Smart abstraction based on iterative ellipsoidal covering

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1 Abstract

Classic abstraction-based techniques consist of discretizing both the input space and the state space and in general, this discretization is done with hyperrectangles [1]. One problem with discretizing the input space is that one needs a growth-bound on the error between the actual state and the quantized state which is used to take into account the discretization error. This yields an over-approximation of the image set of a cell, which increases the level of non-determinism in the symbolic system. Instead, we propose to build an abstraction based on an ellipsoidal covering of the state space, a finite set of local affine controllers, and a lazy/iterative construction of the abstraction. One benefit of this approach is the fact that the state and input spaces need not be discretized and the symbolic-input space is reduced to a finite set of controllers. Moreover, when ellipsoidal cells and affine controllers are considered, it is possible to compute an optimal controller via the solution of a semi-definite program (SDP). Another advantage is that the covering is computed smartly and even designed with the help of classic controller design techniques, unlike classical ones, which use an *a priori* defined, suboptimal, and prone to the curse of dimensionality, grinding approach. To achieve this goal we combine and extend the ideas of the following three papers: [2], [3] and [4].

In the paper [2], the authors provide a strategy to lazily (i.e., postponing heavier numerical operations) build a smart abstraction for a discrete input space. However, in [2], this strategy was based on the use of a growth-bound function on the error between the state and the quantized state. A major drawback is that in the absence of incremental stability, it gives an over-approximation which increases the level of non-determinism in the symbolic system, eventually making the abstract-control synthesis impossible.

More recently, in paper [3], pre-defined ellipsoids covering the state space were used to build a deterministic abstraction and local affine feedback controllers between nearby cells are calculated to define the abstract transitions. This method of calculating transitions requires more computational effort than other methods (such as those based on growth-bound functions used with a discretized input space), but the advantage is that the abstraction is a weighted digraph, instead of a hypergraph. In addition, the state-dependent nature of the controllers allows the use of larger discretization cells, reducing the complexity of the abstraction. To leverage the optimal control solution proposed in [3] while building a smart abstraction, we must design not only the transitions but also the positioning and shape of the ellipsoids along the optimal path, similarly to [2].

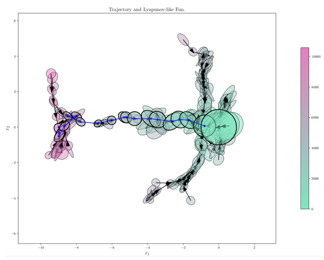

Figure 1: Result of our main algorithm. Ellipsoidal cells and controllers performing transitions among them are simultaneously designed by an RRT-based algorithm. The color-map represents the value of the Lyapunov-like function (i.e., an upper bound on the cost to reach the target from a given state).

To achieve this goal, we propose a strategy using Rapidly-exploring Random Trees (RRT). Building a smart abstraction based on the ellipsoidal cells requires solving ellipsoidal inclusion subproblems, which can be done efficiently by leveraging the results from [4]. An example of the execution of the algorithm is shown in Figure 1: a random tree of cells and the values of a Lyapunov-like function valid inside each of them. This value is also an upper-bound on the worst-case cost for taking any state in a given ellipsoid towards a given goal set.

References

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