

Three Essays on Optimal Fiscal Policy

Guillermo Santos

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Composition du Jury:

Rigas Oikonomou	(UCLouvain, Promoteur)
Francesca Monti	(UCLouvain)
Elisa Faraglia	(University of Cambridge)
Jochen Mankart	(Deutsche Bundesbank)
Romanos Priftis	(European Central Bank)

Président du Jury:

Francesca Monti (UCLouvain)

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Introduction

As a response to the Great Recession and the COVID crisis governments around the world put in place important fiscal stimulus raising quickly the level of debt. This upward trend observed in the debt level raised some concerns on how sustainable fiscal plans are over time. In this context, analysing how fiscal policy should be designed to enhance welfare while respecting the constraints imposed by debt sustainability gained some importance in the economics debate. This thesis analyses *optimal fiscal policy* in three different scenarios that are relevant from the perspective of debt sustainability: the presence of endogenous default risk and household heterogeneity, passive monetary policy and high inflation; and the “new” role for government bonds as a source of liquidity to households.

In the first chapter, I investigate the welfare properties of a model with sovereign default and household heterogeneity. Interestingly, the sovereign default problem followed immediately the effects of the Great Recession in many different countries because the rapid surge in debt raised some doubts on repayment. Within the context of the European Union a government declaring default not only affects foreign investors but also domestic agents, so the theoretical framework analyzed by [Arellano \(2008\)](#) or [Eaton and Gersovitz \(1981\)](#) were extended to include domestic default. Modelling domestic default requires theories explaining the incentives of a government to default on its own citizens. Alternative explanations such as wealth and income inequality as in [D’Erasmus and Mendoza \(2016\)](#) were considered to explain why governments may default on its own residents.

In the model, considered firstly by [D’Erasmus and Mendoza \(2016\)](#), heterogeneous households build precautionary savings by acquiring government debt subject to default risk. On the other side, the government may default on its liabilities to redistribute resources across unequal households. Within this model, I show that the competitive equilibrium allocation is, in general, *constrained inefficient* due to market incompleteness. As observed in similar models with heterogeneity such as [Davila et al. \(2012\)](#) or [Gottardi et al. \(2016\)](#), households chooses savings in a self interest manner not internalizing the effect of their own decisions on the aggregates giving birth to externalities. The presence of those externalities in combination with default risk creates inefficient low levels of domestic debt which incentivizes the government to dictate default more often in the decentralized economy.

Along with the debate on default risk, some theories on the role of inflation to achieve debt sustainability had a revival due to the expansive monetary policies and the recent surge in inflation. From the perspective of the Fiscal Theory of the Price Level (FTPL), when public debt is nominal

and taxes are not perceived to ensure debt repayment, inflation increases to reduce the value of debt (see [Cochrane \(2001\)](#), [Sims \(2011\)](#), among others). [Leeper \(1991\)](#) called this regime active fiscal/passive monetary policy because monetary policy reacts weakly to inflation while taxes does not strongly adjust in response to debt issuance. In standards DSGE models where fiscal policy follows ad hoc rules (as in much of the FTPL), the main macro aggregates shows considerable levels of volatility in the passive monetary policy regime due to high inflation and lower taxes.

However, I show in the second chapter that much of the volatility observed in the passive monetary policy regime is due to fiscal policy following ad hoc rules. If fiscal policy can be optimally chosen by a planner, I show quantitatively there is little difference in the optimal allocations when comparing the active and passive monetary policy regimes. To do so, I set up a standard DSGE model enriched with a large fiscal block and let monetary policy to follow a Taylor rule in which the interest rate reacts weakly to inflation. Later on, I compare the optimal allocations with its polar opposite, in which monetary policy reacts strongly to changes in inflation. Indeed, if the tax rate can be optimally set by a planner, the importance of inflation for debt repayment is considerably reduced and taxes ensures fiscal solvency.

In some economies, the increase in public debt was jointly observed with low interest rates and slow economic growth, yet it was not linked to a high risk of default. To explain this scenario, early works in finance such as [Vayanos and Vila \(2021\)](#), [Krishnamurthy and Vissing-Jorgensen \(2012\)](#) or [Greenwood et al. \(2015\)](#) analyzed the presence of a demand for government debt due to some services government bonds provides to households beyond being a store of value. According to this literature, government debt can be used as a collateral device or as an extra source of liquidity. A relatively simply and understandable way of capturing such property of public debt is by including government bonds in the utility function (as in [Krishnamurthy and Vissing-Jorgensen \(2012\)](#), [Michaillat and Saez \(2021\)](#), among others). When doing so, an endogenous “liquidity premium” arises as a price for the household to pay when holding government bonds over other less liquid assets such as physical capital. As a counterpart for providing those services, the government collects additional revenues which helps to finance the fiscal gap and sustain higher levels of public debt ([Reis \(2022\)](#)).

While this may be seen as a positive scenario for the government, the environment encloses challenges for debt management and fiscal policy. As early identified by [Angeletos et al. \(2020\)](#), issuing new debt means lower collections for the government due to a lower “liquidity premium” which creates tensions for a planner intending to smooth taxes. The third chapter investigates precisely those tensions when government debt enters in the utility function as in [Angeletos et al. \(2020\)](#), but in an environment with sticky prices. The presence of nominal rigidities allows the planner to use distortionary inflation along the business cycle as an extra tool to manage the effect of macroeconomic shocks. When confronted with an unexpected increase in government spending, I show the planner uses inflation to tax wealth at period zero to absorb most of the shock while tax distortions are frontloaded. This is result of the reluctance of the planner to use debt, because debt issuance depress the liquidity premium reducing government revenues and tightening the budget constraint. This behavior departs from the tax smoothing and “near zero” volatility of inflation observed in environments with non-state contingent debt and sticky

prices, such as [Schmitt-Grohé and Uribe \(2004\)](#). Moreover, as a consequence of the high distortionary taxes and inflation, the crowding-out of consumption and investment is higher when households derive utility from government bonds. This is a strike difference from models using non-distortionary taxes or non optimal taxes to study the effect of government expenditure shocks under such preferences (see [Rannenberg \(2021\)](#) or [Bonam \(2020\)](#)).

Chapter 1

Constrained Efficiency, Household Heterogeneity and Sovereign Default

This paper analyzes the normative implications of a model with uninsurable idiosyncratic risk, domestic and external sovereign debt and default. More precisely, it investigates the *constrained efficient* allocation, according to which the planner is constrained by market structure, but is able to correct for potential externalities arising from decentralized equilibrium household choices.

We show analytically that the market outcome in this model is in general *constrained inefficient*. When households do not consider the effects that own savings choices have on aggregates, externalities emerge affecting the bond price and the default propensity. A quantitative two-period model shows that domestic debt is lower in the competitive equilibrium than in the social optimum when income risk is low and debt is risky. When income risk increases, domestic debt also increases due to a precautionary savings motive, but remains lower in the competitive equilibrium than in the constrained planner.

Moreover, for lower levels of wealth inequality, aggregate debt is lower in the competitive equilibrium mainly due to lower levels of domestic debt. The gap between the efficient equilibrium and the market economy, however, shrinks with wealth inequality and total debt becomes higher in the competitive equilibrium for higher levels of wealth inequality. As a consequence of the externalities, the relatively lower levels of domestic debt observed in the competitive equilibrium makes the government to exercise the default option more frequently than what would be socially desirable.

Keywords: sovereign default; foreign and domestic debt; household inequality; externalities.

1.1 Introduction

A recent literature (e.g. [D’Erasmus and Mendoza \(2016\)](#), [Jeon and Kabukcuoglu \(2018\)](#), or [Ferriere \(2014\)](#) among others), has explored on the relationship between household heterogeneity and sovereign default risk. According to these models, when households are heterogeneous and use public debt as a vehicle for self-insurance against income shocks, debt default can be considered by the government as a tool for wealth and income redistribution. While most of the papers in this recent literature perform a positive evaluation of the policy tradeoffs emerging in economies with sovereign default and heterogeneity, this paper takes a normative approach in analysing optimal policy in this class of models.

In particular, I explore the constrained efficiency of the equilibrium allocation in a model with incomplete financial markets, heterogeneous households and government default. Following [Diamond \(1967\)](#), I define the constrained efficient optimum as the solution to an optimal policy program in which a fictitious planner is constrained by the structure of financial markets, but is able to correct potential externalities arising from decentralized household choices. Under incomplete markets the competitive equilibrium allocation may be different than the optimal policy allocation in which households (and the policy authority) face the same market structure but internalize the aggregate impact of their optimal choices.

For tractability, I restrict the analysis to a two-period economy. However my model features heterogeneous households, and both domestic and external default and so it is a rich environment in crucial dimensions. I show that generically the competitive equilibrium allocation is *constrained inefficient* and debt can be either too high in the competitive equilibrium or too low depending on the specification of the models parameters, most notably the degree of heterogeneity in income and wealth.

The competitive equilibrium allocation in my model where default is optimally chosen by the government is defined as follows: The benevolent government exercises default on domestic and foreign debt optimally, taking into account the full set of household optimality conditions in competitive asset markets. Crucially, this means that the government is constrained by the Euler equations according to which households price domestic debt. In contrast, in the efficient policy, the government is not constrained by the Euler equations, being instead allowed to violate private saving rules from domestic households. In turn, it always respect the price implied by the foreign investor.

I first show analytically that the constrained optimal policy is characterized by a savings condition which is a modified Euler equation, featuring additional terms relative to the households privately optimal savings schedule. These terms emerge because in the competitive equilibrium households do not internalize the effects of their own savings decisions on the debt price and on the default policy. The magnitudes of these externalities depend on: i- the initial wealth distribution across agents; ii- the net foreign asset position of the economy; iii-the sensitivity of the bond price and default schedule to changes in domestic debt.

Moreover, the externalities turn out to affect through different channels the optimal quantity of

debt. Whereas the externality linked to the bond price implies a higher marginal cost of buying debt for the household, the one linked to the default propensity involves a higher marginal benefit. These two effects clearly reinforce one another implying, for most of the quantitative experiments lower levels of domestic debt in the competitive equilibrium. To counteract such a result, the government issues foreign debt and consequently dictates default more often in the private equilibrium.

The presence of the externalities in combination with risk averse domestic consumers leads to a second channel in which the higher default probability reduces the bond price, making government debt cheaper for residents. This effect incentivizes the demand for domestic debt to smooth consumption over time. Thus equilibrium debt levels depend on the relative strength of the two channels, which in turn are related to wealth and income inequality.

One key assumption for the presence of externalities besides heterogeneity is the inability of the government to select among foreign and domestic residents which prevents a complete market segmentation. This assumption, in line with most of the literature on default, can be justified by assuming that sovereign debt contracts have a sort of strict *pari-passu* clause in which the creditors have equal treatment by the debtor (Olivares-Caminal (2013)). It can also be sustained in the terms of Broner et al. (2010), in which a secondary market for sovereign debt would make impossible for the government to discriminate among its creditors ¹. As a consequence of non-selective default, sovereign debt has the same price across both foreign and domestic markets and the government is unable to distinguish between them. ²

The inability of the government to select among creditors gives birth to a similar externality in a representative agent economy, as first noted by Mallucci (2021). A representative domestic household will not internalize that, up to a certain threshold, higher levels of domestic debt reduces the probability of default because the government avoids to default in their own residents. I investigate how this externalities changes in the presence of household heterogeneity in the domestic market when the government uses the default option as a redistribution tool among residents.

A quantitative experiment from a two-period economy gives two main results. First, it suggests that the constrained efficient outcome can have lower or higher debt level and this hinges crucially on the level of wealth inequality. Second, the presence of the externalities leads unambiguously to ex-post higher default probabilities in the competitive equilibrium allocation. Regarding the first result, domestic debt is lower in the competitive equilibrium than in the constrained optimum for all levels of income risk when the government decides to issue risky debt. As a consequence, foreign debt and taxes are higher in the decentralized economy, leading to a higher default propensity in equilibrium.³ When idiosyncratic income risk increases, domestic debt raises in both, but it does so more in the competitive equilibrium since the precautionary savings motive (which is of course encapsulated by the household Euler equations) becomes stronger. As a consequence

¹With a secondary market for debt, domestic households can buy defaulted debt in hands of foreign investors making impossible for the government to ultimately distinguish among them

²Besides non-selective default, I also assume partial default is not available for the government. Despite partial default will allow the planner to complete the markets in a better way, the competitive equilibrium will still be inefficient, as we argue in section 1.12

³As one might expect a benevolent government, concerned for the well being of domestic residents, is more likely to default on foreign debt.

of the relatively larger domestic demand for debt, external debt falls, and the default option may be less frequently used. When households are faced with a more volatile income process, the demand for precautionary savings is stronger which incentivizes the demand for domestic debt both in the competitive equilibrium and in the constrained efficient allocation. The gap however, (the excess of debt in the constrained optimum) shrinks. This is so because the government exercises less frequently the default option and so the Euler equation constraints have a less severe effect on the optimal allocation in the competitive equilibrium.⁴

Eventually, for high enough levels of income risk, the demand for domestic debt is large enough so the government issues risk free debt. At this point, default does not occur as an optimal choice in equilibrium and both allocations display similar levels of debt. Of course, last effect is observed because government debt is priced through default probabilities, so externalities vanishes without risky debt, or they do not show quantitatively an important effect.

Moreover, for higher levels of wealth inequality, aggregate debt (the sum across domestic and foreign debt) is higher in the competitive equilibrium than in the constrained optimum. This is explained by the presence of the externalities themselves. The externality on the bond price and the default set creates incentives to underborrow, thus the government turn to foreign debt increasing the incentives for dictating default in the second period. On the other hand, a higher default probability, in turn reduces bond prices, incentivizing domestic demand for debt, especially for the wealthy household. When wealth inequality increases, the wealthy household acquires more debt to smooth consumption taking advantage of the lower price of the bond. At this point, default probability is driven not only by the foreign/domestic debt ratio but also due to wealth inequality, which is considerably high. The optimizing government then issues more debt in the competitive equilibrium to subsequently exercise the default option in the high spending state.

The central finding of the paper is that the presence of externalities due to market incompleteness, i.e. household heterogeneity, is a key component for debt composition, affecting the relative quantities of external and domestic debt, which ultimately affects default probabilities. Lower levels of domestic debt in the competitive equilibrium allocation induce the government to default more often than in the constrained efficient allocation, hence default is mostly external. In addition, the competitive equilibrium allocation shows higher levels of wealth inequality than the constrained optimum, leading the government to default more often, since default is a tool for wealth redistribution across domestic households.

1.1.1 Literature Review

The presence of pecuniary externalities in debt markets has been considered before in the literature. [Jeske \(2006\)](#) and [Wright \(2006\)](#) show the presence of externalities with state-contingent debt markets, when households can privately borrow from abroad in a decentralized manner. They show decentralized borrowing lead unambiguously to over-borrowing with respect to a centralized borrowing decision. [Kim and Zhang \(2012\)](#) studies a similar model to [Jeske \(2006\)](#)

⁴Intuitively private agents are less willing to hold domestic debt at higher default risk. Thus a lower incidence of default tomorrow 'slackens' the participation/ Euler constraints.

and Wright (2006), but considering incomplete markets and conclude that a model with decentralized borrowing and centralized default leads to overborrow and high default probabilities with respect to canonical model by Arellano (2008) in which borrowing and default decision are centralized.⁵ This paper, instead analyze efficiency in a model in which the government can borrow from domestic and foreign agents but it can also default on its debt motivated by redistribution within domestic households. In such a framework, households do not internalize the effect of its own savings choices on the price of the bond and default probability.

Mallucci (2021) analyzes a model with domestic and foreign debt in which the government may default on domestic households. The main result is that the internal/external mix of government debt matters for default decisions. In addition, Mallucci (2021) shows the presence of a pecuniary externality because the foreign and domestic markets for government debt are not segmented and domestic investors does not take into account the effect of their own choices on the bond price. In his model, this is a direct consequence of the inability of the government to dictate selective default. With selective default, the price of the bond would be different across both markets and the competitive equilibrium would remain efficient again. This paper adds an extra dimension to Mallucci (2021) by considering wealth heterogeneity within domestic investors which affects the size and direction of the externalities. In the model by Mallucci (2021), the competitive equilibrium is efficient if foreign debt is removed from the model, since it becomes the usual representative agent model in which debt is financed by lump-sum transfers. In this paper, foreign debt has two main purposes: one the one hand brings the model close to reality as most default episodes involves external debt. On the other hand, the presence of foreign investors helps to price government debt in absence of the Euler equation for domestic agents.

Because I use foreign investors to price debt in the efficient equilibrium, the inability of the government to discriminate among creditors is a key assumption for the result. With some exception like in Erce and Mallucci (2018), the presence of non-selective default is an assumption in line with most of the literature on sovereign debt markets (D'Erasmus and Mendoza (2016), D'Erasmus and Mendoza (2021), Bocola et al. (2019) among others) that can be justified by performing a strict interpretation of the *pari-passu* clause contained in some sovereign debt contracts meaning all investors should have equal treatment (Olivares-Caminal (2013)). Alternatively, the inability of the government to dictate selective default can be thought in the terms of Broner et al. (2010), in which a secondary market for debt would make impossible for the government to discriminate among creditors.

The constrained efficient allocation this paper is interested in follows the setup used in Davila et al. (2012) who studies constrained efficiency in the neoclassical growth model with uninsurable idiosyncratic income risk, without any government default option. In the neoclassical model, individual savings affects aggregate capital and change the equilibrium interest rates and wages. When agents save for precautionary purposes, the competitive equilibrium capital stock may be too high and a planner not constrained by savings schedules can target a lower level of capital to increase welfare upon the competitive equilibrium. Other work analysing the efficiency of

⁵The previous papers aimed to understand the 1980s Latin American debt crisis where some governments guaranteed foreign private debt and eventually defaulted on it

the competitive equilibrium allocation within the context of heterogeneous agent economies are [Gottardi et al. \(2016\)](#) and [Park \(2018\)](#). In this paper I consider a similar analysis but in an economy in which the price of debt depends on a default decision by the government.

The paper also contributes to the literature studying optimal debt and default policies in economies with heterogeneous agents. [D'Erasmus and Mendoza \(2021\)](#) and [D'Erasmus and Mendoza \(2016\)](#) shows that a benevolent optimizing government has incentives to default on debt to reduce income and wealth inequality. In this paper, we consider the normative side of such a model and study the presence of externalities usually present in economies with heterogeneous agents.

Finally, this paper is also related to the literature analysing the optimal mix between foreign and domestic debt. [Guembel and Sussman \(2009\)](#) explores the political economy of a domestic default to conclude that default is less likely when the domestic debt level is larger. [Sunder-Plassmann \(2018\)](#) explains the choice between foreign and domestic debt in a sovereign default model with inflation and debt denominated in local and foreign currency while [Ottonello and Perez \(2019\)](#) finds government issues larger levels of debt in foreign currency due to the lack of commitment to its monetary policy. In this paper, the optimal combination between domestic and foreign debt, depends on externalities arising in the model due to market incompleteness.

The rest of the paper proceeds as follows: next section describes the model economy. Section 2 describes optimal policies for a competitive equilibrium and the constrained planner problem, showing the main analytical results and comparing the result in this paper with the representative agent case. Section 3 presents a quantitative analysis for both allocations, while section 4 describes the result of different sensitivity analysis with respect to key parameters of the model. Section 5 concludes.

1.2 The economy

This section describes the model economy. The model is similar to the two period model considered [D'Erasmus and Mendoza \(2016\)](#), the main difference being that here I will assume that agents face idiosyncratic labour income risk in the second period. The focus is also different since I am interested in comparing the competitive equilibrium outcome with the constrained efficient allocation.

I first describe the problem of domestic households, choosing optimally consumption and savings, and the problem of foreign investors, who choose their desired holdings of government debt. After characterizing these choices I then describe the optimal debt and default problem for the government.

Households

The economy lasts for two periods and is inhabited by I types of agents, $i \in [1, 2, \dots, I]$. Type i agents are of mass μ^i . Moreover it holds that $\sum_i \mu^i = 1$.

Households can have different amount of initial wealth, and let Γ be the initial distribution of wealth across households. Households have the same level of income in period 1.⁶ In the second period household income is uncertain and may be either low or high $\{\epsilon^L, \epsilon^H\}$ with probability π^ϵ and $(1 - \pi^\epsilon)$ respectively. The income shock is independent across agents and aggregate income is assumed to be constant in both periods.

Aside from the idiosyncratic income risk, households face aggregate risk that stems from fluctuations in the level of government spending, g . I assume that though the spending level is known for the first period, in the second model period spending is random. Moreover, spending is uncorrelated with household income.

Agents maximize a twice differentiable and concave utility function satisfying the usual Inada conditions. Preferences are given by:

$$u^i(c_0^i) + \beta E_{g,\epsilon}[u^i(c_1^i)]$$

where $u(\cdot)$ is an utility function, β is a discount factor between 0 and 1, c_t^i is consumption for household i in period t . $E_{g,\epsilon}$ denotes the conditional expectation over period 2 quantities. The subscripts (g, ϵ) are used to make clear that the expectation is a sum over period 2 spending and idiosyncratic income states.

The only asset in the economy is public debt. Thus initial wealth, denoted b_0^i , is government bonds held by households. Households income and wealth are used to pay for lump-sum taxes T_0 (the sign of the taxes is unrestricted), consumption expenditures and to purchase new debt, b_1^i at price q_0 . The budget constraint for the agent i in period 0 is given by:

$$c_0^i + q_0 b_1^i = \epsilon_0 + b_0^i - T_0 \tag{1.1}$$

The budget constraint for the household in the second period differs according to whether or not the government has defaulted. In case of repayment, the household budget constraint in the second period would be:

$$c_1^i = \epsilon_1^i + b_1^i - T_1 \tag{1.2}$$

Thus households consume their labour income and wealth stock net of taxes.

Default brings about penalties to the government. The literature on sovereign default has included different types of costs linked to the default decision, for example losing access to financial markets (financial autarky), exogenous output costs, and reputation costs. In a two-period model, neither assuming reputation costs of default nor financial autarky is meaningful, therefore the penalty of government default will be in the form of an exogenous output loss. Since in this model aggregate income is constant through time, I will assume that the cost of default becomes function of the government spending level in period 2. Finally, in line with most of the literature on sovereign default, I make two additional assumptions regarding the default choice:

⁶This is without loss of generality since initial endowments can differ according to wealth.

i- default is not selective; and ii- it is not partial. Both assumption and their consequences will be explained in the following section when discussing the default decision by the government.

Given the above, I express the household budget constraint under the default case as:

$$c_1^i = \epsilon_1^i(1 - \phi(g)) - T_1 \quad 1.3$$

where ϕ is a convex function of the government expenditure and $\phi(g) \geq 0$. This penalty function is identical to that of [D'Erasmus and Mendoza \(2016\)](#) and [Arellano \(2008\)](#). The difference here is that period 2 income is random. Naturally the higher is income, the larger is the loss of resources due to default, because costs are assumed to be proportional, but the less important this loss will be for the agent's welfare under curved utility.

I make three further assumptions regarding structure of the financial market. First, I assume that the government can not lend money to the private sector, thus focusing on cases where the government issues positive or zero debt. This assumption is common in the literature on optimal fiscal policy with domestic debt, e.g. [Lustig et al. \(2008a\)](#), [Nosbusch \(2008\)](#) and [Faraglia et al. \(2018\)](#) and it is meant to capture (in a reduced form manner) scenarios where public savings are ruled out due to the possibility of private sector default. I do not want to consider the possibility of default by private agents and thus like the above papers I also assume a no lending constraint for the government. The second assumption is that households can not save in foreign assets; only the government can borrow from non residents. Finally, I assume that households cannot borrow privately, in other words they face a no borrowing constraint, a standard assumptions in heterogeneous agents models. Here this assumption also allows me not to have to model private assets, which in principle should be treated differently than government debt.

Foreign investors

The government can issue debt to deep pocket and risk neutral foreign investors, similar to [Arellano \(2008\)](#). Foreign debt is sold by the government at price q_0 . Note that this is also the price of debt in the domestic market since foreign and domestic debt are for the government perfect substitutes.

Foreign investors have access to an international capital market in which they can borrow at a constant interest rate r . To maximize their profits, they will buy debt issued by the government B_1^f . In the second period they expect to receive an amount which will depend on the government default/repayment decision. Discounted (initial) foreign investor profits are given by:

$$\Phi = -q_0 B^f + \frac{(1 - p_1)}{1 + r} B^f$$

Where p_1 is the probability of a sovereign default in the second period. Obviously the term $-q_0 B^f$ represents the expenditure from debt purchases. $\frac{(1 - p_1)}{1 + r} B^f$ denotes the expected present value of

the second period payoff to investors.⁷

The government

In the first period, the government finances its expenditures g_0 and the repayment of the initial debt B_0 through lump sum taxes T_0 and the issuance of new debt. The government budget constraint in the first period is given by:

$$g_0 + B_0 = T_0 + q_0 B_1 \quad 1.4$$

B_1 is the sum of domestic and foreign debt $B_1 = B^d + B^f$ Using the probability density function μ^i , the mass of agents of type i we have

$$\sum_i^I \mu^i b_t^i = B_t^d$$

$$B_t = B_t^d + B_t^f$$

In the second period, government spending is exogenous and stochastic. If the government decides to repay, lump sum taxes are used to pay for the government spending and debt. In case of default, lump sum taxes need to cover only for the government spending level. The government's budget constraint in the second period is given by:

$$T_1 = g_1 + (1 - d_1) B_1 \quad 1.5$$

d_1 is binary variable that takes the value 1 if the government defaults and is 0 otherwise.

1.2.1 Competitive equilibrium

In this section I describe optimal policies under no commitment and define a competitive equilibrium for the previous economy. I describe optimal policies for each agent and then define a competitive equilibrium.

I begin with the maximization problem of the foreign investors, followed by that of domestic households. Finally, I then describe the government debt and default policy. I select a competitive equilibrium in which the benevolent government chooses debt and default optimally, thus selecting the best outcome from the (infinite dimensional) set of competitive equilibria.

The aggregate state variables for this economy (in period 1) are the government spending shock g_1 , the level of foreign debt, B_1^f and the wealth distribution of domestic debt across agents, Γ_1 .

⁷Clearly linearity and the non-randomness of the value of debt B^f in period 2, enable me to aggregate all default contingencies in object p_1 . It should be understood that default will depend on period 2 spending, and also on the other state variables, most notably the distribution of wealth and labour income of households. This dependence will be made explicit later on.

Γ_1 is a state variable (the mean level of households assets equal to domestic debt is generically not sufficient) because the planner will in principle take into account all moments of the wealth distribution, (mean, variance skewness etc), to design optimal policy wealth. Notice however that in the case of default, all household wealth will be zero and hence also trivially Γ_1 will have a mass of 1 at zero wealth because I assume that the government is constrained to default on its entire debt.

Accordingly, in period 0, the state variables that need to be remembered to solve the household, foreign investors and government programs include the distribution of household initial endowments (call it Γ_0), the initial level of foreign debt and the level of spending at time 0. Notice also objects Γ_0 and Γ_1 are linked through a standard law of motion that accounts for households optimal savings decisions and government policy. I will describe this object later on.

Foreign investors

As it is standard in the literature, foreign investors buy debt to maximize their payoff. When the probability of default is zero the price of one unit of debt equals $\frac{1}{1+r}$. In case the probability of a default is positive, the price for the bond reflects the risk of default/no repayment. Standard results imply that foreign investors are indifferent between purchasing an additional unit of debt if the following condition holds:

$$q_0(B_1^f, g_1, \Gamma_1) = \frac{(1 - p_1(B_1^f, g_1, \Gamma_1))}{1 + r} \quad 1.6$$

where $p_1(B_1^f, g_1, \Gamma_1)$ explicitly acknowledges the dependence of the defaulting in the second period on the vector of state variables.

Household problem

Households choose savings to maximize utility over the two periods, taking as given the price for the bond and the government policies: taxes, foreign debt and default. The optimal decision for the household i is the solution to the following problem:

$$v^i(B_0^f, \Gamma_0, b_0^i, g_0) = \max_{\{b_1^i\}} u^i(\epsilon_0^i + b_0^i - T_0 - q_0(B_1^f, g_1, \Gamma_1)b_1^i) + \beta E_{g,\epsilon}[(1 - d_1(B_1^f, g_1, \Gamma_1))u^i(\epsilon_1^i + b_1^i - T_1^{d_1=0}) + d_1(B_1^f, g_1, \Gamma_1)u^i(\epsilon_1^i(1 - \phi(g)) - T_1^{d_1=1})]$$

If the value function is differentiable, the first order condition for savings is :

$$q_0(B_1^f, g_1, \Gamma_1)u_c^i(\epsilon_0^i + b_0^i - T_0 - q_0(B_1^f, g_1, \Gamma_1)b_1^i) = \beta E_{g,\epsilon} \left[(1 - d_1(B_1^f, g_1, \Gamma_1))u_c^i(\epsilon_1^i + b_1^i - T_1^{d_1=0}) \right] \quad 1.7$$

where the expectations are taken with respect to the distribution of the government spending g shock and income shock ϵ . $u_c^i(\cdot)$ is the derivative of the utility function with respect to consumption, hence marginal utility of consumption.

The equation indicates that for those states in which the government chooses to default, the marginal benefit of an extra unit of savings assigned by the household is zero. This result comes from the fact that when the government declares default it does it on the entire debt. Formally, utility under default is given by $u^i(\epsilon_1^i(1 - \phi(g)) - T_1)$ and does not depend on domestic assets b_i^i .

Using the foreign investor condition to substitute for prices, the Euler equation can be written as

$$u_c^i(c_0^i) = \beta(1 + r)E_{g,\epsilon} \left[\frac{(1 - d_1(B_1^f, g_1, \Gamma_1))}{(1 - p_1(B_1^f, g_1, \Gamma_1))} u_c^i(c_1^i) \right] \quad 1.8$$

An increase in the number of states for which default is chosen by the government reduces the demand for bonds since household assigns zero value to those savings. However, for those states in which the government chooses to repay, the value of additional savings increases due to a default risk premium. This shapes the demand for bonds: the larger the default probability, the larger the risk premium that the bonds pay.

The default decision

In the second period, the government chooses to default or to repay the debt by comparing the social welfare under both circumstances. In case the government chooses to repay, taxes in the second period are higher than under default which may hurt low income-wealth agents. Let R and D be the supra indexes for "repayment" and "default" respectively, the default choice by the government, $d_1(B_1^f, g_1, \Gamma_1) \in \{0, 1\}$, is given by the solution to the following problem:

$$\max_{d_1 \in \{0,1\}} [W_1^R(B_1^f, g_1, \Gamma_1), W_1^D(g_1, \Gamma_1)] \quad 1.9$$

s.t.

$$c_1^{i,R} = \epsilon_1^i + b_1^i - T_1$$

$$c_1^{i,D} = \epsilon_1^i(1 - \phi(g)) - T_1$$

$$T_1 = g_1 + (1 - d_1(B_1^f, g_1, \Gamma_1))B_1$$

Where W_1^R is the social welfare under repayment and W_1^D is the social welfare under default:

$$W_1^R = \sum_i \mu_i u^i(c_1^{i,R})$$

and W_1^D is the welfare attained by the society under default. Since in case of default, government debt is set to zero, the state space is reduced to g_1 and ϵ^i

$$W_1^D = \sum_i \mu_i u^i(c_1^{i,D})$$

The default decision allow to compute the probability of a default, which is given by the sum of those states in which the government chooses to default across all the possible realizations of the shocks. Thus, $p_1(x)$ is given by

$$p_1(B_1^f, g_1, \Gamma_1) = \sum_{g'} \sum_{\epsilon'} d_1 F(g') F(\epsilon')$$

where $F(g')$ and $F(\epsilon')$ are the cumulative distribution function of the government spending shock and the idiosyncratic income shock respectively. As in [Arellano \(2008\)](#), default occurs as an equilibrium outcome because markets are incomplete and there is no such a contract that can insure the government in some bad states. In this case, default allows the government to redistribute income across agents ex-post, thus enabling the government to complete the markets along the idiosyncratic shock.

Finally, I made two additional assumptions regarding default policy by the government. First, the government can not discriminate among creditors. Second, the government can not partially default on debt.

The first assumption rules out the government's ability to discriminate between domestic and foreign investors, which prevents the government to segment the market for public debt. As a consequence, the bond price is the same across local and foreign markets. This assumption can be seen as a sort of strict *pari-passu* clause in which the creditors have equal treatment by the debtor.⁸ At the same time, it also can be sustained by considering a secondary market for debt as in [Broner et al. \(2010\)](#). With the existence of a secondary market for sovereign debt, domestic investors could buy defaulted debt in hands of foreign investors making impossible for the government to distinguish among them.

The second assumption indicates that whenever the government decides to default, it does it on the entire amount of debt issued. Bringing partial default to the analysis means the government will have access to a better instrument to reduce consumption dispersion across households. Taking into account the model by [Arellano et al. \(2023\)](#), if the government can partially default on

⁸See [Olivares-Caminal \(2013\)](#) for an analysis of the *pari-passu* clause and its impacts in sovereign debt markets.

its obligations, the default costs could be lower, as the government takes only part of domestic households asset and it could be an option more frequently used by the government than total default.

Definition of the competitive equilibrium

Definition CE: A competitive equilibrium is a value function $v^i(B_0^f, g_0, \Gamma_0, b_0^i)$ and decisions rules for savings, $b_1^i(B_1^f, g_1, \Gamma_1)$ for each agent, a bond pricing function $q_0(B_1^f, g_1, \Gamma_1)$ and a set of government policies $B_1^f(B_1^f, g_1, \Gamma_1)$, $T_0(B_1^f, g_1, \Gamma_1)$, $T_1(B_1^f, g_1, \Gamma_1)$ and $d_1(B_1^f, g_1, \Gamma_1)$ such that:

1. b_1^i and v_1^i solves agent's i problem given the government policies and the bond price, i.e. satisfies the Euler equation 1.10
2. d_1 solves optimal default problem 1.9
3. the bond price q_0 satisfies the foreign arbitrage condition

$$q_0(B_1^f, g_1, \Gamma_1) = \frac{(1 - p_1(B_1^f, g_1, \Gamma_1))}{1 + r}$$

4. the wealth distribution in case of repayment evolves according to

$$\Gamma'(b, \epsilon) = \sum_{\epsilon} \sum_{b^i = b(B_1^f, \Gamma, g, \epsilon)} \Gamma(b, \epsilon) F(\epsilon')$$

while in case of default evolve as

$$\Gamma'(\epsilon) = \sum_{\epsilon} \Gamma(b, \epsilon) F(\epsilon')$$

5. T_0, T_1 are such that government budget constraint are satisfied
6. Bond's market clears in both periods

$$\sum_i \mu^i b_t^i + B_t^f = B_t$$

7. Aggregate resource constraint is satisfied:

(a) in the first period:

$$C + g = Y - B_0 + q_0 B_1$$

(b) in the second period:

$$\begin{aligned} C + g &= Y - B_1 & \text{if } d_1 &= 0 \\ C + g &= Y(1 - \phi(g)) & \text{if } d_1 &= 1 \end{aligned}$$

Competitive equilibrium with optimal debt choice

The government chooses at period zero, the level of debt to be issued to foreign investors, B_1^f and to domestic households b_1^i , respecting private savings rules (equation 3.5) and the foreign investors condition 1.6. Notice that when respecting the private savings condition, the planner ensures the domestic households has their chosen level level of consumption and savings.

Importantly, the government does not have access to agent specific transfers (individual transfers), and instead it is restricted to lump-sum transfers, T_0 , that are equal for all households (average transfers).

A competitive equilibrium allocation is characterized by the Euler equation for households, the private savings rules. This equation 3.5 contains the expected marginal utility of consumption for the second period, which from the perspective of the planner at time zero contains a forward-looking component depending on the default decision to be taken in the second period.

Since the planner can not commit to a default policy at time zero, it only can manipulate households decisions by choosing debt and taxes in the first period.⁹ So, as usual in the literature on discretionary policies (see [Klein et al. \(2008\)](#), for instance) we write the expectation contained in the Euler equation as a function of the state variables.

Considering that in the event of a default, the marginal value of an extra unit of savings is zero, we can define the expectations contained in the Euler equation to be a function $h^i(\cdot)$ of the state variables (B_1^f, Γ_1, g_1) such that:

$$h^i(B_1^f, \Gamma_1, g_1) = \beta E_{g,\epsilon}[(1 - d_1(B_1^f, \Gamma_1, g_1))u_c^i(c_1^i(B_1^f, \Gamma_1, g_1))] \quad 1.10$$

which represents the discounted expected marginal utility of consumption over the repayment set as a function of the state variables. Notice that we do not include the utility under default, as it does not depend on government debt (instead is totally exogenous). Last equation indicates that since the government lacks commitment to a default policy, future consumption it is not in direct control by the planner and instead, it can only be affected by manipulating debt at time zero.

The problem for the government supporting a competitive equilibrium allocation can be stated as follows,

⁹In this paper, we assume no commitment for the government, though [Adam and Grill \(2017\)](#) describes the Ramsey problem for government that can default on its obligations

$$\begin{aligned}
& \max_{\{B_1^f, b_1^i\}} \sum_i \mu^i u^i(\epsilon_0^i + b_0^i - B_0 - g_0 - q_0(B_1^f, g_1, \Gamma_1)(b_1^i - B_1)) + \\
& + \beta E_{g, \epsilon}[(1 - d_1(B_1^f, g_1, \Gamma_1))u_c^i(\epsilon_1^i + b_1^i - B_1 - g_1) + d_1(B_1^f, g_1, \Gamma_1)u_c^i(\epsilon_1^i(1 - \phi(g)) - g_1)] \\
& s.t. \\
& q_0(B_1^f, g_1, \Gamma_1) = \frac{(1 - p_1(B_1^f, g_1, \Gamma_1))}{(1 + r)} \tag{1.11} \\
& q_0(B_1^f, g_1, \Gamma_1)u_c^i(c_0^i) = h^i(B_1^f, g_1, \Gamma_1) \\
& \sum_i \mu^i b_1^i + B_1^f = B_1
\end{aligned}$$

The objective function is the indirect utility across the two periods in which lump-sum transfers has been substituted out by the government budget constraint. The planner respecting a competitive equilibrium allocation is constrained by the bond pricing equations for domestic agents (equation 3.5), the foreign investor condition (1.6) and aggregation in the market for government bonds.

1.2.2 Constrained efficient equilibrium

This section describes the allocation arising from a planner who chooses savings on behalf of the households, but it is not constrained by private savings rules. The kind of efficiency I am interested in this paper is the one described by [Davila et al. \(2012\)](#) or [Diamond \(1967\)](#), who were looking for a Pareto improving allocation without changing the structure of the markets when agents are heterogeneous. This means that financial markets will continue to be incomplete and default will remain optimal in some states.

Importantly, the constrained optimum does not involve eliminating welfare losses derived from discretionary policies (no commitment) or the presence of sovereign default. Instead, it involves finding a welfare improving allocation when the planner can control for the presence of effects in the bond market derived from household optimizing in a self interest manner.

To define the constrained optimum, we take a planner perspective and we let the planner to choose savings on behalf of the domestic agent. Formally, this means the planner is no longer constrained by the Euler equation for savings and it respects only the condition for the foreign investor to price the bond.

The constrained efficient optimum, can be described as the solution to the following problem:

$$\begin{aligned}
& \max_{\{B_1^f, b_1^i\}} \sum_i \mu^i \left[u^i(\epsilon_0^i + b_0^i - B_0 - g_0 - q_0(B_1^f, g_1, \Gamma_1)(b_1^i - B_1)) + \right. \\
& \left. \beta E_{g, \epsilon} [(1 - d_1(B_1^f, g_1, \Gamma_1)) u^i(\epsilon_1^i + b_1^i - B_1 - g_1) + d_1(B_1^f, g_1, \Gamma_1) u^i(\epsilon_1^i(1 - \phi(g)) - g_1)] \right] \\
& s.t. \\
& q_0(B_1^f, g_1, \Gamma_1) = \frac{(1 - p_1(B_1^f, g_1, \Gamma_1))}{1 + r} \\
& \sum_i \mu^i b_1^i + B_1^f = B_1
\end{aligned} \tag{1.12}$$

In problem 1.12, the planner chooses savings on behalf of each household respecting the bond pricing equation given by the foreign condition 1.6 and feasibility. In this problem the feasible set of choices is given by the households budget constraints 1.1, 1.2 and 1.3 in which taxes has been substituted out by the government budget constraint 1.4 and 1.5.

Notice despite the two periods, the model features also discretionary policies due to default decision to be taken in the second period. Thus, in the constrained optimum, the government still faces the optimal default problem stated in 1.9. Of course, this time debt levels are chosen according to the new planner problem.

The FONC with respect to individual savings for agent i is given by:

$$\begin{aligned}
& \overbrace{q_0(B_1^f, g_1, \Gamma_1) u_c^i(c_0^i) - \beta E[(1 - d_1(B_1^f, g_1, \Gamma_1)) u_c^i(c_1^{i,R})]}^{\text{Euler equation}} + \overbrace{\frac{\partial q_0(B_1^f, g_1, \Gamma_1)}{\partial b_1^i} \sum_j (b_1^j - B_1) u_c^j(c_0^j)}^{\text{Price change}} \\
& - \overbrace{q_0(B_1^f, g_1, \Gamma_1) \sum_j u_c^j(c_0^j) \frac{\partial B_1}{\partial b_1^i}}^{\text{taxes}} = - \beta E \left\{ \underbrace{(1 - d(B_1^f, g_1, \Gamma_1)) \sum_j u_c^j(c_1^j) \frac{\partial B_1}{\partial b_1^i}}_{\text{taxes}} \right\} + \\
& + \beta E \left\{ \underbrace{\Delta d_1(B_1^f, g_1, \Gamma_1) \left[\sum_j u^j(\epsilon_1^j(1 - \phi(g)) - g_1) - u^j(\epsilon_1^j + b_1^j - B_1 - g_1) \right]}_{\text{default set change}} \right\}
\end{aligned} \tag{1.13}$$

Notice that in the last equation there are agents specific term (for each i agent) and aggregate effects which considers the summation across all agents. These terms are denoted by $\sum_j$ and considers an aggregate effect across all the agents in the economy $j = [1, 2, \dots, I]$.

The equation dictating savings in the constrained optimum features extra terms to the euler equation defining a competitive equilibrium from section 3.3.4. The effects are due to changes in average taxes (due to debt issuance), the price of the bond and the default probability. All the previous decisions were taken as given by the households at the time of making their own savings choices. In the LHS of the equation, the first additional term to equation 3.5 is a marginal utility weighted-average of the individual effect of the change in the bond price, which amounts to put higher weights on low wealth agents. The effect of taxes in the LHS of the equation is indeed an effect due to the decrease in period zero taxes due to debt issuance, which has an unambiguous negative sign. In the RHS, there are also two additional terms to the household euler equation 3.5. The

first one is a change due to higher expected taxes in the second period due to debt issuance; taxes increases only for those states in which debt repayment is guaranteed by the government. The second effect is a welfare gain (or loss) due to changes in those states for which the government choose to default. Since default is a discrete choice variable, the term $\Delta d(x)$ can be expressed as $\Delta d(B^f, g, \Gamma) = d(B^f, b^i, \Gamma, g) - d(B^f, b^i + h, \Gamma, g)$, when considering an increment of a small amount h in household debt.

Meanwhile, the FONC for foreign debt is the following:

$$\underbrace{\beta E \left\{ (1 - d_1(B_1^f, g_1, \Gamma_1)) \sum_j u_c^j(c_1^j) \right\}}_{\text{taxes}} - \underbrace{\frac{\partial q_0(B_1^f, g_1, \Gamma_1)}{\partial B_1^f} \sum_j (b_1^j - B_1) u_c^j(c_0^j)}_{\text{Price change}} = \underbrace{\Delta d_1(B_1^f, g_1, \Gamma_1) \left[\sum_j u^j(\epsilon_1^i(1 - \phi(g)) - g_1) - u^j(\epsilon_1^i + b_1^i - B_1 - g_1) \right]}_{\text{default set change}} \quad 1.14$$

When the planner chooses foreign debt it considers the effect that debt issuance has on the prices of the bond, taxes in the first and the second period and the change in the welfare gains in the second period due to an increase in the default propensity.

Household heterogeneity and externalities

Clearly, the first order condition for the two problems are different leading to different allocations. We start by comparing the condition for the planner under the constrained optimum to that of the competitive equilibrium, namely the Euler equation.

When debt is subject to default risk there are two additional effects in the equation 1.13 for the constrained planner not appearing in equation 3.5 for the domestic household. In the LHS there is a term capturing the change in the bond prices due to debt issuance, while in the RHS there is an extra term considering the change in future marginal utility due to a higher default propensity.

The first effect is a pecuniary externality and emerges because the household does not take into account the change in the bond price due to own savings decisions. The size and sign of the externality depends on: i) the sensitivity of the bond price to changes in individual savings; ii) the wealth distribution across households; and iii) the distribution of debt between foreign and domestic agents, or the net foreign asset position. The latter is present because the foreign and domestic market of governments bonds are not segmented. To see this, we can rewrite the term as:

$$+ \frac{\partial q_0(B_1^f, g_1, \Gamma_1)}{\partial b_1^i} \left[\sum_j \overbrace{\left(b_1^j - B_1^d \right) u_c^j(c_0^j)}^{\text{wealth distribution}} - \underbrace{B_1^f \sum_j u_c^j(c_0^j)}_{\text{net foreign asset position}} \right]$$

The presence of foreign investors condition allows to link the bond price directly with default probabilities. By replacing the price q_0 by the foreign investor condition 1.6, the derivative of the

price with respect to individual debt can be written as:

$$\frac{\partial q_0(B_1^f, g_1, \Gamma_1)}{\partial b_1^i} = -(1+r)^{-1} \frac{E(\partial p_1(B_1^f, g_1, \Gamma_1))}{\partial b_1^i} \geq 0$$

The equation says that the derivative of the bond price with respect to individual savings is equal to the derivative of the expected default probabilities, which is the change in the expected default probability, or those states for which default is chosen by the government.

Assuming the term has a positive sign, higher domestic savings increases the cost of a default for the government because it will be taking resources from the households, so default probabilities falls and the bond price increases.

The sign of the externality depends also on the other two factors: i)- the wealth distribution across households and; ii)- the distribution between domestic and foreign residents or the net foreign asset/debt position. Note that the first term is a marginal utility weighted average of the individual effects of an increase in debt. If we assume concavity of $u(\cdot)$, the term places higher weights on the low wealth households. Therefore, the more unequal the wealth distribution, the larger and negative is the effect of the externality. When the planner takes the externality into account, it reduces the marginal cost of additional savings, hence households saves more than in the constrained equilibrium allocation. As for the foreign asset position, the larger the net foreign debt, the more negative becomes the externality, reducing even more domestic savings in the competitive equilibrium allocation.

The last additional term on the RHS of equation 1.13 encloses the marginal welfare gain in the second period due to a change in the default propensity for the government, or the change in the default set. The effect considers not only the change due to default costs, but also due to the fact that the planner is taking assets away from households since it is defaulting on domestic debt.

Let us denote with a supra index R , those states in which the government decides to repay and with a supra index D those states in which the government decides to default. We can write the last term in the LHS of equation 1.13 as

$$+\beta E \left[\Delta d_1(B_1^f, g_1, \Gamma_1) \left[\overbrace{\sum_j u^j(c_1^{j,D}) - u^j(c_1^{j,R})}^{\text{marginal welfare gain}} \right] \right] > 0$$

Households fails to internalize the previous effect when choosing savings according to the euler equation 3.5. This effects amounts to changes in default set due to changes in domestic debt levels.

This term is directly linked to the marginal welfare gain due to changes in the default propensity of the government, i.e. the default set. In case there is a reduction in those states for which default is dictated, $\Delta d_1(\cdot) < 0$, the marginal welfare gain of a default, the difference between utilities under repayment and default, becomes negative and the overall sign of the externality becomes positive. The latter would means households will save more under the constrained optimum because involves a higher marginal benefit of saving.

However, if households save less in the competitive equilibrium due to the presence of externalities, the relative lower levels of domestic debt conducts the government to default more often, reducing the price of the bond. If debt is relatively cheaper, domestic households may demand more debt to smooth consumption. This effect may induce the government to take more risk and issue more debt. This effect will become specially important when we introduce wealth inequality because with a lower bond price, the wealthy households can demand larger levels of debt to smooth consumption. This mechanism is also identified by [D'Erasmus and Mendoza \(2016\)](#).

In this model, wealth distribution and the idiosyncratic income risk play significant roles not only through the default decision, but also in the welfare gain of default. This is because the largest the wealth dispersion, the more attractive is for the government to default in the second period and the larger in magnitude is the social welfare gain of a default.

The representative agent case

In order to better understand the effects of wealth and income heterogeneity in the model, this section characterizes the rules for savings in a competitive equilibrium and in the constrained optimum in a representative agent economy.

We assume, as in much of the literature on sovereign default, that the government can not select among the creditors and instead it defaults on foreign and domestic debt simultaneously. In other words, the government is unable to segment the bond market, so the price for debt is the same for domestic and foreign investors. As a consequence, an externality affecting the bond price arises as noted by [Mallucci \(2021\)](#).

When considering a representative agent economy, the Euler equation 3.5 would be the rule dictating savings in a competitive equilibrium allocation. Meanwhile, the problem for the constrained planner would be the same as problem 1.12, except that now there is only one representative agent in the economy and no wealth heterogeneity. As was the case with household inequality, the constrained planner chooses debt on behalf of domestic households taking into account the effect of its action on prices and default schedule.¹⁰

Let B_1^d , B_1^f , be the level of domestic and foreign debt for the second period and g_1 the level of government spending for the second period. Consider the vector composed by (B_1^d, B_1^f, g_1) as the vector of aggregate state variables, the constrained efficient equilibrium will be the solution to the following planner problem:

¹⁰When income shocks are shut down the only source of uncertainty comes from the government spending shock. Default will still occur as a way to avoid higher taxes in the second period, but this time it becomes only a resource transfer from foreign to domestic investors and not within residents.

$$\begin{aligned}
& \max_{\{B_1^f, B_1^D\}} u(\epsilon_0 + B_0^D - B_0 - g_0 - q_0(B_1^f, B_1^d, g_1)(B_1^D - B_1)) \\
& + \beta E_{g, \epsilon} [(1 - d_1(B_1^f, B_1^d, g_1))u(\epsilon_1 + B_1^D - B_1 - g_1) + d_1(B_1^f, B_1^d, g_1)u(\epsilon_1(1 - \phi(g)) - g_1)] \\
& s.t. \\
& q_0(B_1^f, B_1^d, g_1) = \frac{(1 - p_1(B_1^f, B_1^d, g_1))}{1 + r} \\
& B_1^D + B_1^f = B_1 \\
& c_t \geq 0
\end{aligned}$$

where lump-sum transfers has been substituted out by the government budget constraint. The first order condition with respect to domestic debt B_1^d reads:

$$\begin{aligned}
u_c(c_0) \left[q_0(B_1^f, g_1, B_1^d) + \overbrace{\frac{\partial q_0(B_1^f, g_1, B_1^d)}{\partial B_1^d} [B_1^d - B_1]}^{\text{price change}} \right] = \\
\beta E[(1 - d_1(B_1^f, g_1, B_1^d))u_c(c_1^R) + \underbrace{\Delta d_1(B_1^d, B_1^f, g_1)[u(c_1^D) - u(c_1^R)]}_{\text{default set}}]
\end{aligned} \tag{1.15}$$

The equation above is a version of equation 1.13 but for a representative agent economy, hence the effect coming from households redistribution vanishes and instead the redistribution takes place only between foreign and domestic agents.

In the competitive equilibrium, the equation ruling savings, B_1^d for the representative agent would be the usual one:

$$u_c(c_0)q_0(B_1^f, g_1, B_1^d) = \beta E[(1 - d_1(B_1^f, g_1, B_1^d))u_c(c_1^R)]$$

As can be seen in equation 1.15 there is wedge between the two equations because the planner can take into account the effect of domestic debt on the bond price. As was the case with household heterogeneity, the effect of the externality depends on i) the sensitivity of the bond price to changes in domestic debt; and ii) the foreign asset position of the economy. If the government is a net foreign debtor, the sign of the externality becomes negative, meaning that the marginal cost of domestic savings (domestic debt) is lower than in the competitive equilibrium allocation. Therefore, the private equilibrium will show lower levels of domestic debt and higher levels of foreign debt than what would be socially desirable.

When considering heterogeneity, equation 1.15 becomes equation 1.13, which shows several differences with respect to the representative agent case. First, the effect of domestic savings choices on the price of the bond depends on the entire wealth distribution across agents. Second, there is also a distribution effect within domestic households which shapes default probabilities and the bond price. We will show in the quantitative section that the presence of heterogeneity in combination with a foreign investor can lead to higher or lower debt levels, unlike the case of [Mallucci \(2021\)](#) who unambiguously finds lower levels of domestic and foreign debt.

1.3 Quantitative Analysis

This section explores quantitatively the properties of the competitive equilibrium and the constrained optimum. Since the government makes the default decision by comparing welfare across repayment and default, the wealth distribution across agents becomes a state variable. In case of a continuum of agents, the wealth distribution is an infinite dimensional object, thus I restrict the analysis to an economy populated by two residents households.

Aiming to analyze the role of labor income risk and wealth inequality for optimal policies, I perform two experiments. First, to capture the effect of income risk, I let both agents to hold the same level of initial wealth and perform a mean preserving spread over income distribution in the second period. In the second experiment, individual income stays constant over the two periods, but initial wealth inequality varies from an economy with a representative agent to an economy with 'perfect inequality', in which one agent holds the total amount of government debt.

[Table 1.1 approximately here]

Table 1.1 in appendix 1.B shows the value for the main parameters of the model used to conduct the numerical experiment. We set the values for the parameters to mimic some relevant targets for the European Union countries at 2007, the year before the Great Recession and closely following [D'Erasmus and Mendoza \(2016\)](#), who calibrates the model for a subset of economies of the European Union. Importantly, the numerical experiment do not attempt to replicate data on any particular economy or set of economies due to potential inaccuracies of a two-period model to account for time-series dynamic observed in data. Instead, the exercises attempts to show the main mechanisms driving inefficiencies in the competitive equilibrium allocation due to wealth and income heterogeneity.

Households are assumed to have the following preferences over consumption:

$$u(c) = \log(c)$$

I normalize aggregate income to be 1 across both periods. The discount factor is assumed to be 0.96, as in [D'Erasmus and Mendoza \(2016\)](#) and the real international interest rate, r is set to 2%, in line with the average interest rate in Germany between 2000-2007 (this value is also used by [D'Erasmus and Mendoza \(2016\)](#)). The initial level of aggregate debt is set in 0.50, a close figure to the 56% of central government net debt to GDP for the European Union at the beginning of the Great Recession in 2007. From total debt, we fix 80%, so 0.4 as domestic debt being the rest foreign. Government expenditure is 0.18, close to the observed value for general government consumption as percentage of GDP for the European Union in 2007.¹¹ The government spending shock is modelled as an i.i.d. process with 20 states and a standard deviation of 0.015, a bit above the standard deviation of 0.01 registered for the government consumption expenditure

¹¹Data for debt and government consumption expenditure comes from the World Development Indicators database. Data on government consumption excludes military spending and government capital formation

for the European Union between 1970-2020. This helps to create more variability in the default probabilities.

Default costs are modelled as a convex function of government expenditure and takes the form of $\phi(\hat{g} - g_1)$ in which $\hat{g}$ is the upper bound of the government spending distribution for the second period. A higher level of ϕ increases the cost of default for each value of the government spending and makes default less attractive when the government spending is low. This form of the cost function is in the same spirit of [D'Erasmus and Mendoza \(2016\)](#) and [Arellano \(2008\)](#) and aims to avoid the government declaring default in states with low government spending. We calibrate this figure to be 4, which gives a probability of default of 10% in the competitive equilibrium for an economy with a representative agent. Note that this is higher than the usual 3%-5% estimated by [Arellano \(2008\)](#) for an external default, but the probability of default drops to range between 0% and 5% when adding idiosyncratic income risk, a more real situation.

Despite borrowing constraints are not explicitly modelled, the government can not lend money to households, thus they are not allowed to run in debt. A similar framework as [Nosbusch \(2008\)](#) and [Lustig et al. \(2008a\)](#). As a consequence, in equilibrium, household will hold either positive or zero assets. Different from [D'Erasmus and Mendoza \(2016\)](#), we do not allow for the government to accumulate foreign assets.

In the first experiment I allow for changes in the standard deviation of the idiosyncratic income shock without changing average income (mean preserving spread). The shock is transitory and uncorrelated with the government spending process. The income shock may take two values: $\{\epsilon_H, \epsilon_L\}$, high and low. The standard deviation ranges from zero, a representative agent economy, to 0.2, an arbitrary value in which debt issued by the government is risk free.

In the second experiment I change the initial distribution of wealth across the two agents without changing the mean (the aggregate debt level). Inequality ranges from zero, thus a representative agent economy to an economy with extreme inequality in which an agent holds all debt and the other does not hold debt at all.

Both problems are solved backwards using grids with equidistant nodes for the foreign debt and households individual savings. In both cases, I start in the second period solving the government's default decision and obtaining the probability of a default event for each node in the state space. I then compute the bond price and default probability using the optimizing condition for the foreign investor. Afterwards, I approximate the price of the bond using a polynomial function of the state space.

Once the price is parameterized as a function of the state vectors, to solve for the constrained efficient equilibrium allocation, I perform a grid search over the state space to maximize the social welfare function.

To solve the competitive equilibrium, I first solve the system of equations implied by each household budget constraint, the government budget constraint and the Euler equation 3.5 for the agents to find the level of aggregate taxes at period zero supporting the optimal competitive equilibrium. Notice that the Euler equation 3.5 contains next period marginal utility of consumption,

a forward looking component which is a function of the government's optimal policies. I approximate the marginal utility of consumption using a polynomial function of the state space. Finally, to find the policy rule for aggregate debt and individual savings, I maximize the social welfare function over the grid of nodes satisfying the competitive equilibrium. More details about both algorithm to solve the problem can be found in Appendix 1.A.

1.3.1 Results

Ex-ante identical consumers and precautionary savings

First, I explore the role of income uncertainty for optimal policies by performing a mean preserved spread over income distribution in the second period. Assuming that $u'(\cdot)$ is a convex function, a mean preserving spread of ϵ^i , the idiosyncratic shock, increases the demand for savings in both allocations due to precautionary motives when considering lower levels of "risky" debt.¹²

Table 1.2 in appendix 1.B shows the result for the main variables under both allocations for some values of the variance of the idiosyncratic income. The mean of the income distribution is 1 while the standard deviation ranges from 0, the representative agent case, to 0.2, a level in which the government choose to issue risk free debt, hence the government chooses to repay in all states.¹³ The table shows the main results for aggregate debt (the sum across domestic and foreign debt), foreign and domestic debt, transfers at period zero T_0 , the policy rule for default at period 1 and the bond price.

[Table 1.2 approximately here]

In this environment consumers are ex-ante identical and face an idiosyncratic income shock in the second period. Since all agents face the same maximization problem and have the same level of wealth, they will make identical choices. When comparing the representative agent economy for both allocations, one can see that the efficient equilibrium displays higher levels of domestic debt, lower levels of foreign debt and overall higher levels of aggregate debt. When looking at the representative agent economy, we can also see that default is more likely in the competitive equilibrium allocation. Domestic households underborrows and consequently the planner dictates default more often, this time on the foreign investor.

As a counterpart of the lower levels of domestic debt observed in the competitive equilibrium, transfers at period zero are considerable higher being together with foreign debt the main source of funding for the government. This is not surprising given the fact that higher default probabilities translates into a lower bond price which reduces revenues from debt issuance. This leaves the government to choose higher taxes at period zero and higher levels of foreign debt to finance the stream of government spending.

¹²Notice that as in [D'Erasmus and Mendoza \(2016\)](#), the equilibrium may not exist if the government dictates default in all the states.

¹³I choose randomly 6 different values for the variance of ϵ and compute the equilibrium.

On the other hand, when looking at the constrained optimum, transfers at period zero are lower for the representative agent economy, which allows residents to save more which results in higher levels of domestic debt in the efficient equilibrium.

[Figure 1.1 approximately here]

Figure 1.1 in the appendix 1.B shows the same aggregates but for a range of values of the standard deviation of the income shock. The blue line is the constrained optimum allocation and the dashed red line shows optimal policies for the competitive equilibrium. There is an increase in the level of domestic debt under both allocations, which for high values of the standard deviation of the income shock (high income uncertainty) is large enough to cause domestic debt in the competitive equilibrium allocation to be very close to the one in the efficient equilibrium. This is due to the precautionary savings motive. Despite the presence of default risk, in those states in which the government does not declare default (repayment set), the household gives a larger marginal value to additional savings due to precautionary motives. This is especially observed under the competitive equilibrium allocation in which the participation constraint given by the Euler equation binds. When the precautionary savings motive becomes relevant, domestic demand for debt picks up and the government relays more on debt and not in lump-sum taxes at period zero to finance government expenditure. On the contrary, due to the higher domestic appetite for debt, foreign debt drops when the idiosyncratic income risk increases.

As a result of the increase in domestic debt, and despite the reduction in foreign debt, aggregate debt increases with income risk. Eventually, as depicted in figure 1.1, for high enough levels of income risk, neither in the constrained optimum nor in the competitive equilibrium, the planner dictates default, which means the government issues risk free debt. If default does not occur in equilibrium, the differences between the constrained optimum and the competitive equilibrium are minimal: both domestic and foreign debt becomes virtually the same and since debt is priced according to default probabilities, if default is not optimally chosen, the quantitative effect of the externalities vanishes or it gets considerably reduced (this result is similar to the one observed in [Kim and Zhang \(2012\)](#)).

In a nutshell, default policy reflects a debt composition effect with the government exercising less frequently the default option in the competitive equilibrium when precautionary savings becomes more relevant. As a result, default becomes less attractive for the government as debt is largely held by domestic households. Interestingly, despite the increase in domestic debt levels and the reduction in foreign debt, we do not observe a total crowding-out of foreign debt, and the government continues to issue both, domestic and foreign debt.

Initial wealth heterogeneity

In this section, I analyse the effect of wealth inequality on optimal policies for both allocations. To do so, I abstract from uncertainty in the labor market (shut down the labor income risk from previous section) and focus only on changes in initial wealth heterogeneity. The experiment is

as follows: I keep constant the level of aggregate initial wealth (domestic debt) and I change the initial distribution wealth across the two domestic households. Thus, we range from an economy in which both agents are identical -the representative agent case- to an economy in which one agent holds everything and the other start with zero wealth. Formally, for the calibration chosen, standard deviation of initial wealth ranges from 0 (a representative agent) to 0.4, (the opposite pole).

Table 1.3 in appendix 1.B shows the main result for both allocations for some selected values of initial wealth inequality. Figures 1.2 and 1.3 in appendix 1.B shows the optimal allocations for a range of values for initial wealth dispersion. Like in the previous section, the blue line represents the constrained optimum and the dashed red line is the competitive equilibrium.

One can see in the tables and figures that domestic debt is lower in the competitive equilibrium allocation than in the efficient equilibrium. On the contrary, foreign debt is lower in the efficient equilibrium than in the decentralized one. Meanwhile, aggregate debt is lower in the competitive equilibrium allocation except for higher values of wealth inequality, when the contrary is observed. Finally, regarding default policies, one can see that default probabilities are higher in the competitive equilibrium allocation for all values of wealth dispersion.

[Table 1.3 approximately here]

[Figure 1.2 approximately here]

Interestingly, when initial wealth inequality increases, the competitive equilibrium allocation shows an increase in domestic and foreign debt, although this effect is more pronounced in domestic market. Foreign debt increases by 30% and domestic debt does so by 45% when comparing the representative agent economy with an economy in which one household owns the total amount of domestic debt. The raise in the domestic demand for debt can be explained by the lower price of the bond observed in the competitive equilibrium allocation, which makes cheaper the demand for domestic debt. In the competitive equilibrium, where the participation constraint given by the Euler equation becomes binding, the lower price of the bond increases the demand for savings, especially for the high-wealth household which fuels the domestic demand for government bonds. A similar effect is found in [D'Erasmus and Mendoza \(2016\)](#).

[Figure 1.3 approximately here]

This effect can be seen for each one of the households in figure 1.3, which shows optimal policies for the two agents for different levels of initial wealth. For both households, assets (government debt) in the constrained optimum are higher than in the competitive equilibrium allocation. For the high-wealth agent, in both equilibria there is an increase in bond holdings, whereas for the low-wealth agent, bonds holdings are decreasing. Not surprisingly, this is because the low-wealth agent starts with lower levels of initial wealth, while the high-wealth agent starts with higher level of debt as wealth dispersion increases.

As a consequence of the relatively higher levels of domestic debt, -foreign debt remains stable for most of the values of wealth dispersion- aggregate debt is in general higher in the efficient allocation which together with the low proportion of foreign debt makes the government to default less frequently than in the competitive equilibrium. However, as wealth becomes more concentrated, aggregate debt increases in the competitive equilibrium until it becomes higher than in the constrained optimum for the case of 'extreme inequality' (one household holds the entire level of wealth).

As a consequence of the relatively lower levels of debt, lump-sum transfers at period zero, T_0 , are always higher in the competitive equilibrium and become the main source of funding for the government. As wealth becomes more concentrated, lump-sum taxes slightly decreases in the competitive equilibrium due to higher levels of domestic debt. Meanwhile, in the constrained optimum they slightly increase, reducing the need for debt issuance. Clearly, while in the competitive equilibrium, the government issues less debt and collects more revenues through taxes at period zero, in the efficient equilibrium, taxes are lower allowing for higher levels of domestic debt.

Lets turn to default policies and the bond price in the two lower right panels of figure 1.2. Default is more likely under the competitive equilibrium allocation than in the constrained optimum for all levels of wealth dispersion. This depends on two factors: the mix between foreign and domestic debt and wealth dispersion. First, the proportion of external debt is higher in the competitive equilibrium. Looking at the last column of table 1.3, we can see that foreign debt accounts for 27% of total debt in the competitive equilibrium and 11% in the constrained optimum. Second, the level of ex-post wealth inequality displayed in the last row of table 1.3 and computed as the ratio between debt owned by the poor households and the one owned by the rich is 4 times higher in the competitive equilibrium when one agent owns initially all the domestic debt. This means that the government has higher default incentives in the market economy due to wealth distribution purposes, or in other words, the social value of default increases faster in the competitive equilibrium than in the constrained optimum.

As one can see from the experiments, wealth inequality plays an important role in the sign and magnitude of the externalities. For an economy with a representative agent, domestic debt is near 30% lower than in the constrained planner, while foreign debt is 60 % higher. As a consequence, default rate is around 10% in the competitive equilibrium, while is 5% in the efficient equilibrium. When wealth dispersion increases, default start to be a more relevant option, both for the planner and in the private economy, this pushes the bond price down, specially in the competitive equilibrium. Eventually, domestic debt raises due to a larger demand for government bonds by the wealthy household. This effect becomes larger under the competitive equilibrium, where the participation constraint given by the Euler equation is binding.

1.4 Sensitivity analysis

In this section, I perform sensitivity analysis with respect to three parameters of the model to test for the robustness of the results described in the previous section. In one experiment I allow for a lower value for the default cost parameter. I do this because, a lower default parameter may increase the social value of the default option with respect to the previous scenario, making default more attractive to the government as a tool for redistribution. Secondly, I reduce the initial level of foreign debt which reduces the level of aggregate debt at period zero (initial debt). This should play the opposite role than the previous exercise and reduce the incentives for the government to default on debt. Finally, I increase the level of government spending tightening the government budget constraint and making the default option more interesting for the government.

For the three scenarios, I repeat the experiments done in the previous section: investigate the role of idiosyncratic income risk by increasing the variance of the income shock and study how wealth inequality changes optimal policies by increasing initial wealth dispersion.

1.4.1 Lower cost of default

In this section, the parameter governing the default cost, ϕ is set to 3 instead of 4. A lower cost of default enlarges the social value of default, incentivizing the government to take that option more often. As usual in models with sovereign default and no-commitment, if the cost for default is lower, the government is able to issue less debt and consequently becomes harder for the government to provide households with liquidity for savings. Once I set $\phi = 3$, I perform the same exercises from the previous section: I increase the level of idiosyncratic income risk and the level of initial wealth inequality.

The main result of the paper does not change and when the cost of default is lower, the competitive equilibrium continues to be constrained inefficient as is shown in figure 1.4, 1.5 and 1.6 in appendix 1.B.

[Figure 1.4 approximately here]

As was the case before, the competitive equilibrium allocation displays lower levels of domestic debt than the constrained optimum, except for higher levels of income risk, in which domestic debt levels are similar in both allocations. Under this calibration, the government relies more on foreign debt, which is higher in the competitive equilibrium. Interestingly, aggregate debt is slightly higher in the competitive equilibrium than in the constrained optimum due to relatively higher foreign debt levels. When considering domestic debt, not surprisingly, under the competitive equilibrium, the household demands higher levels of debt due to the reduction in default probabilities.

I verify also the other results from previous section: when domestic debt is lower in the competitive equilibrium, the planner finances its expenditures with lump-sum transfers at period zero

and foreign debt. Meanwhile, default is always more likely in the competitive equilibrium than in the constrained optimum. Interestingly, when the cost of default is lower, the government in the competitive equilibrium does not find optimal to issue risk free debt.

[Figure 1.5 approximately here]

[Figure 1.6 approximately here]

Figure 1.5 shows the main result for the increase in wealth inequality. The blue line is the optimal allocation under the constrained optimum and the dashed red line is the allocation under the competitive equilibrium. As was the case in the previous section, the competitive equilibrium displays lower levels of domestic debt for all levels of initial wealth inequality as can be seen at the top right panel. Foreign debt, on the other hand is lower in the constrained optimum than in the competitive equilibrium. Of course, since the cost of default is lower, the government finds optimal to dictate default more often than in the baseline economy from the previous section, especially in the competitive equilibrium allocation. For instance, for the representative agent economy, default probability was 0.1 in the baseline economy and it climbs to 0.35 when the cost of default is lower.

As was the case before, aggregate debt depicted in the top left panel is lower for most of the levels of wealth inequality, except the scenario of “extreme inequality” in which is higher. Interestingly, different from the baseline economy in which foreign debt increases, in this scenario foreign debt decreases in the competitive equilibrium, which could be due to the considerably higher default probability. At this point, the government finance debt with transfers at period zero and domestic debt, which as shown in figure 1.6, is driven by the wealthy household, cause the poor household is constrained and holds zero wealth.

1.4.2 Lower initial foreign debt

In this section, I reduce the level of initial foreign debt by 10% as a different way of stressing the results from the baseline economy, thus initial foreign debt $B_0^f = 0.09$. Intuitively, lowering initial debt levels relax the government budget constraint, since it needs to collect less resources from the private sector. Notice, the latter allows the government to reduce taxes at period 0, with respect to the baseline economy, leaving room for a higher demand for debt from domestic households. Despite foreign debt is lower in this scenario, the domestic households still holds the same amount of debt than in the baseline scenario.

I repeat the experiments already done in the previous section: Increase labor income risk and increase the level of initial wealth inequality.

Results for the experiments are depicted in figures 1.7, 1.8 and 1.9 in the appendix 1.B. As was the case before, the blue line is the constrained optimum and the dashed red line is the competitive equilibrium allocation.

[Figure 1.7 approximately here]

Under this calibration, there is some changes with respect to the baseline scenario when considering optimal policies in figure 1.7. First, domestic debt is very close in both allocations, being almost the same in the competitive equilibrium than in the constrained planner for high levels of income risk. In the case of the representative agent, on the contrary, when the competitive equilibrium registers high default probability, the difference between the two allocations is larger.

Foreign debt, on the other hand is higher in the competitive equilibrium than in the constrained planner as was the case before. Interestingly, in the constrained optimum, the government choose optimally to issue risk free debt for most of the experiment regarding the level of income risk.

[Figure 1.8 approximately here]

[Figure 1.9 approximately here]

Results do not dramatically change when the government departs from lower debt levels. The competitive equilibrium allocation displays lower levels of domestic debt and higher levels of foreign debt than in the constrained optimum. As a consequence, the government dictates default more often in the competitive equilibrium than in the constrained optimum. If so, in relation to the benchmark scenario, default is less likely for both allocations for all levels of wealth inequality.

As was the case before, aggregate debt, at the top left panel, is lower in the competitive equilibrium than in the constrained optimum except for the scenario of “extreme wealth inequality” in which one household concentrates all government debt.

1.4.3 Higher initial government spending

In this section, I increase the initial government spending from the previous value. Intuitively, a higher level of government spending at period zero, g_0 , will have two main direct consequences. First, it will raise the level of lump-sum transfers at period zero, which reduces the demand for domestic debt. Second, since government expenditure in the second period, g_1 , is a function of g_0 , a raise in g_0 automatically translates into higher spending in the second period.¹⁴ Under this scenario, default could be more likely in the second period, which could enlarge the differences between the competitive equilibrium and the constrained optimum. I set the new value for the government spending at 0.21, thus 16% larger than in the baseline scenario. Then, I perform the two experiments from the main section: increase idiosyncratic income risk and increase initial wealth inequality.

Figure 1.10 in the appendix 1.B depicts the result when government spending is 0.21 instead of 0.18. The blue line is the constrained optimum, the dashed red line is the competitive equilibrium allocation. The main result of the paper still holds, the competitive equilibrium displays lower

¹⁴Government spending is a stochastic i.i.d process with mean equal to g_0

levels of domestic debt, though the gap shrinks with income risk. As was the case before, foreign debt is higher in the competitive equilibrium, and unambiguously default is more likely in the competitive equilibrium. Aggregate debt is similar in both allocations, if so slightly higher in the competitive equilibrium allocation, except for the representative agent economy in which is significantly lower than in the constrained optimum. This is the case because foreign debt is higher relatively to the base scenario and is used to finance the higher government spending.

[Figure 1.10 approximately here]

I turn now to the other experiment in which I increase the level of initial wealth inequality. Result is shown in figures 1.11 and 1.12, in which the blue line is the constrained optimum and the dashed red line is the competitive equilibrium allocation. As can be seen in the figures, results do not change with respect to the previous section: domestic debt is lower for most of the levels of wealth inequality and default probability is still higher in the competitive equilibrium allocation than in the constrained optimum.

[Figure 1.11 approximately here]

[Figure 1.12 approximately here]

1.5 Discussion of the main assumptions

In this section we discuss the implications of the main assumptions made in the model, i) non-selective default; ii-) no partial default; iii) no savings abroad.

1.5.1 Selective default

The assumption of non-selective default is an important assumptions for the result. This is because in absence of market segmentation, the price of the bond is the same in the foreign and domestic market. Notice I use foreign investors condition to price debt in the efficient equilibrium, meaning the government can violate the Euler equation from domestic households but not the foreign sector.

With some exception like in [Erce and Mallucci \(2018\)](#) or [Di Casola and Sichlimiris \(2014\)](#), non-selective default is an assumption in most of the literature on sovereign debt markets ([D'Erasmus and Mendoza \(2016\)](#), [D'Erasmus and Mendoza \(2021\)](#), [Bocola et al. \(2019\)](#) among others) in line with a strict interpretation of the *pari-passu* clause contained in some sovereign debt contracts meaning all investors should have equal treatment ([Olivares-Caminal \(2013\)](#)). Alternatively, the inability of the government to dictate selective default can be thought in the terms of [Broner et al. \(2010\)](#), in which a secondary market for debt would make impossible for the government to discriminate among creditors.

Interestingly, if the government can perform selective default and segment the market for government bonds, the competitive equilibrium in the domestic economy will still be inefficient due to the presence of heterogeneous agents as in [Diamond \(1967\)](#) or [Davila et al. \(2012\)](#). Nevertheless, the inclusion of foreign debt, not only brings the model closer to reality but helps to price government debt in the efficient equilibrium where the domestic Euler equation is violated. The introduction of a second asset helps to price government debt by simple arbitrage. An interesting case is [Aguiar et al. \(2023\)](#) who explores potential Pareto improving policies in an economy with public debt and physical capital.

1.5.2 Partial default

I do not allow for partial default, and instead the government defaults on the entire debt level. Partial default gives an extra degree of freedom for the government as it can reduce the cost of default (the government does not take the total amount of assets from households). However, the competitive equilibrium allocation in this two-period economy will continue to be inefficient. To see this suppose default costs are zero, then an utilitarian government will maximize social welfare when ex-post consumption dispersion is zero. This is attained when the government defaults on the entire amount of debt, so partial default will not be useful. If default becomes costly, the government can attain some tolerated level of consumption dispersion by partially defaulting on debt, but it will not eliminate household heterogeneity so they will not internalize the effect of their own savings choice on the aggregate.

Nevertheless, introducing partial default could be interesting when heterogeneous agents are present in the economy. As noted by [Arellano et al. \(2023\)](#), partial default allows the government to reduce debt temporally by postponing debt repayment without exclusion of the financial markets. The planner then can smooth consumption more effectively over time. However, defaulted coupons adds up to the legacy debt and the new borrowing affecting future costs of default. This rich environment, which can not be captured in a two period model, could be analyzed in future work.

1.5.3 No savings abroad

As mentioned in section 1.2.1 we do not allow domestic households to save abroad, meaning they can not accumulate foreign assets. Allowing domestic residents to buy foreign assets means they have an alternative vehicle for saving to the risky government debt. Under this scenario, the appetite for domestic debt will be lower than the one observed in the baseline economy because the high-wealth household will be a net foreign saver. As a consequence, the benevolent government will issue foreign debt as a way to relax the budget constraint for the low-wealth household. In that environment, default will be mostly external. Notice, however that a higher default probability is translated into a lower price for the bond, which may make the rich households to acquire domestic debt. This makes default a valid tool for redistribution within domestic agents.

Despite the changes in debt composition, the competitive equilibrium with positive domestic debt levels will still be inefficient, but the gap between the two (efficient equilibrium and market economy) will shrink with respect to the one shown in the main section.

This assumption is sensible for an emerging market economy because usually the presence of defaultable sovereign debt incentivizes the private sector to save in foreign assets. Nevertheless, as pointed out by [Di Casola and Sichlimiris \(2014\)](#) or [Perez \(2018\)](#), financial repression by the government together with a relatively undeveloped capital market sometimes forces the domestic sector to buy risky government debt creating conditions for positive domestic debt.

1.6 Conclusion

This paper investigates the welfare properties of a model with heterogeneous agents and sovereign default as in [D’Erasmus and Mendoza \(2021\)](#). In the model, households save in non-state contingent debt to be insured against idiosyncratic and aggregate risk. The government issues debt subject to default risk to domestic households and risk neutral foreign investors. In such an environment, I compare the competitive equilibrium with an allocation resulting from a planner not able to overcome the frictions derived from market incompleteness. In a competitive equilibrium, households chooses the level of savings in a self interest manner and does not internalize the effect of own decisions neither on the bond price nor on the default policy. As a consequence, the competitive equilibrium is in general constrained inefficient. The efficient optimum can be restored by a direct mandate of the planner on how much to save, by a planner correcting for market incompleteness or by tax incentives. A tax on debt holdings, contingent on individual wealth, accompanied with a lump-transfer such that the net resource transfer to the agent is zero can restore the social optimum.

A quantitative experiment for a two-period economy reveals that the effect of externalities on optimal policies tend to reinforce one each other and the competitive equilibrium allocation displays lower levels of domestic savings, relatively higher levels of foreign debt and in consequence higher default probability.

When considering income uncertainty, debt composition in the competitive equilibrium changes and due to precautionary savings domestic debt increases crowding out foreign debt. For high levels of income risk, debt becomes risk free due to the higher demand on domestic debt. At this point, both allocations display similar levels of debt, being the reason the fact that debt is priced according to default probabilities.

On the other hand, wealth dispersion leads unambiguously to a lower domestic debt and higher external debt in the competitive equilibrium allocation than in the social optimum. This debt structure makes default more appealing for an utilitarian government.

1.A Appendix: Algorithm

Competitive equilibrium

Period 1: Solve for the default decision given $(B_1^f, b_1^i, g_1, \epsilon^i)$, for $i = 1, 2$

1. Draw g_1 and ϵ^i from their respective distribution function. Both shocks are assumed to be i.i.d. The government spending shock is assumed to have 20 states, while the idiosyncratic shock has 2: high and low. All realizations of the shock have the identical probability.
2. Build a grid for B_1^f and b_1^i . The grids are formed by 70 nodes for each state variable. We allocate a higher numbers of knots around 0. Compute Total debt as $B_1 = B_1^f + \sum_i \mu^i b_1^i$
3. Compute the default decision $d(B_1^f, b_1^i, g_1, \epsilon^i)$ as the solution to the following problem

$$\begin{aligned}
 V_1(g_1, B_1^f, b_1^i, \epsilon^i) &= \max_{d_1 \in [0,1]} [V_1^R(g_1, B_1^f, b_1^i, \epsilon^i), V_1^D(g_1, B_1, b_1^i, \epsilon^i)] \\
 s.t. \\
 c_1^{i,R} &= \epsilon_1^i + b_1^i - T_1 \quad \text{if } d_1 = 0 \\
 c_1^{i,D} &= \epsilon_1^i(1 - \phi(g)) - T_1 \quad \text{if } d_1 = 1 \\
 T_1 &= g_1 + (1 - d(x))B_1
 \end{aligned}$$

Where

$$\begin{aligned}
 V_1^R(g_1, B_1^f, b_1^i, \epsilon^i) &= \sum_i \mu_i u^i(c_1^{i,R}) \\
 V_1^D(g_1, B_1^f, b_1^i, \epsilon^i) &= \sum_i \mu_i u^i(c_1^{i,D})
 \end{aligned}$$

Period 0: Find the policy function and the correspondent consumption rule for given $B_0^f, g_0, \epsilon_0^i, b_0^i$
 Compute the price of the bond q_0 by computing the default probabilities $p(g_1, B_1^f, b_1^i, \epsilon^i)$

$$p(B_1^f, b_1^i) = \sum_{\epsilon_1} \sum_{g_1} d(g_1, B_1^f, b_1^i, \epsilon^i) F(g'|g) F(\epsilon'|\epsilon)$$

and the foreign investor condition

$$q_0 = \frac{1 - p(B_1^f, b_1^i)}{1 + r}$$

Compute next period marginal utility using decision rules from period 1

$$E_{g,\epsilon}[(1 - d_1)u'(c_1^{i,R})]$$

and parametrize the price for the bond and the expected marginal utility for each agent as flexible polynomials of the state space, for example

$$E_{g,\epsilon}[(1 - d_1)u'(c_1^{i,R})] = h^i(B_1^f, b_1^i, \alpha) = \alpha_0^i + \alpha_1^i B_1^f + \alpha_2^i b_1^1 + \alpha_3^i b_1^2$$

To obtain the parameters, run a regression using k vectors of state variables. Then, we can estimate marginal utility as the fitted value for the regression:

$$h^i(B_1^f, b_1^i, \alpha) = \hat{\alpha}_0^i + \hat{\alpha}_1^i B_1^f + \hat{\alpha}_2^i (B_1^f)^2 + \dots + \hat{\alpha}_7^i b_1^2$$

For the case of prices

$$\Phi(B_1^f, b_1^i, \varphi) = \hat{\varphi}_0 + \hat{\varphi}_1 B_1^f + \dots + \hat{\varphi}_2 b_1^1 + \hat{\varphi}_3 b_1^2$$

At time 0, an allocation satisfying the competitive equilibrium must verify the euler equation for each agent, the foreign investor condition and the government budget constraint. The system form,

1.

$$\begin{aligned} T_0 &= g_0 + B_0 - \Phi(B_1^f, b_1^i, \varphi)[B_1^f + \sum_i \mu^i b_1^i] \\ \Phi(B_1^f, b_1^i, \varphi) &= u'(\epsilon_0^i + b_0^i - \Phi(B_1^f, b_1^i, \varphi)b_1^i - T_0) = h^i(B_1^f, b_1^i, \alpha) \end{aligned}$$

2. substitute $\Phi(B_1^f, b_1^i, \varphi)$ and $h^i(B_1^f, b_1^i, \alpha)$ by the estimated parameters

3. solve the previous system to find the T_0 for each point in the grid for B_1^f, b_1^i

At time 0, the problem for the government enforcing a competitive equilibrium allocation is given by:

$$\begin{aligned} V_0(B_0^f, b_0^i) &= \max_{\{c_0^i, b_1^i, B_1^f, T_0\}} \sum_i \mu^i u^i(c_0^i) + \beta E[V_1(B_1^f, b_1^i, g_1, \epsilon_1^i)] \\ s.t. \\ c_0^i &= \epsilon_0^i + b_0^i - \Phi(B_1^f, b_1^i, \varphi)b_1^i - T_0 \\ T_0 &= g_0 + B_0 - \Phi(B_1^f, b_1^i, \varphi)[B_1^f + \sum_i \mu^i b_1^i] \\ \Phi(B_1^f, b_1^i, \varphi)u'(c_0^i) &= h^i(B_1^f, b_1^i, \alpha) \end{aligned}$$

Where taxes T_0 are the ones supporting a competitive equilibrium allocation

Constrained planner problem

Period 1: Solve for the default decision given x_1 , a vector containing the state variables $(B_1^f, b_1^i, g_1, \epsilon^i)$.

1. Draw g_1 and ϵ^i from a probability distribution. Both shocks are i.i.d. The government spending shock is assumed to have 20 states, while the idiosyncratic income shocks has two: high and low. All realizations of the shock have the identical probability.
2. Build a grid for B_1^f and b_1^i . The grids are formed by 70 nodes for each state variable. We allocate a higher number of knots close to zero. Aggregate debt is $B_1 = B_1^f + \sum_i \mu^i b_1^i$
3. Compute the default decision $d(B_1^f, b_1^i, g, \epsilon^i)$ as the solution to the following problem

$$V_1(g_1, B_1^f, b_1^i, \epsilon^i) = \max_{d_1 \in [0,1]} [V_1^R(g_1, B_1^f, b_1^i, \epsilon^i), V_1^D(g_1, B_1^f, b_1^i, \epsilon^i)]$$

s.t.

$$c_1^{i,R} = \epsilon_1^i + b_1^i - T_1 \quad \text{if } d_1 = 0$$

$$c_1^{i,D} = \epsilon_1^i(1 - \phi(g)) - T_1 \quad \text{if } d_1 = 1$$

$$T_1 = g_1 + (1 - d_1)B_1$$

Where

$$V_1^R(g_1, B_1^f, b_1^i, \epsilon^i) = \sum_i \mu_i u^i(c_1^{i,R})$$

$$V_1^D(g_1, B_1^f, b_1^i, \epsilon^i) = \sum_i \mu_i u^i(c_1^{i,D})$$

Period 0: Find the policy function and the correspondent consumption rule $B_0^f, g_0, \epsilon_0^i, b_0^i$
 Compute the price of the bond q_0 by computing the default probabilities $p(g_1, B_1^f, b_1^i, \epsilon^i)$

$$p(B_1^f, b_1^i) = \sum_{\epsilon_1} \sum_{g_1} d(g_1, B_1^f, b_1^i, \epsilon^i) F(g'|g) F(\epsilon'|\epsilon)$$

and using the foreign investor condition

$$q_0 = \frac{1 - p(B_1^f, b_1^i)}{1 + r}$$

where $F(g'|g)$ and $F(\epsilon'|\epsilon)$ are the cumulative distribution function for the government and the idiosyncratic income shock respectively. Approximate the price for the bond by performing a projection over the state space

$$\Phi(B_1^f, b_1^i, \varphi) = \hat{\varphi}_0 + \hat{\varphi}_1 B_1^f + \dots + \hat{\varphi}_2 b_1^1 + \hat{\varphi}_3 b_1^2$$

At time 0, the problem for the constrained planner is given by:

$$\begin{aligned}
 V_0(B_0^f, b_0^i) &= \max_{\{c_0^i, b_1^i, B_1^f, T_0\}} \sum_i \mu^i u^i(c_0^i) + \beta E[d_1 V_1^D(x) + (1 - d(x)) V_1^R(x)] \\
 s.t. \\
 c_0^i &= \epsilon_0^i + b_0^i - \Phi(B_1^f, b_1^i, \varphi) b_1^i - T_0 \\
 T_0 &= g_0 + B_0 - \Phi(B_1^f, b_1^i, \varphi) [B_1^f + \sum_i \pi^i b_1^i]
 \end{aligned}$$

The grid search

1. fix a level for B_0^f and b_0^i
2. compute T_0 using the government budget constraint and the function $\Phi(B_1^f, b_1^i, \varphi)$ for the price
3. compute consumption for each agent using household budget constraint and the function $\Phi(B_1^f, b_1^i, \varphi)$ for the price
4. compute $\sum_i u^i(c_0^i)$
5. compute $E_{g, \epsilon}[V_1(x)]$ using the default decision and taxes computed in the previous stage
6. search the max over the grid

1.B Appendix: Tables and figures

TABLE 1.1: Model parameters

Description	Parameter	Value
Discount factor	β	0.96
Risk aversion		1
Average Income	ϵ	1
Average initial bondholdings	B_0^d	0.40
Initial foreign debt	B_0^f	0.10
Aggregate debt	$B_0^f + B_0^D$	0.50
Govt expenditure (t=0)	g_0	0.18
Std Deviation for govt spending shock	σ_g	0.015
Risk free interest rate	r	0.02
Output cost of default	ϕ	4.0

Notes: β is the discount factor. Income is normalized to one. Initial aggregate debt is 0.50 and stands for the sum of domestic and foreign debt. r is the international interest rate for the foreign investors.

TABLE 1.2: Equilibrium for different levels of the standard deviation of the idiosyncratic shock

	Competitive equilibrium			Constrained Planner		
	Standard deviation income shock σ_y					
Variable	0 (RA)	0.1	0.2	0 (RA)	0.1	0.2
Aggregate debt	0.21	0.28	0.29	0.29	0.29	0.29
Foreign debt	0.06	0.06	0.03	0.04	0.04	0.03
Domestic debt	0.15	0.22	0.26	0.25	0.25	0.26
T_0	0.47	0.40	0.37	0.40	0.40	0.40
Default probability	0.10	0.10	0.0	0.05	0.05	0.0

Note: The table shows the allocation under the competitive equilibrium and the constrained planner for three different levels of the standard deviation of the the idiosyncratic income shock. Average income in both periods is fixed to one. RA stands for representative agent

TABLE 1.3: Equilibrium for different levels of initial wealth inequality

	Competitive equilibrium			Constrained Planner		
	Standard deviation of the initial wealth distribution					
Variable	0.0 (RA)	0.20	0.40	0.0 (RA)	0.20	0.40
Aggregate debt	0.21	0.24	0.30	0.29	0.29	0.27
Foreign debt	0.06	0.06	0.08	0.04	0.04	0.03
Domestic debt	0.15	0.18	0.22	0.25	0.25	0.24
T_0	0.47	0.45	0.45	0.40	0.41	0.42
Default probability	0.10	0.15	0.40	0.05	0.05	0.10
(b_R^1/b_1^2)	1.00	0.25	0.02	1.00	0.41	0.08

Notes: The table shows the allocation under the competitive equilibrium and the constrained planner for three different levels of initial wealth inequality measured as the standard deviation of the wealth distribution. RA stands for representative agent. Aggregate initial wealth is not fixed in 0.2 through all the experiments.

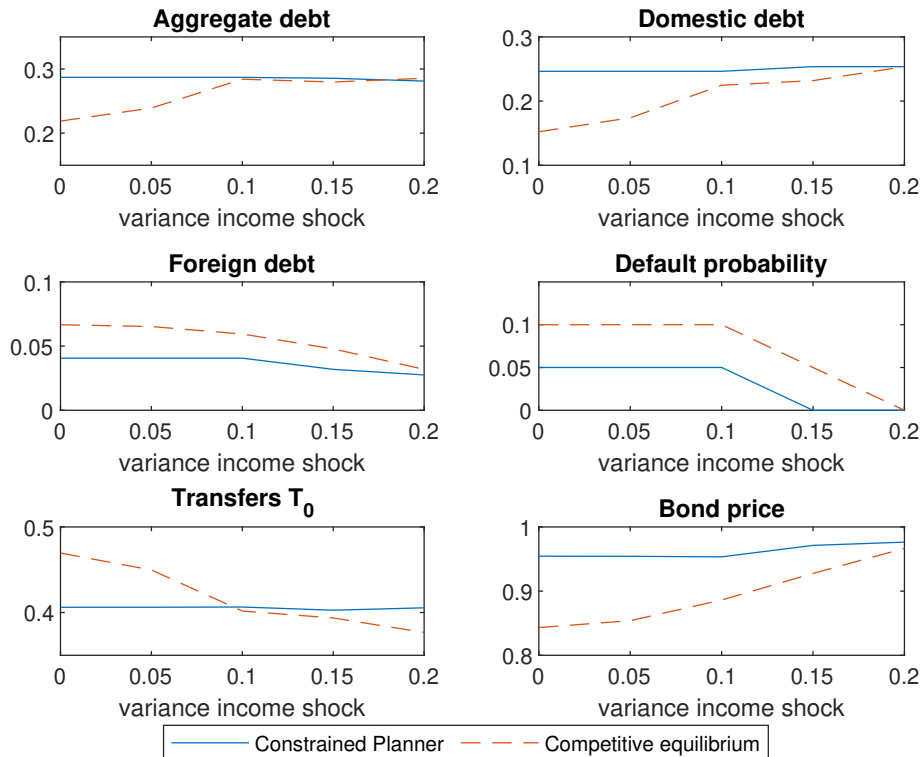


FIGURE 1.1: Optimal policies and the standard deviation of the idiosyncratic income shock

Note: the figure shows the the main equilibrium results for different values of the standard deviation of the idiosyncratic income shock. The dotted red line is the competitive equilibrium allocation, while the blue line is the constrained efficient allocation. Aggregate debt is the sum of domestic and foreign debt. Domestic debt is the weighted average across residents. Transfers correspond to the first period.

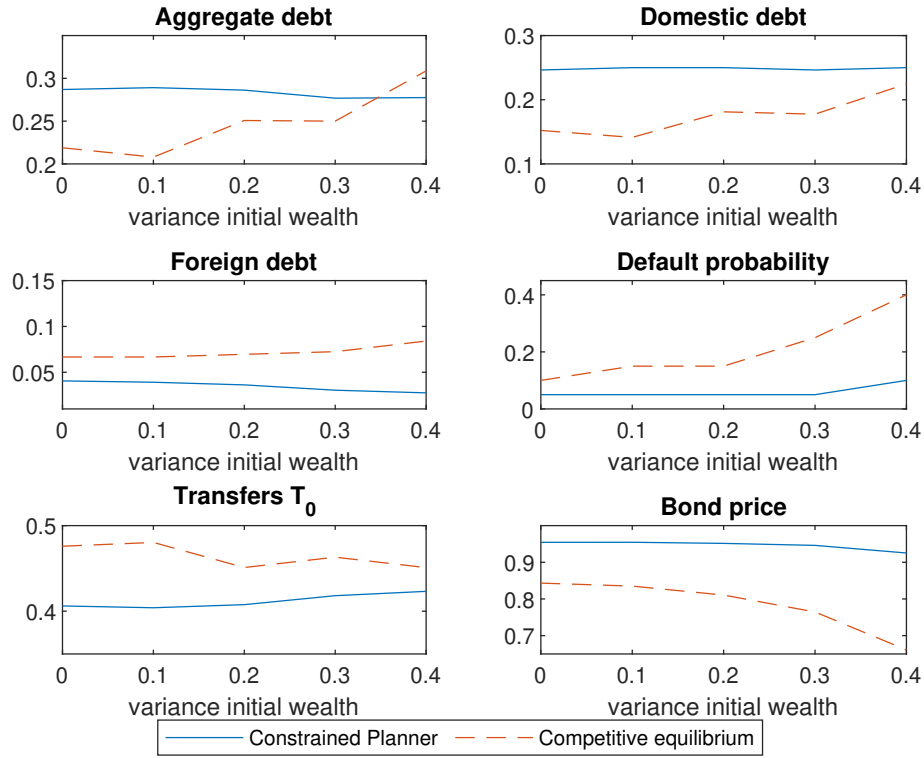


FIGURE 1.2: Optimal policies and the standard deviation of initial wealth

Note: the figure shows the the main equilibrium results for different values of the standard deviation of the initial wealth endowments. The dashed red line is the competitive equilibrium allocation, the blue line is the constrained optimum. Aggregate debt is the sum of domestic and foreign debt. Domestic debt is the weighted average across residents.

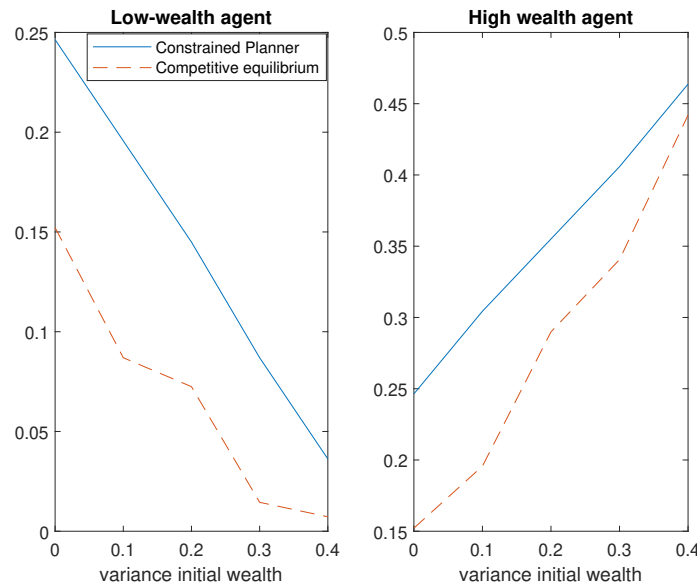


FIGURE 1.3: Optimal policies and the standard deviation of initial wealth: low and high wealth household

Note: the figure shows the main equilibrium result for different values of the standard deviation of the initial wealth endowment. The dashed red line is the competitive equilibrium allocation, the blue line is the constrained optimum.

1.B.1 Lower default costs

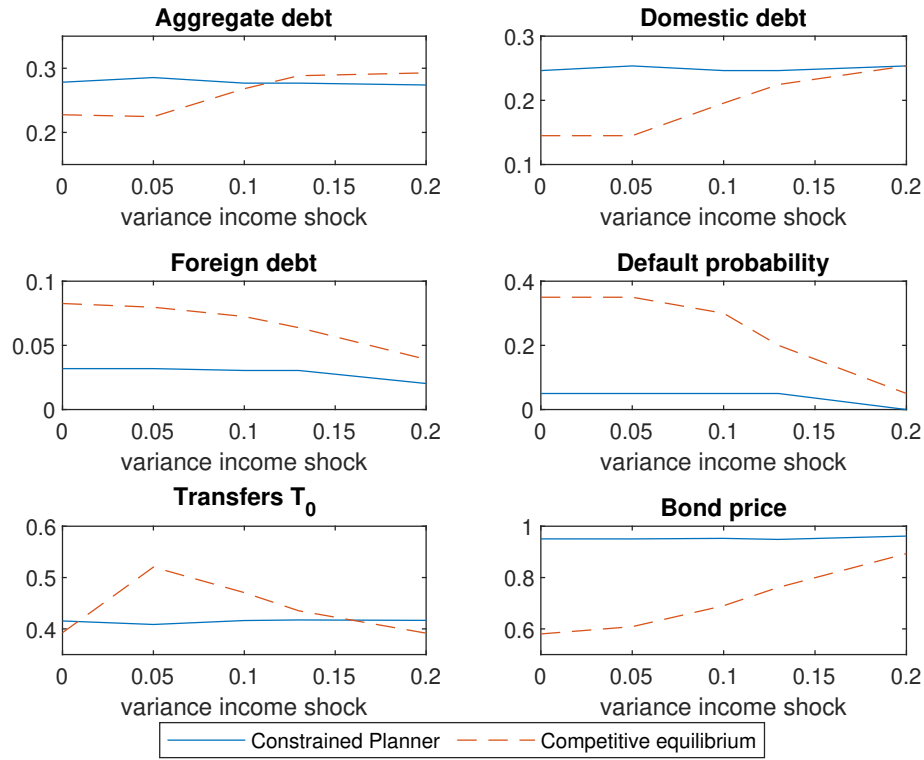
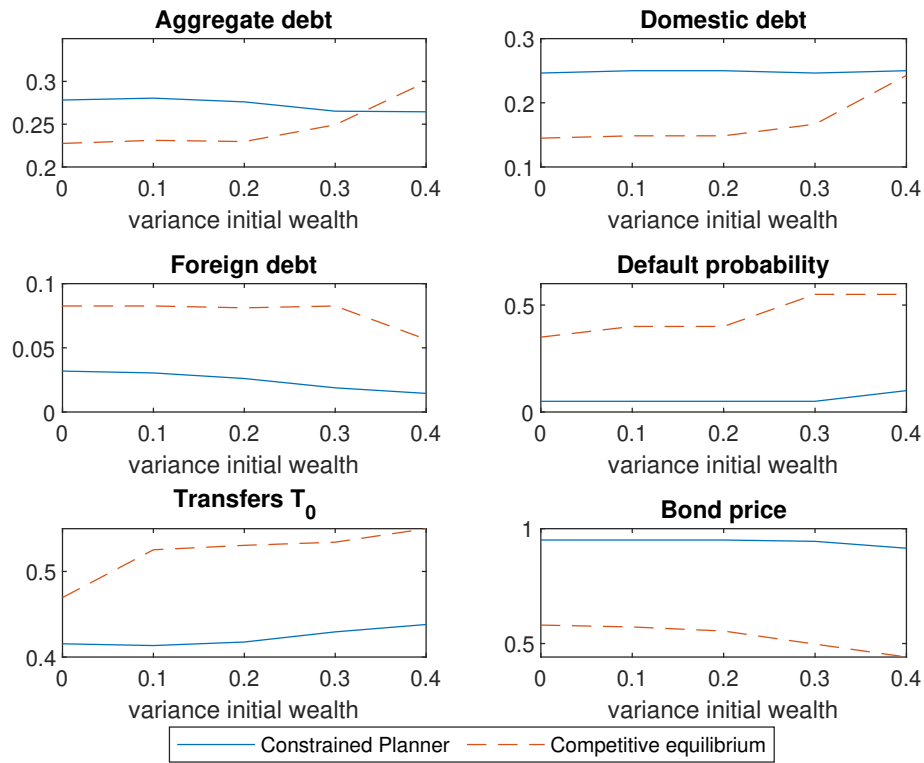
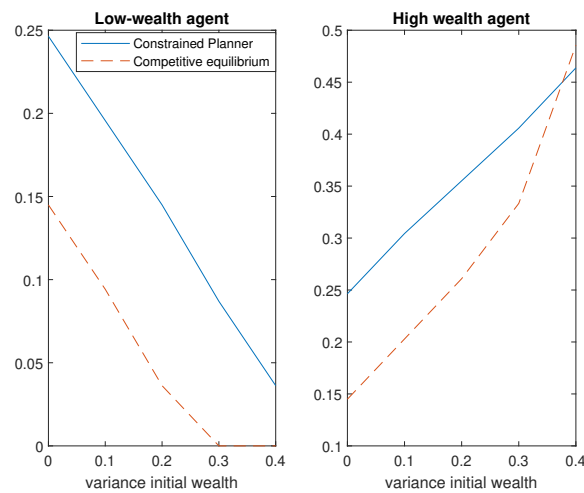


FIGURE 1.4: Optimal policies and the standard deviation of the idiosyncratic income shock, $\phi = 3$

Note: the figure shows the the main equilibrium results for different values of the standard deviation of the idiosyncratic income shock. The dotted red line is the competitive equilibrium allocation, while the blue line is the constrained efficient allocation. Aggregate debt is the sum of domestic and foreign debt. Domestic debt is the weighted average across residents. Transfers correspond to the first period.

FIGURE 1.5: Optimal policies and the standard deviation of initial wealth, $\phi = 3$

Note: the figure shows the the main equilibrium results for different values of initial wealth inequality. The dashed red line is the competitive equilibrium, the blue line is the constrained optimum. Aggregate debt is the sum of domestic and foreign debt. Domestic debt is the weighted average across residents

FIGURE 1.6: Optimal policies low and high wealth household, $\phi = 3$

Note: the figure shows the main equilibrium result for different levels of initial wealth inequality. The dashed red line is the competitive equilibrium, the blue line is the constrained optimum.

1.B.2 Lower initial foreign debt

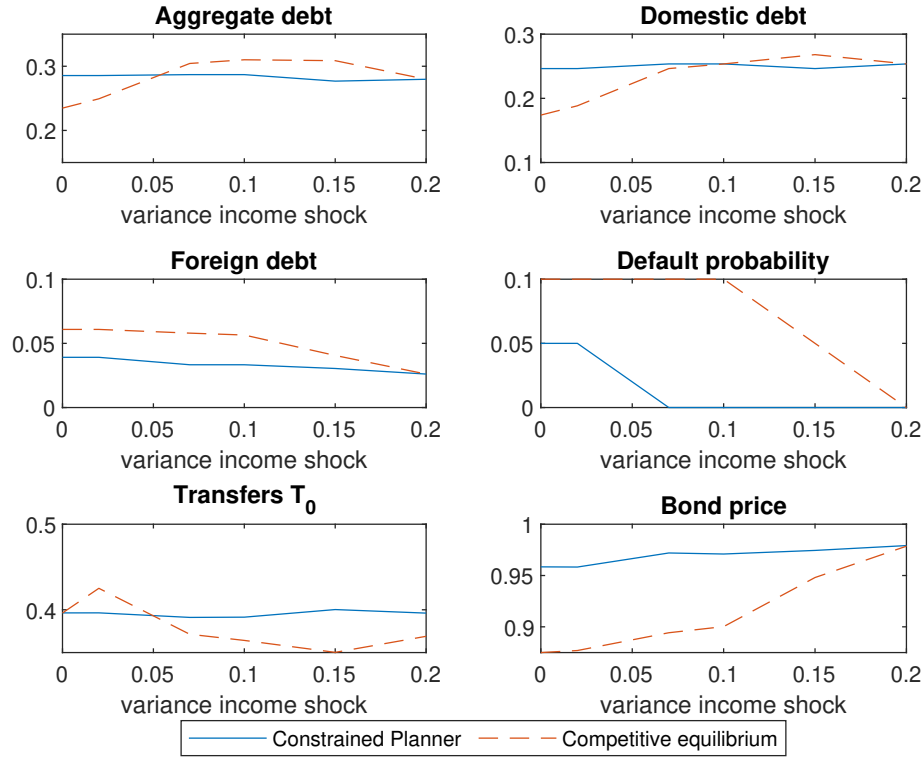


FIGURE 1.7: Optimal policies and the standard deviation of the idiosyncratic income shock, $B_0^f = 0.09$

Note: the figure shows the the main equilibrium results for different values of the standard deviation of the idiosyncratic income shock. The dotted red line is the competitive equilibrium allocation, while the blue line is the constrained efficient allocation. Aggregate debt is the sum of domestic and foreign debt. Domestic debt is the weighted average across residents. Transfers correspond to the first period.

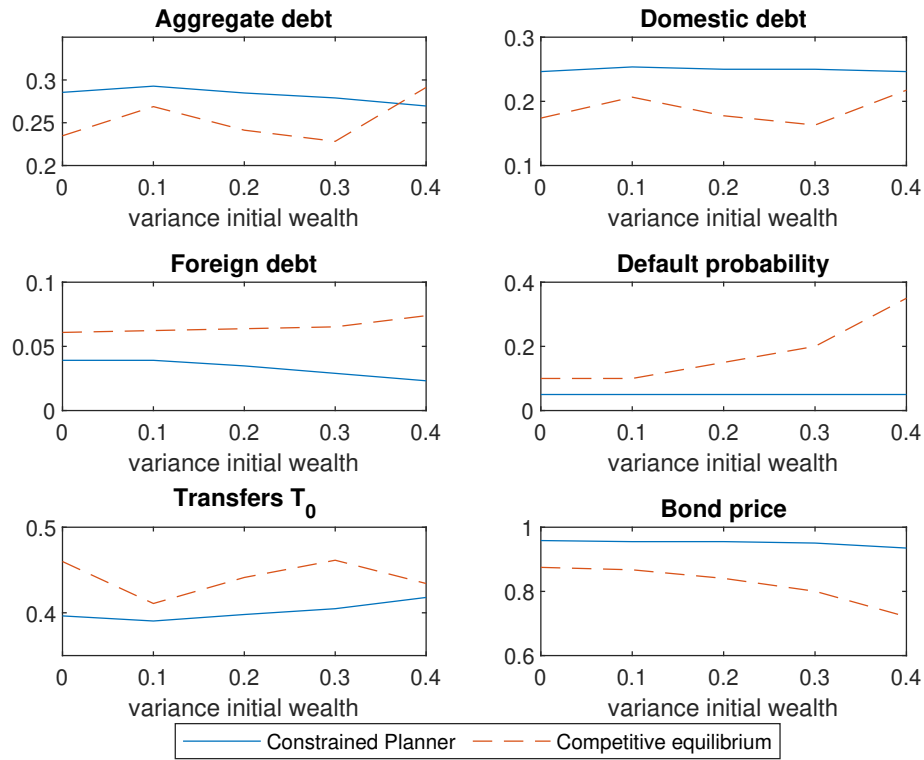


FIGURE 1.8: Optimal policies and the standard deviation of initial wealth, $B_0^f = 0.09$

Note: the figure shows the the main equilibrium results for different values of initial wealth inequality. The dashed red line is the competitive equilibrium, the blue line is the constrained optimum. Aggregate debt is the sum of domestic and foreign debt. Domestic debt is the weighted average across residents

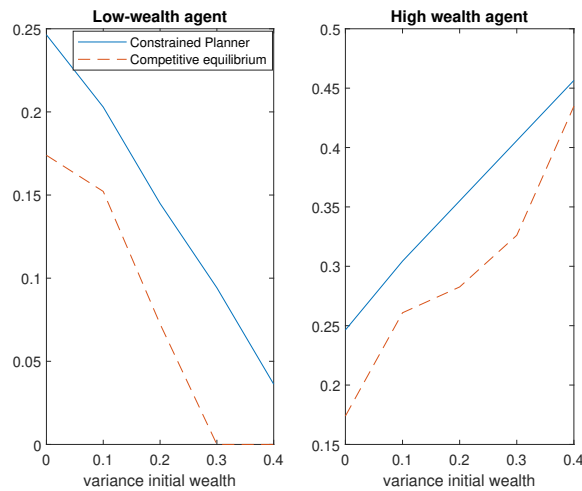


FIGURE 1.9: Optimal policies low and high wealth household, $B_0^f = 0.09$

Note: the figure shows the main equilibrium result for different levels of initial wealth inequality. The dashed red line is the competitive equilibrium, the blue line is the constrained optimum.

1.B.3 Higher initial government spending

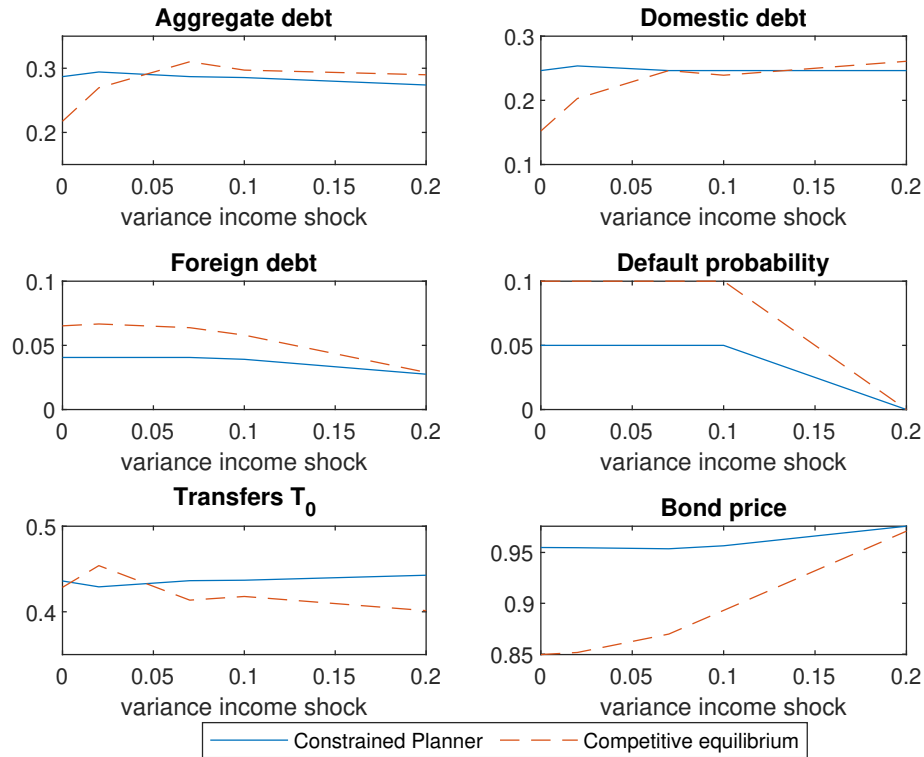


FIGURE 1.10: Optimal policies and the standard deviation of the idiosyncratic income shock, $g_0 = 0.21$

Note: the figure shows the the main equilibrium results for different values of the standard deviation of the idiosyncratic income shock. The dotted red line is the competitive equilibrium allocation, while the blue line is the constrained efficient allocation. Aggregate debt is the sum of domestic and foreign debt. Domestic debt is the weighted average across residents. Transfers correspond to the first period.

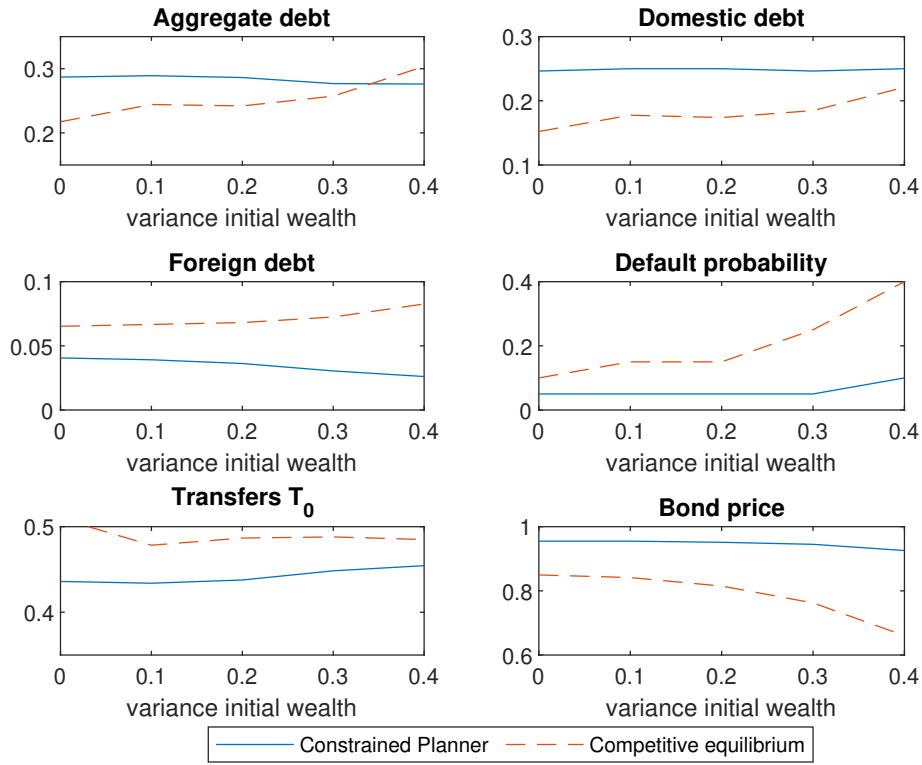


FIGURE 1.11: Optimal policies and the standard deviation of initial wealth, $g_0 = 0.21$

Note: the figure shows the the main equilibrium results for different values of initial wealth inequality. The dashed red line is the competitive equilibrium, the blue line is the constrained optimum. Aggregate debt is the sum of domestic and foreign debt. Domestic debt is the weighted average across residents

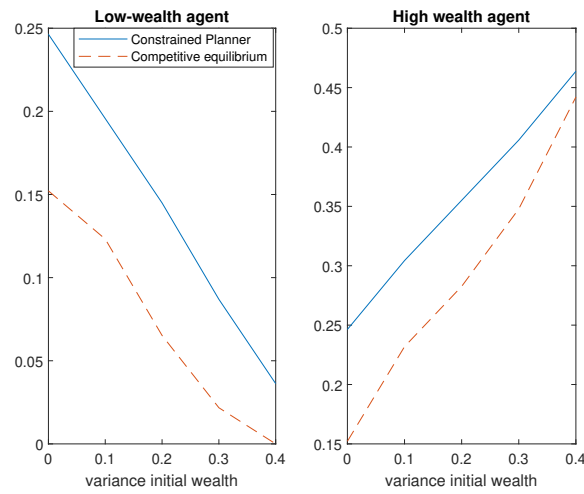


FIGURE 1.12: Optimal policies low and high wealth household, $g_0 = 0.21$

Note: the figure shows the main equilibrium result for different levels of initial wealth inequality. The dashed red line is the competitive equilibrium, the blue line is the constrained optimum.

Chapter 2

Optimal Fiscal Policy in the Fiscal Theory of the Price Level

It is well known that in standard DGSE models where fiscal policy follows ad hoc rules, switching from an active to a passive monetary regime, brings about a considerable increase in macroeconomic volatility, mainly due to inflation fluctuations reflect debt sustainability. We ask whether optimal fiscal policy would choose to set fiscal variables irresponsibly when debt is subject to fiscal inflation in the passive money case.

We do so by studying optimal fiscal policy in a standard New-Keynesian model in which the optimizing government can issue nominal non-contingent debt and is subject to an independent monetary policy setting the nominal interest rate according to an inflation targeting rule. The fiscal authority can stabilize the economy having several tools at its disposal, including government consumption, public investment and distortionary taxes. We focus on the case where the monetary authority sets the nominal interest rate to respond weakly to inflation, the so called passive money regime [Leeper \(1991\)](#). We compare the outcome of optimal fiscal policy in that case with the polar opposite, when the monetary authority aggressively responds to inflation.

We find that the differences in the optimal policy allocation under active/passive monetary policies are small when the fundamental disturbances that hit the economy are standard demand shocks. We show that this result holds both under full commitment and in the case where a time-consistent policy maker cannot commit to the future path of its fiscal variables. In both cases changing the monetary regime from active to passive only has a small effect on equilibrium outcomes.

Keywords: passive monetary policy, optimal fiscal policy, time-consistent equilibrium, public investment

JEL Classification: E31, E52, E62, H21, H54

2.1 Introduction

One of the main consequences of the Great Recession and the COVID-19 crisis has been the increase in public debt observed in most of the developed economies. In the US, for instance, figure 2.1 shows a sharp increase in the ratio of debt to GDP, which surpassed the 100% on December 2020. At the same time, as depicted in figure 2.2, the US has been experiencing a rapid surge in the inflation rate, which has been observed along with a weak reaction in the monetary policy rate.

A theoretical interpretation for this scenario can be found in the Fiscal Theory of the Price Level (FTPL). According to it, if forward-looking agents believe that government debt is not entirely backed by future tax revenues, debt issuance becomes inflationary (Cochrane (2001), Cochrane (2021)).¹

[Figure 2.1 approximately here]

[Figure 2.2 approximately here]

The benchmark model of the FTPL studies the interactions between monetary and fiscal policies assuming ad-hoc rules for interest rates and taxes. A well known property of this model is that switching from the active/passive (monetary/fiscal) scenario to the passive/active regime, brings about a considerable increase in macroeconomic volatility.² Shocks that hit the economy are filtered through the consolidated budget constraint and this produces additional variability in macroeconomic aggregates, inflation and output. Furthermore, the weak reaction of monetary policy to inflation has a pervasive effect on the economy, exacerbating inflation and increasing its persistence over time along with output and consumption volatility (Sims (2011), Leeper et al. (2010)).

This paper asks whether this conclusion is robust towards assuming that a benevolent government can set fiscal variables optimally and monetary policy backs the optimal plan by tolerating inflation deviations from target, through not responding aggressively to inflation. In other words, we ask whether in the case where monetary policy is committed to be passive (setting the inflation coefficient in the interest rate rule to be below unity) is fiscal policy is bound to choose fiscal variables more irresponsibly, leading to considerable variability in macroeconomic aggregates, output, inflation etc.

To answer this question we use a model in the spirit of Schmitt-Grohé and Uribe (2004) and Lustig et al. (2008b) assuming that fiscal policy variables in the economy are set by a Ramsey planner

¹A growing literature has tried to explain past inflationary episodes in the US, such as the '70s using the FTPL. See, for instance Bianchi (2013), Bianchi and Ilut (2017) and further references below.

²In his influential paper Leeper (1991) considers two types of equilibria. In one case monetary policy is active (it responds aggressively to inflation) and fiscal policy is passive (adjusts taxes to make debt solvent intertemporally) in another case monetary/fiscal policies are passive/active, inflation adjusts to make debt solvent and taxes are (roughly) constant through time. Following Leeper (1991), numerous papers have extended this framework in different directions, a literature which we summarize in the next section.

in an environment where prices are sticky and bond markets are incomplete, the government is restricted to issue nominal long term non- state contingent debt.

The optimizing authority can set distortionary labor income taxes, public consumption and public investment spending. Considering public capital as a policy instrument, is quite novel in the literature on optimal fiscal policy (see later on for a few references) and makes our framework more realistic.³

To solve for optimal policy we use a second order approximation to the household utility function and compare the properties of the business cycle when monetary policy is active and passive by allowing for different responsiveness of the Taylor rule to inflation. When monetary policy is active, we set the coefficient of the Taylor rule responding to inflation in 1.5 as in [Woodford \(2001\)](#) and the majority of the literature on New-Keynesian DSGE models.⁴ When monetary policy is passive, we perform the analysis under two values for the coefficient of the Taylor rule targeting inflation: 0.95 and 0.8. The first value is close to the level in which monetary policy becomes active and follows estimations from DSGE models for the period starting in 2005 as in [Bianchi \(2013\)](#). The second is an arbitrary value in which monetary policy reacts even less to inflation and serves as a more strict scenario for passive monetary policy.

Using this setup, we consider separately the case where the fiscal authority can commit to the entire path of fiscal variables (the full commitment allocation) and the time-consistent optimal policy where the planner cannot commit to the path of future taxes and spending levels. Our key finding is that under both of these well known setups of optimal policy, the equilibrium does not change considerably when monetary policy is passive, relative to the case where it is active, and explicitly targets inflation. Ultimately, with passive monetary policy, the planner adjusts the tax rate and debt issuance abandoning the tax smoothing behavior just as we observe with the active monetary policy case. By adjusting taxes, the planner avoids the costs linked to a high and volatile inflation rate usually seen with passive monetary policy and an active rule for taxes.

Under commitment, the optimal path for most of the variables in response to a negative demand shock is similar regardless interest rate response to inflation. Taking as reference the economy under rules, with passive monetary policy, taxes do not respond to debt, thus the government resorts on inflation to relax the budget constraint. When the government can optimally set the tax rate, the latter increases to finance the fiscal gap, while public debt adjust by more under the passive rule. Meanwhile, government consumption and investment increases slightly by more under passive monetary policy to close the output gap and stabilize private consumption. The more noticeable difference between active and passive monetary policy when fiscal policy is optimal is the response of inflation, which picks up faster when monetary policy is passive, a behaviour also observed with rules but into a much larger extent.

Considering the previous result, we study optimal fiscal policy in a time-consistent environment. An experiment, we do mainly for two reasons. First, a time-consistent planner is, in general,

³For instance in the American Recovery and Reinvestment Act of 2009, public investment accounted for nearly 40 % of the the non-transfer spending.

⁴We used the value from [Woodford \(2001\)](#) despite some works estimate the coefficient to be well above 1.5 ([Carvalho \(2019\)](#), [Mehra and Minton \(2007\)](#)) during the Greenspan era, a period in which monetary policy was considered to be active

less able to spread the burden of taxation across time, leading to higher volatility in the economy. Secondly, from a more empirical perspective, one can think of fiscal stimulus plans, such as the ones approved during the Great Recession or the COVID-19 recession, as the result of time-consistent policies. Indeed, in a time-consistent equilibrium, the planner chooses uncontingent policies from which future policy makers will not deviate from, given the path for state variables and under the assumption no future shocks will hit the economy. The stimulus packages approved for instance, during the Great Recession or the COVID-19 recession, could resemble the outcome of time-consistent decisions, if one accepts that they were the result of an agreement between political parties who agree not to revise once in power.

The main result does not change with discretion and the volatility of macro aggregates is very similar under active and passive monetary policy when the economy is hit by a negative demand shock. Nevertheless, optimal policies under discretion are qualitatively very different from the ones prescribed by a committed planner. After the arrival of the shock, the planner cuts distortionary taxes and public investment while increases public consumption and debt. By increasing debt and cutting public capital, the planner increases inflation expectations. An increase in debt raises concerns about repayment, and private sector expects higher inflation to relax the government budget constraint. A cut in public capital, increases future real marginal cost and prices, thus the planner relies on the supply-side of such a policy.⁵

We find our main result to be robust when the economy is hit by a liquidity premium shock and when monetary policy targets not only inflation but also output. Results are slightly different, however, when we confront the economy to a positive shock in the preference for public goods (government consumption). We find that private consumption falls by more under active monetary policy mainly due to higher nominal interest rates.

We show that when monetary policy is passive the planner optimally chooses to adjust the tax rate and debt issuance to fight back a negative demand shock, avoiding the costs of distortionary inflation coming along with passive monetary policy and usually seen with rules. Although, the planner uses actively public investment, there is little difference between the optimal path for public capital across both regimes. One reason could be that the real interest rate, pinned down by the marginal productivity of public capital, is dependent on the Taylor rule, not in direct control by the planner. Along the paper, we restrict the planner to issue long-term debt to capture debt maturity structure in developed countries, but we argue in a separate section that results will not change dramatically if the planner issues short-term debt instead. This is because the planner is constrained by the Taylor rule, thus some channels through which the planner can affect borrowing terms are silent. Still, results could change if the planner has access to a more flexible maturity structure as in [Chafwehé et al. \(2022\)](#).

⁵See [Bouakez et al. \(2017\)](#) and [Bouakez et al. \(2019\)](#) for a discussion regarding the supply side and demand side effects of government investment

2.2 Related Literature

This paper is related to the literature on optimal fiscal policy and the Fiscal Theory of the Price Level (FTPL). This theory argues that since the real value of government debt must equal the sum of real future fiscal surplus, if current debt is not perceived to be backed by future taxes, inflation may raise (Cochrane (2001), Cochrane (2021)) to relax the budget constraint of the government.

In much of the literature on the FTPL, such as Cochrane (2001), Sims (2011), Cochrane (2001), among others, monetary and fiscal policy obeys ad-hoc rules. Monetary policy is usually characterized by a Taylor rule in which nominal interest rate reacts weakly to inflation, while fiscal policy is resumed by a rule in which taxes or fiscal surplus reacts to lagged values of debt. Within this framework, the FTPL could be characterized using Leeper (1991)s' terminology by an Active Fiscal Policy/Passive Fiscal policy mix. This literature has shown that the response of the economy to demand or fiscal shocks under passive and active monetary policy could be very different. Although, we can not compare directly the results with rules and optimal fiscal policy, we try instead to highlight the role for optimal taxation when monetary policy is passive. In other words, we show that the common assumption of using rules to model monetary and fiscal policy interactions is not innocuous at the moment of describing the economic behaviour.

The paper is also related to the literature on the interaction between monetary and fiscal policy. Along these lines, there is strand of the literature analyzing joint optimal fiscal and monetary policy, such as Schmitt-Grohé and Uribe (2004), Leith and Wren-Lewis (2013), Eggertsson (2011). In all this papers, the planner has the ability to manipulate the bond price by setting optimally the interest rate. In this paper, in turn, the planner rely on taxes and debt to stabilize the economy foregoing price stability and allowing instead a higher and persistent inflation rate. This environment can capture, for instance, the institutional arrangement in which an independent central bank has its different objectives from those of the government. It can also describe an environment in which the government has little influence in bond prices, for instance, when the country has debt management units unable to control fiscal deficit. In such a world, shocks have an impact on the government budget constraint affecting inflation. We show, that if fiscal policy is optimal, there is little difference at the time of stabilizing the economy with respect to an active Taylor rule.

On a more positive side, there is a literature showing that past episodes of macroeconomic volatility such those observed in the '60s or '70s in the US, were linked to passive monetary policy and lack of responsiveness of fiscal variables to changes in debt (see Bianchi and Melosi (2019), Bianchi and Melosi (2017), Nicolò (2020), among others). In other words, the behaviour of the FED funds rate together with fiscal policy in the US resembles the passive monetary-active fiscal policy regime identified by Leeper (1991). In this paper, we take a normative approach by showing that when fiscal policy becomes optimal, the aggressiveness of monetary policy to fight inflation back is a second concern for the planner.

Jia (2020) analyzes fiscal policy under passive monetary policy rules, though the paper is more concerned with the implications for debt maturity. Jia (2020) also derives optimised simple rules

to mimic the outcome of a joint optimal monetary and fiscal policy choice. In this paper, we do not address joint optimal policies, which are extensively reviewed by the literature, but we focus on comparing the business cycle properties under active and passive monetary policy when the planner can optimally choose a thorough set of fiscal variables such as government consumption, investment and distortionary taxes.

Finally, this paper contributes also to the literature studying public investment as a stabilization tool. This is the first paper to consider public capital as a stabilization tool in a time-consistent environment. The recently approved fiscal packages in Europe and the US gave rise to a literature, such as [Leeper et al. \(2010\)](#), [Drautzburg and Uhlig \(2015\)](#) or [Coenen et al. \(2013\)](#) analyzing the effectiveness of public capital to boost the economy. None of these papers deals with optimal government investment. [Bouakez et al. \(2019\)](#) investigates optimal public investment as a stabilization tool, though it does not analyze neither the way to finance such a policy nor the behaviour of optimal policies under discretion as this paper analyzes. Another strand of the literature on public capital is concerned on the properties of optimal public capital for long-term growth ([Ambler and Paquet \(1996\)](#), [Glomm and Ravikumar \(1994\)](#)), but not as a tool for stabilizing the economy in response to macroeconomic shocks.⁶

Section 3.3 contains the model environment while 3.3.4 defines the a private equilibrium. Section 2.5 describes the optimal policy problems under commitment and no-commitment. Section 3.5.3 contains the numerical results for the main model. In section 2.7, we consider several extensions: A Taylor rule tracking inflation and output, a liquidity premium shock increasing the demand for bonds and a shock in preferences for government consumption in which public expenditure becomes quasi-exogenous. We also discuss some of the main assumptions of the model, such as the relevance of public capital and the presence of long-term debt to drive the results in section 2.8. Finally, section 3.9 concludes.

2.3 The economy

The framework is a standard New-Keynesian model, with imperfect competition in which firms set prices as in [Rotemberg \(1982\)](#). Households choose labor to supply to firms, consumption and savings. The model is enriched with a large fiscal block including public consumption, from which households enjoy utility, and public investment, which serves as an external input for the monopolistically competitive firm.

The government issues non-state contingent nominal long-term bonds and collects revenues from a distortionary tax, levied on labor income. We model an economy with long-term bonds to capture the maturity structure in developed economies. In line with the literature on optimal fiscal policy, we assume that the government does not have access to lump-sum transfers, which makes the optimal policy program non-trivial as Ricardian equivalence does not hold.

⁶[Azzimonti et al. \(2009\)](#) analyzes optimal public investment in a time-consistent environment, though there is no debt, nor nominal rigidity.

Monetary policy follows a Taylor rule for the nominal interest rate targeting inflation only as in e.g. [Leeper \(1991\)](#).

The presence of nominal rigidities and distortionary taxes precludes an inefficient steady-state which we correct by a subsidy in the same way as [Leith and Wren-Lewis \(2013\)](#) and [Matveev \(2021\)](#). The derivation of the efficient steady-state and the subsidy is in Appendix 2.A.

2.3.1 The environment

Households

The economy is inhabited by identical agents who have the following preferences:

$$E_t \sum_{t=0}^{\infty} \beta^t \left(\epsilon_t \frac{c_t^{1-\sigma}}{1-\sigma} - \chi \frac{n_t^{1+\varphi}}{1+\varphi} + \chi_g \frac{G_t^{c1-\sigma}}{1-\sigma} \right) \quad (2.1)$$

where β denotes the usual discount factor, c_t is consumption, n_t are hours worked and G_t^c is a non-storable public good. We assume, σ , φ , χ and χ_g are all positive parameters.

ϵ_t is a shock to preferences that changes the relative valuation of current and future consumption utility flows. A negative shock induces agents to postpone consumption. Moreover, the ϵ_t changes the (intrapersonal) marginal rate of substitution between consumption and the two other arguments of utility, hours and government consumption. Therefore a negative shock also induces the household to exert less work effort and to desire relatively more public consumption.

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Households enter each period t with $B_{t-1,S}$ units of a short-term bond and $B_{t-1,L}$ units of a long-term bond. We consider that both types of debt are risk-free assets, meaning that debt is default free. Households receive wages from supplying labor and rental payments for owning firms. The household budget constraint can be written as:

$$P_t c_t + q_{t,S} B_{t,S} + q_{t,L} B_{t,L} = (1 - \tau_t) W_t n_t + P_t D_t + B_{t-1,S} + (1 + \rho q_{t,L}) B_{t-1,L} + T_t \quad (2.2)$$

where W_t is the nominal wage and τ_t are labor taxes, both taken as given by households. D_t are dividends.⁸ We denote with $q_{t,S}$ the price for the short-term government bond and $q_{t,L}$ the price

⁷Notice that this is a different approach to modelling a demand shock than many other papers. It is common in the literature, to model demand shocks as shocks to the discount factor (thus a shock enters multiplicatively to utility and concerns all arguments, not just C).

This modelling approach follows closely [Fraglia et al. \(2013\)](#), who study optimal monetary and fiscal policy in a model where government spending is exogenous. Relative to them we use a setup where public spending is endogenous. Moreover, in section 2.7.2 we will follow a different approach and model the demand shock as a liquidity premium shock as in for example [Smets and Wouters \(2007\)](#). Using this alternative setup we will show that our results are robust.

⁸ T_t are lump-sum transfers by the government. Recall that we will not treat T_t as a policy instrument for the optimizing government. T_t is used here because to derive a second order approximation to the household's welfare (see below) we must subsidize firms to reach the efficient level of production. T_t will be assumed to finance that subsidy.

for $B_{t,L}$, a perpetuity with coupon payments decaying at rate $0 \leq \rho \leq 1$ as in [Woodford \(2001\)](#). When $\rho = 0$, we assume the government issues only short-term debt.

Firms

The economy produces a final good that can be used for consumption and investment. The firm that produces this final good is assumed to operate in a perfectly competitive market. The technology is a standard CES function whose inputs are intermediate products:

$$Y_t = \left(\int_0^1 Y_t(z)^{1-1/\theta} dz \right)^{\frac{\theta}{\theta-1}}$$

where $Y_t(z)$ is the quantity of intermediate good z used in the production of the final good and $\theta \geq 1$ denotes the elasticity of substitution between intermediate goods.

Firms producing intermediate goods have market power, every firm produces a differentiated good, $Y_t(z)$. Denoting by $P_t(z)$ the price of the generic intermediate goods, we can use standard arguments to show that the demand for $Y_t(z)$ is:

$$Y_t(z) = \left(\frac{P_t(z)}{P_t} \right)^{-\theta} Y_t \quad (2.3)$$

The generic firm operating in the intermediate goods sector uses labor and public capital to produce output. Notice that this is non-standard for the literature on optimal fiscal policy which typically abstracts from modelling public capital. As discussed previous, we consider public capital in this paper as an additional policy margin for fiscal policy because we seek to consider a broader range of government instruments that in the previous literature. Public investment is obviously an important part of government policy; for example the stimulus packages for the Great Recession and the COVID-19 recession in the US entailed large increases of public investment.

We consider public capital to be an externality to production, in other words, a input that is not chosen by firms, but increases firm output and the productivity of labour. The production function is Cobb-Douglas with constant returns to scale as in [Bouakez et al. \(2019\)](#):

$$Y_t(z) \leq F(N_t(z), K_{G,t-1}(z)) = N_t^\alpha(z) K_{G,t-1}^{1-\alpha}(z) \quad (2.4)$$

Firms set prices subject to adjustment costs as in [Rotemberg \(1982\)](#). The cost function for firm z is given by,

$$\Sigma_t(z) = \frac{\psi}{2} \left(\frac{P_t(z)}{P_{t-1}(z)} - 1 \right)^2 Y_t$$

Moreover, a further reason for subsidizing firm production is the following: [Leith and Wren-Lewis \(2013\)](#) showed that when considering time-consistent policies, the presence of a non-efficient steady-state creates incentives for the planner to distort the steady state by reducing debt. The subsidy allows a positive amount of sustainable debt at the steady-state. We thus follow [Leith and Wren-Lewis \(2013\)](#) to derive the subsidy.

which implies a resource cost when the price in t differs from the price in $t - 1$. Parameter $\psi \geq 0$ governs the magnitude of the price adjustment cost.

Dividends paid out by firm z are given by:

$$D_t(z) = \frac{P_t(z)}{P_t} Y_t(z) - w_t N_t(z)(1 - s) - \Sigma_t(z)$$

where s is a time-invariant employment subsidy used to eliminate distortions arising at the steady-state due to monopolistic competition and distortionary taxation as in [Leith and Wren-Lewis \(2013\)](#) and [Matveev \(2021\)](#). This subsidy is constant over time and as discussed previously it is financed with lump-sum transfer, T_t .

Government

Government policy sets the path of the following variables. First, the public consumption good G_t^c in a non-storable public good that impacts directly household utility. Second, public investment determines the quantity of public capital $K_{G,t}$ in the economy, which influences production and marginal production costs of firms (hence also firm pricing decisions). Third, to finance this spending, the government levies distortionary taxes and issues nominal debt.

Notice that we will assume that government debt is only long term. In other words, short-term debt is in zero net supply and is used to only to price the short term asset. For simplicity, we omit from the government budget constraint which we now state.

Let $b_{t,L}$ be the real value for the stock of long-term debt, the budget constraint in real terms can be written as:

$$q_t b_t = (1 + \rho q_t) \frac{b_{t-1}}{\pi_t} + (G_t^i + G_t^c - \tau_t w_t n_t) \quad (2.5)$$

Where G_t^i and G_t^c stands for expenditures in public investment and consumption respectively. We denote by π_t the gross inflation rate. Notice that the subsidy of the government to firms (discussed previously) does not appear in the budget constraint because it is completely compensated by a lump sum tax.

The law of motion for public capital is standard. For simplicity, we do not allow for delays in the accumulation of public capital (the common time to build assumption as in for example [Kydland and Prescott \(1982\)](#), or [Bouakez et al. \(2019\)](#)). Instead, we assume that capital is build immediately. For simplicity, we also abstract from investment adjustment costs. The law of motion for public capital is thus given by:

$$K_{G,t} = G_t^i + (1 - \delta_G) K_{G,t-1} \quad (2.6)$$

where $0 < \delta_G < 1$ is the depreciation rate.

Monetary policy

Monetary policy is assumed to follow a simple inflation targeting rule of the form:

$$R_t = (\pi_t / \pi^*)^{\phi_\pi} \quad (2.7)$$

Coefficient ϕ_π is a crucial object. As is well known, when $\phi_\pi > 1$ (the Taylor principle is satisfied), monetary policy is 'active' e.g. Leeper (1991). Under this condition, in equilibrium inflation is independent of government debt and is only driven by shocks to the Euler equation and the Phillips curve. In contrast, in the case where $\phi_\pi < 1$ then policy is passive and inflation is set to satisfy the intertemporal solvency of debt. In this equilibrium fiscal variables can affect inflation also due to their influence on the consolidated budget constraint.

2.4 Competitive equilibrium

We now define the competitive equilibrium for this economy given the sequence of government policies $\{G_t^c, G_t^i, \tau_t, B_{t,S}, B_{t,L}, K_{G,t}, R_t\}$ and the path of the exogenous demand shock $\{\epsilon_t\}$. We first derive the optimality conditions of households and firms. Subsequently, using the optimality conditions we provide a formal definition of the equilibrium.

2.4.1 Household problem

Households choose sequences $\{c_t, n_t, B_{t,S}, B_{t,L}\}$ to maximize utility subject to their budget constraints and the usual boundary No-Ponzi condition. As usual, households take government policies, wages and prices as given. The generic household's program (where the relevant vector of state variables at period t is $\{K_{G,t-1}, b_{L,t-1}, b_{S,t-1}, \epsilon_t\}$) can be written as:

$$V(B_{L,t-1}, B_{S,t-1}, K_{G,t-1}, \epsilon_t) = \max_{\{n_t, c_t, B_{L,t}, B_{S,t}\}} \left\{ U(c_t, n_t, G_t^c) + \beta E[V_{t+1}(B_{L,t}, B_{S,t}, K_{G,t}, \epsilon_{t+1})] \right\}$$

st.

$$P_t c_t + q_{t,S} B_{t,S} + q_{t,L} B_{t,L} = (1 - \tau_t) W_t n_t + P_t D_t + B_{t-1,S} + (1 + \rho q_{t,L}) B_{t-1,L} + T_t$$

Standard arguments imply that optimality conditions for households are given by

$$\frac{W_t}{P_t} (1 - \tau) = \frac{\chi n_t^\varphi}{\epsilon_t c_t^{-\sigma}} \quad (2.8)$$

$$q_{t,S} \epsilon_t c_t^{-\sigma} = \beta E_t \left[\epsilon_{t+1} \frac{c_{t+1}^{-\sigma}}{\pi_{t+1}} \right] \quad (2.9)$$

$$q_{t,L} \epsilon_t c_t^{-\sigma} = \beta E_t \left[\epsilon_{t+1} \frac{c_{t+1}^{-\sigma}}{\pi_{t+1}} (1 + \delta q_{t+1,L}) \right] \quad (2.10)$$

Equation 2.8 is the optimal labor supply condition while equations 2.9 and 2.10 are the Euler equations for short and long-term bonds respectively. Note also that for brevity we expressed these optimality conditions in terms of real debt holdings. π_{t+1} denotes the gross inflation rate. Thus equation 2.9 gives us the usual indifference condition between investing an additional unit of income in the short term asset and receiving the appropriately discounted random payoff $\frac{1}{\pi_{t+1}}$. Equation 2.10 in turn gives us the payoff of investing in the long term bond for one period and then acquiring the value of the investment, measured in terms of the resale price $q_{t+1,L}$ and discounted by inflation. In appendix 2.C we derive the complete problem for the household.

2.4.2 Firm's problem

The generic firm z in the intermediate sector sets prices and inputs to maximize the present value of dividends, appropriately discounted using the household real discount factor. We therefore have the following program:

$$\max_{P(z), N(z)} E_t \sum_{l=0}^{\infty} \beta^l \frac{U_{c,t+l+1}}{U_{c,t+l}} \left[\frac{P_{t+l}(z)}{P_{t+l}} Y_{t+l}(z) - (1-s) w_{t+l} N_{t+l}(z) - \frac{\psi}{2} \left(\frac{P_{t+l}(z)}{P_{t+l-1}(z)} - 1 \right)^2 Y_{t+l} \right]$$

subject to the constraints

$$Y_{t+l}(z) = \left(\frac{P_{t+l}(z)}{P_{t+l}} \right)^{-\theta} Y_{t+l}$$

$$Y_{t+l}(z) \geq F(N_{t+l}(z), K_{G,t+l})$$

$U_{c,t+l+1}/U_{c,t+l} = \epsilon_{t+l+1} c_{t+l+1}^{-\sigma} / \epsilon_{t+l} c_{t+l}^{-\sigma}$ is the discount factor of the households that own the shares of firm z .

We will focus on a symmetric equilibrium. Thus, all firms solve the same program and we can drop z from the notation from now on. The first order conditions with respect to prices P_t , labor, N_t are the following:

$$\frac{w_t(1-s)}{F_{N,t}} - \frac{\theta-1}{\theta} = \frac{\psi}{\theta} \left[\pi_t(\pi_t-1) - \beta E_t \left(\frac{\epsilon_{t+1}}{\epsilon_t} \frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}} \frac{Y_{t+1}}{Y_t} (\pi_{t+1}-1) \pi_{t+1} \right) \right]$$

$$-w_t(1-s) + \mu_t F_{N,t} = 0$$

$$Y_t = F(N_t, K_{G,t})$$

where we made use of the fact that all firms set the same price level in the symmetric equilibrium (equal by definition to the aggregate price level) and thus for brevity we expressed optimality with respect to P as a function of gross inflation $\pi_t = (P_t/P_{t-1})$ (appendix 2.C derives the complete problem for the firm). $\mu_t = \frac{w_t(1-s)}{F_{N,t}}$ is the real marginal cost of production, w_t are real wages, and $F_{N,t}$ is the marginal productivity of labor. Note that the first equation is the standard Phillips curve, determining the inflation output tradeoff in the model.

2.4.3 Definition of the competitive equilibrium

Definition CE: Given an exogenous process for the shock, ϵ_t , and initial levels of public debt B_{-1} , and public capital $K_{G,-1}$, a private sector equilibrium for this economy is a sequence of stochastic processes $\{c_t, n_t, y_t, \pi_t, w_t, b_t, k_{G,t}, g_t^i, g_t^c, \tau_t, R_t\}_{t=0}^{\infty}$ such that: (1) $\{c_t, n_t, b_t\}_{t=0}^{\infty}$ solves the household problem for given prices and government policies, (2) $\{\pi_t\}_{t=0}^{\infty}$ conforms the optimal pricing for the firms, (3) the government budget constraint is satisfied, (4) Monetary policy follow the inflation targeting rule and (5) the market for goods and labor clear.

The above is formalized by the following conditions:

$$\begin{aligned}
(1 - \tau_t) \frac{W_t}{P_t} &= \frac{\chi n_t^\varphi}{\epsilon_t c_t^{-\sigma}} \\
q_{t,L} \epsilon_t C_t^{-\sigma} &= \beta E_t \left[\epsilon_{t+1} \frac{C_{t+1}^{-\sigma}}{\pi_{t+1}} \right] (1 + \delta q_{t+1,L}) \\
q_{t,S} \epsilon_t C_t^{-\sigma} &= \beta E_t \left[\epsilon_{t+1} \frac{C_{t+1}^{-\sigma}}{\pi_{t+1}} \right] \\
\frac{w_t(1-s)}{F_{N,t}} - \frac{\theta-1}{\theta} &= \frac{\psi}{\theta} \left[\pi_t(\pi_t-1) - \beta E_t \left(\frac{\epsilon_{t+1}}{\epsilon_t} \frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}} \frac{Y_{t+1}}{Y_t} (\pi_{t+1}-1) \pi_{t+1} \right) \right] \\
K_{G,t} &= (1 - \delta_g) K_{G,t-1} + G_t^i \\
R_t &= (\pi_t / \pi^*)^{\phi_\pi} \\
q_{t,L} b_{t,L} &= G_t^i + G_t - \tau_t w_t n_t + (1 + \delta q_{t,L}) \frac{b_{L,t-1}}{\pi_t} + s(w_t n_t - w^* n^*) \\
Y_t &= C_t + G_t^c + G_t^i + \frac{\psi}{2} (\pi_t - 1)^2 Y_t
\end{aligned}$$

together with the condition $b_{t,S} = 0$ stating that short term debt is in zero net supply.

A few comments are worth making. First, note that we repeated equations we derived previously from the households and firms programs for convenience. Second, the last line represents the economy wide resource constraint, expressing that output is divided between private and public consumption C_t and G_t^c respectively, public investment G_t^i and the resource costs of inflation $\frac{\psi}{2} (\pi_t - 1)^2 Y_t$. To derive this equation we combined the household and government budget constraints. Therefore, we dispensed with the household constraint in the definition of the equilibrium.

Last, we imposed that private assets are equal to the debt issued by the government, which is the market clearing condition.

2.5 Optimal fiscal policy

We now present the optimal policy program that the government solves. To analyze optimal fiscal policy, we linearize model equations and derive a second order approximation to the utility function around the efficient steady-state. This is a standard approach to optimal policy (as in e.g. [Benigno and Woodford \(2012\)](#)). Using the linearized model we consider both the case where

optimal policy operates under commitment and the optimal time consistent policy under discretion. We refer the reader to appendix 2.B for the complete derivation of the welfare function and the constraint set of the government in each case.

2.5.1 The Welfare Criterion

In the appendix we show that the second order approximation of household utility, delivers the following policy objective function:

$$-E_0 \sum_{t=0}^{\infty} \beta^t L_t$$

where

$$L_t = \omega_{g^i} \frac{\tilde{g}_t^2}{2} + \omega_y \frac{\tilde{y}_t^2}{2} + \omega_{\pi} \frac{\tilde{\pi}_t^2}{2} + \omega_{g^c} \frac{\tilde{g}_t^2}{2} + \omega_{k_{g-1}} \frac{\tilde{k}_{G,t-1}^2}{2} + \omega_{yg^i} \tilde{y}_t \tilde{g}_t^i + \omega_{yg^c} \tilde{y}_t \tilde{g}_t^c + \omega_{g^i g^c} \tilde{g}_t^i \tilde{g}_t^c + \omega_{yk_g} \tilde{y}_t \tilde{k}_{G,t-1} \\ + \omega_{\epsilon g^i} \tilde{g}_t^i \tilde{\epsilon}_t + \omega_{\epsilon g^c} \tilde{g}_t^c \tilde{\epsilon}_t + \omega_{\epsilon y} \tilde{y}_t \tilde{\epsilon}_t$$

where the variables are expressed in deviation from the efficient steady state (we use tildes to denote this) and the coefficients ω are given by:

$$\begin{aligned} \omega_{g^i} &= \frac{g^{i,*}}{c^{*\sigma}} \left[g^{i,*} - \frac{g^{i,*}\sigma}{c^*} \right] & \omega_{yg^c} &= \frac{\sigma g^{c,*} y^*}{c^{*1+\sigma}} \\ \omega_y &= -\frac{y^*}{c^{*\sigma}} \left[\frac{\sigma y^*}{c^*} + \frac{1+\varphi}{\alpha} - 1 \right] & \omega_{g^i g^c} &= -\frac{\sigma g^{c,*} g^{i,*}}{c^{*1+\sigma}} \\ \omega_{\pi} &= -\frac{y^* \psi}{c^{*\sigma}} & \omega_{\epsilon g^i} &= -\frac{g^{i,*}}{c^{*,\sigma}} \\ \omega_{g^c} &= -\sigma \left[\frac{g^{c,*2}}{c^{*,\sigma+1}} + \frac{g^{c,*}}{c^{*\sigma}} \right] & \omega_{yk_g} &= -\frac{(1+\varphi)(\alpha-1)y^*}{c^{*\sigma}\alpha} \\ \omega_{yg^i} &= \frac{\sigma g^{i,*} y^*}{c^{*1+\sigma}} & \omega_{g^c \epsilon} &= -\frac{g^{c,*}}{c^{*\sigma}} \\ \omega_{y\epsilon} &= \frac{y^*}{c^{\sigma}} & \omega_{k_{g-1}} &= -\frac{(\alpha-1)y^*}{c^{*\sigma}} \left[\frac{(\alpha-1)}{\alpha} (1+\varphi) - 2 \right] \\ & & & + \frac{k_g^{*2}}{c^{*\sigma}} (1-\delta) \end{aligned}$$

and the complete derivation of these expressions can be found in the Appendix.

As is evident from the above expression for L , the household has preferences for smoothing the variability of output, inflation, (the usual arguments), and also for smoothing public consumption and investment. Recall that public consumption affects welfare directly. Public investment and capital, however, also exert an (indirect) influence on welfare through affecting the production possibilities of the economy. The coefficients corresponding to these terms measure the magnitude of these indirect effects on household utility.

Finally, it is interesting to note that the presence of the cross terms ($\tilde{y}_t \tilde{g}_t^i$, $\tilde{y}_t \tilde{k}_{t-1}$ etc) suggests a degree of complementarity (substitutability) between the arguments of the objective.

2.5.2 The Ramsey planner under commitment

We now turn to the optimal policy program. We first consider the case of full commitment. To solve this program we utilize the methodological approach of [Aiyagari et al. \(2002\)](#), [Schmitt-Grohé and Uribe \(2004\)](#) and other papers solving for optimal Ramsey policy equilibria under incomplete financial markets.

As is typical in the literature, we dispense with some of the variables of the competitive equilibrium and build the constraint set of the government using only the sufficient implementability conditions. We thus use the primal approach and substitute out from the government budget constraint taxes and the bond prices. Then, given the optimal policy allocation, these variables can be found to satisfy the first order condition for labor and the bond price Euler equation. We can therefore dispense also with these equations as constraints.

Making these substitutions, the log-linear government budget constraint can be written as:

$$\begin{aligned} q^* b^* \tilde{b}_t = & q^* b^* \beta^{-1} (\tilde{b}_{t-1} - \tilde{\pi}_t) + q^* b^* (1 - \rho) (-\sigma \tilde{c}_t + \tilde{\epsilon}_t + E_t[\sigma c_{t+1} + \pi_{t+1} - \epsilon_{t+1}] - \\ & - \beta \rho E_t[q_{t+1}]) + g^{i,*} \tilde{g}_t^i + g^{c,*} \tilde{g}_t^c - \tau^* n^* w^* \left(\tilde{n}_t + \tilde{w}_t + \frac{1 - \tau^*}{\tau^*} \left(\tilde{w}_t - \varphi \tilde{n}_t - \sigma \tilde{c}_t + \epsilon_t \right) \right) \end{aligned} \quad (2.11)$$

For simplicity, we also substitute out private consumption by using the aggregate resource constraint of the economy.

$$\tilde{c}_t = \frac{y^*}{c^*} \tilde{y}_t - \frac{g^{i,*}}{c^*} \tilde{g}_t^i - \frac{g^{c,*}}{c^*} \tilde{g}_t^c$$

We solve for optimal policies that maximize the household welfare function defined previously. Since the objective features output and public capital we cannot dispense with either of this variables. Instead we can use the production technology to substitute out the labour input. We keep the real wage in the system of equations that define the planner's constraints set.

The planner chooses output, inflation, short-term interest rate, government consumption and investment, public capital, wages and public debt. The vector of control variables thus is $\{y_t, \pi_t, R_t, g_t^i, g_t^c, k_{G,t}, w_t, b_t\}$. The problem can be solved by taking first order conditions from the following Lagrangian:

$$\begin{aligned}
\max_{\{y_t, \pi_t, R_t, g_t^i, g_t^c, k_{G,t}, w_t, b_t, \rho\}} &= \sum_{t=0}^{\infty} \beta^t E_0 \left\{ -L_t \right. \\
&+ \phi_{1,t} \left[\tilde{R}_t + \sigma \left(\frac{y^*}{c^*} \tilde{y}_t - \frac{g^{i,*}}{c^*} \tilde{g}_t^i - \frac{g^{c,*}}{c^*} \tilde{g}_t^c \right) - \tilde{\epsilon}_t - E_t \left[\sigma \left(\frac{y^*}{c^*} y_{t+1} - \frac{g^{i,*}}{c^*} g_{t+1}^i - \frac{g^{c,*}}{c^*} g_{t+1}^c \right) + \pi_{t+1} - \epsilon_{t+1} \right] \right] \\
&+ \phi_{2,t} \left[\tilde{\pi}_t - \frac{\theta}{\psi} \left(\tilde{w}_t - \tilde{y}_t + \frac{\tilde{y}_t - (1-\alpha) \tilde{k}_{G,t}}{\alpha} \right) - \beta E_t [\pi_{t+1}] \right] \\
&+ \phi_{3,t} \left[\tilde{k}_{G,t} - (1 - \delta_G) \tilde{k}_{G,t-1} - \delta_G \tilde{g}_t^i \right] \\
&+ \phi_{4,t} \left[\tilde{R}_t - \phi_{\pi} \tilde{\pi}_t \right] \\
&+ \phi_{5,t} \left[q^* b^* \tilde{b}_t - q^* b^* \beta^{-1} \tilde{b}_{t-1} + q^* b^* \beta^{-1} \tilde{\pi}_t - q^* b^* (1 - \rho) \left(-\sigma \left(\frac{y^*}{c^*} \tilde{y}_t - \frac{g^{i,*}}{c^*} \tilde{g}_t^i - \frac{g^{c,*}}{c^*} \tilde{g}_t^c \right) + \tilde{\epsilon}_t \right. \right. \\
&+ \left. E_t \left[\sigma \left(\frac{y^*}{c^*} y_{t+1} - \frac{g^{i,*}}{c^*} g_{t+1}^i - \frac{g^{c,*}}{c^*} g_{t+1}^c \right) + \pi_{t+1} - \epsilon_{t+1} \right] - \beta \rho E_t [q_{t+1}] \right) - g^{i,*} \tilde{g}_t^i - g^{c,*} \tilde{g}_t^c \\
&+ \left. \left. \tau^* n^* w^* \left(\frac{\tilde{y}_t - (1-\alpha) \tilde{k}_{G,t}}{\alpha} + \tilde{w}_t + \frac{1 - \tau^*}{\tau^*} \left(\tilde{w}_t - \varphi \frac{\tilde{y}_t - (1-\alpha) \tilde{k}_{G,t}}{\alpha} - \sigma \left(\frac{y^*}{c^*} \tilde{y}_t - \frac{g^{i,*}}{c^*} \tilde{g}_t^i - \frac{g^{c,*}}{c^*} \tilde{g}_t^c \right) + \tilde{\epsilon}_t \right) \right) \right] \right\}
\end{aligned}$$

Where $\phi_{i,t} \forall i = 1, \dots, 5$ are Lagrange multipliers attached to the constraints.

The first order optimality conditions are:

$$\begin{aligned}
\tilde{y}_t : & -\omega_y \tilde{y}_t - \omega_{yg^i} \tilde{g}_t^i - \omega_{yg^c} \tilde{g}_t^c - \omega_{yk_g} k_{G,t-t} - \omega_{ey} \tilde{e}_t + \sigma \frac{y^*}{c^*} \left(\phi_{1,t} - \beta^{-1} \phi_{1,t-1} \right) + \phi_{2,t} \frac{\theta}{\psi} \left(1 - 1/\alpha \right) \\
& + \frac{\beta b^*}{1 - \beta \rho} (1 - \rho) \sigma \frac{y^*}{c^*} \left(\phi_{5,t} - \beta^{-1} \phi_{5,t-1} \right) + \phi_{5,t} \left[w^* n^* \tau^* \left(\frac{1}{\alpha} - \frac{1 - \tau^*}{\tau^*} \left(\frac{\varphi}{\alpha} + \sigma \frac{y^*}{c^*} \right) \right) \right] = 0 \\
\tilde{\pi}_t : & -\omega_\pi \tilde{\pi}_t - \beta^{-1} \phi_{1,t-1} + \phi_{2,t} - \phi_{2,t-1} - \phi_{4,t} \phi_\pi + \frac{b^*}{1 - \beta \rho} (\phi_{5,t} - (1 - \rho) \phi_{5,t-1}) = 0 \\
\tilde{R}_t : & \phi_{1,t} + \phi_{4,t} = 0 \\
\tilde{g}_t^i : & -\omega_{g^i} \tilde{g}_t^i - \omega_{yg^i} \tilde{y}_t - \omega_{g^c g^i} \tilde{g}_t^c + \sigma \frac{g^{i,*}}{c^*} \left(\beta^{-1} \phi_{1,t-1} - \phi_{1,t} \right) - \phi_{3,t} \delta_G \\
& + \frac{\beta b^*}{1 - \beta \rho} (1 - \rho) \sigma \frac{g^{i,*}}{c^*} \left(\beta^{-1} \phi_{5,t-1} - \phi_{5,t} \right) + \phi_{5,t} \left(-g^{i,*} + n^* w^* (1 - \tau^*) \sigma \frac{g^{i,*}}{c^*} \right) = 0 \\
\tilde{g}_t^c : & -\omega_{g^c} \tilde{g}_t^c - \omega_{yg^c} \tilde{y}_t - \omega_{g^c g^i} \tilde{g}_t^i + \sigma \frac{g^{c,*}}{c^*} \left(\beta^{-1} \phi_{1,t-1} - \phi_{1,t} \right) + \\
& \frac{\beta b^*}{1 - \beta \rho} (1 - \rho) \sigma \frac{g^{c,*}}{c^*} \left(\beta^{-1} \phi_{5,t-1} - \phi_{5,t} \right) + \phi_{5,t} \left(-g^{c,*} + n^* w^* (1 - \tau^*) \sigma \frac{g^{c,*}}{c^*} \right) = 0 \\
\tilde{w}_t : & -\frac{\theta}{\psi} \phi_{2,t} + \phi_{5,t} \left[w^* n^* \tau^* \left(1 + \frac{1 - \tau^*}{\tau^*} \right) \right] = 0 \\
\tilde{k}_{G,t} : & -\beta E_t (\omega_{k_{g-1}} k_{G,t} + \omega_{yk_g} y_{t+1}) + \beta \frac{\theta}{\psi} \frac{(1 - \alpha)}{\alpha} E_t \phi_{2,t+1} + \phi_{3,t} - (1 - \delta_G) \beta E_t [\phi_{3,t+1}] + \\
& + \beta E_t \phi_{5,t+1} \left(-n^* w^* \tau^* \frac{(1 - \alpha)}{\alpha} + (1 - \tau^*) n^* w^* \frac{\varphi(1 - \alpha)}{\alpha} \right) = 0 \\
\tilde{b}_{t,L} : & \frac{\beta b^*}{1 - \beta \rho} (\phi_{5,t} - E_t \phi_{5,t+1}) = 0
\end{aligned}$$

For convenience next to each of the FONC we have written the corresponding variable.

A couple of remarks are important here. Note that the above system of first order conditions (together with the constraints) form the system of equations that needs to be resolved (numerically) to find the optimal policy equilibrium. As usual in the case of full commitment policies, the first order conditions feature lags and leads of Lagrange multipliers. The lagged multipliers reveal a well known property of this class of models, ‘history dependence’ (see [Aiyagari et al. \(2002\)](#)). Since in the constraint set we have expectations of future economic variables (e.g. inflation) then optimal policy makes promises about future variables and ‘manipulates’ expectations. The lags of the multipliers capture essentially promises that were made in past periods about the period t allocation.

Moreover, note that the lead terms that appear in the system above, concern the optimality conditions for debt and public capital. These conditions can be read as the Euler equations that derive under optimal public policy. For example, consider the last line of the system, which can be simplified to $\phi_{5,t} = E_t \phi_{5,t+1}$. According to this condition, the multiplier on the consolidated budget constraint, $\phi_{5,t}$, evolves as a pure random walk under optimal policy. This property is standard (e.g. [Aiyagari et al \(2002\)](#)). Under optimal policy, the multiplier measures the burden to society from distortionary taxation (or distortionary inflation) that finances debt and the random walk

property means that the planner desires to spread the distortions evenly across periods. The optimality condition with respect to capital has an analogous interpretation.

Finally, note that the above system can be easily solved with standard techniques for solving linear stochastic systems of difference equations. Since there is nothing new here, we do not provide a description of the numerical procedure used.

2.5.3 Time-consistent problem

The previous section described the optimal policy under full commitment. We now consider the opposite scenario, assuming that the optimizing government cannot commit to future policies at all, in other words we solve the problem of time-consistent optimal policy. There are several reasons why we want to consider both policy environments in this paper. First, it is well known that for a class of models similar to ours, Ramsey policy under commitment makes much less use of inflation to finance debt than the no-commitment policy. For example [Faraglia et al. \(2013\)](#) study optimal monetary and fiscal policy with exogenous spending and conclude that inflation's role in stabilizing debt is only limited in the commitment case, whereas it is considerably more important in the time consistent allocation. Though our policy environment is different from theirs (ours is a model of endogenous public consumption and investment) we can motivate the time consistent scenario based on their finding.

Second, the assumption that fiscal policy cannot commit to future allocations can perhaps be seen as extreme, but so is the full commitment assumption. In reality governments may be able to partially commit to policies for a short horizon (enact a spending plan that determines the path of fiscal variables for a few months or years) but they cannot commit to an entire path of state contingent policies (over the infinite horizon) as is the case in the full commitment model laid out previously. We thus follow much of the literature in considering the no-commitment scenario, to study policy under the two polar opposite assumptions, being aware that reality may be somewhere in between.

As it is standard in the literature, we solve for optimal policy under no-commitment focusing on Markov perfect equilibria where the actions of the government and the private sector are assumed to be well behaved functions of a vector of state variables determining the relevant payoffs. We assume that this state vector is $(K_{G,t-1}, b_{t-1,L}, \epsilon_t)$ ($(\tilde{k}_{G,t-1}, \tilde{b}_{t-1,L}, \tilde{\epsilon}_t)$ in terms of the notation of the linearized model). Note also that in a Markov perfect equilibrium, private sector expectations are to be replaced by linear functions of $(\tilde{k}_{G,t}, \tilde{b}_{t,L}, \tilde{\epsilon}_t)$ as we elaborate below.⁹

We now provide the definition of the optimal policy Markov Perfect Equilibrium.

Definition: Markov Perfect Equilibrium The equilibrium consists of a value function V , and decision rules for quantities $\{y_t, \pi_t\}$ and policy variables $\{R_t, g_t^i, g_t^c, k_{G,t}, b_{t,L}, \tau_t\}$ which are all func-

⁹It should be noted that New-Keynesian models under time-consistent policies can lead to multiple equilibria even in the linear quadratic setup, as pointed out by [Blake and Kirsanova \(2012\)](#). According to [Blake and Kirsanova \(2012\)](#) there are two types of equilibria: the “point-in-time”, which arises as a consequence of different decisions of the private sector for a given government policy, and the ones arising from different discretionary rules. We do not address this here.

tions of the state vector $(k_{G,t-1}, b_{t-1,L}, \tilde{\epsilon}_t)$ and such that for given initial conditions $k_{G,-1}, b_{-1,L}$, the quantities and policies solve the following problem:

$$\begin{aligned} & \max_{\{y_t, \pi_t, R_t, g_t^i, k_{G,t}, w_t, b_t\}} V(k_{G,t-1}, b_{t-1}, \epsilon_t) = -L_t + \beta E_t V(k_{G,t}, \tilde{b}_t, \epsilon_{t+1}) \\ & + \phi_{1,t} \left[\tilde{R}_t + \sigma \left(\frac{y^*}{c^*} \tilde{y}_t - \frac{g^{i,*}}{c^*} \tilde{g}_t^i - \frac{g^{c,*}}{c^*} \tilde{g}_t^c \right) - \tilde{\epsilon}_t + h_1(k_{G,t}, \tilde{\epsilon}_t, \tilde{b}_t) \right] \\ & + \phi_{2,t} \left[\tilde{\pi}_t - \frac{\theta}{\psi} \left(\tilde{w}_t - \tilde{y}_t + \frac{\tilde{y}_t - (1-\alpha)k_{G,t}}{\alpha} \right) - \beta h_2(k_{G,t}, \tilde{\epsilon}_t, \tilde{b}_t) \right] \\ & + \phi_{3,t} \left[\tilde{k}_{G,t} - (1-\delta_G) \tilde{k}_{G,t-1} - \delta_G \tilde{g}_t^i \right] \\ & + \phi_{4,t} \left[\tilde{R}_t - \phi_\pi \tilde{\pi}_t \right] \\ & + \phi_{5,t} \left[q^* b^* \tilde{b}_t - q^* b^* \beta^{-1} b_{t-1} + q^* b^* \beta^{-1} \tilde{\pi}_t - q^* b^* (1-\rho) \left(-\sigma \left(\frac{y^*}{c^*} \tilde{y}_t - \frac{g^{i,*}}{c^*} \tilde{g}_t^i - \frac{g^{c,*}}{c^*} \tilde{g}_t^c \right) + \tilde{\epsilon}_t \right. \right. \\ & \left. \left. - h_1(k_{G,t}, \tilde{\epsilon}_t, \tilde{b}_t) - \beta \rho h_3(k_{G,t}, \tilde{\epsilon}_t, \tilde{b}_t) \right) - g^{i,*} \tilde{g}_t^i - g^{c,*} \tilde{g}_t^c \right. \\ & \left. \left. + \tau^* n^* w^* \left(\frac{\tilde{y}_t - (1-\alpha)k_{G,t}}{\alpha} + \tilde{w}_t + \frac{1-\tau^*}{\tau^*} \left(\tilde{w}_t - \varphi \frac{\tilde{y}_t - (1-\alpha)k_{G,t}}{\alpha} - \sigma \left(\frac{y^*}{c^*} \tilde{y}_t - \frac{g^{i,*}}{c^*} \tilde{g}_t^i - \frac{g^{c,*}}{c^*} \tilde{g}_t^c \right) + \tilde{\epsilon}_t \right) \right) \right] \right\} \end{aligned}$$

As before, we use $\phi_{i,t}$ for $i = 1, \dots, 5$ to denote the multipliers attached to the constraints.

Note that the objects included in the above definition are the variables that appear in the sufficient implementability conditions for a competitive equilibrium under optimal time consistent policies. Therefore, using the same primal approach as in the commitment model of the previous section we have substituted out the labour input and the marginal cost of firms. Having solved for the optimal policy equilibrium these quantities can be easily recovered from:

$$\begin{aligned} \tilde{n}_t &= \frac{(\tilde{y}_t - (1-\alpha)k_{G,t-1})}{\alpha} \\ \tilde{m}c_t &= \tilde{w}_t - \tilde{y}_t + \tilde{n}_t \end{aligned}$$

which are the production function and the firm's optimality condition with respect to labor respectively.

Finally, let us explain the terms denoted with an h that appear in the constraint set in the above program. As is well known, under time-consistent public policies the government cannot make promises for future variables. Therefore terms containing expectations about future variables, such as $E_t \tilde{\pi}_{t+1}$ or $E_t \tilde{y}_{t+1}$, need to be replaced by functions of the state vector, to acknowledge that the only influence that optimal policy can have on expectations is through setting future state variables.

Define the expectations contained in Euler equations for short and long-term bonds and the Phillips curve as $h_1(\cdot), h_2(\cdot), h_3(\cdot)$ respectively. Since we are assuming a Markov Perfect Equilibrium, these expectations are functions of the state variables, $k_{G,t}, \tilde{\epsilon}_t, \tilde{b}_t$.

We therefore have the following definitions:

$$\begin{aligned} h_{1,t}(k_{G,t}, \tilde{\epsilon}_t, \tilde{b}_t) &= E_t \left[-\sigma \left(\frac{y^*}{c^*} y_{t+1} - \frac{g^{i,*}}{c^*} \tilde{g}_{t+1}^i - \frac{g^{c,*}}{c^*} \tilde{g}_{t+1}^c \right) - \pi_{t+1} + \tilde{\epsilon}_t \right] \\ h_{2,t}(k_{G,t}, \tilde{\epsilon}_t, \tilde{b}_t) &= E_t[\pi_{t+1}] \\ h_{3,t}(k_{G,t}, \tilde{\epsilon}_t, \tilde{b}_t) &= E_t[q_{t+1}] \end{aligned}$$

Finally, at the equilibrium obviously, the future variables $q_{t+1}, g_{t+1}^{\tilde{c}}$ etc, are also functions of the state vector as the definition of the Markov equilibrium makes clear. These functions, as well as the functions h are assumed to be well behaved (continuously differentiable). In this linear quadratic program we obviously assume linear functions of the state variables.

Solving the value function above gives us the following set of first order conditions in the time consistent optimal policy model:

$$\begin{aligned} \tilde{y}_t : & -\omega_y \tilde{y}_t - \omega_{yg^i} \tilde{g}_t^i - \omega_{yg^c} \tilde{g}_t^c - \omega_{ykg} k_{G,t-t} - \omega_{ey} \tilde{\epsilon}_t + \sigma \frac{y^*}{c^*} \phi_{1,t} + \phi_{2,t} \frac{\theta}{\psi} \left(1 - 1/\alpha \right) \\ & + \phi_{5,t} \left[\frac{\beta b^*}{1 - \beta \delta} (1 - \rho) \sigma \frac{y^*}{c^*} + w^* n^* \tau^* \left(\frac{1}{\alpha} - \frac{1 - \tau^*}{\tau^*} \left(\frac{\varphi}{\alpha} + \sigma \frac{y^*}{c^*} \right) \right) \right] = 0 \\ \tilde{\pi}_t : & -\omega_\pi \tilde{\pi}_t + \phi_{2,t} - \phi_{4,t} \phi_\pi - \frac{b^*}{1 - \beta \rho} \phi_{5,t} = 0 \\ \tilde{R}_t : & \phi_{1,t} + \phi_{4,t} = 0 \\ \tilde{g}_t^i : & -\omega_{g^i} \tilde{g}_t^i - \omega_{yg^i} \tilde{y}_t - \omega_{g^c g^i} \tilde{g}_t^c - \sigma \frac{g^{i,*}}{c^*} \phi_{1,t} - \phi_{3,t} \delta_G \\ & + \phi_{5,t} \left(-\frac{\beta b^*}{1 - \beta \delta} (1 - \rho) \sigma \frac{g^{i,*}}{c^*} - g^{i,*} + n^* w^* (1 - \tau^*) \sigma \frac{g^{i,*}}{c^*} \right) = 0 \\ \tilde{g}_t^c : & -\omega_{g^c} \tilde{g}_t^c - \omega_{yg^c} \tilde{y}_t - \omega_{g^c g^i} \tilde{g}_t^i - \sigma \frac{g^{c,*}}{c^*} \phi_{1,t} \\ & + \phi_{5,t} \left(-\frac{\beta b^*}{1 - \beta \delta} (1 - \rho) \sigma \frac{g^{c,*}}{c^*} - g^{c,*} + n^* w^* (1 - \tau^*) \sigma \frac{g^{c,*}}{c^*} \right) = 0 \\ k_{G,t} : & \beta E_t \frac{\partial V(\cdot)}{\partial \tilde{b}_t} - \phi_{1,t} \frac{\partial h_1(\cdot)}{\partial k_{G,t}} - \phi_{2,t} \beta \frac{\partial h_2(\cdot)}{\partial k_{G,t}} + \phi_{3,t} - \phi_{5,t} \frac{\beta b^*}{1 - \beta \rho} (1 - \rho) \left(\frac{\partial h_1(\cdot)}{\partial k_{G,t}} - \beta \rho \frac{\partial h_3(\cdot)}{\partial k_{G,t}} \right) = 0 \\ \tilde{w}_t : & -\frac{\theta}{\psi} \phi_{2,t} + \phi_{5,t} \left[w^* n^* \tau^* \left(1 + \frac{1 - \tau^*}{\tau^*} \right) \right] = 0 \\ \tilde{b}_{t,\rho} : & \beta E_t \frac{\partial V_{t+1}}{\partial \tilde{b}_t} - \phi_{1,t} \frac{\partial h_1(\cdot)}{\partial \tilde{b}_t} - \phi_{2,t} \beta \frac{\partial h_2(\cdot)}{\partial \tilde{b}_t} + \phi_{5,t} \frac{\beta b^*}{1 - \beta \rho} \left(1 - (1 - \rho) \left(\frac{\partial h_1(\cdot)}{\partial \tilde{b}_t} - \beta \rho \frac{\partial h_3(\cdot)}{\partial \tilde{b}_t} \right) \right) = 0 \end{aligned}$$

Using the envelope theorem, we can replace for the derivative of the value function with respect to the states variables: $k_{G,t}$ and $\tilde{b}_t$ in the first order condition for debt and public capital

$$\begin{aligned} E_t \frac{\partial V_{t+1}}{\partial k_{G,t}} &= -\beta E_t (\omega_{k_{g-1}} k_{G,t}) + \beta \frac{\theta}{\psi} \frac{(1 - \alpha)}{\alpha} E_t \phi_{2,t+1} - (1 - \delta_G) \beta E_t \phi_{3,t+1} \\ &+ \beta E_t \phi_{5,t+1} \left(-n^* w^* \tau^* \frac{(1 - \alpha)}{\alpha} + (1 - \tau^*) n^* w^* \frac{\varphi(1 - \alpha)}{\alpha} \right) = 0 \end{aligned}$$

$$E_t \frac{\partial V_{t+1}}{\partial b_t} = \frac{\partial L_t}{\partial b_t} = -\frac{\beta b^*}{1 - \beta \rho} E_t \phi_{5,t+1}$$

and as a consequence, we can write the first order condition for public capital, $\tilde{k}_{G,t}$, and government debt, $\tilde{b}_t$ as

$$\begin{aligned} \tilde{k}_{G,t} : & -\beta E_t(\omega_{k_g-1} \tilde{k}_{G,t}) - (1 - \delta_G) \beta E_t \phi_{3,t+1} + \beta E_t \phi_{5,t+1} \left(-n^* w^* \tau^* \frac{(1 - \alpha)}{\alpha} + (1 - \tau^*) n^* w^* \frac{\varphi(1 - \alpha)}{\alpha} \right) \\ & - \phi_{1,t} \frac{\partial h_1(\cdot)}{\partial \tilde{k}_{g,t}} + \beta E_t \phi_{2,t+1} \frac{\theta(1 - \alpha)}{\psi} - \phi_{2,t} \beta \frac{\partial h_2(\cdot)}{\partial \tilde{k}_{G,t}} + \phi_{3,t} - \phi_{5,t} \frac{\beta b^*}{1 - \beta \rho} (1 - \rho) \left(\frac{\partial h_1(\cdot)}{\partial \tilde{k}_{g,t}} - \beta \rho \frac{\partial h_3(\cdot)}{\partial \tilde{k}_{g,t}} \right) = 0 \\ \tilde{b}_{t,\rho} : & -\frac{\beta b^*}{1 - \beta \rho} E_t \phi_{5,t+1} - \phi_{1,t} \frac{\partial h_1(\cdot)}{\partial \tilde{b}_t} - \phi_{2,t} \beta \frac{\partial h_2(\cdot)}{\partial \tilde{b}_t} + \phi_{5,t} \frac{\beta b^*}{1 - \beta \rho} \left(1 - (1 - \rho) \left(\frac{\partial h_1(\cdot)}{\partial \tilde{b}_t} - \beta \rho \frac{\partial h_3(\cdot)}{\partial \tilde{b}_t} \right) \right) = 0 \end{aligned}$$

It is worthwhile to briefly compare this system of first order conditions with the analogous system we derived in the previous subsection, under commitment. Note first, in contrast to the commitment case the system now does not feature any lagged multipliers. Instead the FONC features the partial derivatives of the functions h with respect to their arguments. The interpretation is however analogous. Whereas under commitment the lags were capturing the promises that the planner made with regard to future macroeconomic variables, the derivatives h capture the planner's incentive to manipulate expectations by setting the path of the next periods state variables. Under commitment the lags of the multipliers were state variables of the model, here (by the definition of the equilibrium) the state vector is $(\tilde{k}_{G,t-1}, \tilde{b}_{t-1,L}, \tilde{\epsilon}_t)$.

Note also that just like in the commitment model the system of FONC in the time consistent solution features expectations for the multipliers, $\phi_{i,t+1}$. Again these are related to the intertemporal choices the planner has to make with respect to public capital and debt. In the time consistent solution these expectations can also be expressed as functions of the state vector.

Lastly, note that solving for optimal policies in this case is not trivial (as it was under commitment). To find the solution we need to parameterize the conditional expectations and guess the values of the coefficients of the linear functions. Then, an iterative procedure needs to be followed, to update these coefficients until they converge (do not change with additional iterations). More details on this solution technique can be found in appendix 2.D.

2.6 Quantitative experiments.

2.6.1 Calibration

Table 2.1 in appendix 2.E contains the main values for the parameters used to perform the quantitative experiments. A model period represents one quarter. We set β , the discount factor, equal to 0.99, thus the interest rate is at annual horizon equal to 4%.

[Table 2.1 approximately here]

We assume the production function is a Cobb-Douglas. The output elasticity with respect to public capital is 0.06, which is within the range of 0.03 - 0.1 estimated by the relevant literature (see [Ramey \(2020\)](#) for a review). The elasticity with respect to labour is thus 0.94.

According to data from the Federal Reserve Bank of St. Louis, total government expenditure for the US as percentage of GDP for the period 1990-2018 has been at around 20%, of which 4.5% represents public investment. So, we set government consumption G_t^c to be 0.15 of output at the steady-state. δ , the depreciation rate for public capital, is set to 0.03 which, given β , α and the ratio for government consumption to GDP gives public investment as percentage of total government expenditure at 24% in line with the data.

Moreover, the elasticity of substitution between intermediate goods θ , is set at 6, to generate a mark-up of 20% in the steady state. The parameter ψ governing the price adjustment cost is 200. Given this value the slope coefficient of the linearized Phillips curve (θ/ψ) is 0.03, as in [Bouakez et al. \(2019\)](#). Gross steady state inflation is assumed to be 1.

We set the risk aversion parameter, σ , equal to 1.5 and the inverse Frisch elasticity, φ to 2. [Drautzburg and Uhlig \(2015\)](#) and [Leeper et al. \(2010\)](#) estimate DSGE models with public capital and debt. They obtain mean values for risk aversion coefficient equal to 1.5 and 1.75 respectively, and an inverse of the Frisch elasticity equal to 2.

We set the debt level to be 60% of steady-state output as much of the literature on optimal fiscal policy. Together with government spending as percentage of output, this figure gives a labor tax rate in the steady-state close to 17%. We fix ρ , the decaying coupon factor for long-term debt to be 0.95. This allows to match an average duration for the public debt of 20 quarters, which is consistent with US data as well as data of many OECD economies.

Furthermore, we calibrate the autocorrelation coefficient of the preference shock to be 0.95 and the variance of the shock is 0.07. With these numbers the shock to preferences produces a fall in output of around 2% with respect to the steady state at the trough of a recession when monetary policy is active. This number is chosen to be comparable to the growth of output in the US between Q3-2008 and Q4-2008 (2.2%). The process of the shock is given by:

$$\log(\epsilon_t) = \rho_\epsilon \log(\epsilon_{t-1}) + \nu_t, \quad \nu_t \sim \mathcal{N}(0, \sigma_\nu^2)$$

For each of the two regimes for monetary policy, active and passive, we need to assign a value of the inflation coefficient in the interest rate rule. When monetary policy is active, the parameter of the rule is fixed in 1.5 as in e.g. [Woodford \(2001\)](#) and many others in the New Keynesian model literature. When considering a passive regime for monetary policy, we investigate optimal policy under two calibrations for the parameter ϕ_π : 0.8 and 0.95. These values are consistent with relevant empirical evidence regarding the specification of the interest rate rule in the US in the pre-Volcker era, 1955-1979; ([Carvalho \(2019\)](#)) estimates the coefficient to be between 0.8 and 0.9. The DSGE model used in [Bianchi \(2013\)](#) produces a similar estimate for this parameter. We consider two calibrations of ϕ_π in our experiments to tease out the influence of this parameter on optimal policy.

2.6.2 Optimal policies under commitment

We first focus on the optimal policy under commitment. Figure 2.3 in appendix 2.E shows the optimal response of the economy to a negative consumption shock. The blue line shows the economy under an active monetary policy rule while the red and orange lines show optimal policies under the passive monetary policy rule with $\phi_\pi = 0.95$ and $\phi_\pi = 0.8$, respectively. As can be seen from the figure, the optimal paths for private consumption, labor and output are very similar in both regimes. In contrast, inflation, interest rates and public debt respond differently in the two models.

[Figure 2.3 approximately here]

In all cases the shock reduces private consumption, output and hours. Government consumption increases since as discussed previously, the shock also affects the marginal rate of substitution between private and government consumption. Therefore a negative shock, shifts the household's preference more towards government provided services and the benevolent planner accounts for this. This is the primary force that lies behind the fluctuations observed in the figure.

Moreover, an economic recession induced by a rise in household target saving rate (a consumption slump) is clearly a good time to invest in public capital. As can be seen from the Figure, most of the household excess savings are channelled in public investment and not to public debt. The reason is that public capital (from a societal, not necessarily a private point of view) is preferable since it increases firm productivity and can thus mitigate output losses at longer horizons. In consequence, public investment grows from 4.5% of GDP (the steady state value) to peak at almost 6%. Meanwhile, public consumption rises from 15% of steady state output to a peak of 15.5%. Thus the response of government spending to the recession is mainly concentrated at building new infrastructure, which increases productivity.

It is also worth stressing that though these forces (shifts in marginal utilities ratios within and between periods) are the primary forces behind optimal policies, there are also non trivial effects that derive from the government budget constraint and the household Euler equation. For example consider again the FONC for $\tilde{g}_t^c$ derived previously and repeated here for convenience.

$$-\omega_{g^c} \tilde{g}_t^c - \omega_{y g^c} \tilde{y}_t - \omega_{\epsilon g^c} \tilde{\epsilon}_t + \sigma \frac{g^{c,*}}{c^*} \left(\beta^{-1} \phi_{1,t-1} - \phi_{1,t} \right) + \frac{\beta b^*}{1 - \beta \rho} (1 - \rho) \sigma \frac{g^{c,*}}{c^*} \left(\beta^{-1} \phi_{5,t-1} - \phi_{5,t} \right) + \phi_{5,t} \left(-g^{c,*} + n^* w^* (1 - \tau^*) \sigma \frac{g^{c,*}}{c^*} \right) = 0$$

In the case where the government budget and the Euler equations are not relevant constraints¹⁰ we get $\phi_{1,t} = \phi_{5,t} = 0$ and therefore $-\omega_{g^c} \tilde{g}_t^c - \omega_{y g^c} \tilde{y}_t - \omega_{\epsilon g^c} \tilde{\epsilon}_t$ so that the only force determining the

¹⁰For example when the government can levy lump sum taxes we would have $\phi_{5,t} = 0$ for all t . Analogously if we had assumed state contingent debt, then the multiplier would be zero (in deviation from steady state which is what matters here). The Euler equation would not be relevant if the planner could tax bond returns, or in a constrained efficient allocation.

optimal path of spending is substitution between the two arguments in utility. However, in the presence of binding constraints $\phi_{1,t}, \phi_{5,t} \neq 0$ and are time varying we have the relevant terms in the FONC also affect the optimal allocation. We can think that terms which involve ϕ_1 affect the optimal path of spending, since spending impacts private consumption and if it were too erratic it would violate the household's intertemporal substitution of consumption as given by the Euler equation. Similarly terms that involve ϕ_5 reflect the impact on the government budget constraint, the market value of the debt issued and the government surplus. Generally, the government will use spending as a shock absorber in this model. If the shock results in a drop in ϕ_5 (the constraint is relative more slack) then higher spending can stabilize the government's budget. In contrast if the shock results in an increase in ϕ_5 spending will be lower (than what is desired by the marginal utility considerations) to relax the constraint.

The former happens in the model. A negative shock to preferences, increases the present discount value of the government's surplus relative to the real market value of debt. This means that the constraint slackens a drop in the surplus (through higher spending) can absorb the shock.

Consider now the behavior of taxes over time. As can be seen from the Figure, taxes increase on impact and remain above the steady state level for at least 20 quarters (i.e. in the case of active policy) or more (in the case of passive policies). Notice that the reaction of distortionary taxes to the shock, is not (obviously) related to the reaction of debt. In standard models of the FTPL, taxes follow rules which are essentially functions of lagged debt. Under active monetary policy a higher debt level would need to be compensated by higher taxes, and partially compensated in the passive monetary policy. Yet, in the Figure taxes seem to move opposite to the debt level. The reason for this is that under optimal policy tax rates are functions of current and lagged values of Lagrange multipliers not debt per se. We should thus not expect to find that ad hoc rules, such as the ones that are commonly used in the literature fit the shape of the tax schedule in the Ramsey model.

The fact that taxes drop in the long run, when public capital has settled to its steady state value, is indicative of the effect of the shock on the intertemporal budget constraint, discussed previously. Since the constraint slackens, taxes have to (eventually) become permanently lower (i.e. the standard random walk property of the Ramsey model) to sustain a lower level of debt in the long run. This is obviously consistent with the Figure. However, taxes may increase in the short run because the accumulation of public capital reduces the marginal costs of production and the planner compensates with higher distortionary taxes to mitigate the negative inflationary pressures. These responses are thus easy to rationalize with the standard features of the Ramsey model.

Let us now turn to variables whose responses differ between the active and passive monetary policy regimes, most notably inflation and the nominal interest rate. Whereas inflation is higher than steady state in the passive money regime, it is lower under active policy. What explains this?

Note that a disturbance to preferences is a shock to demand and therefore the Euler equation. In the case of active policy, this negative demand shock will result to deflation, and this is not surprising. What is surprising perhaps is that the shocks produces the opposite in the case of passive policy.

To clarify this, let us assume that $\sigma = 0$ (preferences are linear in consumption) and combining the interest rate rule with the Euler equation we can get:

$$\phi_\pi \tilde{\pi}_t - \epsilon_t = E_t \tilde{\pi}_{t+1}$$

¹¹ Under active monetary policy this difference equation can be solved forward to obtain:

$$\tilde{\pi}_t = \sum_{j=0}^{\infty} E_t \frac{1}{\phi_\pi^{j+1}} \epsilon_{t+j}$$

Thus a negative ϵ shock will result in deflation consistent with the Figure.

Under passive fiscal policy we get

$$E_t \tilde{\pi}_{t+1} = \phi_\pi \tilde{\pi}_t - \epsilon_t \rightarrow \tilde{\pi}_{t+1} = \phi_\pi \tilde{\pi}_t - \epsilon_t + \theta_{t+1} \rightarrow \tilde{\pi}_{t+1} = \sum_{j=0}^t \phi_\pi^j (-\epsilon_{t-j} + \theta_{t+1-j})$$

The θ shock is i.i.d and it is pinned down by the government budget and the Euler equation constraints.¹² Generically, the sequence θ will be negative since the constraint slackens following the shock. This produces deflation. However, the shock itself exerts a positive inflationary purpose on the economy, as is evident from the above equation. This more than compensates for the effect deriving from the budget constraint.

This argument also applies in the case where $\sigma > 0$. Obviously, in that case, movements in the real rate of interest will also result in inflation dynamics under passive and active policies.

Note finally, that the positive inflation we get under passive monetary policy destabilizes government debt. Deflation makes debt even lower along the transition and implies that taxes need to be further decreased in the long run for intertemporal solvency.

We can draw several interesting implications from this exercise. However, let us not lose sight of what the aim is, to evaluate whether the different specifications of monetary policies imply drastically different fiscal policies along the transition path. The conclusion that we can draw from our experiments in this subsection is that fiscal variables behave quite similarly, though we do find long term differences in the level of taxes, which can be attributed to the different paths followed by debt.

2.6.3 Optimal policies under discretion

We now turn to analyze optimal policies when the planner can not commit. Figure 2.4 in appendix 2.E shows the response of the economy to a negative shock on private consumption (preferences shock). The blue line corresponds to the active monetary policy regime while the red and orange

¹¹For simplicity we impose that the shock is i.i.d.

¹²This is a standard FTPL argument, the budget constraint pins down inflation and multiple equilibria can be avoided.

lines shows the economy under passive monetary policy regime, for $\phi_\pi = 0.95$ and $\phi_\pi = 0.8$ respectively.

Focusing on the differences in the adjustment of fiscal variables after the shock, notice, first, that as was the case under commitment, the paths of taxes government spending and investment are similar across active and passive monetary policies. The differences in Figure 2.4 are even smaller than in Figure 2.3, under commitment.

However, we do observe a considerable difference for all models relative to the commitment case, with regard to the behavior of taxes and debt. Whereas under commitment the shock induced a drop in debt and in taxes in the long run, here we observe the opposite, long run levels of taxes are above steady state. We attribute this significant different to the much sharper rise in government consumption we get under discretion. The key difference here is that the planner is not able to manipulate expectations, by committing to future policies and can only use debt and public capital to influence the expected path of consumption and inflation. Apparently, for the Euler equation to hold along the transition path, government consumption must increase considerably when the shock hits and rapidly (within five periods or so) settle to a lower path to transition smoothly to the steady state. But this sharp increase in spending implies tighter budget constraints for the government, under any specification of monetary policy. Therefore taxes need to eventually settle to a higher long run value.

Another interesting point to make has to do with the behavior of inflation. Notice that inflation rates are now very similar in the passive and active regimes. In all cases we observe a strong deflation initially, which contributes to the rise in the debt level. We suspect that this has again to do with the need to satisfy the Euler equation along the transition path. Since the behavior of all other model aggregates is similar, the path of inflation also needs to be similar.¹³

We conclude that assuming discretion rather than commitment, changes considerably the optimal policy implications we can draw from the model. However, the conclusion that assuming passive / active monetary policy does not affect significantly the course of the fiscal policy variables in response to the shock to preferences, remains.

[Figure 2.4 approximately here]

2.7 Extensions

We now consider several robustness exercises to verify that our results continue to hold under alternative modelling assumptions. First, we allow for the interest rate rule to target not only inflation, as was in the previous sections, but also the output gap. Second, we allow for other types of demand shock considered in the literature: a shock in preference for government consumption, which makes public consumption expenditure quasi exogenous, and a liquidity premium shock, which affects the return for government bonds.

¹³Note that in terms of previous derivations, deflation under passive policy requires that θ is negative enough to compensate for the ϵ shock.

2.7.1 Taylor rule targeting inflation and output

Assume that monetary policy sets interest rates according to a rule of the form:

$$\tilde{R}_t = \phi_\pi \tilde{\pi}_t + \phi_y \tilde{y}_t$$

where now an explicit output target is added. Notice that under this specification it is simple to show that monetary policy is active if

$$\phi_\pi + \frac{\phi_\pi(1 - \beta)}{\kappa} > 1$$

where κ is the output gap slope in the Phillips curve. Conversely, monetary policy is passive if

$$\phi_\pi + \frac{\phi_\pi(1 - \beta)}{\kappa} \leq 1$$

. This result is standard, and a formal proof is omitted for brevity.

Let us maintain the values ϕ_π at 0.95 and 0.8 for the passive money regime. To now have a passive policy we need to set ϕ_y less than 0.3, given the calibration of the model's parameters. We set ϕ_y at 0.05 as in [Bouakez et al. \(2019\)](#). In the active policy scenario we maintain $\phi_\pi = 1.5$ as in the previous paragraphs and keep ϕ_y at 0.05.

Figures 2.5 and 2.6 show the results under commitment and discretion respectively, under the new interest rate rule. Notice that the patterns of adjustment of fiscal variables do not change qualitatively. We do however, witness a reduction in macroeconomic volatility, both public consumption and investment are now less responsive to shocks. The intuition is clear. Since monetary policy now reacts to the output gap, there is a strong adjustment of the policy rate aiming to mitigate the effect of the shock. In the case of commitment this implies that a smaller increase in spending is needed to absorb the shock, whereas under discretion the expansionary monetary policy changes the path of government consumption needed to satisfy the Euler equation and the new equilibrium features less of a reaction at the fiscal margin.

[Figure 2.5 approximately here]

[Figure 2.6 approximately here]

These differences aside, the key result of the paper, that differences between active and passive policies with regard to fiscal variables are small, remains.

2.7.2 Liquidity premium shock

We now replace the preference shock with a shock to the liquidity premium, an alternative way to model a the negative demand shock considered in the literature. Introducing this new source

of uncertainty requires to modify some of the model equations and so we firstly spell out the new competitive equilibrium relations, focusing on what is important here.

The budget constraint of the household now becomes

$$P_t c_t + \zeta_t q_{t,S} B_{t,S} + \zeta_t q_{t,L} B_{t,L} = (1 - \tau_t) W_t n_t + P_t D_t + B_{t-1,S} + (1 + \rho q_{t,L}) B_{t-1,L} + T_t$$

where ζ_t , (liquidity premium) is an exogenous shock to the returns on government bonds (short and long-term) The modelling assumption is similar to [Smets and Wouters \(2007\)](#). A negative shock to ζ_t increases the returns on government bonds, making them more attractive. Private consumption and hours fall as a consequence .

We the same quantitative experiments as before , assuming that the shock evolves according to:

$$\log(\zeta_t) = \rho_\zeta \log(\zeta_{t-1}) + \nu_t, \quad \nu_t \sim \mathcal{N}(0, \sigma_\nu^2)$$

We calibrate the shock to have persistence, $\rho_\zeta = 0.8$ and variance $\sigma_\nu = 0.01$. We chose these numbers to get a drop in consumption of similar magnitude as in the baseline model.

The optimality conditions for the households are now given by:

$$\begin{aligned} (1 - \tau_t) w_t &= \frac{\chi n_t^\varphi}{c_t^{-\sigma}} \\ q_{t,S} \zeta_t C_t^{-\sigma} &= \beta E_t \left[\frac{C_{t+1}^{-\sigma}}{\pi_{t+1}} \right] \\ q_{t,L} \zeta_t C_t^{-\sigma} &= \beta E_t \left[\frac{C_{t+1}^{-\sigma}}{\pi_{t+1}} \right] (1 + \delta q_{t+1,L}) \end{aligned}$$

while the rest of the constraints of the competitive equilibrium do not change.

In log-Linear format the above equations can be written as:

$$\begin{aligned} \tilde{w}_t &= \varphi \tilde{n}_t + \sigma \tilde{c}_t + \frac{\tau^*}{(1-\tau^*)} \tilde{\tau}_t \\ \tilde{R}_t &= -\sigma \tilde{c}_t + \tilde{\zeta}_t + E_t[\sigma c_{t+1} + \pi_{t+1}] \\ q_{t,L} &= -\tilde{R}_t + \beta \rho E_t[q_{t+1,L}] \end{aligned}$$

Notice that differently from the previous model, the liquidity premium shock does not affect the labor supply condition and since this is not a shock to the discount fact of consumption the future value of ζ will not show up in the Euler equation.

Moreover, now the second order approximation to the utility function produces a different objective .

$$-E_0 \sum_{t=0}^{\infty} \beta^t L_t$$

$$L_t = \omega_{g^i} \frac{\tilde{g}_t^i{}^2}{2} + \omega_y \frac{\tilde{y}_t^2}{2} + \omega_{\pi} \frac{\tilde{\pi}_t^2}{2} + \omega_{g^c} \frac{\tilde{g}_t^c{}^2}{2} + \omega_{k_{g-1}} \frac{k_{G,t-1}^2}{2} + \omega_{yg^i} \tilde{y}_t \tilde{g}_t^i + \omega_{yg^c} \tilde{y}_t \tilde{g}_t^c + \omega_{g^i g^c} \tilde{g}_t^i \tilde{g}_t^c + \omega_{yk} \tilde{y}_t k_{G,t-1} \quad (2.12)$$

The most significant difference is that the interactions terms between the shock and the variables $\tilde{g}_t^i, \tilde{g}_t^c, \tilde{y}_t$ and $k_{G,t}$ are no longer there. Hence the shock will not show up in the first order conditions for these variables, unless there are interactions in the constraints.¹⁴

Figure 2.7 shows the responses of the economy to a liquidity premium shock under commitment. Again there is little evidence of significant differences between the monetary policy regimes. Unsurprisingly consumption drops in response to the shock, because households demand larger levels of government debt. Due to the lower demand, output and inflation also fall.

The planner compensates with an government expenditure (public consumption and investment). Notice that this again absorbs the shock impact from the government constraint, and simultaneously keeps government debt from falling to accommodate the higher demand by households.

[Figure 2.7 approximately here]

Once again, in this model, the behavior of inflation across time is key to understand the differences in terms of the paths of debt. Under passive policies we obtain positive inflation rates, whereas under active policy the opposite holds, inflation turns negative in response to the shock. The intuition was spelled out previously and we will not repeat the argument here.

[Figure 2.8 approximately here]

Under discretion (Figure 2.8) we obtain a similar image as with the preference shock. Again there is no significant difference across the two monetary policy regimes in terms of the fiscal variables. Our results are robust to the liquidity shock we considered in this subsection.

2.7.3 Shock to preferences on government consumption

We now investigate the behavior of the model following a shock to preferences over the consumption of public goods. Assume that preferences are:

$$E_t \sum_{s=0}^{\infty} \beta^s \left(\frac{c_t^{1-\sigma}}{1-\sigma} - \chi \frac{n_t^{1+\varphi}}{1+\varphi} + \xi_t \frac{\chi_g G_t^{c,1-\sigma}}{1-\sigma} \right)$$

where ξ_t is a shock to preferences on government consumption expenditure. A positive shock increases households demand for public goods and government consumption expenditure thus

¹⁴We refer the reader to appendix 2.B for complete derivations of what is shown here.

leading to an optimal fiscal expansion. Note that in a sense government consumption expenditure become quasi-exogenous here, the planner still has the ability to choose a smaller or bigger reaction of spending to the disturbance, depending on the policy specification and the constraints she faces.

The new welfare loss is:

$$-E_0 \sum_{t=0}^{\infty} \beta^t L_t$$

$$L_t = \omega_{g^i} \frac{\tilde{g}_t^i{}^2}{2} + \omega_y \frac{\tilde{y}_t^2}{2} + \omega_{\pi} \frac{\tilde{\pi}_t^2}{2} + \omega_{g^c} \frac{\tilde{g}_t^c{}^2}{2} + \omega_{k-1} \frac{\tilde{k}_{t-1}^2}{2} + \omega_{yg^i} \tilde{y}_t \tilde{g}_t^i + \omega_{yg^c} \tilde{y}_t \tilde{g}_t^c + \omega_{g^i g^c} \tilde{g}_t^i \tilde{g}_t^c + \omega_{yk} \tilde{y}_t \tilde{k}_{t-1} + \omega_{\xi g^c} \tilde{g}_t^c \tilde{\xi}_t$$
(2.13)

where the last term, $\omega_{\xi g^c} \tilde{g}_t^c \tilde{\xi}_t$ is an interaction between the shock and the government consumption expenditure. We assume the following AR(1):

$$\log(\xi_t) = \rho_{\xi} \log(\xi_{t-1}) + \nu_t, \quad \nu_t \sim \mathcal{N}(0, \sigma_{\nu}^2)$$

We calibrate the parameters, $\rho_{\xi} = 0.95$ and $\sigma_{\nu} = 0.07$, so the shock has the same variance and persistence as the preference shock on private consumption analysed in the main section. This calibration is not far from the one used in the literature on optimal fiscal policy, such as [Faraglia et al. \(2013\)](#) or [Schmitt-Grohé and Uribe \(2004\)](#), to calibrate government spending shocks that does not provide utility to household. For instance in [Faraglia et al. \(2013\)](#) the government spending shock is modeled as an auto correlated process with a coefficient ρ_{ξ} of 0.95 and a standard deviation, σ_{ν} , of 0.08. So despite the nature of the shock we are modelling is not the same as in [Faraglia et al. \(2013\)](#) or [Schmitt-Grohé and Uribe \(2004\)](#), we calibrate it in a close way.

[Figures 2.9 and Figure 2.10 approximately here]

Figures 2.9 and Figure 2.10 repeat the optimal policy experiments. Unsurprisingly a positive shock to spending crowds out private consumption. However, and in contrast to the previously considered scenarios, total output and hours increase following the shock. As discussed, there is no effect of the shock on the labour market condition, in other words the shock is basically a standard spending shock.

Notice that government consumption increases by virtually the same amount in all models. And in all cases considered the positive spending shock leads to a drop in investment. This margin is used as a shock absorber. Through the combined effect of total government spending, debt does not change significantly relative to the steady state (at least we compared with the cases studied in previous subsections) and taxes also do not need to strongly adjust.

From the graphs we can easily conclude that optimal fiscal policy does not change when we move from an active monetary policy regime to a passive regime.

2.7.4 Inflationary shock

As a final experiment, we model an inflationary shock which we append to the linearized Phillips curve. The shock increases current inflation triggering an immediate response of the interest rate, given that Taylor rule targets inflation. As well as with the preference shock, the inflationary shock filters through the government budget constraint, creating a fiscal response.

$$\tilde{\pi}_t = \frac{\theta}{\psi} \left(\tilde{w}_t - \tilde{y}_t + \frac{\tilde{y}_t - (1 - \alpha)\tilde{k}_{G,t}}{\alpha} \right) + \beta E_t[\tilde{\pi}_{t+1}] + \varsigma_t$$

$$\log(\varsigma_t) = \rho_\xi \log(\varsigma_{t-1}) + \nu_t, \quad \nu_t \sim \mathcal{N}(0, \sigma_\nu^2)$$

We calibrate the shock as in [Leeper et al. \(2021a\)](#) who analyses optimal debt maturity under no commitment in the presence of a cost-push shock. The shock is considerably persistent with an auto correlation coefficient of 0.953 and a variance of 0.05.

[Figures 2.11 and Figure 2.12 approximately here]

As was the case with the previous shock, there is little difference in the optimal response of the economy to the shock. The shock filters through the budget constraint, and the planner reduces taxes and government consumption in an effort to counteract the effect on prices. On the other hand, public investment grows at the beginning, which helps to contain output drop, but as soon as the inflationary effects picks up, the planner cuts public capital. Public debt raises persistently, reaching a new level.

Under discretion, we can observe also little differences between optimal policies. Perhaps the increase in current interest triggered by a higher current inflation, reduces consumption and hours by more when monetary policy is active. Qualitatively, the shock shows some differences, though: the planner cuts debt to avoid future inflation, it increases public investment to increase output and cut expected inflation back.

2.8 Discussion: the role of debt maturity and public capital

2.8.1 Discussion: the role of debt maturity

Throughout the paper, we restricted the government to issue only long-term debt to reflect the fact that debt maturity is typically long in developed economies. However, it would be interesting to know whether our results would carry through if the planner has access to short-term instead of long-term debt.

Basically there are two margins via which debt maturity can affect optimal policy in the Ramsey plan: first inflation may adjust differently and second, with long term debt, economic shocks can be partly absorbed by changes in long bond prices.

The second channel is a standard fiscal insurance argument. Consider the baseline demand shock to the marginal utility of consumption. In the commitment case this shock resulted in a fall in taxes and the debt level; the intuition behind this is that it increased the present value of government surpluses. Even though real long term bond prices also increased (which partly restored the intertemporal solvency of debt) the rise of the surpluses was larger. With short term, the market value of debt cannot adjust to the shock and therefore the impact on the government budget constraint is larger. We would therefore observe a stronger adjustment in fiscal variables but qualitatively the results will not change.

Turning to the impact of maturity on inflation, we note that in standard Ramsey plans (in which the planner can freely set inflation without needing to adhere to any explicit interest rate rule) a longer maturity of debt, implies a more persistent reaction of inflation to shocks. Basically, when debt is short term, committing to a persistent change in inflation will not contribute towards making debt sustainable, since only current inflation matters for the real debt level.¹⁵

However, in our model, where the optimal policy has to abide by the interest rate rule, inflation is not a freely chosen variable, its dynamics are basically determined by the rule. (This is a point we made earlier). Maturity will have no effect whatsoever under active monetary policy, and under passive policy inflation persistence will be only governed by parameter ϕ_π , that is it will hinge completely on the interest rate rule.

Thus our results will not change if we assume short term instead of long term debt.

2.8.2 Discussion: the importance of public capital

The inclusion of public capital in the New Keynesian model is interesting through several facts. Firstly, marginal productivity of public capital pins down the real interest rate. This is often not the case in models with optimal fiscal policy in which the real interest rate is given only by the ratio of marginal utilities of consumption. Secondly, public capital exerts a downward pressure on marginal costs, becoming an extra tool to affect optimal inflation, which could be relevant specially when monetary policy is passive.¹⁶

Notice, however, that if the planner uses public capital to stimulate the economy, it needs to finance the additional expenditure either by new debt issuance, an increase in taxes and/or some inflation. With passive monetary policy, real interest rate is higher initially (the nominal interest rate tracks inflation less than one to one) and so it is marginal productivity of public capital, which is an incentive for the planner to use public investment. At the same time, it also becomes more

¹⁵Under no-commitment maturity should have an insignificant impact on the optimal inflation path.

¹⁶Some literature such as Bouakez et al. (2018) and Bouakez et al. (2019) shows that the planner optimally increases public investment when the interest rate is constrained by the zero lower bound; an extreme scenario of passive monetary policy. Nevertheless, Bouakez et al. (2018) and Bouakez et al. (2017) does not address how to finance public investment as we do in this paper.

costly to finance public investment by debt issuance. This result in addition to the Taylor rule which limits the planner ability to manipulate the borrowing terms makes likely the planner will process the adjustment through taxes. Furthermore, they raise a bit more to finance the optimal expenditure as much as with the active regime (see section 3.5.3). Finally, as soon as inflation picks up, the real interest rate becomes lower, thus public capital becomes less effective and so less interesting for the planner. We expect our results then to hold if the model abstracts from public capital, however the recession could be harder to handle for the planner.

Finally, the effectiveness of public capital has been debated in the literature due to the presence of delays in the outlays execution (see [Kydland and Prescott \(1982\)](#), [Ramey \(2020\)](#)). Under commitment and active monetary policy, we can expect the planner to use public capital less intensively with time-to-build as the ex-ante real interest rates becomes higher for a longer period of time. Intuitively with a longer time-to-build, the dissinflationary effect of public capital shows up further in the future, thus the real long-term interest becomes higher, reducing the effectiveness of public investment.¹⁷ On the other hand, the time to build in combination with passive monetary policy will likely mean a higher long-term interest rate at the moment the negative shock arrives, but an even lower and more persistent one after the effect of the time-to-build comes to play. This reduces the scope for public capital under both, active and passive monetary policy. Thus, we do not expect much difference in the optimal outcome from section 3.5.3 if time-to-build is added to the model.

Under discretion, the presence of time-to-build means the planner has incentives to increase public investment in the short run to boost the economy as the deflationary effect of public capital occurs far in the future. Thus optimal policies will resemble more the outcome under commitment. Nevertheless, these incentives will show up, both under active and passive monetary policy.

2.9 Conclusion

In this paper we analyze optimal fiscal policy in a New-Keynesian model when monetary policy is passive in the sense of [Leeper \(1991\)](#). We conclude that when fiscal policy is optimal, there is little change in consumption and output volatility in comparison to the active monetary policy regime, thoroughly analyzed by the literature. We endow the planner with several fiscal tools such as public consumption and investment, distortionary taxes and long-term bonds. Monetary policy follows a Taylor rule targeting inflation to acknowledge for the fact that central banks in developed economy shows some degree of independence.

With ad-hoc rules for monetary and fiscal policy, the literature on the FTPL has shown that within the passive monetary/active fiscal policy regime, the tax rate reacts weakly to changes in debt, letting inflation to do most of the job at the time of debt repayment. As a consequence, output and consumption volatility is higher due to the effects of higher inflation and interest rate. In this paper, we show that if the planner can optimally choose the tax rate, the importance of this mechanism is reduced and taxes ensures debt repayment. The planner uses also public consumption

¹⁷See [Bouakez et al. \(2018\)](#) for a discussion on the time-to-build and the use of public capital.

and investment to close the output gap and compensate for the fall in private consumption after the negative demand shock.

We found the same result under discretion: the planner chooses optimally the path for state variables (public capital and debt) and tax rates so there is practically no difference between the active and passive monetary policy regime. We show that if taxes and debt can be optimally chosen by a planner, the way in which monetary policy is set does not affect the path for private consumption and output when the economy faces demand shocks.

Qualitatively, results under commitment and discretion are very different at the moment of using the fiscal tools to fight back a demand driven recession. While in the commitment allocation the stimulus the government increases both, public consumption and investment, under discretion, public capital drops to increase future prices. The way in which the stimulus is financed is considerably different under both scenarios. While a committed planner increases taxes, cuts public debt as percentage of output and promises a future reduction of fiscal deficit, a no committed planner increases debt and cuts public capital to manipulate inflation expectations.

Finally, we show the following result holds in a scenario in which the Taylor rule tracks inflation and output and the economy is hit by a liquidity premium shock. Perhaps if there is some difference between the two models, is when there is a fiscal shock. Nevertheless, differences in optimal policies are not of the magnitude observed with rules.

2.A Appendix: First Best and flexible price steady state

First best

In this scenario, the planner is choosing $\{C_t, N_t, K_{G,t}, G_t^i, G_t^c\}$.

$$\begin{aligned} & \max_{\{C_t, N_t, K_{G,t}, G_t^i, G_t^c\}} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ U(C_t, N_t, G_t^c) \right. \\ & + \lambda_{1,t} \left[F(N_t, K_{G,t-1}) - C_t - G_t^c - G_t^i \right] \\ & \left. + \lambda_{2,t} \left[(1 - \delta_g) K_{G,t-1} + G_t^i - K_{G,t} \right] \right\} \end{aligned}$$

The efficient allocation is the solution to the following set of FONC:

$$\begin{aligned} 0 &= U_{c,t} - \lambda_{1,t} \\ 0 &= -U_{n,t} + \lambda_{1,t} F_N(N_t, K_{G,t-1}) \\ 0 &= U_{g,t} - \lambda_{1,t} \\ 0 &= -\lambda_{1,t} + \lambda_{2,t} \\ 0 &= \beta \lambda_{1,t} F_K(K_{G,t}, N_t) - \lambda_{2,t} + \beta E_t \lambda_{2,t+1} (1 - \delta_g) \\ 0 &= F(N_t, K_{G,t-1}) - C_t - G_t^c - G_t^i \\ 0 &= (1 - \delta_g) K_{G,t-1} + G_t^i - K_{G,t} \end{aligned}$$

Steady-state Using 2.14, we can get the following:

$$F_K(K_G, N) = \frac{1 - \beta(1 - \delta)}{\beta}$$

Calibrating hours in the steady-state to be 1/3 of disposable time, we can use pat equation together with the production function to solve for K_G in the steady state.

$$(1 - \alpha) N^\alpha K_G^{-\alpha} = \frac{1 - \beta(1 - \delta)}{\beta}$$

which in the steady state is:

$$K_G^* = \left[\frac{1 - \beta(1 - \delta)}{\beta} \frac{N^{-\alpha}}{1 - \alpha} \right]^{-1/\alpha}$$

and using the production function:

$$Y^* = N^\alpha K_G^{1-\alpha}$$

Using equation 2.14, we can recover government investment (where δ_g is taken from the literature):

$$G^{i,*} = \delta_g K_G^*$$

Using 2.14, the value for σ and φ from the literature, we can recover χ , so the household devotes 1/3 of her disposable time working:

$$\begin{aligned}\chi \frac{n^{*\varphi}}{c^{*-\sigma}} &= \alpha \frac{y^*}{n^*} \\ \chi &= \alpha \frac{y^* c^{*-\sigma}}{n^{*1+\varphi}}\end{aligned}$$

We can combine 2.14 and 2.14 to find for χ_g

$$\chi_g = \frac{c^{-\sigma}}{g^{-\sigma}}$$

Finally, we calibrate the value for $G^{c,*}$ in the steady state to be 0.2, hence using aggregate resource constraint we can recover C^* .

Flexible price steady-state

In a flexible price steady-state, the planner is not constrained by the Phillips curve, thus the equation can be dropped from the set of constraints along with the attached multiplier ϕ_2 . The euler equation for short term debt has to hold, hence dropping time subscripts from euler equation and in a zero inflation steady state and assuming $\zeta^* = 1, \pi^* = 1$:

$$\begin{aligned}\frac{1}{c^\sigma} q_{t,S} &= \beta \frac{1}{c^\sigma} \frac{1}{\pi^*} \\ q_S &= \frac{1}{R^*} = \beta \\ R^* &= \frac{1}{\beta}\end{aligned}$$

From the euler equation from long term bonds

$$\begin{aligned}\frac{1}{c^\sigma} q_{t,L} &= \beta \frac{1}{c^\sigma} \frac{1}{\pi^*} (1 + \delta q_{t,L}) \\ q_L &= \frac{\beta}{1 - \beta\delta} \\ R_L^* &= \frac{1 + \delta q_L^*}{q_L^*}\end{aligned}$$

From the profit maximization problem of the firm, the marginal costs are

$$mc_t = 1 - \frac{1}{\theta}$$

The behavior of firms imply they will produce until marginal cost equals marginal revenues, and substituting the definition from marginal cost in the the household FONC for labor, we can obtain the optimal subsidy for labor

$$\begin{aligned} \log(\mu_t) &= -mc_t \\ mc^* &= \left(1 - \frac{1}{\theta}\right) \\ \left(1 - \frac{1}{\theta}\right) &= \frac{(1-s)}{(1-\tau_t)} \chi \frac{n_t^\varphi}{c_t^{1-\sigma}} \frac{\alpha Y_t}{n_t} \\ (1-s^*) &= \left(1 - \frac{1}{\theta}\right) (1-\tau^*) \end{aligned}$$

Taxing the households accordingly we arrive to the same labor supply as in the efficient steady state. From the first order condition with respect to labor, eliminating time subscripts:

$$\begin{aligned} \chi \frac{n^{*\varphi}}{c^{*1-\sigma}} &= \alpha \frac{y^*}{n^*} \\ \chi &= \alpha \frac{y^* c^{*1-\sigma}}{n^{*1+\varphi}} \end{aligned}$$

Note that the equation is the same one obtained in the first best equilibrium.

From the FONC for the firm w.r.t labor, we can obtain the wage by replacing the production function and the optimal subsidy:

$$\begin{aligned} mc_t &= \frac{w_t(1-s)}{F_{N,t}} \\ w^* &= \frac{mc^*}{(1-s^*)} \alpha \frac{Y^*}{n^*} \\ w^* &= \frac{\alpha Y^*}{(1-\tau^*)n^*} \end{aligned}$$

Taking the households budget constraint, and dividing through price P_t , and imposing short term bonds are in zero net supply

$$\begin{aligned} P_t c_t + b_{t,L} + q_{t,S} B_{t,S} &= (1-\tau_t) W_t n_t + q_t D_t + (1+\rho q_{t,L}) B_{t-1,L} + B_{t-1,S} + T_t \\ c_t + b_{t,L} &= (1-\tau_t) W_t n_t + D_t + (1+\rho q_{t,L}) b_{t-1,L} + T_t \\ c_t + b_{t,L} &= (1-\tau_t) W_t n_t + D_t + (1+\rho q_{t,L}) b_{t-1,L} + T_t \end{aligned}$$

replacing dividends and the function for $Y_t(z)$

$$c_t + b_{t,L} = (1 - \tau_t)W_t n_t + (1 + \rho q_{t,L})b_{t-1,L} + \frac{P_t(z)}{P_t} Y_t(z) - w_t N_t(z)(1 - s) - \frac{\psi}{2} \left(\frac{P_t(z)}{P_{t-1}(z)} - 1 \right)^2 Y_t + T_t$$

$$c_t + b_{t,L} = W_t n_t (s - \tau_t) + (1 + \rho q_{t,L})b_{t-1,L} + \frac{P_t(z)}{P_t} \left(\frac{P_t(z)}{P_t} \right)^{-\theta} Y_t - \frac{\psi}{2} \left(\frac{P_t(z)}{P_{t-1}(z)} - 1 \right)^2 Y_t + T_t$$

Imposing a symmetric equilibrium in which each firm chooses the same price, $P_t(z) = P_t$

$$c_t + b_{t,L} = W_t n_t (s - \tau_t) + (1 + \rho q_{t,L})b_{t-1,L} + Y_t - \frac{\psi}{2} \left(\pi_t - 1 \right)^2 Y_t + T_t$$

replacing c_t with aggregate resource constraint and canceling the terms we arrive to the govt budget constraint and the subsidy in the steady state

$$Y_t - G_t^i - G_t^c - \frac{\psi}{2} \left(\pi_t - 1 \right)^2 Y_t + b_{t,L} = W_t n_t (s - \tau_t) + (1 + \rho q_{t,L})b_{t-1,L} + Y_t - \frac{\psi}{2} \left(\pi_t - 1 \right)^2 Y_t + T_t$$

$$- G_t^i - G_t^c + b_{t,L} = W_t n_t (s - \tau_t) + (1 + \rho q_{t,L})b_{t-1,L} + T_t$$

$$W_t n_t s = T_t$$

For the steady-state value of the tax rate, we calibrate a value so that debt to gdp ratio is 60% of output in the steady-state and we consider the value for wages in the steady state. From the govt budget constraint, first replacing the price for the long term bond:

$$\tau_t w_t n_t - g_t^i - g_t^c = (1 + \rho q_{t,L})b_{t,L} - q_{t,L}b_{t,L}$$

$$\tau_t w_t n_t - g_t^i - g_t^c = \left(\frac{1 - \beta}{1 - \beta \rho} \right) b_{t,L}$$

Replace w_t for the value in the steady state:

$$\tau_t \frac{\alpha y_t}{(1 - \tau) n_t} n_t - g_t^i - g_t^c = \left(\frac{1 - \beta}{1 - \beta \rho} \right) b_{t,L}$$

$$\tau_t = \frac{(1 - \tau_t)}{\alpha y_t} \left(\frac{1 - \beta}{1 - \beta \rho} \right) b_{t,L} + g_t^i + g_t^c$$

$$\tau_t = \frac{\frac{1}{\alpha y_t} \left[\left(\frac{1 - \beta}{1 - \beta \rho} \right) b_{t,L} + g_t^i + g_t^c \right]}{\left[1 + \frac{1}{\alpha y_t} \left(\frac{1 - \beta}{1 - \beta \rho} \right) b_{t,L} + g_t^i + g_t^c \right]}$$

So in the steady state, the tax rate can be the one to sustain certain level of debt to output ratio,

calibrated to 60%

$$\tau^* = \frac{\frac{1}{\alpha} \left[\left(\frac{1-\beta}{1-\beta\rho} \right) \frac{b_L^*}{y^*} + \frac{g^{*,i}}{y^*} + \frac{g^{c,*}}{y^*} \right]}{\left[1 + \frac{1}{\alpha} \left(\frac{1-\beta}{1-\beta\rho} \right) \frac{b_L^*}{y^*} + \frac{g^{i*}}{y^*} + \frac{g^{c,*}}{y^*} \right]}$$

2.B Appendix: Second order approximation to the utility function

In this appendix we derive the second order approximation for the utility function (see [Benigno and Woodford \(2012\)](#)). We first replace consumption by the aggregate resource constraint and labor by the production function. Variables denoted with a $\tilde{x}$ represents the percentage deviation from the steady-state, while variables denoted as x^* denoted the steady-state value for variable x .

The utility function is:

$$E_t \sum_{s=0}^{\infty} \beta^s \left(\epsilon_t \frac{c_t^{1-\sigma}}{1-\sigma} - \chi \frac{n_t^{1+\varphi}}{1+\varphi} + \chi_g \frac{G_t^{c,1-\sigma}}{1-\sigma} \right)$$

Taking the first term on consumption and substituting by the aggregate resource constraint the:

$$E_t \sum_{s=0}^{\infty} \beta^s \left(\epsilon_t \frac{[Y_t(1 - \psi/2(\pi_t - 1)^2) - G_t^i - G_t^c]^{1-\sigma}}{1-\sigma} \right)$$

taking derivatives:

$$F_{\pi} = -\psi(\pi_t - 1)y_t c_t^{-\sigma} \epsilon_t$$

$$F_y = c_t^{-\sigma} \left(1 - \frac{\psi}{2}(\pi_t - 1)^2 \right) \epsilon_t$$

$$F_{g^i} = -c_t^{-\sigma} \epsilon_t$$

$$F_{g^c} = -c_t^{-\sigma} \epsilon_t$$

$$F_{\epsilon_t} = \frac{c_t^{1-\sigma}}{1-\sigma}$$

$$F_{yy} = -\sigma c_t^{-\sigma-1} \left(1 - \frac{\psi}{2}(\pi_t - 1)^2 \right)^2 \epsilon_t$$

$$F_{\pi\pi} = -\frac{y^* \psi}{c^{*,\sigma}}$$

$$F_{g^i g^i} = -\sigma c_t^{-\sigma-1} \epsilon_t$$

$$F_{g^c g^c} = -\sigma c_t^{-\sigma-1} \epsilon_t$$

$$F_{\epsilon\epsilon} = 0$$

$$F_{\pi y} = \psi(\pi_t - 1)y_t \sigma c_t^{-\sigma-1} \epsilon_t \left[1 - \frac{\psi}{2}(\pi_t - 1)^2 \right]$$

$$F_{y g^i} = \left(1 - \frac{\psi}{2}(\pi_t - 1)^2 \right)^2 \epsilon_t \sigma c_t^{-\sigma-1}$$

$$F_{y g^c} = \left(1 - \frac{\psi}{2}(\pi_t - 1)^2 \right)^2 \epsilon_t \sigma c_t^{-\sigma-1}$$

$$F_{\pi g^i} = \psi(\pi_t - 1)y_t \sigma c_t^{-\sigma-1} \epsilon_t$$

$$F_{\pi g^c} = \psi(\pi_t - 1)y_t \sigma c_t^{-\sigma-1} \epsilon_t$$

$$F_{g^i \pi} = \sigma c_t^{-\sigma-1} \epsilon_t (-\psi(\pi_t - 1)y_t)$$

$$F_{g^c \pi} = \sigma c_t^{-\sigma-1} \epsilon_t (-\psi(\pi_t - 1)y_t)$$

$$F_{g^i y} = \sigma c_t^{-\sigma-1} \epsilon_t \left(1 - \frac{\psi}{2}(\pi_t - 1)^2 \right)$$

$$F_{g^c y} = \sigma c_t^{-\sigma-1} \epsilon_t \left(1 - \frac{\psi}{2}(\pi_t - 1)^2 \right)$$

$$F_{g^c g^i} = -\sigma c_t^{-\sigma-1} \epsilon_t$$

$$F_{g^i g^c} = -\sigma c_t^{-\sigma-1} \epsilon_t$$

$$F_{g^i \epsilon} = -c_t^{-\sigma}$$

$$F_{g^c \epsilon} = -c_t^{-\sigma}$$

$$F_{\epsilon g^c} = -c_t^{-\sigma}$$

$$F_{\epsilon g^i} = -c_t^{-\sigma}$$

$$F_{\epsilon y} = \left(1 - \frac{\psi}{2}(\pi_t - 1)^2 \right) c_t^{-\sigma}$$

$$F_{y \epsilon} = \left(1 - \frac{\psi}{2}(\pi_t - 1)^2 \right) c_t^{-\sigma}$$

$$F_{\epsilon \pi} = -\psi(\pi_t - 1)c_t^{-\sigma} y_t \epsilon_t$$

$$F_{\pi \epsilon} = -\psi(\pi_t - 1)c_t^{-\sigma} y_t$$

$$F_{y \pi} = \epsilon_t \psi(\pi_t - 1) \left[\sigma c_t^{-\sigma-1} y_t \left(1 - \frac{\psi}{2}(\pi_t - 1)^2 \right) - c_t^{-\sigma} \right]$$

Considering that in the steady-state $\pi_t = 1$ and $\epsilon_t = 1$, the second order approximation is:

$$F = \frac{y^*}{c^{*\sigma}} \tilde{y}_t - \frac{g^{i,*}}{c^{*\sigma}} \tilde{g}_t^i - \frac{\sigma y^{*2}}{c^{*1+\sigma}} \frac{1}{2} \tilde{y}_t^2 - \frac{y^* \psi}{c^{*\sigma}} \frac{1}{2} \tilde{\pi}_t^2 - \frac{\sigma g^{i,*2}}{c^{*1+\sigma}} \frac{1}{2} \tilde{g}_t^{i2} + \frac{\sigma g^{i,*} y^*}{c^{*1+\sigma}} \tilde{y}_t \tilde{g}_t^i + \frac{\sigma g^{c,*} y^*}{c^{*1+\sigma}} \tilde{y}_t \tilde{g}_t^c \\ - \frac{\sigma g^{c,*} g^{i,*}}{c^{*1+\sigma}} \tilde{g}_t^i \tilde{g}_t^c - \frac{g^{i,*}}{c^{*,\sigma}} \tilde{g}_t^i \tilde{\epsilon}_t + \frac{y^*}{c^{*,\sigma}} \tilde{y}_t \tilde{\epsilon}_t - \frac{g^{c,*}}{c^{*,\sigma}} \tilde{g}_t^c - \sigma \frac{g^{c,*2}}{c^{*,\sigma+1}} \frac{1}{2} \tilde{g}_t^{c2} - \frac{g^{c,*}}{c^{*,\sigma}} \tilde{g}_t^c \tilde{\epsilon}_t + t.i.p$$

Taking disutility from labor and replacing by the production function

$$E_t \sum_{s=0}^{\infty} \beta^s \left(\chi \frac{(y_t^{\frac{1}{\alpha}} k_{g,t-1}^{\frac{\alpha-1}{\alpha}})^{1+\varphi}}{1+\varphi} \right)$$

taking derivatives:

$$H_y = \frac{1}{\alpha} n_t^{1+\varphi} \frac{\chi}{y_t} \quad H_{y k_g} = (1+\varphi) \frac{1}{\alpha} \frac{\alpha-1}{\alpha} n_t^{1+\varphi} \frac{\chi}{k_{g,t-1} y_t} \\ H_{k_g} = \frac{\alpha-1}{\alpha} n_t^{1+\varphi} \frac{\chi}{k_{g,t-1}} \quad H_{k_g y} = (1+\varphi) \frac{1}{\alpha} \frac{\alpha-1}{\alpha} n_t^{1+\varphi} \frac{\chi}{k_{g,t-1} y_t} \\ H_{yy} = \frac{1}{\alpha} n_t^{1+\varphi} \frac{\chi}{y_t^2} \left[\frac{1}{\alpha} (\varphi+1) - 1 \right] \\ H_{k_g k_g} = \frac{\alpha-1}{\alpha} n_t^{1+\varphi} \frac{\chi}{k_{g,t-1}^2} \left[\frac{\alpha-1}{\alpha} (\varphi+1) - 1 \right]$$

At the steady state, total hours worked are 1/3 of disposable time, hence $\chi = \frac{\alpha y^* c^{*\sigma}}{n^{*1+\varphi}}$, making a second order approximation to the disutility of labor

$$H = \frac{y^*}{c^{*\sigma}} \tilde{y}_t + \frac{(\alpha-1)y^*}{c^{*\sigma}} k_{g,t-1} \tilde{y}_t + \frac{y^*}{c^{*\sigma}} \left[\frac{(1+\varphi)}{\alpha} - 1 \right] \frac{1}{2} \tilde{y}_t^2 + \frac{(\alpha-1)y^*}{c^{*\sigma}} \left[\frac{(\alpha-1)}{\alpha} (1+\varphi) - 1 \right] \frac{1}{2} k_{g,t-1}^2 \tilde{y}_t^2 \\ + \frac{(1+\varphi)(\alpha-1)y^*}{c^{*\sigma} \alpha} \tilde{y}_t k_{g,t-1} + t.i.p$$

Taking government consumption

$$E_t \sum_{s=0}^{\infty} \beta^s \left(\chi_g \frac{G_t^{c,1-\sigma}}{1-\sigma} \right)$$

$$G_g = \chi_g G_t^{-\sigma} \\ G_{gg} = -\chi_g \sigma G_t^{-\sigma-1}$$

$$G = \chi_g g^{c,*-\sigma+1} \tilde{g}_t^c - \chi_g \sigma g^{c,*-\sigma+1} \frac{1}{2} \tilde{g}_t^{c2} + t.i.p$$

At the steady state: $\chi_g = \frac{c^{-\sigma}}{g^{-\sigma}}$, hence the equation becomes

$$G = \frac{g^{c,*}}{c^{*,\sigma}} \tilde{g}_t^c - \frac{g^{c,*}\sigma}{c^{*,\sigma}} \frac{1}{2} \tilde{g}_t^{c^2} + t.i.p$$

Putting the terms together: $F + G - H$

$$\begin{aligned} U = & \frac{y^*}{c^{*\sigma}} \tilde{y}_t - \frac{g^{i,*}}{c^{*\sigma}} \tilde{g}_t^i - \frac{\sigma y^{*2}}{c^{*1+\sigma}} \frac{1}{2} \tilde{y}_t^2 - \frac{y^* \psi}{c^{*\sigma}} \frac{1}{2} \tilde{\pi}_t^2 - \frac{\sigma g^{i,*2}}{c^{*1+\sigma}} \frac{1}{2} \tilde{g}_t^{i^2} + \frac{\sigma g^{i,*} y^*}{c^{*1+\sigma}} \tilde{y}_t \tilde{g}_t^i + \frac{\sigma g^{c,*} y^*}{c^{*1+\sigma}} \tilde{y}_t \tilde{g}_t^c \\ & - \frac{\sigma g^{c,*} g^{i,*}}{c^{*1+\sigma}} \tilde{g}_t^i \tilde{g}_t^c - \frac{g^{i,*}}{c^{*,\sigma}} \tilde{g}_t^i \tilde{\epsilon}_t + \frac{y^*}{c^{*,\sigma}} \tilde{y}_t \tilde{\epsilon}_t - \frac{g^{c,*}}{c^{*,\sigma}} \tilde{g}_t^c - \frac{\sigma g^{c,*2}}{c^{*,\sigma+1}} \frac{1}{2} \tilde{g}_t^{c^2} - \frac{g^{c,*}}{c^{*\sigma}} \tilde{g}_t^c \tilde{\epsilon}_t + \frac{g^{c,*}}{c^{*\sigma}} \tilde{g}_t^c - \frac{\sigma g^{c,*}}{c^{*\sigma}} \frac{1}{2} \tilde{g}_t^{c^2} \\ & - \frac{y^*}{c^{*\sigma}} \tilde{y}_t - \frac{(\alpha-1)y^*}{c^{*\sigma}} k_{g,t-1} - \frac{y^*}{c^{*\sigma}} \left[\frac{(1+\varphi)}{\alpha} - 1 \right] \frac{1}{2} \tilde{y}_t^2 \\ & - \frac{(\alpha-1)y^*}{c^{*\sigma}} \left[\frac{(\alpha-1)}{\alpha} (1+\varphi) - 1 \right] \frac{1}{2} k_{g,t-1}^2 - \frac{(1+\varphi)(\alpha-1)y^*}{c^{*\sigma}\alpha} \tilde{y}_t k_{g,t-1} + t.i.p \end{aligned}$$

Canceling some terms,

$$\begin{aligned} U = & -\frac{g^{i,*}}{c^{*\sigma}} \tilde{g}_t^i - \frac{\sigma y^{*2}}{c^{*1+\sigma}} \frac{1}{2} \tilde{y}_t^2 - \frac{y^* \psi}{c^{*\sigma}} \frac{1}{2} \tilde{\pi}_t^2 - \frac{\sigma g^{i,*2}}{c^{*1+\sigma}} \frac{1}{2} \tilde{g}_t^{i^2} + \frac{\sigma g^{i,*} y^*}{c^{*1+\sigma}} \tilde{y}_t \tilde{g}_t^i + \frac{\sigma g^{c,*} y^*}{c^{*1+\sigma}} \tilde{y}_t \tilde{g}_t^c - \frac{\sigma g^{c,*} g^{i,*}}{c^{*1+\sigma}} \tilde{g}_t^i \tilde{g}_t^c - \frac{g^{i,*}}{c^{*,\sigma}} \tilde{g}_t^i \tilde{\epsilon}_t \\ & - \frac{\sigma g^{c,*2}}{c^{*,\sigma+1}} \frac{1}{2} \tilde{g}_t^{c^2} - \frac{\sigma g^{c,*}}{c^{*\sigma}} \frac{1}{2} \tilde{g}_t^{c^2} - \frac{g^{c,*}}{c^{*,\sigma}} \tilde{g}_t^c \tilde{\epsilon}_t + \frac{y^*}{c^{*,\sigma}} \tilde{y}_t \tilde{\epsilon}_t \\ & - \frac{(\alpha-1)y^*}{c^{*\sigma}} k_{g,t-1} - \frac{y^*}{c^{*\sigma}} \left[\frac{(1+\varphi)}{\alpha} - 1 \right] \frac{1}{2} \tilde{y}_t^2 - \frac{(\alpha-1)y^*}{c^{*\sigma}} \left[\frac{(\alpha-1)}{\alpha} (1+\varphi) - 1 \right] \frac{1}{2} k_{g,t-1}^2 \\ & - \frac{(1+\varphi)(\alpha-1)y^*}{c^{*\sigma}\alpha} \tilde{y}_t k_{g,t-1} + t.i.p. \end{aligned}$$

There are some linear terms remaining, taking a second order approximation to the law of motion for capital and reordering, we can replace for government investment (see Hansen (2018), which indeed is an application of Benigno and Woodford (2012))¹⁸:

$$\tilde{g}_t^i = \left[k_{g,t} + k_g^* \frac{k_{g,t}^2}{2} - (1-\delta) \left(k_{g,t-1} + k_g^* \frac{k_{g,t-1}^2}{2} \right) \right] \frac{1}{\delta} - g^{*,i} \frac{\tilde{g}_t^{i^2}}{2}$$

¹⁸Hansen (2018) proposes a New-Keynesian model with private capital and Calvo pricing. They derive a second order approximation as in Benigno and Woodford (2012) for the utility function using a second order approximation for the aggregate resource constraint and the law of motion for capital to eliminate linear terms from the objective function: consumption, and investment. In this problem there is only one linear variable, which is public investment.

Replacing in the second order approximation for the utility function:

$$\begin{aligned}
U = & -\frac{g^{i,*}}{c^{*\sigma}} \left[\left[\tilde{k}_{g,t} + k_g^* \frac{\tilde{k}_{g,t}^2}{2} - (1-\delta) \left(\tilde{k}_{g,t-1} + k_g^* \frac{\tilde{k}_{g,t-1}^2}{2} \right) \right] \frac{1}{\delta} - g^{i,*} \frac{\tilde{g}_t^2}{2} \right] - \frac{\sigma y^{*2}}{c^{*1+\sigma}} \frac{1}{2} \tilde{y}_t^2 - \frac{y^* \psi}{c^{*\sigma}} \frac{1}{2} \tilde{\pi}_t^2 \\
& - \frac{\sigma g^{i,*2}}{c^{*1+\sigma}} \frac{1}{2} \tilde{g}_t^2 - \frac{\sigma g^{i,*} y^*}{c^{*1+\sigma}} \tilde{y}_t \tilde{g}_t^i + \frac{\sigma g^{c,*} y^*}{c^{*1+\sigma}} \tilde{y}_t \tilde{g}_t^c + \frac{\sigma g^{c,*} g^{i,*}}{c^{*1+\sigma}} \tilde{g}_t^i \tilde{g}_t^c - \frac{g^{i,*}}{c^{*,\sigma}} \tilde{g}_t^i \tilde{\zeta}_t - \frac{\sigma g^{c,*2}}{c^{*,\sigma+1}} \frac{1}{2} \tilde{g}_t^{c2} - \frac{\sigma g^{c,*}}{c^{*\sigma}} \frac{1}{2} \tilde{g}_t^{c2} - \frac{g^{c,*}}{c^{*\sigma}} \tilde{g}_t^c \tilde{\zeta}_t \\
& - \frac{(\alpha-1)y^*}{c^{*\sigma}} \tilde{k}_{g,t-1} - \frac{y^*}{c^{*\sigma}} \left[\frac{(1+\varphi)}{\alpha} - 1 \right] \frac{1}{2} \tilde{y}_t^2 - \frac{(\alpha-1)y^*}{c^{*\sigma}} \left[\frac{(\alpha-1)}{\alpha} (1+\varphi) - 1 \right] \frac{1}{2} \tilde{k}_{g,t-1}^2 \\
& - \frac{(1+\varphi)(\alpha-1)y^*}{c^{*\sigma}\alpha} \tilde{y}_t \tilde{k}_{g,t-1} + \frac{y^*}{c^{*,\sigma}} \tilde{y}_t \tilde{\epsilon}_t + t.i.p.
\end{aligned}$$

Focusing on the linear terms....

$$-\frac{g^{i,*}}{c^{*\sigma}} \left[\tilde{k}_{g,t} - (1-\delta) \left(\tilde{k}_{g,t-1} \right) \right] \frac{1}{\delta} - \frac{(\alpha-1)y^*}{c^{*\sigma}} \tilde{k}_{g,t-1}$$

In the steady state $g^{i,*} = \delta k_g^*$, production function is a Cobb-Douglas so $F_{k_g} = \frac{1-\beta(1-\delta)}{\beta} = \frac{(1-\alpha)y^*}{k_g^*}$,

$$-\frac{k_g^*}{c^{*\sigma}} \left[\tilde{k}_{g,t} - (1-\delta) \tilde{k}_{g,t-1} + (\alpha-1)y^* \tilde{k}_{g,t-1} \right]$$

recalling the summation ¹⁹

$$\begin{aligned}
& -\frac{k_g^*}{c^{*\sigma}} \sum_{t=0}^{\infty} \beta^t \left[\tilde{k}_{g,t} - (1-\delta) \tilde{k}_{g,t-1} + (\alpha-1)y^* \tilde{k}_{g,t-1} \right] \\
& -\frac{k_g^*}{c^{*\sigma}} \sum_{t=0}^{\infty} \beta^t \left[\tilde{k}_{g,t} - \beta^{-1} \tilde{k}_{g,t-1} \right] \\
& -\frac{k_g^*}{c^{*\sigma}} k_{g,0}
\end{aligned}$$

hence there is no linear terms in the second order approximation

Adding and subtracting: $\frac{(\alpha-1)y^*}{c^{*\sigma}} k_{g,t-1}^2$,

$$\begin{aligned}
U = & \frac{g^{i,*2}}{c^{*\sigma}} \frac{\tilde{g}_t^2}{2} - \frac{\sigma y^{*2}}{c^{*1+\sigma}} \frac{1}{2} \tilde{y}_t^2 - \frac{y^* \psi}{c^{*\sigma}} \frac{1}{2} \tilde{\pi}_t^2 - \frac{\sigma g^{i,*2}}{c^{*1+\sigma}} \frac{1}{2} \tilde{g}_t^2 - \frac{\sigma g^{i,*} y^*}{c^{*1+\sigma}} \tilde{y}_t \tilde{g}_t^i + \frac{\sigma g^{c,*} y^*}{c^{*1+\sigma}} \tilde{y}_t \tilde{g}_t^c + \frac{\sigma g^{c,*} g^{i,*}}{c^{*1+\sigma}} \tilde{g}_t^i \tilde{g}_t^c - \frac{g^{i,*}}{c^{*,\sigma}} \tilde{g}_t^i \tilde{\epsilon}_t \\
& - \left[\frac{\sigma g^{c,*2}}{c^{*,\sigma+1}} + \frac{\sigma g^{c,*}}{c^{*\sigma}} \right] \frac{1}{2} \tilde{g}_t^{c2} - \frac{g^{c,*}}{c^{*\sigma}} \tilde{g}_t^c \tilde{\epsilon}_t - \frac{y^*}{c^{*\sigma}} \left[\frac{(1+\varphi)}{\alpha} - 1 \right] \frac{1}{2} \tilde{y}_t^2 + \frac{y^*}{c^{*,\sigma}} \tilde{y}_t \tilde{\epsilon}_t \\
& - \frac{(1+\varphi)(\alpha-1)y^*}{c^{*\sigma}\alpha} \tilde{y}_t \tilde{k}_{g,t-1} - \frac{(\alpha-1)y^*}{c^{*\sigma}} \left[\frac{(\alpha-1)}{\alpha} (1+\varphi) - 1 \right] \frac{1}{2} \tilde{k}_{g,t-1}^2 - \frac{k_g^{*2}}{c^{*\sigma}} \frac{\tilde{k}_{g,t}^2}{2} \\
& + \frac{k_g^{*2}}{c^{*\sigma}} (1-\delta) \frac{1}{2} \tilde{k}_{g,t-1}^2 - \frac{(\alpha-1)y^*}{c^{*\sigma}} k_{g,t-1}^2 + \frac{(\alpha-1)y^*}{c^{*\sigma}} k_{g,t-1}^2 + t.i.p.
\end{aligned}$$

¹⁹Expanding the summation one can see that, in the second line, it adds up to $k_{g,0}$

Then:

$$-\frac{k_g^*}{c^{*\sigma}} \left[k_{g,t}^2 - (1 - \delta)k_{g,t-1}^2 + (\alpha - 1)y^*k_{g,t-1}^2 \right]$$

recalling the summation

$$-\frac{k_g^*}{c^{*\sigma}} \sum_{t=0}^{\infty} \beta^t \left[k_{g,t}^2 - (1 - \delta)k_{g,t-1}^2 + (\alpha - 1)y^*k_{g,t-1}^2 \right]$$

Which as before can be proved independent to policy, hence:

$$\begin{aligned} U = & \frac{g^{i,*2} \tilde{g}_t^2}{c^{*\sigma} 2} - \frac{\sigma y^{*2}}{c^{*1+\sigma}} \frac{1}{2} \tilde{y}_t^2 - \frac{y^* \psi}{c^{*\sigma}} \frac{1}{2} \tilde{\pi}_t^2 - \frac{\sigma g^{i,*2}}{c^{*1+\sigma}} \frac{1}{2} \tilde{g}_t^2 - \frac{\sigma g^{i,*} y^*}{c^{*1+\sigma}} \tilde{y}_t \tilde{g}_t^i + \frac{\sigma g^{c,*} y^*}{c^{*1+\sigma}} \tilde{y}_t \tilde{g}_t^c + \frac{\sigma g^{c,*} g^{i,*}}{c^{*1+\sigma}} \tilde{g}_t^i \tilde{g}_t^c - \frac{g^{i,*}}{c^{*,\sigma}} \tilde{g}_t^i \tilde{\epsilon}_t \\ & - \left[\frac{\sigma g^{c,*2}}{c^{*,\sigma+1}} + \frac{\sigma g^{c,*}}{c^{*\sigma}} \right] \frac{1}{2} \tilde{g}_t^c^2 - \frac{g^{c,*}}{c^{*\sigma}} \tilde{g}_t^c \tilde{\epsilon}_t - \frac{y^*}{c^{*\sigma}} \left[\frac{(1 + \varphi)}{\alpha} - 1 \right] \frac{1}{2} \tilde{y}_t^2 - \frac{(1 + \varphi)(\alpha - 1)y^*}{c^{*\sigma} \alpha} \tilde{y}_t k_{g,t-1} \\ & - \frac{(\alpha - 1)y^*}{c^{*\sigma}} \left[\frac{(\alpha - 1)}{\alpha} (1 + \varphi) - 2 \right] + \frac{y^*}{c^{*,\sigma}} \tilde{y}_t \tilde{\epsilon}_t + t.i.p \end{aligned}$$

Last equation can be written as

$$\begin{aligned} L_t = & \omega_{g^i} \frac{\tilde{g}_t^2}{2} + \omega_y \frac{\tilde{y}_t^2}{2} + \omega_{\pi} \frac{\tilde{\pi}_t^2}{2} + \omega_{g^c} \frac{\tilde{g}_t^2}{2} + \omega_{k_{g-1}} \frac{k_{g,t-1}^2}{2} + \omega_{y g^i} \tilde{y}_t \tilde{g}_t^i + \omega_{y g^c} \tilde{y}_t \tilde{g}_t^c + \omega_{g^i g^c} \tilde{g}_t^i \tilde{g}_t^c + \omega_{\epsilon g^i} \tilde{g}_t^i \tilde{\epsilon}_t \\ & + \omega_{y k_g} \tilde{y}_t k_{g,t-1} + \omega_{\epsilon g^c} \tilde{g}_t^c \tilde{\epsilon}_t + \omega_{\epsilon y} \tilde{y}_t \tilde{\epsilon}_t \end{aligned}$$

where:

$$\begin{aligned} \omega_{g^i} &= \frac{g^{i,*}}{c^{*\sigma}} \left[g^{i,*} - \frac{g^{i,*} \sigma}{c^*} \right] \\ \omega_y &= -\frac{y^*}{c^{*\sigma}} \left[\frac{\sigma y^*}{c^*} + \frac{1 + \varphi}{\alpha} - 1 \right] \\ \omega_{k_g} &= -\frac{k_g^{*2}}{c^{*\sigma}} \\ \omega_{\pi} &= -\frac{y^* \psi}{c^{*\sigma}} \\ \omega_{g^c} &= -\sigma \left[\frac{g^{c,*2}}{c^{*,\sigma+1}} + \frac{g^{c,*}}{c^{*\sigma}} \right] \\ \omega_{y g^i} &= \frac{\sigma g^{i,*} y^*}{c^{*1+\sigma}} \\ \omega_{y g^c} &= \frac{\sigma g^{c,*} y^*}{c^{*1+\sigma}} \end{aligned}$$

$$\begin{aligned} \omega_{\epsilon y} &= \frac{y^*}{c^{*,\sigma}} \\ \omega_{g^i g^c} &= -\frac{\sigma g^{c,*} g^{i,*}}{c^{*1+\sigma}} \\ \omega_{\epsilon g^i} &= -\frac{g^{i,*}}{c^{*,\sigma}} \\ \omega_{y k_g} &= -\frac{(1 + \varphi)(\alpha - 1)y^*}{c^{*\sigma} \alpha} \\ \omega_{\epsilon k_g} &= -\frac{(\alpha - 1)y^*}{c^{*\sigma}} \\ \omega_{g^c \epsilon} &= -\frac{g^{c,*}}{c^{*\sigma}} \\ \omega_{k_{g-1}} &= -\frac{(\alpha - 1)y^*}{c^{*\sigma}} \left[\frac{(\alpha - 1)}{\alpha} (1 + \varphi) - 1 \right] \\ &+ \frac{k_g^{*2}}{c^{*\sigma}} (1 - \delta) \end{aligned}$$

2.C Appendix: Households and firm's problem

We now state the household's and firms problems considering preferences defined in the main text.

Households: Household maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\epsilon \frac{C_t^{1-\sigma}}{1-\sigma} - \chi \frac{n_t^{1+\varphi}}{1+\varphi} + \chi_g \frac{G_t^{c,1-\sigma}}{1-\sigma} \right)$$

subject to,

$$P_t c_t + q_{t,S} + q_{t,L} B_{t,L} = (1 - \tau_t) W_t n_t + P_t D_t + B_{t-1,S} + (1 + \rho q_{t,L}) B_{t-1,L} + T_t \quad (2.14)$$

where C_t denotes consumption, n_t denotes hours worked. D_t represents firm's profits redistributed to households, and P_t denotes the aggregate price level. $B_{t,L}$ is a long-term government bond, a perpetuity with coupons payment decaying at rate $0 \leq \rho < 1$ and price $q_{t,L}$. $B_{t,S}$ denotes a short-term government bond with price $q_{t,S}$. We assume that short-term debt is in zero net supply. ϵ_t is a preference shock affecting private consumption.

The first order conditions for the household's problem are:

$$\begin{aligned} q_{t,S} \epsilon_t c_t^\sigma &= \beta E_t \epsilon_{t+1} \frac{c_{t+1}^\sigma}{\pi_{t+1}} \\ q_{t,L} \epsilon_t c_t^\sigma &= \beta E_t \epsilon_{t+1} \frac{c_{t+1}^\sigma}{\pi_{t+1}} (1 + \rho q_{t+1,L}) \\ (1 - \tau_t) \frac{W_t}{P_t} &= \frac{\chi n_t^\varphi}{\epsilon_t c_t^{-\sigma}} \end{aligned}$$

where $\pi_t = \frac{P_t}{P_{t-1}}$ is the gross inflation rate

Firms

Production takes place in monopolistically competitive firms z which operates technologies with labor and public capital. The final good is a CES aggregate of the intermediate goods, $Y_t(z)$ of the form.

$$Y_t = \left(\int_0^1 Y_t(z)^{1-1/\theta} dz \right)^{\frac{\theta}{\theta-1}}$$

Intermediate goods seeks to set prices and labor input in order to maximize profits and subject to the following demand curve for each intermediate input z ,

$$Y_t(z) = \left(\frac{P_t(z)}{P_t} \right)^{-\theta} Y_t$$

and the price adjustment costs, which are given by,

$$\Sigma_t(z) = \frac{\psi}{2} \left(\frac{P_t(z)}{P_t} - 1 \right)^2 Y_t$$

Consider the dynamic profit maximization problem, in which the firm chooses price $P_t(z)$,

$$\max_{P(z)} E_t \sum_{l=0}^{\infty} \beta^l \frac{U_{c,t+l+1}}{U_{c,t+l}} \left[\frac{P_{t+l}(z)}{P_{t+l}} Y_{t+l}(z) - (1-s) \frac{mc_{t+l}}{P_{t+l}} Y_{t+l}(z) - \frac{\psi}{2} \left(\frac{P_{t+l}(z)}{P_{t+l-1}(z)} - 1 \right)^2 Y_{t+l} \right]$$

subject to the constraints

$$Y_{t+l}(z) = \left(\frac{P_{t+l}(z)}{P_{t+l}} \right)^{-\theta} Y_{t+l}$$

$$Y_{t+l}(z) \geq F(N_{t+l}(z), K_{G,t+l})$$

$U_{c,t+l+1}/U_{c,t+l} = \epsilon_{t+l+1} c_{t+l+1}^{-\sigma} / \epsilon_{t+l} c_{t+l}^{-\sigma}$ is the discount factor of the households that own the shares of firm z .

The firms's first order condition with respect to $P_t(z)$ are given by,

$$\begin{aligned} & \frac{1}{P_t} Y_t \left(\frac{P_t(z)}{P_t} \right)^{-\theta} + \frac{P_t(z)}{P_t^2} Y_t (-\theta) \left(\frac{P_t(z)}{P_t} \right)^{-\theta-1} + \theta \frac{(1-s)mc_t(z)}{P_t} Y_t - \psi \left(\frac{P_t(z)}{P_{t-1}(z)} - 1 \right) \frac{Y_t}{P_{t-1}(z)} \\ & + \beta E_t \frac{\epsilon_{t+1} c_{t+1}^{-\sigma}}{\epsilon_t c_t^{-\sigma}} \psi \left(\frac{P_{t+1}(z)}{P_t(z)} - 1 \right) \frac{P_{t+1}(z)}{P_t^2(z)} Y_{t+1} = 0 \end{aligned}$$

Imposing a symmetric equilibrium in which all firms set the same price, $P_t(z) = P_t$, all the firms consider the same productivity and real wage:

$$\frac{Y_t}{P_t} - \theta \frac{Y_t}{P_t} + \theta \frac{mc_t(1-s)}{P_t} Y_t - \psi \left(\frac{P_t}{P_{t-1}} - 1 \right) \frac{Y_t}{P_{t-1}} + \beta E_t \frac{\epsilon_{t+1} c_{t+1}^{-\sigma}}{\epsilon_t c_t^{-\sigma}} \psi \left(\frac{P_{t+1}}{P_t} - 1 \right) \frac{P_{t+1}}{P_t^2} Y_{t+1} = 0$$

Multiply by P_t and dividing by Y_t :

$$1 - \theta - \psi \left(\frac{P_t}{P_{t-1}} - 1 \right) \frac{P_t}{P_{t-1}} + \theta(1-s)mc_t + \beta E_t \frac{\epsilon_{t+1} c_{t+1}^{-\sigma}}{\epsilon_t c_t^{-\sigma}} \psi \left(\frac{P_{t+1}}{P_t} - 1 \right) \frac{P_{t+1}}{P_t} \frac{Y_{t+1}}{Y_t} = 0$$

Define gross inflation rate, π_t as P_t/P_{t-1} , then

$$\begin{aligned}
 1 - \theta + \theta(1 - s)mc_t - \psi \left[(\pi_t - 1)\pi_t - \beta E_t \frac{\epsilon_{t+1} c_{t+1}^{-\sigma}}{\epsilon_t c_t^{-\sigma}} (\pi_{t+1} - 1)\pi_{t+1} \frac{Y_{t+1}}{Y_t} \right] &= 0 \\
 \theta(1 - s)mc_t &= \theta - 1 + \psi \left[(\pi_t - 1)\pi_t - \beta E_t \frac{\epsilon_{t+1} c_{t+1}^{-\sigma}}{\epsilon_t c_t^{-\sigma}} (\pi_{t+1} - 1)\pi_{t+1} \frac{Y_{t+1}}{Y_t} \right] \\
 (1 - s)mc_t &= \frac{\theta - 1}{\theta} + \frac{\psi}{\theta} \left[(\pi_t - 1)\pi_t - \beta E_t \frac{\epsilon_{t+1} c_{t+1}^{-\sigma}}{\epsilon_t c_t^{-\sigma}} (\pi_{t+1} - 1)\pi_{t+1} \frac{Y_{t+1}}{Y_t} \right]
 \end{aligned}$$

Replacing by marginal cost by the definition, we get the same equation as in text

$$(1 - s) \frac{w_t}{F_{N,t}} = \frac{\theta - 1}{\theta} + \frac{\psi}{\theta} \left[(\pi_t - 1)\pi_t - \beta E_t \frac{\epsilon_{t+1} c_{t+1}^{-\sigma}}{\epsilon_t c_t^{-\sigma}} (\pi_{t+1} - 1)\pi_{t+1} \frac{Y_{t+1}}{Y_t} \right]$$

2.D Appendix: Numerical strategy for the time-consistent problem

We first solve the linearized version of the model under commitment using Dynare. By doing this, we obtain the policy function for each endogenous variable. Afterwards, we parameterize each variable contained in the expectations described in section 2.5.3 as linear polynomials of the state variables. Notice that since we are solving a linear model, the derivatives of each variable contained in the expectations term with respect to the state variables are directly the coefficients from the policy function.

We then solve the set of equations for the time-consistent model by using as initial guess for the parameters contained in the expectations, the coefficients from the policy function under commitment. We then update the parameters until the change in the parameters with the previous iteration become less than $1e^{-6}$. To sum up:

- Run a linear version of the model under commitment to get the initial guess for the coefficients of the policy function
 1. The model is linear so policy functions would be of the form $x_t = \alpha_1 s_{t-1} + \alpha_2 s_{t-1} + \alpha_3 z_{t-1}$, where x_t is the endogenous variable, s_t z_t are the endogenous and exogenous state variables
 2. Parameterize the conditional expectations contained in the time-consistent problem as linear function of the state variables. Eg: $E_t c_{t+1} = \alpha_1 E_t b_t + \alpha_2 E_t k g_t + \alpha_3 E_t \zeta_{t+1}$, under consistency, is : $E_t c_{t+1} = \alpha_1 b_t + \alpha_2 k g_t + \alpha_3 \rho \zeta_t$
 3. Once the expectations are parameterized solve the system for the time-consistent problem using Dynare and the coefficients from the policy function obtained in the commitment problem
 4. Update the coefficients until the change with previous iteration is less than $1e^{-6}$

Notice that a linear solution would be of the form:

$$\begin{aligned}
 c_t &= a_k(k_{G,t-1} - k_G^*) + a_b(b_{t-1} - b^*) + a_z(z_t - z^*) + a_e e_t \\
 E_t[c_{t+1} - c^*] &= E[a_k(k_{G,t} - k_G^*) + a_b(b_{t-1} - b^*) + a_z(z_{t+1} - z^*) + a_e e_{t+1}] \\
 E e_{t+1} &= 0 \\
 E_t[\tilde{c}_{t+1}] &= a_k \tilde{k}_{g,t} + a_b \tilde{b}_t + a_z \rho \tilde{z}_t
 \end{aligned}$$

2.E Appendix: Tables and figures

TABLE 2.1: Model parameters

Description	Parameter	Value	Source/Target
Discount factor	β	0.959	$r=0.04$
Risk aversion	σ	1.5	Leeper et al. (2010)
Frisch elasticity	φ	2	Leeper et al. (2010)
Labor intensity	a	0.954	Leeper et al. (2010)
Elasticity of substitution intermediate goods	θ	6	target mark-up 20%
Price-adjustment cost parameter	ψ	200	target Phillips curve slope 0.03
Autocorrelation preference shock	ρ_ϵ	0.955	
Autocorrelation liquidity premium shock	ρ_ζ	0.8	
Autocorrelation preference government consumption shock	ρ_z	0.955	
Decaying coupon rate	ρ	0.95	target debt maturity 16 quarters
Depreciation rate public capital	δ	0.03	target $g^{i,*}/y^* = 4.5\%$
Debt to output (market value)	$q_L^* b_L^*/4y^*$	60%	US data
Public consumption to output	$g^{c,*}/y^*$	15%	US data
Monetary policy			
Feedback to inflation: active	ϕ_π	1.5	Woodford (2011)
passive		0.95	see text
passive		0.8	see text

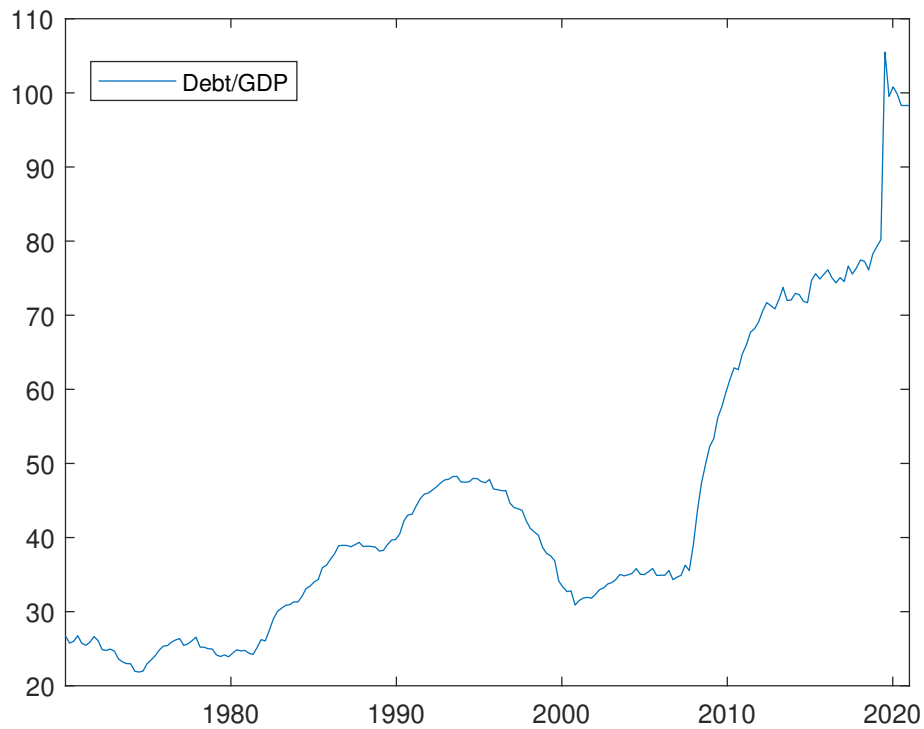


FIGURE 2.1: US public debt held by the public as % of GDP

Note: the figure shows US public debt held by the public as percentage of GDP. Source: Federal Reserve Bank of St. Louis

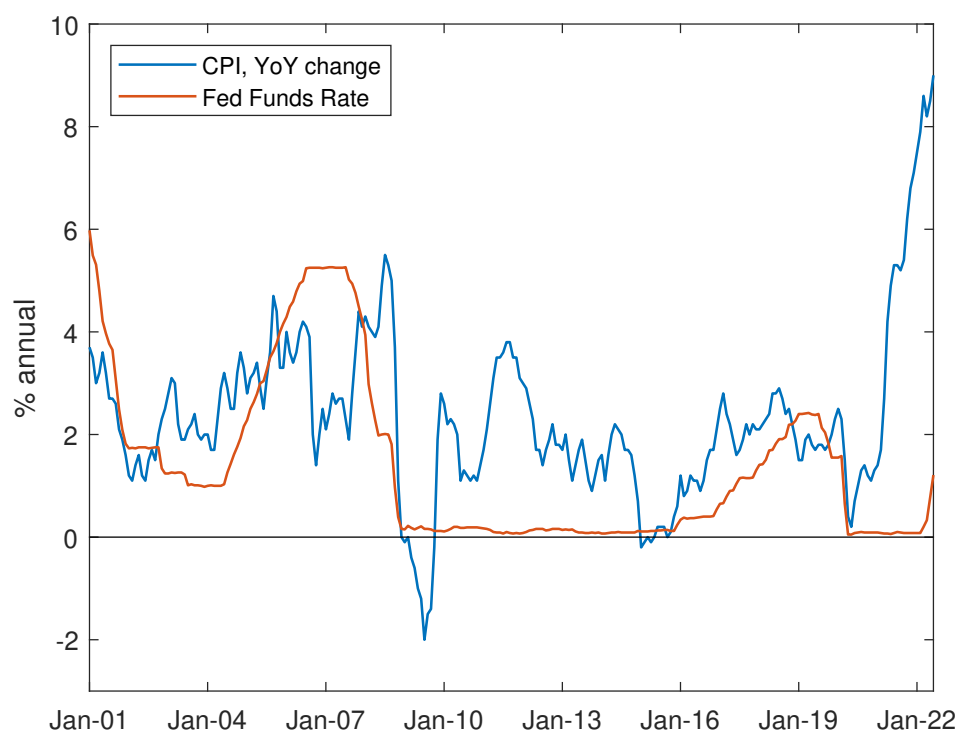


FIGURE 2.2: US inflation YoY (%) and FED funds interest rate

Note: the figure shows the annual change of the CPI for the US (in percentage) as measured by the Bureau of Labor Statistics and the Effective Federal Funds interest rate. Source: Bureau of Labor Statistics and Federal Reserve Bank of St. Louis

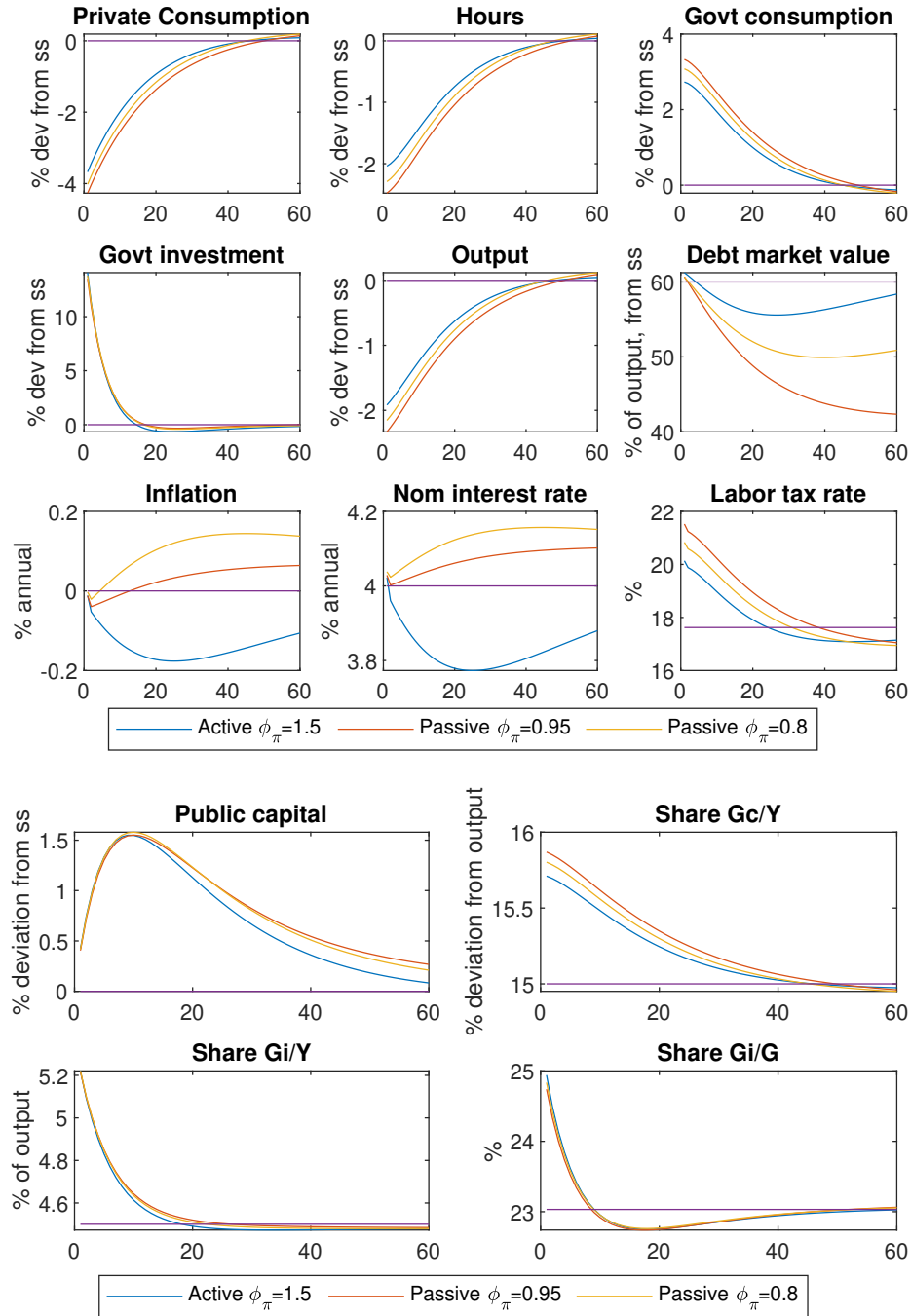


FIGURE 2.3: Ramsey optimal response to a negative preference shock on private consumption:
Active vs Passive monetary policy

Note: the figure shows the optimal response of the main macroeconomic variables to a negative consumption shock for a committed Ramsey planner. The blue line represents the economy under an active monetary policy regime ($\phi_\pi = 1.5$), where the red and the orange lines represent the optimal response to the economy under passive monetary policy ($\phi_\pi = 0.8$) ($\phi_\pi = 0.95$) respectively

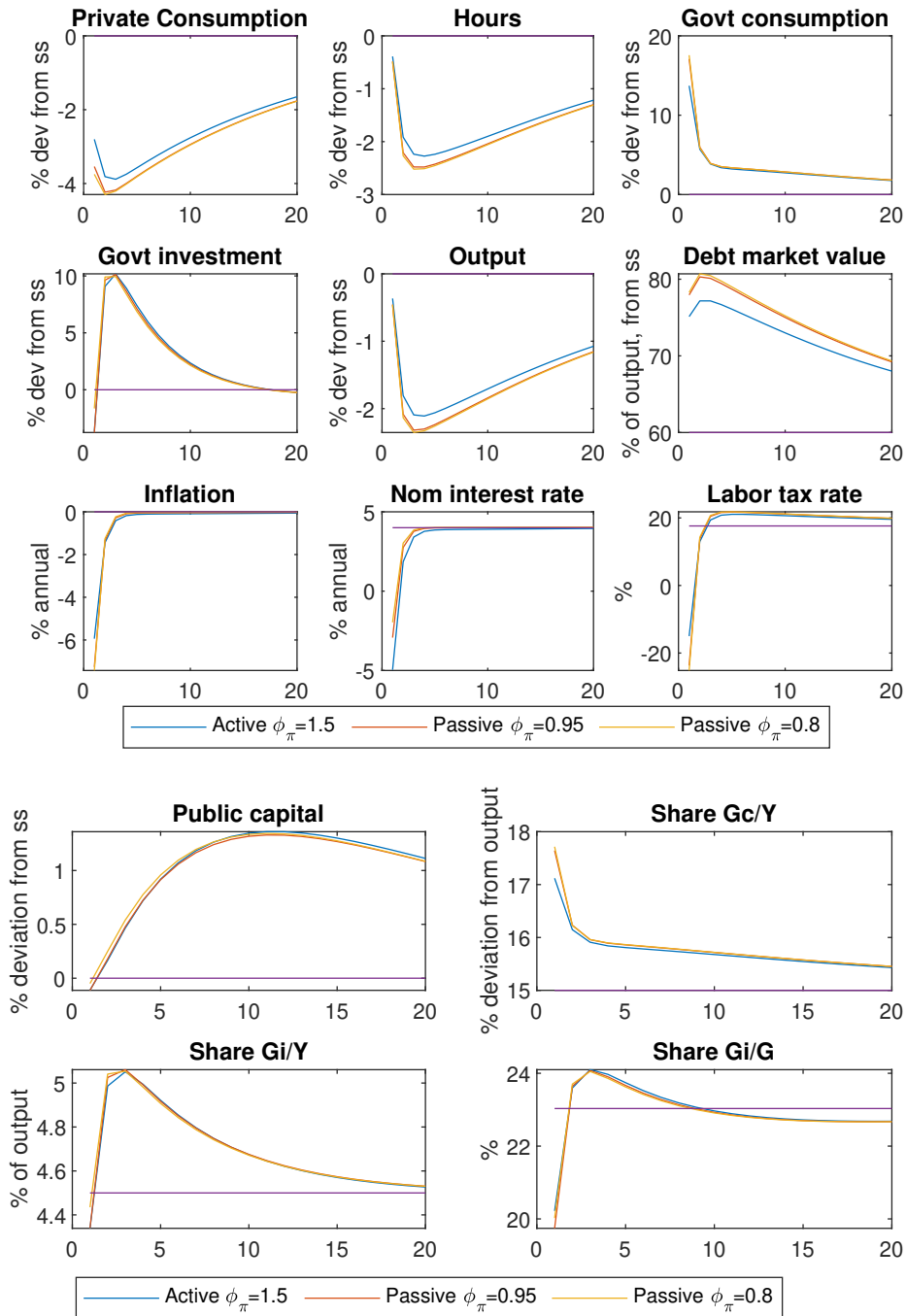


FIGURE 2.4: Time-consistent response to a negative preference shock on private consumption:
Active vs passive monetary policy

Note: the figure shows the optimal response of the main macroeconomic variables to a negative consumption shock for a time-consistent planner. The blue line represents the economy under an active monetary policy regime ($\phi_\pi = 1.5$), where the red and the orange lines represent the optimal response to the economy under passive monetary policy ($\phi_\pi = 0.8$) ($\phi_\pi = 0.95$) respectively

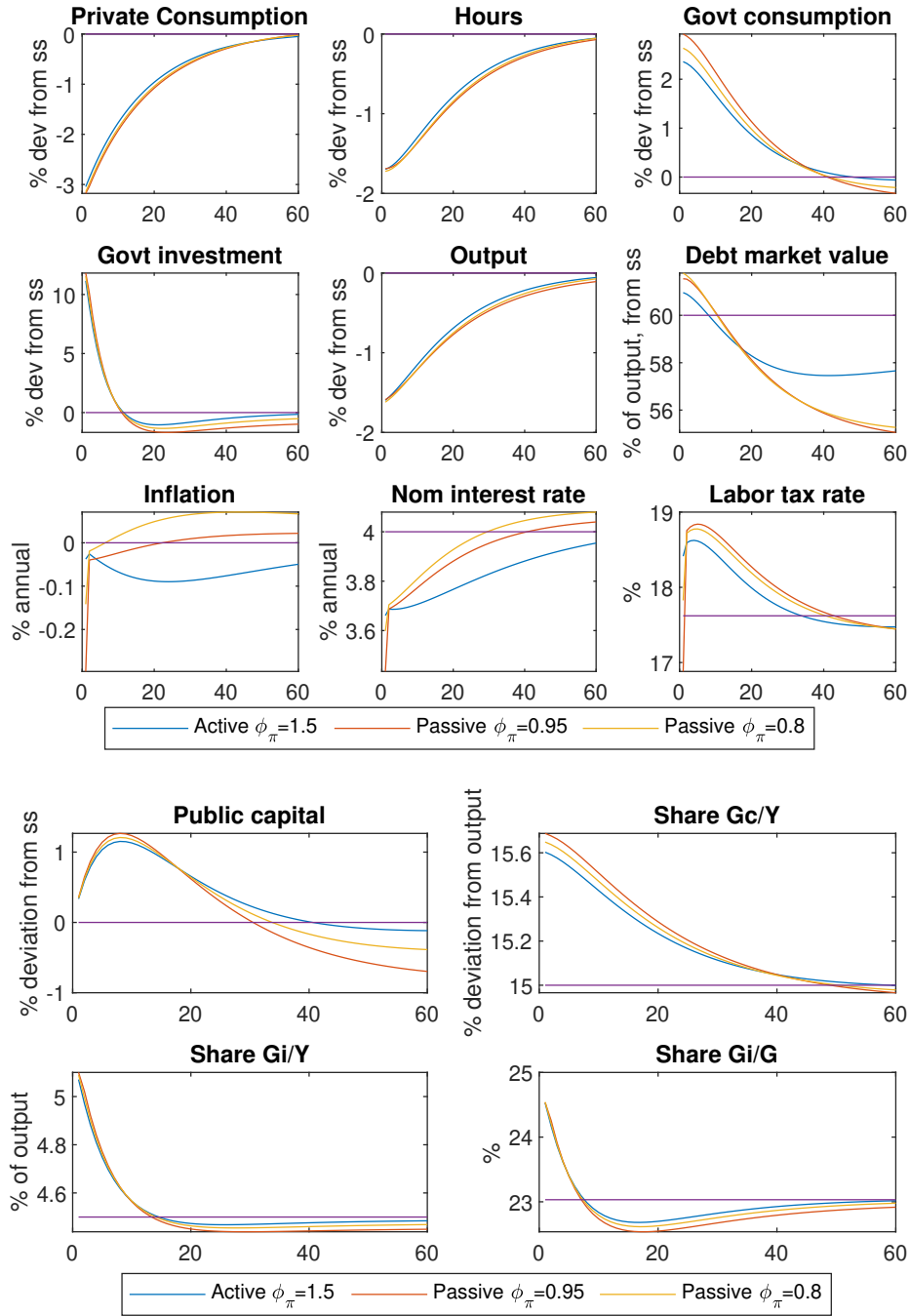


FIGURE 2.5: Commitment: Optimal response to a negative preference shock on private consumption-Taylor rule targeting inflation and output

Note: the figure shows the optimal response of the main macroeconomic variables to a liquidity premium shock for a committed Ramsey planner. The blue line represents the economy under an active monetary policy regime ($\phi_\pi = 1.5$), where the red and the orange lines represent the optimal response to the economy under passive monetary policy ($\phi_\pi = 0.8$) ($\phi_\pi = 0.95$) respectively. In all cases, $\phi_y = 0.05$

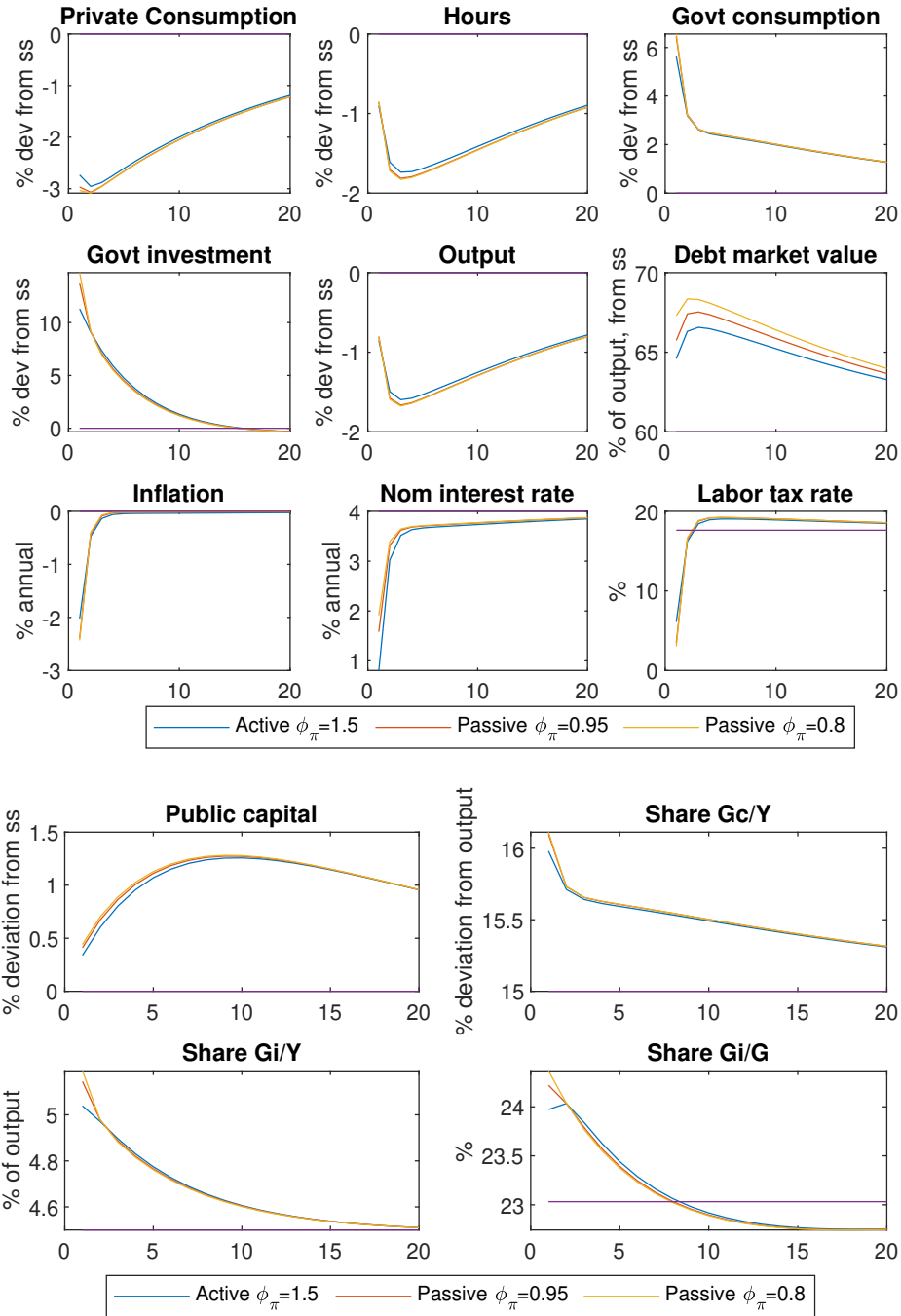


FIGURE 2.6: Discretion: Optimal response to a negative preference shock on private consumption-Taylor rule targeting inflation and output

Note: the figure shows the optimal response of the main macroeconomic variables to a negative consumption shock for a time-consistent planner. The blue line represents the economy under an active monetary policy regime ($\phi_\pi = 1.5$), where the red red and the orange lines represents the optimal response to the economy under passive monetary policy ($\phi_\pi = 0.8$) ($\phi_\pi = 0.95$) respectively. In all cases, $\phi_y = 0.05$

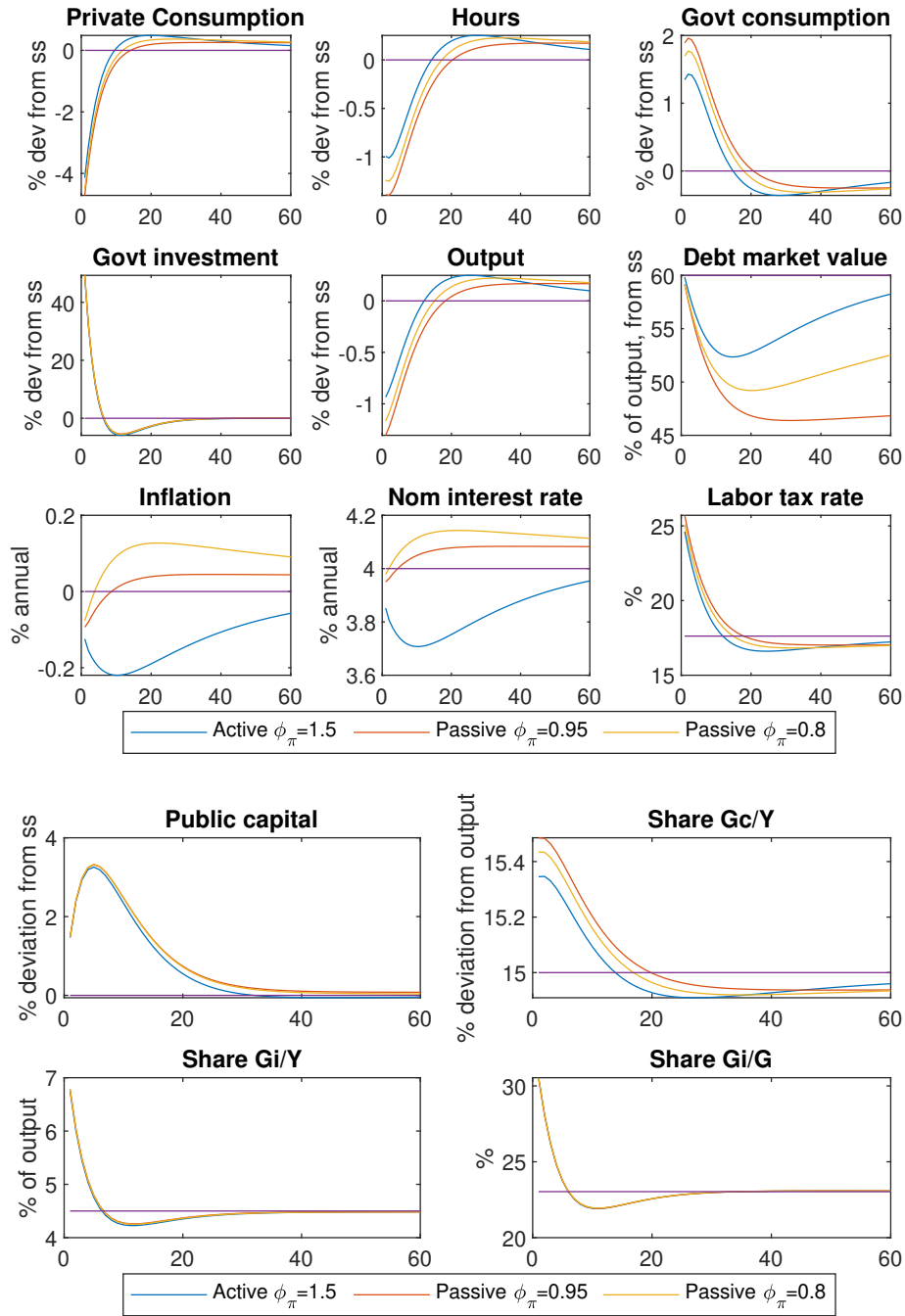


FIGURE 2.7: Commitment: Optimal response to a liquidity premium shock

Note: the figure shows the optimal response of the main macroeconomic variables to a liquidity premium shock for a committed Ramsey planner. The blue line represents the economy under an active monetary policy regime ($\phi_\pi = 1.5$), where the red and the orange lines represent the optimal response to the economy under passive monetary policy ($\phi_\pi = 0.95$) ($\phi_\pi = 0.8$) respectively

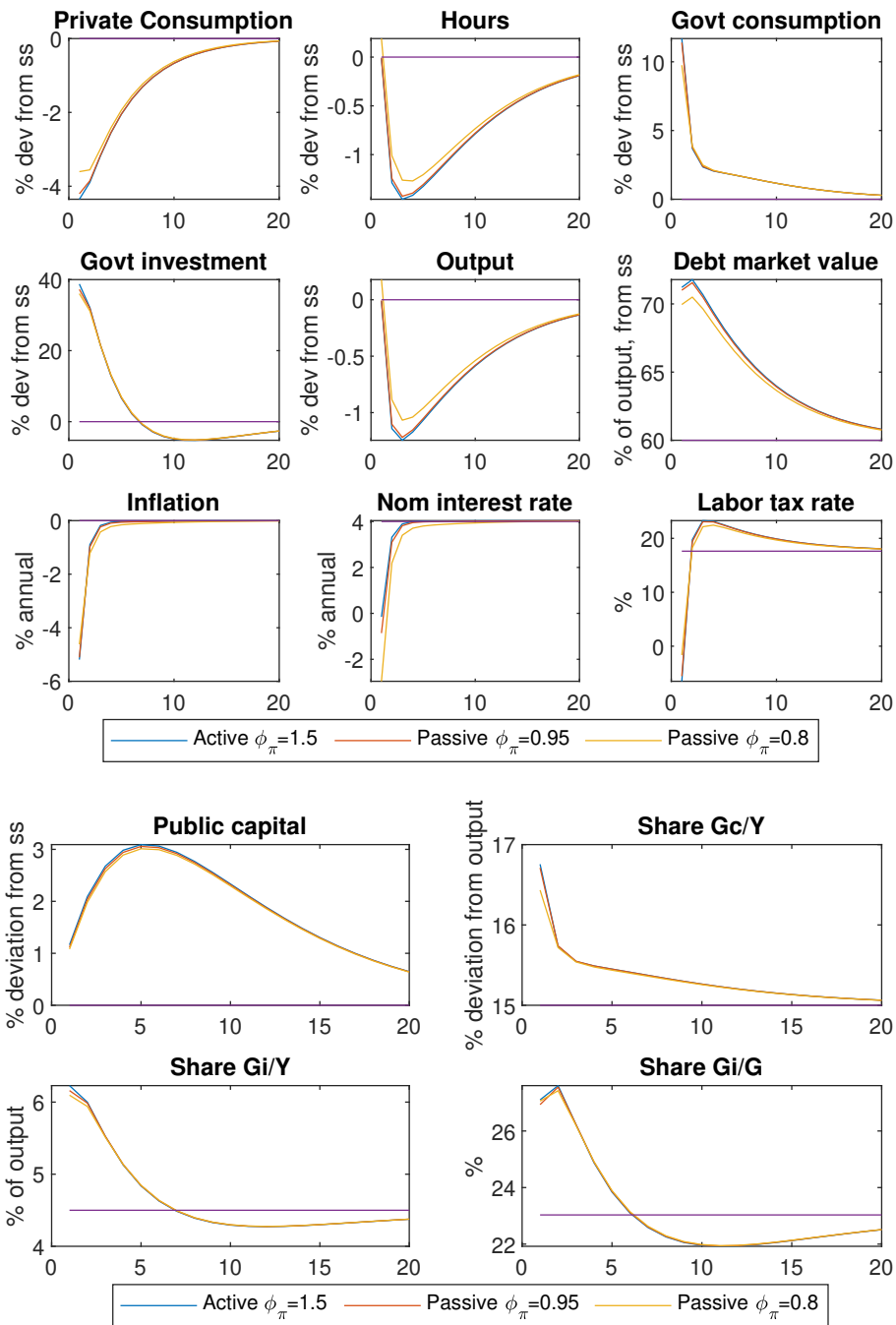


FIGURE 2.8: Discretion: Optimal response to a liquidity premium shock

Note: the figure shows the optimal response of the main macroeconomic variables to a negative consumption shock for a time-consistent planner. The blue line represents the economy under an active monetary policy regime ($\phi_\pi = 1.5$), where the red and the orange lines represent the optimal response to the economy under passive monetary policy ($\phi_\pi = 0.8$) ($\phi_\pi = 0.95$) respectively

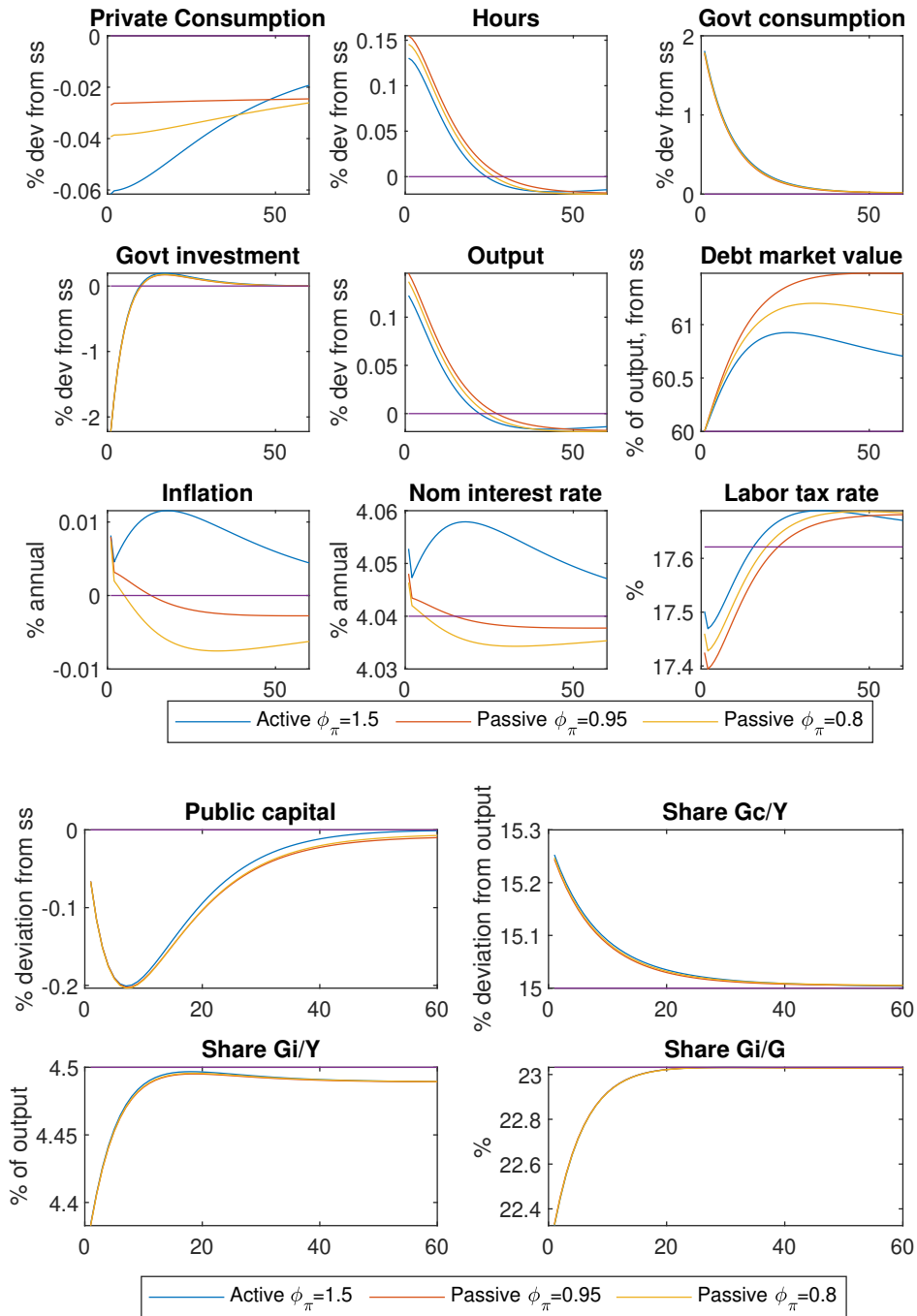


FIGURE 2.9: Commitment: Optimal response to a positive shock in preferences for government consumption

Note: the figure shows the optimal response of the main macroeconomic variables to a shock in preferences for government consumption. The blue line represents the economy under an active monetary policy regime ($\phi_\pi = 1.5$), where the red and the orange lines represent the optimal response to the economy under passive monetary policy ($\phi_\pi = 0.8$) ($\phi_\pi = 0.95$) respectively

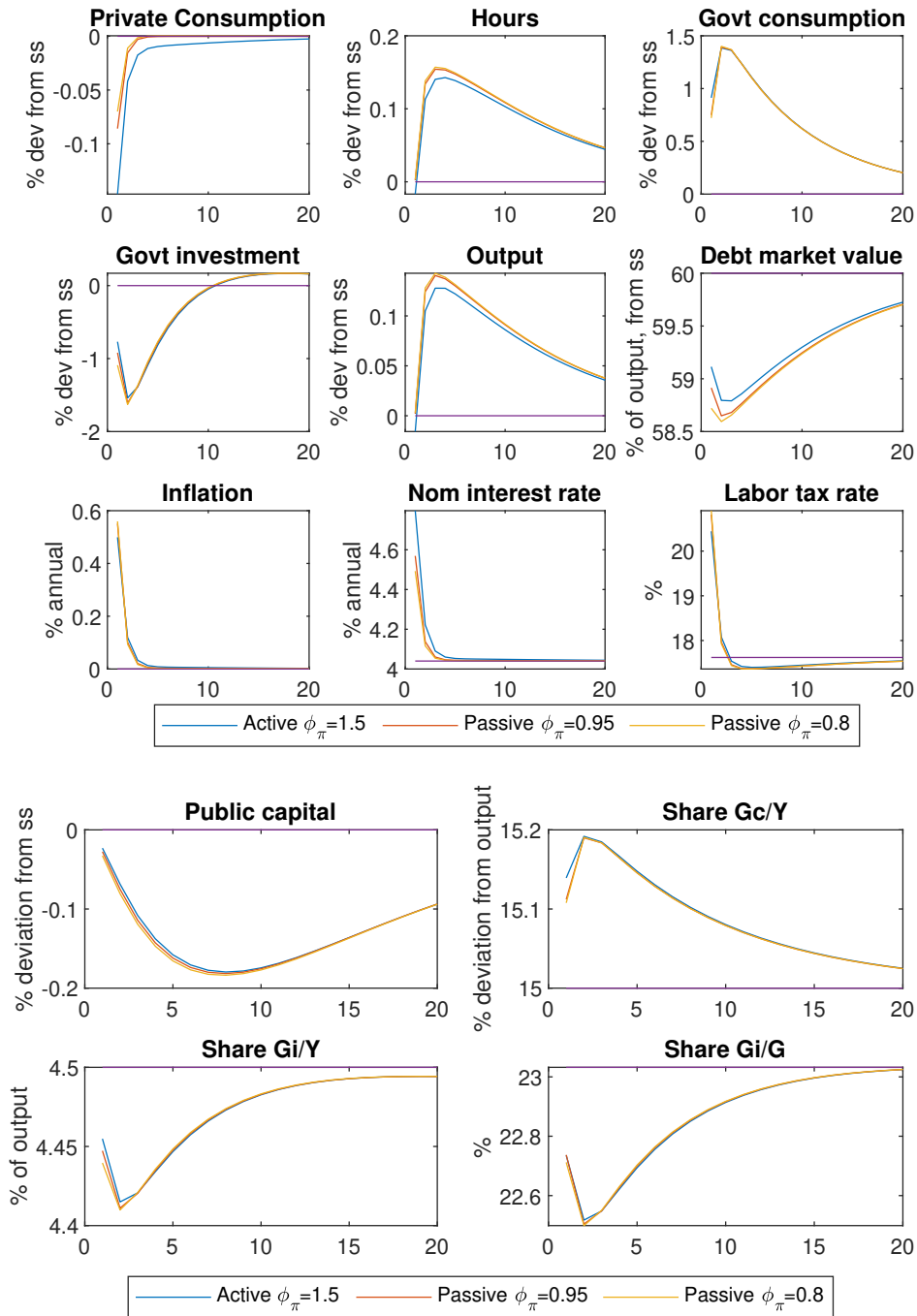


FIGURE 2.10: Discretion: Optimal to a positive shock in preferences for government consumption

Note: the figure shows the optimal response of the main macroeconomic variables to a shock in preferences for government consumption. The blue line represents the economy under an active monetary policy regime ($\phi_\pi = 1.5$), where the red and the orange lines represent the optimal response to the economy under passive monetary policy ($\phi_\pi = 0.8$) ($\phi_\pi = 0.95$) respectively

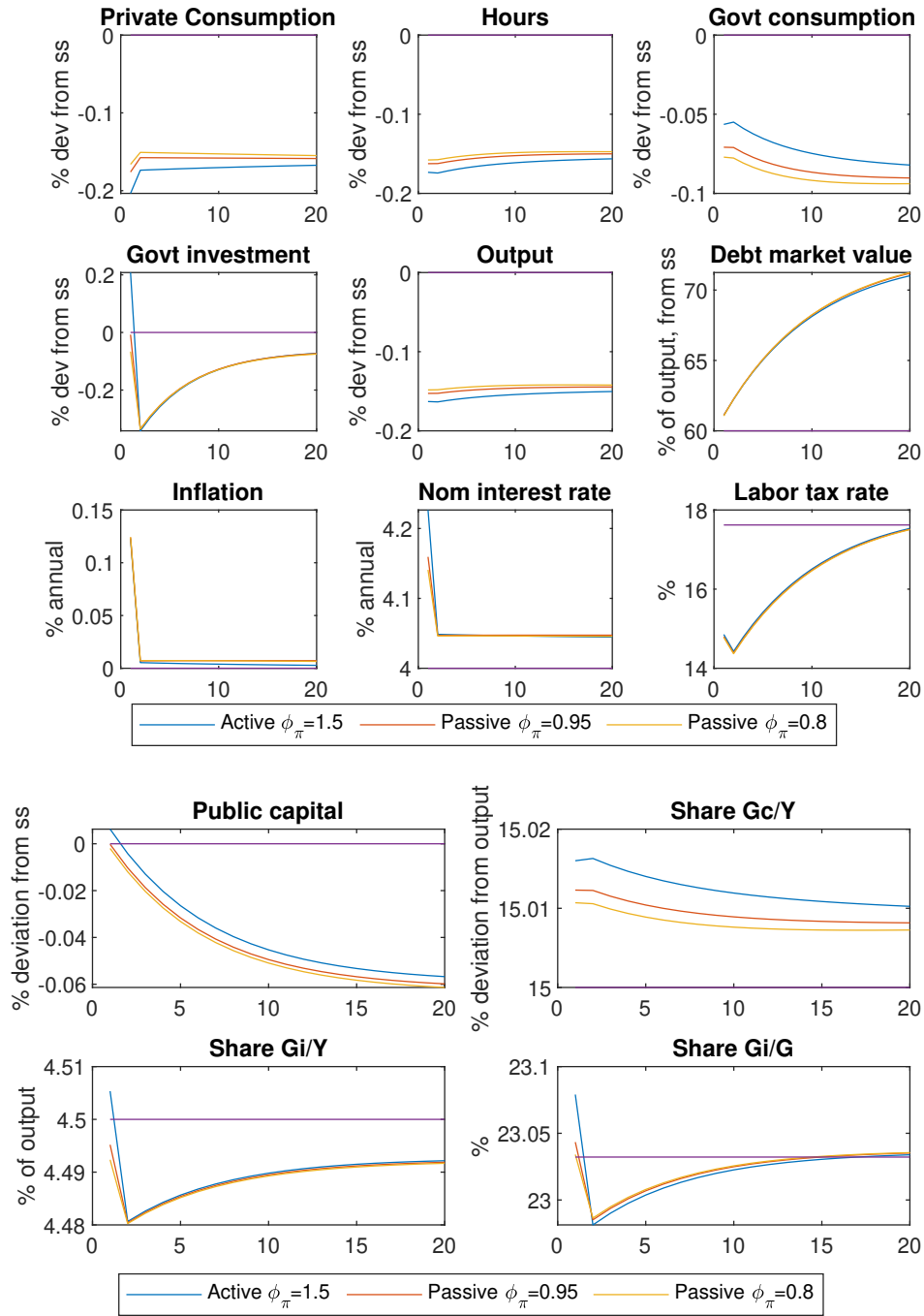


FIGURE 2.11: Commitment: Optimal response to an inflationary shock

Note: the figure shows the optimal response of the main macroeconomic variables to a shock in the Phillips curve increasing current inflation. The blue line represents the economy under an active monetary policy regime ($\phi_{\pi} = 1.5$), where the red and the orange lines represent the optimal response to the economy under passive monetary policy ($\phi_{\pi} = 0.8$) ($\phi_{\pi} = 0.95$) respectively

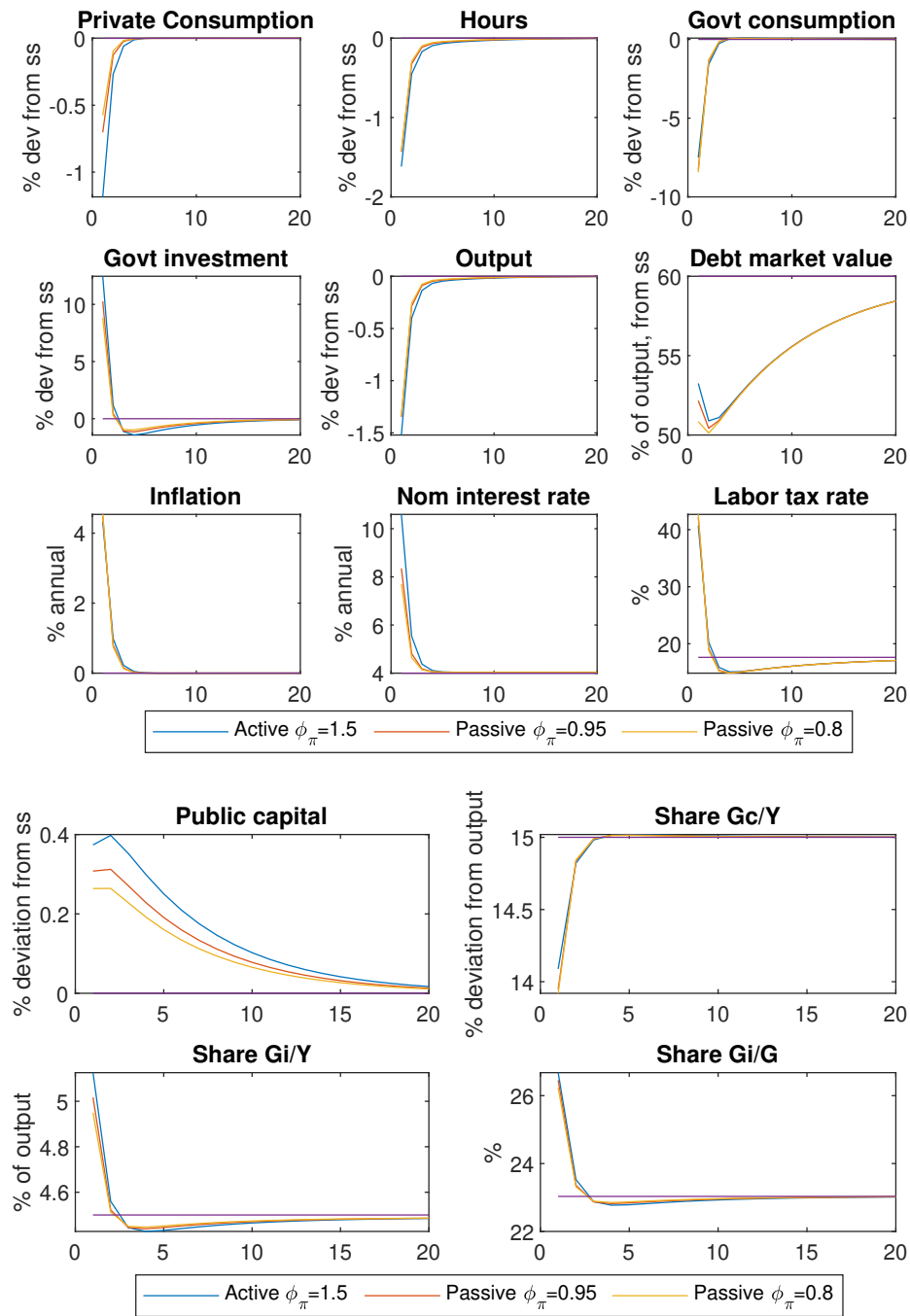


FIGURE 2.12: Discretion: Optimal response to an inflationary shock

Note: the figure shows the optimal response of the main macroeconomic variables to a shock in the Phillips curve increasing current inflation. The blue line represents the economy under an active monetary policy regime ($\phi_\pi = 1.5$), where the red and the orange lines represent the optimal response to the economy under passive monetary policy ($\phi_\pi = 0.8$) ($\phi_\pi = 0.95$) respectively

Chapter 3

Optimal Fiscal and Monetary Policy with Preferences Over Safe Assets

This paper investigates the optimal fiscal and monetary policies in a New-Keynesian model with preferences over safe assets (POSA). Relative to a model with standard preferences, the Ramsey planner facing POSA uses inflation more actively to absorb fiscal and demand shocks despite inflation being costly. The optimal response of inflation to the shocks departs from the traditional prescription observed in standard New-keynesian models with sticky prices in which inflation volatility is near zero. Moreover, under POSA taxes are not as smooth, as they are under standard preferences, and are more frontloaded, an outcome that brings the model closer to optimal policy with flexible prices. With POSA, debt issuance depresses the liquidity premium, reducing revenues collected by the government and tightening the budget constraint. Therefore the planner is much less willing to issue debt in response to (say) a fiscal shock, which explains the excess tax volatility observed.

These results do not dramatically change when private capital (a non-liquid asset), is introduced to the economy, the planner stills finds optimal to use inflation to absorb the shocks. Moreover, in spite of the fact that optimal debt issuance is now lower, private investment is crowded out more under POSA due to the stronger reaction of distortionary taxes to shocks. Finally, the planner faced with POSA outperforms the New-Keynesian planner (with standard preferences) in terms of stabilizing the economy to a negative demand disturbance, but underperforms in terms of the government spending shock. The negative demand shock increases the demand for government debt and relaxes the tradeoff facing the planner. The opposite holds in the case of a spending shock.

Keywords: optimal fiscal and monetary policy, bonds in the utility function, distortionary taxes, liquidity premium

JEL Classification: E31, E52, E62, H21

3.1 Introduction

Since the 1980s public debt as percentage of GDP has been showing an upward trend in most developed economies. This trend in government liabilities has been observed along with a persistent drop in the return on these assets as well as slow economic growth. Most developed countries do not seem to face any significant risk of having a debt crisis in spite of the considerable growth in public debt. One plausible explanation is that the demand for public debt has been growing too, due to particular services provided by government debt and which are valued by private investors. Indeed a large number of recent papers (see below for an extensive review of the literature) argue that public debt besides a store of value, provides liquidity and safety to private investors and is also valuable as collateral. As a counterpart for providing those services, government bonds pay a lower return than private assets, which in turn implies an extra revenue for the government as the provider of those services.

From the perspective of debt management, fiscal and monetary policies this must surely have implications. First, the rents that the government can derive by being the provider of liquidity in the economy will relax the intertemporal budget constraint, implying that a lower surplus can be targeted intertemporally to finance outstanding liabilities. Second, over the business cycle, debt management and fiscal policies will face tradeoffs when debt provides liquidity/safety services to investors which are not present in canonical models (where public debt does not carry safety and liquidity attributes). In the latter models, debt issuance is determined only by the desire of the optimizing government to smooth taxes across time. The price of debt is a function of consumption growth and inflation, the path of debt has essentially no bearing on equilibrium interest rates. In a world where debt provides liquidity, however, the quantity of debt issued determines the liquidity premium, and thus determines the interest rate. The government will not only consider tax smoothing when designing its optimal policy, but also the cost of financing, which is now a function of the issuance.

Aiming to investigate these margins, we characterize optimal policy when government bonds provide liquidity/safety services to private agents and compare with optimal policy in a model where they do not. To model liquidity/safety attributes we assume, as many recent papers do, that bonds enter in the utility function, thus explicitly assuming that agents have ‘preferences over safe assets’ (POSA). Recent literature motivates this modelling assumption in several contexts. For instance in [Guibaud et al. \(2013\)](#) it is the idea that long term government debt provides liquidity to young households to finance retirement, whereas in [Vayanos and Vila \(2021\)](#) investors have preferred habitat for certain type of bonds, some types of agents value the money like liquidity services provided by short term whereas other investors value long term liquidity and focus on long term assets. In [Krishnamurthy and Vissing-Jorgensen \(2012\)](#) POSA is assumed to rationalize the empirical finding that the supply of US treasuries is a key driver of the spread between government and corporate bonds.¹

With liquidity provision by government bonds, the classical tradeoff of issuing debt to postpone

¹See also [Angeletos et al. \(2020\)](#), [Greenwood et al. \(2015\)](#), [Rannenberg \(2021\)](#), [Michaillat and Saez \(2021\)](#) among others.

tax increases, and thus smooth tax distortions across time, may change drastically. Issuing debt to target a constant tax path after a shock (a key feature of optimal policy in canonical models) causes a drop in the liquidity premium (a sharp rise in the interest rate) and consequently a rise in debt servicing costs. In response to this new element, an optimizing government may desire to give up on tax smoothing, and invoke intertemporal distortions (depart from constant taxes) in order to stabilize the debt level and avoid the sharp increase in the interest rates.

Similarly, the role of inflation in achieving debt sustainability can also differ considerably in the POSA model relative to the canonical New Keynesian model. In the canonical model, the policy authority will use inflation to reduce the real value of debt in response to a shock, relaxing the need to raise distortionary taxes. The extent to which this policy is optimal is governed by the slope of the Phillips curve; less costly inflation makes it optimal to rely more on inflation and less on taxes to finance shocks. In the presence of liquidity services of government debt, however, an inflation tax maybe optimal even under sticky prices, because interest rates (the cost of debt) will not increase one for one with inflation. Since agents derive utility from debt, they accept not to be compensated fully for higher inflation risk. This then gives an additional motive to use inflation as a shock absorber, beyond the usual cost benefit consideration- the resource costs of inflation vs the tax smoothing benefits.

We consider an economy with two types of shocks: First, shocks to government spending can increase the deficit and require an increase in taxes and/or inflation to make government debt sustainable and second, shocks to demand which induce agents to postpone consumption and hence change real bond prices, can affect the intertemporal budget constraint through shifting the market value of outstanding debt relative to the present value of surpluses that compensate for debt, requiring again a change in monetary and fiscal policy variables.

Our baseline model assumes that government debt is short term, and there is no capital, households can only use government bonds to save. We rely on this simplistic setup for tractability, but also because it has been documented in the recent literature that short term government bonds have a money like premium, they provide liquidity to investors over and above the liquidity provided by longer maturities.² Using this setup we study optimal policy assuming that a benevolent planner, maximizes the preferences of households subject to the competitive equilibrium constraints, under full commitment.

Our results show that when faced with the additional tradeoffs implied by POSA, the planner departs significantly from the optimal policies of the canonical New-keynesian model. Specifically, under POSA, it becomes optimal to use unanticipated inflation to a greater extent in order to tax nominal private assets (reduce the real payout of public debt), and changes in the price level absorb a large part of the shocks. The optimal rate of inflation is more volatile than in the canonical model, where we obtain the standard near zero volatility of inflation outcome. Moreover, taxes under POSA are frontloaded, rather than smooth across time, which shows the reluctance of the planner to increase the bond supply, deviating from the target debt level over the business cycle.

²See for example [Greenwood et al. \(2015\)](#). Though treating explicitly the case of long term debt, when short and long bonds may have different liquidity attributes is beyond the scope of this paper, we discuss the implications of this alternative setup when we present our results.

A well known result from the literature (see for example [Aiyagari et al. \(2002\)](#)) is that under optimal policy in the canonical model, the Lagrange multiplier on the government budget constraint, (which measures the burden of distortions) evolves according to a random walk. Since the multiplier is a state variable, taxes inherit its properties and as shown in [Aiyagari et al. \(2002\)](#), following a shock that increases the debt burden, taxes become permanently higher. This is the celebrated tax smoothing á la Barro result. We derive analytically an analogous process of the Lagrange multiplier under POSA, and show that in contrast to the usual random walk, we now get a mean reverting process. This property is at the heart of the results we obtain, which run against tax smoothing and the limited contribution of inflation to debt solvency.

The optimal policy under POSA hinges crucially on the parameter that measures the curvature of the utility function with respect to the liquid asset, in other words the elasticity according to which households are willing to substitute liquidity services intertemporally. The higher is this elasticity, the closer is optimal policy to that of the canonical model since liquidity premia are less responsive to fluctuations in the supply of debt. Arguably, this elasticity is not easy to measure from the data, but we experiment with the range of parameter values that the literature views as plausible and the key qualitative results continue to hold. The paper however highlights this new margin as being important quantitatively for the optimal monetary/fiscal policy mix, complementing the existing literature which has focused mainly on degree of price stickiness in the economy as the most crucial element.

After flexing out these results in the model where debt is the only asset available to households, we then turn to a model in which households can also save in capital. We model capital as an illiquid asset (not an argument in utility and subject to the usual installation costs). This in turn allows to interpret the liquidity/safety premium as the wedge between the return on real capital and the real return obtained from public debt, but more importantly it allows households to substitute debt for capital along the transition path, following a shock to the macroeconomy.

Our results regarding optimal policy continue to hold in this extension of the baseline model. The optimal tax schedule is again more frontloaded under POSA whereas in the canonical model, we continue to find that perfect tax smoothing is desirable.

Interestingly, the reaction of fiscal monetary policy to spending shocks can rationalize an unexpected pattern along the transition: Under POSA we find a stronger crowding out effect on private investment, even though the shock induces a drop in the liquidity premium. We would expect the opposite to hold: Since debt and capital carry different returns a fiscal shock may not induce a strong reaction of the return to capital, but only a reduction in the liquidity premium, which ought to stabilize private investment. We obtain more crowding out, however, under optimal policy driven by the excess volatility of taxes and inflation which reduce the real resources available to the private sector to invest.

As a final experiment we contrast POSA and standard preferences in a model where monetary and fiscal policies are not jointly optimal (as is usually assumed in the Ramsey literature), and rather monetary policy is independent policy and follows a Taylor rule setting the nominal interest rate as a function of macroeconomic conditions, inflation and output. Otherwise fiscal variables are set optimally by the planner.

In this version of the model we find that the differences between POSA and standard preferences are much smaller. In line with previous results in the literature, standard preferences produce a larger volatility of inflation and taxes that are serially uncorrelated (giving up on the assumption that monetary policy is optimal typically has an important effect on the optimal allocation in this class of models). Analogously, POSA produces serially uncorrelated taxes and slightly more responsive inflation to the shocks. The differences are then not considerable. From this experiment we conclude that assuming an unconstrained and optimal monetary policy is key to get nontrivial implications regarding optimal policy when public debt provides liquidity/safety services.

We then conclude the paper with a brief discussion of extensions of our model left to future work.

3.2 Literature review

This paper is related to the considerable literature on optimal fiscal and monetary policies with incomplete financial markets (non-contingent debt), for example [Aiyagari et al. \(2002\)](#), [Schmitt-Grohé and Uribe \(2004\)](#), [Faraglia et al. \(2013\)](#) and [Lustig et al. \(2008a\)](#). Two common results in this literature are firstly that debt and taxes evolve according to random walks and secondly when prices are sticky and hence inflation is distortionary the optimal adjustment to shocks burdens mostly taxes, inflation does not adjust to make debt solvent. In this paper, we maintain the assumption that government debt is non-state contingent but assume that debt is net wealth for private investors (its value exceeds that of the present value of taxes). Our key finding is that in such an environment a benevolent Ramsey planner finds optimal to use inflation to a much greater extent and taxes do not increase permanently in response to shocks, rather the optimal path is frontloaded.

There is a growing literature that acknowledges the importance of public debt in providing liquidity services and this of course is relevant here. Firstly, in the relevant finance literature [Krishnamurthy and Vissing-Jorgensen \(2012\)](#) and [Greenwood et al. \(2015\)](#) showed that larger supply of public debt suppresses the premium of Treasuries over private debt instruments. Like in this paper, their modelling assumes that public debt provides utility to private investors. This helps to explain the relatively low interest rate of short-term debt.

Second, a couple of recent papers study optimal policy when debt is net wealth analysing the implications for tax smoothing, as we do here. [Angeletos et al. \(2020\)](#) builds a model in which debt can be used as collateral in transactions. In a special case of their model in which preferences are linear in consumption, the authors are able to derive an equivalence to a bonds in utility framework and analytically derive implications for optimal fiscal policy using the nonlinear model. In their model the Ramsey planner does not utilize inflation to stabilize debt.

Here, we take a more reduced form approach to a similar policy problem [Angeletos et al. \(2020\)](#) and our results are quantitative derived from a linear model. However, we do not need constrain the model to quasi-linear preferences and consider both taxes and inflation as policy instruments available to the planner. Moreover, though the focus of [Angeletos et al. \(2020\)](#) lies primarily in characterizing transitional dynamics to the Ramsey steady state, here the focus is mainly on the

business cycle properties of taxes, debt and inflation. The two approaches thus complement one another.

[Canzoneri et al. \(2016\)](#) study optimal Ramsey policy in a model with cash, credit and bond goods. Public debt can be used to purchase bond goods and so it facilitates transactions for the private sector. However, in the baseline model debt is state contingent and the authors consider how the introduction of an illiquid public debt instrument affects optimal policy allocations. The main result is that with such a bond, the optimality of the Friedman rule and that of perfect tax smoothing holds. In the absence of the illiquid asset, full tax smoothing is not optimal as this leads to excess volatility of the liquid public bonds supply.

The result is similar to the one obtained here, however the framing of the paper is different. We consider the Ramsey solution under POSA (only liquid debt) but do not assume complete financial markets (state-contingent assets). This has implications regarding the timing of debt taxes and inflation; under incomplete markets these variables show considerable persistence through time, one of the points made by this paper is that this result hinges also on the liquidity services of public debt. Moreover, whereas [Canzoneri et al. \(2016\)](#) build a very stylized model to derive analytical results, we explore the quantitative aspects of optimal policy in a more standard New-Keynesian model and also consider a version in which capital is available to investors. The assumption of state contingent assets is thus replaced with a more realistic asset structure, this makes the environment considered in this paper more relevant empirically.

On a more positive side (papers that do not study optimal policies) are [Rannenberg \(2021\)](#) or [Bonam \(2020\)](#) [Berentsen and Waller \(2018\)](#) and [Bayer et al. \(2020\)](#). [Rannenberg \(2021\)](#) shows that with POSA, the fiscal multiplier of a permanent increase in government expenditures is larger when interest rates are at the zero lower bound, however finds only a small difference when interest rates are positive and follow a Taylor rule. [Bonam \(2020\)](#) and [Berentsen and Waller \(2018\)](#) study the effect of the “convenience yield” on the fiscal theory of the price level. Lastly, [Bayer et al. \(2020\)](#) analyses the liquidity channel of fiscal policy showing both empirically and theoretically that there less crowding out of investment. An expansionary fiscal policy lowers the liquidity premium over private illiquid capital and does not reduce investment. As discussed previously we obtain the opposite result under optimal distortionary taxation.

The paper proceeds as follows. Section 3.3 contains the model environment while 3.3.4 defines the private equilibrium and the liquidity premium. Section 3.4 describes the optimal policy problems under commitment and the calibration. Section 3.5.3 contains the numerical results, while section 3.9 concludes.

3.3 The baseline model with Capital

3.3.1 Households/ Firms and the Government

We model an economy in which households derive utility from holding safe assets as in [Rannenberg \(2019\)](#). Safe assets are short-term government bonds. To save space we will consider

in our baseline model that households can also invest in physical capital, a non-liquid asset. We will however also separately consider experiments in which private capital is not available to households, or, equivalently, the capital stock is held constant through time.

Including short term government bonds in the utility function can be motivated in several ways. We dedicate a short section to motivate the departing from the standard preferences before describing the main model. Among those motivations there is the idea that household values safe and liquid assets ([Krishnamurthy and Vissing-Jorgensen \(2012\)](#), [Greenwood et al. \(2015\)](#)), some papers showing that properties of the heterogeneous agents model are similar to those of models with wealth in utility (e.g. [Kaplan and Violante \(2018\)](#), [Hagedorn \(2018\)](#) or [Angeletos et al. \(2020\)](#), or investors with a preferred habitat for certain types of bond ([Guibaud et al. \(2013\)](#), [Vayanos and Vila \(2021\)](#)).

The model is otherwise essentially a standard New-Keynesian model. We assume that firms with price adjustment cost as in [Rotemberg \(1982\)](#), which gives rise to a standard NEw-Keynesian Phillips curve. The government collects revenues through a distortionary labor tax, levied on labour income. It issues debt to smooth taxes across time, financing its primary deficit.

Before expliciting the main model, we provide a subsection to motivate the inclusion of bonds in the utility function. Later on, in section 3.8 we also discuss some limitations of such an approach for optimal fiscal policy.

Bonds in the utility function: some micro foundations

The inclusion of bonds in the utility function maybe seen as non-standard, however it has been motivated by the literature in different directions. From the perspective of optimal fiscal policy, the idea that government bonds provide liquidity services to households seems compatible with two observation we see in data: the drop in short-term interest rate seen in the last years and the negative comovement between interest rate and the supply of US treasuries (see [Reis \(2022\)](#) for a recent survey regarding the interest rate)

Some works along this line are [Krishnamurthy and Vissing-Jorgensen \(2012\)](#) and [Greenwood et al. \(2015\)](#), both include government bonds in the utility function to capture liquidity services government debt provides. [Greenwood et al. \(2015\)](#) in particular argues that short-term debt has money-like properties that are valuable from the perspective of the household as opposed to long-term bonds which carry the risk of roll-over. By extending the model, these works were able to derive a demand for government bonds and explain the negative comovement between short-term interest rate and the supply of US treasuries.³

In a different direction, models based on the preferred habitat like [Vayanos and Vila \(2021\)](#) and the ones with *cliente* for specific type of bonds such as [Guibaud et al. \(2013\)](#) can also justify the inclusion of bonds in the utility function. In the first one, investors who have a preferred habitat for bonds of different maturities coexists with indifferent households. The model can explain the

³Note that this could be also done by introducing a borrowing constraint to a households saving in government bonds. A borrowing constraint restricts the ability of the household to smooth consumption over time, thus public debt could ease that friction. It is a model considerable harder to solve numerically.

term structure of the interest rate by comparing returns across investors. [Guibaud et al. \(2013\)](#) develops an OLG model in which the youth saves in long-term bond for retirement purposes, thus one can argue that long-term bonds could be included in the utility function representing those young household.

Besides liquidity purposes, there are other alternatives to microfoundate the inclusion of bonds in the utility function. [Bansal and Coleman \(1996\)](#), for instance, explains the equity premium puzzle and the term-structure of the interest rate by setting up a model in which short-term debt is a valuable asset for banks because it can be used to back up household deposit accounts. In that framework, adding bonds in the utility function can be seen as shortcut to capture the fact that interest rate depends on bond supply decided by the government.

From a more general perspective, including not only bond but also other kind of wealth in the utility function could help to explain the behavior of rich households. The permanent income hypothesis from standard macro models show some difficulties to explain the right tail of the wealth distribution or the relatively higher savings rates observed in rich households. As pointed out by [Carroll \(2000\)](#), one simple way to capture such behavior could be a model in which households save as and end itself, which can be easily represented by including wealth as an argument of the utility function.

Finally, an interesting feature of this approach is that it can mimic relatively well some results obtained in the literature of heterogeneous agents (see [Kaplan and Violante \(2018\)](#), [Campbell et al. \(2017\)](#), [Hagedorn \(2018\)](#)) At equilibrium, those models show a lower interest rate in comparison to the standard representative agent model due to precautionary savings. This can also be seen in this paper as a result of bonds in the utility function due to the presence of the *convenience yield*. Along this line, some papers like [Angeletos et al. \(2020\)](#) or [Hagedorn \(2018\)](#) models with heterogeneous agent can behave similar to a model in which bonds enters the utility function ⁴

Households

The economy is inhabited by identical households with the following preferences:

$$E_t \sum_{t=0}^{\infty} \beta^t \left(z_t \frac{c_t^{1-\sigma}}{1-\sigma} - \chi \frac{n_t^{1+\varphi}}{1+\varphi} + \chi_b \frac{b_t^{1-\sigma_b}}{1-\sigma_b} \right) \quad (3.1)$$

where β denotes the usual discount factor, c_t is consumption, n_t are hours worked and b_t is the real value of risk-free short-term government bonds.⁵ We assume $\chi, \sigma, \varphi, \chi_b, \sigma_b$ are all positive parameters.

z_t is an exogenous shock to the preferences for consumption. A drop in z_t makes the household value future consumption more, relative to current consumption. Notice that this is a different

⁴[Angeletos et al. \(2020\)](#) show that the two-period OLG economy with uninsurable idiosyncratic risk and an infinite horizon variant with collateral frictions can also be represented by a model in which household derive utility from holding government debt.

⁵Thus households value real debt (the ratio of the dollar value of debt they hold divided by the general price level) as opposed to nominal bonds. This is a standard modelling assumption.

approach to modelling a demand shock than in many other papers. It is common in the literature to model demand shocks as shocks to the discount factor (thus a shock enters multiplicatively to utility and concerns all arguments, not just C). In models of optimal policy such discount factor shocks turn out have only a small effect on the economy, since monetary policy can very effectively track the real rate and neutralize them. Modelling a demand shock like z_t on the other hand will have non-trivial effects on the economy. This type of shock will change the intertemporal preferences for consumption but also the intratemporal allocation between consumption, leisure and bonds. A lower z_t will also imply that the household desires more leisure and enjoys a relatively higher utility from liquid wealth. Both seem plausible for a household that desires to postpone consumption. Finally, note that [Faraglia et al. \(2013\)](#), among others model demand shocks in this way.

Households have at the beginning of period t , b_{t-1} units of real bonds and k_t units of physical capital. They receive a wage for supplying labor and a rent payment from supplying capital to firms. The household budget constraint in real terms can be written as:

$$c_t + b_t + I_t = (1 - \tau_t)w_t n_t + r_t^k k_t + D_t + \frac{R_{t-1}}{\pi_t} b_{t-1} + T_t \quad (3.2)$$

where w_t is the real wage, r_t^k is the real return rate for non-liquid private capital units, D_t and T_t are dividends coming from firm ownership and lump-sum transfers collected by the government. The household also pays τ_t distortionary labor taxes to the government which together with the wage and the rental rate from capital are taken as given. We denote by π_t the gross rate of inflation (P_t/P_{t-1} where P_t is the price of the final good produced in the economy) and R_t is the nominal gross interest rate on public debt.

Private capital accumulation is subject to the following law of motion, which allows for investment adjustment costs:

$$k_{t+1} = \left(1 - S\left(\frac{I_t}{I_{t-1}}\right)\right) I_t + (1 - \delta)k_t$$

where δ is the depreciation rate and the function $S(\cdot)$ satisfies $S'(\cdot) \geq 0$; $S''(\cdot) \geq 0$, $S(1) = 0$; $S'(1) = 0$, standard properties for adjustment cost functions.

Notice that adjustment costs effectively make private capital non-liquid. Thus the household derives liquidity services from purchasing government debt (entering in the utility function) and can also invest in non-liquid capital.

Firms

The economy produces a final good that can be used for consumption and investment. The firm that produces this final good is assumed to operate in a perfectly competitive market. The tech-

nology is a standard CES function whose inputs are intermediate products:

$$Y_t = \left(\int_0^1 Y_t(j)^{1-1/\theta} dj \right)^{\frac{\theta}{\theta-1}}$$

where $Y_t(j)$ is the quantity of intermediate good j used in the production of the final good and $\theta \geq 1$ denotes the elasticity of substitution between intermediate goods.

Intermediate goods firms have market power, since the elasticity of substitution across $Y_t(j)$ is finite. Denoting by $P_t(j)$ the price of the intermediate good, standard arguments can be applied to show that the demand for j is given by:

$$Y_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\theta} Y_t$$

The intermediate differentiated good $Y_t(j)$ is produced using labor and capital. We assume that the production function is of the following form

$$Y_t(j) \geq F(n_t(j), k_t(j)) = n_t(j)^a k_t(j)^b$$

Notice that we will impose constant returns to scale, that is $a + b = 1$.

Given the demand and production functions and factor prices for capital and labour, the firm sets its price level to maximize profits. We assume that price setting is subject to Rotemberg (quadratic) adjustment costs of the form:

$$\Sigma_t(j) = \frac{\psi}{2} \left(\frac{P_t(j)}{P_{t-1}(j)} - 1 \right)^2 Y_t$$

Thus a firm that charges a different price in t than in $t-1$ will bear a resource cost $\Sigma_t(j)$. Parameter $\psi \geq 0$ governs the magnitude of these price adjustment costs. when $\psi = 0$ then resource costs are zero and in effect prices become fully flexible. In contrast, when $\psi = \infty$, then prices are fully sticky, in the sense that a firm will never find it optimal to change its price level.

Since we assume that firms have monopoly power, optimal prices will exceed marginal production costs. Thus, firms will realize profits and letting $D_t(j)$ denote the dividends paid out to firm owners we have:

$$D_t(j) = (1 + \text{Sub}) \frac{P_t(j)}{P_t} Y_t(j) - w_t n_t(j) - r_t^k k_t(j) - \Sigma_t(j)$$

The term $\text{Sub} = \frac{1}{(\theta-1)}$ denotes a subsidy given by the government to the firm. As is standard in the literature, the purpose of including this policy is to eliminate distortions stemming from monopolistic competition. We further assume that the subsidy $\text{Sub} = \frac{1}{(\theta-1)}$ is financed through a nondistortionary tax T .

Government

Government spending, G_t^c , is exogenous to the model and it is uncertain. As in [Aiyagari et al. \(2002\)](#), we assume that households do not derive utility from the level of government expenditures.

To finance spending, the government can levy distortionary labor income taxes and issue short-term nominal debt.⁶ Let B_t denote the quantity of nominal bonds issued by the government in t . The government budget constraint is:

$$B_t = B_{t-1}R_{t-1} + P_t(G_t^c - \tau_t w_t n_t - T_t) \quad (3.3)$$

where the term $(G_t^c - \tau_t w_t n_t - T_t)$ is the real deficit of the government. Dividing the RHS and LHS of the above equation with P_t and letting $b_t \equiv \frac{B_t}{P_t}$ denote the real debt in t we have:

$$b_t = \frac{B_{t-1}}{P_{t-1}} \frac{P_{t-1}}{P_t} R_{t-1} + (G_t^c - \tau_t w_t n_t - T_t) = b_{t-1} \frac{R_{t-1}}{\pi_t} + (G_t^c - \tau_t w_t n_t - T_t)$$

3.3.2 Household program

We now define the optimization problems of households and firms. Households choose sequences for consumption, next period capital, bonds, labor and investment $\{c_t, n_t, b_t, k_{t+1}, I_t\}$ to maximize utility 3.1 subject to the budget constraint, the law of motion for capital and a No-Ponzi game condition. As usual, households take government policies, wages, rental rate for capital and prices as given. We can state the household program as follows:

$$\begin{aligned} & \max_{\{b_t, k_{t+1}, c_t, n_t, I_t\}} E_0 \sum_{t=0}^{\infty} \beta^t \left(z_t \frac{c_t^{1-\sigma}}{1-\sigma} - \chi \frac{n_t^{1+\varphi}}{1+\varphi} + \chi_b \frac{b_t^{1-\sigma_b}}{1-\sigma_b} \right) \\ & st. \\ & k_{t+1} = \left(1 - S \left(\frac{I_t}{I_{t-1}} \right) \right) I_t + (1 - \delta) k_t \\ & c_t + b_t + I_t = (1 - \tau_t) w_t n_t + r_t^k k_t + D_t + \frac{R_{t-1}}{\pi_t} b_{t-1} + T_t \end{aligned}$$

Solving the above program with standard techniques, yields the following conditions character-

⁶In other words we constraint the lump sum tax T to finance the firm subsidies. As is evident, allowing the government to use the lump sum instrument to finance spending, will make the optimal policy program trivial, as any desired (constant) debt level can be financed with zero distortions.

izing the optimum:

$$(1 - \tau_t)w_t = \frac{\chi n_t^\varphi}{z_t c_t^{-\sigma}} \quad (3.4)$$

$$z_t c_t^{-\sigma} = \beta E_t \left[z_{t+1} \frac{R_t c_{t+1}^{-\sigma}}{\pi_{t+1}} \right] + \chi_b b_t^{-\sigma_b} \quad (3.5)$$

$$1 = \left[1 - S \left(\frac{I}{I_{t-1}} \right) - S' \left(\frac{I}{I_{t-1}} \right) \frac{I}{I_{t-1}} \right] q_t + \beta E_t \left[\frac{z_{t+1}}{z_t} \frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}} S'_I \left(\frac{I_{t+1}}{I_t} \right) \left(\frac{I_{t+1}}{I_t} \right)^2 q_{t+1} \right] \quad (3.6)$$

$$q_t = \beta E_t \left[\frac{z_{t+1}}{z_t} \frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}} ((1 - \delta)q_{t+1} + r_{t+1}^k) \right] \quad (3.7)$$

which together with the household budget constraint, the law of accumulation for private capital and the usual boundary (No-Ponzi game) condition form the system of equations that needs to be resolved to determine the optimal allocation.

Let us briefly discuss the above equations. 3.4 is the usual optimal labor supply condition; at the margin the household equates the utility cost of exerting additional work effort to the benefit it receives, stated in terms of the net wages and the increment in utility derived from an increase in consumption.

Equation 3.5 is the Euler equation for bonds. Notice that since the household derives utility from the real quantity of bonds, b_t , this is now different than the usual condition equating the utility cost of savings $z_t c_t^{-\sigma}$ with the discounted marginal utility benefit of gaining return R next period (in other words the term $\beta E_t \left[z_{t+1} \frac{R_t c_{t+1}^{-\sigma}}{\pi_{t+1}} \right]$). What is different, obviously, is the term $\chi_b b_t^{-\sigma_b}$ on the RHS of 3.5. The household in this model will not hold government debt only for its return properties, but also because it derives utility from debt. Equation 3.5 then equates the cost of savings, to the benefit measured both in terms of the expected return and the services that government bonds yield to the household.

Equations 3.6 and 3.7 are to be considered jointly. 3.6 is the first order condition with respect to investment and 3.7 is an Euler equation for the optimal capital accumulation. Object q_t is the price of installed capital whose dynamics are defined in 3.6. Then, 3.7 has the usual interpretation, whereby the household equates the cost q_t of acquiring an extra unit of capital to the (appropriately discounted) benefit $((1 - \delta)q_{t+1} + r_{t+1}^k)$, given that capital pays a dividend r_{t+1}^k and can be sold at price q_{t+1} tomorrow.

To explore further these conditions it is worthwhile contrasting the Euler equations for debt and for capital. On the basis of these objects we can then define the liquidity/safety premium for government debt over private capital, the latter can be seen as a non-liquid/non-safe asset in this model.

For simplicity and to make concepts clear, we will momentarily abstract from investment adjustment costs. This enables to get rid of equation 3.6, and focus only on equations 3.5 and 3.7, since trivially $q_t = 1$. What we will derive here, will however also be relevant for the quantitative model of Section 5 where the presence of investment costs is motivated by the observation that without

them one cannot get a realistic hump shaped response of investment to shocks (see for example [Christiano et al. \(2011\)](#)).

Assuming $q_t = 1$, we can define the gross real rental rate of capital as $R_t^k = r_t^k + 1 - \delta$. Merging the Euler equations 3.5 and 3.7, we arrive to the following equation on the basis of which we can define the liquidity/safety premia in this version of the model:

$$\beta E_t[z_{t+1}c_{t+1}^{-\sigma}R_{t+1}^k] - \beta E_t\left[\frac{z_{t+1}c_{t+1}^{-\sigma}R_t}{\pi_{t+1}}\right] = \chi_b b_t^{-\sigma_b}$$

Standard results enable to write this as:

$$\beta E_t[z_{t+1}c_{t+1}^{-\sigma}]E_t[R_{t+1}^k] + \beta Cov_t(z_{t+1}c_{t+1}^{-\sigma}, R_{t+1}^k) - \beta E_t[z_{t+1}c_{t+1}^{-\sigma}]E_t\left[\frac{R_t}{\pi_{t+1}}\right] + \beta Cov_t\left[z_{t+1}c_{t+1}^{-\sigma}, \frac{R_t}{\pi_{t+1}}\right] = \chi_b b_t^{-\sigma_b} \quad (3.8)$$

where the covariance terms capture the asset returns covariances with the marginal utility of consumption. Rearranging this expression we can derive the excess return of capital over government debt as:

$$\left(E_t R_{t+1}^k - E_t \frac{R_t}{\pi_{t+1}}\right) = \frac{1}{\beta E_t(z_{t+1}c_{t+1}^{-\sigma})} \left[-\beta Cov_t\left(z_{t+1}c_{t+1}^{-\sigma}, R_{t+1}^k\right) + \beta Cov_t\left(z_{t+1}c_{t+1}^{-\sigma}, \frac{R_t}{\pi_{t+1}}\right) + \chi_b b_t^{-\sigma_b}\right] \quad (3.9)$$

According to 3.9, the difference in the expected rates of return of capital and debt (the latter defined as the real return, scaled by inflation) is proportional to the difference between the two covariance terms leading the RHS of 3.9, and proportional also to the marginal utility derived from bond holdings.

Note that in a standard fashion we can utilize the leading difference in the covariance to define a safety premium for bonds. When the return to capital covaries positively with the marginal utility of consumption $z_{t+1}c_{t+1}^{-\sigma}$ then the asset is a poor hedge against consumption risk. A low return is delivered when the consumption is low (and/or when the z is high so that the household does not desire to postpone consumption) and so private capital should compensate the household with a high gross return R^k in expectation. Analogously, the term $Cov_t\left(z_{t+1}c_{t+1}^{-\sigma}, \frac{R_t}{\pi_{t+1}}\right)$ measures the effectiveness of holding government bonds for consumption smoothing purposes. If this covariance is negative then the real return to government debt is low in times of high consumption needs. This increases the expected return $E_t \frac{R_t}{\pi_{t+1}}$ required to compensate the investor for bearing the risk.

In this non-linear model therefore the covariances can be interpreted as a premium to safety of

government bonds when the second covariance is less negative than the first. Such a property will normally hold in standard models, though it is perhaps worth mentioning that an entire literature has been devoted to explain why the equity premium predicted by the models is typically too low relative to the data. (For the sake of brevity we will not revisit the arguments here.)

Furthermore, note that the last term on the RHS of 3.9 also contributes towards a positive difference in expected returns $E_t R_{t+1}^k - E_t \frac{R_t}{\pi_{t+1}}$. In this non-linear version of the model (when the covariance terms measure relative safety) this term can be interpreted as a liquidity premium: Government debt provides services to the private sector (modelled here in reduced form through a direct utility effect) whereas private capital does not provide any services. Then, our modelling assumptions tell us that the higher is the supply of debt, the smaller is the premium and hence the difference between the expected returns of the two assets. This pattern is consistent with recent empirical evidence (e.g. [Krishnamurthy and Vissing-Jorgensen \(2012\)](#)).

Importantly, let us note that up to a first order approximation of this model, the covariance terms will be equal to 0. We can show that

$$\left[R^{k*} U_c^* \right] E_t [\tilde{R}_{t+1}^k] - \left[\frac{R^* U_c^*}{\pi^*} \right] [\tilde{R}_t - E_t [\pi_{t+1}]] = \frac{U_b^*}{\beta} [-\sigma_b \tilde{b}_t + \sigma E[c_{t+1}] - E[z_{t+1}]]$$

where we let tildes denote that a variable is in log-deviation from its steady state value and U_c^* and U_b^* are, respectively, the marginal utility of consumption and government bonds at the steady-state.

On the LHS of the above equation we have the differences in the expected real returns of the two assets, and on the RHS we have the supply of government debt and the marginal utility of consumption. Notice that since the covariances will not appear in this linear equation then we have to interpret the RHS of this equation as reflecting both liquidity and safety premia for bonds. This is a shortcut taken by many papers.⁷ Moreover, since without the bonds in utility assumption we made here, private capital and government debt would have the same expected return in the linear model, parameters U_b^*, σ_b could be calibrated so that the difference of expected returns in the model is equal to the equity premium one finds in the data. We return to this below.

3.3.3 Firms problem

Let us now turn to the firm's program. Consider a generic producer in the intermediate sector indexed by j . This firm chooses prices and inputs to maximize the discounted flow of net profits

⁷For example, in the recent QE literature, where the focus is on long term over short term government bonds, the excess risk of holding long term debt is also considered to be a linear function of the relative long bond supply. The modelling assumption here is similar.

It is perhaps worth discussing how a safety premium can vary with the supply of bonds. Note that if indeed bonds are safer than stocks then an increase in their relative supply should make the households consumption stream less volatile. This could in turn mean that the household is more willing to bear the additional risk (measured in terms of the covariance with consumption) of holding stocks, or commands a lower risk premium for doing so. If this non-linear effect is translated to the linear model, then the naturally, the risk premium will be considered a function of b .

(dividends):

$$E_t \sum_{s=0}^{\infty} \beta^t \frac{z_{t+s} U_{c,t+s}}{z_t U_{c,t}} \left[(1 + \text{Sub}) \frac{P_{t+s}(j)}{P_{t+s}} Y_{t+s}(j) - w_{t+s} N_{t+s}(j) - r_{t+s}^k k_{t+s}(j) - \frac{\psi}{2} \left(\frac{P_{t+s}(j)}{P_{t+s-1}(j)} - 1 \right)^2 Y_{t+s} \right]$$

where the appropriate discount factor is the households asset pricing kernel and the firm's profits are net of price adjustment costs. Moreover, as explained previously, object $\text{Sub} = 1/(\theta - 1)$ is the subsidy correcting for the steady state distortions from monopolistic competition in the goods market.

Optimization for firm j is subject to the following equations:

$$Y_{t+s}(j) = \left(\frac{P_{t+s}(j)}{P_{t+s}} \right)^{-\theta} Y_{t+s}$$

$$Y_{t+s}(j) \geq F(n_{t+s}(j), k_{t+s}(j))$$

For brevity we will not derive explicitly the first order conditions from this program.

It is well known, that in the context of this model where firms bear a resource cost of adjusting prices the equilibrium is symmetric. This will also be the focus here. Thus all firms will solve the same program and we can, hereafter, drop j from notation. Imposing the symmetric equilibrium, we can show that the firm's optimality conditions can be rearranged to obtain the following condition governing the dynamics of inflation in the model:

$$\pi_t(\pi_t - 1) = \frac{\theta}{\psi} (mc_t - 1) + \beta E_t \left(\frac{z_{t+1}}{z_t} \frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}} \frac{Y_{t+1}}{Y_t} (\pi_{t+1} - 1) \pi_{t+1} \right)$$

which is essentially the Phillips curve at the heart of our model. Variable mc_t denotes the marginal costs of production. In terms of the standard cost minimization program solved by firms, mc_t reflects the marginal cost of using labour to produce an additional unit of output $\frac{w_t}{F_{N,t}}$ (where F_N denotes the marginal product of that factor) and the cost of producing with capital $\frac{r_t^k}{F_{K,t}}$ (F_K is the marginal product of K). At the optimum firms will equate $\frac{w_t}{F_{N,t}}$ and $\frac{r_t^k}{F_{K,t}}$. (See appendix 3.A for the complete derivations leading up to this equation).

3.3.4 Definition of the Competitive equilibrium

Having described the firms' and households' programs we now provide the definition of the competitive equilibrium in this economy. In standard fashion we define the equilibrium for a given sequence of policy variables, taxes and interest rates (and also given the exogenous random realizations of the government spending process). The sequence of policy variables will then be determined as the outcome of the optimization program of a benevolent planner, that will select policies to implement the welfare maximizing competitive equilibrium. We will later turn to the formal description of that program.

Definition CE: Given exogenous processes for the preference shock z_t , and the government spending shock G_t^c , given initial levels for the public debt b_{-1} and private capital, k_{-1} , and government policies $\{\tau_t, R_t\}$ a competitive equilibrium is a sequence $\{c_t, n_t, y_t, \pi_t, w_t, b_t, r_t^k, I_t, k_t\}_{t=0}^\infty$ such that: (1) $\{c_t, n_t, b_t, k_t, I_t\}_{t=0}^\infty$ solves the household problem given prices and government policies, (2) $\{\pi_t\}_{t=0}^\infty$ derives from the optimal pricing program for the firms, (3) the government budget constraint is satisfied and (4) the goods and factor markets clear.

Thus, the following conditions need to hold for a competitive equilibrium:

$$\begin{aligned}
(1 - \tau_t)w_t &= \frac{\chi n_t^\varphi}{z_t c_t^{-\sigma}} \\
z_t c_t^{-\sigma} &= \beta E_t \left[z_{t+1} \frac{R_t c_{t+1}^{-\sigma}}{\pi_{t+1}} \right] + \chi_b b_t^{-\sigma_b} \\
1 &= \left[1 - S_I \left(\frac{I_t}{I_{t-1}} \right) - S'_I \left(\frac{I_t}{I_{t-1}} \right) \frac{I_t}{I_{t-1}} \right] q_t + \beta E_t \left[\frac{z_{t+1}}{z_t} \frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}} S'_I \left(\frac{I_{t+1}}{I_t} \right) \left(\frac{I_{t+1}}{I_t} \right)^2 q_{t+1} \right] \\
q_t &= \beta E_t \left[\frac{z_{t+1}}{z_t} \frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}} ((1 - \delta)q_{t+1} + r_{t+1}^k) \right] \\
mc_t &= 1 + \frac{\psi}{\theta} \left[\pi_t(\pi_t - 1) - \beta E_t \left(\frac{z_{t+1}}{z_t} \frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}} \frac{Y_{t+1}}{Y_t} (\pi_{t+1} - 1) \pi_{t+1} \right) \right] \\
k_{t+1} &= (1 - \delta)k_t + \left(1 - S_I \left(\frac{I_t}{I_{t-1}} \right) \right) I_t \\
Y_t &= F(n_t, k_t) = n_t^a k_t^b \\
b_t &\equiv \frac{B_t}{P_t} = \frac{R_{t-1}}{\pi_t} b_{t-1} + G_t^c - \tau_t w_t n_t \\
\left(1 - \frac{\psi}{2} \pi_t^2 \right) Y_t &= c_t + I_t + G_t^c
\end{aligned}$$

(For the sake of the exposition we repeated equations derived previously). The final equation in the above system is the economy wide resource constraint, stating that total output net of price adjustment costs can be used to finance, consumption, investment and government expenditures.

3.4 Optimal fiscal and monetary policy

3.4.1 The Ramsey problem

We now present the optimal policy problem that the planner solves. As standard in the literature on optimal policy, we assume that a benevolent Ramsey planner will set policy variables to select from the set of competitive equilibria defined previously, the one that maximizes household utility 3.1. Notice that this practically means that optimal policy will involve treating the competitive equilibrium equations in Definition CE, as constraints in the planner's program. However, applying standard results, we can reduce the dimensionality of the program, leave aside some of the variables and corresponding constraints when the latter will be slack in optimization. For example, substituting out wages, using the labour supply condition, we have one less constraint to treat. Analogously variable r_t^k can be substituted out of the system when we make use

of the Euler equation for capital. Given w and r^k will be recovered as residuals (when we have solved the planning program we can use the labour supply and Euler equations to find their exact values) the marginal cost mc will also not need to be explicitly considered in optimization.

Formally, after these substitutions, we can state the planner's program as:

$$\begin{aligned}
 \max_{\{c_t, n_t, k_{t+1}, I_t, q_t, R_t, b_t, \tau_t, \pi_t\}} &= E_0 \sum_{t=0} \beta^t \left\{ \left(z_t \frac{c_t^{1-\sigma}}{1-\sigma} - \chi \frac{n_t^{1+\varphi}}{1+\varphi} + \chi_b \frac{b_t^{1-\sigma_b}}{1-\sigma_b} \right) \right. \\
 \text{st.} & \left[z_t c_t^{-\sigma} - \beta E_t \left[z_{t+1} \frac{c_{t+1}^{-\sigma} R_t}{\pi_{t+1}} \right] - \chi_b b_t^{-\sigma_b} \right] \\
 & \left[\frac{-\chi n_t^\varphi}{(1-\tau_t) z_t c_t^{-\sigma} F_n(k_t, n_t)/n_t} - 1 - \frac{\psi}{\theta} \left[\pi_t (\pi_t - 1) - \beta E_t \left(\frac{z_{t+1} c_{t+1}^{-\sigma}}{z_t c_t^{-\sigma}} \frac{F(k_{t+1}, n_{t+1})}{F(k_t, n_t)} (\pi_{t+1} - 1) \pi_{t+1} \right) \right] \right] \\
 & \left[\left(1 - \frac{\psi}{2} \pi_t^2 \right) F(k_t, n_t) - c_t - I_t - G_t^c \right] \\
 & \left[(1-\delta)k_t + \left(1 - S_I \left(\frac{I_t}{I_{t-1}} \right) \right) I_t - k_{t+1} \right] \\
 & \left[\left[1 - S_I \left(\frac{I_t}{I_{t-1}} \right) - S'_I \left(\frac{I_t}{I_{t-1}} \right) \frac{I_t}{I_{t-1}} \right] q_t + \beta \frac{z_{t+1}}{z_t} \frac{z_{t+1}}{z_t} \frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}} S'_I \left(\frac{I_{t+1}}{I_t} \right) \left(\frac{I_{t+1}}{I_t} \right)^2 q_{t+1} - 1 \right] \\
 & \left[\beta \frac{z_{t+1}}{z_t} \frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}} ((1-\delta) + \frac{-\chi n_{t+1}^\varphi}{(1-\tau_{t+1}) z_{t+1} c_{t+1}^{-\sigma} F_n(k_{t+1}, n_{t+1})/n_{t+1}} F_k(k_{t+1}, n_{t+1})) - 1 \right] \\
 & \left. \left[b_t - \frac{R_{t-1}}{\pi_t} b_{t-1} - G_t^c + \tau_t \left[\frac{-\chi n_t^\varphi}{(1-\tau_t) z_t c_t^{-\sigma}} \right] n_t \right] \right\}
 \end{aligned}$$

To solve the above program we write the following Lagrangian function:

$$\begin{aligned}
 \mathcal{L} = & E_0 \sum_{t=0} \beta^t \left\{ \left(z_t \frac{c_t^{1-\sigma}}{1-\sigma} - \chi \frac{n_t^{1+\varphi}}{1+\varphi} + \chi_b \frac{b_t^{1-\sigma_b}}{1-\sigma_b} \right) \right. \\
 & + \phi_{1,t} \left[z_t c_t^{-\sigma} - \beta E_t \left[z_{t+1} \frac{c_{t+1}^{-\sigma} R_t}{\pi_{t+1}} \right] - \chi_b b_t^{-\sigma_b} \right] \\
 & + \phi_{2,t} \left[\frac{-\chi n_t^\varphi}{(1-\tau_t) z_t c_t^{-\sigma} F_n(k_t, n_t)/n_t} - 1 - \frac{\psi}{\theta} \left[\pi_t (\pi_t - 1) - \beta E_t \left(\frac{z_{t+1} c_{t+1}^{-\sigma}}{z_t c_t^{-\sigma}} \frac{F(k_{t+1}, n_{t+1})}{F(k_t, n_t)} (\pi_{t+1} - 1) \pi_{t+1} \right) \right] \right] \\
 & + \phi_{3,t} \left[\left(1 - \frac{\psi}{2} \pi_t^2 \right) F(k_t, n_t) - c_t - I_t - G_t^c \right] \\
 & + \phi_{4,t} \left[(1-\delta)k_t + \left(1 - S_I \left(\frac{I_t}{I_{t-1}} \right) \right) I_t - k_{t+1} \right] \\
 & + \phi_{5,t} \left[\left[1 - S_I \left(\frac{I_t}{I_{t-1}} \right) - S'_I \left(\frac{I_t}{I_{t-1}} \right) \frac{I_t}{I_{t-1}} \right] q_t + \beta \frac{z_{t+1}}{z_t} \frac{z_{t+1}}{z_t} \frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}} S'_I \left(\frac{I_{t+1}}{I_t} \right) \left(\frac{I_{t+1}}{I_t} \right)^2 q_{t+1} - 1 \right] \\
 & + \phi_{6,t} \left[\beta \frac{z_{t+1}}{z_t} \frac{c_{t+1}^{-\sigma}}{c_t^{-\sigma}} ((1-\delta) + \frac{-\chi n_{t+1}^\varphi}{(1-\tau_{t+1}) z_{t+1} c_{t+1}^{-\sigma} F_n(k_{t+1}, n_{t+1})/n_{t+1}} F_k(k_{t+1}, n_{t+1})) - 1 \right] \\
 & + \phi_{7,t} \left[b_t - \frac{R_{t-1}}{\pi_t} b_{t-1} - G_t^c + \tau_t \left[\frac{-\chi n_t^\varphi}{(1-\tau_t) z_t c_t^{-\sigma}} \right] n_t \right] \left. \right\}
 \end{aligned}$$

where ϕ_i denote the Lagrange multipliers we attach to the constraints and we further asset $\phi_{i,-1}$, $i = 1, \dots, 7$ are given. For brevity, we leave to the appendix to derive the first-order conditions for the nonlinear model (see appendix 3.B). In what follows we will work with a log-linear approximation of the model and the first order conditions and therefore state the optimality conditions in this form.

3.4.2 The log-linear system of equations.

The system of equations that we need to resolve to characterize optimal policies, comprises of the competitive equilibrium constraints and the first order conditions. The log-linear competitive equilibrium constraints are:

$$\begin{aligned}
& -\sigma c^{*- \sigma} \tilde{c}_t + \tilde{z}_t = c^{*- \sigma} \beta R^* [E_t z_{t+1} + \tilde{R}_t - \sigma E_t \tilde{c}_{t+1} - E_t \tilde{\pi}_{t+1}] - \sigma_b b^* \tilde{b}_t \\
& \tilde{\pi}_t = \frac{\theta}{\psi} \left(\frac{\tau^*}{1 - \tau^*} \tilde{\tau}_t + \varphi \tilde{n}_t + \sigma \tilde{c}_t - (a \tilde{n}_t + b \tilde{k}_t) + \tilde{n}_t - \tilde{z}_t \right) + \beta E_t [\pi_{t+1}] \\
& a \tilde{n}_t + b \tilde{k}_t = \frac{c^*}{y^*} \tilde{c}_t + \frac{i^*}{y^*} \tilde{i}_t + \frac{g^{c,*}}{y^*} \tilde{g}_t^c \\
& \tilde{k}_{t+1} = (1 - \delta) \tilde{k}_t + \delta \tilde{i}_t \\
& \tilde{i}_t = \frac{1}{1 + \beta} \tilde{i}_{t-1} + \frac{\beta}{1 + \beta} E_t [\tilde{i}_{t+1}] + \frac{1}{\omega(1 + \beta)} \tilde{q}_t \\
& \tilde{q}_t = E_t z_{t+1} - \tilde{z}_t + \beta(1 - \delta) E_t [\tilde{q}_{t+1}] + \sigma [E_t [\tilde{c}_{t+1}] - \tilde{c}_t] + \\
& + \left[1 - \beta(1 - \delta) \right] E_t \left[\frac{\tau^*}{1 - \tau^*} \tau_{t+1} + \varphi n_{t+1} + \sigma c_{t+1} - (a \tilde{n}_{t+1} + b \tilde{k}_{t+1}) + n_{t+1} - z_{t+1} + a \tilde{n}_{t+1} + b \tilde{k}_{t+1} - \tilde{k}_{t+1} \right] \\
& b^* \tilde{b}_t = R^* b^* [R_{t-1} - \tilde{\pi}_t + \tilde{b}_{t-1}] + g^{c,*} \tilde{g}_t^c - \tau^* \frac{\chi n^{*\varphi}}{(1 - \tau^*) c^{*\sigma}} n^* \left(\tilde{\tau}_t + \frac{\tau^*}{1 - \tau^*} \tilde{\tau}_t + \varphi \tilde{n}_t + \sigma \tilde{c}_t - \tilde{z}_t + \tilde{n}_t \right)
\end{aligned}$$

where these equations represent, respectively, the (log-linear) Euler equation, the Phillips curve the resource constraints and the law of motion of capital. We then have the optimality conditions for invest and the Euler equation for capital and finally the government budget constraint.

The linearized the first order conditions derived from the Lagrangian are:

$$\begin{aligned}
c_t : & U_c^*(-\sigma\tilde{c}_t + \tilde{z}_t) + U_{cc}^*\hat{\phi}_{1,t} - R^*U_{cc}^*\hat{\phi}_{1,t-1} + \sigma(1/c^*)\hat{\phi}_{2,t} - \phi_3^*\hat{\phi}_{3,t} + n^*\frac{\chi n^{*\varphi}}{(1-\tau^*)c^{*\sigma}}\tau^*\sigma(1/c^*)\hat{\phi}_{7,t} \\
& + \beta((1-\delta) + F_k^*)(-U_{cc}^*/U_c^*)\hat{\phi}_{6,t} - ((1-\delta) + F_k^*)(-U_{cc}^*/U_c^*)\hat{\phi}_{6,t-1} + \phi_{6,t-1}(-U_{cc}^*/U_c^*)F_k^* = 0 \\
n_t : & \varphi U_n^*\tilde{n}_t + (1+\varphi-a)\frac{1}{n^*}\hat{\phi}_{2,t} + \phi_3^*Y_n^*(\hat{\phi}_{3,t} - \tilde{n}_t + a\tilde{n}_t + b\tilde{k}_t) + ((1+\varphi)\frac{F_k^*}{n^*})\hat{\phi}_{6,t-1} \\
& + (\tau^*\frac{\chi n^{*\varphi}}{(1-\tau^*)c^{*\sigma}}n^*(1+\varphi))\hat{\phi}_{7,t} = 0 \\
I_t : & -\hat{\phi}_{3,t} + \hat{\phi}_{4,t} - \omega(1+\beta)\tilde{i}_t + \omega\tilde{i}_{t-1} + \beta\omega E_t\tilde{i}_{t+1} + \beta\omega/(\phi_4^*I^*)E_t\hat{\phi}_{5,t+1} + \omega/(\phi_4^*I^*)\hat{\phi}_{5,t-1} - \\
& - \omega(1+\beta)/(\phi_4^*I^*)\hat{\phi}_{5,t} = 0 \\
q_t : & \hat{\phi}_{5,t} - \hat{\phi}_{6,t} + (1-\delta)\hat{\phi}_{6,t-1} = 0 \\
k_{t+1} : & -\beta(b/k^*)E_t\hat{\phi}_{2,t+1} + \phi_3^*\beta F_k^*E_t(\phi_{3,t+1} + a\tilde{n}_{t+1} + b\tilde{k}_{t+1} - k_{t+1}) - \phi_4^*\hat{\phi}_{4,t} + \beta(1-\delta)\phi_4^*E_t\hat{\phi}_{4,t+1} \\
& - \hat{\phi}_{6,t}((1-\beta(1-\delta))/k^*) = 0 \\
\pi_t : & (R^*/\beta)\hat{\phi}_{1,t-1} + (\psi/\theta)(\hat{\phi}_{2,t-1} - \hat{\phi}_{2,t}) - \phi_3^*Y^*\psi\hat{\pi}_t = 0 \\
R_t : & -\beta U_c^*\hat{\phi}_{1,t} - \beta b^*E_t\hat{\phi}_{7,t+1} = 0 \\
b_t : & -\sigma_b U_b^*\tilde{b}_t - U_{bb}^*\hat{\phi}_{1,t} + \hat{\phi}_{7,t} - \beta R^*E_t\hat{\phi}_{7,t+1} = 0 \\
\tau_t : & (1/(1-\tau^*))\hat{\phi}_{2,t} + (F_k^*/(k^*(1-\tau^*)))\hat{\phi}_{6,t-1} + n^*\frac{\chi n^{*\varphi}}{(1-\tau^*)c^{*\sigma}}((2-\tau^*)/(1-\tau^*))\hat{\phi}_{7,t} = 0
\end{aligned}$$

where for convenience, in front of each equation we have added the variable to which the FONC corresponds. As previously, we use $\tilde{x}_t$ to indicate that variable x is in log-deviation relative to the steady state, x^* . Variables denoted with $\hat{x}_t$ are in level deviations from steady state. We use both types of approximations (in levels and in logs), since some of the variables (e.g. the multipliers) could be zero in the steady state (i.e some of the constraints are not relevant for the steady state allocation). This is a standard property of Ramsey programs.

It is perhaps too costly in terms of space to inspect in detail every single one of the above equations. Let us therefore focus on a couple of noteworthy features of the optimal allocation, to conserve space.

A central object in optimal policy models, is the multiplier attached to the government budget constraint, $\hat{\phi}_{7,t}$. Under POSA, we obtain the following expression showing the dynamics of this variable:

$$-\sigma_b U_b^*\tilde{b}_t - U_{bb}^*\hat{\phi}_{1,t} + \hat{\phi}_{7,t} - \vartheta E_t\hat{\phi}_{7,t+1} = 0 \quad (3.10)$$

which is a first order difference equation with time varying forcing terms $-\sigma_b U_b^*\tilde{b}_t - U_{bb}^*\hat{\phi}_{1,t}$. U_{bb}^* is the second order derivative of utility with respect to government bonds evaluated at the steady state. $\hat{\phi}_{1,t}$ denotes the Lagrange multiplier on the bond pricing (Euler) equation. Moreover, for simplicity, we wrote $\vartheta \equiv \beta R$.

Let us compare this expression with the analogous object obtained in the standard model without POSA preferences; In this case we have that $\vartheta = 1$ (the familiar equilibrium condition setting the interest rate equal to the inverse of the discount rate) and $U_b^*, U_{bb}^* = 0$. Then, the above equation

reduces to the well-known result in the literature of Ramsey optimal policy under incomplete markets, in which the multiplier attached to the government budget constraint follows a random walk (see [Schmitt-Grohé and Uribe \(2004\)](#), [Aiyagari et al. \(2002\)](#) and [Faraglia et al. \(2013\)](#) among others). The intuition behind this well known property is the following: Since the planner issues debt to smooth tax distortions over time, she will optimally target to increase taxes permanently following a shock (to either spending or preferences). Making taxes permanently increase, essentially means that debt should also follow a random walk, hence the process for the multiplier.

In the presence of POSA, this equation changes. There is now an additional term reflecting the contribution of debt to household utility (the leading term in the above equation) and another term (measured by the multiplier ϕ_1) which captures the change in the price of government debt for a given change in the quantity of bonds outstanding. Finally, (besides these two forcing terms) the stochastic difference equation does not imply that ϕ_7 will need to follow a random walk, since also $\vartheta < 1$.

Let us briefly explain the intuition behind these results. First, note that it is obvious that the planner needs to account for the direct effect that b has on household utility. Therefore, the inclusion of the term $-\sigma_b U_b^* \tilde{b}_t$ is not surprising, as this term essentially pins down the optimal target level for debt. Over the business cycle, optimal policy will issue debt to smooth taxes, but insofar as $\tilde{b}_t = 0$ is optimal for households, debt will not deviate permanently from its target level. It is evident that this feature of optimal policy has implications for the optimal path of taxes. Simply put, if debt is to converge back to its target level in the long run, then taxes (that compensate for debt in steady state) will also return to the steady state value.

Moreover, the term $-U_{bb}^* \hat{\phi}_{1,t}$ basically captures the effect of a change in the bond quantity of the price of bonds. As discussed previously, the safety/liquidity premium in this model is a function of b , with higher debt supply implying a smaller premium. Thus, issuing more debt to smooth tax distortions across time, entails an increase in the interest payments of debt. This evidently implies a higher cost measured in terms of distortionary taxes, and therefore $-U_{bb}^* \hat{\phi}_{1,t}$ also invokes a penalty for higher debt issuances. The planner needs to account for the effect of b on the price.

If the leading two terms in 3.10 were set equal to 0, we would obtain: $\hat{\phi}_{7,t} - \vartheta E_t \hat{\phi}_{7,t+1} = 0$. It is then evident that $\hat{\phi}_{7,t} = 0$ for all t .⁸ The government budget is then not a relevant constraint for the planning program.

To understand this property, note that since debt in steady state trades at a higher average price than β , then government debt can in principle be 'self financed'; financing government debt can be done simply through issuing more debt. To put differently, since the government provides liquidity services to the private sector, it obtains rents (in the form of lower interest payments) that can be used to finance spending without resorting to distortionary taxes. If the government desired to maximize the rents, and exploit this self financing opportunity, then it would see the debt level decrease over time, savings could accumulate to the point where spending could be financed without any distortions.

⁸For $\vartheta < 1$ $\hat{\phi}_{7,t} = \vartheta E_t \hat{\phi}_{7,t+1}$ is solved forward to yield a unique non-explosive solution $\hat{\phi}_{7,t} = 0$.

However, such a path cannot be optimal as household utility depends on the bond supply (the term $-\sigma_b U_b^* \tilde{b}_t$) and hence the planner will not target a debt level that maximizes rents. In general, under optimal policy we will have $\hat{\phi}_{7,t} \neq 0$.

In short, the dynamics of debt, taxes and the multiplier under POSA preferences are going differ considerably from the usual analogues in the canonical Ramsey model. We next calibrate and solve the models to discern how the differences we have noted here, translate into paths for macroeconomic and policy variables.

3.5 Quantitative experiments

3.5.1 Calibration

We first describe the calibration of the model's parameters.

In our numerical simulations we will analyze the optimal response of the economy to two types of shocks: a positive government spending shock and a negative preference shock. For both shocks, we will compare the response of the economy under optimal policy with POSA and the response with standard (NOPOSA) preferences to assess the effect of introducing liquid debt on optimal fiscal policy.

We calibrate the model with quarterly data from the US. We assume the economy is at the steady-state in period 0 and moreover, we assume a zero inflation steady-state, thus $\pi^* = 1$. We fix the debt to output ratio to be 60%, a standard value in the literature on optimal fiscal policy. Government spending is set to 20% of output broadly consistent with the data on the US economy for the period 1987-2018. With these parameter values, the labour income tax is 24.7% at steady-state.

[Table 3.1 approximately here]

The gross nominal interest rate, R^* at the steady state is set to 1.005. In annual terms, this gives an interest rate of 2,02%. With NOPOSA, this value for the interest rate implies $\beta = 1/R^*$ is equal to 0.995. For the POSA model we set the value for ϑ in 0.99 following [Campbell et al. \(2017\)](#). Then $\beta = \frac{\vartheta}{R^*} = 0.9851$.

Given these calibrated values, we can compute the value for parameter χ_b at the steady-state. Considering two alternative calibrations for the curvature parameter $\sigma_b \in \{0.2, 1\}$ We obtain χ_b equal to 0.0261 when $\sigma_b = 0.2$ and equal to 0.03 when $\sigma_b = 1$. We treat as our baseline the case where the curvature parameter σ_b is 0.2 following in [Rannenberg \(2021\)](#).

As discussed previously, we will present results from a model where households can purchase government bonds and invest in private capital, but also explore a model where investments in

private capital are ruled out and government debt is the only asset available to households. The above calibrated values correspond to both models.⁹

Finally, the intertemporal elasticity of substitution of consumption is set to 1, and Frisch elasticity of labour supply is also 1 following much of the related literature. We choose the value for χ such that the household spends 1/3 of its unitary time endowment working, at the steady state.

Given these parameter values the POSA model predicts an excess return of capital over government debt equal to 4.2% (at the annual horizon). [Damodaran \(2015\)](#) computes the implied equity premium as the return differential on S&P500 and 10-year Treasury bond, to be equal to 4.1% in the US for the period 1987-2018. Thus our model is consistent with this crucial data moment.

The production technology is Cobb-Douglas with constant returns to scale. We assume that the labor share in output is 0.64, while the elasticity of output with respect to private capital is 0.36 (see [Christiano et al. \(2005\)](#)). The elasticity of substitution between intermediate inputs is 6, so the steady-state mark-up is 20% (in the absence of any subsidy by the government). The parameter governing price adjustment costs, ψ is set to be 200, and is calibrated together with θ to have a slope of the linearized Phillips curve equal to 0.03. Following [Christiano et al. \(2005\)](#), investment adjustment costs are of the form:

$$S\left(\frac{I_t}{I_{t-1}}\right) = \frac{\omega}{2} \left(\frac{I_t}{I_{t-1}} - 1\right)^2$$

and we set the parameter ω to 5.5 (e.g. [Rannenberg \(2019\)](#)).

Finally, government purchases G_t^c and the shock to preferences over private consumption, z_t are exogenous AR(1) process of the form:

$$\log(z_t) = \rho_z \log(z_{t-1}) + \epsilon_t, \quad \epsilon_t \sim \mathcal{N}(0, \sigma_z^2)$$

$$\log(g^c) = \rho_g \log(g_{t-1}^c) + \nu_t, \quad \nu_t \sim \mathcal{N}(0, \sigma_{g^c}^2)$$

where ρ_z and ρ_g are set to 0.8 and ϵ_t and ν_t are i.i.d. disturbances with a standard deviation of 1%. Since the focus of our exercise is normative, it should be clear that we do not aim to match data moments of the US business cycle, and rather focus in analyzing optimal fiscal policy in the context of the two persistent shocks. That said, it is worth noting that spending and demand shocks are key ingredients of any DSGE model that matches the US data and therefore the implications we can draw from our exercise will continue to apply in medium scale models.

Table 2.1 in appendix 3.E reports the calibrated parameter values discussed above as well as their counterparts in the US data.

⁹With the calibrated values for β and R^* , we get private investment equal to 17.3% of output in the model with NOPOSA, at steady state. This is consistent with US data extracted from the Federal Reserve Bank of St. Louis for the period 1987-2018. We set the depreciation rate for private capital to be 0.02. In the model with POSA, when $\vartheta = 0.99$ we get a ratio of investment over output equal to 16% not far from the data average.

Note that, [Rannenberg \(2021\)](#) calibrates ϑ to be 0.96 in his model, which gives $\beta = 0.9557$. If we choose these values instead, then ratio I/Y falls to 10%, much lower than the 17.3% observed in data. For the model without capital, the alternative calibration used in [Rannenberg \(2021\)](#) do not affect our results (see appendix).

3.5.2 Quantitative results: the economy without capital

Fiscal expenditure shock

We first study optimal policy assuming that government debt is the only asset in the economy. Figure 3.1 shows the response of the economy to an unexpected increase of 1 percentage point in government spending. The blue line depicts the optimal response under standard preferences, the red line and the green dashed lines are the responses with POSA for $\sigma_b = 0.2$ and $\sigma_b = 1$, respectively.

In all of the models considered, the government spending shock increases the interest rate and crowds out private consumption. Output, the sum of consumption and spending increase because total demand is higher. Since firms need to increase production, to meet the higher demand, hours worked increase. Moreover, since households have to be on their labour supply curves higher hours worked can only be induced if wages (marginal costs) increase and due to standard wealth effect of lower consumption on labour supply. These standard responses are clearly evident from Figure 3.1.

[Figure 3.1 approximately here]

Note that the increase in government spending can be financed partly by issuing more debt or by higher labour income taxes and higher inflation (the inflation tax channel), or both. The extent to which the planner makes use of debt vs taxes to finance the shock, however, differs considerably across the models.

Taxes are more volatile and more responsive to the shock with POSA than under NOPOSA and therefore, in the former case a larger fraction of the shock is financed through the immediate adjustment of taxes. The POSA models predict a more frontloaded response of the tax schedule to the shock. Notice that under POSA, taxes are strongly mean reverting and therefore they should converge to the steady state at finite time. This convergence is, however, slow and even after the 40 model periods shown in Figure taxes have not yet reached the steady state value.

Clearly the driving force behind the response of taxes is the fact that with POSA the planner will resist increasing government debt considerably after the shock. This is evident from the response of real bonds shown in the bottom left panel. The reason is twofold; firstly, debt is an argument in utility and so a large reaction to the shock would increase the supply of bonds by too much relative to the target level implied by the household utility; secondly, since increasing bond supply lowers the bond price (increases the nominal interest rate) the government has to pay a higher cost of financing debt. It then becomes optimal not to rely too much on debt issuance to smooth taxes across time, and the frontloaded path shown in the Figure obtains. These forces are not present in the NOPOSA model debt and taxes essentially follow a random walk.¹⁰

¹⁰Note that there is an initial blip in the tax schedule, taxes are higher when the shock hits than in other periods. This effect is explained in Faraglia et al. (2016) and it represents an interest rate twisting channel of optimal policy. In response to an adverse spending shock, the planner has the incentive to distort taxes intertemporally in order to lower the real cost of financing debt. This effect is however small in the NOPOSA model and for the case of short term bonds, it concerns only the initial period.

The responses in the POSA model vary with σ_b as expected. Higher σ_b implies a more rapid increase in the cost of issuing debt when the debt level rises, and this provides a stronger incentive to frontload taxes to finance the shock. Analogously, assuming a higher σ_b implies that the household is less willing to tolerate large swings in the real value of debt and the response of debt is more muted also for this reason.

The top right panel in 3.1 shows the liquidity/safety premium's response to the shock. Consistent what we said previously, the premium falls more in the case where $\sigma_b = 1$, in spite of the fact that the bond supply changes less.

Let us now turn to how inflation reacts to the shock. Under all models considered, inflation increases after the increase in spending, but the rise of inflation will only last for one model period. This is not surprising. Since debt is short-term, the only margin via which inflation can erode the real value of debt and stabilize the government's budget following the shock, is the contemporaneous one. Had we assumed that government bonds are of longer maturity, then the inflation channel could be used over the longer term by the planner (see for example [Leeper et al. \(2021b\)](#), [Chafwehé et al. \(2022\)](#) among others).

Nevertheless we see a significant difference between the POSA and NOPOSA models in terms of how the planner will make use of inflation, in spite of the response of inflation being shortlived. In particular, assuming POSA and $\sigma_b = 1$ leads to a rise of inflation in response to the shock which is about 6 times larger than the analogous rise under NOPOSA. This is an interesting finding. Many papers on optimal monetary/fiscal policies have highlighted that in the presence of sticky prices the government will not resort to inflation to finance fiscal shocks and instead will utilize debt and taxes to do so (e.g. [Schmitt-Grohé and Uribe \(2004\)](#), [Faraglia et al. \(2013\)](#)). The rationale behind this finding, is that the costs of inflation in standard New Keynesian models (when the Phillips curve is calibrated to match the US data) are considerably higher than the implicit costs of using distortionary taxes, intertemporally.

Under POSA, the planner is willing to bear the high costs of inflation in order to avoid having to increase debt considerably. Therefore, the welfare costs of adjusting debt become an important margin leading to the higher inflation variability observed. Moreover, that the higher volatility of inflation implies that more of the burden of the fiscal adjustment falls on inflation under POSA than under NOPOSA.

Demand shock

We now consider the response of the economy to a negative demand shock under optimal policy. Figure 3.2 shows the behavior of macroeconomic variables to the shock. Both, with and without POSA, private consumption and hours fall as a consequence of the shock. As explained previously, a negative shock to preferences implies two effects: First, the households will desire to postpone consumption and second, hours worked will fall, the shock acts as a labour wedge

Note that this channel should also be operative in the POSA model and contributes to the higher volatility of the tax schedule we observe.

shock in the labour supply condition. This triggers simultaneously a reduction in output and consumption.

[Figure 3.2 approximately here]

How does the planner react to the shock? From Figure 3.2 we see that the shock leads to a reduction in taxes, inflation and the debt level. Note that this may seem odd given that the shock leads in a recession, and thus to a drop in the revenue of the government. However, simultaneously, the shock persistently lowers real interest rates and through this channel, it increases the present value of surpluses that finance government debt. Due to the rise in the value of the surpluses, the planner will optimally reduce taxes and inflation in response to the shock (see for example [Chafwehé et al. \(2022\)](#)).¹¹

The fact that interest rates fall (bond prices increase) helps to clarify the reaction of the real value of debt shown in the bottom left panel of the figure. At higher bond prices, the quantity of debt that needs to be issued is lower and so the real value of debt outstanding in every period after the initial one, is lower. Initially, however, the negative rate of inflation, pushes up the real value of debt outstanding. Thus debt is higher for one period, then becomes negative (in deviation from the target) in all periods. Under NOPOSA, debt is a random walk. With POSA, debt follows a hump shaped pattern that will converge to the steady state after many model periods.

Consider now the behavior of taxes and inflation in the two models. As is shown clearly in the Figure, the differences noted in the previous paragraph remain; POSA entails a much stronger reaction to the shock through taxes and inflation and these differences increase in σ_b . At the same time, whereas taxes in the NOPOSA model evolve according to a random walk under POSA the optimal tax schedule is more frontloaded, echoing the incentive of the planner to not let debt get far off target. Thus, we obtain a similar adjustment of taxes and inflation to the preference shock as we did to the spending shock.

The main takeaways from this simple model without capital is that POSA, introduce a strong incentive to frontload taxes and use inflation to react to shocks to spending and preferences. The usual random walk property of optimal taxation in canonical models does not obtain under POSA and neither the near zero volatility of inflation. The rationale behind these patterns is that with POSA, the planner has a strong incentive to keep debt anchored near the target value, dictated by the preferences of the households.

We next turn to see whether these results continue to hold in the case where households can allocate their savings in an illiquid asset as well as in government debt, considering optimal policy in the model with capital.

¹¹It turns out that for preference shocks the maturity of debt being issued is important. For example, assuming long term debt, means that long bond prices will increase simultaneously with the governments surplus. This can then shield the budget constraint of the government and the direct effects on output can dominate. In this case we could even observe a rise in taxes following the negative demand shock. Though this is an important qualitative difference in terms of the optimal policy, as discussed previously, considering separately long term bonds is beyond the scope of this paper. We however provide a brief discussion of the implications of adding long bonds to the model before concluding the paper.

3.5.3 The economy with private capital

We first focus on the case of a rise in government spending.

Note that in the economy with capital, a government spending shock will not only crowd consumption out, but it will also exert a negative effect on private investment. In the standard New Keynesian model where bonds and capital earn effectively the same rate of return, this is evident. A rise in the real interest rate, following a shock to spending, will induce households to reduce consumption and induce them to reduce investment.

With POSA this margin becomes even more interesting to consider: Since the return of short term government debt differs from the return to capital accumulation, the adverse spending shock will force the government to issue debt and this may only lead to a reduction in the liquidity/safety premium, without impinging a big effect on the price of capital. Under this condition a spending shock may exert a weaker effect on investment than in the NOPOSA model (see for example [Bayer et al. \(2020\)](#)).

We find the opposite under optimal policy, however. Though the liquidity premium does indeed react to the shock (middle left panel), investment drops by more under POSA. We attribute this to the response of policy variables which is again different across the models. Once again, the POSA model predicts considerable tax volatility and also inflation is more responsive to the shock. Under NOPOSA we obtain a weaker reaction of both taxes and inflation. Our previous findings thus continue to hold.

[Figure 3.3 approximately here]

Let us now turn to the demand shock and notice that this adds a further interesting margin to account for. Given that this shock induces households to postpone consumption, it strengthens the incentive to save. As such, we anticipate to see private consumption and investment comove negatively, and this is indeed confirmed by figure 3.4. As before, consumption drops after the shock, however, now private investment increases.

Notice that this additional effect (relative to the no capital model studied in the previous subsection) leads to a significant change in terms of the reactions of taxes to the shock, relative to our previous findings. We now see that while taxes decrease under POSA, they slightly increase under NOPOSA.

What explains this? At higher investment rates, the economy gradually builds up its capital stock. More capital increases the productivity of labour and therefore lowers the marginal costs of production. To avoid a persistent drop in the inflation rate (which anyway does not contribute towards making debt sustainable, when debt is short term) the planner increases in labour income taxes for a few periods after the shock in the NOPOSA model. Since distortionary taxes act like a cost push shock in the Phillips curve, this policy keeps inflation at bay. Notice however that after a few periods taxes turn permanently negative, suggesting that the demand shock has an analogous impact as in the no capital model investigated previously; it increases the present value of surpluses and lowers taxes and inflation.

Next consider the POSA model. Comparing Figure 4 with Figure 2 (no capital) it is evident that the incentive to distort taxes intertemporally to avoid a larger deflation than desired, continues to apply. However, under POSA, taxes are frontloaded, to manage the path of debt, and thus we do not observe a switch in the sign of the tax response a few periods after the shock. With this additional effect accounted for, the responses we observe in Figure 4 are not different qualitatively than the analogous channels in the no capital model.

It is also worthwhile noting that in the presence of capital as an additional asset, the government faces a different tradeoff than in the model without capital. As explained previously, a negative demand shock makes savings relatively more attractive. The price of debt increases as investors are willing to receive a lower return to hold the asset, which now yields additional utility services to them. In a model without capital, the optimizing government has the incentive to lower taxes in order to keep the supply of debt from dropping sharply in the transition. But in the presence of capital, an alternative asset is available for savings, excess households savings can be channeled to the capital market and this relieves the tension between lowering taxes for intertemporal debt solvency and keeping the bond supply at a high level to accommodate household desired savings.

To sum up, the key features of optimal policy in response to the two shocks we found in the model without capital continue to hold when we allow for the illiquid asset. In what follows we consider additional exercises to investigate the robustness of our findings.

[Figure 3.4 approximately here]

3.6 Extensions

We first investigate the impact of price stickiness in our model. In particular, we solve for the optimal fiscal and monetary policy plan allowing for different degrees of price stickiness making inflation more costly for the planner. We will show, that regardless of the degree of price stickiness, optimal inflation and taxes continue being more volatile under POSA. This key result of our paper is also robust to changes in parameters ϑ and σ_b .

In a second experiment, we contrast POSA with NOPOSA, but instead of analyzing jointly optimal fiscal and monetary policy, we let the planner to choose optimal fiscal policies while monetary policy follows a Taylor rule. Under this experiment we obtain that taxes and inflation behave quite similarly in the POSA and NOPOSA models a finding that suggests that a key ingredient of our previous analysis is that monetary policy is not constrained by an interest rate rule.

3.6.1 Degree of price adjustment costs

A well known result in the Ramsey literature is that when prices are sticky the planner makes little use of inflation to stabilize the economy against fiscal shocks (e.g. [Schmitt-Grohé and Uribe \(2004\)](#)). The intuition is that the welfare costs of adjusting distortionary taxes are typically smaller

than the costs associated with inflation volatility, when the Phillips curve is calibrated to the US data.

Of course this implication regarding the optimal inflation/tax mix hinges on the specification of the resource costs of inflation. The latter can be measured by the slope of the Phillips curve, but as is well known, empirically, Phillips curve slopes are not easily estimated.¹² Assuming a different calibration of the model, with smaller resource costs of inflation, may close the gap between the POSA and NOPOSA models with regard to the optimal use of inflation.

To argue this let us consider the limit when prices are fully flexible. In [Chari and Kehoe \(1999\)](#) the optimal fiscal policy under flexible prices in the canonical model is one where taxes are constant through time and inflation fully absorbs spending shocks. Given the nominal debt, higher inflation reduces the real value of bonds outstanding to satisfy the intertemporal budget of the government. [Chari and Kehoe \(1999\)](#) show that this model produces excessive volatility of fiscal inflation.

Such an outcome will obviously not occur in the POSA model. If the planner were to use inflation to such an extent, that is to fully stabilize debt, then the real debt level would systematically deviate from the level desired by households. Since the variability of real debt is penalized in the household's objective, the optimal policy mix will feature both tax and inflation variability.

This seems an interesting property. Recent literature has drawn a distinction between funded and unfunded fiscal shocks, the former are financed through adjustment of taxation and thus lead to persistent increases in the debt level (when positive) whereas the latter entail an adjustment of inflation and a drop in the real debt burden holding taxes constant. What we can learn from the above discussion is that fully unfunded shocks are not optimal regardless of the degree of price stickiness under POSA.

Assuming flexible prices is not credible however, and for this reason we focus the exercise of this section to a plausible calibration of the model. In particular we lower the resource cost of inflation by a factor of 10. Whereas our baseline Phillips curve has a slope of 0.03 (it is very flat consistent with recent estimates) we now assume a slope of .3 which more accurately corresponds to estimates including US data from the 1980s and early 1990s. We solve again the model, focusing for simplicity on the environment without capital.

[Figure 3.5 approximately here]

[Figure 3.6 approximately here]

Figures 3.5 and 3.6 in the appendix 3.E, show the optimal responses to the government expenditure and the negative demand shocks for the baseline calibration and for the model with a smaller degree of price stickiness. By and large, the results that we have shown till now do not change.

¹²In recent literature several authors have claimed that the US Phillips curve has become flatter in the 1990s and 2000s. Others contest this finding. Generically, estimating Phillips curves requires to accurately measure inflation expectations, but the different measures that have been used in the literature did not produce a narrow range for the slope estimates.

We continue finding a frontloaded response in the model with POSA, whereas in the canonical model again obtain the random walk result. Moreover, the excess inflation variability under POSA remains and so does the near zero inflation response in the canonical model. This holds for both the case of the spending and the case of the demand shock. It thus seems that when we calibrate the slope of the Phillips curve to the higher end of the estimates found in the literature, our conclusions continue to hold.

3.6.2 Optimal fiscal policy with a Taylor rule

Consider now that monetary policy cannot be chosen arbitrarily by the planner, rather it has to follow an interest rate rule setting the nominal interest rate as a function of inflation and the output gap. In this scenario we may consider that monetary policy is independent and commits to an instrument rule, the fiscal authority which again is represented by the Ramsey planner has to account for this rule in optimization. This type of optimal policy scenario has been considered for example by [Leeper and Leith \(2016\)](#).

We saw previously that since debt is short term in the model, it becomes optimal to use inflation only contemporaneously with the shock. The intuition was that inflation after the shock has occurred will not contribute anything towards making debt sustainable and on the other hand inflation is costly in the model, so the optimal policy keeps a constant price level after the shock.

Implementing such a path for inflation, may not be easy when the nominal interest rate is tied down by an instrument rule. In the previous sections the planner was able to set interest rates independently from inflation and it is unlikely that optimal policies could be described by a simple rule setting the nominal rate to respond to the deviation of inflation from its target, as commonly independent central banks have been using. In the POSA model, this interaction between an independent monetary authority and an optimizing fiscal authority may be even more interesting, if we consider that the nominal interest rate will influence the liquidity /safety premium of government debt, a margin that directly relates to household utility.

We now consider as an additional constraint to the planners program a rule of the form

$$R_t = \left[\left(\frac{\pi_t}{\pi^*} \right)^{\phi_\pi} \left(\frac{Y_t}{Y^*} \right)^{\phi_y} \right] \quad (3.11)$$

where we set $\phi_\pi > 1$ and $\phi_y \geq 0$ as is commonly assumed in the literature. Starred variables are the target levels of inflation and output and these are the steady-state values.

The Ramsey problem and its first order conditions are similar to the problem described in section 3.4, but now the planner takes into account the Taylor rule. For brevity, we do not state here the problem, but refer the reader to appendix 3.D, which contains the non-linear Ramsey problem and the associated first order conditions.

To study the responses to the fiscal and the negative preference shocks we use the same calibration as before. We further assign the value of 1.5 to the inflation coefficient ϕ_π (a common assumption

in the literature) and let 0.15 be the coefficient ϕ_y . Under this calibration the modified Ramsey program yield a determinate equilibrium solution to the system of first order conditions.

Consider first the case of a fiscal shock in the model without capital. Figure 3.7 shows the responses. Notice that now, the shapes of the responses of policy variables to the shock are similar across the POSA and NOPOSA models. Indeed under both scenarios taxes are serially uncorrelated, the random walk properly no long holds even for NOPOSA. It has been noted that assuming an independent monetary policy has a considerable effect on how the government will time taxes [Leeper and Leith \(2016\)](#). To understand this intuitively, note that distortionary taxation not only affects the government's budget constraint in the model but also it exerts an influence on the Phillips curve (through the marginal cost) and further more it affects aggregate output and consumption, thus affecting also the real rate of interest. Thus, a government that is constrained to not tolerate a deviation of inflation from target or a deviation of output from its respective target level, is obliged to also constrain the response of taxes to the shock. (It is then not surprising that the only effects present are contemporaneous to the shock, as inflation and taxes will need to adjust anyway to satisfy the government budget constraint, and as we have seen for inflation this is only through a contemporaneous channel.)

Remarkably, however, we continue to find a higher volatility of inflation and taxes under POSA.

[Figure 3.7 approximately here]

Consider now the case of the negative preference shock shown in Figure 3.8. Essentially, the policy variables react in the same manner (though the sign is opposite of course). We again find that the planner limits the response of taxes (essentially) to the period the shock occurs, even though POSA continues to give more volatility to taxes and inflation. Once again the constraint imposed by the instrument rule, affects the optimal policy solution profoundly.

[Figure 3.8 approximately here]

[Figures 3.9 and 3.10 approximately here]

Lastly, the same finding emerges from the models with capital shown in Figures 3.9 and 3.10. We thus conclude that assuming an explicit interest rate rule has a considerable bearing on optimal policies. Note that the results shown here concern only one calibration of the interest rate rule. Admittedly, it would also be important to consider different functional forms for the instrument rule for example allowing for interest rate inertia or (perhaps more importantly) assuming that the monetary authority targets the deviation of output not from its steady state level, but from the efficient level which accounts for distortionary taxes. In these cases the policy rule could allow the planner more room to manipulate taxes intertemporally and this could bring back some of the results highlighted in the previous subsections. We have chosen not to consider these cases, because specifying rules that target efficient output, though optimal in some contexts, is generically not a realistic assumption. In any case this paragraph confirms a previous finding in the literature, that the Ramsey policy can be constrained by an independent monetary authority, also in the case of POSA. Whether there are plausible specifications of monetary policies under which this constraint is not significant remains to be explored.

3.7 Discussion: short vs. long-term government bonds

We now briefly discuss how our results would change if we were to allow the government to issue long term debt. Consider first the case where the government debt is only long term (i.e we replace short with long maturity bonds). Assuming long bonds in the canonical model implies more tax smoothing and more use of inflation to absorb shocks to the government budget. The intuition is clear. Long bonds enable the planner to spread the costs of inflation over a longer horizon, by promising a persistent increase in inflation following a positive spending shock. Thus inflation becomes less costly and the policy mix shifts away from taxes and towards inflation as a shock absorber.

This margin will also be available to the planner in the POSA model. However, there is no reason to believe that our previous results regarding the response of fiscal and monetary policies to shocks will not continue to hold. In particular, since the debt level will continue being part of the welfare objective, the planner will adhere to the target level of debt when deciding optimal policy. Thus taxes will be frontloaded as our previous findings suggest.

Perhaps a more interesting setup is one that allows the government to issue both short and long term debt simultaneously. It has been argued (see [Greenwood et al. \(2015\)](#) and [Krishnamurthy and Vissing-Jorgensen \(2012\)](#)) that short term debt provides liquidity/safety services over and above what long bonds provide. Perhaps at a limiting case we can consider that short bonds enter in the utility function while long bonds are only used to transfer resources across periods (as in the canonical model). Under this assumption it is probable that a government could vary the issuance of long bonds over the business cycle to absorb fiscal shocks, leaving the quantity of short term debt constant to the level desired by the private sector. We can speculate that in this version of the model, optimal policy will behave similarly to the canonical model, restoring the random walk tax schedule.

Of course, this conclusion is premature and derives from extrapolating from the linear quadratic problem we have solved. In nonlinear models, the optimal policy could differ considerably from the canonical Ramsey outcome. This is so because in the presence of nonlinearities, the optimizing government will face a non-trivial tradeoff, as long term bonds provide *fiscal insurance* (see [Faraglia et al. \(2012\)](#)) whereas short bonds do not. The government can then trade off the insurance benefit of issuing long term debt, against the lower cost of the more liquid short term asset. This tradeoff is studied in [Mankart et al. \(2022\)](#) who solve a non-linear optimal debt management model. The focus of that paper is however, on optimal portfolios and not on the optimal tax inflation mix. Extending the approach of our paper to model long term debt as an imperfect substitute for short maturities is interesting and left to future work.

3.8 Discussion: limitations from the approach

We close the paper with a brief comment on the further extensions of the approach taken this paper. In particular, we critically assess the assumption that bonds enter in the utility function.

Including bonds in the utility function is arguably a simplistic environment to explain the behavior of short-term interest rates, the liquidity premium. However, though this is indeed a simplistic mechanism to motivate that government debt offers money like services to the private sector, recent research has shown that this reduced form model is able to capture well the properties of more complicated environments in which agents face idiosyncratic risks and have a strong preference for safe and liquid assets (see [Mankart et al. \(2022\)](#) and references therein).

Since in these models government debt is essential to households to accumulate precautionary savings and selfinsure against idiosyncratic risks, it follows that a target level of debt will obtain and in principle the incentive to frontload taxes will carry through.

We cannot claim however, that the insights we can draw from our optimal policy exercise can be transferred completely to a model with idiosyncratic shocks. In the optimal policy context, a planner will design policies not only to smooth taxes and inflation over time but also to redistribute across households and hence optimal policy variables will be driven by forces that are not present in the representative agent model we analyzed here.

As discussed previously, a further shortcoming of the approach we adopted here, is our focus on short term debt. In practice, households can save in both short and long term assets and as we argued previously, the implications of including long bonds in the model can be far from trivial.

Instead of adding long term bonds, however, this paper experimented with a model with capital as an alternative asset. We showed that our results are robust to this inclusion, however, aggregate capital is not a government provided asset, and hence does not appear in the government budget constraint to directly affect policy.

One could also investigate a model in which capital can provide partial liquidity services to households, for example when a portfolio of stocks can be used to borrow and finance consumption.¹³ This modelling assumption could also capture the idea that when the government provides too little liquidity to the private sector, financial intermediaries compensate by creating more private liquidity in line with the empirical findings in [Greenwood et al. \(2015\)](#).

Under this scenario, households could more easily substitute bonds for private capital and some of the results that we showed in section 5.3 could be overturned. More specifically, if reducing government debt in response to a negative demand shock is not as detrimental to household welfare, when households can invest in capital, then a Ramsey planner may opt for smoothing taxes as in the canonical model. We leave this extension of our model to future work.

3.9 Conclusion

In this paper we studied optimal monetary and fiscal policy when households has preferences over government bonds. When this is the case, there is a wedge between the household discount factor and the nominal interest rate allowing the government to collect extra revenues from

¹³Alternatively, [Rannenberg \(2021\)](#) presents an extension of including not only debt but also physical capital as an argument of the utility function. In this case, it is assumed that the household derives utility from wealth, as wealth is implicitly a variable determining social status.

providing liquidity services. In other words, the government can borrow cheaper than in the standard New-Keynesian model due to the presence of the liquidity premium.

In such a world, the planner faces different forces in addition to the tax smoothing behavior proposed by Barro: debt issuance allows to smooth taxes, though at the same time reduces the liquidity premium and constrains the government. This force prevents the planner from issuing larger levels of debt to finance an unexpected increase in government spending. Instead, the planner uses inflation as a non-distortionary tax at period zero to absorb most of the shock when monetary policy is optimal. These behaviour departs from the “near zero” volatility of inflation usually present in an environment with sticky prices.

Moreover, due to the tensions found by the planner at the moment of issuing debt, its become more difficult to smooth taxes and tax distortions are front-loaded making the tax rate more volatile than in the standard model. As a consequence of the high distortionary taxes and inflation, the crowding-out of consumption and investment is higher under POSA. This is a strike difference from models using non-distortionary taxes to model the effect of government expenditure shocks under POSA.

3.A Appendix: Firm's problem

Production takes place in monopolistically competitive firms j which operates technologies with labor and public capital. The final good is a CES aggregate of the intermediate goods, $Y_t(j)$ of the form.

$$Y_t = \left(\int_0^1 Y_t(j)^{1-1/\theta} dj \right)^{\frac{\theta}{\theta-1}}$$

Intermediate goods seeks to set prices and labor input in order to maximize profits and subject to the following demand curve for each intermediate input j ,

$$Y_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\theta} Y_t$$

and the price adjustment costs, which are given by,

$$\Sigma_t(j) = \frac{\psi}{2} \left(\frac{P_t(j)}{P_t} - 1 \right)^2 Y_t$$

Consider the cost minimization problem for the firm, the firm chooses labor such that,

$$\min W_t n_t - r_t k_t$$

subject to,

$$F(n_t(j), k_{t-1}(j)) \geq \left(\frac{P_t(j)}{P_t} \right)^{-\theta} Y_t$$

The Lagrangian for this problem can be written as,

$$\mathcal{L} = -W_t n_t(j) - r_t k_t(j) + \mu_t(j) \left(F(n_t(j), k_{t-1}(j)) - \left(\frac{P_t(j)}{P_t} \right)^{-\theta} Y_t \right)$$

The FONC is:

$$W_t = \mu_t(j) F_n(n_t(j), k_{t-1}(j))$$

$$r_t = \mu_t(j) F_k(n_t(j), k_{t-1}(j))$$

Define $\mu_t(j) = \frac{W_t}{F_n(n_t(j), k_{t-1}(j))} = \frac{r_t}{F_k(n_t(j), k_{t-1}(j))}$ is the marginal cost for the firm j , $mc_t(j)$

Considering that $F(n_t, k_{t-1})$ is a Cobb-Douglas we can write the marginal productivity of labor for each firm j as $F_n(n_t(j), k_{t-1}(j)) = \frac{\alpha Y_t(j)}{n_t(j)}$ and the marginal productivity of capital as $F_k(n_t(j), k_{t-1}(j)) = \frac{(1-\alpha)Y_t(j)}{k_t}$. The latter allow us to write, $w_t = mc_t \alpha \frac{Y_t}{N_t}$ and $r_t = mc_t(1-\alpha) \frac{Y_t}{k_t}$.

Total costs given by $w_t N_t(j) + r_t k_t(j)$ can be written as $mc_t(j) \alpha Y_t(j) + mc_t(j)(1-\alpha)Y_t(j) = mc_t(j)Y_t(j)$

Considering now the problem in which the firm seeks prices, $P_t(j)$ to maximize profits,

$$\max_{P(j)} E_t \sum_{l=0}^{\infty} \beta^l \frac{U_{c,t+l+1}}{U_{c,t+l}} \left[(1 + sub) \frac{P_{t+l}(j)}{P_{t+l}} Y_{t+l}(j) - \frac{mc_{t+l}(j)}{P_{t+l}} Y_{t+l}(j) - \frac{\psi}{2} \left(\frac{P_{t+l}(j)}{P_{t+l-1}(j)} - 1 \right)^2 Y_{t+l} \right]$$

subject to the constraints

$$Y_{t+l}(j) = \left(\frac{P_{t+l}(j)}{P_{t+l}} \right)^{-\theta} Y_{t+l}$$

$$Y_{t+l}(j) \geq F(n_{t+l}(j), k_{t+l}(j))$$

$U_{c,t+l+1}/U_{c,t+l} = z_{t+l+1} c_{t+l+1}^{-\sigma} / z_{t+l} c_{t+l}^{-\sigma}$ is the discount factor of the households that own the shares of firm j .

The firm's first order condition with respect to $P_t(j)$ are given by,

$$(1 + sub) \frac{1}{P_t} Y_t \left(\frac{P_t(j)}{P_t} \right)^{-\theta} + (1 + sub) \frac{P_t(j)}{P_t^2} Y_t (-\theta) \left(\frac{P_t(j)}{P_t} \right)^{-\theta-1} + \theta \frac{mc_t(j)}{P_t} Y_t - \psi \left(\frac{P_t(j)}{P_{t-1}(j)} - 1 \right) \frac{Y_t}{P_{t-1}(j)} \\ + \beta E_t \frac{\epsilon_{t+1} c_{t+1}^{-\sigma}}{\epsilon_t c_t^{-\sigma}} \psi \left(\frac{P_{t+1}(j)}{P_t(j)} - 1 \right) \frac{P_{t+1}(j)}{P_t^2(j)} Y_{t+1} = 0$$

Imposing a symmetric equilibrium in which all firms set the same price, $P_t(j) = P_t$, all the firms consider the same productivity and real wage:

$$(1 + s) \frac{Y_t}{P_t} - (1 + sub) \theta \frac{Y_t}{P_t} + \theta \frac{mc_t}{P_t} Y_t - \psi \left(\frac{P_t}{P_{t-1}} - 1 \right) \frac{Y_t}{P_{t-1}} + \beta E_t \frac{\epsilon_{t+1} c_{t+1}^{-\sigma}}{\epsilon_t c_t^{-\sigma}} \psi \left(\frac{P_{t+1}}{P_t} - 1 \right) \frac{P_{t+1}}{P_t^2} Y_{t+1} = 0$$

Multiply by P_t and dividing by Y_t :

$$(1 + sub)1 - (1 + sub)\theta - \psi \left(\frac{P_t}{P_{t-1}} - 1 \right) \frac{P_t}{P_{t-1}} + \theta mc_t + \beta E_t \frac{\epsilon_{t+1} c_{t+1}^{-\sigma}}{\epsilon_t c_t^{-\sigma}} \psi \left(\frac{P_{t+1}}{P_t} - 1 \right) \frac{P_{t+1}}{P_t} \frac{Y_{t+1}}{Y_t} = 0$$

Define gross inflation rate, π_t as P_t/P_{t-1} , then

$$\begin{aligned}
(1 + sub)(1 - \theta) + \theta mc_t - \psi \left[(\pi_t - 1)\pi_t - \beta E_t \frac{\epsilon_{t+1} c_{t+1}^{-\sigma}}{\epsilon_t c_t^{-\sigma}} (\pi_{t+1} - 1)\pi_{t+1} \frac{Y_{t+1}}{Y_t} \right] &= 0 \\
\theta mc_t &= (\theta - 1)(1 + sub) + \psi \left[(\pi_t - 1)\pi_t - \beta E_t \frac{\epsilon_{t+1} c_{t+1}^{-\sigma}}{\epsilon_t c_t^{-\sigma}} (\pi_{t+1} - 1)\pi_{t+1} \frac{Y_{t+1}}{Y_t} \right] \\
mc_t &= \frac{(1 + sub)(\theta - 1)}{\theta} + \frac{\psi}{\theta} \left[(\pi_t - 1)\pi_t - \beta E_t \frac{\epsilon_{t+1} c_{t+1}^{-\sigma}}{\epsilon_t c_t^{-\sigma}} (\pi_{t+1} - 1)\pi_{t+1} \frac{Y_{t+1}}{Y_t} \right]
\end{aligned}$$

Replacing by marginal cost by the definition and making $sub = 1/(\theta - 1)$ we get the same equation as in text

$$\frac{w_t}{F_{N,t}} = 1 + \frac{\psi}{\theta} \left[(\pi_t - 1)\pi_t - \beta E_t \frac{z_{t+1} c_{t+1}^{-\sigma}}{z_t c_t^{-\sigma}} (\pi_{t+1} - 1)\pi_{t+1} \frac{Y_{t+1}}{Y_t} \right]$$

3.B Appendix: FONC for the Ramsey planner problem. Joint optimal fiscal and monetary policy

The first-order condition for the problem described in section 3.4 are:

Consumption, C_t :

$$\begin{aligned}
0 = E_t \left\{ z_t U_{C,t} + \left(z_t \phi_{1,t} - \beta^{-1} \phi_{1,t-1} \beta \frac{R_{t-1}}{\pi_t} \right) \left(U_{CC,t}(\cdot) \right) - \phi_{2,t} \left(\frac{U_{cc,t}}{U_{c,t}} \right) mc_t \right. \\
- \frac{\psi}{\theta} \left(\frac{U_{CC,t}(\cdot)}{U_{C,t}(\cdot)} \right) \left(\phi_{2,t} \beta \frac{U_{C,t+1}(\cdot)}{U_{C,t}(\cdot)} \frac{Y_{t+1}}{Y_t} \pi_{t+1} (\pi_{t+1} - 1) - \beta^{-1} \phi_{2,t-1} \beta \frac{U_{C,t}(\cdot)}{U_{C,t-1}(\cdot)} \frac{Y_t}{Y_{t-1}} \pi_t (\pi_t - 1) \right) - \phi_{3,t} \\
+ \phi_{6,t} \beta \frac{U_{C,t+1}(\cdot)}{U_{C,t}(\cdot)} \left((1 - \delta) q_{t+1} + mc_{t+1} F_{K,t+1} \right) \left(- \frac{U_{CC,t}(\cdot)}{U_{C,t}(\cdot)} \right) \\
- \beta^{-1} \phi_{6,t-1} \beta \frac{U_{C,t}(\cdot)}{U_{C,t-1}(\cdot)} \left((1 - \delta) q_t + mc_t F_{K,t} \right) \left(- \frac{U_{CC,t}(\cdot)}{U_{C,t}(\cdot)} \right) + \beta^{-1} \phi_{6,t-1} \beta \frac{U_{C,t}}{U_{C,t-1}} \left(- \frac{U_{cc,t}}{U_{c,t}} \right) mc_t F_{K,t} \\
\left. + \phi_{7,t} \tau_t N_t mc_t \left(\frac{-U_{CC}}{U_C} \right) \right\}
\end{aligned} \tag{3.12}$$

Labor, N_t :

$$\begin{aligned}
0 = E_t \left\{ U_{N,t} + \phi_{2,t} \left(\frac{U_{NN,t}}{U_{N,t}} - \frac{F_{NN,t}}{F_{N,t}} \right) mc_t \right. \\
- \frac{\psi}{\theta} \left(\phi_{2,t} \beta \frac{U_{C,t+1}(\cdot)}{U_{C,t}(\cdot)} \frac{Y_{t+1}}{Y_t} \pi_{t+1} (\pi_{t+1} - 1) - \beta^{-1} \phi_{2,t-1} \beta \frac{U_{C,t}(\cdot)}{U_{C,t-1}(\cdot)} \frac{Y_t}{Y_{t-1}} \pi_t (\pi_t - 1) \right) \left(\frac{F_{N,t}}{Y_t} \right) \\
+ \phi_{3,t} F_{N,t} \left(1 - \psi \frac{\pi_t^2}{2} \right) + \beta^{-1} \phi_{6,t-1} \beta \frac{U_{C,t}(\cdot)}{U_{C,t-1}(\cdot)} mc_t F_{K,t} \left(\frac{U_{NN,t}}{U_{N,t}} - \frac{F_{NN,t}}{F_{N,t}} + \frac{F_{KN,t}}{F_{K,t}} \right) \\
\left. + \phi_{7,t} \tau_t \left(1 + N_t mc_t \left(\frac{U_{NN,t}}{U_{N,t}} - \frac{F_{NN,t}}{F_{N,t}} \right) \right) \right\}
\end{aligned} \tag{3.13}$$

Private investment, I_t

$$\begin{aligned}
0 = E_t \Bigg\{ & -\phi_{3,t} + \phi_{4,t} \left[1 - S_I \left(\frac{I_t}{I_{t-1}} \right) - \left(\frac{I_t}{I_{t-1}} \right) S'_I \left(\frac{I_t}{I_{t-1}} \right) \right] + \beta \phi_{4,t+1} \left(\frac{I_{t+1}}{I_t} \right)^2 S'_I \left(\frac{I_{t+1}}{I_t} \right) \\
& - \phi_{5,t} \left[\frac{q_t}{I_{t-1}} \left[2S'_I \left(\frac{I_t}{I_{t-1}} \right) + \left(\frac{I_t}{I_{t-1}} \right) S''_I \left(\frac{I_t}{I_{t-1}} \right) \right] + \beta \frac{U_{C,t+1}(\cdot)}{U_{C,t}(\cdot)} \frac{q_{t+1}}{I_t} \left(\frac{I_{t+1}}{I_t} \right)^2 \left[2S'_I \left(\frac{I_{t+1}}{I_t} \right) + \left(\frac{I_{t+1}}{I_t} \right) S''_I \left(\frac{I_{t+1}}{I_t} \right) \right] \right. \\
& + \beta \phi_{5,t+1} \frac{q_{t+1}}{I_t} \left(\frac{I_{t+1}}{I_t} \right) \left[2S'_I \left(\frac{I_{t+1}}{I_t} \right) + \left(\frac{I_{t+1}}{I_t} \right) S''_I \left(\frac{I_{t+1}}{I_t} \right) \right] \\
& \left. + \beta^{-1} \phi_{5,t-1} \beta \frac{U_{C,t}(\cdot)}{U_{C,t-1}(\cdot)} \left(\frac{I_t}{I_{t-1}} \right) \frac{q_t}{I_{t-1}} \left[2S'_I \left(\frac{I_{t+1}}{I_t} \right) + \left(\frac{I_{t+1}}{I_t} \right) S''_I \left(\frac{I_{t+1}}{I_t} \right) \right] \right\}
\end{aligned} \tag{3.14}$$

Private capital, K_{t+1}

$$\begin{aligned}
0 = E_t \Bigg\{ & -\beta \phi_{2,t+1} \left(\frac{F_{NK,t+1}}{F_{N,t+1}} \right) mc_{t+1} - \\
& - \frac{\psi}{\theta} \left(\beta \phi_{2,t+1} \beta \frac{U_{C,t+2}(\cdot)}{U_{C,t+1}(\cdot)} \frac{Y_{t+2}}{Y_{t+1}} \pi_{t+2} (\pi_{t+2} - 1) - \phi_{2,t} \beta \frac{U_{C,t+1}(\cdot)}{U_{C,t}(\cdot)} \frac{Y_{t+1}}{Y_t} \pi_{t+1} (\pi_{t+1} - 1) \right) \left(\frac{F_{K,t+1}}{Y_{t+1}} \right) + \\
& + \beta \phi_{3,t+1} \left(1 - \psi \frac{\pi_{t+1}^2}{2} \right) F_{K,t+1} - \phi_{4,t} + \beta(1 - \delta) \phi_{4,t+1} - \phi_{6,t} \beta \frac{U_{C,t+1}(\cdot)}{U_{C,t}(\cdot)} \left(\frac{F_{NK,t+1}}{F_{N,t+1}} - \frac{F_{KK,t+1}}{F_{K,t+1}} \right) mc_{t+1} F_{K,t+1} \Bigg\}
\end{aligned} \tag{3.15}$$

Price of private capital, q_t

$$\begin{aligned}
0 = E_t \Bigg\{ & \phi_{5,t} \left[1 - S_I \left(\frac{I_t}{I_{t-1}} \right) - \left(\frac{I_t}{I_{t-1}} \right) S'_I \left(\frac{I_t}{I_{t-1}} \right) \right] + \beta^{-1} \phi_{5,t-1} \beta \frac{U_{C,t}(\cdot)}{U_{C,t-1}(\cdot)} \left(\frac{I_t}{I_{t-1}} \right)^2 S'_I \left(\frac{I_t}{I_{t-1}} \right) \\
& - \phi_{6,t} + \beta^{-1} \phi_{6,t-1} \beta \frac{U_{C,t}(\cdot)}{U_{C,t-1}(\cdot)} (1 - \delta) \Bigg\}
\end{aligned} \tag{3.16}$$

Interest rate, R_t

$$0 = E_t \Bigg\{ -\phi_{1,t} \beta U_{C,t+1} \frac{1}{\pi_{t+1}} - \beta \phi_{7,t+1} \frac{b_t}{\pi_{t+1}} \Bigg\} \tag{3.17}$$

Inflation rate, π_t

$$\begin{aligned}
0 = E_t \Bigg\{ & \beta^{-1} \frac{\phi_{1,t-1} R_{t-1}}{\pi_t^2} \frac{U_{C,t}(\cdot)}{U_{C,t-1}(\cdot)} - \frac{\psi}{\theta} \left[\phi_{2,t} - \beta^{-1} \phi_{2,t-1} \beta \frac{U_{C,t}(\cdot)}{U_{C,t-1}(\cdot)} \frac{Y_t}{Y_{t-1}} \right] (1 + 2\pi_t) - \phi_{3,t} \psi Y_t \pi_t \\
& + \phi_{7,t} \frac{R_{t-1}}{\pi_t^2} b_{t-1} \Bigg\}
\end{aligned} \tag{3.18}$$

Public debt, b_t

$$0 = E_t \Bigg\{ U_{B,t} - \phi_{1,t} U_{BB,t} + \phi_{7,t} - \beta \phi_{7,t+1} \frac{R_t}{\pi_{t+1}} \Bigg\} \tag{3.19}$$

taxes, τ_t

$$0 = E_t \Bigg\{ \phi_{2,t} \left[\frac{1}{(1 - \tau_t)} \right] mc_t + \beta^{-1} \phi_{6,t-1} \beta \frac{U_{Ct}}{U_{C,t-1}} \frac{F_{K,t}}{(1 - \tau)} mc_t + \phi_{7,t} N_t w_t \left[\frac{2 - \tau}{1 - \tau} \right] mc_t \Bigg\} \tag{3.20}$$

Where the production function, real wages and marginal cost are defined as:

$$\begin{aligned}
 Y_t &= F(k_t, K_G, N_t) \\
 w_t &= \frac{-U_{N,t}}{(1 - \tau_t)U_{C,t}} \\
 mc_t &= \frac{w_t}{F_{N,t}}
 \end{aligned}$$

And have been substituted out in the log-linear version in the main text for exposition motives.

3.C Appendix: Flexible prices steady-state

From the first order conditions for the households, the euler equation for short term bonds in the steady-state reads:

$$c^{*-\sigma} = \beta \frac{R^* c^{*-\sigma}}{\pi^*} + \chi_b (b^*)^{-\sigma_b}$$

$$1 = \beta \frac{R^*}{\pi^*} + \chi_b \frac{b^{*-\sigma_b}}{c^{*-\sigma}}$$

Considering a zero inflation steady state ($\pi^* = 1$),

$$1 = \beta R^* + \chi_b \frac{b^{*-\sigma_b}}{c^{*-\sigma}}$$

$$\beta R^* = 1 - \chi_b \frac{b^{*-\sigma_b}}{c^{*-\sigma}} < 1$$

We can perform the same transformation as [Campbell et al. \(2017\)](#) and [Rannenberg \(2021\)](#) and express the euler equation for bonds as:

$$\tilde{c}_t = \vartheta [\tilde{R}_t - E_t \tilde{\pi}_{t+1}] - \vartheta E_t \tilde{c}_{t+1} + (1 - \vartheta) \frac{\sigma_b}{\sigma} \tilde{b}_t \quad (3.21)$$

whenever $\vartheta = \beta R^*$. In absence of POSA, $\vartheta = 1$ and the equation is the standard Euler equation for bonds.

From the household first order condition for labor, eliminating time subscripts, imposing $n^* = 1/3$ we arrive to the following:

$$\chi \frac{n^{*\varphi}}{(1 - \tau^*) c^{*-\sigma}} = \alpha \frac{y^*}{n^*}$$

$$\chi = \alpha \frac{y^* c^{*-\sigma} (1 - \tau^*)}{n^{*1+\varphi}}$$

From the FONC for the firm w.r.t labor, we can obtain the wage by replacing the production function and the optimal subsidy:

$$mc_t = \frac{w_t}{F_{N,t}}$$

$$w^* = mc^* \alpha \frac{Y^*}{n^*}$$

$$w^* = \frac{mc^* \alpha Y^*}{(1 - \tau^*) n^*}$$

Given the assumption for the investment adjustment costs, $S(1) = 0$ and $S'(1) = 0$, from the FONC from the households, in the steady-state we have:

$$1 = [1 - S(1) - S'(1)]q^* + \beta E \left[\frac{U'^*}{U'^*} S'(1)q \right]$$

$$q^* = 1$$

Substituting last result in the last FONC for the household,

$$1 = \beta \left[\frac{U'^*}{U'^*} (1 - \delta) + mc^* F_k^* \right]$$

$$F_k^* = \frac{1 - \beta(1 - \delta)}{\beta mc^*}$$

Calibrating hours in the steady-state to be 1/3 of disposable time, we can use last equation together with the production function to solve for K in the steady state.

$$(1 - \alpha)N^\alpha K^{-\alpha} = \frac{1 - \beta(1 - \delta)}{\beta}$$

which in the steady state is:

$$K^* = \left[\frac{1 - \beta(1 - \delta)}{\beta} \frac{N^{-\alpha}}{1 - \alpha} \right]^{-1/\alpha}$$

and using the production function:

$$Y^* = N^\alpha K^{1-\alpha}$$

Using Law of motion for private capital, we can recover investment:

$$I^* = \delta K^*$$

An initial steady-state level of debt, and assuming a zero inflation steady state can be obtained from the government budget constraint for given values of government consumption. We impose the value for g^c from the steady-state according to data, while labor hours are set to 1/3.

$$b^* = \frac{\tau^* w^* n^* - g^{c,*}}{R^*}$$

For the level of the tax rate at the steady state, we calibrate a value so that debt to output ratio is 60% in the steady-state (on a quarterly calibration this correspond to 240% of output). From the govt budget constraint, first replacing the price for the bond:

$$\tau_t w_t n_t - g_t^c = b_t - q_t b_t$$

$$\tau_t w_t n_{tt} - g_t^c = (1 - \beta) b_t$$

Replace w_t for the value in the steady state:

$$\begin{aligned}\tau_t \frac{mc_t \alpha y_t}{(1 - \tau_t) n_t} n_t - g_t^c &= (1 - \beta) b_t \\ \tau_t &= \frac{(1 - \tau_t)}{mc_t \alpha y_t} (1 - \beta) b_t + g_t^c \\ \tau_t &= \frac{\frac{1}{mc_t \alpha y_t} \left[(1 - \beta) b_t + g_t^c \right]}{\left[1 + \frac{1}{mc_t \alpha y_t} (1 - \beta) b_t + g_t^c \right]}\end{aligned}$$

So in the steady state, the tax rate can be the one to sustain certain level of debt to output ratio, calibrated to 60%

$$\tau^* = \frac{\frac{1}{\alpha mc^*} \left[(1 - \beta) \frac{b^*}{y^*} + \frac{g^{c,*}}{y^*} \right]}{\left[1 + \frac{1}{\alpha mc^*} (1 - \beta) \frac{b^*}{y^*} + \frac{g^{c,*}}{y^*} \right]}$$

In a flexible price steady-state, the planner is not constrained by the Phillips curve, thus the equation and the multiplier can be dropped from the system, thus $\phi_2 = 0$. Now, we impose a zero inflation steady-state such that,

$$\pi^* = 1$$

3.D Appendix: Ramsey planner problem with a Taylor rule

In this section we describe the Ramsey planner problem when the economy can accumulate private capital and it is constrained by a Taylor rule for the nominal interest rate. When consider an economy without capital, one just need to remove the constraints linked to private capital: the Euler equation for capital, the household first order condition with respect to investment and the law of motion for private capital, together with the variables I_t, k_t, q_t, r_t^k

Consider $H_t = \{c_t, n_t, k_{t+1}, I_t, R_t, \pi_t, b_t, \tau_t, q_t\}$, the vector of control variables. The Lagrangian for a competitive equilibrium with monetary policy following a Taylor rule:

$$\begin{aligned}
\max_{\{H_t\}} L_0 = E_0 \sum_{t=0}^{\infty} \beta^t & \left\{ U(c_t, n_t, b_t, z_t) \right. \\
& + \phi_{1,t} \left[z_t U_{C,t} - \beta E_t \left[z_{t+1} \frac{U_{C,t+1} R_t}{\pi_{t+1}} \right] - U_{b,t} \right] \\
& + \phi_{2,t} \left[mc_t - 1 - \frac{\psi}{\theta} \left(\pi_t (\pi_t - 1) - \beta \frac{z_{t+1}}{z_t} \frac{U_{C,t+1}(\cdot)}{U_{C,t}(\cdot)} \frac{Y_{t+1}}{Y_t} \pi_{t+1} (\pi_{t+1} - 1) \right) \right] \\
& + \phi_{3,t} \left[\left(1 - \frac{\psi}{2} \pi_t^2 \right) Y_t - C_t - I_t - G_t^c \right] \\
& + \phi_{4,t} \left[(1 - \delta) k_t + I_t - k_{t+1} \right] \\
& + \phi_{5,t} \left[\left[1 - S_I \left(\frac{I_t}{I_{t-1}} \right) - S'_I \left(\frac{I_t}{I_{t-1}} \right) \frac{I_t}{I_{t-1}} \right] q_t + \beta \frac{z_{t+1}}{z_t} \frac{U_{C,t+1}(\cdot)}{U_{C,t}(\cdot)} S'_I \left(\frac{I_{t+1}}{I_t} \right) \left(\frac{I_{t+1}}{I_t} \right)^2 q_{t+1} - 1 \right] \\
& + \phi_{6,t} \left[\beta \frac{z_{t+1}}{z_t} \frac{U_{C,t+1}}{U_{C,t}} ((1 - \delta) + mc_{t+1} F_{K,t+1}) - 1 \right] \\
& + \phi_{7,t} \left[R_t - (\pi_t)^{\phi_\pi} \left(\frac{Y_t}{Y} \right)^{\phi_y} \right] \\
& + \phi_{8,t} \left[b_t - \frac{R_{t-1}}{\pi_t} b_{t-1} - G_t^c + \tau_t w_t N_t \right] \left. \right\}
\end{aligned}$$

Where the production function, real wages and marginal cost are defined as:

$$\begin{aligned}
Y_t &= F(k_t, n_t) \\
w_t &= \frac{-U_{N,t}}{(1 - \tau_t) z_t U_{C,t}} \\
mc_t &= \frac{w_t}{F_{N,t}}
\end{aligned}$$

The first-order condition for the problem are:

Consumption, C_t :

$$\begin{aligned}
0 = E_t \Big\{ & z_t U_{C,t} + \left(z_t \phi_{1,t} - \beta^{-1} \phi_{1,t-1} \beta \frac{R_{t-1}}{\pi_t} \right) \left(U_{CC,t}(\cdot) \right) - \phi_{2,t} \left(\frac{U_{cc,t}}{U_{c,t}} \right) m_{C,t} \\
& - \frac{\psi}{\theta} \left(\frac{U_{CC,t}(\cdot)}{U_{C,t}(\cdot)} \right) \left(\phi_{2,t} \beta \frac{U_{C,t+1}(\cdot)}{U_{C,t}(\cdot)} \frac{Y_{t+1}}{Y_t} \pi_{t+1} (\pi_{t+1} - 1) - \beta^{-1} \phi_{2,t-1} \beta \frac{U_{C,t}(\cdot)}{U_{C,t-1}(\cdot)} \frac{Y_t}{Y_{t-1}} \pi_t (\pi_t - 1) \right) - \phi_{3,t} \\
& + \phi_{6,t} \beta \frac{U_{C,t+1}(\cdot)}{U_{C,t}(\cdot)} \left((1 - \delta) q_{t+1} + m_{C,t+1} F_{K,t+1} \right) \left(- \frac{U_{CC,t}(\cdot)}{U_{C,t}(\cdot)} \right) \\
& - \beta^{-1} \phi_{6,t-1} \beta \frac{U_{C,t}(\cdot)}{U_{C,t-1}(\cdot)} \left((1 - \delta) q_t + m_{C,t} F_{K,t} \right) \left(- \frac{U_{CC,t}(\cdot)}{U_{C,t}(\cdot)} \right) + \beta^{-1} \phi_{6,t-1} \beta \frac{U_{C,t}}{U_{C,t-1}} \left(- \frac{U_{cc,t}}{U_{c,t}} \right) m_{C,t} F_{K,t} \\
& + \phi_{8,t} \tau_t N_t m_{C,t} \left(\frac{-U_{CC}}{U_C} \right) \Big\}
\end{aligned} \tag{3.22}$$

Labor, N_t :

$$\begin{aligned}
0 = E_t \Big\{ & U_{N,t} + \phi_{2,t} \left(\frac{U_{NN,t}}{U_{N,t}} - \frac{F_{NN,t}}{F_{N,t}} \right) m_{C,t} \\
& - \frac{\psi}{\theta} \left(\phi_{2,t} \beta \frac{U_{C,t+1}(\cdot)}{U_{C,t}(\cdot)} \frac{Y_{t+1}}{Y_t} \pi_{t+1} (\pi_{t+1} - 1) - \beta^{-1} \phi_{2,t-1} \beta \frac{U_{C,t}(\cdot)}{U_{C,t-1}(\cdot)} \frac{Y_t}{Y_{t-1}} \pi_t (\pi_t - 1) \right) \left(\frac{F_{N,t}}{Y_t} \right) \\
& + \phi_{3,t} F_{N,t} \left(1 - \psi \frac{\pi_t^2}{2} \right) + \beta^{-1} \phi_{6,t-1} \beta \frac{U_{C,t}(\cdot)}{U_{C,t-1}(\cdot)} m_{C,t} F_{K,t} \left(\frac{U_{NN,t}}{U_{N,t}} - \frac{F_{NN,t}}{F_{N,t}} + \frac{F_{KN,t}}{F_{K,t}} \right) \\
& - \phi_{7,t} \phi_y (\pi_t)^{\phi_y} \left(\frac{Y_t}{Y} \right)^{\phi_y} \frac{F_{N,t}}{Y_t} + \phi_{8,t} \tau_t \left(1 + N_t m_{C,t} \left(\frac{U_{NN,t}}{U_{N,t}} - \frac{F_{NN,t}}{F_{N,t}} \right) \right) \Big\}
\end{aligned} \tag{3.23}$$

Private investment, I_t

$$\begin{aligned}
0 = E_t \Big\{ & - \phi_{3,t} + \phi_{4,t} \left[1 - S_I \left(\frac{I_t}{I_{t-1}} \right) - \left(\frac{I_t}{I_{t-1}} \right) S'_I \left(\frac{I_t}{I_{t-1}} \right) \right] + \beta \phi_{4,t+1} \left(\frac{I_{t+1}}{I_t} \right)^2 S'_I \left(\frac{I_{t+1}}{I_t} \right) \\
& - \phi_{5,t} \left[\frac{q_t}{I_{t-1}} \left[2S'_I \left(\frac{I_t}{I_{t-1}} \right) + \left(\frac{I_t}{I_{t-1}} \right) S''_I \left(\frac{I_t}{I_{t-1}} \right) \right] + \beta \frac{U_{C,t+1}(\cdot)}{U_{C,t}(\cdot)} \frac{q_{t+1}}{I_t} \left(\frac{I_{t+1}}{I_t} \right)^2 \left[2S'_I \left(\frac{I_{t+1}}{I_t} \right) + \left(\frac{I_{t+1}}{I_t} \right) S''_I \left(\frac{I_{t+1}}{I_t} \right) \right] \right] \\
& + \beta \phi_{5,t+1} \frac{q_{t+1}}{I_t} \left(\frac{I_{t+1}}{I_t} \right) \left[2S'_I \left(\frac{I_{t+1}}{I_t} \right) + \left(\frac{I_{t+1}}{I_t} \right) S''_I \left(\frac{I_{t+1}}{I_t} \right) \right] \\
& + \beta^{-1} \phi_{5,t-1} \beta \frac{U_{C,t}(\cdot)}{U_{C,t-1}(\cdot)} \left(\frac{I_t}{I_{t-1}} \right) \frac{q_t}{I_{t-1}} \left[2S'_I \left(\frac{I_{t+1}}{I_t} \right) + \left(\frac{I_{t+1}}{I_t} \right) S''_I \left(\frac{I_{t+1}}{I_t} \right) \right] \Big\}
\end{aligned} \tag{3.24}$$

Capital, K_{t+1}

$$\begin{aligned}
0 = E_t \Bigg\{ & -\beta\phi_{2,t+1} \left(\frac{F_{NK,t+1}}{F_{N,t+1}} \right) mc_{t+1} - \\
& -\frac{\psi}{\theta} \left(\beta\phi_{2,t+1} \beta \frac{U_{C,t+2}(\cdot)}{U_{C,t+1}(\cdot)} \frac{Y_{t+2}}{Y_{t+1}} \pi_{t+2} (\pi_{t+2} - 1) - \phi_{2,t} \beta \frac{U_{C,t+1}(\cdot)}{U_{C,t}(\cdot)} \frac{Y_{t+1}}{Y_t} \pi_{t+1} (\pi_{t+1} - 1) \right) \left(\frac{F_{K,t+1}}{Y_{t+1}} \right) + \\
& + \beta\phi_{3,t+1} \left(1 - \psi \frac{\pi_{t+1}^2}{2} \right) F_{K,t+1} - \phi_{5,t} + \beta(1 - \delta)\phi_{4,t+1} \\
& - \phi_{6,t} \beta \frac{U_{C,t+1}(\cdot)}{U_{C,t}(\cdot)} \left(\frac{F_{NK,t+1}}{F_{N,t+1}} - \frac{F_{KK,t+1}}{F_{K,t+1}} \right) mc_{t+1} F_{K,t+1} - \\
& - \phi_{7,t+1} \phi_y (\pi_{t+1})^{\phi_y} \left(\frac{Y_{t+1}}{Y} \right)^{\phi_y} \frac{F_{K,t+1}}{Y_{t+1}} + \beta\phi_{8,t+1} \tau_{t+1} N_{t+1} mc_{t+1} \left(\frac{F_{NK,t+1}}{F_{N,t+1}} \right) \Bigg\}
\end{aligned} \tag{3.25}$$

Price of private capital, q_t

$$\begin{aligned}
0 = E_t \Bigg\{ & \phi_{5,t} \left[1 - S_I \left(\frac{I_t}{I_{t-1}} \right) - \left(\frac{I_t}{I_{t-1}} \right) S'_I \left(\frac{I_t}{I_{t-1}} \right) \right] + \beta^{-1} \phi_{5,t-1} \beta \frac{U_{C,t}(\cdot)}{U_{C,t-1}(\cdot)} \left(\frac{I_t}{I_{t-1}} \right)^2 S'_I \left(\frac{I_t}{I_{t-1}} \right) \\
& - \phi_{6,t} + \beta^{-1} \phi_{6,t-1} \beta \frac{U_{C,t}(\cdot)}{U_{C,t-1}(\cdot)} (1 - \delta) \Bigg\}
\end{aligned} \tag{3.26}$$

Interest rate, R_t

$$0 = E_t \Bigg\{ -\phi_{1,t} \beta U_{C,t+1} \frac{1}{\pi_{t+1}} + \phi_{7,t} - \beta\phi_{8,t+1} \frac{b_t}{\pi_{t+1}} \Bigg\} \tag{3.27}$$

Inflation rate, π_t

$$\begin{aligned}
0 = E_t \Bigg\{ & \beta^{-1} \frac{\phi_{1,t-1} R_{t-1}}{(\pi_t)^2} \frac{U_{C,t}(\cdot)}{U_{C,t-1}(\cdot)} - \frac{\psi}{\theta} \left[\phi_{2,t} - \beta^{-1} \phi_{2,t-1} \beta \frac{U_{C,t}(\cdot)}{U_{C,t-1}(\cdot)} \frac{Y_t}{Y_{t-1}} \right] (1 + 2\pi_t) - \phi_{3,t} \psi Y_t \pi_t \\
& - \phi_{7,t} \phi_\pi \left(\frac{Y_{t+1}}{Y} \right)^{\phi_y} (1 + \pi_t)^{\phi_\pi - 1} + \phi_{8,t} \frac{R_{t-1}}{(\pi_t)^2} b_{t-1} \Bigg\}
\end{aligned} \tag{3.28}$$

Public debt, b_t

$$0 = E_t \Bigg\{ -\sigma_b U_B b_t - \phi_{1,t} U_{BB,t} + \phi_{8,t} - \beta\phi_{8,t+1} \frac{R_t}{\pi_{t+1}} \Bigg\} \tag{3.29}$$

taxes, τ_t

$$0 = E_t \Bigg\{ \phi_{2,t} \left[\frac{1}{(1 - \tau_t)} \right] mc_t + \beta^{-1} \phi_{6,t-1} \beta \frac{U_{C,t}}{U_{C,t-1}} \frac{F_{K,t}}{(1 - \tau)} mc_t + \phi_{8,t} N_t w_t \left[\frac{2 - \tau}{1 - \tau} \right] mc_t \Bigg\} \tag{3.30}$$

The linear version of the model is similar to the previous one, but one needs to add the Linear version of the Taylor rule, and consider the multiplier ϕ_8 attached to it.

3.E Appendix: tables and figures

TABLE 3.1: Model parameters

Description	Parameter	POSA	NOPOSA	Source/Target
Preferences parameters				
Discount factor	β	0.985	0.99	r=0.005
Consumption risk aversion	σ		1	standard
Inverse Frisch elasticity	φ		1	standard
Curvature safe assets	σ_b	0.2; 1	-.-	Rannenberg (2021)
Discounting wedge	ϑ	0.99	1	Campbell et al. (2017)
Firms				
Labor intensity	a	0.64		Christiano et al. (2005)
Price adjustment cost	ψ	200		target Phillips curve slope 0.03
Elasticity substitution intermediate goods	θ	6		target mark-up 20%
Depreciation rate	δ	0.02		target $K/4Y = 2.7\%$
Private investment adjustment costs	ω	5.5		Rannenberg (2021)
Taylor rule				
Taylor rule inflation	ϕ_π	1.5		Woodford (2011)
Taylor rule output	ϕ_y	0.15		
Government				
Government expenditure share	g/y	20%		US data
Government debt to output	$b/4y$	60%		US data

Notes: The table shows the main calibration parameters to meet the data in steady-state considering standard preferences, ie. NOPOSA. US data for the period 1987-2018 from the Federal Reserve Bank of St. Louis

3.E.1 Figures

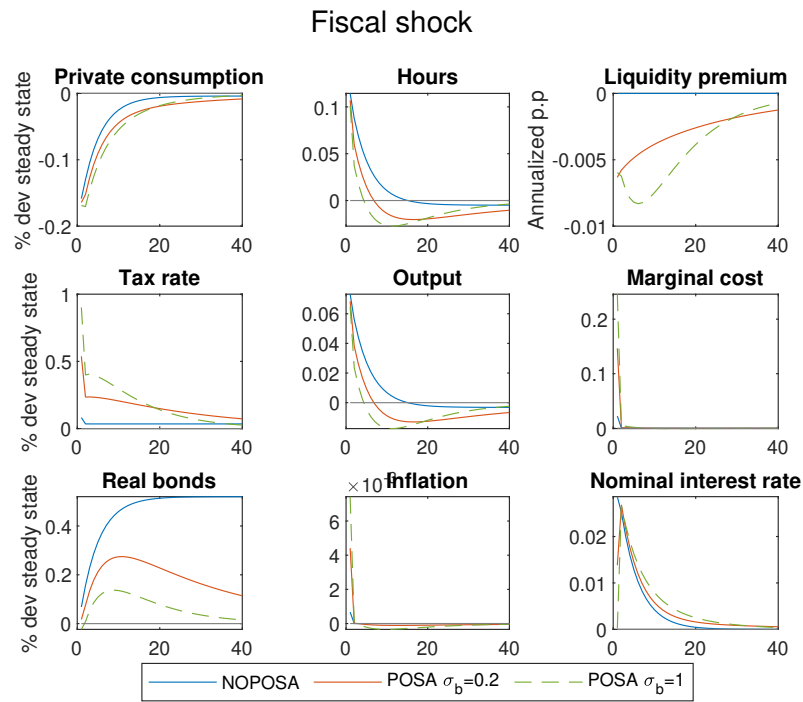


FIGURE 3.1: Optimal fiscal and monetary policy in response to a fiscal shock

Note: the figure shows the optimal response of the economy to a government spending shock, which increases expenditures by 1% above the steady-state level in an economy without capital. The blue line is the economy with standard preferences. The red and green dashed line are the economy with POSA, $\sigma_b = 0.2$ and $\sigma_b = 1$

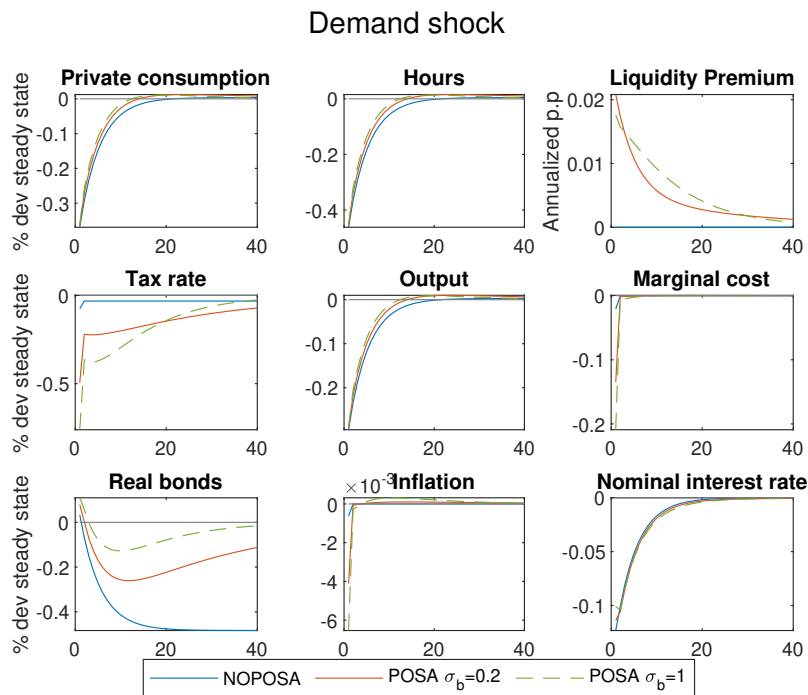


FIGURE 3.2: Optimal fiscal and monetary policy in response to a demand shock

Note: the figure shows the optimal response of the economy to a negative demand shock z_t reducing consumption by 1% in an economy without capital. The blue line is the economy with standard preferences. The red and green dashed line are the economy with POSA, $\sigma_b = 0.2$ and $\sigma_b = 1$.

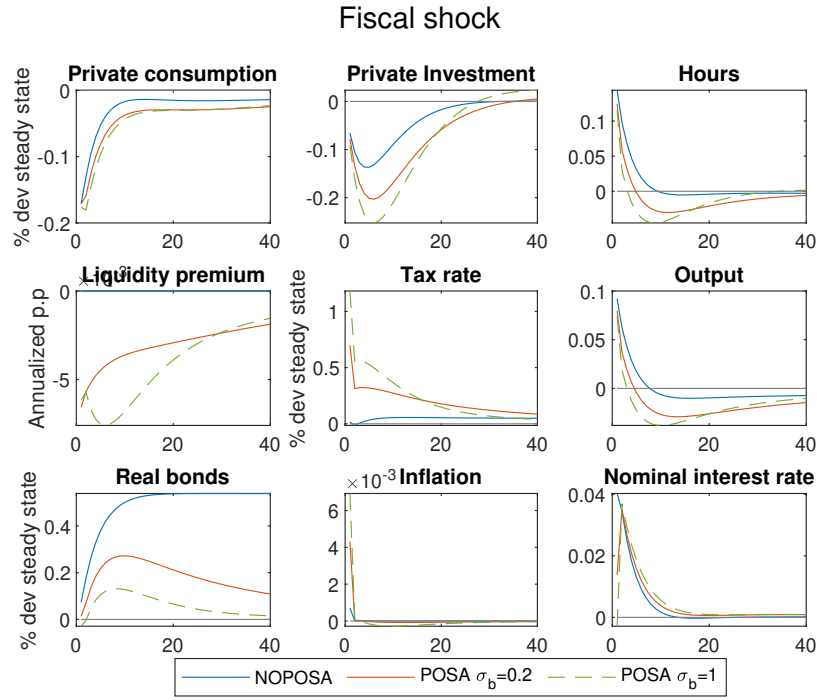


FIGURE 3.3: Optimal monetary and fiscal response to a government spending shock

Note: the figure shows the optimal response of the economy to a negative demand shock z_t reducing consumption in 1% in an economy with private capital. The blue line is the economy with standard preferences. The red and green dashed line are the economy with POSA, $\sigma_b = 0.2$ and $\sigma_b = 1$

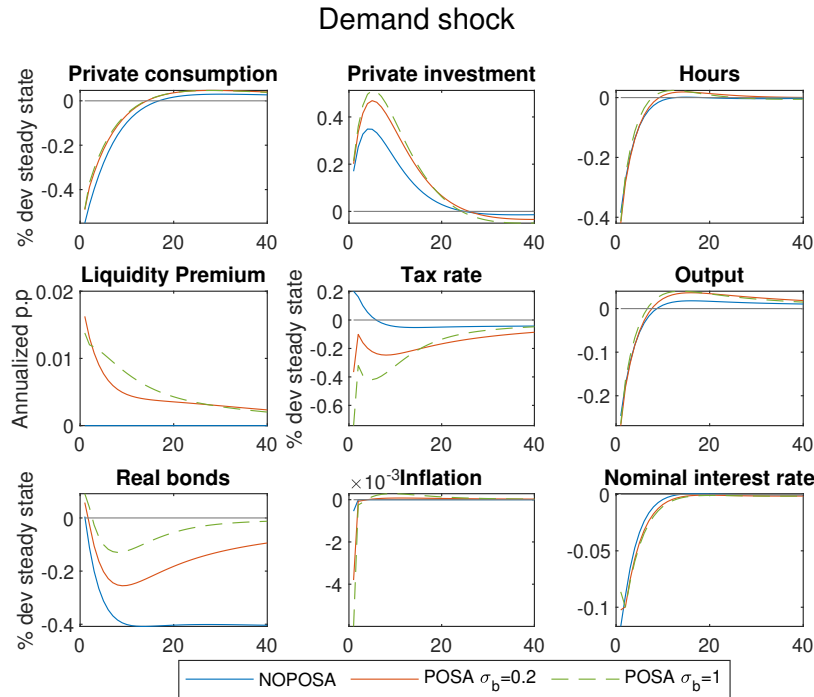


FIGURE 3.4: Optimal monetary and fiscal response to a demand shock

Note: the figure shows the optimal response of the economy to a government spending shock, which increases expenditures by 1% above the steady-state level in an economy with private capital. The blue line is the economy with standard preferences. The red and green dashed line are the economy with POSA, $\sigma_b = 0.2$ and $\sigma_b = 1$

3.E.2 Extension: Degree of price stickiness

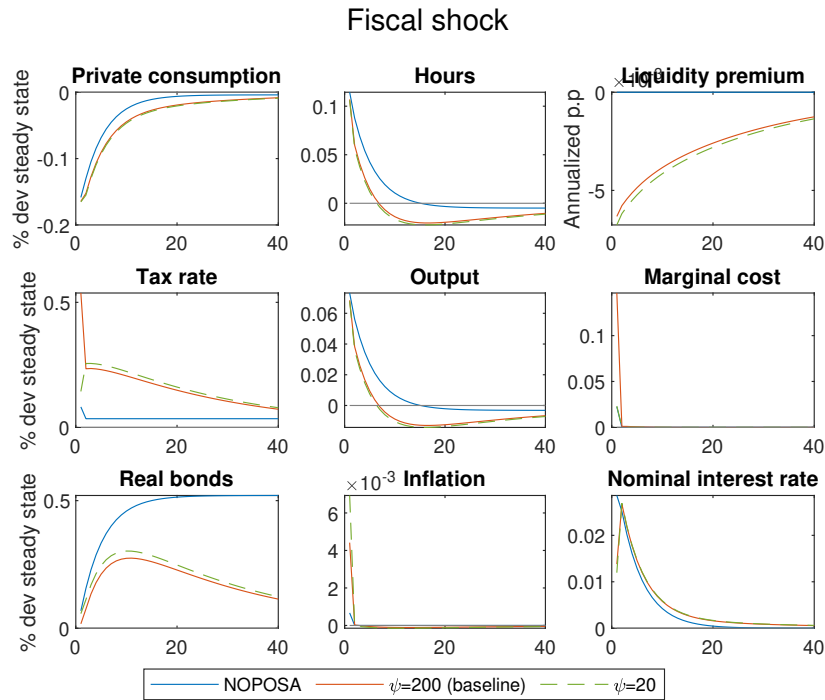


FIGURE 3.5: Degree of price stickiness and optimal inflation and tax rate volatility

Note: the figure shows the optimal response of the economy to a government expenditure shock of 1% in an economy without capital. The blue line stands for the economy under NOPOSA (standard model). The red line is the baseline economy with $\psi = 200$ and the green dashed line is the economy with $\psi = 20$ with a slope for the Phillips curve of 0.3, ten times larger than the baseline economy

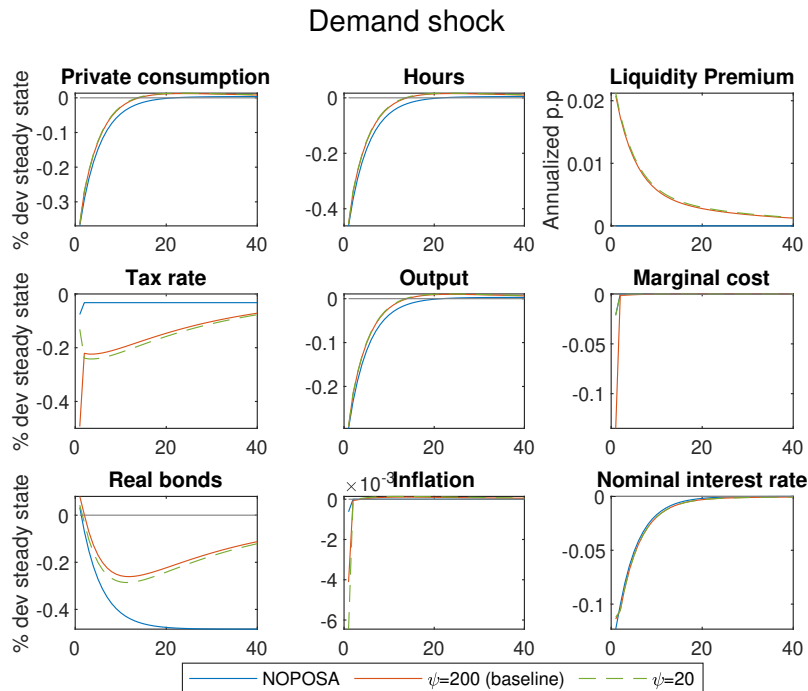


FIGURE 3.6: Degree of price stickiness and optimal inflation and tax rate volatility

Note: the figure shows the optimal response of the economy to a negative demand shock z_t reducing consumption in 1% in an economy without capital. The blue line stands for the economy under NOPOSA (standard model). The red line is the baseline economy with $\psi = 200$ and the green dashed line is the economy with $\psi = 20$ representing a slope for the Phillips curve of 0.3, ten times larger than the baseline economy

3.E.3 Extension: Economy with Taylor rule

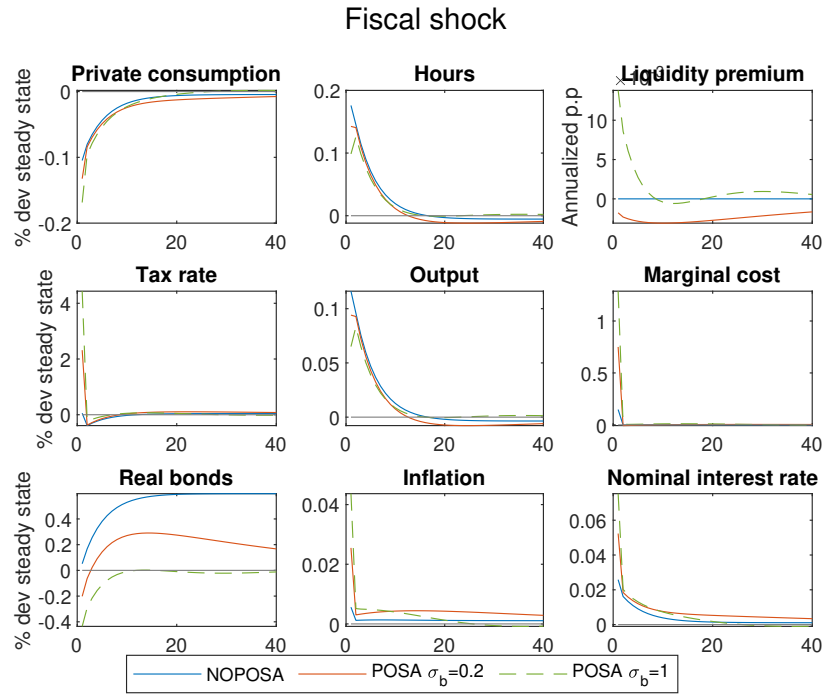


FIGURE 3.7: Optimal fiscal policy in response to a fiscal shock - Taylor rule

Note: the figure shows the optimal response of the economy to a government spending shock, increasing expenditures by 1% above the steady-state level in an economy without capital. Nominal interest rate follows a Taylor rule of the form $\tilde{R}_t = \phi_\pi \tilde{\pi}_t + \phi_y \tilde{y}_t$. The blue line is the economy with standard preferences. The red and green dashed line are the economy with POSA, $\sigma_b = 0.2$ and $\sigma_b = 1$

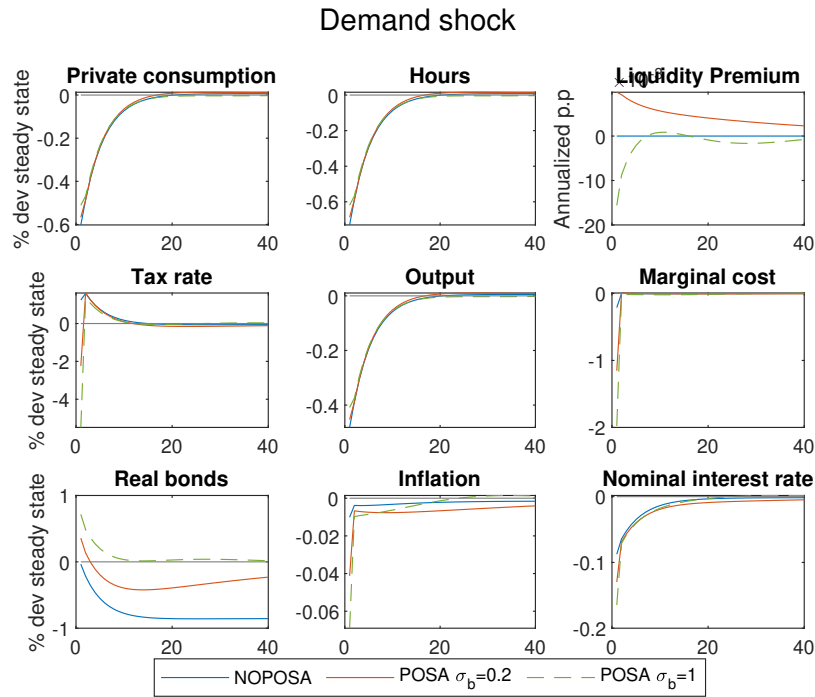


FIGURE 3.8: Optimal fiscal policy in response to a demand shock-Taylor rule

Note: the figure shows the optimal response of the economy to a negative demand shock z_t reducing consumption by 1% in an economy without capital. Nominal interest rate follows a Taylor rule, $R_t = \phi_\pi \tilde{\pi}_t + \phi_y \tilde{y}_t$. The blue line is the economy with standard preferences. The red and green dashed line are the economy with POSA, $\sigma_b = 0.2$ and $\sigma_b = 1$

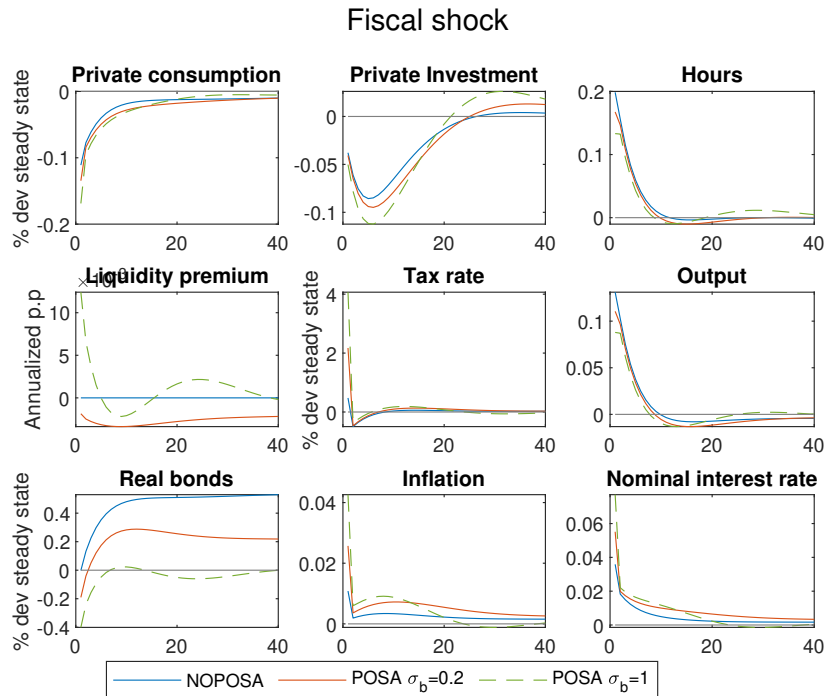


FIGURE 3.9: Optimal response to a fiscal shock: Taylor rule

Note: the figure shows the optimal response of the economy to a government spending shock, which increases expenditures by 1% above the steady-state level in an economy with private capital. Nominal interest rate follows a rule of the form $\tilde{R}_t = \phi_\pi \tilde{\pi}_t + \phi_y \tilde{y}_t$. The blue line is the economy with standard preferences. The red and green dashed line are the economy with POSA, $\sigma_b = 0.2$ and $\sigma_b = 1$

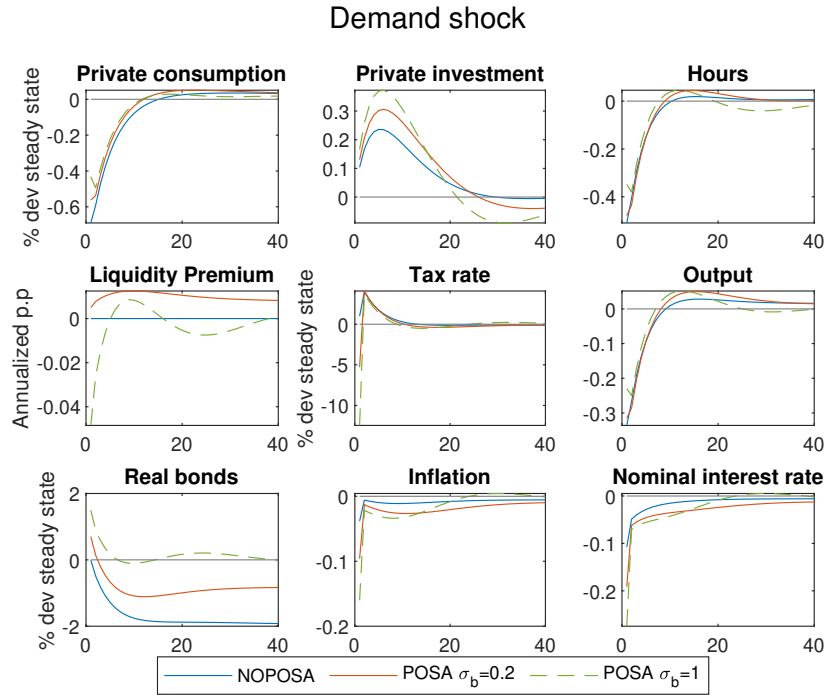
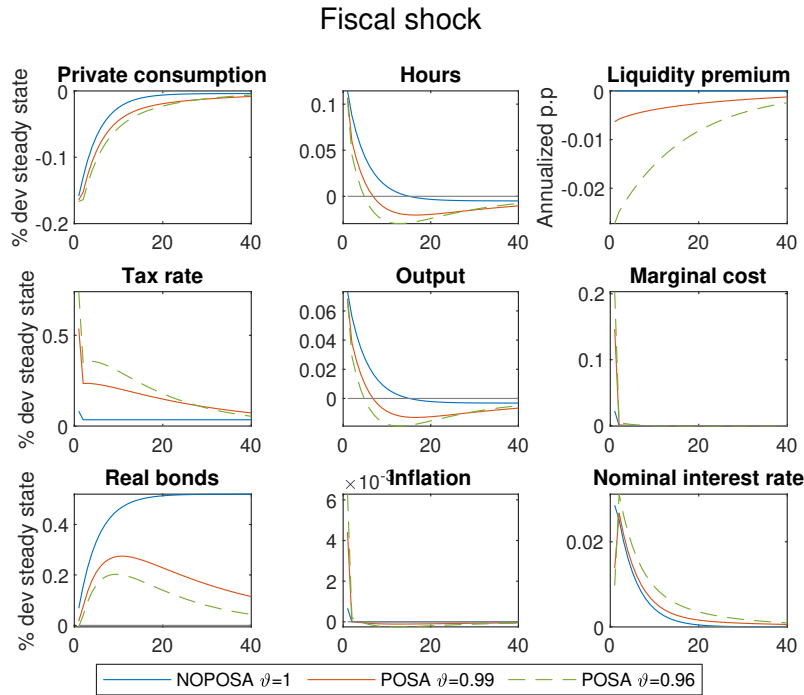


FIGURE 3.10: Optimal fiscal policy in response to a demand shock-Taylor rule

Note: the figure shows the optimal response of the economy to a negative demand shock z_t reducing consumption by 1% in an economy with capital. Nominal interest rate follows a Taylor rule, $R_t = \phi_\pi \tilde{\pi}_t + \phi_y \tilde{y}_t$. The blue line is the economy with standard preferences. The red and green dashed line are the economy with POSA, $\sigma_b = 0.2$ and $\sigma_b = 1$

3.E.4 Model with no capital and $\vartheta = 0.96$

FIGURE 3.11: Optimal monetary and fiscal response to a fiscal shock. $\vartheta = 0.99; 0.96$

Note: the figure shows the optimal response of the economy to a government spending shock, which increases expenditures by 1% above the steady-state level in an economy without private capital. The blue line is the economy with standard preferences. The red and green dashed line are the economy with POSA, $\vartheta = 0.99$ and $\vartheta = 0.96$

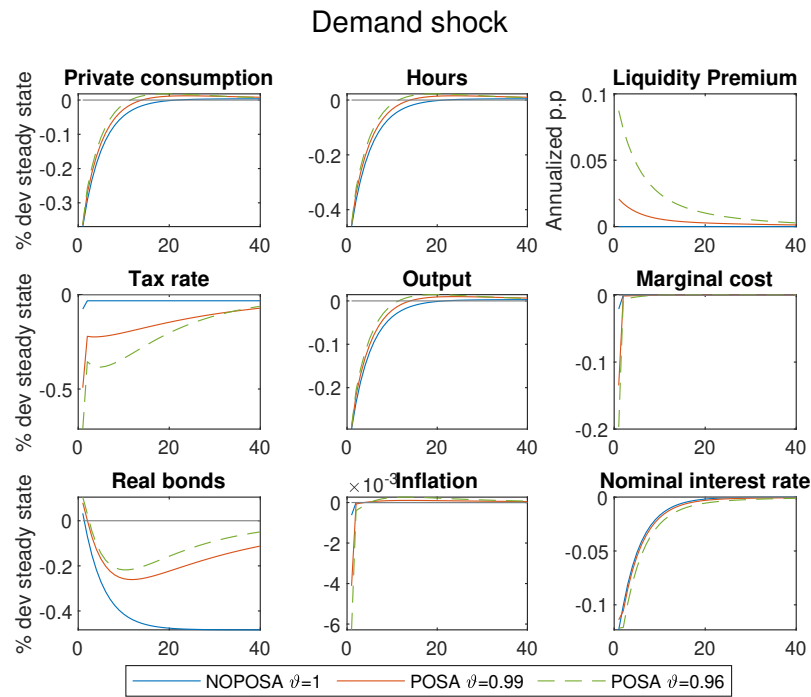


FIGURE 3.12: Optimal monetary and fiscal response to a demand shock $\vartheta = 0.99; 0.96$

Note: the figure shows the optimal response of the economy to a government spending shock, which reduces consumption by 1% below the steady-state level in an economy without private capital. The blue line is the economy with standard preferences. The blue line is the economy with standard preferences. The red and green dashed line are the economy with POSA, $\vartheta = 0.99$ and $\vartheta = 0.96$

Conclusions

This thesis proposes three different chapters to study optimal fiscal policy. Each one of them presents scenarios that are relevant for the design of fiscal policy particularly attending the recent episodes in the world economy such as the Great Recession and the COVID crisis. The nature and magnitude of such episodes tenses the boundaries for fiscal policy and challenges the traditional ways it should be conducted.

In the first chapter, I study the sovereign default problem in an environment with wealth and income inequality. Once affecting mostly emerging markets, sovereign default has been recently concerning also some advanced economies due to the quick rise in public debt levels observed after the Great Recession. In environments with wealth and income inequality, public debt eases some financial restrictions by providing a vehicle for saving, but it also raises the concerns for a default as it can be used to redistribute resources from rich to poor households. In this chapter, I show that the presence of externalities due to self interest maximization can exacerbate default risk and reduce overall welfare.

In environments with sticky prices, a government issuing nominal debt can be tempted to use inflation to dilute the value of debt when a shock that tenses its budget constraint arrives. This incentives grow when the central bank reacts weakly to inflation and taxes are perceived not enough to back up the fiscal plan. This is the environment considered in the second chapter of the thesis which aims to reflect the “zero lower bound” episode and the years before the COVID crisis where we observed a shy reaction of the policy rate. I show that when fiscal policy is optimal, there is little difference in the equilibrium outcomes regarding monetary policy response to inflation.

Finally, while the increase in public debt levels have confronted some economies with an scenario of augmented default risk, in other economies there has been little (if no any) concerns regarding debt sustainability. The “new role” for government bonds as a source of liquidity and safety has been considered by many to be behind this scenario. This environment can be seen convenient for the government because it collects revenues from those services, but at the same time it poses different challenges for fiscal policy. The third chapter of the thesis presents the tensions and tradeoffs for fiscal and monetary policy when government bonds are valued by households due to the services they provide.

Bibliography

- Adam, K. and Grill, M. (2017). Optimal sovereign default. *American Economic Journal: Macroeconomics*, 9(1):128–64.
- Aguiar, M., Amador, M., and Arellano, C. (2023). Micro risks and (robust) pareto improving policies. *mimeo*.
- Aiyagari, S., Marcet, A., Sargent, T., and Seppälä, J. (2002). Optimal taxation without state contingent debt. *Journal of Political Economy*, 110(6):1220–1254.
- Ambler, S. and Paquet, A. (1996). Fiscal spending shocks, endogenous government spending, and real business cycles. *Journal of Economic Dynamics and Control*, 20(1):237 – 256.
- Angeletos, G.-M., Collard, F., and Dellas, H. (2020). Public debt as private liquidity: Optimal policy. *mimeo*, pages 167–94.
- Arellano, C. (2008). Default risk and income fluctuations in emerging economies. *American Economic Review*, 98(3):690–712.
- Arellano, C., Mateos-Planas, X., and Rios-Rull, J.-V. (2023). Partial default. *Journal of Political Economy*, forthcoming.
- Azzimonti, M., Sarte, P.-D., and Soares, J. (2009). Distortionary taxes and public investment when government promises are not enforceable. *Journal of Economic Dynamics and Control*, 33(9):1662 – 1681.
- Bansal, R. and Coleman, W. J. (1996). A monetary explanation of the equity premium, term premium, and risk-free rate puzzles. *Journal of Political Economy*, 104(6):1135–1171.
- Bayer, C., Born, B., and Luetticke, R. (2020). The liquidity channel of fiscal policy. CEPR Discussion Papers 14883.
- Benigno, P. and Woodford, M. (2012). Linear-quadratic approximation of optimal policy problems. *Journal of Economic Theory*, 147(1):1 – 42.
- Berentsen, A. and Waller, C. (2018). Liquidity premiums on government debt and the fiscal theory of the price level. *Journal of Economic Dynamics and Control*, 89:173–182. Fed St. Louis-JEDC-SCG-SNB-UniBern Conference, titled: “Fiscal and Monetary Policies”.
- Bianchi, F. (2013). Regime switches, agents’ beliefs, and post-world war ii u.s. macroeconomic dynamics. *The Review of Economic Studies*, 80(2 (283)):463–490.

- Bianchi, F. and Ilut, C. (2017). Monetary/fiscal policy mix and agents' beliefs. *Review of Economic Dynamics*, 26:113–139.
- Bianchi, F. and Melosi, L. (2017). Escaping the great recession. *American Economic Review*, 107(4):1030–58.
- Bianchi, F. and Melosi, L. (2019). The dire effects of the lack of monetary and fiscal coordination. *Journal of Monetary Economics*, 104:1–22.
- Blake, A. P. and Kirsanova, T. (2012). Discretionary Policy and Multiple Equilibria in LQ RE Models. *The Review of Economic Studies*, 79(4):1309–1339.
- Bocola, L., Bornstein, G., and Dovis, A. (2019). Quantitative sovereign default models and the european debt crisis. *Journal of International Economics*, 118:20–30.
- Bonam, D. (2020). A convenient truth: The convenience yield, low interest rate and implications for fiscal policy. *Working paper - Central Bank of Netherlands*.
- Bouakez, H., Guillard, M., and Roulleau-Pasdeloup, J. (2017). Public investment, time to build, and the zero lower bound. *Review of Economic Dynamics*, 23:60 – 79.
- Bouakez, H., Guillard, M., and Roulleau-Pasdeloup, J. (2019). The optimal composition of public spending in a deep recession. *Journal of Monetary Economics*.
- Bouakez, H., Oikonomou, R., and Priftis, R. (2018). Optimal debt management in a liquidity trap. *Journal of Economic Dynamics and Control*, 93:5–21. Monetary and Fiscal Policy Stabilization amid a Debt Crisis.
- Broner, F., Martin, A., and Ventura, J. (2010). Sovereign risk and secondary markets. *American Economic Review*, 100(4):1523–55.
- Campbell, J. R., Fisher, J. D. M., Justiniano, A., and Melosi, L. (2017). Forward guidance and macroeconomic outcomes since the financial crisis. *NBER Macroeconomics Annual*, 31:283–357.
- Canzoneri, M., Cumby, R., and Diba, B. (2016). Optimal money and debt management: Liquidity provision vs tax smoothing. *Journal of Monetary Economics*, 83:39–53.
- Carroll, D. (2000). Why do the rich save so much? *The Economic Consequences of Taxing the Rich*, Harvard University Press.
- Carvalho, Carlos, F. N. T. T. (2019). Taylor rule estimations by ols. *Federal Reserve Bank of San Francisco Working Paper*.
- Chafwehé, B., de Beaufort, C., and Oikonomou, R. (2022). Optimal monetary policy rules in the fiscal theory of the price level. *mimeo*.
- Chari, V. and Kehoe, P. J. (1999). Chapter 26 optimal fiscal and monetary policy. volume 1 of *Handbook of Macroeconomics*, pages 1671–1745. Elsevier.

- Christiano, L., Eichenbaum, M., and Evans, C. (2005). Nominal rigidities and the dynamic effects of a shock to monetary policy. *Journal of Political Economy*, 113(1):1–45.
- Christiano, L., Eichenbaum, M., and Rebelo, S. (2011). When is the government spending multiplier large? *Journal of Political Economy*, 119(1):78–121.
- Cochrane, J. H. (2001). Long-term debt and optimal policy in the fiscal theory of the price level. *Econometrica*, 69(1):69–116.
- Cochrane, J. H. (2021). The fiscal roots of inflation. *Review of Economic Dynamics*.
- Coenen, G., Straub, R., and Trabandt, M. (2013). Gauging the effects of fiscal stimulus packages in the euro area. *Journal of Economic Dynamics and Control*, 37(2):367 – 386. Fiscal Policy in the Aftermath of the Financial Crisis.
- Damodaran, A. (2015). Equity risk premiums (erp): Determinants, estimation and implications – the 2015 edition. *SSRN*, (6).
- Davila, J., Hong, J. H., Krusell, P., and Ríos-Rull, J.-V. (2012). Constrained efficiency in the neo-classical growth model with uninsurable idiosyncratic shocks. *Econometrica*, 80(6):2431–2467.
- D’Erasmus, P. and Mendoza, E. (2021). History remembered: Optimal sovereign default on domestic and external debt. *Journal of Monetary Economics*, 117:969–989.
- D’Erasmus, P. and Mendoza, E. G. (2016). Distributional incentives in an equilibrium model of domestic sovereign default. *Journal of the European Economic Association*, 14(1):7–44.
- Di Casola, P. and Sichlimiris, S. (2014). Domestic and external sovereign debt. *Sveriges Riskbank Working papers*, (345).
- Diamond, P. A. (1967). The role of a stock market in a general equilibrium model with technological uncertainty. *The American Economic Review*, 57(4):759–776.
- Dräutzbürg, T. and Uhlig, H. (2015). Fiscal stimulus and distortionary taxation. *Review of Economic Dynamics*, 18(4):894 – 920.
- Eaton, J. and Gersovitz, M. (1981). Debt with potential repudiation: Theoretical and empirical analysis. *The Review of Economic Studies*, 48(2):289–309.
- Eggertsson, G. (2011). What fiscal policy is effective at zero interest rates? *NBER Macroeconomics Annual*, 2010, 25:59–112.
- Erce, A. and Mallucci, E. (2018). Selective sovereign default. *International Finance Discussion Papers*, (1239).
- Faraglia, E., Marcet, A., Oikonomou, R., and Scott, A. (2013). The impact of debt levels and debt maturity on inflation. *The Economic Journal*, 123(566):F164–F192.
- Faraglia, E., Marcet, A., Oikonomou, R., and Scott, A. (2016). Long-term government bonds. *mimeo*.

- Faraglia, E., Marcet, A., Oikonomou, R., and Scott, A. (2018). Government Debt Management: The Long and the Short of It. *The Review of Economic Studies*, 86(6):2554–2604.
- Faraglia, E., Marcet, A., and Scott, A. (2012). Debt management and optimal fiscal policy with long bonds. *BIS Paper Chapters*, in “Threat of Fiscal Dominance”, 65:177–212.
- Ferriere, A. (2014). Sovereign default, inequality and progressive taxation. *Mimeo*.
- Glomm, G. and Ravikumar, B. (1994). Public investment in infrastructure in a simple growth model. *Journal of Economic Dynamics and Control*, 18(6):1173 – 1187.
- Gottardi, P., Kajii, A., and Nakajima, T. (2016). Constrained inefficiency and optimal taxation with uninsurable risks. *Journal of Public Economic Theory*, 18(1):1–28.
- Greenwood, R., Hanson, S. G., and Stein, J. C. (2015). A comparative-advantage approach to government debt maturity. *Journal of Finance*, LXX(4):1683–1722.
- Guembel, A. and Sussman, O. (2009). Sovereign Debt without Default Penalties. *The Review of Economic Studies*, 76(4):1297–1320.
- Guibaud, S., Nosbusch, Y., and Vayanos, D. (2013). Bond market clienteles, the yield curve, and the optimal maturity structure of government debt. *The Review of Financial Studies*, 26(8):1913–1961.
- Hagedorn, M. (2018). Prices and inflation when government bonds are net wealth. *mimeo*.
- Hansen, J. (2018). Optimal monetary policy with capital and a financial accelerator. *Journal of Economic Dynamics and Control*, 92:84 – 102.
- Jeon, K. and Kabukcuoglu, Z. (2018). Income inequality and sovereign default. *Journal of Economic Dynamics and Control*, 95:211 – 232.
- Jeske, K. (2006). Private international debt with risk of repudiation. *Journal of Political Economy*, 114(3):576–593.
- Jia, P. (2020). The macroeconomic impact of monetary-fiscal policy in a “fiscal dominance” world. *Macroeconomic Dynamics*, 24(3):670–707.
- Kaplan, G. and Violante, G. L. (2018). Microeconomic heterogeneity and macroeconomic shocks. *Journal of Economic Perspectives*, 32(3):167–94.
- Kim, Y. J. and Zhang, J. (2012). Decentralized borrowing and centralized default. *Journal of International Economics*, 88(1):121 – 133.
- Klein, P., Krusell, P., and Ríos-Rull, J.-V. (2008). Time-Consistent Public Policy. *The Review of Economic Studies*, 75(3):789–808.
- Krishnamurthy, A. and Vissing-Jorgensen, A. (2012). The aggregate demand for treasury debt. *Journal of Political Economy*, 120(2):233–267.

- Kydland, F. E. and Prescott, E. C. (1982). Time to build and aggregate fluctuations. *Econometrica*, 50(6):1345–1370.
- Leeper, E. M. (1991). Equilibria under ‘active’ and ‘passive’ monetary and fiscal policies. *Journal of Monetary Economics*, 27(1):129–147.
- Leeper, E. M. and Leith, C. (2016). Understanding inflation as a joint monetary-fiscal phenomenon. Working Paper 21867, National Bureau of Economic Research.
- Leeper, E. M., Leith, C., and Liu, D. (2021a). Optimal time-consistent monetary, fiscal and debt maturity policy. *Journal of Monetary Economics*, 117:600–617.
- Leeper, E. M., Leith, C., and Liu, D. (2021b). Optimal time-consistent monetary, fiscal and debt maturity policy. *Journal of Monetary Economics*, 117:600–617.
- Leeper, E. M., Walker, T. B., and Yang, S.-C. S. (2010). Government investment and fiscal stimulus. *Journal of Monetary Economics*, 57(8):1000 – 1012.
- Leith, C. and Wren-Lewis, S. (2013). Fiscal sustainability in a new keynesian model. *Journal of Money, Credit and Banking*, 45(8):1477–1516.
- Lustig, H., Sleet, C., and Şevin Yeltekin (2008a). Fiscal hedging with nominal assets. *Journal of Monetary Economics*, 55(4):710 – 727.
- Lustig, H., Sleet, C., and Şevin Yeltekin (2008b). Fiscal hedging with nominal assets. *Journal of Monetary Economics*, 55(4):710–727.
- Mallucci, E. (2021). Domestic debt and sovereign defaults. *Journal of Money, Credit and Banking*.
- Mankart, J., Oikonomou, R., and Priftis, R. (2022). The long and the short of financing government spending. *mimeo*.
- Matveev, D. (2021). Time-consistent management of a liquidity trap with government debt. *Journal of Money, Credit and Banking*, 53(8):2129–2165.
- Mehra, Y. P. and Minton, B. (2007). A taylor rule and the greenspan era. *FRB Richmond Economic Quarterly*, 93(3):229–250.
- Michaillat, P. and Saez, E. (2021). Resolving New Keynesian Anomalies with Wealth in the Utility Function. *The Review of Economics and Statistics*, 103(2):197–215.
- Nicolò, G. (2020). Monetary policy, expectations and business cycles in the u.s. post-war period. *mimeo*.
- Nosbusch, Y. (2008). Interest costs and the optimal maturity structure of government debt. *The Economic Journal*, 118(527):477–498.
- Olivares-Caminal, R. (2013). The pari passu clause on sovereign debt instruments: developments in recent litigation. *Tech. rep., Bank for International Settlements Paper No. 72, 72*.

- Ottonello, P. and Perez, D. J. (2019). The currency composition of sovereign debt. *American Economic Journal: Macroeconomics*, 11(3):174–208.
- Park, Y. (2018). Constrained efficiency in a human capital model. *American Economic Journal: Macroeconomics*, 10(3):179–214.
- Perez, D. (2018). Sovereign debt, domestic banks and the provision of public liquidity. *mimeo*.
- Ramey, V. (2020). The macroeconomic consequence of infrastructure investment. *NBER*.
- Rannenberg, A. (2019). Forward guidance with preferences over safe assets. *National Bank of Belgium*, 364.
- Rannenberg, A. (2021). State-dependent fiscal multipliers with preferences over safe assets. *Journal of Monetary Economics*, 117:1023–1040.
- Reis, R. (2022). Has monetary policy cared too much about a poor measure of r^* ? *mimeo*.
- Rotemberg, J. J. (1982). Sticky prices in the united states. *Journal of Political Economy*, 90(6):1187–1211.
- Schmitt-Grohé, S. and Uribe, M. (2004). Optimal fiscal and monetary policy under sticky prices. *Journal of Economic Theory*, 114(2):198–230.
- Sims, C. A. (2011). Stepping on a rake: The role of fiscal policy in the inflation of the 1970s. *European Economic Review*, 55(1):48–56. Special Issue on Monetary and Fiscal Interactions in Times of Economic Stress.
- Smets, F. and Wouters, R. (2007). Shocks and frictions in us business cycles: A bayesian dsge approach. *American Economic Review*, 97(3):586–606.
- Sunder-Plassmann, L. (2018). Inflation, default, and sovereign debt: The role of denomination and ownership. *Mimeo*.
- Vayanos, D. and Vila, J. (2021). A preferred-habitat model of the term structure if interest rates. *Econometrica*, 89(1):77–112.
- Woodford, M. (2001). Fiscal requirements for price stability. *Journal of Money, Credit and Banking*, 33(3):669–728.
- Woodford, M. (2011). Simple analytics of the government expenditure multiplier. *American Economic Journal: Macroeconomics*, 3(1):1–35.
- Wright, M. L. (2006). Private capital flows, capital controls, and default risk. *Journal of International Economics*, 69(1):120 – 149. Emerging Markets.

