

Contractually Stable Alliances

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Abstract

We analyze how different rules for exiting an alliance affect the formation of strategic alliances. We adopt the concept of contractual stability to predict the alliances that are going to emerge in the long run. We find that any asymmetric alliance structure consisting of two alliances is contractually stable under the unanimity decision rule. In addition, the grand alliance which is the efficient structure is stable. If we allow for side payments to compensate former partners, then some less efficient structures that were stable without side payments are no more stable. Moreover, we show that the stability of alliances under the unanimity rule to exit is robust to the type of firms, myopic or farsighted. Finally, there is no contractually stable alliance structure under the simple majority decision rule.

Key words: Strategic alliances; Coalition formation; Contractual stability; Farsightedness; Exit rules.

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1 Introduction

A common practice for firms is to pool their expertise in partnerships such as joint ventures and strategic alliances.¹ Strategic alliances refer to agreements characterized by the commitment of two or more firms to reach a common goal entailing the pooling of their resources and activities. The formation of an alliance usually creates negative externalities for nonmembers and alliance agreements contain mechanisms to regulate exit. Three rules of exit are commonly used in alliances: (i) exit without breach via a deadlock implemented by the contractual board where only unanimous decisions are taken (a unanimity decision rule),² (ii) exit via breach of the agreement subject to damages (a unanimity decision rule with side payments), (iii) exit at the will of the larger party subject to forewarning (a simple majority decision rule).³ We analyze how different rules for exiting an alliance will affect the formation of strategic alliances.

We adopt the concept of contractual stability to predict the alliances that are going to emerge at equilibrium. A new partner enters an alliance only if she wishes to come in, her new partners wish to accept her, and she obtains the consent from her former partners to withdraw if she was before member of another alliance. Under the unanimity rule, she needs the consent of all members of her initial alliance. Under the simple majority rule, she needs the consent of at least half of the members of her initial alliance. Under the unanimity rule with side payments, she still needs the unanimous consent of her former partners but she can now make side payments to her former partners in order to reach their approval. Side payments can only be made to members of the initial alliance she wants to leave.

We investigate in Bloch's (1995) model of associations of firms whether requiring the consent of former partners may help to sustain the emergence of more efficient alliances in the long run. We find that any asymmetric alliance structure consisting of two alliances is contractually stable under the unanimity decision rule. Requiring the consent of all partners to exit reverts to give each partner a veto power to any change made to the alliance. As a consequence, from any asymmetric alliance structure consisting of two alliances, we have that (i) any deviation where an alliance is divided in two or more

¹Hagedoorn (2002) has reported that in 2000 there were 199 strategic alliances in the biotechnology industry out of 575 strategic alliances counted overall, making biotechnology the first industry in the ranking followed by the information technology (184 alliances) and automotive (53 alliances). See also Baker, Gibbons and Murphy (2008). However, Link and Bauer (1989) have reported that the number of alliances and the number of firms forming an alliance varies widely across industries and research projects.

²The contractual board monitors the alliance activities and shapes ongoing developments. Representatives of each partner belong to the contractual board and unanimity is the norm for taking decisions. Alliances cannot be easily dissolved and deadlocks are usually solved thanks to a dispute resolution mechanism. See Smith (2005).

³Forewarning is usually required for an alliance agreement to be terminable at the will of the larger party. For instance, the Exclusive License and Collaboration Agreement between MedImmune Inc. and Critical Therapeutics Inc. (July 30, 2003) provided that MedImmune had the right to terminate on six months notice. See Smith (2005).

alliances is blocked, (ii) any deviation to the grand alliance is blocked, and (iii) any deviation where some members of the smallest alliance leave their alliance to join the largest one is blocked. In addition, the grand alliance which is the efficient structure (i.e. the structure that maximizes social surplus) is stable. If we allow for side payments only among former partners in addition to the unanimity rule, then some less efficient structures that were stable without side payments are no more stable. The grand alliance remains contractually stable. Moreover, we find that the grand alliance and all asymmetric alliance structures are contractually stable under the unanimity decision rule when firms are farsighted in the sense that they forecast how others might react to their decisions. Hence, the stability of alliances under the unanimity rule to exit is robust to the type of firms, myopic or farsighted.⁴

We find that there is no contractually stable alliance structure under the simple majority decision rule when the industry consists of more than seven firms. Hence, our analysis shows that in a coalition formation game with negative externalities, the simple majority rule to exit fails to stabilize the efficient coalition structure. However, in a coalition formation game with positive externalities, like the formation of cartels on oligopolistic markets, the simple majority rule achieves the stability of the efficient structure. The intuition behind this result has to do with the size of the deviating coalitions. In the formation of strategic alliances, large coalitions have incentives to deviate. The simple majority rule is unsuccessful in blocking such deviations because the former partners of the deviating firms are a minority in their previous alliances. In the formation of cartels, small coalitions have incentives to deviate, and so the simple majority rule allows the former partners of the deviating firms to block deviations.

Finally, we show that different rules of exit may coexist in different alliances at equilibrium. For instance, the asymmetric alliance structure where half-plus-one of the total number of firms are in one alliance and all other firms are in a second alliance is contractually stable when simple majority is in effect in the first alliance while unanimity is in effect in the second alliance.

Joining an alliance requires the unanimous consent of all current members of the alliance. Obviously, a different rule for joining an alliance could lead to different predictions about stable alliance structures. In the open membership game, all firms simultaneously announce a message and alliances are then formed by all firms who have announced the same message. In fact, any firm can form an alliance with another firm simply by announcing the same message. Yi (1997) has shown that the only Nash equilibrium of the open membership game is the grand alliance.⁵ However, there is no strong Nash equi-

⁴These results are coherent with properties of associations of firms that we observe in practice. For instance, Bekkers, Duysters and Verspagen (2002) have reported that an alliance of five firms (Ericsson, Nokia, Siemens, Motorola and Alcatel) dominated the market for GSM infrastructures and terminals in the 1990s. This alliance had a total market share of 85 percent in 1996 thanks to having made their GSM patents available to their partners and unavailable to the rest of firms (Phillips, Bull and Telia).

⁵Belleflamme (2000) has studied games of coalition formation with open membership where firms form

librium of the open membership game. Bloch (1995) has proposed a sequential game for forming associations of firms. One firm proposes an alliance. All the prospective members of the alliance respond in turn to the offer. If all the firms accept to join the alliance, the proposed alliance is formed and all members benefit from the reduction in marginal cost. If one of the firms does not accept to join the alliance, the proposed alliance is not formed and the firm that did not accept becomes the firm that makes a proposal in the next period. Bloch has shown that in equilibrium, firms form two asymmetric alliances, with the largest one comprising roughly three-quarters of industry members.⁶

The paper is organized as follows. In Section 2 we set up the model. In Section 3 we introduce the notion of contractual stability under the unanimity decision rule and we characterize the stable alliance structures. In Section 4 we introduce side payments. In Section 5 we study the stable alliance structures when firms are farsighted. In Section 6 we first deal with the simple majority decision rule. We next allow different alliances to use different rules for consent and we discuss the role of positive versus negative externalities. Finally, we conclude and give some directions for further research.

2 Strategic Alliances of Firms

Cooperation among competing firms is increasingly common in oligopolistic markets. More and more often, competing firms agree to share information, build common facilities or launch common research programmes in order to decrease their production costs. Bloch (1995) proposed a simple model to analyze the formation of alliances of firms where the benefits from cooperation increase linearly in the size of the alliance.

Consider a market with n symmetric firms indexed by $i = 1, 2, \dots, n$, where $n \geq 4$. The interactions among firms are modelled as a two-stage game. In stage one, alliances are formed. In stage two, given the alliance structure, firms compete on the market. Once alliances are formed, firms behave as competitors on the market and maximize individual profits. Demand is linear and given by $p = \alpha - \sum_{i=1}^n q_i$, where α measures the absolute size of the market. Firms have a constant marginal cost of production, which is decreasing in the size of the alliance they belong to. Formally, the cost of a firm i in an alliance A of size a is given by $c_i = \lambda - \mu a$. The parameters α , λ and μ are chosen in such a way that, for any alliance structure, all firms are active in a Cournot equilibrium. Once alliances are formed on the market, firms select non-cooperatively the quantities they

associations or alliances in order to decrease their costs before competing on the market. Once firms are asymmetric, the grand alliance may not be a Nash equilibrium outcome of the game. See also Yi and Shin (2000).

⁶Espinosa and Macho-Stadler (2003) have analyzed the formation of associations of firms when effort to reduce costs is not verifiable. Roketskiy (2012) has developed a model of collaboration between farsighted firms competing in a tournament. Firms collaborate in pairs, and a structure of collaboration is represented by a network. He has found that farsightedly stable networks consist of two asymmetric mutually disconnected complete components.

offer on the market. Let $N = \{1, 2, \dots, n\}$ be the set of firms. An alliance structure $S = \{A_1, A_2, \dots, A_m\}$ is a partition of the set of firms such that $A_i \cap A_j = \emptyset$ for $i \neq j$ and $\bigcup_{i=1}^m A_i = N$. Let s be the cardinality of S (i.e. the number of alliances in S). Let $A(i) \in S$ be the alliance to which firm i belongs. An alliance structure A is symmetric if and only if $a_i = a_j$ for all $A_i, A_j \in S$, where a_i is the size of alliance A_i . Let $S^* = \{N\}$ be the grand alliance.

For any given alliance structure $S = \{A_1, A_2, \dots, A_m\}$, one can easily show that there exists a unique Cournot equilibrium on the market, and that each firm's profit $\Pi_i(S)$ is a monotonically increasing function of the following valuation,

$$V_i(S) = \alpha - \lambda + \mu (n + 1) a(i) - \mu \sum_{j=1}^m (a_j)^2,$$

where $a(i)$ denotes the size of the alliance firm i belongs to. In fact, $V_i(S) = (n + 1) \sqrt{\Pi_i(S)}$.

3 Contractual Stability under Unanimity

A simple way to analyze the alliances that one might expect to emerge in the long run is to examine an equilibrium requirement that no group of firms benefits from altering the alliance structure. What about possible deviations? An alliance structure S' is obtainable from S via A , $A \subseteq N$, if (i) $\{A'_i \in S' \mid A'_i \subseteq N \setminus A\} = \{A_i \setminus A \mid A_i \in S, A_i \setminus A \neq \emptyset\}$ and (ii) $\exists \{A'_1, \dots, A'_k\} \subseteq S'$ such that $\bigcup_{j=1}^k A'_j = A$. Condition (i) means that if the firms in A leave their respective alliance(s) in S , the non-deviating firms do not move. Condition (ii) allows the deviating firms in A to form one or several alliances in the new alliance structure S' . Non-deviating firms do not belong to those new alliances.

Once identified all possible deviations from an existing alliance structure, different stability concepts could be studied. We adopt the concept of contractual stability under unanimity to predict the alliances that are going to emerge at equilibrium. As in Drèze and Greenberg (1980), we assume that alliances are contracts binding all members and that modifying an alliance requires the unanimous consent of the members of the alliance. That is, a new partner will enter an alliance only if she wishes to come in, her new partners wish to accept her, and she obtains from her former partners permission to withdraw. The unanimity decision rule reflects an existing rule of exit in alliances: exit without breach via a deadlock implemented by the contractual board where only unanimous decisions are taken.⁷

Definition 1. An alliance structure S is contractually stable under the unanimity decision rule if for any $A \subseteq N$, S' obtainable from S via A and $i \in A$ such that $V_i(S') > V_i(S)$, there exists $k \in A(j)$ with $A(j) \in S$ and $j \in A$ such that $V_k(S') \leq V_k(S)$.

⁷In Section 6 we deal with the case of the simple majority decision rule where deviating firms need the consent of more than half members of each initial alliance of the deviating firms.

Under the unanimity decision rule, the move from an alliance structure S to any obtainable alliance structure S' needs the consent of every deviating firm and the consent of every member of the initial alliances of the deviating firms. Then, an alliance structure is contractually stable under the unanimity decision rule if any deviating firm or any member of the former alliances of the deviating firms is not better off from the deviation to any obtainable alliance structure S' .

From Proposition 2 in Bloch (1995) we have that any two alliances always have incentives to merge if both alliance sizes are smaller than $(n+1)/2$. That is, $V_i(S \setminus \{A_1, A_2\} \cup \{A_1 \cup A_2\}) > V_i(S)$ for all $i \in A_1 \cup A_2$ if $a_1 < (n+1)/2$ and $a_2 < (n+1)/2$. Hence, alliance structures consisting of more than two alliances cannot be contractually stable whatever the decision rule for consent since there always exists two alliances having a size smaller than $(n+1)/2$ and this merger does not request the consent of any other firm than those involved in the merger. Moreover, any symmetric alliance structure consisting of two alliances cannot be contractually stable since both alliances have a size smaller than $(n+1)/2$. Therefore, the only candidates for being contractually stable under unanimity are asymmetric alliance structures consisting of two alliances and the grand alliance structure.

We now show that, if the exit of an alliance requires the consent of all members of the alliance, then any asymmetric alliance structure consisting of two alliances is contractually stable. Lemma 1 tells us that, from any alliance structure consisting of two or more alliances, any deviation where an alliance is divided in two or more alliances is blocked under the unanimity rule.

Lemma 1. *Take any alliance structure $S \neq S^*$. Then, any deviation from S to S' where an alliance $A \in S$ is divided into two (or more) alliances is blocked under the unanimity decision rule.*

Proofs can be found in the appendix. Lemma 2 tells us that, from any asymmetric alliance structure consisting of two alliances, any deviation to the grand alliance structure is blocked under the unanimity rule.

Lemma 2. *Take any asymmetric alliance structure S with $s = 2$. Then, any deviation from S to S^* is blocked under the unanimity decision rule.*

Lemma 3 tells us that, from any asymmetric alliance structure consisting of two alliances, any deviation where some members of the smallest alliance leave their alliance to join the largest alliance is blocked under the unanimity rule.

Lemma 3. *Take any asymmetric alliance structure $S = \{A, N \setminus A\}$ with $a \leq n-1$. Then, any deviation from $S = \{A, N \setminus A\}$ to $S' = \{A \cup B, (N \setminus A) \setminus B\}$ with $B \subset N \setminus A$ is blocked under the unanimity decision rule.*

Using Lemma 1 to Lemma 3 we have that any asymmetric alliance structure consisting of two alliances is contractually stable under the unanimity decision rule since all possible profitable deviations are blocked.

Proposition 1. *Any asymmetric alliance structure S such that $s = 2$ is contractually stable under the unanimity decision rule.*

Moreover, once the exit of an alliance requires the consent of all members of the alliance, the grand alliance which is the efficient structure becomes contractually stable. Indeed, any deviation from S^* to any S is blocked by at least one member of N who will be worse off in S than in S^* .

Proposition 2. *The grand alliance $S^* = \{N\}$ is always contractually stable under the unanimity decision rule.*

We now compare the outcomes obtained under the notion of contractual stability with those obtained under a sequential game of coalition formation proposed by Bloch (1996).⁸ A fixed protocol is assumed and the sequential game proceeds as follows. Firm 1 proposes the formation of an alliance A_1 to which she belongs. Each prospective firm answers the proposal in the order fixed by the protocol. If one prospective firm rejects the proposal, then she makes a counter-proposal to which she belongs. If all prospective firms accept, then the alliance A_1 is formed. All firms in A_1 withdraw from the game, and the game proceeds among the firms belonging to $N \setminus A_1$. This sequential game has an infinite horizon, but the firms do not discount the future. The firms who do not reach an agreement in finite time receive a payoff of zero. Once some firms have agreed to form an alliance they are committed to remain in that alliance. Bloch (1995) has shown that the inefficient alliance structure $S = \{A^*, N \setminus A^*\}$ where A^* is the integer closest to $(3n + 1)/4$ of the industry forms around three quarters (i.e. a^* is the integer closest to $(3n + 1)/4$) of the industry forms and the remaining firms form a smaller alliance is the unique symmetric stationary perfect equilibrium outcome of the sequential game. Contractual stability under the unanimity decision rule not only sustains this inefficient alliance structure but also stabilizes the efficient grand alliance.

4 Unanimity Decision Rule with Side Payments

Beside exit without breach via a deadlock implemented by the contractual board where only unanimous decisions are taken (unanimity rule), another rule is commonly used in alliances to govern exit: exit via breach of the agreement subject to damages. This rule can be modelled by allowing for side payments among partners in addition to the unanimity decision rule.

Definition 2. An alliance structure S is contractually stable under side payments and the unanimity decision rule if for any $A \subseteq N$, S' obtainable from S via A and $i \in A$ such that $V_i(S') > V_i(S)$, there is $j \in A$ such that either $V_j(S') \leq V_j(S)$ or

$$\sum_{k \in A(j) \in S} V_k(S') \leq \sum_{k \in A(j) \in S} V_k(S).$$

⁸See also Ray and Vohra (1999).

When a group of firms deviate by leaving some alliance, they can now compensate their former partners to obtain their consent. Obviously, an alliance structure that is contractually stable under the unanimity decision rule with side payments is contractually stable under the unanimity decision rule without side payments.⁹

Proposition 3. *Allowing for side payments among partners, the contractually stable alliance structures under the unanimity decision rule are*

- (i) *any asymmetric alliance structure $S = \{A, N \setminus A\}$ with $(3n - 1)/4 < a \leq n - 1$.*
- (ii) *the grand alliance structure $S^* = \{N\}$.*

Once we allow for side payments among partners when breaking up an alliance, some asymmetric alliance structures consisting of two alliances are no more contractually stable under the unanimity rule. When the largest alliance in $S = \{A, N \setminus A\}$ (i.e. A) is not too large (i.e. $a \leq (3n - 1)/4$) the deviation from S to $S' = \{A \cup B, (N \setminus A) \setminus B\}$ where b firms leave the alliance $N \setminus A$ to join the alliance A to form a new alliance $A \cup B$ is not blocked since the net gains made by the b firms in S' are large enough to make side payments to the members of $(N \setminus A) \setminus B$ for getting their consent in breaking up the alliance $N \setminus A$. Notice that $a \leq (3n - 1)/4$ is the condition that guarantees the profitability for members of A of welcoming $b \geq 1$ new partners. However, once the largest alliance is large enough (i.e. $(3n - 1)/4 < a$), any asymmetric alliance structure $S = \{A, N \setminus A\}$ is still contractually stable even if it is possible to compensate former partners. Indeed, some members of A would like to drop some partners but they cannot compensate those partners to reach their consent. Finally, the grand alliance structure remains contractually stable because it is efficient and all firms belong to the same alliance (hence, each firm can veto any change to the alliance structure).

We now illustrate our main results by means of an example with eight firms. In Table 1 we give the payoffs for $\alpha - \lambda = 42$ and $\mu = 1$. We make a slight abuse of notation. For instance, $\{5, 2, 1\}$ should not be interpreted as a single alliance structure but as the alliance structures, composed by three alliances of size 5, 2 and 1, that can be formed by eight firms. The alliance structure $\{5, 3\}$ is no more contractually stable once side payments are allowed since the deviation to $\{7, 1\}$ is not blocked because the two firms that are changing of alliance can compensate their former partner. Notice that $\{5, 3\}$ is the less efficient structure among the contractually stable ones under the unanimity rule without side payments. Hence, allowing for side payments and requiring unanimity helps to improve efficiency. In addition, the efficient structure $\{8\}$ is still stable but is never the outcome of Bloch's sequential game of coalition formation that selects $\{6, 2\}$.¹⁰

⁹Suppose that only a share $z \in [0, 1]$ of the additional profits after a deviation can be distributed to compensate former partners. Then, a share $z' > z$ refines stability. That is, the set of contractually stable alliance structures under z' is (weakly) included in the set of contractually stable alliance structures

Coalitions:	{8}	{7, 1}	{6, 2}	{5, 3}	{4, 4}	{6, 1, 1}	{5, 2, 1}
Payoffs:	(50)	(55, 1)	(56, 20)	(53, 35)	(46, 46)	(58, 13, 13)	(57, 30, 21)
Stability:							
1. Unanimity	yes	yes	yes	yes	no	no	no
2. Side payments	yes	yes	yes	no	no	no	no
3. Farsighted	yes	yes	yes	yes	no	no	no

Table 1: The 8-firm case with $\alpha - \lambda = 42$, $\mu = 1$, and all payoffs in 1/9-th's.

5 Contractual Stability with Farsighted Firms

Up to now firms are not farsighted in the sense that they do not forecast how others might react to their decisions.¹¹ The notion of farsighted improving path captures the fact that farsighted alliances consider the end alliance structure that their deviation(s) may lead to. That is, a farsighted improving path is a sequence of alliance structures that can emerge when firms form alliances based on the improvement the end alliance structure offers relative to the current alliance structure.

Definition 3. A farsighted improving path under the unanimity decision rule from an alliance structure S to an alliance structure $S' \neq S$ is a finite sequence of alliance structures $S_1, \dots, S_K$ with $S_1 = S$ and $S_K = S'$ such that for any $k \in \{1, \dots, K-1\}$, S_{k+1} is obtainable from S_k via some coalition A_k and $V_i(S_K) > V_i(S_k)$ for all $i \in \{A(j) \in S_k \mid j \in A_k\}$.

A farsighted improving path under the unanimity decision rule is an improving path where not only the deviating firms but also any member of the initial alliances of the deviating firms benefit at the end alliance structure. For a given alliance structure S , let $F(S)$ be the set of alliance structures that can be reached by a farsighted improving path under the unanimity decision rule from S . The farsighted core under the unanimity decision rule to exit an alliance is simply $\{S \in \mathbb{S} \mid F(S) = \emptyset\}$.¹² Obviously, any alliance structure that is not contractually stable under the unanimity decision rule does not belong to the farsighted core. Hence, the only candidates for belonging to the farsighted core are any asymmetric alliance structure S such that $s = 2$ and the grand alliance structure.

under z . Indeed, the probability of blocking a deviation is smaller the higher the share z . The case $z = 0$ ($z = 1$) reverts to contractual stability without (with) side payments.

¹⁰For the equilibrium binding agreements game of Ray and Vohra (1997), the most concentrated stable alliance structure in our model is $\{(n+3)/2, (n-3)/2\}$ (see Yi, 1997). It reverts to $\{5, 3\}$ in our example with eight firms. Moreover, none of the contractually stable alliances with side payments are stable in the equilibrium binding agreements game.

¹¹Chwe (1994), Konishi and Ray (2003) or Herings, Mauleon and Vannetelbosch (2004, 2010) have developed notions to predict which coalitions or groups are likely to be formed among farsighted agents.

¹²Diamantoudi and Xue (2003) have studied the notion of farsighted core in hedonic games.

Proposition 4. *The farsighted core under the unanimity decision rule only consists of all asymmetric alliance structures S such that $s = 2$ and of the grand alliance structure $S^* = \{N\}$.¹³*

First, the grand alliance structure $S^* = \{N\}$ belongs to the farsighted core because $\{N\}$ is the efficient alliance structure and any sequence of deviations from $\{N\}$ requires the consent of all firms. Second, take any $S = \{A, N \setminus A\}$ such that $n/2 < a < n$. Any sequence of deviations where the first deviation involves firms from both alliances is blocked since it requires the consent of all firms and S is not Pareto dominated. For any other sequence of deviations, the members of the alliance, say A , that is first splitting obtain lower payoffs while the members of the other alliance, say $N \setminus A$, benefit from this first deviation. Members of the alliance A that is splitting will deviate first only if they expect to become all of them better off at the end alliance structure. So, further deviations along the sequence involve some change in the alliance $N \setminus A$, hence the consent of all members of $N \setminus A$ is required. But since the end alliance structure does not Pareto dominate the initial one, all members of N cannot be made better off at the end alliance structure, and this sequence cannot be a farsighted improving path. To conclude, Proposition 4 tells us that the stability of alliances under the unanimity rule to exit an alliance is robust to the type of firms, myopic or farsighted.

6 Discussion

6.1 Simple majority decision rule

We now analyze the stability of alliances when the exit of an alliance requires the consent of a majority of the members of the alliance. This rule reflects a third rule of exit often used in alliances: exit at the will of the larger party subject to forewarning.

Definition 4. An alliance structure S is contractually stable under the simple majority decision rule if for any $A \subseteq N$, S' obtainable from S via A and $i \in A$ such that $V_i(S') > V_i(S)$, there exists (i) $l \in A$ such that $V_l(S') \leq V_l(S)$, or (ii) $\hat{A} \subseteq A(j)$ with $A(j) \in S$ and $j \in A$ such that $V_k(S') \leq V_k(S)$ for all $k \in \hat{A}$ and $\hat{a} \geq a(j)/2$.

Under the simple majority decision rule, the move from an alliance structure S to any obtainable alliance structure S' needs the consent of every deviating firm and the consent of more than half members of each initial alliance of the deviating firms. Then, an alliance structure S is contractually stable under the simple majority decision rule if any deviating firm or at least half members of some former alliance of the deviating firms

¹³Our result is robust to other farsighted concepts. A set $\mathcal{S}$ is a von Neumann-Morgenstern farsightedly stable set if (i) $F(S) = \emptyset$ for all $S \in \mathcal{S}$, (ii) $F(S) \cap \mathcal{S} \neq \emptyset$ for all $S \notin \mathcal{S}$. The set consisting of all asymmetric alliance structures S such that $s = 2$ and of the grand alliance structure $S^* = \{N\}$ is the unique vNM (farsightedly) stable set. This set is also the unique farsightedly stable set defined in Herings, Mauleon and Vannetelbosch (2010).

are not better off from the deviation to any obtainable alliance structure S' . Obviously, an alliance structure that is contractually stable under the simple majority decision rule is contractually stable under the unanimity decision rule. A deviation that is blocked under simple majority is also blocked under unanimity.

We find that there is no contractually stable alliance structure under the simple majority rule when the size of the industry is not too small. The intuition behind the proof is as follows. First, we already know that the only candidates for being stable are asymmetric alliance structures consisting of two alliances and the grand alliance structure. Second, any asymmetric alliance structure $S = \{A, N \setminus A\}$ where the size of the largest alliance in $S = \{A, N \setminus A\}$, say A , is small (i.e. $n/2 < a \leq (2n-1)/3$), cannot be contractually stable since there always exists a profitable deviation from S to $S' = \{A \cup B, (N \setminus A) \setminus B\}$ where a majority of firms in $N \setminus A$ (i.e. $2b > n - a$) leaves their partners in $N \setminus A$ to join the alliance A . Third, any asymmetric alliance structure $S = \{A, N \setminus A\}$ where the size of the largest alliance in $S = \{A, N \setminus A\}$, say A , is large (i.e. $(2n-1)/3 < a \leq n-1$), cannot be contractually stable since there always exists a profitable deviation from $S = \{A, N \setminus A\}$ to $S'' = \{A', B, N \setminus A\}$ where a majority of firms in A (i.e. $a > 2b$) drops b former partners. Fourth, the grand alliance structure cannot be contractually stable under the simple majority decision rule since there always exists a majority of firms in N (i.e. $2a > n$) who has incentives to leave their partners in N to form a new alliance A .

Proposition 5. *There is no contractually stable alliance structure under the simple majority rule for $n \geq 8$.¹⁴*

Hart and Kurz (1983) have introduced the notion of Δ -stability.¹⁵ Since an alliance structure that is Δ -stable is contractually stable under the simple majority decision rule, we have that there is no Δ -stable alliance structure for $n \geq 8$. In addition, since contractual stability with side payments refines contractual stability without side payments, we have that there is no contractually stable alliance structure with side payments under the simple majority rule for $n \geq 8$.

6.2 Heterogeneous decision rules

Gulati, Sytch and Mehrotra (2008) have provided evidence that different rules of exit are used in different alliances that are competing in related markets. We denote by $S = \{(A, q_u), (N \setminus A, q_m)\}$ an alliance structure where, in alliance A , the rule of exit is the

¹⁴A complete proof can be found in Mauleon, Sempere-Monerris and Vannetelbosch (2013). For $n = 3$, $\{N\}$ is the unique contractually stable alliance structure under the simple majority decision rule. For $4 \leq n \leq 5$, there is a unique contractually stable alliance structure under the simple majority decision rule: $\{A, N \setminus A\}$ with $a = n - 1$. For $n = 6$, the unique contractually stable alliance structures under the simple majority decision rule are $\{A, N \setminus A\}$ with $n - 2 \leq a \leq n - 1$. For $n = 7$, the unique contractually stable alliance structures under the simple majority decision rule are $\{A, N \setminus A\}$ with $a = n - 2$.

¹⁵An alliance structure S is Δ -stable if for any $A \subseteq N$, S' obtainable from S via A and $i \in A$ such that $V_i(S') > V_i(S)$, there is $j \in A$ such that $V_j(S') \leq V_j(S)$.

unanimity decision rule (q_u) while, in alliance $N \setminus A$, the rule of exit is the simple majority rule (q_m). Can different rules co-exist? Asymmetric alliance structures $\{(A, q_u), (N \setminus A, q_m)\}$ where the size of A is not too large and the unanimity rule is in effect in A are not contractually stable since the profitable deviation for a majority of members of $N \setminus A$ to join the alliance A is not blocked. Once the size of A becomes large, there is a profitable deviation for a majority of firms in A to reduce the size of their alliance by excluding some partners, but those targeted partners can veto any change made to the alliance thanks to the unanimity rule in effect in A . Hence, any asymmetric alliance structure $S = \{(A, q_u), (N \setminus A, q_m)\}$ with $(2n - 1)/3 < a \leq n - 1$ is contractually stable.

Any asymmetric alliance structure $\{(A, q_m), (N \setminus A, q_u)\}$ with $n/2 < a \leq (n + 3)/2$ is contractually stable when simple majority is in effect in the largest alliance while unanimity is in effect in the smallest one. When the size of A becomes larger then the deviation where a majority of A excludes some partners is not blocked. Notice that such asymmetric alliance structures are not stable if simple majority was in effect in both alliances. Then, a majority of firms in $N \setminus A$ would leave $N \setminus A$ to join the alliance A and this deviation would not be blocked by their former partners in $N \setminus A$ because of the simple majority rule.

6.3 Negative versus positive externalities

Bloch's (1995) model of formation of alliances in an oligopoly with linear demand is a model where the formation of an alliance creates negative externalities for nonmembers. Another example of negative externalities is the formation of customs unions. There are economic models where coalition formation creates positive externalities instead. See for instance, Yi (1997). Examples of positive externalities include cartels on oligopolistic markets and economies with pure public goods (e.g. environmental agreement).

Contractual stability under the unanimity decision rule achieves to stabilize the efficient alliance structure, while contractual stability under the simple majority decision rule fails to do it. The reason is that members of larger alliances are better off than members of smaller ones in any alliance structure. Hence, profitable deviations are likely to involve large coalitions of deviating firms and such deviations can be blocked under the unanimity rule to exit an alliance but not under the simple majority rule. In a cartel formation game, on the contrary, members of smaller cartels are better off than members of larger ones in any cartel structure. Hence, it becomes more likely that even the simple majority decision rule can achieve the emergence of the efficient cartel structure in the long run.

6.4 Concluding comments

We have analyzed how different rules for exiting an alliance affect the formation of strategic alliances in Bloch's (1995) linear model. The linear model is too specific to bring any

definitive conclusion. Nevertheless, our analysis can be useful to provide some insights and intuitions on what would happen with more general functional forms.

Minehart and Neeman (1999) have analyzed two termination contracts that are widely used in practice (the shotgun rule and price competition) to dissolve partnerships. Under the shotgun rule (also known as the Texas Shootout), one partner proposes a price and the other decides whether to buy or sell at that price. Under the price competition, both partners submit bids and the high bidder buys the shares of the low bidder at a price equal to the higher bid. Minehart and Neeman have evaluated the performance of each termination contract to achieve the success of the partnership. They have found that although these contracts do not achieve full efficiency, they both perform well.¹⁶ While rules governing an alliance are mostly designed to guarantee the success of the alliance, we have shown that rules for exiting an alliance are important to determine the size and the number of alliances that will be formed in the industry.

An interesting extension is to allow for the existence of overlapping alliances. For instance, it may happen that firms A, B and C may decide to form an alliance altogether while firms A and D form a partnership. A first step is Goyal and Joshi (2003) who have studied networks of collaboration between oligopolistic firms where a collaboration link between two firms involves a fixed cost and leads to an exogenously specified reduction in marginal production cost. Recently, Caulier, Mauleon, Sempere-Monerris and Vannetelbosch (2013) have introduced the framework of coalitional networks that can be applied to improve the predictions of existing economic models studying separately the formation of R&D collaboration networks and of research joint ventures.¹⁷

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¹⁶Brooks, Landeo and Spier (2010) have provided theoretical and experimental explanations for why Texas Shootouts are included in business agreements.

¹⁷See also Ray (2007) for alternative overlapping coalition formation models and Myerson (1980) who has studied, by means of hypergraphs, situations in which communication is possible in conferences that can consist of an arbitrary number of players.

Appendix

Proof of Lemma 1.

Take any alliance structure $S \neq S^*$. The deviation from S to $S' = S \setminus \{A\} \cup \{A_1, A_2\}$ with $A_1 \cup A_2 = A$ is blocked because (i) at least one of the new alliances A_1 or A_2 has a size strictly smaller than $(n+1)/2$ and so the members of this alliance are worse off in S' than in S , and (ii) unanimity of members of A is required. ■

Proof of Lemma 2.

For all $i \in N$, we have $V_i(\{N\}) = \alpha - \lambda + \mu[(n+1)(n) - (n)^2]$. Take any asymmetric alliance structure $S = \{A, N \setminus A\}$. Without loss of generality, let $a \geq (n+1)/2$. For all $i \in A$, we have $V_i(\{A, N \setminus A\}) = \alpha - \lambda + \mu[(n+1)a - (a)^2 - (n-a)^2]$. For all $i \in N \setminus A$, we have $V_i(\{A, N \setminus A\}) = \alpha - \lambda + \mu[(n+1)(n-a) - (a)^2 - (n-a)^2]$. Comparing those expressions and given that $a \geq (n+1)/2$, members of A block the deviation from $S = \{A, N \setminus A\}$ to $S^* = \{N\}$ because they are not better off in S^* (i.e., $V_i(\{N\}) \leq V_i(\{A, N \setminus A\})$, for all $i \in A$). ■

Proof of Lemma 3.

[Case 1.] Suppose that $(n+1)/2 \leq a \leq n-1$.

Take any asymmetric alliance structure $S = \{A, N \setminus A\}$ and consider the deviation from S to $S' = \{A \cup B, (N \setminus A) \setminus B\}$ with $B \subset N \setminus A$. For all $i \in N \setminus A$, we have $V_i(\{A, N \setminus A\}) = \alpha - \lambda + \mu[(n+1)(n-a) - (a)^2 - (n-a)^2]$. For all $i \in (N \setminus A) \setminus B$, we have $V_i(\{A \cup B, (N \setminus A) \setminus B\}) = \alpha - \lambda + \mu[(n+1)(n-a-b) - (a+b)^2 - (n-a-b)^2] = \alpha - \lambda + \mu[(n+1)(n-a) - (a)^2 - (n-a)^2 + b(n-1-4a-2b)]$. Thus, members of $(N \setminus A) \setminus B$ block the deviation from $S = \{A, N \setminus A\}$ to $S' = \{A \cup B, (N \setminus A) \setminus B\}$ if and only if $(n-1-4a)/2 < b$. This condition is always satisfied since $(n+1)/2 \leq a$.

[Case 2.] Suppose that $a < (n+1)/2$.

Take any asymmetric alliance structure $S = \{A, N \setminus A\}$ and consider the deviation from S to $S' = \{A \cup B, (N \setminus A) \setminus B\}$ with $B \subset N \setminus A$. For all $i \in (N \setminus A) \setminus B$, we have $V_i(\{A \cup B, (N \setminus A) \setminus B\}) = \alpha - \lambda + \mu[(n+1)(n-a) - (a)^2 - (n-a)^2 + b(n-1-4a-2b)]$, and $V_i(\{A, N \setminus A\}) = \alpha - \lambda + \mu[(n+1)(n-a) - (a)^2 - (n-a)^2]$. Thus, each $i \in (N \setminus A) \setminus B$ blocks the deviation if and only if $(n-1-4a)/2 < b$.

For all $i \in B$, we have $V_i(\{A \cup B, (N \setminus A) \setminus B\}) = \alpha - \lambda + \mu[(n+1)(a+b) - (a+b)^2 - (n-a-b)^2]$, and $V_i(\{A, N \setminus A\}) = \alpha - \lambda + \mu[(n+1)(n-a) - (a)^2 - (n-a)^2]$. Thus, each $i \in B$ has incentives to deviate if and only if $b(3n+1-4a) - 2b^2 - (n-2a)(n+1) > 0$.

For all $i \in A$, we have $V_i(\{A \cup B, (N \setminus A) \setminus B\}) = \alpha - \lambda + \mu[(n+1)(a+b) - (a+b)^2 - (n-a-b)^2]$, and $V_i(\{A, N \setminus A\}) = \alpha - \lambda + \mu[(n+1)a - (a)^2 - (n-a)^2]$. Thus, each $i \in A$ has incentives to accept members of B if and only if $3n+1-4a-2b > 0$. Notice that $V_i(\{A \cup B, (N \setminus A) \setminus B\}) - V_i(\{A, N \setminus A\}) > 0$ for $i \in A$ whenever $V_i(\{A \cup B, (N \setminus A) \setminus B\}) - V_i(\{A, N \setminus A\}) > 0$ for $i \in B$. Then, we need to find the conditions for $b(3n+1-4a) - 2b^2 - (n-2a)(n+1) > 0$. Since the second derivative with respect to b is negative, $b(3n+1-4a) - 2b^2 - (n-2a)(n+1) > 0$ between the

two roots for which $b(3n + 1 - 4a) - 2b^2 - (n - 2a)(n + 1) = 0$. Solving this equation we find the roots for b ; that is, $b^- = n - 2a$ and $b^+ = (n + 1)/2$. Comparing the roots we have the following: if $a < (n - 1)/4$, then b should be such that $b \in [(n + 1)/2, n - 2a]$, while if $a > (n - 1)/4$, then b should be such that $b \in [n - 2a, (n + 1)/2]$, in order for $V_i(\{A \cup B, (N \setminus A) \setminus B\}) - V_i(\{A, N \setminus A\}) > 0$ for $i \in B$.

But then the condition for $i \in (N \setminus A) \setminus B$ to block the deviation from S to S' is satisfied. That is, we have that $(n - 1 - 4a)/2 < b$. Indeed, if $a > (n - 1)/4$, we have that $(n - 1 - 4a)/2 < 0$ and, hence, $(n - 1 - 4a)/2 < b$. Moreover, if $a < (n - 1)/4$ and $b \in [(n + 1)/2, n - 2a]$, then we have that $b > (n + 1)/2 > (n - 1 - 4a)/2$. ■

Proof of Proposition 2.

The grand alliance $S^* = \{N\}$ is the efficient alliance structure. We have that $nV_i(\{N\}) > \sum_{j=1}^m a_j V_i(S)$ for any $S = \{A_1, A_2, \dots, A_m\}$ such that $S \neq \{N\}$. Under the unanimity decision rule, any deviation from S^* to any S requires the approval of all members of N . Therefore, any deviation from S^* to any S is blocked by at least one member of N who will be worse off in S than in S^* . ■

Proof of Proposition 3.

First, we show that the b members of B can compensate the $n - a - b$ members of $(N \setminus A) \setminus B$ when they deviate jointly with members of A from $S = \{A, N \setminus A\}$ to $S' = \{A \cup B, (N \setminus A) \setminus B\}$. Indeed, we have $\sum_{l \in N \setminus A, N \setminus A \in S} V_l(S') > \sum_{l \in N \setminus A, N \setminus A \in S} V_l(S)$ where $\sum_{l \in N \setminus A, N \setminus A \in S} V_l(S') = (n - a)(\alpha - \lambda) + \mu b[(n + 1)(a + b) - (a + b)^2 - (n - a - b)^2] + \mu(n - a - b)[(n + 1)(n - a - b) - (a + b)^2 - (n - a - b)^2]$ and $\sum_{l \in N \setminus A, N \setminus A \in S} V_l(S) = (n - a)(\alpha - \lambda) + \mu(n - a)[(n + 1)(n - a) - (a)^2 - (n - a)^2]$. Then, $\sum_{l \in N \setminus A, N \setminus A \in S} V_l(S') > \sum_{l \in N \setminus A, N \setminus A \in S} V_l(S)$ if and only if $b > [n(a + 2) - a(3 + 4a)]/[2(a + 1)]$. This condition is always satisfied since $1 > [n(a + 2) - a(3 + 4a)]/[2(a + 1)]$ for $a \geq n/2$. For $a \leq (3n - 1)/4$ there exists $b \geq 1$ that makes members of A accepting the deviation from S to S' .

Second, we show that the $a - b$ members of A who deviate cannot compensate the other b members of A when they deviate from $S = \{A, N \setminus A\}$ to $S' = \{A \setminus B, B, N \setminus A\}$. Indeed, we have $\sum_{k \in A \in S} V_k(S') \leq \sum_{k \in A \in S} V_k(S)$ where $\sum_{k \in A \in S} V_k(S) = a(\alpha - \lambda) + \mu a[(n + 1)a - (a)^2 - (n - a)^2]$ and $\sum_{k \in A \in S} V_k(S') = a(\alpha - \lambda) + \mu(a - b)[(n + 1)(a - b) - (a - b)^2 - (b)^2 - (n - a)^2] + \mu b[(n + 1)b - (a - b)^2 - (b)^2 - (n - a)^2]$. Then, $\sum_{k \in A \in S} V_k(S') \leq \sum_{k \in A \in S} V_k(S)$ if and only if $(n + 1) - a > 0$, a condition which is always satisfied. ■

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