Structured Illumination and Variable Density Sampling for Compressive Fourier Transform Interferometry

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***Abstract*—Fourier Transform Interferometry (FTI) is an appealing Hyperspectral (HS) imaging modality for many applications demanding high spectral resolution, *e.g.*, in fluorescence microscopy. However, the effective resolution of FTI is limited by the durability (or photobleaching) of biological elements when exposed to illuminating light. We propose two variants of the FTI imager, *i.e.*, Coded Illumination-FTI (CI-FTI) and Structured Illumination FTI (SI-FTI), based on the theory of compressive sensing (CS). These schemes efficiently modulate light exposure temporally (in CI-FTI) or spatiotemporally (in SI-FTI). Leveraging a variable density sampling strategy recently introduced in CS, we provide optimal illumination strategies, so that the light exposure imposed on a biological specimen is minimized while the spectral resolution is preserved. In particular, under certain conditions, the biological HS volumes can be reconstructed with a three-to-ten-fold reduction in the light exposure.**

In a nutshell, the operating principle of a conventional FTI, called hereafter *Nyquist FTI*, is based on Michelson interferometry [1]. As shown in Fig. 1 (top), a parallel beam of coherent light obtained from the 2D image of a thin, still, and almost transparent biological specimen magnified by a (confocal) microscope (not represented here), is divided into two beams by a Beam-Splitter (BS), *e.g.*, made of two birefringent prisms. The two beams are reflected back by a fixed and a moving mirror (controlling the optical path difference, or OPD, between the beams), and then recombined by the BS. The intensity of the interfering beams is next acquired by a standard imaging sensor (camera); each image captured by this camera corresponds to a given OPD, and each pixel records a specific interference pattern over the time domain.

In a simplified discrete setting with $N_{\rm p}$ spatial locations (or camera pixels) and N_ξ OPD samples (achieved from a time sampling of the mirror motion), if $\boldsymbol{Y}^{\rm nyq} \in \mathbb{R}^{N_\xi \times N_{\rm p}}$ and $\boldsymbol{X} \in \mathbb{R}^{N_\xi \times N_{\rm p}}$ denote the matrix *unfolding* of the cubes representing the interference patterns and the hyperspectral volume, respectively, physical optics shows that the (noiseless) acquisition process of Nyquist-FTI can be formulated in matrix form as $\boldsymbol{y}^{\rm nyq} = \boldsymbol{\Phi}^*\boldsymbol{x}$, where $\boldsymbol{\Phi} := \boldsymbol{I}_{N_{\rm p}} \otimes \boldsymbol{F}$, "$\otimes$" is the Kronecker product, $\boldsymbol{F} \in \mathbb{C}^{N_\xi \times N_\xi}$ is the 1D discrete Fourier transform, $\boldsymbol{y}^{\rm nyq} := {\rm vec}(\boldsymbol{Y}^{\rm nyq})$, and $\boldsymbol{x} := {\rm vec}(\boldsymbol{X})$ [2].

In this work, we study two different compressive sensing strategies for FTI. In coded illumination FTI (CI-FTI), we consider an easily integrable temporal coding of the specimen illumination, *e.g.*, by programming the global activation of the light source. In structured illumination FTI (SI-FTI), we adopt a more complex scheme that spatially structures the illumination of the specimen, *e.g.*, thanks to a spatial light modulator (SLM). Both acquisition systems can be modeled as

$$\boldsymbol{y} = \boldsymbol{\Phi}_\Omega^*\boldsymbol{x} + \boldsymbol{n},$$

where $\boldsymbol{\Phi}_\Omega$ stands for a selection of the $M = |\Omega|$ columns of $\boldsymbol{\Phi}$ indexed by the multiset $\Omega \subset [N]$ ($N := N_{\rm p}N_\xi$). In CI-FTI, by construction, $\Omega = \Omega_\xi \times [N_{\rm p}]$, where $\Omega_\xi \subset [N_\xi]$ selects the M_ξ OPD samples where the light source is active; all spatial locations share thus the same OPD subsampling and $M = M_\xi N_{\rm p}$. This limitation is removed in SI-FTI by allowing for a general subsampling of $[N]$.

Our work leverages the variable sampling strategy of Krahmer and Ward [3] to optimize the subsampling multiset Ω with respect to both the sensing basis $\boldsymbol{\Phi}$ and a sparsity basis $\boldsymbol{\Psi}$ of $\boldsymbol{x}$. In particular, for CI-FTI, if $\boldsymbol{\Psi} = \boldsymbol{I}_{N_{\rm p}} \otimes \boldsymbol{\Psi}_{\rm 1D}$ (with $\boldsymbol{\Psi}_{\rm 1D}$ the Haar wavelet basis) then Ω_ξ can be randomly generated by gathering M_ξ independent draws of a random variable $\beta_\xi \in [N_\xi]$ whose *pmf* is inversely proportional to the distance to the OPD origin. For SI-FTI, with $\boldsymbol{\Psi} = \boldsymbol{\Psi}_{\rm 2D} \otimes \boldsymbol{\Psi}_{\rm 1D}$ combining $\boldsymbol{\Psi}_{1D}$ with the 2D Haar wavelet transform $\boldsymbol{\Psi}_{\rm 2D}$, Ω is generated from M independent draws of the bivariate random variable $(\beta_{\rm p}, \beta_\xi)$, with $\beta_{\rm p}$ uniform in the spatial domain.

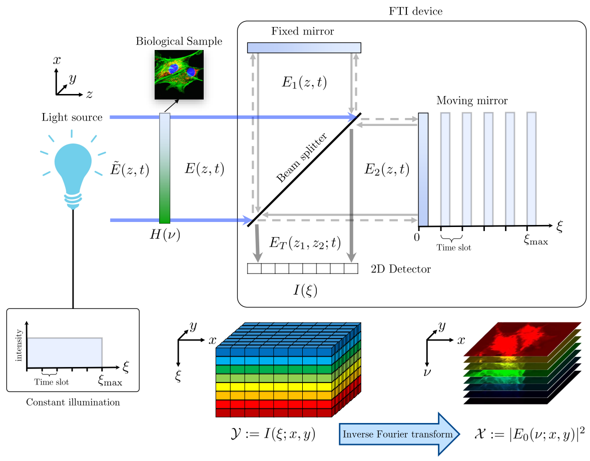

Fig. 1: Operating principle of Fourier transform interferometry.

An accurate estimate $\hat{\boldsymbol{x}}$ of the HS volume $\boldsymbol{x}$ is obtained from the solution of a Basis Pursuit DeNoise algorithm [4], whose ℓ_2-fidelity term is weighted according to the previous pmfs [3]. In particular, if $\boldsymbol{x}$ is well represented by a K-sparse approximation in $\boldsymbol{\Psi}$, provided that $M = O(K)$ (up to log factors in the dimensions), with high probability, $\|\boldsymbol{x} - \hat{\boldsymbol{x}}\|$ is bounded by the addition of a sparsity error term and the noise power estimated in the specific weighted-norm imposed by the VDS scheme (for which we provide novel bounds). For CI-FTI, the sparsity error term integrates the $N_{\rm p}$ best $(K/N_{\rm p})$-term ℓ_1-approximation errors of the N_p spectra in $\boldsymbol{\Psi}_{\rm 1D}$; and SI-FTI simply uses the best K-term ℓ_1-approximation error of $\boldsymbol{x}$ in $\boldsymbol{\Psi}$.

Our work then takes advantage of this error bound to first study a trade-off between exposure intensity and the quality of the reconstructed HS volume for a given spectral resolution; and, second, to maximize the HS volume quality for a fixed spectral resolution and constrained light exposure budget. Our predictions are conclusively supported by experiments on synthetic data, and on interferometric observations acquired under various illumination conditions with an actual FTI device on which post-acquisition subsamplings were applied.

References

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