $V_{\rho k}(z)$, $S_{\rho p}(z)$ and $I_p(z)$ are the z-transformed sequences $v_{\rho k}(n)$, $s_{\rho p}(n)$ and $I_p(n)$, respectively. Let $\underline{\underline{S}}(z)$ be the $N \times N$ FB polyphase matrix obtained with the N polyphase sequences of the N synthesis filters, i.e.

$$\underline{\underline{S}}(z) = \left[\begin{array}{ccc} \underline{S}_1(z) & \cdots & \underline{S}_N(z) \end{array}\right] \tag{4.6}$$

$$\underline{S}_p(z) = \left[\begin{array}{ccc} S_{1p}(z) & \cdots & S_{Np}(z) \end{array}\right]^T \tag{4.7}$$

and let $\underline{I}(z)$ be the $N \times 1$ vector obtained with the N symbol sequences, i.e.

$$\underline{I}(z) \triangleq [I_1(z), \cdots, I_N(z)]^T . \tag{4.8}$$

The synthesis operation in (4.4) can be expressed shortly with a matrix notation as :

$$\underline{V}_k(z) = \underline{\underline{S}}_k(z)\, \underline{I}_k(z) \tag{4.9}$$

where

$$\underline{V}_k(z) \triangleq [V_{1k}(z), \cdots, V_{Nk}(z)]^T \tag{4.10}$$

$$\underline{\underline{S}}_k(z) \triangleq \left[\underline{\underline{S}}(z)\right]_{p \in \mathcal{C}_k} \tag{4.11}$$

$$\underline{I}_k(z) \triangleq [\underline{I}(z)]_{p \in \mathcal{C}_k} . \tag{4.12}$$

In the sequel, we focus on a specific type of synthesis filter banks that fulfill the paraunitary property :

$$\underline{\underline{S}}(z)\, \underline{\underline{S}}^H(1/z^*) = \underline{\underline{E}}_N = \underline{\underline{S}}^H(1/z^*)\, \underline{\underline{S}}(z) \tag{4.13}$$

where $\underline{\underline{E}}_N$ stands for the unit matrix of size N. With this interesting property, a bank of N analysis filters matched to the synthesis bank provides the original symbol streams at its N outputs. All signatures in a paraunitary FB can be shown to have the same unit energy :

$$\sum_{n=-\infty}^{+\infty} s_p(n)^2 = \int_0^{2\pi} |S_p\left(e^{j\Omega}\right)|^2 \frac{\mathrm{d}\Omega}{2\pi} = 1 \tag{4.14}$$

where the signature Fourier transform is defined as :

$$S_p\left(e^{j\Omega}\right) = \sum_{n=-\infty}^{+\infty} s_p(n)\, e^{-jn\Omega}. \tag{4.15}$$

The transmitted power is thus unaffected by the FB modulation. Finally, paraunitary filter banks can be shown to preserve the total power spectral density as

$$\sum_{p=1}^{N} |S_p\left(e^{j\Omega}\right)|^2 = N \qquad \forall\ \Omega. \tag{4.16}$$

This implies that, for independent input data symbols with constant variance σ_I^2 on *all signatures*, the resulting transmitted power spectrum is flat.

4.2.3 Examples of filter banks

The choice of a specific set of signatures $s_p(n)$ to implement the multiple access scheme is widely arbitrary. Based on the knowledge of the user channels $c_k(t)$, the optimization of the transmitter could be investigated, leading to the choice of an optimal set of signatures according to some performance criterion in relation with a specific receiver structure (see [55]). This optimization is not considered in this work : the signatures are assumed to be fixed. The only degree of freedom considered here lies in the allocation of the predefined signature set to the different users. The only assumptions made here concerning the signatures is that the associated filter bank satisfies the paraunitary property and that the signatures are limited to a length of $N' \geq N$. For the purpose of illustration and comparison, four kinds of filter banks are considered, with increasing frequency selectivity :

- The **Single Carrier** modulation ($N' = N$) is obtained by letting $\underline{\underline{S}}(z) = \underline{\underline{E}}_N$, which is equivalent to a simple parallel-to-serial conversion. The N signatures correspond here to N successive time slots within a periodic frame of T seconds. The allocation of one or more time slots to each user is equivalent to Time-Division Multiple-Access (TDMA). Here the whole frequency spectrum is covered by all signatures. Nevertheless, the signature allocation has an impact on the system performance as it determines the interference structure of the received signal (relative origin of neighbouring symbols).

- The **Walsh-Hadamard codes** ($N' = N$) are binary : $s_p(n) = \pm\sqrt{\frac{1}{N}}$, and such an orthogonal set of codes exists for N being a power of 2. The synthesis operation is extremely fast for these binary signatures. Figure 4.2 illustrates the Hadamard signatures $s_5(n)$, $s_6(n)$, $s_{21}(n)$ and $s_{22}(n)$ for $N = 64$. The corresponding signature frequency responses $S_p(e^{j\Omega})$ are given in Figure 4.4. The implementation of multiple-access through binary signature codes is actually equivalent to Code-Division Multiple-Access (CDMA).

- The **Discrete Multi Tone (DMT)** modulation ($N' = N$) is based on the (Inverse) Discrete Fourier Transform or (I)DFT. For baseband transmission, the equivalent signatures are given by

$$s_0(n) = \sqrt{\frac{1}{N}} \qquad s_{N-1}(n) = \sqrt{\frac{1}{N}}(-1)^n \tag{4.17}$$

$$s_{2p-1}(n) = \sqrt{\frac{2}{N}}\cos\left(\frac{2\pi np}{N}\right) \qquad s_{2p}(n) = -\sqrt{\frac{2}{N}}\sin\left(\frac{2\pi np}{N}\right) \tag{4.18}$$

with $n \in [1 \cdots N]$ and $p \in [1 \cdots N/2 - 1]$. The synthesis operation is implemented efficiently by means of the Inverse Fast Fourier Transform (IFFT) whose basis functions are

$$s_{p,DMT}(n) = \frac{1}{\sqrt{N}} e^{\frac{2\pi jnp}{N}}, \tag{4.19}$$

with the following complex sequences $J_p(m)$ at the N inputs of the IFFT operator :

$$J_0(m) = I_0(m) \qquad J_{N/2}(m) = I_{N-1}(m) \tag{4.20}$$

$$J_p(m) = \frac{1}{\sqrt{2}}\left(I_{2p-1}(m) + jI_{2p}(m)\right) \qquad J_{N-p}(m) = J_p^*(m). \tag{4.21}$$

Figure 4.3 illustrates the DMT signatures $s_5(n)$, $s_6(n)$, $s_{21}(n)$ and $s_{22}(n)$ for $N = 64$. The corresponding signature frequency responses

$S_p(e^{j\Omega})$ are given in Figure 4.5. The implementation of multiple-access through IDFT basis functions is equivalent to Orthogonal Frequency-Division Multiple-Access (OFDMA).

- The **Perfect Reconstruction Modulated Filter Bank (PRMF)** ($N' = 2N$) is based on a single prototype $p_0(n)$, that is modulated around different frequencies. The prototype can be optimized to provide good spectral properties, e.g. a very high frequency selectivity. The basis functions are given by

$$s_p(n) = p_0(n)\,\cos\left[\frac{\pi}{N}(p+\frac{1}{2})(n-N-\frac{1}{2}) - (-1)^p\frac{\pi}{4}\right] \tag{4.22}$$

with $n \in [1 \cdots 2N]$. Unlike the first three examples, the signatures given by (4.22) have a length of 2 symbol periods. This means that successive modulated symbols overlap in time. This is the price to pay to obtain an increased frequency selectivity. The polyphase matrix associated with this set of signatures can be written as $\underline{\underline{S}}(z) = \underline{\underline{S}}_0 + \underline{\underline{S}}_1 z^{-1}$, while for the first three examples the matrix expansion is limited to the zero-order term $\underline{\underline{S}}_0$. The multiple access scheme associated with PRMF is more or less equivalent to Frequency-Division Multiple-Access (FDMA).

4.2.4 Transmitted signals

The signal $x_k(t)$ transmitted by user k is obtained after digital-to-analog conversion (DAC) through a chip shaping filter $p(t)$:

$$x_k(t) = \sum_{n=-\infty}^{+\infty} v_k(n)\,p(t-nT_c). \tag{4.23}$$

A realistic shaping filter $p(t)$ should include the effect of the analog front end circuitry of the transmitter. A convenient model used for theoretical investigations is the square-root raised cosine Nyquist filter with a roll-off α and a bandwidth given by $B = 0.5\,(1+\alpha)/T_c$. Continuous-time signature waveforms $g_p(t)$ are obtained by combining the signature codes $s_p(n)$ with the shaping filter $p(t)$:

$$g_p(t) = \sum_{n=-\infty}^{+\infty} s_p(n)p(t-nT_c). \tag{4.24}$$

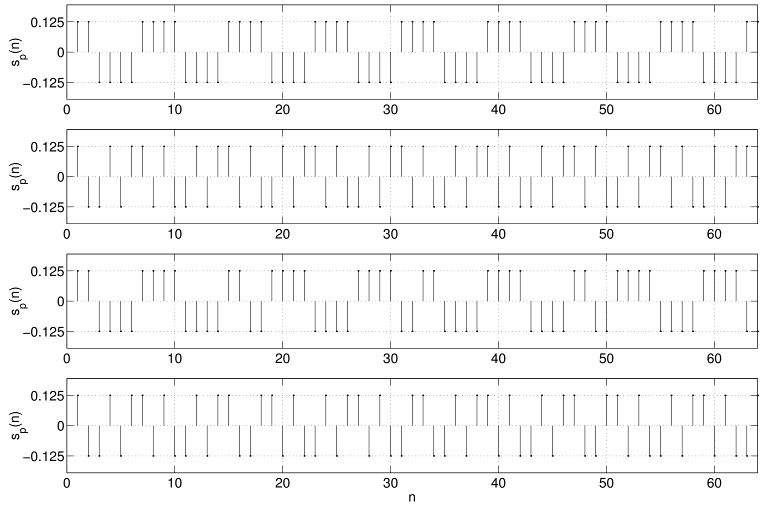

Figure 4.2: *Examples of signatures for the Hadamard filter bank ($N = 64$)*

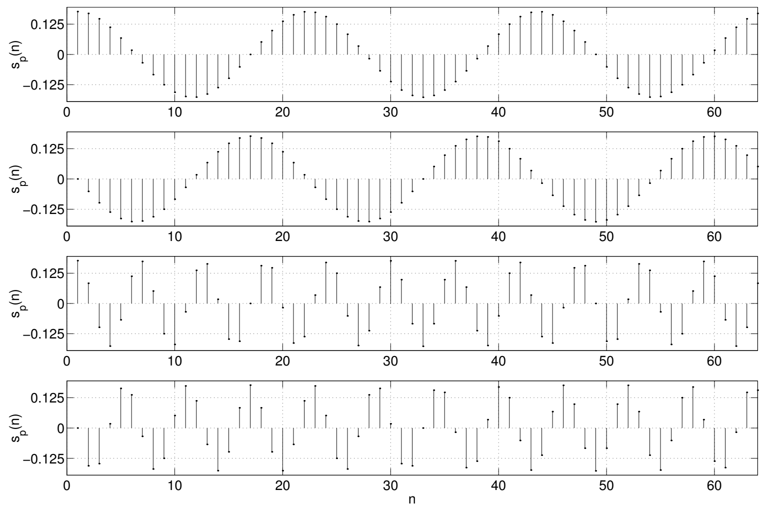

Figure 4.3: *Examples of signatures for the DMT filter bank ($N = 64$)*

The transmitted signals can be expressed as sums of modulated signature waveforms :

$$x_k(t) = \sum_{m=-\infty}^{+\infty} \sum_{p \in \mathcal{C}_k} I_p(m) g_p(t - mT). \tag{4.25}$$

The power spectral density of the transmitted signal is given by

$$\gamma_{x_k}(f) = \frac{\sigma_I^2}{T} \sum_{p \in \mathcal{C}_k} |G_p(f)|^2 \tag{4.26}$$

with $G_p(f) = \mathcal{F}\{g_p(t)\}$ and $\mathcal{F}\{.\}$ denotes the Fourier transform. Thanks to property (4.16) of the paraunitary filter banks, the power spectral density of $x_k(t)$ can be shown to be upper bounded as follows :

$$\gamma_{x_k}(f) \leq \frac{\sigma_I^2}{T_c} |P(f)|^2 \tag{4.27}$$

where $P(f) = \mathcal{F}\{p(t)\}$ and equality holds if all signatures are allocated to user k, with symbol sequences of equal variance σ_I^2 at the N inputs. This property allows to control the transmitted power spectral density when a limitation (power mask) $\gamma_{x_k}(f) \leq \overline{\gamma}(f)$ is imposed.

4.2.5 Multiuser channel and receiver front-end

At the output of the multiple access channel, the transmitted signals are filtered by the user-specific channels $c_k(t)$ and corrupted by additive noise $n(t)$:

$$r(t) = \sum_{k=1}^{K} [x_k(t) \otimes c_k(t)] + n(t) \tag{4.28}$$

$$= \sum_{n=-\infty}^{+\infty} \sum_{k=1}^{K} v_k(n) \tilde{c}_k(t - nT_c) + n(t) \tag{4.29}$$

$$= \sum_{m=-\infty}^{+\infty} \sum_{k=1}^{K} \sum_{p \in \mathcal{C}_k} I_p(m) h_{pk}(t - mT) + n(t) \tag{4.30}$$

with $\tilde{c}_k(t) = p(t) \otimes c_k(t)$, $h_{pk}(t) = g_p(t) \otimes c_k(t)$ and $\otimes$ denotes convolution. The channel impulse responses $c_k(t)$ include the effect of multipath propagation and that of cable losses. The derivation of the impulse responses from the cable characteristics and the network topology was discussed in

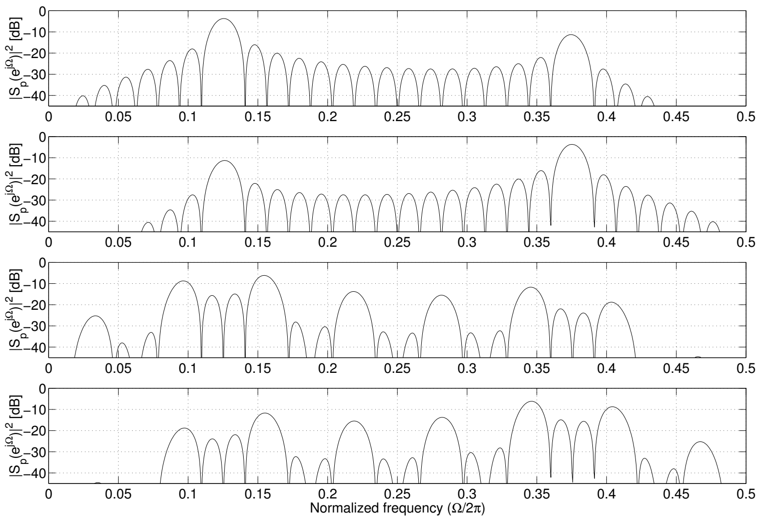

Figure 4.4: *Examples of signature frequency responses for the Hadamard filter bank ($N = 64$)*

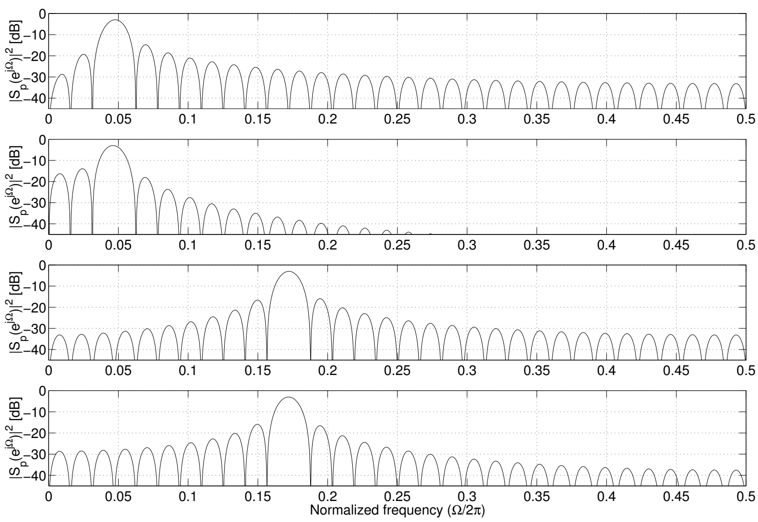

Figure 4.5: *Examples of signature frequency responses for the DMT filter bank ($N = 64$)*

Chapter 2. The additive noise $n(t)$ is assumed to be a white Gaussian noise (AWGN) with double-sided power spectral density $N_0/2$.

The equivalent channel equations for the downlink are

$$r_k(t) = \left[\sum_{l=1}^{K} x_l(t)\right] \otimes c_k(t) + n_k(t) \qquad (k \in \{1, \cdots, K\}) \tag{4.31}$$

where $r_k(t)$ and $n_k(t)$ denote the received signal and the additive noise in the k-th receiver.

The *energy* and the *mean square bandwidth* of the received symbols $I_p(m)$ associated with the signature waveform $g_p(n)$ and the user channel $c_k(t)$ are given by:

$$\mathcal{E}_{pk} = \sigma_I^2 \int_{-\infty}^{+\infty} |h_{pk}(t)|^2 \, dt = \sigma_I^2 \int_{-\infty}^{+\infty} |H_{pk}(f)|^2 \, df \tag{4.32}$$

$$W_{pk}^2 = \frac{\int_{-\infty}^{\infty} (2\pi f)^2 |H_{pk}(f)|^2 \, df}{\int_{-\infty}^{\infty} |H_{pk}(f)|^2 \, df} \tag{4.33}$$

These quantities depend both on the spectral characteristics of the signature and on the effect of the considered channel on that signature.

The received signal is filtered by means of a presampling filter $f(t)$ with cutoff $M/2T$ and sampled at the fractional rate $1/T_s = M/T$ where $M \geq N(1+\alpha)$ is an integer chosen large enough in order to cover the whole bandwidth of the signal spectrum. The filtered received signal and the filtered noise are denoted by $\overline{r}(t) = r(t) \otimes f(t)$ and $\overline{n}(t) = n(t) \otimes f(t)$, respectively. The filtered channel impulse responses are $\overline{c}_k(t) = p(t) \otimes c_k(t) \otimes f(t)$.

We define M polyphase components $r_\rho(m)$ (resp. $n_\rho(m)$) of the received signal (resp. the filtered noise), with $\rho \in [0, M-1]$, as follows :

$$r_\rho(m) = \overline{r}\left(mT + \rho T_s\right). \tag{4.34}$$

The M received polyphase sequences $r_\rho(m)$ can be related to the KN transmitted polyphase sequences $v_{\rho' k}(n)$ as follows :

$$\begin{aligned} r_\rho(m) &= \sum_{n=-\infty}^{+\infty} \sum_{k=1}^{K} v_k(n) \overline{c}_k(mT + \rho T_s - nT_c) + \overline{n}\left(mT + \rho T_s\right) \\ &= \sum_{n=-\infty}^{+\infty} \sum_{k=1}^{K} \sum_{\rho'=0}^{N-1} c_{\rho\rho' k}(n-m) v_{\rho' k}(n) + n_\rho(m) \end{aligned} \tag{4.35}$$

where the MN polyphase channel sequences $c_{\rho\rho'\,k}(m)$ are

$$c_{\rho\rho'\,k}(m) = \overline{c}_k\left(mT + \rho T_s - \rho' T_c\right). \tag{4.36}$$

The received sequences can also be related to the N initial symbol sequences $I_p(m)$ by

$$\begin{aligned} r_\rho(m) &= \sum_{n=-\infty}^{+\infty}\sum_{k=1}^{K}\sum_{p\in\mathcal{C}_k} I_p(n)\overline{h}_{pk}(mT+\rho T_s - nT) + \overline{n}\left(mT+\rho T_s\right) \\ &= \sum_{n=-\infty}^{+\infty}\sum_{k=1}^{K}\sum_{p\in\mathcal{C}_k} h_{\rho\, pk}(m-n)I_p(n) + n_\rho(m) \end{aligned} \tag{4.37}$$

The $h_{\rho\, pk}(m)$ polyphase component represents the cumulative effect of the signature waveform $g_p(t)$, the channel $c_k(t)$ and the filter $f(t)$, sampled at the symbol rate T with a phase ρT_s :

$$\begin{aligned} h_{\rho\, pk}(m) &= \overline{h}_{pk}\left(mT+\rho T_s\right) \\ &= \sum_{n=\infty}^{+\infty} s_p(n)\overline{c}_k(mT+\rho T_s - nT_c) \\ &= \sum_{n=\infty}^{+\infty}\sum_{\rho'=0}^{N-1} s_{\rho' p}(n)c_{\rho\rho',k}(m-n). \end{aligned} \tag{4.38}$$

The noise variance after sampling at M/T is $\sigma_n^2 = N_0 M/2T$.

4.2.6 IIR observation model

Taking the z-transform of equation (4.35), we write

$$R_\rho(z) = \sum_{\rho'=0}^{N-1}\sum_{k=1}^{K} V_{\rho' k}(z)C_{\rho\rho',k}(z) + N_\rho(z) \tag{4.39}$$

where $R_\rho(z)$, $C_{\rho\rho',k}(z)$ and $V_{\rho',k}(z)$ are the z-transformed sequences $r_\rho(m)$, $c_{\rho\rho',k}(m)$ and $v_{\rho',k}(n)$, respectively. Let $\underline{\underline{C}}(z)$ be the $M\times N$ channel polyphase matrix obtained with the MN polyphase channel sequences, i.e.

$$\underline{\underline{C}}(z) = \begin{bmatrix} C_{11}(z) & \cdots & C_{1N}(z) \\ \vdots & \ddots & \vdots \\ C_{M1}(z) & \cdots & C_{MN}(z) \end{bmatrix} \tag{4.40}$$

and let $\underline{\mathrm{R}}(z)$ (resp. $\underline{\mathrm{N}}(z)$) be the $M \times 1$ vector obtained with the M received (resp. noise) polyphase sequences, i.e.

$$\begin{aligned} \underline{\mathrm{R}}(z) &\triangleq [R_1(z), \cdots, R_M(z)]^T \\ \underline{\mathrm{N}}(z) &\triangleq [N_1(z), \cdots, N_M(z)]^T . \end{aligned} \tag{4.41}$$

The multiuser channel equations (4.35) and (4.37) can be expressed shortly with a matrix notation as :

$$\begin{aligned} \underline{\mathrm{R}}(z) &= \sum_{k=1}^{K} \underline{\underline{\mathrm{C}}}_k(z)\, \underline{\mathrm{V}}_k(z) + \underline{\mathrm{N}}(z) \\ &= \sum_{k=1}^{K} \underline{\underline{\mathrm{H}}}_k(z)\, \underline{\mathrm{I}}_k(z) + \underline{\mathrm{N}}(z) \end{aligned} \tag{4.42}$$

where the IIR equivalent channel matrix $\underline{\underline{\mathrm{H}}}_k(z) = \underline{\underline{\mathrm{C}}}_k(z)\, \underline{\underline{\mathrm{S}}}_k(z)$ can be obtained with the adequate z-transformed sequences $h_{\rho pk}(m)$. The IIR channel matrix $\underline{\underline{\mathrm{C}}}_k(z)$ is Toeplitz in the special case where $T_s = T_c$. In the general case, however, i.e. when the receiver operates at a fractional rate, this property is not satisfied. Equation (4.42) provides an IIR observation model for the general multiple access channel depicted in Figure 4.1. This model is associated with a continuous mode of transmission, with symbol sequences $I_p(m)$ transmitted continuously at a rate $1/T$.

4.2.7 FIR observation model

Even in the case of a continuous mode of transmission, it is often desirable to build an FIR observation model that relates a finite-length observation window of the received signal to a finite set of transmitted symbols. This is possible if the channel impulse responses $\bar{c}_k(t)$ and the signature codes $s_p(n)$ have a finite size.

Let $N_f T = (n_1 + n_2 + 1)T$ be the length of the observation window, i.e. the received signal is observed in the time interval $[(n - n_2)T, (n + n_1 + 1)T]$. With the sampling rate $1/T_s = M/T$, a vector of $N_f M$ samples is sufficient to represent the received signal in the selected time interval. Grouping together the M polyphase components of the received signal (resp. the additive noise), we define received signal vectors $\underline{\mathrm{r}}(n)$ (resp. noise vectors $\underline{\mathrm{n}}(n)$) as follows :

$$\underline{\mathrm{r}}(n) = \begin{bmatrix} r_{M-1}(n) \\ \vdots \\ r_0(n) \end{bmatrix} \qquad \underline{\mathrm{n}}(n) = \begin{bmatrix} n_{M-1}(n) \\ \vdots \\ n_0(n) \end{bmatrix} \tag{4.43}$$

The signal observed in a time interval of length $N_f T$ is obtained by stacking N_f received signal vectors as defined in equation (4.43).

The user channel impulse responses $c_k(t)$ are assumed to be causal and to have a length of $L_c T_c$ with $L_c \geq 0$. The length of the composite channels $h_{pk}(t) = \sum_n s_p(n)\bar{c}_k(t - nT_c)$ is then upper-bounded by $(L+1)T$ with

$$L = \lceil \frac{L_c + N'}{N} \rceil - 1 \geq 0. \tag{4.44}$$

The minimum length of T corresponds to ideal Dirac channels $\bar{c}_k(t) = \delta(t)$ associated with signatures $s_p(n)$ of length N.

Grouping the M equivalent channel polyphase sequences associated with the N signature waveforms in a $M \times N$ rectangular matrix, we define channel blocks $\underline{\underline{\mathrm{h}}}(n)$ and $\underline{\underline{\mathrm{h}}}_k(n)$ as follows :

$$\underline{\underline{\mathrm{h}}}(n) = \begin{bmatrix} h_{M-1,1}(n) & \cdots & h_{M-1,N}(n) \\ \vdots & \ddots & \vdots \\ h_{0,1}(n) & \cdots & h_{0,N}(n) \end{bmatrix} \qquad \underline{\underline{\mathrm{h}}}_k(n) = \left[\underline{\underline{\mathrm{h}}}(n)\right]_{\{p \in \mathcal{C}_k\}}. \tag{4.45}$$

With FIR channels, $L+1$ channel blocks are non-zero. Channel matrices $\underline{\underline{\mathrm{H}}}$ and $\underline{\underline{\mathrm{H}}}_k$ are defined by stacking the $L+1$ non-zero channel blocks as follows :

$$\underline{\underline{\mathrm{H}}} = \begin{bmatrix} \underline{\underline{\mathrm{h}}}(L) \\ \vdots \\ \underline{\underline{\mathrm{h}}}(0) \end{bmatrix} \qquad \underline{\underline{\mathrm{H}}}_k = \left[\underline{\underline{\mathrm{H}}}\right]_{\{p \in \mathcal{C}_k\}}. \tag{4.46}$$

The N columns of $\underline{\underline{\mathrm{H}}}$ actually contain the N composite impulse responses $h_{pk}(t)$ sampled at a rate M/T and stored in reverse order. User-specific channel matrices $\underline{\underline{\mathrm{H}}}_k$ are just obtained by selecting the columns corresponding to the signatures allocated to user k.

Finally, the global channel equation relating received samples to transmitted symbols can be written as follows :

$$\begin{bmatrix} \underline{r}(n+n_1) \\ \vdots \\ \underline{r}(n) \\ \vdots \\ \underline{r}(n-n_2) \end{bmatrix} = \begin{bmatrix} \underline{\underline{h}}(0) & \underline{\underline{h}}(1) & \cdots & \underline{\underline{h}}(L) & & & & \\ & \underline{\underline{h}}(0) & \cdots & \underline{\underline{h}}(L-1) & \underline{\underline{h}}(L) & & & \\ & & \ddots & \ddots & \ddots & \ddots & & \\ & & & \underline{\underline{h}}(0) & \underline{\underline{h}}(1) & \cdots & \underline{\underline{h}}(L) & \\ & & & & \underline{\underline{h}}(0) & \cdots & \underline{\underline{h}}(L-1) & \underline{\underline{h}}(L) \end{bmatrix}$$

$$\cdot \begin{bmatrix} \underline{I}(n+n_1) \\ \vdots \\ \underline{I}(n) \\ \vdots \\ \underline{I}(n-n_2-L) \end{bmatrix} + \begin{bmatrix} \underline{n}(n+n_1) \\ \vdots \\ \underline{n}(n) \\ \vdots \\ \underline{n}(n-n_2) \end{bmatrix}. \quad (4.47)$$

Using the following notation to denote the concatenation of successive vectors coming from an arbitrary sequence $\underline{x}(n)$:

$$\underline{x}_{n_2}^{n_1}(n) \triangleq \left[\ \underline{x}(n+n_1)^T \quad \cdots \quad \underline{x}(n-n_2)^T\ \right]^T, \quad (4.48)$$

the channel equation (4.47) can be rewritten shortly as

$$\underline{r}_{n_2}^{n_1}(n) = \underline{\underline{\mathcal{H}}}_{N_f}\, \underline{I}_{n_2+L}^{n_1}(n) + \underline{n}_{n_2}^{n_1}(n). \quad (4.49)$$

The global channel matrix $\underline{\underline{\mathcal{H}}}_{N_f}$, defined by identification with equation (4.47), is a *block-Toeplitz* matrix of size $N_f M \times (N_f + L)N$, generated with the $L+1$ channel blocks $\underline{\underline{h}}(n)$ defined in (4.45). It can be obtained by stacking N_f cyclic shifts of the first row of $M \times N$ channel blocks. The relative contribution of the different users can be put forward by writing the global channel equation as the sum of user-specific contributions :

$$\underline{r}_{n_2}^{n_1}(n) = \sum_{k=1}^{K} \underline{\underline{\mathcal{H}}}_{N_f,k}\, (\underline{I}_k)_{n_2+L}^{n_1}(n) + \underline{n}_{n_2}^{n_1}(n) \quad (4.50)$$

where the block-Toeplitz matrices $\underline{\underline{\mathcal{H}}}_{N_f,k}$ are generated with the user channel blocks $\underline{\underline{h}}_k(n)$ defined in (4.45).

4.3 Bit rates computation

As shown in Figure 4.1, a joint detector (JD) located in the receiver has the task to build estimates $\hat{I}_p(n)$ of the N symbol sequences transmitted by the K users. Different kinds of joint detectors are analyzed and compared in Chapter 5. Whatever the detector implemented in the receiver, the obtained symbol estimates can be expressed as

$$\hat{I}_p(n) = \sum_{m=-\infty}^{+\infty} \sum_{k=1}^{K} \sum_{p' \in \mathcal{C}_k} x_{pp'}(m)\, I_{p'}(n-m) + \nu_p(n). \tag{4.51}$$

Expression (4.51) includes five types of terms :

- A contribution of the useful symbol $x_{pp}(0) I_p(n)$.
- A weighted sum of past and future symbols associated with the same signature waveform $g_p(t)$. This is the residual ISI (Inter-Symbol-Interference).
- A weighted sum of symbols associated with other signature waveforms $g_{p'}(t)$ with $p' \neq p$ that are allocated to user k. This is the residual ICI (Inter-Code-Interference).
- A weighted sum of symbols transmitted by the other users $k' \neq k$. This is the residual MAI (Multiple-Access-Interference).
- The contribution of the additive Gaussian noise $\nu_p(n)$.

The performance index for the p-th detector output is the Signal-to-Interference-plus-Noise Ratio (SINR) denoted by ρ_p and given by

$$\rho_p = \frac{x_{pp}^2(0)\,\sigma_I^2}{\left(\sum_{(m,p') \neq (0,p)} x_{pp'}^2(m)\right)\,\sigma_I^2 + \sigma_{\nu_p}^2}. \tag{4.52}$$

For a PAM constellation of size $N_p = 2^{b_p}$, the symbol error probability p_s is known to be

$$p_s = \frac{N_p - 1}{N_p} \operatorname{erfc}\left(\sqrt{\frac{3\,\rho_p}{2(N_p^2 - 1)}}\right). \tag{4.53}$$

This expression is valid only if all sources of noise (including ISI, ICI and MAI) are Gaussian. It remains approximately valid, however, if a high number of independent PAM symbols contribute to the expression of the composite interference. It is also assumed that Gray mapping is used for the PAM constellations, which implies that an error on one symbol generates an error on one single bit. The bit error probability is then $p_b = p_s/b_p$.

For a given target bit-error-rate p_b, the maximum constellation size can be derived from equation (4.53) as

$$b_p = \frac{1}{2}\log_2\left(1+\frac{\rho_p}{\Gamma(p_b)}\right) \tag{4.54}$$

where Γ is called the 'SNR gap' and can be approximated by

$$\Gamma(p_b) \approx \frac{2}{3}\left[\mathrm{erfc}^{-1}(p_b)\right]^2. \tag{4.55}$$

The effective SNR gap should be modified to take into account the effect of error correcting codes. Actually, equation (4.53) does not guarantee that an integer number of bits is obtained for the PAM constellation. In a practical system, the effective bit error rate associated with the different user signatures can be slightly higher or lower than the reference probability p_b, as long as the average bit error rate over all signatures allocated to a given user is equal to p_b. The selected constellation size is then rounded to the upper or lower nearest integer value, depending on the signature under consideration. For the bit rate computations presented in this work, the number of bits are given by (4.54) and are not rounded, which has a negligible impact on the accuracy of the presented results. Finally, the global bit rate for a given user is given by

$$R_k = \frac{1}{T}\sum_{p\in\mathcal{C}_k} b_p = \frac{1}{T}\log_2\left[\prod_{p\in\mathcal{C}_k}\left(1+\frac{\rho_p}{\Gamma}\right)\right] \approx \frac{N_k}{T}\left[\log_2(\overline{\rho}_k)-\log_2(\Gamma)\right] \tag{4.56}$$

where the geometrical SINR for user k is defined by

$$\overline{\rho}_k = \sqrt[N_k]{\prod_{p\in\mathcal{C}_k}\rho_p} \tag{4.57}$$

and the last approximation in (4.56) is valid for high values of the SINR's.

4.4 CAP-FB transmission

A common constraint in high-speed communications over wired channels (like DSL and PLC) is that the lower part of the frequency spectrum (from 0 to F_0 MHz) should remain unaffected by the broadband signals. As a consequence, the broadband signals must be of the passband type, but with a carrier frequency f_c of the same order as the available bandwidth $B-F_0$.

In these conditions, it is desirable to make use of a carrierless modulation technique. This is obtained by directly producing a digital passband signal at the input of the DAC. Two options are available to build a passband signal in the digital domain : either the signatures corresponding to the forbidden frequencies are not loaded, or all signatures are loaded but the obtained baseband signal is filtered by means of a well-chosen passband digital filter.

The first option assumes the use of a *frequency-selective filter bank*. With the DFT or PRMF filters, for example, the different signatures can be associated with specific subbands. The signatures located in the lower subbands are not modulated. The total number of signatures to be allocated to the K users is then $\overline{N} < N$. Figure 4.6 (upper figure) gives the power spectral density $\gamma_{x_k}(f)$ of the transmitted signal obtained with an ideal frequency-selective FB with $N = 16$ subbands, a raised-cosine filter with roll-off α, and signatures 3 to 16 allocated to the same user k. With a practical FB, of course, sidelobes have to be taken into account and an analog highpass filter should be used to attenuate the signal spectrum in the forbidden frequency band.

The second option uses the *Carrierless Amplitude-Phase modulation (CAP)*. Basically, the CAP modulation is equivalent to a single carrier QAM modulation at low frequency, which implies that the modulation operation can be obtained directly by the use of sine- and cosine-modulated shaping filters $p_a(t)$ and $p_b(t)$ operating on two distinct symbol sequences $I_a(n)$ and $I_b(n)$, at a rate $1/T'_c = B - F_0$. The shaping filters form a Hilbert pair :

$$\begin{aligned} p_a(t) &= \sqrt{2}\,p'(t)\,\cos(\omega_c t) = \tilde{p}_b(t) \qquad (4.58)\\ p_b(t) &= \sqrt{2}\,p'(t)\,\sin(\omega_c t) = \tilde{p}_a(t) \qquad (4.59)\end{aligned}$$

where f_c is an arbitrary carrier frequency with the constraint $f_c \geq \frac{1+\alpha'}{2T'_c}$, and $p'(t)$ is a baseband half root Nyquist filter with roll-off α'. Practically, the CAP modulation can be performed with two digital filters $p_a(\frac{nT'_c}{3})$ and $p_b(\frac{nT'_c}{3})$ followed by the digital to analog conversion. The oversampling factor of 3 is generally sufficient when $F_0 \ll B$.

The CAP technique can be extended to filter bank modulation. In the CAP-FB scheme, two baseband digital signals $v_{ka}(n)$ and $v_{kb}(n)$ are produced by each user, with an arbitrary FB, at a rate $1/T'_c$. Let $\underline{\mathrm{V}}_{ka}(z)$ and $\underline{\mathrm{V}}_{kb}(z)$ be the respective polyphase sequences obtained at the output of a filter bank $\underline{\underline{\mathrm{S}}}(z)$ with inputs $\underline{\mathrm{I}}_{ka}(z)$ and $\underline{\mathrm{I}}_{kb}(z)$:

$$\left[\ \underline{\mathrm{V}}_{ka}(z) \quad \underline{\mathrm{V}}_{kb}(z)\ \right] = \underline{\underline{\mathrm{S}}}_k(z)\left[\ \underline{\mathrm{I}}_{ka}(z) \quad \underline{\mathrm{I}}_{kb}(z)\ \right] \qquad (4.60)$$

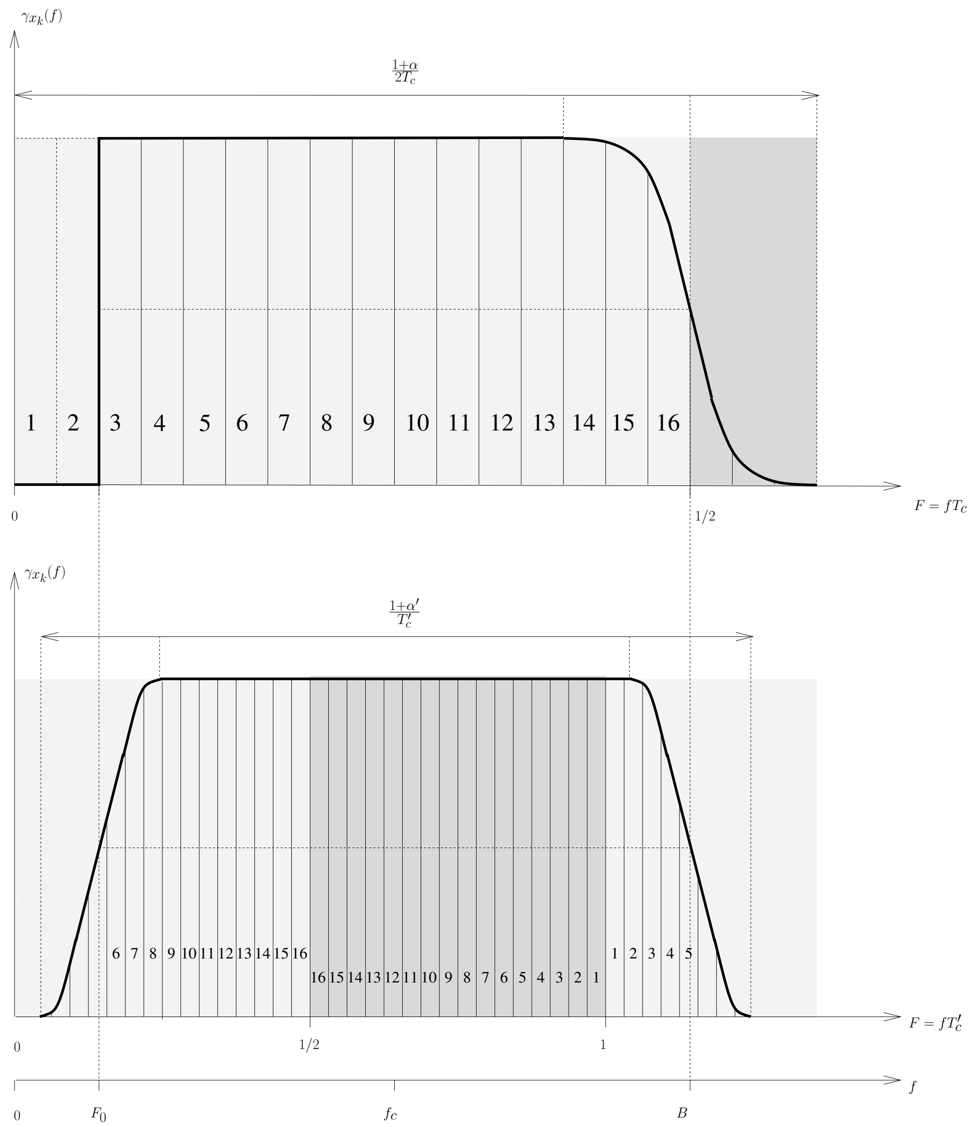

Figure 4.6: *Power spectrum of baseband-FB and CAP-FB modulated signals*

These sequences are used to modulate the two quadrature shaping filters.

The received signal becomes :

$$
\begin{aligned}
r(t) &= \sum_{n=-\infty}^{+\infty} \sum_{k=1}^{K} \left[v_{ka}(n)\, c_{ka}(t - nT'_c) + v_b(n)\, c_{kb}(t - nT'_c) \right] + n(t) && (4.61) \\
&= \sum_{m=-\infty}^{+\infty} \sum_{k=1}^{K} \sum_{p \in \mathcal{C}_k} \left[I_{p,a}(m) h_{pk,a}(t - mT') + I_{p,b}(m) h_{pk,b}(t - mT') \right] \\
&+ \; n(t) && (4.62)
\end{aligned}
$$

with $c_{ka}(t) = p_a(t) \otimes c_k(t)$ and $c_{kb}(t) = p_b(t) \otimes c_k(t) = \tilde{c}_{ka}(t)$ and two series of equivalent channels :

$$
\begin{aligned}
h_{pk,a}(t) &= \sum_{n=-\infty}^{+\infty} s_p(n) c_{ka}(t - nT'_c) && (4.63) \\
h_{pk,b}(t) &= \sum_{n=-\infty}^{+\infty} s_p(n) c_{kb}(t - nT'_c) = \tilde{h}_{pk,a}(t). && (4.64)
\end{aligned}
$$

The received signal is sampled at a fractional rate M/T'. The constraint on the oversampling factor M is now :

$$
M \geq 2 f_c T' + N(1 + \alpha'). \qquad (4.65)
$$

The IIR observation model (4.42) relating the M received polyphase sequences to the $2N$ sequences of transmitted symbols becomes :

$$
\begin{aligned}
\underline{\mathrm{R}}(z) &= \sum_{k=1}^{K} \left[\underline{\underline{\mathrm{C}}}_{ka}(z)\, \underline{\mathrm{V}}_{ka}(z) + \underline{\underline{\mathrm{C}}}_{kb}(z)\, \underline{\mathrm{V}}_{kb}(z) \right] + \underline{\mathrm{N}}(z) && (4.66) \\
&= \sum_{k=1}^{K} \left[\underline{\underline{\mathrm{H}}}_{ka}(z)\, \underline{\mathrm{I}}_{ka}(z) + \underline{\underline{\mathrm{H}}}_{kb}(z)\, \underline{\mathrm{I}}_{kb}(z) \right] + \underline{\mathrm{N}}(z) && (4.67)
\end{aligned}
$$

with $\underline{\underline{\mathrm{H}}}_{ka}(z) = \underline{\underline{\mathrm{C}}}_{ka}(z)\underline{\underline{\mathrm{S}}}_{k}(z)$ and $\underline{\underline{\mathrm{H}}}_{kb}(z) = \underline{\underline{\mathrm{C}}}_{kb}(z)\underline{\underline{\mathrm{S}}}_{k}(z)$. The extension of the FIR observation model (4.49) is straightforward.

Figure 4.6 (lower figure) gives the power spectral density $\gamma_{x_k}(f)$ of the CAP-FB signal obtained with an ideal frequency-selective FB with $N = 16$ subbands. Actually, the CAP-FB scheme can be used with an arbitrary set of signatures. The difference between QAM and CAP is also visible in

Figure 4.6. The spectrum of the different signatures is not moved by f_c as in the classical QAM modulation. With CAP, the periodic spectrum of the signatures $|S_p(e^{2\pi jF'})|^2$ with $F' = fT'_c$ is fixed, and the pulse shaping filters select the frequencies in the range $F' \in \left[f_cT'_c - \frac{1}{2}, f_cT'_c + \frac{1}{2}\right]$.

The complex envelope of the transmitted signals $x_k(t)$ is given by

$$\begin{aligned} e_{x_k}(t) &= \sum_{p\in\mathcal{C}_k} \sum_{n=-\infty}^{+\infty} \sum_{m=-\infty}^{+\infty} \overline{I}_p(m) \,.\, s_p(n-mN) \,.\, e^{-2\pi jnf_cT'_c}\, p'(t-nT'_c) \\ &= \sum_{p\in\mathcal{C}_k} \sum_{n=-\infty}^{+\infty} \sum_{m=-\infty}^{+\infty} \left[\overline{I}_p(m)\, e^{-2\pi jmf_cT'}\right] \,.\, \overline{s}_p(n-mN) \\ &\qquad\qquad .\, p'(t-nT'_c) \end{aligned} \tag{4.68}$$

where the complex QAM symbols $\overline{I}_p(m)$ and the complex modulated signatures $\overline{s}_p(n)$ are defined by

$$\overline{I}_p(m) = I_{p,a}(m) + jI_{p,b}(m) \tag{4.69}$$

$$\overline{s}_p(n) = s_p(n)\, e^{-2\pi jnf_cT'_c}. \tag{4.70}$$

From (4.68), it appears that the analysis of the CAP-FB transmission scheme could be achieved with a lowpass equivalent model including complex symbol sequences $\overline{I}_p(m)$ and a set of complex modulated signatures $\overline{s}_p(n)$, by rotating the complex symbols $\overline{I}_p(m)$ with an angle $-2\pi mf_cT'$ that varies according to the symbol index m. In the sequel, however, the bandpass formalism will be used.

Finally, it should be noted that the total bit rates offered by the two passband schemes displayed in Figure 4.6 are roughly the same. In the first scheme (baseband FB transmission), $\overline{N}$ signatures are modulated at a baud rate $1/T$. In the second scheme (CAP-FB transmission), $2N$ signatures are modulated at a baud rate $1/T'$. As $\frac{\overline{N}}{N} = \frac{2T}{T'} = \frac{B-F_0}{B}$, the potential bit rates are roughly equivalent.

4.5 CAP-CDMA with long codes

In the case of *binary signature codes of length* N, the CAP-FB multiple access technique described in the last section is referred to as CAP-CDMA. The basic CAP-CDMA system uses short codes, that is to say codes which have a length equal to the symbol period. The code used for all the symbols of a given user is then the same. It may happen that the amount of interference experienced by this code is high and it will stay so. A careful code allocation algorithm must be implemented in order to obtain the best matching between the allocated codes and the user channel responses. Moreover, several signature codes should be allocated to the weakest users to offer them a fair level of performance.

Many CDMA-based communications standards utilize aperiodic or longer period sequences instead of short codes. In WCDMA [56], for example, a long scrambling code truncated to the 10 msec frame length is used. The long scrambling code $a(n)$ is superimposed on the short spreading code $s_k(n)$, by modulo-2 addition. This means that successive symbols of a given user are spread by different code segments, and a certain code segment only repeats after, say, P symbols or PN chips where the spreading factor is denoted by N and P is the code period measured in symbol periods. The long code is split in P segments $a_p(n)$ of length N. One period of the long code can be obtained from the concatenation of the P sequences $a_p(n)$ with $p \in [0, P-1]$. The resulting signature associated with the p-th segment is given by

$$s_{k,p}(n) = s_k(n) \,.\, a_p(n) \tag{4.71}$$

where the product is equivalent to modulo-2 addition with antipodal sequences. In the long code scenario, the interference experienced by successive symbols changes, and on average all users will experience similar levels of interference. The signature allocation algorithm is not needed anymore. Whatever the initial signature code $s_k(n)$ selected for user k, the average level of performance is roughly constant thanks to the effect of the long code. The relative performance of the different users can be controlled in two ways : either by varying the number of parallel signatures allocated to each user (i.e. by using multicode CDMA), or by varying the transmission power of the different user signals $x_k(t)$. To simplify notations, we assume in the sequel that a single signature code $s_k(n)$ is allocated to each user.

The discrete time chip-rate signals $v_{k,q}(n)$ with $q \in \{a, b\}$ produced by user k are given by

$$v_{k,q}(n) = \sum_{m=-\infty}^{\infty} \sum_{p=0}^{P-1} I_{k,q}(mP + p)\, s_{k,p}(n - mPN - pN). \tag{4.72}$$

The received signal can be written as

$$r(t) = \sum_{k=1}^{K} \sum_{m=-\infty}^{\infty} \sum_{p=0}^{P-1} \sum_{q\in\{a,b\}} I_{k,p,q}(m)\, h_{k,p,q}(t - mPT') + n(t) \tag{4.73}$$

where $I_{k,p,q}(m) = I_{k,q}(mP + p)$ and

$$h_{k,p,q}(t) = \sum_{n=1}^{N} s_{k,p}(n)\, c_{k,q}(t - pT' - nT'_c) \tag{4.74}$$

is the composite impulse response resulting from the cascade of the p-th code segment and the channel impulse response $c_{kq}(t) = p_q(t) \otimes c_k(t)$. The advantage of this representation of each user signal by means of P parallel streams is that the code used in each stream is time-invariant.

An important parameter to compare performance is the received energy per symbol. Obviously not all symbols will be received with the same energy if the transmission power is constant. As a matter of fact, each code segment has a specific frequency response and hence the energy left after propagation over the channel will be segment dependent. The received symbol energy for a symbol transmitted with phase p in a packet of P symbols is given by

$$\mathcal{E}_{k,p,q} = \sigma_I^2 \int_{-\infty}^{\infty} |h_{k,p,q}(t)|^2 \,\mathrm{d}t \tag{4.75}$$

where σ_I^2 is the symbol variance. The averaged received symbol energy for a given user is

$$\mathcal{E}_k = \frac{1}{2P} \sum_{p=0}^{P-1} \sum_{q \in \{a,b\}} \mathcal{E}_{k,p,q}. \tag{4.76}$$

The received signal is sampled at a fractional rate $1/T_s = M/T'$ where M has to satisfy the constraint in (4.65). As the symbol sequences in (4.73) are now defined at a baud rate of $1/PT'$, the total number of polyphase sequences needed to represent the received signal is MP :

$$\begin{aligned} r_\rho(n) &= \overline{r}[(nMP + \rho)T_s] & (4.77)\\ &= \sum_{k=1}^{K} \sum_{m=-\infty}^{\infty} \sum_{p=0}^{P-1} \sum_{q \in \{a,b\}} I_{k,p,q}(m) h_{k,p,\rho,q}(n-m) + n_\rho(n) & (4.78) \end{aligned}$$

with $h_{k,p,\rho,q}(n-m) = h_{k,p,q}[((n-m)MP + \rho)T_s]$ and $\rho \in [0, MP-1]$. The IIR observation model relating the MP received polyphase sequences to the $2KP$ transmitted symbol sequences is

$$\underline{\mathrm{R}}(z) = \underline{\underline{\mathrm{H}}}(z)\, \underline{\mathrm{I}}(z) + \underline{\mathrm{N}}(z) \tag{4.79}$$

where $\underline{R}(z)$ and $\underline{N}(z)$ are of size MP, $\underline{I}(z)$ of size $2KP$ and $\underline{\underline{H}}(z)$ of size $MP \times 2KP$.

With finite-length channel impulse responses $c_{k,q}(t)$, the composite channels $h_{k,p,q}(t)$ are non-zero in a time interval of length $(L+1)T'$. A *segment-dependent* FIR observation model can be obtained as follows. Let $\underline{I}_p(n)$ be the block of $2K$ real symbols transmitted by the users on the p-th code segments and $\underline{r}_p(n)$ (resp. $\underline{n}_p(n)$) be the block of M signal (resp. noise) samples received during the p-th segment of duration T' :

$$\underline{I}_p(n) = \begin{bmatrix} I_{1,p,a}(n) \\ I_{1,p,b}(n) \\ \vdots \\ I_{K,p,a}(n) \\ I_{K,p,b}(n) \end{bmatrix} \qquad \underline{r}_p(n) = \begin{bmatrix} r_{pM+M-1}(n) \\ \vdots \\ r_{pM}(n) \end{bmatrix} \tag{4.80}$$

Grouping M adjacent polyphase components of the $2K$ composite impulse responses $h_{k,p,q}(t)$ associated with the p-th code segments in a $M \times 2K$ matrix, we define segment-dependent channel blocks $\underline{\underline{h}}_p(n)$ as follows :

$$\underline{\underline{h}}_p(n) = \begin{bmatrix} h_{M-1,p,1}(n) & \tilde{h}_{M-1,p,1}(n) & \cdots & h_{M-1,p,K}(n) & \tilde{h}_{M-1,p,K}(n) \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ h_{0,p,1}(n) & \tilde{h}_{0,p,1}(n) & \cdots & h_{0,p,K}(n) & \tilde{h}_{0,p,K}(n) \end{bmatrix}. \tag{4.81}$$

With FIR channels, $L+1$ channel blocks are non-zero. Segment-specific channel matrices $\underline{\underline{H}}_p$ are defined by stacking the $L+1$ non-zero channel blocks as follows :

$$\underline{\underline{H}}_p = \begin{bmatrix} \underline{\underline{h}}_p(L) \\ \vdots \\ \underline{\underline{h}}_p(0) \end{bmatrix}. \tag{4.82}$$

The MP samples received during the n-th period of length PT' are given by

$$\underline{r}_0^{P-1}(n) = \overline{\underline{\underline{\mathcal{H}}}} \begin{bmatrix} \underline{I}_0^{P-1}(n) \\ \underline{I}_{P-L}^{P-1}(n-1) \end{bmatrix} + \underline{n}_0^{P-1}(n) \tag{4.83}$$

where the global channel matrix $\overline{\underline{\underline{\mathcal{H}}}}$ is of size $MP \times 2K(P+L)$. This matrix is built with the $2KP$ composite channels as shown in Figure 4.7. Because of the long code, it is not a block-Toeplitz matrix any more. For very long codes, i.e. $P \gg L$, this matrix is sparse as only a few blocks around the

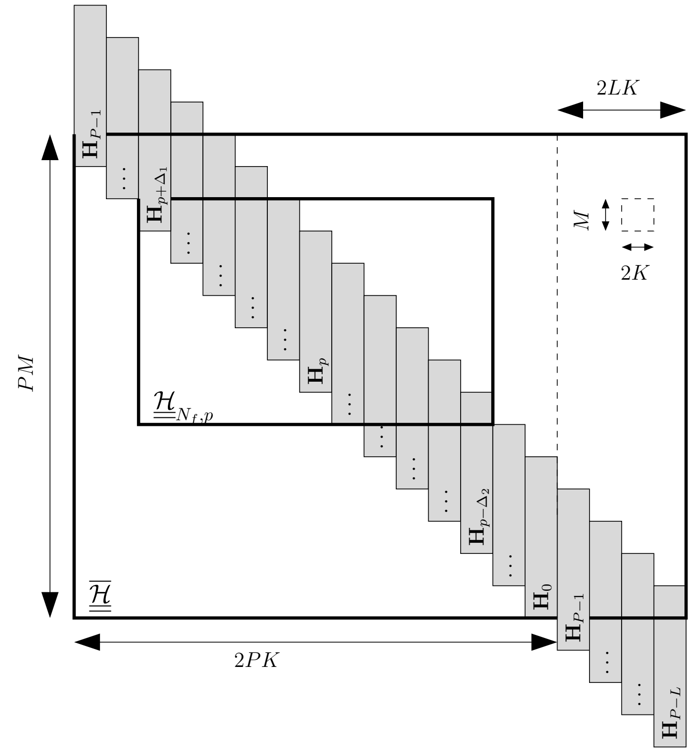

Figure 4.7: *Structure of the global channel matrix*

main diagonal are nonzero. This reflects the fact that only a few symbols $I_{p,k,q}(n)$ have a contribution to a given received block $\underline{\mathrm{r}}_p(n)$. It is useful to propose a shorter-window observation model :

$$\underline{\mathrm{r}}_{p-n_2}^{p+n_1}(n) = \underline{\underline{\mathcal{H}}}_{N_f,p}\,\underline{\mathrm{I}}_{p-n_2-L}^{p+n_1}(n) + \underline{\mathrm{n}}_{p-n_2}^{p+n_1}(n) \tag{4.84}$$

where the segment-dependent channel matrix $\underline{\underline{\mathcal{H}}}_{N_f,p}$ is the size $MN_f \times 2K(N_f+L)$ submatrix of $\overline{\underline{\underline{\mathcal{H}}}}$ centered around $\underline{\underline{\mathrm{H}}}_p$, as illustrated in Figure 4.7. Actually, the segment indices in Equation (4.84) should be defined modulo P. To keep the notation simple, the border effect, corresponding to a case where samples or symbols in blocks $n \pm 1$ would have to be used, has not been considered.

4.6 Cyclic-prefixed block transmission

An alternative to the continuous transmission schemes developed in the previous sections is the block-based multiple access. For a *baseband block transmission* associated with a given FB of size N, the chip-rate sequence transmitted by each user (see Equation (4.3)) becomes :

$$v_k(n) = \sum_{p \in \mathcal{C}_k} I_p\, s_p(n) \quad \leftrightarrow \quad \underline{\mathrm{v}}_k = \underline{\underline{\mathrm{S}}}_k\, \underline{\mathrm{I}}_k, \tag{4.85}$$

where the signature matrix $\underline{\underline{\mathrm{S}}}_k$ is of size $N' \times N_k$ and contains the length-N' signature codes allocated to user k as its N_k columns. Everything happens as if a single block of N_k symbols was transmitted by each user. The next block of symbols can be transmitted after a short idle time of duration νT_c, with the condition that successive blocks do not interfere with each other in the receiver. As the channel impulse responses $\overline{c}_k(t)$ have a length of $L_c T_c$, the duration of the inter-block interval must satisfy :

$$\nu \geq L_c. \tag{4.86}$$

The baud rate associated with the N sequences of symbols $I_p(n)$ is now $1/T_b = \frac{N}{N+\nu} 1/T$. A systematic penalty of $\frac{N}{N+\nu}$ is thus associated with the bit rate obtained with block-based transmission. That is the price to pay in order to suppress the inter-block interference in the receiver.

Rather than transmitting nothing during the inter-block interval, the cyclic prefix (CP) technique can be used to cyclically extend the transmitted sequence $\underline{\mathrm{v}}_k$:

$$\underline{\mathrm{v}}_k^{\nu} = \left(\begin{array}{c|c} \underline{\underline{0}}_{\nu \times N'} & \underline{\underline{\mathrm{E}}}_{\nu} \\ \hline \multicolumn{2}{c}{\underline{\underline{\mathrm{E}}}_{N'}} \end{array} \right) \underline{\mathrm{v}}_k \tag{4.87}$$

where $\underline{\underline{\mathrm{E}}}_x$ denotes a unit matrix of size x. After propagation over the frequency-selective channels, the received signal is sampled at the chip rate $1/T_c$. The received sequence is obtained by linear convolution of the transmitted sequences $v_k^{\nu}(n)$ and the channel response $c_k(n) = \overline{c}_k(nT_c)$:

$$r(n) = \sum_{k=1}^{K} c_k(n) \otimes v_k^{\nu}(n) + n_(n). \tag{4.88}$$

This can be expressed in matrix form as follows :

$$\underline{\mathrm{r}}^{\nu} = \sum_{k=1}^{K} \underline{\underline{\mathcal{C}}}_k^{\nu} \underline{\mathrm{v}}_k^{\nu} + \underline{\mathrm{n}}^{\nu} \tag{4.89}$$

where the $(N'+2\nu)\times(N'+\nu)$ matrix $\underline{\underline{\mathcal{C}}}_k^{\nu}$ is a *Toeplitz matrix* generated with the samples $[c_k(\nu),\cdots,c_k(0)]$, appended by zeros. After removal of the cyclic prefix, the $N'\times 1$ received vectors are finally :

$$\begin{aligned} \underline{\mathrm{r}} &= \left(\underline{\underline{0}}_{N'\times\nu} \;\middle|\; \underline{\underline{\mathrm{E}}}_{N'} \;\middle|\; \underline{\underline{0}}_{N'\times\nu} \right) \underline{\mathrm{r}}^{\nu} && (4.90)\\ &= \sum_{k=1}^{K} \underline{\underline{\mathcal{C}}}_k \underline{\mathrm{v}}_k + \underline{\mathrm{n}} && (4.91)\\ &= \sum_{k=1}^{K} \underline{\underline{\mathrm{H}}}_k \underline{\mathrm{I}}_k + \underline{\mathrm{n}} && (4.92)\end{aligned}$$

where $\underline{\underline{\mathrm{H}}}_k = \underline{\underline{\mathcal{C}}}_k\,\underline{\underline{\mathrm{S}}}_k$ and the $N'\times N'$ matrix $\underline{\underline{\mathcal{C}}}_k$ is now a *circulant matrix.* The effect of the cyclic prefix is thus to turn the linear convolutions associated with time dispersive channels into circular convolutions. This principle is already exploited in the well-known multicarrier modulation technique based on the DFT filter bank. Actually, it can be extended to any type of FB, including single carrier modulation [20]. The circulant matrices $\underline{\underline{\mathcal{C}}}_k$ are known to have the following eigenvalue decomposition :

$$\underline{\underline{\mathcal{C}}}_k = \underline{\underline{\mathrm{W}}}_{N'}\,\underline{\underline{\Lambda}}_k\,\underline{\underline{\mathrm{W}}}_{N'}^{H} \qquad (4.93)$$

where $\underline{\underline{\mathrm{W}}}_{N'}$ is the orthonormal DFT matrix of size N' and $\underline{\underline{\Lambda}}_k$ is a diagonal matrix formed with the values of the DFT of the channel impulse responses $c_k(n)$.

4.7 Conclusion

A general framework for the uplink of a PLC system was introduced in this Chapter. A large number of parallel subchannels are implemented through the modulation of N mutually orthogonal signatures by independent sequences of multilevel information symbols. The signature set can be represented by a filter bank. Classical multiple access methods (OFDMA, CDMA, TDMA) can be obtained by selecting the appropriate FB. Actually, any kind of FB can be investigated, leading to an infinity of multiple access systems. In this work, we restrict our attention to the class of paraunitary FBs.

For a given FB, the number of signatures is assumed to be much larger than the number of users. Each user gets a subset of signatures, according to a subchannel allocation algorithm. The allocation algorithm allows the spectral shaping of the transmitted signals as a function of the user-specific channel responses, with a maximum of flexibility. The amount of information in each subchannel is adapted to the signal to noise ratio measured at the output of the receiver.

IIR and FIR observation models have been established, relating the received signal at the output of the noisy multiuser channel, to the different sequences of information symbols. Practical joint detectors will be derived in Chapter 5 on the basis of these observation models.

Finally, three variants of the initial scheme have been proposed. The first one, CAP-FB transmission, provides a passband multiple access system (which is required for PLC), but without the need for a carrier. The second one, CAP-CDMA with long codes, represents an alternative to the signature allocation algorithm : the signature corresponding to a given subchannel changes from one symbol to another, which provides an identical level of performance for all signatures. Finally, cyclic-prefixed block transmission, which is classically restricted to OFDM, has been extended to the general FB transmission scheme.