Fast Spectral Iterative Technique for the Analysis of Planar Metasurface-type Structures

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***Abstract*—A fast iterative algorithm based on the Krylov subspace is proposed to simulate the current distribution on large printed structures. The acceleration is obtained by computing the matrix-vector product series appearing in the formulation in the spectral domain and by making efficiently use of the Contour-FFT transformation. The resulting algorithm is then very robust, i.e. can be used for any kind of printed structures. The numerical analysis of modulated metasurface antennas based on the proposed method is presented and compared with a pre-validated algorithm, making use of the concept of equivalent "Impedance Boundary Condition" to directly get the current distribution.**

***Index Terms*—Method-of-Moments (MoM), spectral domain methods, metasurfaces (MTSs), fast numerical method, contour-FFT.**

I. Introduction

Many electromagnetic systems require the analysis of large printed element structures. In antennas engineering for example, a full wave simulation of antennas arrays [1], [2], reflectarrays [3], and recently metasurfaces [4] is of great interest in the design process.

A typical metasurface antenna is made of several thousands of sub-wavelength scatterers printed on a substrate [4], implementing an Impedance Boundary Condition (IBC). Fast full-wave methods using full-domain basis functions have been proposed recently to analyze the equivalent IBC distribution [5], [6], [7]. However, a simulation of the physical patches obtained at the final design step is still important in order to validate the synthesis process (mapping between an equivalent IBC and the corresponding physical patches). This simulation is still nowadays a very challenging task. Indeed, the metasurfaces often display tiny unit cells with fine details, and their size typically corresponds to several wavelengths. This leads to a very fine mesh with a large number of unknowns (more than 10^6), which cannot be readily tackled by straightforward methods such as the Method-of-Moments (MoM).

In most practical metasurfaces, the unit cells are very similar to each other, with only the orientation and a small scale factor of the patches changing from one cell to another: that is, the cells exhibit some (quasi) symmetries [8]. Together with their small electrical size, this allows a drastic reduction of the number of unknowns by exploiting symmetries and using higher-order basis functions such as Macro Basis Functions (MBF). In [4], the authors use a combination of the Sparse-Matrix-Adaptive Integral Methods (SM-AIM) [9] and the Synthetic Functions eXpansion method (SFX) [10] to solve the problem iteratively, while constructing appropriate MBFs. The authors of [11] propose a class of basis functions defined on elliptical patches, which take into account the rotation and the axial ratio of the ellipses. The closed form spectrum of these basis functions allows an efficient formulation of the Method-of-Moments (MoM).

However, all these methods cannot be used when no symmetries exist at the level of the printed patches or between different cells. Such a simulation tool should improve the quality of the synthesis by mixing different types of patches. This paper propose the use of the recently developed iterative method based on the Contour-FFT [15] to analyze arbitrary metasurfaces.

The paper is structured as follows. Section II recalls the Contour-FFT method. Section III explains the proposed Iterative technique. Finally, Section IV and V present the designed MTS and some numerical results.

II. The Contour-FFT operator

The Contour-FFT [12] is a recent method that allows the fast and accurate computation of Fourier transforms of singular functions. Starting from the singular function $\widetilde{H}$ in the spectral domain, its Fourier transform is expressed as

$$H(x,y) = \frac{1}{4\pi^2} \iint \widetilde{H}(k_x,k_y)\, e^{-j(k_x x + k_y y)}\, \mathrm{d}k_x\, \mathrm{d}k_y \quad (1)$$

Due to rapid variation near the spectral poles of $\widetilde{H}$, using a simple FFT to evaluate (1) will lead to large numerical errors. To smoothen the integrand, one can modify the integration contour to push the integration path further from the poles (see Fig. 1).

As proven in [12], the Fourier transform expressed in terms of the real spectral variables is written

$$H(x,y) = \frac{1}{4\pi^2} \iint \widetilde{H}(k_x^c,k_y^c)\, e^{-j(k_x x + k_y y)} e^{\left(k_x^i x + k_y^i y\right)} C\left(k_x,k_y\right)\, \mathrm{d}k_x\, \mathrm{d}k_y \quad (2)$$

with k_x, k_y and k_x^i, k_y^i the real and imaginary parts of the complex wavenumbers k_x^c, k_y^c, respectively, and C a function that depends on the chosen contour. Note that the real exponential term in (2) contains both spectral and spatial

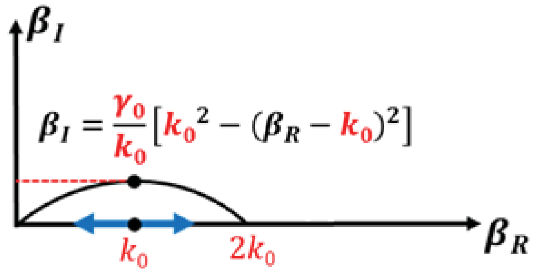


Fig. 1. Quadratic integration contour. Note that the imaginary part is expressed as a function of the real part.

variables; this expression is thus not strictly speaking a Fourier transform, and cannot be evaluated with an FFT. To preserve the advantageous complexity of the FFT, that exponential term is expanded into a two-dimensional Taylor series of (x, y) around $(0, 0)$. By rearranging the terms the following expression is obtained

$$\begin{aligned} H(x,y) \approx K' \Big[& \mathrm{FFT}\left\{\widetilde{H}'\right\} \\ & + x\mathrm{FFT}\left\{\gamma k_x \widetilde{H}'\right\} + y\mathrm{FFT}\left\{\gamma k_y \widetilde{H}'\right\} \\ & + \frac{x^2}{2}\mathrm{FFT}\left\{\gamma^2 k_x^2 \widetilde{H}'\right\} \\ & + xy\ \mathrm{FFT}\left\{\gamma^2 k_x k_y \widetilde{H}'\right\} \\ & + \frac{y^2}{2}\mathrm{FFT}\left\{\gamma^2 k_y^2 \widetilde{H}'\right\} + \dots \Big] \\ & \triangleq \mathrm{CFFT}\left\{\widetilde{H}\right\} \end{aligned} \tag{3}$$

which is the definition of the Contour-FFT operator.

The formulation (3) is a rapidly converging series of FFTs. Thus it combines the attractive $N \log N$ complexity inherent to the FFT, together with a robust treatment of singularities thanks to the integration contour deformation. An inverse Contour-FFT operator has also been defined in a similar way, which evaluates Fourier transforms in complex spectral coordinates.

III. The Contour-FFT iterative method

The solution of the MoM system of equations

$$\mathbf{Zx} = -\mathbf{b} \tag{4}$$

can be approximated as a linear combination of the columns of

$$\mathbf{Q}_n = \begin{bmatrix} \mathbf{b} & \mathbf{Zb} & \mathbf{Z}^2\mathbf{b} & \dots & \mathbf{Z}^{n-1}\mathbf{b} \end{bmatrix} \tag{5}$$

spanning the Krylov subspace, and which is iteratively constructible by successive products with the interaction matrix $\mathbf{Z}$. Candidate solutions for x are then computed iteratively via the reduced matrix $\mathbf{Q}_n$ using the GMRES procedure described in [13].

The matrix-vector products appearing in (5) constitute the complexity bottleneck of such iterative methods. A large spectrum of methods initiated by [14] use the Fast Fourier Transform (FFT) to lower the complexity of that operation. We will summarize here the building blocks of one of these FFT-based iterative methods, that is the Contour-FFT [15].

First the rooftop or Rao-Wilton-Glisson (RWG) [16] basis functions are distributed using a Lagrange interpolation scheme, on a cartesian grid of points that covers the whole domain of the problem. At each iteration, the current distribution is thus distributed on the grid and its convolution with the Green's function (GF) is computed in the spectral domain using the Contour-FFT and its inverse. The key point of the Contour-FFT method is that the spectral GF $\widetilde{\mathbf{G}}$ used in the convolution is analytical for any layered media. The fields obtained that way are tested on each testing function in the spatial domain. These operations can be summarized as

$$[\mathbf{Zb}]_i \approx \iint \hat{\mathbf{f}}_i \cdot \mathrm{CFFT}\left\{\widetilde{\mathbf{G}}\ \mathrm{CFFT}^{-1}\left\{\sum_{j=0}^{N} b_j \hat{\mathbf{f}}_j\right\}\right\} \mathrm{d}\mathbf{r} \tag{6}$$

with $\hat{\mathbf{f}}_i$ the i-th basis function distributed on the grid, b_j the j-th entry of the excitation vector $\mathbf{b}$, $\widetilde{\mathbf{G}}$ the analytic dyadic GF in the spectral domain. Finally, a correction operation is carried out to compensate for inaccurate short-range interactions, using a pre-computed sparse operator [17].

The resulting method is quite accurate and scales well with the number of unknowns [15]. Also, error models are readily available to determine the size of the grid, the shape of the integration contour and the order of the Taylor expansion, which is a convenient way to control the error and the complexity of the overall method [18], [19].

IV. Isotropic metasurface antenna

The analyzed structure is a broadside radiating isotropic MTS antenna similar to the one described in [4]. The impressed impenetrable IBC modulation is defined as:

$$Z_+(\vec{\rho}) = jX_0\ [1 + M_0\ \sin(2\pi\rho/d - \phi)] \tag{7}$$

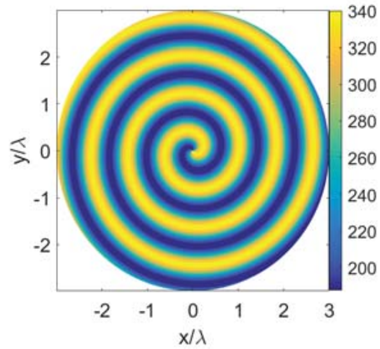
with: $X_0 = 0.71\ \eta_0$; η_0 is the free-space impedance; $M_0 = 0.3$; the dielectric relative permittivity $\epsilon_r = 3.66$; the dielectric thickness $h = 1.524$ mm; $d = \lambda_0/\sqrt{1 + (X_0/\eta_0)^2}$; and $a = 3\lambda$ is the antenna radius. The operating frequency is set at 17 GHz.

The synthesis of the IBC distribution has been carried out using square patches, with the locally periodic assumption as described in [8]. For such a geometry, the equivalent IBC of the infinite array of patches can be approximated in closed form [20], [21] as:

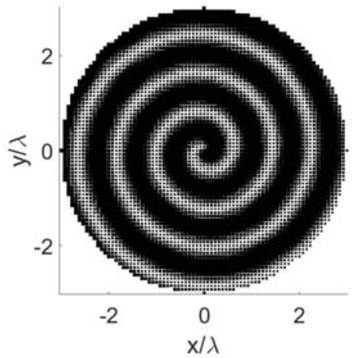
$$Z_s(w, k_t) = \frac{j\eta_0 k_{z1} \tan(hk_{z1})}{k\epsilon_r - k(\epsilon_r + 1)\delta k_{z1} \tan(hk_{z1})} \tag{8}$$

with $k_{z1} = \sqrt{k^2\epsilon_r - k_t^2}$; $\delta = (a/\pi)\ln[1/\sin(\pi w/2a)]$, k_t is the wavenumber transverse to $\hat{z}$; $\hat{z}$ is the unit vector normal to the MTS; k is the free-space wavenumber; and w is the spacing between two adjacent patches. Solving the corresponding TM dispersion equation transverse to $\hat{z}$ (resonance condition) allows the determination of the surface wave wavenumber k_t, and then the corresponding surface impedance.

The final structure is obtained by discretizing the MTS aperture with a rectangular grid made of square cells of size $a = \lambda/13$. In each cell, the side of the square patch is calculated in order to match the desired IBC (7). The IBC modulation and the obtained structure, implementing this IBC are represented in Fig 2.



(a)



(b)

Fig. 2. Synthesis Metasurface.(a): Impedance modulation, (b): Final metasurface implementing the impedance modulation.

V. Numerical results

First, the proposed method is validated with a classical MoM code. For this purpose, we analyze a smaller version (radius equal λ_0) of the MTS described in Fig. 2, as this MTS can not be analyzed with a standard MoM on our computer. The structure is meshed with 17152 Rao-Wilton-Glisson (RWG) basis functions. The brute-force MoM solution took more than 8 hours to complete on a regular desktop computer. The proposed iterative method, with a grid of 256×256 points and a Taylor order of 3, ran for 104 minutes. Fig. 3 shows the co-polar pattern, comparing our method to the standard MoM. One can observe an excellent agreement between simulated radiated fields. We also observed an average error of -33 dB on the currents obtained with the MoM and with the Contour-FFT method, which corresponds to the targeted accuracy (-30 dB).

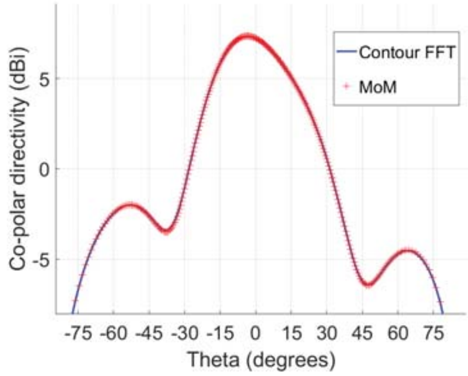
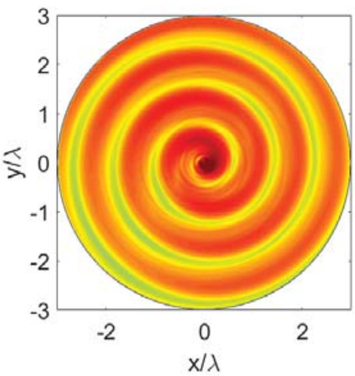
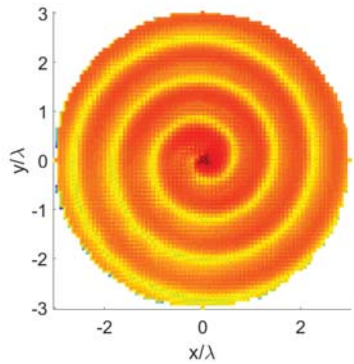


Fig. 3. Co-polar pattern of the smaller version (radius equal λ_0).

Now, we analyze the designed MTS (of radius $3\lambda_0$) presented in Fig. 2 with the Contour-FFT method and compare it with the IBC simulation. This structure is meshed with 154868 RWG basis functions, which is too large for a standard MoM on our desktop computer. The absolute value of the current distribution for both approaches is represented in Fig. 4. We can observe the same trend for both methods.



(a)



(b)

Fig. 4. (a): Current distribution obtained with IBC simulation, (b): Current distribution obtained with the Contour-FFT method on the designed MTS.

The obtained co-polar pattern is illustrated in Fig. 5. One can see a good representation of the main lobe and a preservation of the side lobes level. However, the secondary lobes are not well represented. This difference can be corrected by a more rigorous synthesis. Indeed, expression (8) is an approximation. A rigorous method should consist of extracting the MTS impedance from an infinitely periodic Method-of-Moment simulation, making use of the local periodicity assumption, as described in [22], [23].

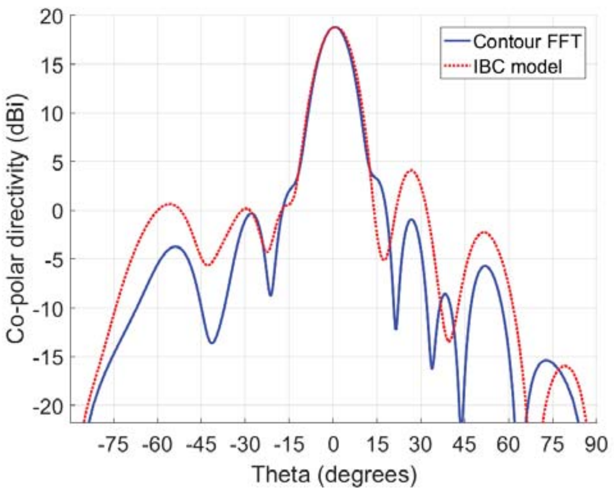


Fig. 5. Co-polar pattern of the designed MTS.

It is important to note that the method is implemented as a combination of MATLAB and C++ codes [15], sharing information through a very simple file-based protocol. We observe that this has a huge influence on the computation of the sparse interaction matrix used for the preconditioner and precorrectionner. This operation is the most time-consuming in the whole process. Indeed, we estimated that the overall simulation time could be divided by at least a factor 3 by improving this protocol.

VI. Conclusion

We proposed to use the recent Contour-FFT iterative method to simulate planar metasurfaces (MTSs). The results from our method is compared with traditional MoM. The algorithm is quite accurate and compares well with the MoM. Discrepancies between realized MTS and ideal IBC emphasize the need for an accurate full-wave solver in the design process and a rigorous implementation of the impressed modulation.

Simulation times with the proposed method are still large compared to other methods, specially designed for metasurfaces. However the Contour-FFT iterative method is designed for arbitrary structures. That is, it should be able to perform just as well with unit cells of very different shapes and dimensions, which other methods could not manage.

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