

Split extension classifiers in the category of cocommutative Hopf algebras

Marino Gran*

Université catholique de Louvain

The category $\mathbf{Hopf}_{K,coc}$ of cocommutative Hopf algebras over an arbitrary field K has been recently shown to be semi-abelian [3, 5], extending the classical theorem by Takeuchi saying that the category of commutative and cocommutative Hopf algebras is abelian. The category $\mathbf{Hopf}_{K,coc}$ is also action representable in the sense of [2], so that the categorical notions of center, centralizer and commutator can be explored in this category. In particular, we shall explain that the categorical notion of center in $\mathbf{Hopf}_{K,coc}$ turns out to coincide with the usual notion of center from Hopf algebra theory [1]. When K is an algebraically closed field of characteristic 0 it is possible to give an explicit description of the *split extension classifier* of a cocommutative Hopf algebra [4]. This universal construction can be seen as the natural counterpart in $\mathbf{Hopf}_{K,coc}$ of both the automorphism group of a group in the category $\mathbf{Grp}$ and of the algebra of derivations in the category $\mathbf{Lie}_K$ of Lie algebras over K .

References

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