



UNIVERSITÉ CATHOLIQUE DE LOUVAIN
FACULTÉ DES SCIENCES, ÉCOLE DE PHYSIQUE
GEORGES LEMAÎTRE CENTRE FOR EARTH AND CLIMATE RESEARCH
EARTH AND LIFE INSTITUTE

SEA ICE RHEOLOGIES FOR LARGE-SCALE MODELS

DOCTORAL DISSERTATION PRESENTED BY

SYLVAIN BOUILLON

FOR THE DEGREE OF
DOCTOR IN SCIENCES

THESIS COMMITTEE:

Prof. Thierry Fichefet (Supervisor)	Université catholique de Louvain
Prof. Vincent Legat (Supervisor)	Université catholique de Louvain
Prof. Hugues Goosse	Université catholique de Louvain
Dr. Jérôme Weiss	Laboratoire de Glaciologie et de Géophysique de l'Environnement, Grenoble
Dr. Miguel Angel Morales Maqueda	National Oceanography Centre, Liverpool
Prof. Marie-Laurence De Keersmaecker (Chair)	Université catholique de Louvain

LOUVAIN-LA-NEUVE
MARCH 2013

Un chercheur est-il heureux quand il cherche ou quand il trouve? Il est certainement heureux quand il trouve de l'écoute et de l'attention. Quand il reçoit des encouragements et des conseils, cela lui permet d'avancer. Aux workshops et conférences, il présente ce qu'il a trouvé mais discute surtout de ce qu'il cherche. Pour la rédaction, c'est l'inspiration qu'il cherche avec parfois quelques coups de pied au ... et quand il trouve des bugs, des erreurs, des fautes de frappe, il se dit qu'il a bien fait de chercher mais qu'il aurait préféré ne rien trouver.

Au final, je crois être un chercheur heureux. J'ai trouvé des réponses mais aussi de nouvelles questions qui me permettent de chercher davantage. J'ai aussi trouvé des collègues sympas, des promoteurs intéressés et des relecteurs exigeants. J'ai pu chercher la petite bête et faire avancer la science dans un domaine qui me passionne. Enfin, j'ai eu l'occasion de partager mon enthousiasme pour d'autres sujets, de l'activisme cyclophile à la réinformation climatique, en passant par l'élevage d'abeilles et autres petites choses bourdonnantes.

Bref, je vous dis merci à tous pour avoir contribué à mes recherches et trouvailles de grands et petits bonheurs lors de ces années de thèse.

*A Stéphanie et nos petits loups,
une raison de se battre pour un monde moins pire.*

Ce travail a été rendu possible par diverses sources de financement. C'est en tant qu'assistant à l'Université catholique de Louvain que j'ai été engagé pendant six ans. Le projet européen EMBRACE a financé trois mois d'engagement comme assistant de recherche. Ce travail est aussi supporté par le programme Tournesol de Wallonie-Bruxelles International et par le projet SLIM, financé par la Communauté Française de Belgique, comme Actions de Recherche Concertées, sous les contrats ARC 04/09-316 et ARC 10/15-028.

CONTENTS

Contents	iii
1 Introduction	1
Sea ice dynamics	2
A better representation of sea ice dynamics	9
2 The elastic-viscous-plastic model	13
2.1 Introduction	14
2.2 Model description	15
2.3 Discretizations	17
2.4 Results and discussion	25
2.5 Conclusions	34
3 The elastic-viscous-plastic model revisited	37
3.1 Introduction	38
3.2 Model description	39
3.3 The elastic-viscous-plastic method revisited	42
3.4 Results and discussion	53
3.5 Conclusions	64
4 The elasto-brittle model	67
4.1 Introduction	68
4.2 Model description	68
4.3 Results and discussion	76
4.4 Conclusions	92
5 Conclusions	95
Achievements	95
Perspectives	98
References	103

CHAPTER 1

INTRODUCTION

Since the first satellite records in 1979, the Arctic sea ice has been declining at a rapid pace. The succession of recent sea ice extent minima (Stroeve et al., 2012b; Comiso et al., 2008; Lindsay et al., 2009) is mainly caused by the continuous decrease of sea ice volume (Rothrock et al., 1999; Wadhams and Davis, 2000; Haas et al., 2008). In the recent years, the portion of perennial ice reduced drastically (Kwok and Rothrock, 2009; Laxon et al., 2013) and some authors even predict summer ice-free Arctic Ocean in the near future (Wang and Overland, 2012).

These changes greatly affect the Arctic populations and ecosystems, but have also large implications on the regional and global climate system through several mechanisms (Porter et al., 2012). The most important interaction with the other components of the climate system is the ice-albedo feedback. The albedo, which is the fraction of solar radiation reflected by a material, is very high for snow and ice and very low for open water (Perovich et al., 2002). When the ice cover is reduced, more shortwave radiation is absorbed by the ocean, inducing further reduction of the ice cover (Stroeve et al., 2012b). This positive feedback is considered to be largely responsible for the high sea ice sensitivity to climate change (Ebert and Curry, 1993; Manabe and Stouffer, 1980; Holland and Bitz, 2003). Another interaction with the ocean is due to salt rejection during sea ice production that plays a crucial role in the formation of deep waters and thus in the global thermohaline circulation (e.g., Aagaard and Carmack, 1989). In addition, sea ice constitutes a natural barrier between the atmosphere and the ocean (e.g., Serreze et al., 2007).

When the ice cover is reduced, the fluxes from the relatively warm ocean waters induce deep heating and moistening of the cold polar atmosphere (Porter et al., 2012). Lower sea ice extents in summer may then modify the general atmospheric circulation and lead to colder and snowier winters in the United States and Europe (Jaiser et al., 2012; Liu et al., 2012).

To understand the complex role played by sea ice in the Earth system and to make reliable projections, scientists use coupled atmosphere-ocean-sea ice models, which are validated against satellites and in-situ observations. The coupling has to take into account the interactions between sea ice and the other components of the climate system. Current climate models underestimate the present reduction rate of the Arctic sea ice and large uncertainties remain in the predictions (Stroeve et al., 2012a; Massonnet et al., 2012). Comparisons with drifting buoys data also show that climate models underestimate the mean ice drift and fail in capturing its recent acceleration (Rampal et al., 2011). Moreover, these models do not reproduce the ice speed seasonal cycle which has a minimum in March and a maximum in September. This cycle is consistent with the seasonal cycle of sea ice thickness. This link between ice speed and thickness is surprisingly not observed in results, whatever the complexity of the rheology used in these models (Rampal et al., 2011). A possible cause of these deficiencies is the use of inadequate rheologies to compute sea ice motion in response to the wind and ocean currents.

The main objective of this thesis is to improve the representation of sea ice dynamics. A better reproduction of sea ice dynamics at all spatial and temporal scales is likely to have a great impact on the simulated mean ice drift, thickness and concentration, either by modifying the response of the ice to the wind and ocean forcing, and then the advection, or by enhancing the mechanical redistribution related to the opening and closing of open water areas in the ice cover. A better representation of sea ice deformation will also help in reformulating models to take into account the effects of small scale features, such as leads and ridges. To consider lead width distributions, for example, is crucial to simulate exchanges between the atmosphere and the ocean in (Steiner et al., 2013; Marcq and Weiss, 2012).

The following section presents the main characteristics of the ice pack dynamics and defines the concepts used to develop a new framework for modelling sea ice.

Sea ice dynamics

In 1982, scientists participating in the Arctic Ice Dynamics Joint Experiment campaign (AIDJEX) already asked: “Isn’t it time to re-examine our models of thickness distribution and stress-strain laws which are formulated in terms of the mean deformation?” (Thorndike and Colony, 1977). They actually raised a difficult issue

which is still not fully understood. Their question was caused by the strong link between the value of sea ice deformation and the spatial and temporal scales considered. In consequence, they suggested to always associate the length and time scales to the definition of the deformation. When applied to large set of observations, this scale analysis reveals the complex nature of the ice pack and the large discrepancy with its representation in models.

A huge amount of data on sea ice dynamics are available thanks to the analysis of satellite images and drifting buoy trajectories. These data cover a wide range of temporal and spatial scales, from 1 hour to several months, and from 10 km to the size of the Arctic basin (~ 1000 km). Statistical analysis reveals the strong heterogeneity and intermittency of sea ice deformation rates (Marsan et al., 2004; Rampal et al., 2008). These characteristics are shared by other geophysical systems, such as the Earth's crust (Marsan and Weiss, 2010), but deformation in the ice pack is an ideal subject as it can be considered as a two-dimensional problem and is observed on much shorter time scales than plate tectonics.

A powerful tool to study sea ice deformation is the radarsat geophysical processor system (RGPS) that tracks sea ice motion from satellite images and generates ice deformation fields at a temporal resolution of 3 days and a spatial resolution of 10 km (Kwok, 2001). As shown in Figure 1.1, the deformation of the initially regular grid is concentrated along linear features (green cells), whereas the rest of the grid remains regular, indicating no deformation (red cells). The localized deformation is associated to the opening and closing of leads (darker zones) and to the creation of massive ice ridges that render expeditions in the Arctic so adventurous.

RGPS data are available for the winter and summer seasons, and provide sea ice deformation rate in the centre of the Arctic. As shown in Figure 1.2, almost all the ice shear and divergence concentrate in narrow, linear features spanning all the Arctic. The activity, persistency and length scales of these patterns are quite remarkable (Kwok, 2006). In addition, their evolution is intermittent, alternating quiet and active periods. These features are associated to discontinuities in the velocity field which are responsible for the spatial and temporal scaling of the ice deformation rate (Nye, 1973).

To study the spatial scaling of the ice deformation from RGPS data, Marsan et al. (2004) developed a coarsening procedure that is illustrated in Figure 1.3. ϵ_L , the deformation rate at length scale L , is obtained by averaging ice velocity derivatives inside boxes of width L to compute the divergence and shear rates, $\dot{\epsilon}_d$ and $\dot{\epsilon}_s$, respectively, for each box. Both invariants are combined to define the total deformation rate as $\epsilon_L = \sqrt{\dot{\epsilon}_d^2 + \dot{\epsilon}_s^2}$, which is shown for length scales ranging from 20 to 640 km in Figure 1.3.

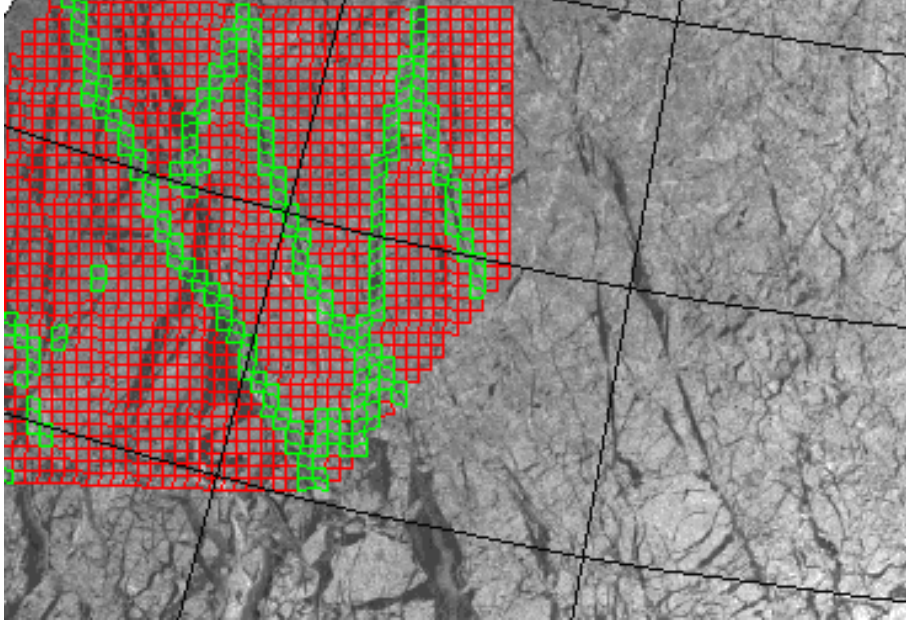


Figure 1.1: Superposition of a Lagrangian RGPS grid on a satellite image. By comparing successive images obtained by the Synthetic Aperture Radar (SAR) and tracking sea ice details such as leads (darker zones), the system computes the displacement of each grid point and the associated deformation for each cell. In this example, cells in green are highly deformed, whereas the rest of the grid (in red) exhibits almost no deformation. Source : <http://psc.apl.washington.edu/Harry/SARworkshop>.

Analysing the dependence of ϵ_L on the length scale L allows to quantify the observed heterogeneity of the ice deformation. Figure 1.4 presents the values of ϵ_L corresponding to Figure 1.3. The red dots correspond to $\langle \epsilon_L \rangle$, the mean value over all boxes of each length scale. The blue line is the best linear fit in the log-log space of the mean deformation and corresponds to the following power-law relationship:

$$\langle \epsilon_L \rangle \sim L^{-\beta(1)}, \quad (1.1)$$

on length scales L ranging from 10 to 1000 km. The exponent $\beta(1)$, corresponds to the slope of the blue line and is about 0.19, which is a typical value for the winter season. The power-law relationship indicates spatial clustering of the deformation. When the velocity gradients are randomly shuffled, there is a strong departure from the power-law relationship with a much more rapid decrease of the mean deformation as a function of the scale (Marsan et al., 2004; Stern and Lindsay, 2009).

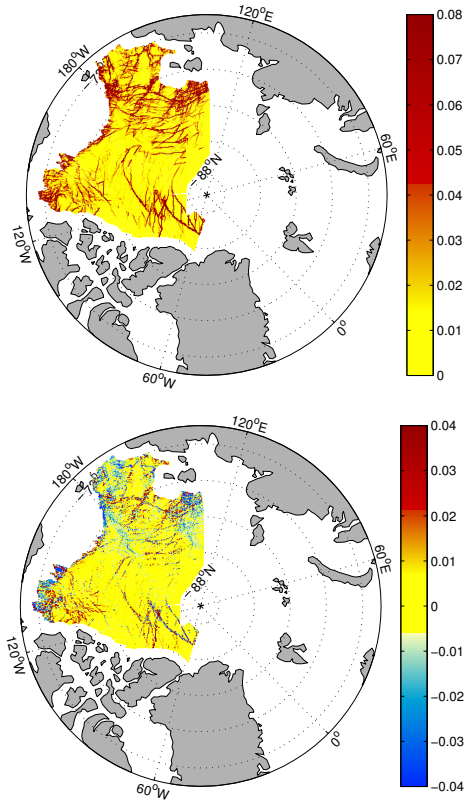


Figure 1.2: Sea ice shear (top) and divergence (bottom) rates in $[day^{-1}]$ on the 6th November 1997 at a spatial scale of 10 km computed from the available RGPS Lagrangian data (same example as in Marsan et al. (2004)). Shear and divergence are well correlated, and are concentrated in long, narrow features.

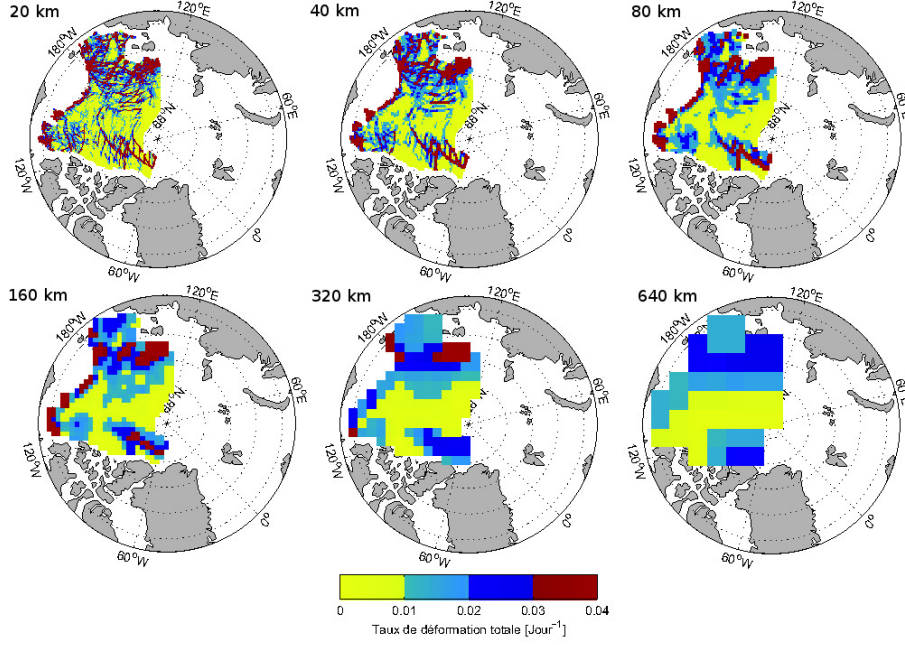


Figure 1.3: Illustration of the coarsening procedure that computes the total sea ice deformation rate in [day^{-1}] at different length scales (here from 20 to 640 km) from the RGPS data available for the 6th November 1997 (same example as in Marsan et al. (2004)). For each length scale L , the domain is covered by overlapping boxes of width L whose centres are located on a regular grid of resolution $\frac{L}{2}$.

There is a strong variability of $\beta(1)$. The exponents are higher during summer ($\beta(1) \sim 0.25$) than during winter ($\beta(1) \sim 0.18$), and higher for first-year ice than for multiyear ice. This effect highlights the link between the strength of the ice and the ability to have mechanical interactions between distant parts of the ice pack (Stern and Lindsay, 2009). Similar statistical properties are found from in-situ experiments, specifically designed to resolve sea ice deformation at several length scales with nested arrays of buoys. In the Beaufort Sea, Hutchings et al. (2011) detected a scaling relationship of the hourly deformation rate, from 10 to 140 km, with a power-law exponent equal to 0.19. This exponent decreases as the timescale increases and is about 0.11 at 3 days. Rampal et al. (2008) found higher values of $\beta(1)$ in evaluating the deformation rate from the variation of the distance between hundreds of pairs of buoys. In the Southern Hemisphere, very few data are available to afford a general view of the ice deformation but Hutchings et al. (2012) also saw a scaling relationship with an array of buoys in the Weddel Sea.

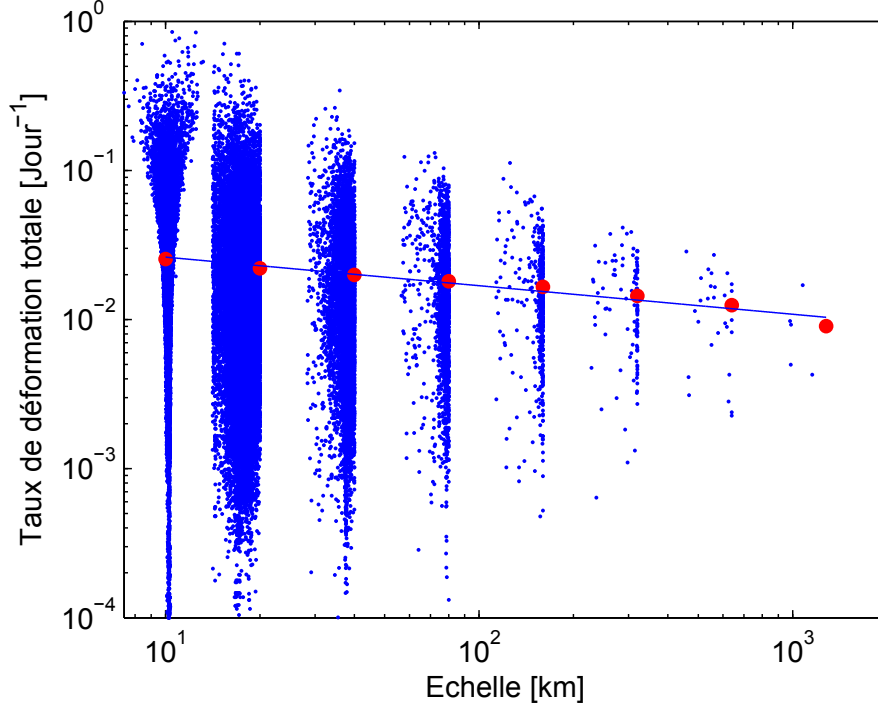


Figure 1.4: Total deformation rate $\dot{\epsilon}_L$ computed for several spatial scales L , ranging from 10 to 1280 km with the coarsening procedure described in the text (same example as in Marsan et al. (2004)). The mean value of the total deformation rate $\langle \dot{\epsilon}_L \rangle$ is given by the red point for each length scale. The blue line is the best linear fit in the log-log space of the mean deformation and corresponds to a power-law relationship between $\langle \dot{\epsilon}_L \rangle$ and L . We observe that decreasing the length scale is associated with an increase of the mean total deformation and a faster increase of the deviation.

Another signature of the localization of the deformation is the rapid increase of the spatial variability when decreasing the length scale. This increase of the standard deviation is seen in the probability density functions (pdf) of $\dot{\epsilon}_L$ which are not Gaussian but are characterized by power-law tails (not shown). The slope of the power-law tails decreases when decreasing the length scale, corresponding to larger deformation rates and to the rapid increase of the standard deviation, which analytically reaches infinity when the slope becomes lower than 3. In consequence, as the length scale decreases, deformation is accommodated by a smaller percentage of the total area where deformation rates are very high. In our example, the highest deformation rates at the 10 km length scale almost reach 1 day^{-1} . Marsan

et al. (2004) estimated that about 15% of the deformation is accommodated by 3.2% of the domain at length scales $L = 160 - 320$ km and by 1.6% at $L = 13 - 20$ km. The scaling properties allow to extrapolate the distribution at smaller length scales. For $L = 1$ m, only 0.2% of the area accommodate 15% of the deformation. In these areas, the deformation rate is higher than 10 day^{-1} , which corresponds to the brittle regime for saline ice.

Changing the spatial scale induces a modification of the shape of the probability distribution. This effect is completely characterized by looking at the scale dependence of the moments of the deformation rate (Marsan et al., 2004):

$$\langle \dot{\epsilon}_L^q \rangle \sim L^{-\beta(q)}, \quad (1.2)$$

The deformation is multi-fractal because $\beta(q)$ is a quadratic function of q . The multi-fractality is also observed for other quantities such as fracture density, and Weiss and Marsan (2004) showed how it can be simulated by a multiplicative cascade process, similarly to what is done for atmospheric turbulence or rainfall. However, a limitation of cascade models is the absence of any directional information. A model only based on this approach would produce scale invariant and correlated fields, but cannot generate quasi-linear features as observed in the ice pack.

Sea ice deformation is also clustered in time and this intermittency is characterized by the power-law relationship between the deformation rate and the temporal scale τ :

$$\langle \dot{\epsilon}_\tau \rangle \sim \tau^{-\alpha}, \quad (1.3)$$

where $\dot{\epsilon}_\tau$ is the deformation rate computed at time scale τ . At small scales (<10 km), Rampal et al. (2008) found this relationship valid over 3 orders of magnitude, from 1 hour to 1 month, with the exponent α close to 1, which corresponds to the accommodation of the deformation by one single deformation event. Hutchings et al. (2011) found the same exponent for spatial scales between 6 and 140 km.

Multi-fractal scaling and intermittency has also been observed for sea ice velocity and acceleration (Rampal et al., 2009). This gives an argument in favour of the central role of the internal stress because none of the other factors playing a role in the ice pack dynamics presents the same statistical property (Weiss, 2013). Sea ice fluctuating velocity distributions are characterized by power-law tails that cannot be explained by Gaussian distributions of wind and ocean velocities (Weiss, 2013). This analysis also leads to discarding the drunkard's walk approach of Colony and Thorndike (1985) and Thorndike (1986) where ice displacement is modelled as a Gaussian random vector with a spatially varying mean and a prescribed variance.

Strong intermittency and spatial variability are also observed in stress measurements (Weiss et al., 2007; Weiss and Marsan, 2004; Weiss, 2008), with noticeable similarities with earthquakes. Weiss (2003) showed by analysing the acoustic emissions generated by sea ice fracturing that the probability density function p of the dissipated energy S during ice-quakes follows a power-law relationship, $p(S) \sim S^{-b}$, with an exponent b similar to that observed in Earth seismicity (Chmel et al., 2005). In a recent study, Chmel and Smirnov (2011) concluded the existence of long-range interactions for the consolidated ice pack, but to a lower level than for tectonic processes.

Another particularity of the ice pack deformation is the strong space-time coupling (Marsan and Weiss, 2010). The exponent $\beta(1)$, which characterizes the heterogeneity of the deformation, strongly depends on the temporal scale. Analysing trajectories of buoys fixed on ice floes reveals a continuous decrease of the exponent $\beta(1)$ as the time scale τ increases (Rampal et al., 2008), with a possible linear relationship between the exponent $\beta(1)$ and the logarithm of τ (Hutchings et al., 2011; Marsan and Weiss, 2010). Symmetrically, the intermittency depends on the spatial scale. At larger spatial scales (~ 300 km), the exponent decreases ($\alpha \sim 0.3$) but intermittency remains remarkable (Rampal et al., 2008). The dependence of the value of α on the spatial scale is explained by the implication of more deformation events as the spatial scale increases. Weiss et al. (2009) proposed a multiscale statistical model of sea ice fracturing to explain how temporal and spatial scaling laws arise from intermittent fracturing events organized in 1D-like features.

A better representation of sea ice dynamics

The characteristics described here above are slowly being considered to develop and validate sea ice models. Girard et al. (2009) compared results from sea ice-ocean models to observations and found very little resemblance in terms of statistical properties of the deformation. The scaling laws actually challenge the assumptions on which classical sea ice rheologies are based, as they preclude the definition of a characteristic size that usually allows homogenization over a representative element. The assumption that sea ice is an isotropic plastic material with no (or little) tensile strength is also questioned (Coon et al., 2007). The debate is not over and some studies showed that classical rheologies can produce deformation fields that look realistic (e.g., Maslowski and Lipscomb, 2003), but still no sea ice-ocean model has reproduced the scaling properties observed in the ice pack (Weiss and Marsan, 2004).

This thesis was dedicated to the development of a better representation of sea ice dynamics for large-scale sea ice-ocean models. In the first part of the thesis, we improved the numerical methods used in classical sea ice models and in the second

one, we implemented a completely new kind of rheology based on the elasto-brittle framework.

Most large-scale sea ice models use the rheology proposed by Hibler (1979), in which sea ice is modelled as a non-linear viscous-plastic (VP) material. To solve the resulting non-linear equations, most numerical methods use pseudo-time steps or sub-iterations within each global time step (Zhang and Hibler, 1997). At each sub-iteration, the equations are linearized and the system of equations is solved. An alternative method for the calculation of the VP dynamics is the elastic-viscous-plastic (EVP) formulation of Hunke and Dukowicz (1997), where regularization terms avoid the need of a linear solver, thus easing parallelization.

In **Chapter 2**, the first integration of the EVP formulation in a coupled sea ice-ocean model discretized on a C-grid is presented. The performances are compared to the previous VP formulation and the influence of the type of grid is discussed.

In **Chapter 3**, we investigate the numerical convergence of the EVP method thanks to its reinterpretation as an iterative solver for the VP non-linear problem. We show that the linear bands of high ice deformation simulated with the original regularization of Hunke and Dukowicz (1997) are clearly related to a lack of convergence of the numerical method. These features aligned with the grid completely disappear with a simple correction of the regularization terms.

In parallel to the improvements of current sea ice rheologies, a new approach has been developed with the elasto-brittle (EB) rheology, which is inspired by progressive damage models used in solid mechanics (Amitrano et al., 1999). With this kind of models, Girard et al. (2010) reproduce the power-law distributions of the dissipated energy and the spatial correlation of damage events. In Girard et al. (2011), we present realistic simulations of the Arctic sea ice deformation at high resolution (10 km), on a temporal scale of 3 days. Statistical analyses of these results highlight the spatial heterogeneity of the simulated ice deformation. Shear and divergence rate distributions produced with the EB model compare much better to observations than those obtained with the VP model in the same configuration.

In **Chapter 4**, we integrate the new EB rheology into a finite element sea ice model. The heterogeneity of sea ice deformation is correctly reproduced and a parameterisation of the healing process allows to study the variability and the persistency of the simulated sea ice deformation patterns.

Chapter 5 presents the general perspectives and conclusions.

The developments related to this thesis are made in two different sea ice models being continuously improved at the Université catholique de Louvain and briefly presented hereafter.

The Louvain-la-Neuve sea Ice Model, LIM¹, has been developed by Fichefet and Maqueda (1997, 1999) and became the default sea ice module of the Nucleus for European Modelling of the Ocean, NEMO². LIM is also widely used outside the NEMO project in numerous laboratories involved in climate studies and operational oceanography. It is the sea ice component of the global coupled sea ice-ocean model CLIO (Goosse and Fichefet, 1999), and has been coupled to the ocean general circulation models OPA (Océan PARallélisé), which is the precursor of NEMO (Madec et al., 1998), and MOM3 (Modular Ocean Model, version 3, Hofmann and Maqueda (2006)), as well as to the Earth system models of intermediate complexity CLIMBER3 α (Montoya et al., 2005) and LOVECLIM (Driesschaert et al., 2007), and to the climate general circulation model IPSL-CM4 (Marti et al., 2010). Simulations with the coupled OPA-LIM model have been analyzed by Timmermann et al. (2005) with the LIM2 version which has only two ice thickness categories. A more elaborated version of the model has lead to major improvements (LIM3, Vancoppenolle et al. (2009)). In order to account for unresolved variations in ice thickness, several thickness categories have been included along with a new multi-layer halo-thermodynamic component.

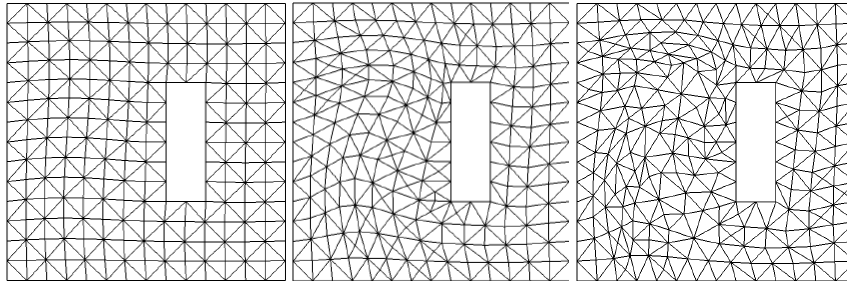


Figure 1.5: Example of the mesh deformation and adaptation. The ice flows around the rectangular island in the clockwise direction with no displacement on the boundaries (Lietaer, 2011).

The Second generation Louvain-la-Neuve Ice-ocean Model, SLIM³, has the ambition of modelling multiscale hydrodynamical processes. This project is based on

¹<http://www.climate.be/LIM>

²<http://www.locean-ipsl.upmc.fr/NEMO/>

³<http://www.climate.be/slim>

the discontinuous finite element method and unstructured grids and features a 1D river model, 2D depth averaged model and a full 3D model (de Brye et al., 2010; Blaise et al., 2010; Comblen et al., 2010). The sea ice component of this project is based on LIM2 and has representations of both dynamic and thermodynamic sea ice processes. The viscous-plastic rheology is included along with parameterisations of the atmospheric fluxes. The natural ability of unstructured meshes to locally adapt the mesh resolution allows to capture the complex oceanic areas formed by coasts and islands such as the Canadian Arctic Archipelago (CAA) (Lietaer et al., 2008). The finite element sea ice model has been used to study the representation of the ice age (Lietaer et al., 2011). A Lagrangian version of the sea ice model allowing the computational grid to move with the ice drift (Figure 1.5) has also been developed (Lietaer, 2011). This Lagrangian version of the model has several interesting applications, such as the dynamical mesh refinement along any region of interest (e.g., the ice edge) or buoy tracking.

CHAPTER 2

THE ELASTIC-VISCOUS-PLASTIC MODEL

This Chapter is based on the following paper, submitted on 26 August 2008:
S. Bouillon, M. A. M. Maqueda, V. Legat, and T. Fichefet. An elastic-viscous-plastic sea ice mode formulated on Arakawa B and C grids. *Ocean Modelling*, 27:174–184, 2009.

Abstract

The elastic-viscous-plastic (EVP) sea ice rheology has been introduced in the large-scale Louvain-la-Neuve sea-Ice Model, version 2 (LIM2), and its performance has been evaluated. Centred difference versions of the rheology have been implemented on both an Arakawa B-grid and a C-grid, and their performances have been intercompared in coupled simulations with the Nucleus for European Modelling of the Ocean (NEMO) model. Integrations with both implementations lead to fairly similar results which compare well with observations and with previous LIM simulations. The C-grid version, however, offers a number of advantages: (a) easier ice coupling with NEMO, which is itself defined on a C-grid; (b) possibility of representing ice transport across one-cell-wide straits and passages; (c) better representation of inertial-plastic compressive waves. For these reasons, the C-grid EVP formulation is now the standard method to solve the ice dynamics in most applications of NEMO-LIM2 and NEMO-LIM3 models.

2.1 Introduction

The dynamical component of most large-scale sea ice models uses some variants of the rheology proposed by Hibler (1979), in which sea ice is assumed to be a non-linear viscous-plastic (VP) material whose resistance to deformation depends on its instantaneous state of motion and on several large-scale scalar properties, such as the mean ice thickness and lead fraction. The VP formulation of Hibler (1979) has known many successes, but it is computationally expensive and not well suited for efficient parallel integrations. The numerical method first used to solve the VP dynamics was a relatively slow implicit point relaxation method (Hibler, 1979). More efficient implicit methods have been proposed subsequently, namely, the line relaxation method (Zhang and Hibler, 1997) and the alternating direction implicit method (Zhang and Rothrock, 2000). However, the most popular alternative for the calculation of the VP dynamics is the elastic-viscous-plastic (EVP) formulation of Hunke and Dukowicz (1997). Distinctive advantages of the EVP dynamics over the VP dynamics is that it is much simpler to program and can be solved explicitly in time, thus easing parallelization. Because of the appealing numerical properties of the EVP dynamics, a growing number of large-scale, coupled ocean-sea ice and atmosphere-ocean-sea ice models have adopted this formulation (e.g., Randall et al., 2007).

Official releases of the Louvain-la-Neuve sea-Ice Model (LIM) have, until now, employed the VP dynamics formulation, although a cavitating fluid approach was also briefly tested by Fichet and Maqueda (1997), and the versions of LIM coupled to MOM3 and CLIMBER3 α do incorporate already implementations of the EVP dynamics (Montoya et al., 2005). However, this work is the first attempt to evaluate the impact of the EVP parameterisation on LIM.

We have tested three versions of the EVP dynamics in LIM, namely, the bilinear discretization of Hunke and Dukowicz (2002) on an Arakawa B-grid, and two simpler discretizations that we will describe here in full, the first of which is also formulated on a B-grid, while the second is for a C-grid. We present results of simulations with the coupled OPA-LIM model, each employing one of the three EVP formulations that we have just referred to. For comparison, a fourth integration was also carried out with the original VP parameterisation. Reassuringly, all three EVP discretizations produce very similar results. However, the C-grid version is the one that has been chosen for use within the NEMO project, as it is faster than any of the other two and affords a more direct coupling with the ocean component of NEMO, which is an updated version of OPA and is also discretized on a C-grid.

The chapter is organized as follows. Section 2.2 succinctly describes LIM. Section 2.3 discusses issues pertaining to the new discretization of the model dynamics,

specially as regards the new implementation on a C-grid. Section 2.4 presents an intercomparison of key results from the numerical simulations.

2.2 Model description

The version of LIM used in this study (LIM2) is described in full detail in Timmermann et al. (2005) and references therein. Therefore, only a brief summary of the model, with emphasis on its dynamics, is given here.

The thermodynamic part of LIM (Fichefet and Maqueda, 1997, 1999) uses a three-layer model for the vertical heat conduction within snow and ice. The storage of latent heat in brine pockets is taken into account, and sea ice growth and decay rates are obtained from the ice energy budget. Air-ice and air-ocean heat fluxes are computed using empirical parameterisations described by Goosse (1997), and the ice-ocean heat flux is computed as in McPhee (1992).

The model dynamics are based on the two-category (consolidated ice plus leads) approach of Hibler (1979). This two-category ice cover is treated as a two-dimensional compressible fluid driven by winds and oceanic currents. Sea ice resists deformation with a strength which increases monotonically with ice thickness and concentration.

The conservation of linear momentum for sea ice is expressed as in Leppäranta (2005) by

$$m\mathbf{u}_t = \nabla \cdot \boldsymbol{\sigma} + A(\boldsymbol{\tau}_a + \boldsymbol{\tau}_w) - mf\mathbf{k} \times \mathbf{u} - mg\nabla\eta, \quad (2.1)$$

where m is the ice mass per unit area, $\mathbf{u}$ is the ice velocity, $\boldsymbol{\sigma}$ is the internal stress tensor, A is the ice area fraction, or concentration, $\boldsymbol{\tau}_a$ is the wind stress, $\boldsymbol{\tau}_w$ is the ocean stress (typically quadratic), f is the Coriolis parameter, $\mathbf{k}$ is an upward pointing unit vector, g is the gravity acceleration and η is the ocean surface elevation with respect to zero sea level. Note that the momentum advection is being ignored in (2.1) and that the wind and ocean stresses are multiplied by the ice concentration as suggested by Connolley et al. (2004).

Calculation of sea ice internal forces in LIM has customarily been done using the VP approach of Hibler (1979), which, in practice, is a particular case of the EVP formulation of Hunke and Dukowicz (1997). A description of the general framework for the VP and EVP formulations of the ice internal stresses is given in Hunke and Dukowicz (2002) and Hunke and Lipscomb (2008). For completeness, we reproduce here the key elements of such a framework. Let us denote σ_{11} , σ_{22} and σ_{12}

the components of the ice internal stress tensor, and let

$$\sigma_1 = \sigma_{11} + \sigma_{22}, \quad (2.2)$$

$$\sigma_2 = \sigma_{11} - \sigma_{22}, \quad (2.3)$$

$$D_D = \frac{1}{h_1 h_2} \left(\frac{\partial}{\partial \xi_1} (h_2 u) + \frac{\partial}{\partial \xi_2} (h_1 v) \right), \quad (2.4)$$

$$D_T = \frac{1}{h_1 h_2} \left(h_2^2 \frac{\partial}{\partial \xi_1} (u/h_2) - h_1^2 \frac{\partial}{\partial \xi_2} (v/h_1) \right), \quad (2.5)$$

$$D_S = \frac{1}{h_1 h_2} \left(h_1^2 \frac{\partial}{\partial \xi_2} (u/h_1) + h_2^2 \frac{\partial}{\partial \xi_1} (v/h_2) \right), \quad (2.6)$$

where D_D , D_T and D_S are the divergence, horizontal tension and shearing strain rates, respectively, ξ_1 and ξ_2 are generalized orthogonal coordinates, and h_1 and h_2 are the associated scale factors. With these definitions, the stress tensor is given by

$$\sigma_1 = \left(\frac{D_D}{\Delta} - 1 \right) P, \quad (2.7)$$

$$\sigma_2 = \frac{D_T}{e^2 \Delta} P, \quad (2.8)$$

$$\sigma_{12} = \frac{D_S}{2e^2 \Delta} P, \quad (2.9)$$

where P is the ice compressive strength, e is the ratio of principal axes of the elliptical yield curve (see below) and Δ , a measure of the deformation rate, is given by

$$\Delta = \sqrt{D_D^2 + \frac{1}{e^2} (D_T^2 + D_S^2)}. \quad (2.10)$$

Note that the tensor given by (2.7) to (2.9) is entirely equivalent to the more classic Reiner-Rivlin formulation used by Hibler (1979).

This rheology links the compressive stress, σ_1 , to the shearing stress,

$$\sigma_s = \sqrt{\sigma_2^2 + 4\sigma_{12}^2}, \quad (2.11)$$

by the following quadratic relation,

$$(\sigma_1/P + 1)^2 + e^2 (\sigma_s/P)^2 = 1 \quad (2.12)$$

which defines an elliptical yield curve. The ice compressive strength P is empirically related to the ice thickness per unit area, h , and ice concentration, A , by $P = P^* h e^{-c^*(1-A)}$, where P^* and c^* are empirical constants.

In (2.7)-(2.9), a regularization is needed when Δ goes to zero. A simple regularization is to set a lower bound, Δ_{min} , for Δ . For values of Δ smaller than Δ_{min} , sea ice behaves like a linear viscous fluid undergoing very slow creep. As we shall see below, if the plastic behavior of sea ice is to be accurately represented, Δ_{min} must be sufficiently small (say 10^{-9} s^{-1} or less). Note that Δ_{min} is directly related to the ζ_{max} parameter used in Hibler (1979) by $\zeta_{max} = P / (2\Delta_{min})$.

An alternative regularization was proposed by Hunke and Dukowicz (1997), and it consists in introducing time dependence and an artificial elastic term in (2.7)-(2.9), leading to the EVP formulation:

$$2T\sigma_{1,t} + \sigma_1 = \left(\frac{D_D}{\Delta} - 1 \right) P, \quad (2.13)$$

$$\frac{2T}{e^2}\sigma_{2,t} + \sigma_2 = \frac{D_T}{e^2\Delta} P, \quad (2.14)$$

$$\frac{2T}{e^2}\sigma_{12,t} + \sigma_{12} = \frac{D_S}{2e^2\Delta} P, \quad (2.15)$$

where T is a time scale that controls the rate of damping of elastic waves. Note that, while (2.13)-(2.15) become (2.7)-(2.9) in the steady state, static flow in the EVP rheology is represented by an elastic deformation, and so imposing a minimum value of Δ is only necessary for numerical reasons. Hunke and Dukowicz (1997) showed that the numerical solution of (2.1) in combination with (2.13)-(2.15) does indeed converge to the VP stationary solution as long as the elastic time scale T is several times smaller than the time scale of variation of the external forcing.

The components of the internal stress force are (Hunke and Dukowicz, 2002):

$$2F_1 = \frac{1}{h_1} \frac{\partial \sigma_1}{\partial \xi_1} + \frac{1}{h_1 h_2^2} \frac{\partial (h_2^2 \sigma_2)}{\partial \xi_1} + \frac{2}{h_1^2 h_2} \frac{\partial (h_1^2 \sigma_{12})}{\partial \xi_2}, \quad (2.16)$$

$$2F_2 = \frac{1}{h_2} \frac{\partial \sigma_1}{\partial \xi_2} - \frac{1}{h_1^2 h_2} \frac{\partial (h_1^2 \sigma_2)}{\partial \xi_2} + \frac{2}{h_1 h_2^2} \frac{\partial (h_2^2 \sigma_{12})}{\partial \xi_1}. \quad (2.17)$$

2.3 Discretizations

Numerical stability analyses show that explicit time integration of the VP dynamics would be very expensive. Indeed, the critical time step for a stable explicit VP scheme is about 1 s for a 100-km grid, and scales as d^2 , where d is the horizontal resolution (Hunke and Dukowicz, 1997). Given such a prohibitively small explicit time step, an implicit method of integration is required. In LIM, the implicit VP solver currently used closely follows the successive relaxation method of Hibler (1979) with under-relaxation. For typical spatial resolutions of 50 km to 100 km, daily wind forcing variability and time step, Δt , of half a day to a quarter of a day, a

few hundred iterations per time step are required for the relaxation scheme to converge. A type of predictor-corrector scheme is used for the ice dynamics, whereby, starting from a solution at time t , an intermediate solution is first evaluated at time $t + \Delta t/2$. The solution at time $t + \Delta t$ is then calculated with the non-linear terms in the internal stress and ice-ocean stress terms centred at $t + \Delta t/2$. For time steps of between 1 day and a few hours, the solution thus obtained may be a rather crude approximation of the real plastic flow, and the resulting stress state is likely to lie away from the yield curve. Accuracy can be improved by repeating the predictor-corrector calculation several times, say n , with a subcycling time step $\Delta t/n$.

In contrast to the VP dynamics, the EVP dynamics can be solved explicitly with time steps that can be a few orders of magnitude larger than the maximum permissible explicit time step for the VP formulation. In addition, this maximum time step scales linearly, rather than quadratically, with d , although it also depends on the time scale T (Hunke and Dukowicz, 1997; Hunke and Lipscomb, 2008). The explicit numerical solution of the EVP equations is thus significantly less expensive than implicit methods for the VP formulation. Moreover, the convergence of the calculated stress state towards the elliptic yield curve is relatively fast because all the non-linear terms in (2.1) are reevaluated every time step. The EVP scheme is also easy to parallelize, and gives a higher speedup factor (measured as the ratio between wall clock serial time and wall clock parallel time) than parallel implementations of the VP method (Hunke and Zhang, 1999).

2.3.1 B-grid discretization

We have introduced in LIM three discretizations of the EVP dynamics. The first discretization is the one formulated by Hunke and Dukowicz (2002). These authors use a sophisticated variational method to calculate the internal stress force in discrete generalized orthogonal curvilinear coordinates. Formally working on a B-grid, and using bilinear approximations for the ice velocities over a given grid cell, the components of the strain rate and internal stress tensors are computed at the corners of each grid cell, thus requiring four computations per tensor component per grid cell. As Hunke and Dukowicz (2002) emphasize, this approach greatly helps to mitigate checkerboard mode solutions that are frequently generated on the B-grid.

The discretization of Hunke and Dukowicz (2002) is computationally expensive because it requires four calculations of each component of the strain rate and internal stress tensors per grid cell. As an alternative, we have also implemented in LIM a naïve, centred difference discretization on the B-grid. This second discretization is constructed as follows. On the B-grid, consider a grid cell whose central, or scalar, point has indexes i, j . As shown in Fig. 2.1, the indexes of the four corners, or velocity points, of the cell are then $i - 1/2, j - 1/2$ (left, bottom corner), $i + 1/2, j - 1/2$

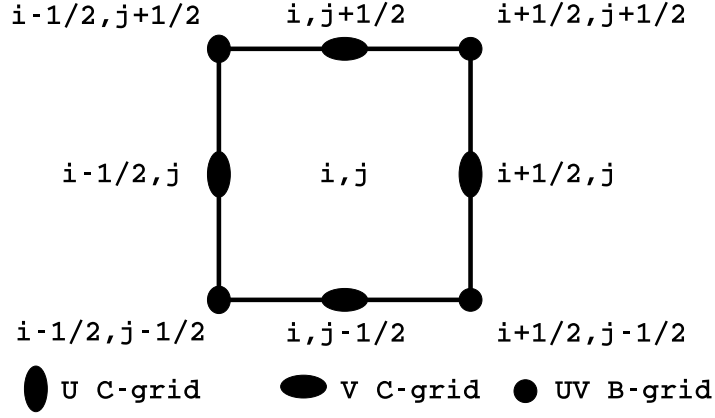


Figure 2.1: Centred difference grid showing the indexing convention used.

(right, bottom corner), $i + 1/2, j + 1/2$ (right, top corner), and $i - 1/2, j + 1/2$ (left, top corner). Analogously, the indexes of the four mid points on the grid cell sides, or transport points, are $i - 1/2, j$ (left, centre point), $i, j - 1/2$ (centre, bottom point), $i + 1/2, j$ (right, centre point), and $i, j + 1/2$ (centre, top point). Denoting the grid elements $e_1 = h_1 \Delta \xi_1$ and $e_2 = h_2 \Delta \xi_2$, where $\Delta \xi_1$ and $\Delta \xi_2$ are the spatial steps in the two orthogonal directions, the components of the strain rate tensor are given by

$$2 e_{1i,j} e_{2i,j} D_{Di,j} = e_{2i+1/2,j} (u_{i+1/2,j+1/2} + u_{i+1/2,j-1/2}) - e_{2i-1/2,j} (u_{i-1/2,j+1/2} + u_{i-1/2,j-1/2}) + e_{1i,j+1/2} (v_{i+1/2,j+1/2} + v_{i-1/2,j+1/2}) - e_{1i,j-1/2} (v_{i+1/2,j-1/2} + v_{i-1/2,j-1/2}), \quad (2.18)$$

$$2 e_{1i,j} e_{2i,j} D_{Ti,j} = e_{2i,j}^2 \left(\frac{u_{i+1/2,j+1/2} + u_{i+1/2,j-1/2}}{e_{2i+1/2,j}} - \frac{u_{i-1/2,j+1/2} + u_{i-1/2,j-1/2}}{e_{2i-1/2,j}} \right) - e_{1i,j}^2 \left(\frac{v_{i+1/2,j+1/2} + v_{i-1/2,j+1/2}}{e_{1i,j+1/2}} - \frac{v_{i+1/2,j-1/2} + v_{i-1/2,j-1/2}}{e_{1i,j-1/2}} \right), \quad (2.19)$$

$$2 e_{1i,j} e_{2i,j} D_{Si,j} = e_{1i,j}^2 \left(\frac{u_{i+1/2,j+1/2} + u_{i-1/2,j+1/2}}{e_{1i,j+1/2}} - \frac{u_{i+1/2,j-1/2} + u_{i-1/2,j-1/2}}{e_{1i,j-1/2}} \right) + e_{2i,j}^2 \left(\frac{v_{i+1/2,j+1/2} + v_{i+1/2,j-1/2}}{e_{2i+1/2,j}} - \frac{v_{i-1/2,j+1/2} + v_{i-1/2,j-1/2}}{e_{2i-1/2,j}} \right). \quad (2.20)$$

All three quantities $D_{Di,j}$, $D_{Ti,j}$ and $D_{Si,j}$ are defined on the centre of the grid cells. The components of the internal stress tensor, also defined on the grid cell centres, can now be evaluated by solving (2.13), (2.14) and (2.15). The internal stress force components, F_1 and F_2 , which are defined on the grid cell corners, can then be calculated as

$$\begin{aligned}
& 4 e_{1i+1/2,j+1/2} e_{2i+1/2,j+1/2} F_{1i+1/2,j+1/2} = \\
& e_{2i+1/2,j+1/2} (\sigma_{1i+1,j+1} + \sigma_{1i+1,j} - \sigma_{1i,j+1} - \sigma_{1i,j}) + \\
& \frac{1}{e_{2i+1/2,j+1/2}} \left(e_{2i+1,j+1/2}^2 (\sigma_{2i+1,j+1} + \sigma_{2i+1,j}) - e_{2i,j+1/2}^2 (\sigma_{2i,j+1} + \sigma_{2i,j}) \right) + \\
& \frac{2}{e_{1i+1/2,j+1/2}} \left(e_{1i+1/2,j+1}^2 (\sigma_{12i+1,j+1} + \sigma_{12i,j+1}) - e_{1i+1/2,j}^2 (\sigma_{12i+1,j} + \sigma_{12i,j}) \right).
\end{aligned} \tag{2.21}$$

$$\begin{aligned}
& 4 e_{1i+1/2,j+1/2} e_{2i+1/2,j+1/2} F_{2i+1/2,j+1/2} = \\
& e_{1i+1/2,j+1/2} (\sigma_{1i+1,j+1} + \sigma_{1i,j+1} - \sigma_{1i+1,j} - \sigma_{1i,j}) - \\
& \frac{1}{e_{1i+1/2,j+1/2}} \left(e_{1i+1/2,j+1}^2 (\sigma_{2i+1,j+1} + \sigma_{2i,j+1}) - e_{1i+1/2,j}^2 (\sigma_{2i+1,j} + \sigma_{2i,j}) \right) + \\
& \frac{2}{e_{2i+1/2,j+1/2}} \left(e_{2i+1,j+1/2}^2 (\sigma_{12i+1,j+1} + \sigma_{12i+1,j}) - e_{2i,j+1/2}^2 (\sigma_{12i,j+1} + \sigma_{12i,j}) \right).
\end{aligned} \tag{2.22}$$

The discretization in time of both the bilinear and centred difference formulations on the B-grid follows that of Hunke and Lipscomb (2008), and is

$$2T \frac{\sigma_1^{k+1} - \sigma_1^k}{\Delta t} + \sigma_1^{k+1} = \left(\frac{D_D^k}{\Delta^k} - 1 \right) P, \tag{2.23}$$

$$\frac{2T}{e^2} \frac{\sigma_2^{k+1} - \sigma_2^k}{\Delta t} + \sigma_2^{k+1} = \frac{D_T^k}{e^2 \Delta^k} P, \tag{2.24}$$

$$\frac{2T}{e^2} \frac{\sigma_{12}^{k+1} - \sigma_{12}^k}{\Delta t} + \sigma_{12}^{k+1} = \frac{D_S^k}{2e^2 \Delta^k} P, \tag{2.25}$$

$$\begin{aligned}
& m \frac{u^{k+1} - u^k}{\Delta t} = \\
& F_1^{k+1} + A \left(\tau_{a1} + c_D \rho_o |\mathbf{u}_o - \mathbf{u}^k| (u_o - u^{k+1}) \right) + m f v^{k+1} - m g \frac{1}{h_1} \frac{\partial \eta}{\partial \xi_1},
\end{aligned} \tag{2.26}$$

$$m \frac{v^{k+1} - v^k}{\Delta t} = F_2^{k+1} + A \left(\tau_{a2} + c_D \rho_o |\mathbf{u}_o - \mathbf{u}^k| (v_o - v^{k+1}) \right) - m f u^{k+1} - m g \frac{1}{h_2} \frac{\partial \eta}{\partial \xi_2}, \quad (2.27)$$

where, for expediency, we have dropped spatial sub-indexes, the super-indexes k and $k+1$ denote variables evaluated at times $k\Delta t$ and $(k+1)\Delta t$, respectively, where Δt is the dynamics time step, c_D is the ice-ocean drag coefficient, ρ_o is the reference density of seawater and $\mathbf{u}_o \equiv (u_o, v_o)$ is the surface oceanic current. The ice compressive strength P is updated only every thermodynamics and ice transport time step, which is normally orders of magnitude larger than Δt .

2.3.2 C-grid discretization

For each sea ice-ocean coupling step, the B-grid discretization requires interpolation of sea ice fields onto the ocean grid, which is of Arakawa C type, and, likewise, surface oceanic fields need also to be interpolated onto the sea ice grid. Within the sea ice model, interpolation of sea ice drift rates onto the midpoint of grid cell sides is also required before transport of sea ice scalars is calculated. Such interpolations would be avoided if a C-grid were used in the calculation of the sea ice dynamics, which is why we have also formulated a centred difference version of the EVP dynamics (see Kantha and Clayson (2000) for a C-grid, centred difference formulation of the VP case). Another important reason why a C-grid is desirable is that, with no slip boundary conditions, transport of scalar properties through narrow straits and passages with a width of just one single grid cell is possible on a C-grid, while it is precluded on a B-grid.

Using the same indexing conventions as for the B-grid, the discretization of the strain rate tensor is

$$e_{1i,j} e_{2i,j} D_{Di,j} = e_{2i+1/2,j} u_{i+1/2,j} - e_{2i-1/2,j} u_{i-1/2,j} + e_{1i,j+1/2} v_{i,j+1/2} - e_{1i,j-1/2} v_{i,j-1/2}, \quad (2.28)$$

$$e_{1i,j} e_{2i,j} D_{Ti,j} = e_{2i,j}^2 \left(\frac{u_{i+1/2,j}}{e_{2i+1/2,j}} - \frac{u_{i-1/2,j}}{e_{2i-1/2,j}} \right) - e_{1i,j}^2 \left(\frac{v_{i,j+1/2}}{e_{1i,j+1/2}} - \frac{v_{i,j-1/2}}{e_{1i,j-1/2}} \right), \quad (2.29)$$

$$e_{1i+1/2,j+1/2} e_{2i+1/2,j+1/2} D_{Si+1/2,j+1/2} = e_{1i+1/2,j+1/2}^2 \left(\frac{u_{i+1/2,j+1}}{e_{1i+1/2,j+1}} - \frac{u_{i+1/2,j}}{e_{1i+1/2,j}} \right) + e_{2i+1/2,j+1/2}^2 \left(\frac{v_{i+1,j+1/2}}{e_{2i+1,j+1/2}} - \frac{v_{i,j+1/2}}{e_{2i,j+1/2}} \right). \quad (2.30)$$

Note that the components D_D , D_T , σ_1 and σ_2 are all defined on the cell centres, while D_S and σ_{12} are defined on the corners. The internal stress force component F_1 is located on u points and F_2 on v points. In a C-grid, the velocity components are ideally located for the calculation of the components of the strain rate and the internal stress tensors, requiring fewer interpolations than on the B-grid. As the invariant Δ is used to compute the internal stress components, it needs, however, to be computed both on cell centres and corners, and so, for the purpose of the calculation of Δ , D_D and D_T must be interpolated onto cell corners, while D_S must be interpolated onto cell centres. The expression for D_D on cell corners is

$$\begin{aligned} & (e_{1i,j+1/2} + e_{1i+1,j+1/2})(e_{2i+1/2,j} + e_{2i+1/2,j+1})D_{Di+1/2,j+1/2} = \\ & e_{2i+1/2,j+1} (e_{1i+1,j+1/2} D_{Di,j} + e_{1i,j+1/2} D_{Di+1,j}) + \\ & e_{2i+1/2,j} (e_{1i+1,j+1/2} D_{Di,j+1} + e_{1i,j+1/2} D_{Di+1,j+1}), \end{aligned} \quad (2.31)$$

with an analogous formula for $D_{Ti+1/2,j+1/2}$, while the centred value of D_S is given by

$$4 D_{Si,j} = D_{Si-1/2,j-1/2} + D_{Si+1/2,j-1/2} + D_{Si+1/2,j+1/2} + D_{Si-1/2,j+1/2}. \quad (2.32)$$

The internal stress force components on the C-grid are

$$\begin{aligned} & 2 e_{1i+1/2,j} e_{2i+1/2,j} F_{1i+1/2,j} = \\ & e_{2i+1/2,j} (\sigma_{1i+1,j} - \sigma_{1i,j}) + \frac{1}{e_{2i+1/2,j}} (e_{2i+1,j}^2 \sigma_{2i+1,j} - e_{2i,j}^2 \sigma_{2i,j}) + \\ & \frac{2}{e_{1i+1/2,j}} (e_{1i+1/2,j+1/2}^2 \sigma_{12i+1/2,j+1/2} - e_{1i+1/2,j-1/2}^2 \sigma_{12i+1/2,j-1/2}), \end{aligned} \quad (2.33)$$

$$\begin{aligned} & 2 e_{1i,j+1/2} e_{2i,j+1/2} F_{2i,j+1/2} = \\ & e_{1i,j+1/2} (\sigma_{1i,j+1} - \sigma_{1i,j}) - \frac{1}{e_{1i,j+1/2}} (e_{1i,j+1}^2 \sigma_{2i,j+1} - e_{1i,j}^2 \sigma_{2i,j}) + \\ & \frac{2}{e_{2i,j+1/2}} (e_{2i+1/2,j+1/2}^2 \sigma_{12i+1/2,j+1/2} - e_{2i-1/2,j+1/2}^2 \sigma_{12i-1/2,j+1/2}). \end{aligned} \quad (2.34)$$

Time stepping of the internal stress tensor on the C-grid is identical to (2.23) to (2.25), but with σ_1 and σ_2 calculated on the centre of the grid cells and σ_{12} calculated on the corners. The momentum equation is now solved using

$$\begin{aligned} & m_u \frac{u^{k+1} - u^k}{\Delta t} = \\ & F_1^{k+1} + A_u (\tau_{a1} + c_D \rho_o |u_o - u^k|_u (u_o - u^{k+1})) + m_u f_u v_u^{k+c} - m_u g \frac{1}{h_1} \frac{\partial \eta}{\partial \xi_1}, \end{aligned} \quad (2.35)$$

$$m_v \frac{v^{k+1} - v^k}{\Delta t} = F_2^{k+1} + A_v \left(\tau_{a2} + c_D \rho_o |\mathbf{u}_o - \mathbf{u}^k|_v (v_o - v^{k+1}) \right) - m_v f_v u_v^{k+1-c} - m_v g \frac{1}{h_2} \frac{\partial \eta}{\partial \xi_2}, \quad (2.36)$$

where the subscripts u and v represent a variable defined on, or interpolated onto, u and v points, respectively, and where c is alternatively equal to 1 or 0. On odd iterations, $c = 0$ and (2.35) is solved first. Then, u^{k+1} is interpolated onto v points and is used to solve (2.36). On even iterations, $c = 1$ and (2.36) is solved first. The updated value of v is then interpolated onto u points to calculate the Coriolis term of (2.35). This procedure is equivalent to solving the Coriolis term semi-implicitly. Note that, unlike on a B-grid, an implicit treatment of the Coriolis term on a C-grid would require the simultaneous solution of (2.35) and (2.36) all across the domain, which would be computationally expensive.

2.3.3 Boundary conditions

The way in which boundary conditions are dealt with also differs from one grid to another. A no-slip condition is prescribed on land boundaries. On the B-grid, both components of the velocity vector are defined on the coast, and their value is therefore simply set to zero on land points. On the C-grid, in contrast, the normal velocity is defined and set to zero at the coast, but the tangential velocity is not defined. To impose a zero tangential velocity at the coast, a mirror velocity point is defined inland of the boundary, and its value is set to the opposite of the tangential velocity component seaward of the coast, thus delivering a no slip condition on the coast.

On a coarse resolution B-grid with a no slip boundary condition, plug flows along lateral boundaries are poorly reproduced. This can adversely affect the transport of sea ice properties if, as it is customary, longshore advective velocities are calculated as the average of the longshore velocity component on the coast, which is zero, and the nearest offshore velocity component. To alleviate this problem, the longshore component of the advective velocity on the B-grid is prescribed to be equal to the nearest offshore velocity component. This boundary condition for advection helps reducing the differences in ice transport and thickness between B-grid and C-grid simulations, and makes integrations carried out on either grid more easily comparable.

2.3.4 Linear plastic wave propagation in B- and C-grids

In ocean modelling, the problem of geostrophic adjustment in finite difference shallow water equations has traditionally provided a useful framework for the intercomparison of discrete staggered grids (Randall, 1994). Interestingly, a parallel

analysis can be carried out for the linearized sea ice momentum equations in the case of the cavitating fluid approximation, which consists in setting the parameter e in (2.8) and (2.9), or (2.14) and (2.15), to infinite, so that sea ice shear stresses vanish (Flato and Hibler, 1992). Assume a motionless sea ice cover of uniform thickness per unit area h_0 and concentration A_0 . Let us assume that the ice is experiencing compression and that all external forcing is zero. For time scales much larger than the damping time scale for sea ice elastic waves, T , the linearized sea ice momentum equations under the cavitating fluid approximation are :

$$m_0 \mathbf{u}'_t = -\nabla P' - m_0 f \mathbf{k} \times \mathbf{u}', \quad (2.37)$$

$$P'_t = -(1 + CA_0) P_0 \nabla \cdot \mathbf{u}', \quad (2.38)$$

where the primed quantities indicate small departures from the unperturbed values and m_0 and P_0 are the unperturbed sea ice mass per unit area and compressive sea ice strength, respectively. Equations (2.37) and (2.38) are formally equivalent to the shallow water equations linearized about a resting state (Randall, 1994). These equations admit free wave solutions with dispersion relation

$$\left(\frac{\sigma}{f}\right)^2 = 1 + \lambda^2 (k^2 + l^2), \quad (2.39)$$

where σ is the frequency of the wave, k and l are wavenumbers in the x and y directions, respectively (assuming the use of Cartesian coordinates), and λ is a deformation radius given by

$$\lambda = c f^{-1} = \left(\frac{(1 + CA_0) P^* e^{-c^*(1-A_0)}}{\rho_i} \right)^{\frac{1}{2}} f^{-1}, \quad (2.40)$$

where c is the phase speed of linear compressive inertia-plastic waves in sea ice and ρ_i is the ice density. For a typical central Arctic sea ice concentration in winter of $A_0 = 1$ and values used in our experiments of $P^* = 20 \times 10^3 \text{ N m}^{-2}$, $c^* = 20$ and $\rho_i = 900 \text{ kg m}^{-3}$, we obtain a phase velocity of plastic waves of 21.6 cm s^{-1} and a deformation radius of about 148 km. A similar analysis for the propagation of uniaxial plastic waves in sea ice has been carried out by Gray (1999).

The numerical equivalent of the dispersion relation (2.39) for the shallow water equations in different types of Arakawa grids was derived by Arakawa and Lamb and published by Randall (1994). Here we include the expressions for the B- and C-grids, which, assuming a square grid of cell size d , are :

$$\left(\frac{\sigma}{f}\right)^2 = 1 + \lambda^2 \frac{1 - \cos(kd) \cos(ld)}{d^2/2} \quad (2.41)$$

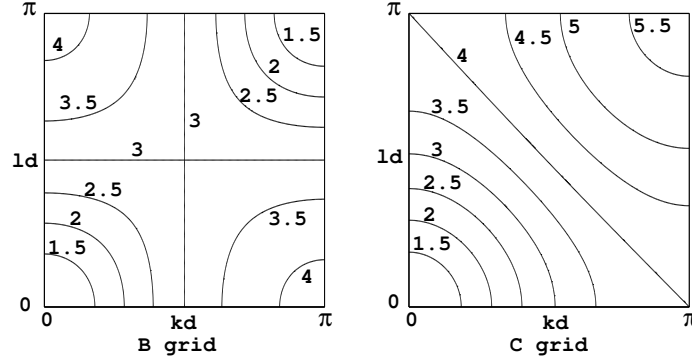


Figure 2.2: Contours of normalized frequency σ/f as a function of kd and ld from equations 2.41 (left) and 2.42 (right) with $\lambda/d = 2$.

and

$$\left(\frac{\sigma}{f}\right)^2 = \frac{1 + \cos(kd) + \cos(ld) + \cos(kd)\cos(ld)}{4} + \lambda^2 \frac{\sin^2(kd/2) + \sin^2(ld/2)}{d^2/4}, \quad (2.42)$$

respectively. For the LIM applications to be discussed below, the ratio of plastic deformation radius to grid size, λ/d , lies between 2 and 3. From Fig. 2.2, which displays contours of normalized frequency σ/f as a function of kd and ld for the case $\lambda/d = 2$, it is easy to see that the latter dispersion relation is closer to the exact relation, (2.39), than the former. In particular, on the C-grid, σ grows monotonically with k and l , and the associated group velocity is always positive, as it is in the continuous case, while, on a B-grid, σ is not monotonic, and group velocity components are negative whenever kd or ld become larger than $\pi/2$. Based on these considerations, the performance of the C-grid appears to be superior to that of the B-grid at resolutions of 75 km or less, thus providing additional motivation for using a C-grid sea ice model.

2.4 Results and discussion

All the simulations presented here have used the 2° global configuration of the model OPA-LIM described by Timmermann et al. (2005), including the forcing. The grid is tripolar, with all three poles located on land, and its minimum resolution is about 65 km in the Arctic and 50 km in the Antarctic. Fram Strait is resolved with 5 grid cells, while Bering Strait had to be artificially widened to two grid cells. Parry Channel, the strait connecting the Beaufort Sea and Baffin Bay, is represented by a two cell-wide passage through the Canadian Archipelago. Wind stress is computed with a drag coefficient of 1.4×10^{-3} , while ice-ocean drag uses a coefficient of $5.5 \times$

10^{-3} . No turning angle is applied for the calculation of air-ice, air-ocean or ice-ocean stresses. The sea ice strength parameter is $P^* = 20 \times 10^3 \text{ N m}^{-2}$, and the lead closing parameter $h_0 = 0.3 \text{ m}$.

The ocean model has a time step of 5760 s (96 minutes), which requires 15 ocean time steps per day, and is coupled to the sea ice model every five time steps. The dynamics time step is 120 s and the elastic time scale T is 40 times larger to ensure stability. The ocean model is initialized with temperature and salinity fields from the World Ocean Atlas (Levitus, 1983), while the sea ice model initial conditions are set to uniform sea ice thickness of 3 m in the Arctic and 1 m in the Antarctic. All experiments started from the same 1948-1959 spin-up integration on a B-grid and with VP ice dynamics used by Timmermann et al. (2005), but with $\Delta_{min} = 2 \times 10^{-9} \text{ s}^{-1}$ instead of $2 \times 10^{-8} \text{ s}^{-1}$.

The control run was conducted with the EVP formulation of the sea ice dynamics on a C-grid presented in Section 2.3.2. The results from this experiment do not greatly differ from the validation presented in Timmermann et al. (2005), and so we restrict our analysis to only a few salient features.

The Arctic sea ice extent, defined as the total oceanic area where the ice concentration is greater than, or equal to, 15%, oscillates from about $16 \times 10^6 \text{ km}^2$ in March to $10.5 \times 10^6 \text{ km}^2$ in September, in good accordance with observations. In summer, however, ice concentrations remain above 90% in the central Arctic, and relatively heavy ice covers persist in the Greenland and Barents Seas at times when observations show ice-free conditions. Ice thickness varies from more than 6 m along the Canadian Arctic Archipelago to 4 m in the central Arctic, and about 2 m over the western Siberian shelf (Fig. 2.3, top panels). A persistent feature of the model is that, compared with the observations of Bourke and Garrett (1987), it tends to somewhat overestimate the ice thickness along the Siberian coast: by about 1 m in the Kara Sea, 1.5 m in the Laptev Sea, and 2 m in the East Siberian Sea.

The two main patterns of the Arctic ice current system, namely the Beaufort gyre and the transpolar ice drift are present (Fig. 2.3, bottom left panel) but they are weaker than in data produced by the CERSAT from QuikSCAT and SSM/I drift vectors (Ezraty and Piollé, 2004). For the comparison, we interpolate monthly drift vectors from a 62.5 km polar stereographic grid to our simulation grid and we average data over all winter seasons (October to April) from 1992 to 2001 (Fig. 2.3, bottom right panel). Simulated velocities along the Alaskan North Coast in winter are about $0 - 2 \text{ cm s}^{-1}$ instead of $1 - 5 \text{ cm s}^{-1}$ in observations, and the maximum is located nearer to Bering Strait. The simulated transpolar drift is also weaker, $1 - 3 \text{ cm s}^{-1}$ instead of $2 - 4 \text{ cm s}^{-1}$, and its location is influenced by the absence of the Franz Joseph Archipelago in our configuration. This weaker drift is partly due to the

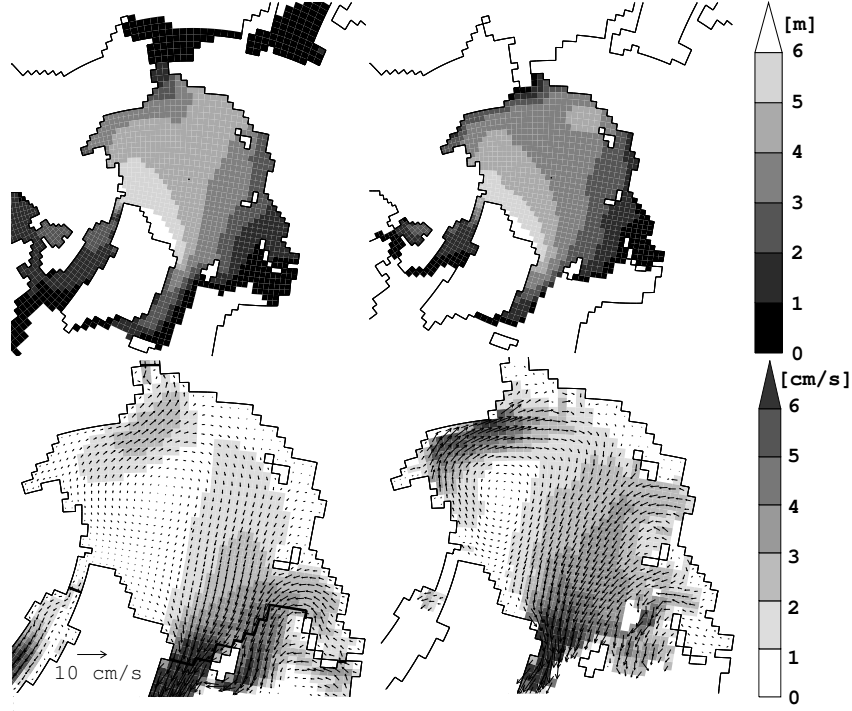


Figure 2.3: Simulated Arctic mean sea ice thicknesses in March (top left) and September (top right) for the period 1965-2001 and winter mean sea ice velocities (October to April) for the period 1992-2001 simulated by the model (bottom left) and from observations (bottom right). The black line in the third panel bounds the region used to calculate velocities distributions.

small ratio between the atmospheric drag coefficient (1.4×10^{-3}) and the ice-ocean drag coefficient (5.5×10^{-3}).

Following Martin and Gerdes (2007), we have computed histograms of monthly mean ice drift speed in the central Arctic basin to characterize the ice motion. For this calculation, the region of interest is the area located north of 70° N between 50° E and 270° E, and north of 80° N between 270° E and 50° E, as shown on the third panel of Fig. 2.3. The left panel of Fig. 2.4 shows histograms for March and September, both of which compare rather well with the observational estimates used by Martin and Gerdes (2007). A mode at very low speed exists for both months. In March, sea ice speeds are mainly below 0.05 m s^{-1} , and 45% of them are actually smaller than 0.01 m s^{-1} . In September, ice tends to be faster but, still, it almost nowhere reaches 0.10 m s^{-1} . While these results are encouraging, a more detailed

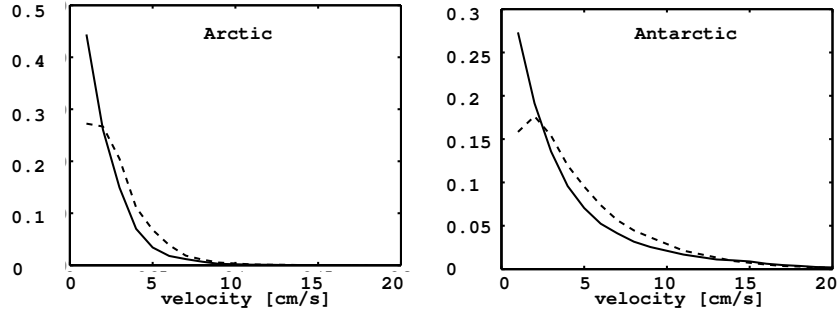


Figure 2.4: Left: Histogram of monthly mean ice velocities in the central Arctic (as delimited by the black line shown in the third panel of Fig. 2.3). Solid line for March and discontinuous line for September. Right: Histogram of monthly mean velocities in the Southern Ocean. Solid line for September and discontinuous line for March. Drift speeds below 0.5 cm s^{-1} have not been taken into account. The bin width is 1 cm s^{-1} , beginning at 0.5 cm s^{-1} . The period of interest is 1965-2001.

analysis should be conducted to ascertain the role that anomalous features of the ice distribution, such as the excess thickness along the Eurasian coast, play in modifying the flow of sea ice. The mean ice export through Fram Strait is about 0.11 Sv , in the upper limit of observational estimates.

In the Southern Ocean, the ice extent oscillates between $5 \times 10^6 \text{ km}^2$ in February and $19 \times 10^6 \text{ km}^2$ in August, values which are close to the observational ones. Sea ice concentrations are also quite realistic, except along coastlines, where polynya formation cannot be appropriately represented because of the coarse resolution of the model (Fig. 2.5, top panels). The winter mean ice velocity field for the period 1979-1997 (March to November) shows a fairly strong westward drift of $2-8 \text{ cm s}^{-1}$ all around Antarctica, and an offshore eastward drift of about $2-6 \text{ cm s}^{-1}$ (Fig. 2.5, bottom left panel). This is similar to the circulation obtained by averaging SSM/I monthly mean ice drift from the Atlas of Antarctic Sea Ice Drift (Schmitt et al., 2004) over the same period (Fig. 2.5, bottom right panel). Observation vectors have been interpolated on the simulation grid for comparison. The Weddell, Ross and Kerguelen gyres, with drift speeds of $2-5 \text{ cm s}^{-1}$, are all represented by the model, although the observed northward ice drifts in the Weddell and Ross regions are wider and somewhat stronger than in our simulation, presumably at least partly because of deficiencies in the surface wind stress. The histogram of velocities for the entire Southern Ocean reveals a sea ice cover that is significantly faster than its central Arctic counterpart. However, the ice speed mode is still in the vicinity of 0.01 m s^{-1} all year round.

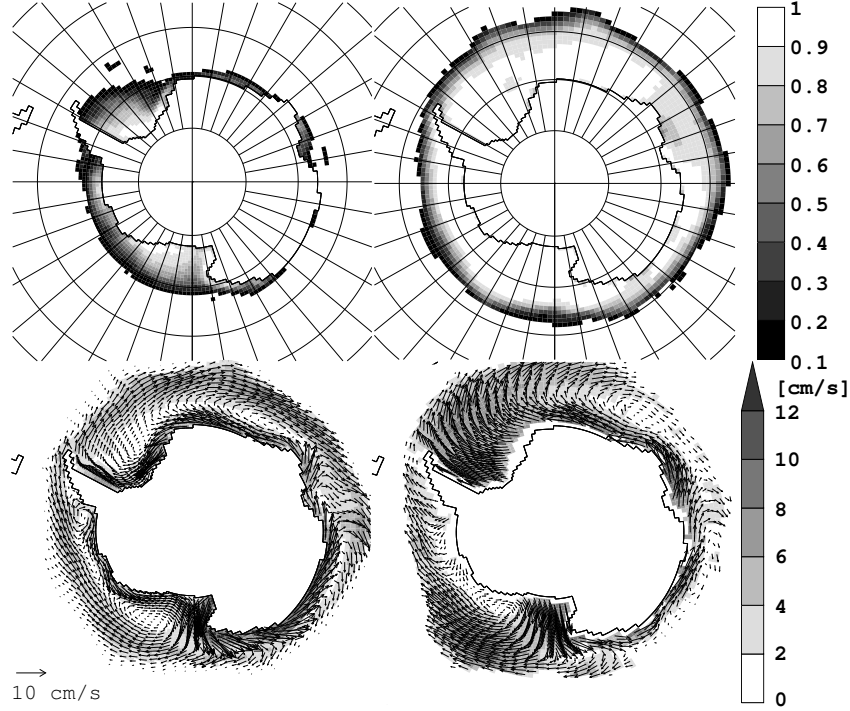


Figure 2.5: Simulated Southern Ocean mean sea ice concentration in March (top left) and September (top right) for the period 1965-2001 and winter mean sea ice velocities (March to November) for the period 1979-1997 simulated by the model (bottom left) and from observations (bottom right).

The results of the EVP simulation on a C-grid are quite encouraging. However, before adopting this formulation as standard, it is important to compare those results with output from integrations that use other discretization methods, as discussed in Section 2.3. To this end, we have conducted a number of 5-year experiments starting in 1960. These experiments are as follows. Experiment EVPB1 uses the EVP discretization of Hunke and Dukowicz (2002) on a B-grid. Experiment EVPB2 is as EVPB1, but with the simpler, centred difference discretization outlined in Section 2.3.1. The third experiment, EVPC1, uses the centred difference, C-grid formulation of Section 2.3.2, the same as in the control run. A second C-grid experiment, denoted EVPC2, was performed in which all straits and passages with a width of only one grid point were closed to ice advection. Finally, three experiments were also carried out using the VP, B-grid discretization of Fichefet and Maqueda (1997), and these are all collectively referred to as VPB experiments. The first VP experi-

ment (VPB1) uses $\Delta_{min} = 2 \times 10^{-9} \text{ s}^{-1}$ and 1 subcycling time step, as in the spin-up integration. The second experiment (VPB2) is like the first, but using 3 subcycling time steps (subcycling consists in solving the momentum equation with a time step smaller than the one used for ice thermodynamics and transport). The third experiment (VPB3) uses $\Delta_{min} = 2 \times 10^{-8} \text{ s}^{-1}$ and 3 subcycling time steps. The impact of the different methods of solution of the sea ice dynamics on the Southern Ocean ice cover are minuscule, and so we will concentrate on results for the Northern Hemisphere.

We start by examining the ability of VP to approximate a plastic flow. From the three simulations with VP, we evaluate the impact of the value of Δ_{min} and of the number of subcycling time steps on sea ice simulations with the VP rheology. To see

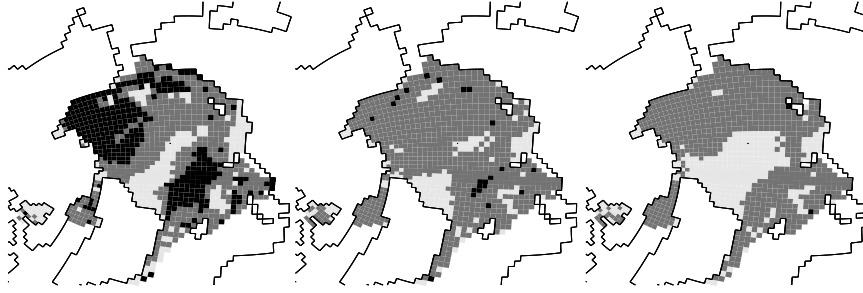


Figure 2.6: Left hand side of (2.12) calculated from instantaneous stress states on a simulation day of September 1964 in experiments VPB1 (left), VPB2 (centre) and VPB3 (right). Values larger than 1.2 (black) correspond to states located out of the ellipse, values between 0.8 and 1.2 (grey) correspond to states near the ellipse and values smaller than 0.8 (light grey) correspond to states inside the ellipse.

how well the elliptic relation between principal stress components is satisfied, we compute the quadratic function $(\sigma_1/P + 1)^2 + e^2(\sigma_s/P)^2$ from (2.12) and check how close it is to 1. A greater value corresponds to a state out of the elliptic yield curve, while a smaller value indicates a state inside the ellipse. With one subcycling time step, the VP formulation does a very poor job at representing the yield curve. In the example shown in the left panel of Fig. 2.6, we can see large regions of the Arctic are in a stress state located outside the ellipse. Ellipticity is partly recovered when 3 subcycling time steps are applied (central panel of Fig. 2.6), but there are still situations when the quadratic expression on the left hand side of (2.12) is greater than 1. Using 3 subcycling time steps and a higher value of Δ_{min} further improves the representation of the elliptical yield curve thanks to a better convergence of the numerical scheme, but it generates more creep flow, where the value of quadratic function is lower than 1, as shown in the right panel of Fig. 2.6. More elaborated

implicit schemes can be used to improve the convergence while using small values of Δ_{min} (Zhang and Hibler, 1997; Zhang and Rothrock, 2000).

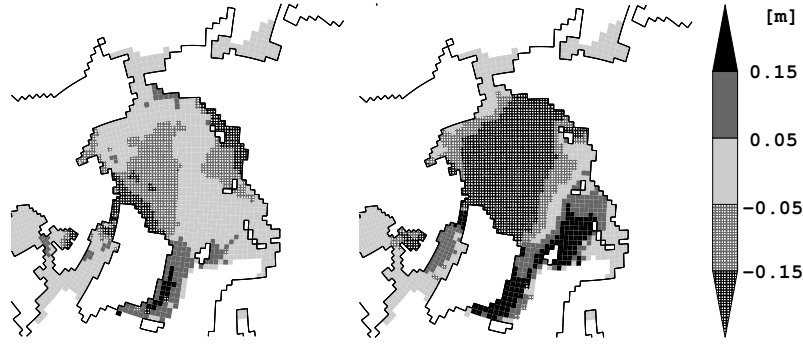


Figure 2.7: Mean ice thickness differences between experiments VPB2 and VPB1 (left) and between experiments VPB3 and VPB2 (right) in winter (October to December average for 1963 and 1964).

In terms of the representation of the Arctic winter mean (December to March) ice thickness geographical distribution, the impact of using 3 subcycling time steps (VPB2) instead of just 1 (VPB1) is a diminution of more than 0.15 m along the Canadian Arctic Archipelago, in the central Beaufort Sea and in the Laptev Sea (Fig. 2.7, left panel). In contrast, there is more ice along the eastern coast of Greenland. Differences are even more marked between experiments VPB3 and VPB2, which differ by the magnitude of Δ_{min} (Fig. 2.7, right panel). There is a sensible decrease in both the horizontal gradients and spatial mean value of the ice thickness across the entire central Arctic basin in VPB3 compared to VPB2. Ice is $\sim 10\%$ thinner in the central Arctic and $\sim 10\%$ thicker in the Greenland and northern Barents seas. This is so because the more viscous dynamics of VPB3 allows for a winter flow in the central Arctic that is 5-10% stronger than in VPB2, and causes as well an easier transport of ice across Fram Strait and Parry Channel.

The differences between the results of the three VPB experiments illustrate the importance of guaranteeing that plastic ice flow is well represented. This requires the use of relatively small values of Δ_{min} in the VP dynamics, which results in a fairly slow numerical convergence towards the plastic solution. In contrast, sensitivity tests with the EVP rheology show that a small value of Δ_{min} ($2 \times 10^{-12} \text{ s}^{-1}$, say) can be used without in any way compromising convergence. The EVP solution is also not affected by the particular value of T chosen, as long as it is several times smaller than the thermodynamics and transport time steps. Thus, with suitable

Δ_{min} and T values, ice internal stress states calculated with the EVP formulation tend to be much closer to the elliptical yield curve than those determined using the VP rheology.

The aim of experiments EVPB1, EVPB2, EVPC1 and EVPC2 was to examine the influence of the spatial discretization on EVP solutions. Overall, there is a reassuring similarity between the results of all four experiments, although we note that the B-grid tends to develop checkerboard patterns in certain variables, such as the ice divergence field. In most cases, this computational mode does not appear to have a direct impact on the ice dynamics, but it could be a problem if deformation rates are used in the calculation of the ice thickness redistribution. The use of a bilinear approach à la Hunke and Dukowicz (2002), rather than the more simple centred difference formulation we have proposed in Section 2.3.1, helps attenuating the checkerboard mode thanks to the spatial averaging involved in the calculation of the internal stress force. This numerical mode cannot appear on a C-grid.

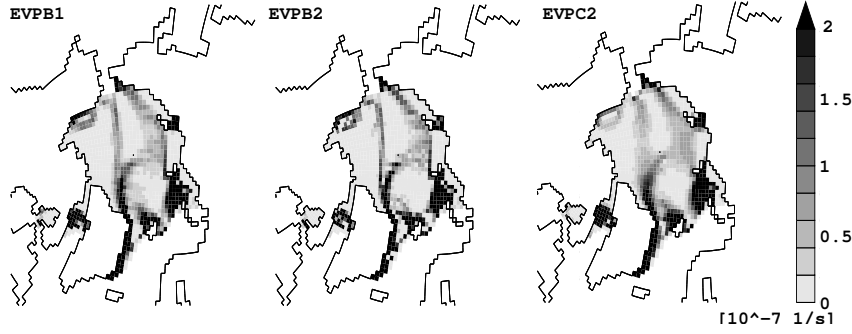


Figure 2.8: Instantaneous shearing strain rates in a simulation day of October 1964 with experiments EVPB1 (left), EVPB2 (centre) and EVPC2 (right).

An example of instantaneous shearing strain rates is shown in Fig. 2.8 for the EVPB1, EVPB2 and EVPC2 simulations. The shearing strain rate patterns look similar in all three experiments and show how the EVP dynamics is capable of creating spatial linear and relatively narrow features along which high ice deformation occurs. These linear features act as divides between regions where the ice motion is nearly rigid. Looking into these features more closely, we note that their detailed structure depends both on the discretization and on the relative orientation of linear features and the numerical grid. The B-grid with the simple EVP discretization (EVPB2) produces the narrowest features (just two cells wide) when they are aligned with the grid. However, as it might have been expected, checkerboard patterns in shear strain rates are very prominent in this discretization, while they are less so in

the cases EVPB1 and EVPC2. Nevertheless, these differences do not seem to influence the ice drift very much. Ice drift patterns are fairly similar in all simulations.

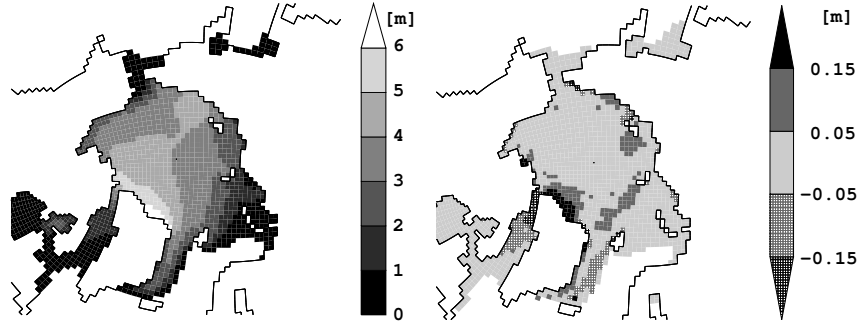


Figure 2.9: Mean ice thicknesses in experiment EVPB1 (left) and mean ice thickness differences between experiments EVPB2 and EVPB1 (right) in winter (October to December average for 1963 and 1964).

Winter mean sea ice thickness differences between EVPB1 and EVPB2 are fairly small except in the Lincoln Sea and along Parry Channel (Fig. 2.9, right panel). These very localized differences do not seem to have a significant large-scale impact, and so we have not investigated their origin in any great detail. The ice volume export through Fram Strait is virtually the same in both experiments: about 0.11 Sv, with a standard deviation of 0.08 Sv.



Figure 2.10: Mean ice thickness differences between experiments EVPC1 and EVPB1 (left) and between experiments EVPC2 and EVPB1 (right) in winter (October to December average for 1963 and 1964).

Of perhaps greater interest are the differences between the B-grid and C-grid integrations. The left panel of Fig. 2.10 shows the difference in winter mean ice thickness between experiments EVPC1 and EVPB1. Ice in the central Arctic tends to be thinner in the EVPC1 experiment, with the exception of a few choke points, such as Parry Channel and the passages between Eurasia and the New Siberian Islands, Severnaya Zemlya and Novaya Zemlya. These passages are represented with just one grid point in our model, and, therefore, no ice can flow through them when a B-grid is used. As a result, ice simulated in EVPB1 tends to accumulate on the upstream side of these passages and, thus, be thicker than it would be on a C-grid, while, downstream of the passages, the ice tends to be generally thinner. This is confirmed by the results of a second C-grid experiment, EVPC2, in which all one cell straits were closed to ice transport. The right panel of Fig. 2.10 shows that the differences in winter sea ice thickness between this experiment and EVPB1 have sensibly decreased on both sides of the narrow central Arctic passages. In both EVPC1 and EVPC2, ice area and volume exports across Fram Strait are about 5% smaller than in EVPB1. In contrast, ice circulation in the Beaufort Gyre is slightly stronger (+5%), and southward velocities along the east Greenland coast are around 10% larger, which is consistent with the existence of the band of thicker ice one can observe all along the eastern Greenland coast, and that counters the larger onshore component of the Coriolis force associated with the 10% higher velocities.

2.5 Conclusions

The elastic-viscous-plastic (EVP) sea ice rheology formulation of Hunke and Dukowicz (2002) has been incorporated in the Louvain-la-Neuve sea-Ice Model, version 2 (LIM2), which has until now used a viscous-plastic (VP) sea ice dynamics (Hibler, 1979). Two centred difference discretizations of the EVP rheology have also been introduced, one defined on a B-grid and the second defined on a C-grid. These centred difference formulations help reducing computation costs for the sea ice dynamics by about a factor of 2 compared to the more sophisticated approach of Hunke and Dukowicz (2002).

Because LIM runs normally coupled to a C-grid ocean model (the Nucleus for European Modelling of the Ocean, NEMO), we have retained the centred difference C-grid formulation of the EVP dynamics as the most suitable for LIM. The performance of the coupled ice-ocean model has been illustrated with results from a long simulation spanning the years 1960 to 2001. The model reproduces well the sea ice concentration, thickness and velocity fields in both hemispheres.

We have conducted a number of short experiments to test the sensitivity of the model to the use of the different implementations of the ice rheology. We have

first compared the performance of the original VP formulation in LIM with the new EVP implementation. Attaining appropriate plastic behavior with the VP dynamics requires the use of subcycling and a fairly small value of Δ_{min} (on the order of $2 \times 10^{-9} \text{ s}^{-1}$ or less), which hinders convergence of the relaxation algorithm used by the VP method. In contrast, computing time is not affected by the value of Δ_{min} when using the EVP formulation.

We have also intercompared EVP simulations with different spatial discretizations. The two B-grid discretizations we have investigated, namely, the one by Hunke and Dukowicz (2002) and the simplified discretization described in Section 2.3.1, lead both to very similar results, at least for relatively coarse spatial model resolutions. Disparities in ice thickness between the centred difference formulations on B-grid and C-grid are largely associated with slower ice transport through narrow straits and along coastlines on the B-grid. There is also a tendency for the C-grid formulation to produce slightly higher ice velocities in the central Arctic, but also a reduced ice export through Fram Strait.

In summary, the new EVP formulation on a C-grid implemented in LIM is able to produce realistic mean ice drift in both hemispheres. In addition, the EVP ice dynamics approaches plasticity better and is numerically more efficiently than the previous VP implementation. The C-grid EVP formulation is now the standard method for solving the ice dynamics in NEMO-LIM.

Motivation for the next chapter

The performances of the VP and EVP rheologies in terms of statistical and scaling properties of the sea ice drift and deformation have been analysed with simulations at high resolution (Girard et al., 2009). We found that results produced by LIM are similar to observations in terms of large-scale velocity field and distributions of velocity fluctuations. However VP and EVP do not generate power law tailed strain rate distributions and are not able to reproduce the spatial and temporal scaling that characterizes the heterogeneity and intermittency of the ice pack deformation.

We also implemented the VP rheology in the finite element model SLIM and showed a strong discrepancy between results and observations in terms of statistical properties of the ice deformation rate (Girard et al., 2011). VP was found unable to reproduce high deformation rates observed within the ice cover, even in simulations at high resolution (10 km).

Other authors found large differences between sea ice deformation fields from classical models and observations (Coon et al., 2007; Kwok et al., 2008). On the other hand, some authors claimed that VP and EVP were able to produce realistic linear kinematic features when used at high resolution (Wang and Wang, 2009;

Maslowski and Lipscomb, 2003). The following chapter investigates this issue and leads to a complete reinterpretation of the EVP model.

CHAPTER 3

THE ELASTIC-VISCOUS-PLASTIC MODEL REVISITED

This Chapter is based on the following paper, submitted on 21 December 2012:
S. Bouillon, T. Fichefet, V. Legat, and G. Madec. The elastic-viscous-plastic model revisited. *submitted to Ocean Modelling*, 2012.

Abstract

The elastic-viscous-plastic method (EVP) is revisited as a point iterative solver dedicated to the non-linear viscous-plastic sea ice model (VP). A clear distinction appears between the numerical and physical parameters. Two numerical parameters entirely determine the stability and convergence of the method and are adjusted to mitigate spurious ice deformation frequently observed with EVP. The changes in the implementation correspond to a new regularization of the internal stress enabling better convergence for all components. Comparison of the old and new regularizations highlights the strong link between the generation of linear bands of high sea ice deformation and numerical problems, characterized by large remaining errors in the approximate solution.

3.1 Introduction

This chapter focus on the accuracy of the solution of the VP equations. In particular, we show that the influence of badly resolved solutions leads to completely different results with artificial linear bands of high deformation that can be confounded with the real ones.

One of the most widely used methods to solve the non-linear VP rheology is the elastic-viscous-plastic (EVP) method thoroughly documented in Hunke and Dukowicz (1997); Hunke and Dukowicz (2002) and Hunke and Lipscomb (2008). We will refer here to the version presented in Hunke (2001) and to the modified version proposed by Lemieux et al. (2012). The trivial parallelization of the EVP method explains its success in coupled ice-ocean models. However, this method presents several drawbacks. The slow convergence to a solution implies a strong impact of the numerical parameters on the results when using a limited number of iterations (Losch et al., 2010; Losch and Danilov, 2011). Moreover, the method is sometimes presented as a new rheology and that may confuse the choice of parameters and the interpretation of results. Another serious problem of the EVP method is the generation of spurious deformation in zones of near rigid ice (Hunke, 2001). This contributes to make multi-category sea ice models unstable at high resolution (Lipscomb et al., 2007) and also causes large biases in the ice thickness by two-level sea ice models (Lemieux et al., 2012). This problem is critical in the Canadian Arctic Archipelago (CAA) where deformation rates are very low and ice thickness is relatively high.

Other methods are also used to solve the non-linear VP equations. Most of them are based on a linearization of the rheology with non-linear terms computed from velocities at the previous time step. The resulting linear system of equations is solved implicitly with various linear solvers. The non-linear equations are supposed to be correctly approximated if the time step is much smaller than the forcing time scale. This constraint is relaxed by using outer-loop iterations (or pseudo-time steps) during which all the terms are considered constant except the rheology (Zhang and Hibler, 1997; Zhang and Rothrock, 2000). In all cases, a linear solver is needed, making the parallelization of the code more complex. All those methods also present poor rates of convergence and large residual errors (Lemieux and Tremblay, 2009).

An interesting approach is the use of general non-linear solvers. Lemieux et al. (2010) have implemented a Jacobian-free Newton-Krylov method which is based on a more sophisticated linearization of the VP equations. The resulting matrix also contains derivatives relative to the unknowns, speeding up the convergence of the non-linear problem. A linear solver is still needed and the efficiency has not been evaluated in parallel environments. Their approach however, based on the

systematic analysis of method convergence and stability as well as the accuracy of the solution, has inspired some aspects of the present study.

The first goal of our study is the mitigation of spurious ice deformation produced by the EVP method in nearly rigid ice regions. By reinterpreting EVP as a point iterative solver, we define clear criteria for stability and local convergence, leading to a better control of spurious ice deformation. This reinterpretation of EVP leads to a new regularization of the internal stress ensuring the same convergence for all stress components. Surprisingly this new regularization does not produce linear bands of high ice deformation aligned with the grid as does the one initially proposed by Hunke and Dukowicz (1997). These features are also observed in other studies and seem to be linked to numerical artefacts.

This chapter is organized in three sections. The first one shortly recalls the VP model equations. The second one presents the spatial discretization of the VP equations on a C-grid and presents a reinterpretation of EVP as an iterative solver, allowing to study the convergence of the method. Several simulations are presented in the third section to illustrate the effects of numerical parameters on the behaviour of the method and their impacts on results. A comparison of results obtained with the old and new regularization of the internal stress shows the strong impact of numerical details on the generation of linear bands of high ice deformation and their correlation with large remaining errors in the velocity field.

3.2 Model description

The viscous-plastic sea ice model proposed by Hibler (1979) is widely used in many large-scale ice-ocean models (e.g., CCSM, MITgcm and NEMO-LIM). It is based on the assumption that sea ice has no (or little) tensile strength but plastically responds to compressive and shear deformations. Combined with a parameterisation of the ice strength as a function of the ice concentration and thickness, this model is able to reasonably well reproduce the mean ice drift and leads to a realistic ice thickness distribution within the Arctic basin (e.g., Zhang and Rothrock, 2000).

The momentum equation obtained by integrating the three-dimensional momentum equation through the snow and ice in the vertical direction is

$$m \partial_t \mathbf{u} = \nabla \cdot \boldsymbol{\sigma} + A(\boldsymbol{\tau}_a + \boldsymbol{\tau}_w) - m f \mathbf{k} \times \mathbf{u} - m g \nabla \eta, \quad (3.1)$$

where m is the total mass of snow and ice per unit area, ∂_t is the temporal derivative, $\mathbf{u}$ is the horizontal ice velocity and $\boldsymbol{\sigma}$ is the internal ice stress tensor. The momentum advection is ignored because inertial forces are small compared to other forces in most applications (Fichefet et al., 1998). The wind stress $\boldsymbol{\tau}_a$ and the ocean

Symbol	Definition	Units
P	ice strength	$N m^{-1}$
m	mass of snow and ice per unit area	$Kg m^{-2}$
h	mean ice thickness	m
A	ice concentration	-
Δ	measure of the deformation rate	s^{-1}
D	rate of internal work	$N m s^{-1}$
$\mathbf{F} = (F_u, F_v)$	stress divergence	$N m^2$
$\dot{\epsilon}_{ij}$	component of the strain rate tensor	s^{-1}
σ_{ij}	component of the stress tensor	$N m^2$
$\mathbf{u} = (u, v)$	ice velocity	$m s^{-1}$
τ_w, τ_a	ocean and atmosphere stresses	$N m^2$
η	sea surface elevation	m

Table 3.1: Definition and units of the variables used in the text.

stress τ_w are multiplied by the ice concentration, A , following the proper area scaling determined by Connolley et al. (2004) and Hunke and Dukowicz (2003). τ_a is an external forcing and τ_w is given by a quadratic expression, $c_D \rho_o |\mathbf{u}_o - \mathbf{u}|(\mathbf{u}_o - \mathbf{u})$, where c_D is the water drag coefficient, ρ_o is the reference density of seawater and $\mathbf{u}_o$ is the oceanic surface velocity. The last two terms of equation (3.1) are related to the Coriolis effect and to the slope of the sea surface elevation η ; f is the Coriolis parameter, $\mathbf{k}$ is a unit vector normal to the reference surface of the Earth and g is gravity. Definitions and units of symbols used in the text are detailed in Table 3.1.

The VP constitutive law relates the internal ice stress tensor σ to the strain rate tensor $\dot{\epsilon} = \frac{1}{2}(\nabla \mathbf{u} + (\nabla \mathbf{u})^T)$ by:

$$\sigma_{ij} = \frac{P}{2(\Delta + \Delta_{min})} \left[(\dot{\epsilon}_{kk} - \Delta) \delta_{ij} + \frac{1}{e^2} (2\dot{\epsilon}_{ij} - \dot{\epsilon}_{kk} \delta_{ij}) \right], \quad (3.2)$$

where P is the ice strength, Δ is a particular measure of the deformation rate and Δ_{min} is a parameter determining a smooth transition from pure viscous flow ($\Delta \ll \Delta_{min}$) to pure plastic flow ($\Delta \gg \Delta_{min}$). $\dot{\epsilon}_{kk}$ is the divergence strain rate, δ_{ij} is the Kronecker unit tensor and e is a parameter giving the ratio between the maximum compressive stress and twice the maximum shear stress. Δ is given by

$$\Delta = \sqrt{\dot{\epsilon}_{kk}^2 + \frac{1}{e^2} \dot{\epsilon}_s^2}, \quad (3.3)$$

where $\dot{\epsilon}_s$ is the shear strain rate given by $\dot{\epsilon}_s = \sqrt{(\dot{\epsilon}_{11} - \dot{\epsilon}_{22})^2 + 4\dot{\epsilon}_{12}^2}$. Note that $\frac{P}{2(\Delta + \Delta_{min})}$ in equation (3.2) corresponds to the non-linear bulk viscosity defined in Hibler (1979). This formulation of the VP rheology is differentiable, well defined, and gives no internal stress when Δ tends to zero.

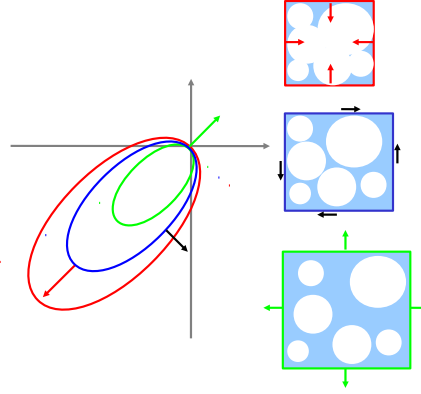


Figure 3.1: Schematic description of the VP rheology, with on the left, the three yield curves corresponding to three different ice pack states on the right. The white disks represent ice floes and the blue color represents the open water area. From an initial yield curve in blue, three different cases are displayed. Under pure convergence (red arrows), the internal ice stress is high and negative, opposing the deformation. The ice concentration and thickness increase (red square), leading to a higher ice strength (red ellipse). Under pure shear (black arrows), the ice resists the deformation but no change occurs in the ice concentration or thickness. Under pure divergence (green arrows), the ice offers no resistance and the ice concentration and thickness decrease (green square), leading to a rapid reduction of the ice strength (green ellipse).

In the pure plastic case, the compressive stress σ_{kk} and the shear stress $\sigma_s = \sqrt{(\sigma_{11} - \sigma_{22})^2 + 4\sigma_{12}^2}$ define an ellipse aligned with compressive stress in the space of the principal stress components (Figure 3.1). This elliptical yield curve is not centred on the origin but translated to be entirely in the $\sigma_{kk} \leq 0$ (compressive) part of the stress space. The length of the major axis gives the maximum compressive stress and is equal to $2P$. The ratio between the major axis and the minor axis is given by e . The response of the ice does not depend on the magnitude of the deformation rate but only on the direction of the strain rate vector defined by $\dot{\epsilon}_{kk}$ and $\dot{\epsilon}_s$. For each possible stress state on the ellipse, principal components of the strain rate are normal to the yield curve if the principal stress and strain rate axes coincide. The ice has then no resistance to pure divergence but resists compression and shearing motion.

According to Hibler (1979), the ice strength P is parameterised as

$$P = P^* h e^{c^*(1-A)}, \quad (3.4)$$

where P^* and c^* are empirical parameters. h is the mean ice thickness, corresponding to the volume of ice per unit area. Other yield curves, flow rules and strength

parameterisations have been proposed (e.g., Zhang and Rothrock, 2005), but this combination of the elliptical yield curve, the normal flow rule and Hibler's parameterisation for the ice strength is very popular.

3.3 The elastic-viscous-plastic method revisited

We reinterpret the EVP method to address the generation of spurious ice deformations in near rigid ice regions which may cause instabilities and ice thickness biases and are not produced with other methods. The persistence of these oscillations is often attributed to a lack of damping (Hunke, 2001), but our analysis shows they also result from numerical instabilities due to the choice of the numerical parameters. Our first attempt to improve EVP leads to a new discretization of the VP equations on C-grids that respects the fundamental property of energy dissipation, following the approach proposed by Hunke and Dukowicz (2002) for B-grids. However, it is the modification of the temporal discretization that leads to real improvements of the EVP method. Our reinterpretation of EVP follows the approach of Lemieux et al. (2012), leading to clear criteria for stability and to better information on the convergence.

3.3.1 The spatial discretization

The VP equations are often solved on the C-grid (e.g., Bouillon et al., 2009; Losch et al., 2010; Lemieux et al., 2012) but the discretized equations do not respect the fundamental property of energy dissipation of the VP rheology. It is important to preserve this property in the discretization such that the discrete system dissipates energy in the same manner as the continuous system (Hunke and Dukowicz, 2002).

The rate of change of the ice kinetic energy, D , summed over S , the area covered by the ice pack, is given by:

$$D = - \int (\sigma_{11} \dot{\epsilon}_{11} + \sigma_{22} \dot{\epsilon}_{22} + 2\sigma_{12} \dot{\epsilon}_{12}) dS. \quad (3.5)$$

Introducing the constitutive law equations and considering Δ_{min} negligible lead to

$$D = - \int \frac{P}{2\Delta} \left[(\dot{\epsilon}_{kk} - \Delta) \dot{\epsilon}_{kk} + \frac{1}{\rho^2} ((\dot{\epsilon}_{11} - \dot{\epsilon}_{22})^2 + 4\dot{\epsilon}_{12}^2) \right] dS. \quad (3.6)$$

Using the definition of Δ gives

$$D = - \int \frac{P}{2} [\Delta - \dot{\epsilon}_{kk}] dS. \quad (3.7)$$

As Δ is obviously larger than or equal to $\dot{\epsilon}_{kk}$ and P is positive, D is always negative and the internal ice force over the ice pack works to dissipate energy.

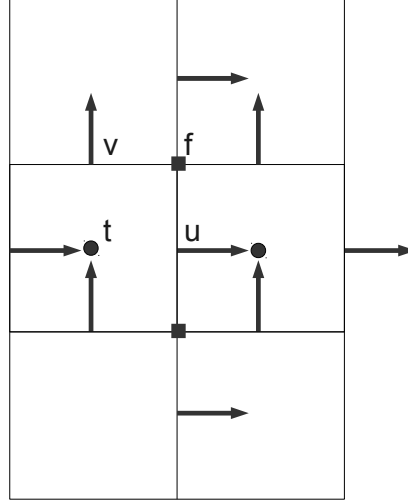


Figure 3.2: Arrangement of variables on the C-grid. Velocities are defined on u - and v -points. Scalars are defined on t -points, as well as σ_1 , σ_2 , $\dot{\epsilon}_1$ and $\dot{\epsilon}_2$. σ_{12} and $2\dot{\epsilon}_{12}$ are defined on f -points.

The spatial discretization is based on the C-grid arrangement of variables (Figure 3.2) with cells centred on scalar points (t -points). Velocity points (u - and v -points) are defined at the centre of each edge of the cells. In order to minimize the number of averaging operations for the evaluation of the different derivatives, $\dot{\epsilon}_1$ and $\dot{\epsilon}_2$ are defined as the values of $\dot{\epsilon}_{kk}$ and $(\dot{\epsilon}_{11} - \dot{\epsilon}_{22})$, respectively, at the centre of each cell (t -points), whereas $\dot{\epsilon}_{12}$ is computed on the corners of the cells (f -points). σ_1 and σ_2 are then naturally defined as σ_{kk} and $(\sigma_{11} - \sigma_{22})$, respectively, on t -points and σ_{12} on f -points by

$$\sigma_1 = \frac{P}{(\Delta + \Delta_{min})} (\dot{\epsilon}_1 - \Delta), \quad (3.8)$$

$$\sigma_2 = \frac{P}{(\Delta + \Delta_{min})} \frac{\dot{\epsilon}_2}{e^2}, \quad (3.9)$$

$$\sigma_{12} = \frac{P}{(\Delta + \Delta_{min})} \Big|_f \frac{\dot{\epsilon}_{12}}{e^2}, \quad (3.10)$$

$$\Delta = \sqrt{\dot{\epsilon}_1^2 + \frac{1}{e^2} (\dot{\epsilon}_2^2 + 4 \dot{\epsilon}_{12}^2|_t)}, \quad (3.11)$$

where $q|_t$ (and $q|_f$) is a finite differences operator computing the average of f -points variables on t -points (and of t -points variables on f -points). The energy

dissipation property of the discretized equations depends on the definition of these operators.

Here we develop the expression of D from the discretized variables to verify the dissipative property. The integral (3.7) becomes a sum of different terms weighted by the grid local area $e_{1t}e_{2t}$ and $e_{1f}e_{2f}$, where e_{1t} , e_{2t} , e_{1f} and e_{2f} are the grid lengths in each direction as defined at t - and f -points:

$$D = - \sum_{i,j} \frac{P}{2\Delta} \left[(\dot{\epsilon}_1 - \Delta) \dot{\epsilon}_1 + \frac{1}{e^2} \dot{\epsilon}_2^2 \right] e_{1t}e_{2t} - \sum_{i,j} \frac{1}{e^2} \mathcal{M} \left(\frac{P}{2\Delta} \right) 4\dot{\epsilon}_{12}^2 e_{1f}e_{2f}, \quad (3.12)$$

where $\mathcal{M}(\cdot)$ is an operator that provides the value of t -point defined variables on a f -point. Using $\mathcal{M}^*$, the inverse operation leads to:

$$D = - \sum_{i,j} \frac{P}{2\Delta} \left[(\dot{\epsilon}_1 - \Delta) \dot{\epsilon}_1 + \frac{1}{e^2} \left(\dot{\epsilon}_2^2 + \frac{1}{e_{1t}e_{2t}} \mathcal{M}^* (4\dot{\epsilon}_{12}^2 e_{1f}e_{2f}) \right) \right] e_{1t}e_{2t}. \quad (3.13)$$

Using the definition of Δ at t -points,

$$\Delta = \sqrt{\dot{\epsilon}_1^2 + \frac{1}{e^2} \left(\dot{\epsilon}_2^2 + \frac{1}{e_{1t}e_{2t}} \mathcal{M}^* (4\dot{\epsilon}_{12}^2 e_{1f}e_{2f}) \right)}, \quad (3.14)$$

gives the same inequality as for the continuous equation:

$$D = - \sum_{i,j} \frac{P}{2} [\Delta - \dot{\epsilon}_1] e_{1t}e_{2t} \leq 0. \quad (3.15)$$

Expression for the only two averaging operations are obtained by defining

$$\mathcal{M}(q) = \overline{\overline{q}}^{i+1/2, j+1/2}, \quad (3.16)$$

with the following auto-adjoint operator:

$$\overline{\overline{q}}^{i,j} = \frac{q_{i,j} + q_{i,j-1} + q_{i-1,j} + q_{i-1,j-1}}{4}. \quad (3.17)$$

Indices i and j cover all of the domain and indicate where a quantity is computed. (i, j) corresponds to t -points, $(i + 1/2, j)$ to u -points, $(i, j + 1/2)$ to v -points and $(i + 1/2, j + 1/2)$ to f -points. This leads to this choice of operators:

$$\dot{\epsilon}_{12}^2|_t = \frac{1}{e_{1t}e_{2t}} \overline{\overline{\dot{\epsilon}_{12}^2}}^{i,j} e_{1f}e_{2f}, \quad (3.18)$$

$$\frac{P}{\Delta}|_f = \frac{\overline{\overline{P}}^{i+1/2, j+1/2}}{\Delta}. \quad (3.19)$$

Another simple solution is possible by defining $\mathcal{M}$ as

$$e_{1f}e_{2f}\mathcal{M}(q) = \frac{\overline{\overline{q}}^{i+1/2,j+1/2}}{e_{1t}e_{2t}}, \quad (3.20)$$

and corresponds to

$$\dot{\epsilon}_{12}^2|_t = \overline{\overline{\dot{\epsilon}_{12}^2}}^{i,j}, \quad (3.21)$$

$$\frac{P}{\Delta}|_f = \frac{P}{e_{1f}e_{2f}} \frac{1}{\Delta} \overline{\overline{1}}^{i+1/2,j+1/2}. \quad (3.22)$$

In the case of a regular grid, the two operators are given by:

$$q|_t = \frac{q_{i+1/2,j+1/2} + q_{i-1/2,j+1/2} + q_{i-1/2,j-1/2} + q_{i+1/2,j-1/2}}{4}, \quad (3.23)$$

$$q|_f = \frac{q_{i,j} + q_{i+1,j} + q_{i+1,j+1} + q_{i,j+1}}{4}, \quad (3.24)$$

and are only used to average $\dot{\epsilon}_{12}^2$ on t -points to compute Δ and to average $\frac{P}{(\Delta + \Delta_{min})}$ on f -points to compute σ_{12} . Note that any linear combination of $\dot{\epsilon}_{12}$ on t -points and any linear combination of Δ on f -points will fail in satisfying the constraint of kinetic energy dissipation.

This averaging method is the only difference in the spatial discretization presented in Bouillon et al. (2009). This change in the spatial discretization has however not led to noticeable change in results. In particular, it does not help in reducing the noise in ice deformation fields nor does it lead to faster convergence. This new discretization is used for other experiments made for this study.

3.3.2 The temporal discretization

Contrary to the correction of the spatial discretization, our modification of the temporal discretization of the VP model has significant impacts on the quality of the simulated fields. Our approach actually leads to a complete reinterpretation of the EVP method and greatly simplifies the choice of numerical parameters and the analysis of simulation results.

In order to illustrate our approach and to ease the analysis, the following simplified one-dimensional version of the VP problem will be used:

$$\sigma = \frac{P}{2\Delta} \partial_x u, \quad (3.25)$$

$$m \partial_t u = \partial_x \sigma + f(u), \quad (3.26)$$

$$\Delta = |\partial_x u|, \quad (3.27)$$

where u and $f(u)$ are the ice velocity and the external stress in the x direction, respectively, and ∂_x is the spatial derivative. f contains terms depending on u , since the Coriolis and ice-ocean drag terms are both function of u .

The simplest temporal discretization is an explicit scheme to advance from time step n to time step $n + 1$ using:

$$m \frac{u_{n+1} - u_n}{\Delta t} = \partial_x \frac{P}{2\Delta_n} \partial_x u_n + f_{n+1}. \quad (3.28)$$

f is solved implicitly to avoid numerical instabilities due to the Coriolis term. The stability criterion scales quadratically with Δx as

$$\Delta t \leq \frac{m\Delta}{P} \Delta x^2. \quad (3.29)$$

If the deformation rate Δ is 10^{-9} s^{-1} , the spatial resolution $\Delta x = 50 \text{ km}$, $h = 1 \text{ m}$, $P = 20000 \text{ N m}^{-1}$ and $m = 1000 \text{ kg m}^{-2}$, the order of magnitude of the required time step would be 10 s . To allow larger time steps, implicit schemes are preferred.

Solving of the full implicit equations is not easy because of the presence of the term $\frac{P}{2\Delta}$ in equation (3.25) where Δ depends on u . First implementations simply used the value of Δ at the previous time step in the term $\frac{P}{2\Delta_n}$ instead of solving the non-linear equations, but Hunke and Zhang (1999) showed that the resulting sea ice slowly responds to changes in the forcing. The same idea is used when solving the non-linear equation by using outer loop iterations, also called pseudo-time steps or sub-iterations (Zhang and Hibler, 1997). Within each global time step, an additional loop of iterations is performed with indices p using a linearized version of the equation where Δ is computed with the velocity from the previous iteration.

$$m \frac{u_{n+1}^{p+1} - u_n}{\Delta t} = \partial_x \frac{P}{2\Delta_{n+1}^p} \partial_x u_{n+1}^{p+1} + f_{n+1}^{p+1}. \quad (3.30)$$

The main drawback of this basic non-linear solver is the slow convergence towards the solution. Even with 40 outer loop iterations, the velocity field can have significant errors as shown by Lemieux and Tremblay (2009). Within each iteration a solver is called to solve the linear system of equations (GMRES, LSOR, ...). Lemieux et al. (2010) proposed to use a Jacobian-free Newton-Krylov method to solve the non-linear set of equations but it still necessitates the use of a linear solver.

EVP has been presented as a way to solve the VP non-linear equations with a method similar to an explicit time discretization (Hunke and Dukowicz, 1997), suppressing the need of a linear solver. In the one-dimensional example, it corresponds to add an elastic term $2T\partial_t\sigma$ to equation (3.25) where T is the elastic time

scale parameter. The EVP method may be interpreted as a supplementary temporal discretization with a smaller time step Δt_e . Within each global step, the following equations are iterated n_{evp} times:

$$2T \frac{\sigma^{p+1} - \sigma^p}{\Delta t_e} + \sigma^p = \frac{P}{2\Delta^p} \partial_x u^p, \quad (3.31)$$

$$m \frac{u^{p+1} - u^p}{\Delta t_e} = \partial_x \sigma^{p+1} + f^{p+1}, \quad (3.32)$$

where Δt_e is the internal time step. u and σ stand for u_{n+1} and σ_{n+1} , but subscripts corresponding to the global time step are not written any more except when equal to n . The parameters T and Δt_e have to be carefully chosen to ensure the stability of the explicit scheme and to damp the elastic waves. During the sub-iterations, other variables are not updated and the forcing is kept constant.

3.3.3 Reinterpretation of EVP

In this section, we completely reinterpret EVP as a point iterative solver for the system of non-linear equations obtained from the implicit time discretization of the VP problem. At each iteration, an approximate solution u^{p+1} and σ^{p+1} is generated from u^p and σ^p . Applied to the one-dimensional case presented in section 3.3.2, the algorithm converges to the solution $\hat{\sigma} = \frac{P}{2\Delta} \partial_x \hat{u}$ and $\hat{u} = u_n + (\partial_x \hat{\sigma} + f(\hat{u})) \frac{\Delta t}{m}$ by successively executing these operations:

$$\sigma^{p+1} - \sigma^p = (\hat{\sigma} - \sigma^p) \frac{1}{\alpha}, \quad (3.33)$$

$$u^{p+1} - u^p = (\hat{u} - u^p) \frac{1}{\beta}, \quad (3.34)$$

where α and β are non-dimensional parameters larger than 1. To avoid the need of a solver, the update of internal stress and velocity components are decoupled. Internal stress is firstly updated towards the target $\hat{\sigma} = \frac{P}{2\Delta^p} \partial_x u^p$, then, velocity is updated towards the target $\hat{u} = u_n + (\partial_x \sigma^{p+1} + f(\hat{u})) \frac{\Delta t}{m}$.

The proposed method closely corresponds to the standard EVP method of Hunke (2001), if the new numerical parameters are defined by:

$$\alpha = \frac{2T}{\Delta t_e}, \quad (3.35)$$

$$\beta = \frac{\Delta t}{\Delta t_e}. \quad (3.36)$$

A crucial difference however, is the treatment of the physical inertial term, $m \frac{u - u_n}{\Delta t}$. In the standard EVP method, it is confounded with the update of the velocity during the iterative process, $m \frac{u^{p+1} - u^p}{\Delta t_e}$. In consequence, the number of iterations n_{evp}

must be equal to $\frac{\Delta t}{\Delta t_e}$, in order to remain consistent with the momentum equation. In the revisited EVP method, as in the modified EVP method proposed by Lemieux et al. (2012), the constraint on the number of iterations disappears, since the velocity update and physical inertial term are treated separately. The extra parameter β^* proposed by Lemieux et al. (2012) relates to our non-dimensional parameter β by the relationship:

$$\beta = \frac{\beta^* \Delta t}{m \Delta t_e}. \quad (3.37)$$

The elimination of the constraint on n_{evp} allows to evaluate the error of the approximate solution during the iterative process and to stop it when a prescribed accuracy is reached. It also allows us to study the convergence towards the solution as a function of the number of iterations.

Another advantage of the algorithm presented here is the reduced number of parameters compared to the modified EVP method of Lemieux et al. (2012). Only two numerical parameters remain, α and β , instead of the three parameters T , Δt_e and β^* . The reduction of the number of parameters, which are now non-dimensional, helps in understanding their effects on stability and convergence properties. The method is consistent, meaning that if (u^p, σ^p) is a solution, (u^{p+1}, σ^{p+1}) given by (3.33) and (3.34) is equal to (u^p, σ^p) .

3.3.4 Convergence and stability of the method

The iterative process corresponds to

$$\sigma^{p+1} = \sigma^p - \left(\sigma^p - \frac{P}{2\Delta^p} \partial_x u^p \right) \frac{1}{\alpha}, \quad (3.38)$$

$$u^{p+1} = u^p - (\hat{f}(u^{p+1}) - \partial_x \sigma^{p+1}) \frac{\Delta t}{m} \frac{1}{\beta}. \quad (3.39)$$

where $\hat{f}(u) = m \frac{u - u_n}{\Delta t} - f(u)$.

As $\beta \gg 1$, the u -dependent part of $\hat{f}(u)$, corresponding to $m \frac{u}{\Delta t}$, the Coriolis and ice-ocean drag terms, is neglected for the following analysis. Combining both equations and considering Δ as a constant leads to:

$$(u^{p+1} - 2u^p + u^{p-1}) + (u^p - u^{p-1}) \frac{1}{\alpha} - \partial_x^2 u^p \frac{\gamma}{\alpha\beta} = \hat{f} \frac{\Delta t}{m} \frac{1}{\alpha\beta}, \quad (3.40)$$

where

$$\gamma = \frac{P}{2\Delta} \frac{\Delta t}{m}. \quad (3.41)$$

Equation (3.40) looks like a wave equation describing the evolution of the approximate solution during the iterative process. By dropping the non homogeneous term and supposing $u^p(x) \sim e^{i(kx - \omega p)}$, one obtains the dispersion relation:

$$\omega^2 + i\omega \frac{1}{\alpha} - k^2 \frac{\gamma}{\alpha\beta} = 0, \quad (3.42)$$

where $i = \sqrt{-1}$, k is the wave number (with condition $k^2 \Delta x^2 \leq 1$) and ω is a non-dimensional complex number. For each term, the followings relations have been used, respectively:

$$e^{-i2\omega} - 2e^{-i\omega} + 1 \approx -\omega^2, \quad (3.43)$$

$$e^{-i\omega} - 1 \approx -i\omega, \quad (3.44)$$

$$e^{-i\omega} \approx 1. \quad (3.45)$$

The damping is controlled by the imaginary part of ω , given by:

$$\omega = \frac{1}{2\alpha} \left(-i + \sqrt{\frac{1}{\xi^2} - 1} \right), \quad (3.46)$$

where ξ is the damping ratio defined by $\xi = \frac{S_v}{S_e}$, where S_e and S_v are two non-dimensional numbers defined by:

$$S_e = \sqrt{\frac{\alpha\beta}{\gamma}} \Delta x, \quad (3.47)$$

$$S_v = \frac{\beta}{\gamma} \frac{\Delta x^2}{2}, \quad (3.48)$$

where γ comes from (3.41). The value of the damping ratio ξ determines the type of damping. Oscillations occur during the approach to the solution if $\xi < 1$ and the damping rate is equal to $\frac{1}{2\alpha}$. If $\xi \geq 1$, there is no oscillation and optimal damping ratio occurs when $\xi = 1$. The condition $\xi^2 > 1$ corresponds to the relation $T < \frac{e^2 m \Delta x^2 \Delta}{8p}$ found by Hunke (2001).

The definitions of S_e and S_v transform equation (3.40) into the following discrete wave equation:

$$(u^{p+1} - 2u^p + u^{p-1})S_e^2 + (u^p - u^{p-1})2S_v - u^p \delta_x^2 = 0, \quad (3.49)$$

with the second order finite difference:

$$\delta_x^2 u = u(x + \Delta x) - 2u(x) + u(x - \Delta x). \quad (3.50)$$

S_e relates to the oscillatory period, which is the number of iterations over which one oscillation occurs. S_v is related to the total number of iterations needed to reach equilibrium.

The stability criterion is developed by assuming that $u^p = a^p e^{ikx}$, where $a^p = \lambda a^{p-1}$. The method is stable if $|\lambda| < 1$. Equation (3.49) becomes:

$$\lambda^2 - \left(2 - \frac{k^2 \Delta x^2}{S_e^2} - \frac{1}{\alpha}\right) \lambda + \left(1 - \frac{1}{\alpha}\right) = 0. \quad (3.51)$$

Using the properties $\alpha > 1$ and $k^2 \Delta x^2 \leq 1$, the stability criterion is given by $S_e^2 > 0.5$. This stability criterion is similar to the condition $T > \frac{P(1+e^2)^2 \Delta t_e^2}{32m\Delta x^2 e^2 \Delta}$ developed by Hunke (2001).

In cases where internal stress is negligible, i.e., under ice divergence or in regions with low ice concentration, the problem reduces to:

$$\beta(u^{p+1} - u^p) = \left(f \frac{\Delta t}{m} + u_n\right) - u^{p+1}. \quad (3.52)$$

The solution is approached as $e^{-\frac{\epsilon}{\beta} p}$ and the convergence is controlled by $\frac{1}{\beta}$. The method is always stable and locally convergent. β is then an estimate of the number of iterations needed to approach the solution when internal stress is low.

Equivalently, for a constant strain rate, the convergence rate of the stress state is given by $\frac{1}{\alpha}$ and α is an estimate of the required number of iterations. Note that being on the ellipse does not imply that the solution has converged to the correct stress state. The correct position for a given strain rate is the unique point on the curve where the normal flow rule is respected. The distance to the yield curve is therefore not always a good diagnostic to evaluate the convergence of the stress state.

3.3.5 Comparison of EVP simulations

To illustrate our reinterpretation of EVP results, we have selected four test case simulations made by Hunke (2001) (cases (a) and (b) from their figure 4) and by Lemieux et al. (2012) (blue and black lines in their figure 1), and thereafter referred to as Hunke_a, Hunke_b, Lemieux_a and Lemieux_b. In Hunke (2001), the variable of interest is the divergence rate near the right edge of the square domain where ice concentration is greater than 0.9. This field is smooth in Hunke_a but spurious in Hunke_b. In Lemieux et al. (2012), the variable of interest is the L2-norm of the non-linear system of equations given for the first time-step. Its value converges in Lemieux_a but stagnates with oscillations in Lemieux_b. The corresponding numerical parameters and the diagnostic numbers S_e , S_v and ξ are presented in Table 3.2.

Name	α	β	S_e	S_ν	ξ
Hunke_a	864	1200	0.23	31×10^{-6}	1×10^{-4}
Hunke_b	86	120	0.02	3×10^{-6}	1×10^{-4}
Lemieux_a	86	120	1.49	0.116	0.08
Lemieux_b	29	40	1.49	0.039	0.03

Table 3.2: Values of the numerical parameters α and β and of the diagnostic numbers S_e , S_ν and ξ for the analysed experiments from Hunke (2001) and Lemieux et al. (2012). The value of the non-dimensional parameters α and β are computed by (3.35) and (3.36), and the values of S_e and S_ν by (3.47) and (3.48), respectively. ξ is given by the ratio $\frac{S_\nu}{S_e}$.

The two test cases Hunke_a and Hunke_b were made on a 80×80 grid, with a resolution of 16 km , a time step of 1 hour and an elastic time scale T equal to 1296 seconds. Ice thickness is fixed at 2 m and ice concentration varies linearly from 0 at the left side to 1 at the right side of the domain. Ocean and wind stresses are smooth and slowly varying. $P^* = 27.5 \times 10^3 \text{ N m}^{-2}$ and Δ_{min} is equal to 10^{-11} s^{-1} . In Hunke_a, the solution after 1200 iterations with $\Delta t_e = 3 \text{ s}$ does not present spurious ice deformations, while in Hunke_b, the solution after 120 iterations with $\Delta t_e = 30 \text{ s}$ is quite noisy with variability in the ice divergence rate of about $6 \times 10^{-8} \text{ s}^{-1}$ in a region where the ice is highly concentrated and nearly rigid. Hunke (2001) explained the noise in Hunke_b by a lack of damping, but our analysis indicates an instability problem. Indeed, the damping ratio ξ in Hunke_a and Hunke_b is low but it has the same value in both experiments. The observed spurious deformation in Hunke_b is then better explained by the smaller value of S_e which makes Hunke_b more unstable than Hunke_a (Table 3.2). In both cases, neither the stability nor the damping criteria is fulfilled because of the very low value of Δ_{min} .

The two test cases Lemieux_a and Lemieux_b were made on a grid covering all the Arctic with a constant resolution of 40 km . The time step is equal to 1200 s and the elastic time scale T is equal to 432 s . Ice thickness is fixed at 1 m and concentration is equal to 100% everywhere. $P^* = 27.5 \times 10^2 \text{ N m}^{-2}$ and Δ_{min} is equal to $2 \times 10^{-9} \text{ s}^{-1}$. Forcing fields come from NCEP-NCAR wind stress and climatological ocean currents. The residual for Lemieux_a ($\Delta t_e = 10 \text{ s}$) reaches the machine accuracy in about 4500 iterations, while the residual for Lemieux_b ($\Delta t_e = 30 \text{ s}$) stagnates rapidly, oscillating around 10^{12} times the machine accuracy. Those oscillations are not due to instabilities as S_e has the same value in both simulations and fulfil the stability criterion ($S_e^2 > 0.5$). These come from a lack of damping due to the lower damping ratio in Lemieux_b.

These examples show the interest of the diagnostics S_e and ξ to understand the cause of numerical problems. For the two examples, γ has been computed for a concentration equal to 1 and with Δ equal to Δ_{min} . In both papers, $P = P^* h e^{-c^*(1-A)}$

and $m \simeq \rho_i h$ where ρ_i is the ice density and h the mean ice thickness. γ is then given by $\gamma = \frac{P^*}{2\Delta_{min}} \frac{\Delta t}{\rho_i}$ and is equal to 4.95×10^{15} for the configuration of Hunke (2001) and to 8.25×10^{11} for the configuration of Lemieux et al. (2012).

3.3.6 Summary and details of implementation

In summary, EVP is an ad-hoc solver for the system of non-linear equations obtained from the implicit time discretization of the VP model. Convergence and stability of the method are controlled by two numerical parameters. Three non-dimensional numbers, independent of the configuration, characterize the behaviour of the solver and offer a simple way to compare different implementations of the EVP method.

In previous implementations of EVP, numerical and physical terms were somewhat confounded, generating some confusion between the numerical method and the physical model. The numerical parameters were interpreted as an artificial elastic time scale T and a sub-iteration time step Δt_e . The number of iterations n_{evp} had to be equal to $\frac{\Delta t}{\Delta t_e}$. The revisited EVP method suppresses the need for a fixed number of iterations and simplifies the choice of the two non-dimensional numerical parameters. The parallelization is still trivial and the computational time is unchanged.

The treatment of the stress update is slightly different from previous EVP implementations, where $2T\partial_t\sigma_1$, $\frac{2T}{\rho^2}\partial_t\sigma_2$, $\frac{2T}{\rho^2}\partial_t\sigma_{12}$, were added to the left hand side of equations (3.8), (3.9), (3.10) to regularize the definition of the stress. The reinterpretation of EVP leads to a new regularization where σ_1 is not treated differently than σ_2 and σ_{12} . The convergence of each stress component will be similar and controlled by α . To reproduce the old regularization, one can use $\alpha = \frac{2T}{\Delta t_e}$ for σ_1 , and $\alpha = \frac{1}{\rho^2} \frac{2T}{\Delta t_e}$ for σ_2 and σ_{12} . Keeping the same α for the three equations has a surprisingly good impact on the convergence as analysed in the next section.

At the first iteration ($p = 1$) of each global time step, an initial guess for $(\mathbf{u}, \sigma)$ has to be defined. We take zero for $\sigma_1^1, \sigma_2^1, \sigma_{12}^1$ and the ice velocity at the previous time step for $\mathbf{u}^1$. This choice allows to evaluate the convergence to the correct stress state by the distance to the ellipse as we restart from zero at the beginning of each time step. Other initial guesses for $\mathbf{u}$ as the free drift velocity or a smoothed version of the previous velocity could be used to limit the propagation of undamped oscillations from one time step to the other.

The entire algorithm is detailed hereafter to ease implementation in sea ice models. Within the iterative process, variables indexed with p are updated, while variables indexed with n are referred to the global time step and stay constant. The

number of iterations, n_{evp} , can be fixed a priori or depend on a termination criterion as a function of the residual. At each solver iteration, the internal stress tensor is updated first by:

$$\alpha(\sigma_1^{p+1} - \sigma_1^p) = \frac{P}{(\Delta^p + \Delta_{min})} (\dot{\epsilon}_1^p - \Delta^p) - \sigma_1^p, \quad (3.53)$$

$$\alpha(\sigma_2^{p+1} - \sigma_2^p) = \frac{P}{(\Delta^p + \Delta_{min})} \frac{\dot{\epsilon}_2^p}{e^2} - \sigma_2^p, \quad (3.54)$$

$$\alpha(\sigma_{12}^{p+1} - \sigma_{12}^p) = \frac{P}{(\Delta^p + \Delta_{min})} \frac{\dot{\epsilon}_{12}^p}{e^2} - \sigma_{12}^p. \quad (3.55)$$

The internal force vector is then computed by

$$\mathbf{F}^{p+1} = (F_u^{p+1}, F_v^{p+1}) = \nabla \cdot \sigma^{p+1} \quad (3.56)$$

and used in the velocity update.

The updates of u and v are decoupled. In odd iterations, (3.58) is solved first with v^* as an interpolation of v^p on u -points and u^* equal to u^{p+1} . Then, (3.60) is solved with $v^* = v^{p+1}$ and u^* is u^{p+1} interpolated on v -points. The reverse order is applied for even iterations. One therefore has:

$$\beta \frac{m}{\Delta t} (u^{p+1} - u^p) = F_u^{p+1} - \frac{m}{\Delta t} (u^{p+1} - u_n) + \quad (3.57)$$

$$A(\tau_{au} + c_D \rho_o |\mathbf{u}_o - (u^*, v^*)|(u_o - u^{p+1})) + m f v^* \quad (3.58)$$

and

$$\beta \frac{m}{\Delta t} (v^{p+1} - v^p) = F_v^{p+1} - \frac{m}{\Delta t} (v^{p+1} - v_n) + \quad (3.59)$$

$$A(\tau_{av} + c_D \rho_o |\mathbf{u}_o - (u^*, v^*)|(v_o - v^{p+1})) - m f u^*. \quad (3.60)$$

3.4 Results and discussion

In this section, different simulations are performed to analyse the impacts of the numerical parameters on the quality of the solution. Convergence analysis and comparison between the old and new stress regularizations raise questions about the real ability of the viscous-plastic model to reproduce sea ice dynamics at high resolution. In particular, linear bands of high ice deformation are shown to be highly correlated to numerical problems and to a strong lack of convergence.

We work in the framework of the Nucleus for European Modelling of the Ocean (NEMO) model (Madec, 2008) with the Louvain-la-Neuve sea Ice Model (LIM) discretized on a C-grid as in Bouillon et al. (2009). The modifications are made to

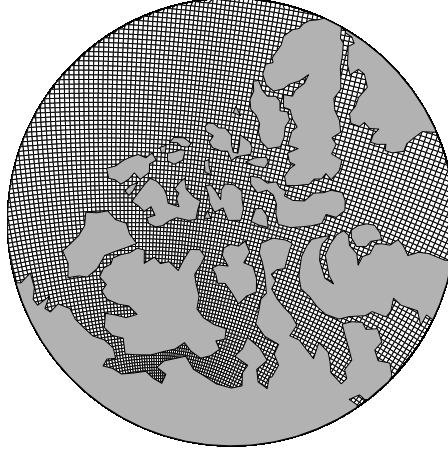


Figure 3.3: ORCA05 mesh over the Canadian Arctic Archipelago (CAA). The resolution varies from 15 km in the CAA to about 25 km in the central Arctic.

version 3.4 of NEMO-LIM (branch /dev_r3322_NOCS09_SAS) with the LIM2 version of the sea ice model on the ORCA05 grid in an Arctic configuration (see Madec et al. (1998) and Fichefet and Maqueda (1997) for a description of LIM2). The ocean is supposed to be at rest with no elevation. The ocean stress is computed by a quadratic formula with a drag coefficient $c_D = 5 \times 10^{-3}$ and no turning angle. The surface wind is obtained from the 6 hourly DFS4.1 10m wind data (Brodeau et al., 2010). Surface boundary conditions (wind stress and other fluxes) are determined by the CORE bulk formulae proposed by Large and Yeager (2004). The time step is equal to 4 hours ($\Delta t = 14400s$) and all simulations start the first of January 1991. To ease the analysis, very simple initial conditions are chosen. There is no ice wherever the sea surface temperature is $2^\circ C$ above the seawater freezing point, 3 m of ice and 50 cm of snow elsewhere. Sea surface temperatures and salinities are derived from the Levitus98 monthly climatology (Boyer et al., 1998). The initial concentration of the ice pack is equal to 1. The ORCA05 tripolar grid has a resolution of about 15 km in the Canadian Arctic Archipelago and around 25 km in the central Arctic (Figure 3.3). The domain is restricted to the ocean surface at latitudes higher than 25° North. The model parameters are the following: $P^* = 10^4 Nm^{-2}$ and $c^* = 20$. We choose a very small value for Δ_{min} ($10^{-20} s^{-1}$) in order to model nearly pure plastic flow.

The first part of this section concerns the choice of the values of the numerical parameters α and β to have a correct approximation of the solution within a limited number of iterations. The effect of these parameters on stability and convergence is checked by looking at three diagnostics: the evolution of the maximum increment

Name	α	β	S_e	S_v	ξ
Exp1	40	3000	0.87	95×10^{-4}	1×10^{-3}
Exp2	98	122	0.27	4×10^{-4}	1.5×10^{-3}
Exp3	126	947	0.86	30×10^{-4}	3.5×10^{-3}
Exp4	40	300	0.27	10×10^{-4}	3.7×10^{-3}

Table 3.3: Values of the two non-dimensional numerical parameters α and β and the different non-dimensional numbers for the four set of parameters used in the present study. $S_e^2 > 0.5$ is the stability criterion, S_v is a measure of the convergence of the velocity field and ξ is the damping ratio.

of the velocity during the iterative process, the distance of stress state to the yield curve after 300 iterations and the divergence field after 300 iterations in the Canadian Arctic Archipelago (CAA), which is a region where the sea ice deformation rate is very low and where stability and damping are then difficult to maintain. An optimal set of parameters is proposed to mitigate spurious ice deformations in the CAA, to maintain stress states on the yield curve, and to avoid oscillations during the iterative process.

In the second step, the error of the approximate solution is analysed. The error is evaluated a posteriori by looking at differences with the best approximate solution obtained with a very large number of iterations. Two slightly different versions are compared, one with the new regularization of the internal stress where the three components use the same α and one with the old regularization that is equivalent to take $\frac{\alpha}{e^2}$ for updating σ_2 and σ_{12} , and α for σ_1 . The lack of convergence with the old regularization is explained and the link with the generation of discontinuities in the velocity field is highlighted in Section 3.4.2.

3.4.1 Selection of the values of the numerical parameters

Here is a brief summary useful for the analysis. α and β are the only two numerical parameters of the solver. α commands the stress update and β the velocity update. Their combination controls the stability of the method and the damping of oscillations during the solving process. S_e , S_v and ξ are three non-dimensional numbers independent of the configuration. The stability is ensured if $S_e^2 > 0.5$ and oscillations are damped if $\xi \geq 1$.

Four simulations of a few days are performed to analyse the effects of α and β (Table 3.3). All simulations have the same number of iterations: $n_{evp} = 300$, which is acceptable for most applications. Only results for the first time step are shown, but the analysis remains valid for the other time steps. Each experiment deals with the same forcing and initial conditions. Exp1 is the optimal choice (Table 3.3). Exp2

is the worst with the lowest values for S_e and ξ . Exp3 has the same stability criterion as Exp1 but a worse damping ratio. Exp4 has the same stability criterion as Exp2 with a better damping ratio. The evolution of the velocity update during the iterative process is illustrated in Figure 3.7. The stress states (Figure 3.5) and the divergence field in the CAA (Figure 3.6) are given at iteration 300 of the first time step.

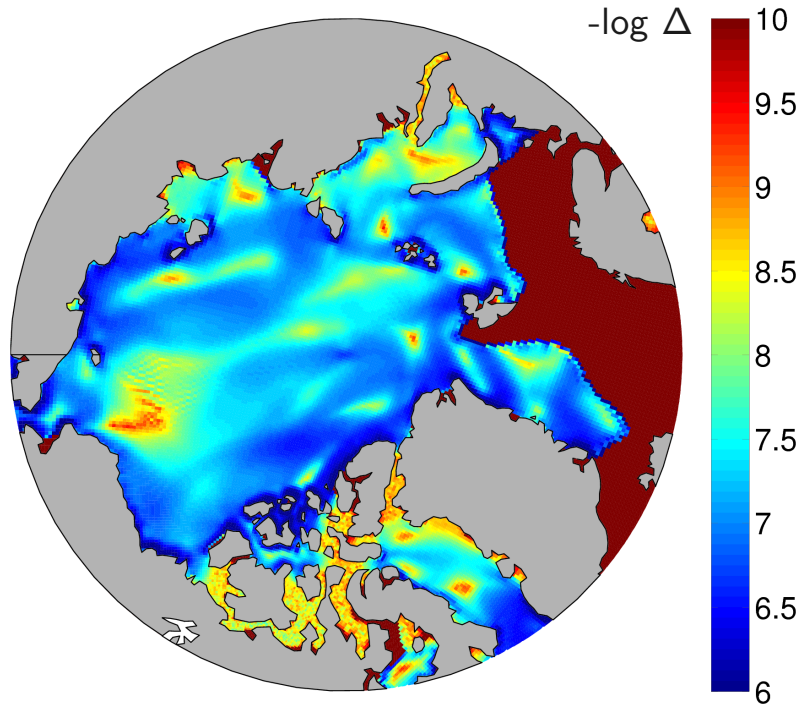


Figure 3.4: Logarithm of Δ after 300 iterations for the first time step with parameters given by Exp1. Noise in the ice deformation field is visible in narrow passages and near rigid ice regions but with magnitude around $10^{-9} s^{-1}$.

To compute the diagnostics presented in Table 3.3, we had to evaluate the value of the non-dimensional number γ , which is a function of Δt , Δ and the strength-to-mass ratio of the ice. In our case, Δ is evaluated a posteriori from the simulated velocity field. The minimum value simulated in a cell that is not at the coast or at the ice edge is around $2 \times 10^{-9} s^{-1}$. Figure 3.4 shows the logarithm of Δ after 300 iterations for Exp1. It is a better estimate than Δ_{min} , which is in our case very small

and never reached ($10^{-20} s^{-1}$). The value $\gamma = 36 \times 10^{12}$ is used as a constant to estimate S_e , S_v and ξ for the different set of parameters. Therefore, those diagnostics characterize the behaviour of the method in near rigid ice regions. In regions with larger deformation rate, stability and damping conditions are relaxed since S_e and ξ increase as $\sqrt{\Delta}$.

Exp1 is conducted with an optimal set of parameters for avoiding oscillations during the approach to the solution. After 300 iterations, the stress states are well located on the yield curve. The stability criterion is ensured for $\Delta > 1.5 \times 10^{-9} s^{-1}$. The damping ratio is the highest of the four experiments but is still lower than 1. Oscillations are damped where $\Delta \geq 4 \times 10^{-6} s^{-1}$. Spurious ice deformations are low (Figure 3.7) and only visible in Figure 3.4 where the logarithm of Δ is shown. The convergence towards the ellipse is good and the maximum increment of the velocity monotonically decreases (Figure 3.5 and 3.6).

Exp2 with parameters corresponding to the standard case Hunke_b leads to the poorest performances. The convergence criterion is only valid where $\Delta > 1.5 \times 10^{-8} s^{-1}$ and the damping ratio is the lowest. The stress state has not converged to the ellipse after 300 iterations (Figure 3.5) and large spurious ice deformations are observed as in Hunke (2001) (Figure 3.7). Large and rapid oscillations are noticed in the evolution of the maximum increment of the velocity (red line in Figure 3.6), corresponding to instabilities generated by this set of parameters.

Exp3 has the same stability criterion as Exp1. The damping properties are affected by the higher values of α and the lower value of ξ . At first, the velocity update is higher than for Exp1 (red dashed line in Figure 3.6), but it decreases more rapidly and starts to slightly oscillate in the last 20 iterations. Spurious ice deformations are still observed in the CAA to a lesser extent and with a smaller magnitude than in Exp2 and Exp4 (Figure 3.7). The distance of the stress state to the ellipse is still large after 300 iterations (Figure 3.5).

In Exp4, the damping properties are restored to the detriment of the stability criteria S_e , which drops down to the same value as in Exp2. Exp4 has a good convergence towards the ellipse due to the smaller value of α (Figure 3.5) but experiences an oscillatory behaviour similar to that seen in Exp2 (blue line in Figure 3.6). Large spurious divergence rates are observed in the CAA (Figure 3.7).

To summarize, Exp2 and Exp4 present instabilities characterized by large oscillations in the maximum increment of the velocity and stagnation of its value around $0.1 cm s^{-1}$. Exp1 and Exp3 do not exhibit instabilities, but Exp3 exhibits weak oscillations in the maximum increment of the velocity and small spurious ice deformation in the CAA. This is associated with a lack of convergence towards the ellipse

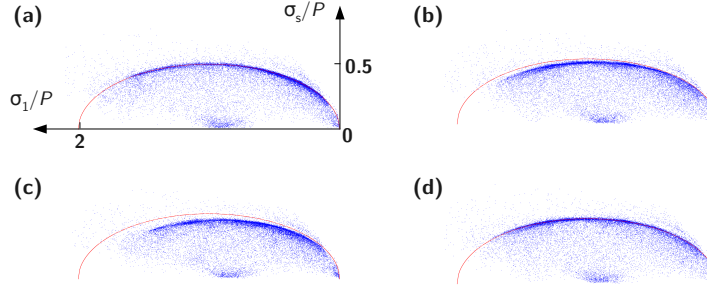


Figure 3.5: Internal ice stress normalized by the ice strength P for the entire domain after 300 iterations for the first time step of experiments Exp1 (a), Exp2 (b), Exp3 (c) and Exp4 (d).

and is caused by an insufficient damping during the iterative process. The spurious oscillations in the CAA have two origins: either an insufficient damping of the oscillations during the iterative process or instabilities of the method. Both problems are tackled in Exp1 with a well reproduced yield curve and limited noise in the ice deformation in rigid ice regions (less than 10^{-3} day^{-1}). The counterpart is a lower convergence of the ice velocity field for Exp1. This explains the large differences with Exp2 and Exp4 observed in almost all the domain. Large differences in the simulated ice divergence rate are visible in Figure 3.7 north of the CAA.

Solutions obtained after 300 iterations are then influenced by the numerical parameters because of the slow convergence of the solver. The choice of the numerical parameters determines the quality of the ice velocity, deformation and stress state, but one cannot have the three fields correct at the same time with a limited number of iterations of the EVP method. This lack of convergence and its impacts are analysed in the following section.

3.4.2 Error analysis

An advantage of the revisited EVP method is the possibility of evaluating the error by comparing the approximate solutions after a different number of iterations. We will use this error analysis to evaluate the differences between the old and new regularizations of the internal stress.

The effect of using the old regularization is clearly visible on Figure 3.8, which shows the stress states after 300 iterations obtained with the set of parameters Exp2. Contrary to the solution obtained with the new regularization shown in Figure 3.5(b), the stress state of the old regularization does not have the expected aspect ratio,

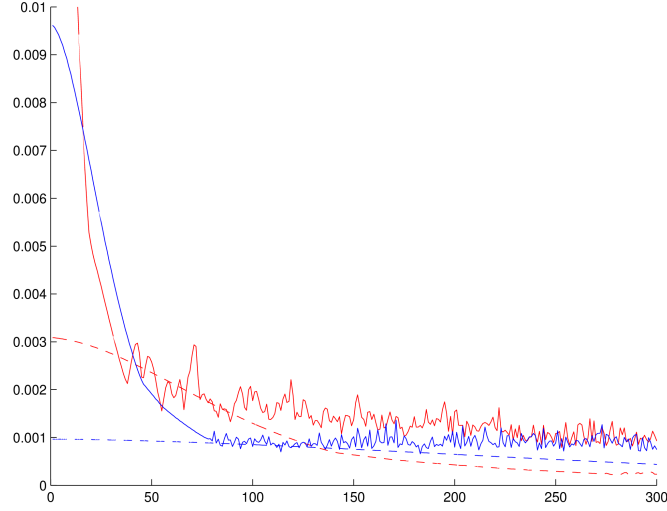


Figure 3.6: Evolution of the maximum increment of the velocity [m/s] during the first 300 iterations of the first time step. Exp1 corresponds to the dashed blue curve, Exp2 to the red curve, Exp3 to the dashed red curve and Exp4 to the blue curve.

the ellipse is too wide. This indicates a faster convergence in the shear stress direction than in the compressive stress direction. With the new regularization, α is the same for all stress components and the aspect ratio remains near the prescribed value e during all the solving process. Both methods converge to the correct yield curve with more iterations but produce very different deformation and velocity error fields as we show hereafter.

Results with the two regularizations are compared in terms of convergence, remaining error, and quality of the approximate solution. The difference between the two versions only concerns the division of α by e^2 for updating σ_2 and σ_{12} to mimic the original formulation of EVP used in previous studies (Hunke and Dukowicz, 2002; Hunke, 2001; Bouillon et al., 2009; Lemieux et al., 2012). This modification does not change the equations for which the solver is called. The rest of the code is exactly the same, as are the parameters, initial conditions and external forcing, which are the same as in Exp1. The analysis presented here has been verified for all sets of parameters, with no noticeable change.

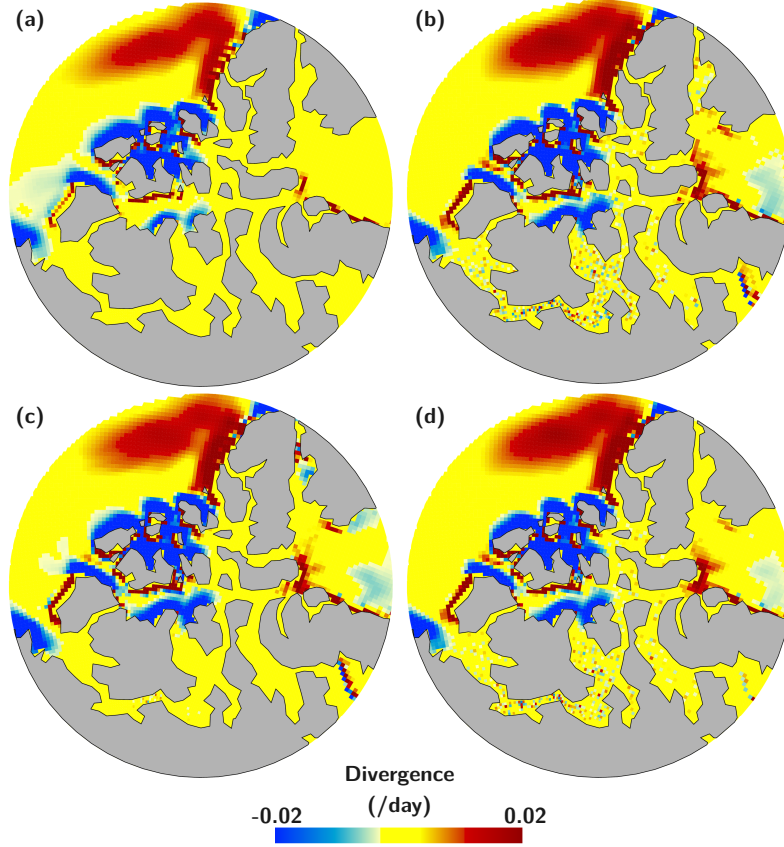


Figure 3.7: Sea ice divergence rate within the Canadian Arctic Archipelago after 300 iterations for the first time step of experiments Exp1 (a), Exp2 (b), Exp3 (c) and Exp4 (d). 0.02 day^{-1} is approximately equal to $2 \times 10^{-7} \text{ s}^{-1}$.

Figure 3.9 displays the ice divergence and shear rates obtained after 30000 iterations with the two versions of the solver. The old regularization produces linear bands of high ice deformation which are not present with the new regularization. High positive values for the divergence are observed in those bands which are aligned with the grid in both directions and are one cell wide. In the rest of the domain, deformation and velocity fields are similar between the two versions of the code. No spurious deformation is observed in the CAA with both versions and the stress states are well located on the ellipse (not shown).

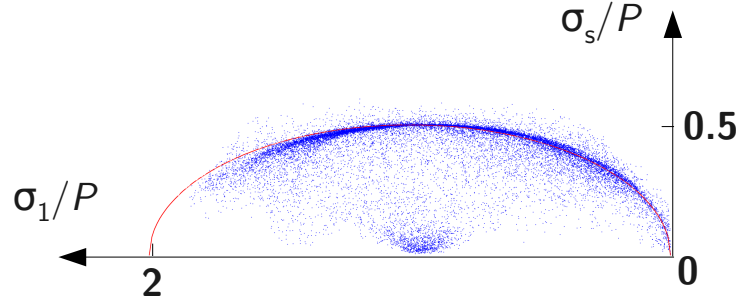


Figure 3.8: Internal ice stress normalized by the ice strength P for all the domain after 300 iterations for the first time step of the same experiment as Exp2 but with the old regularization instead of the new one.

The error at iteration 300 (or 3000) is measured by computing the differences in velocity between the approximate solutions at iteration 300 (or 3000) and 30000. The error after 300 iterations is of the same order of magnitude for both versions (Figure 3.10). The maximum error in the velocity components is 0.13 cm s^{-1} for the old regularization and 0.08 cm s^{-1} for the new one. After 3000 iterations, the maximum error is divided by 2 for the old regularization and by 40 for the new one (0.06 and 0.002 cm s^{-1} , respectively) (Figure 3.10). In both cases, the average error decreases, but locally, the error remains significant with the old regularization in large parts of the domain. The linear bands of high ice deformation are correlated with large up-and-down variations in the profile of the velocity error in both directions (up to 0.06 cm s^{-1}) (not shown). Such structures in the velocity error are not observed with the new regularization except in narrow passages (e.g., Nares Strait), but with a much lower magnitude of 0.0005 cm s^{-1} .

Our explanation of the problematic behaviour of the old regularization is that the different treatment of the stress components excites oscillations and slows down the convergence to the solution. This small difference in the solving method causes the production of high deformation bands aligned with the grid.

Similar characteristics have been observed in other studies. Wang and Wang (2009) present several simulations of the VP model showing linear bands of high ice deformation rate aligned with the grid which were compared to the linear kinematic features observed in the Arctic ice pack. Their numerical method, even if it is not EVP, may also be assimilated to a fixed-point iterative method. Each of their 5-minutes time steps corresponds to one iteration toward the solution of the non-linear problem. The response of the ice is not directly expressed in terms of

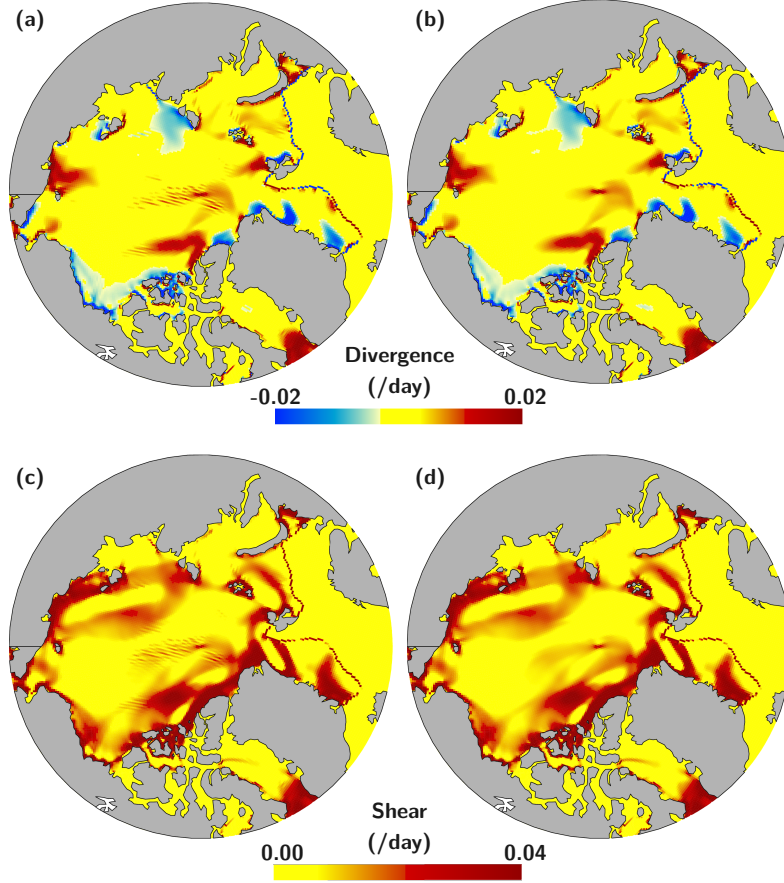


Figure 3.9: Sea ice divergence rate (top) and shear rate (bottom) after 30000 iterations for the first time step with two stress regularizations. Results with the old regularization are on the left and with the new one on the right. The values of the numerical parameters are the same as in Exp1. The linear bands of high ice deformation are aligned with the grid in both directions and are one cell wide. 0.02 day^{-1} is approximately equal to $2 \times 10^{-7} \text{ s}^{-1}$.

internal stresses but derived from non-linear viscosities computed from the deformation rate of the previous iteration. The so-called linear kinematic features are produced with different yield curves and their number increases with the resolution. The error has been evaluated by comparing a reference solution to a solution with 100 times more iterations. The average and the maximum differences in the velocity field are 0.13 cm s^{-1} and 0.51 cm s^{-1} , respectively. The stress states are also given for each yield curve after ~ 1000 and ~ 2000 iterations (Figure 18 of Wang

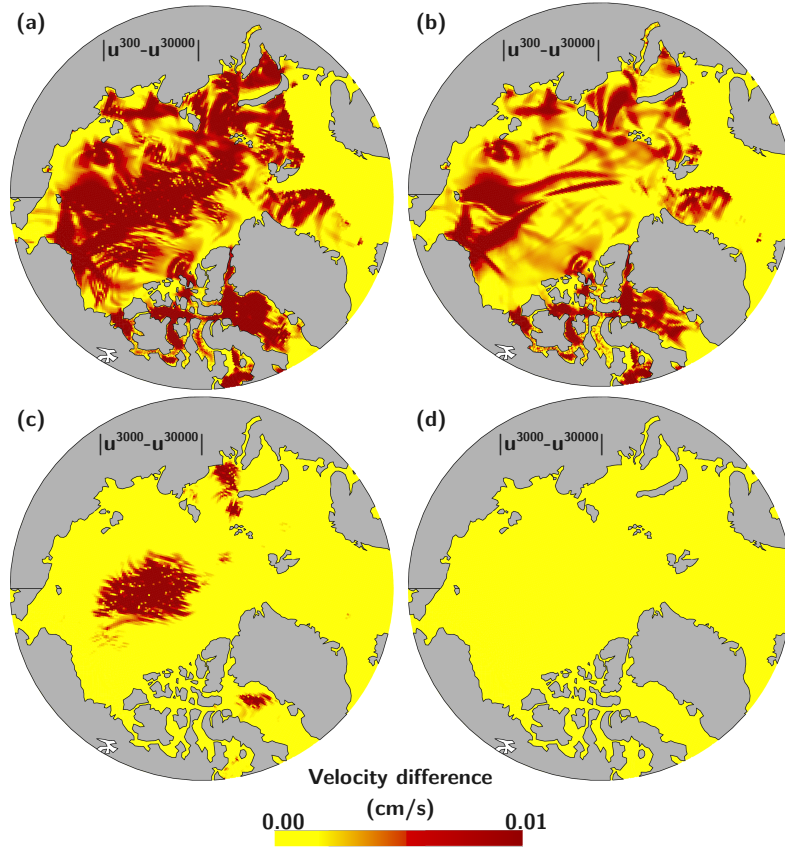


Figure 3.10: Norm of the ice velocity difference between the solutions after 300 and 30000 iterations (top) and between the solutions after 3000 and 30000 iterations (bottom) for the first time step with the two versions of the method. Results from the old regularization are on the left and from the new one on the right. The values of the numerical parameters are the same as in Exp1. For the old regularization, the maximum values of the error is divided by 2 going from 0.13 cm s^{-1} for iteration 300 to 0.06 cm s^{-1} at iteration 30000. For the new one, it is divided by 40 going from 0.08 cm s^{-1} for iteration 300 to 0.002 cm s^{-1} at iteration 30000.

and Wang (2009)), showing a very slow convergence toward the yield curve for the modified Coulombic curve (between 1000 and 2000 iterations required) and for the curved diamond (more than 2000 iterations required). The inability to converge for those two yield curves coincides with the generation of numerous linear kinematic features.

Lemieux et al. (2012) compared solutions given by the classical EVP method to a fully converged solution obtained with a Picard scheme and a tight non-linear convergence criterion. The EVP solver with ~ 2000 iterations captures the general pattern of ice deformation but produces additional shear lines and zones of strong positive or negative divergence. Those features persist with increasing the number of iterations and are more pronounced as the grid is refined. Increasing the number of iterations from 120 to ~ 2000 improves the agreement with the reference solution, but some regions still have errors of $O(1) \text{ cm s}^{-1}$. Similar arch-like deformations were also obtained by Maslowski and Lipscomb (2003) with EVP at a 9 km resolution with the standard parameters of Hunke_b.

The similarities with those studies and the simulation with the old regularization presented here may help in clarifying the origins of these linear bands of high ice deformation. We suggest that the cause of the generation of such features is the poor numerical behaviour of the methods used to solve the non-linear VP problem. Their appearance is characterized by large remaining errors in the velocity field and a strong lack of convergence. In addition, the link with the resolution indicates stability problems. The alignment with the grid has not been observed in all studies but is further evidence of poor numerical behaviour. The small correction of the EVP iterative solver completely filters out those features, while maintaining the same solution for the rest of the domain.

3.5 Conclusions

The reinterpretation of the elastic-viscous-plastic method (EVP) as an iterative solver defines a clear distinction between numerical and physical parameters. The constraint on the number of iterations is removed, which enables convergence analysis. The interpretation of results and the choice of parameters are simplified and clear criteria are defined to ensure stability of the method and to avoid oscillations during the iterative solving process.

The new numerical parameters are non-dimensional and entirely control the convergence of the stress states and the ice velocity towards the solution. Their combinations give clear criteria, independent of the configuration, to control the stability of the method and the damping of oscillations during the iterative process. An optimal set of parameters is proposed to mitigate spurious oscillations that often arise in near rigid ice regions as in the Canadian Arctic Archipelago (CAA). The new parameters and diagnostics have also proven useful in comparing simulations from other configurations to distinguish if the real cause of numerical problems was instabilities or a lack of damping.

A new spatial discretization for the viscous-plastic (VP) equations on a C-grid has also been proposed to ensure that the dissipative property of the continuous equations is respected for the discrete equations. However, it does not lead to significant improvement of the results.

The comparison between the old and new regularizations of the internal stress illustrates the strong impact of numerical details when using unconverged results. The old regularization produces linear bands of high deformation aligned with the grid that are not present with the new one. These features may be confounded with real linear kinematic features but are in fact the results of a numerical artefact.

Motivation for the next chapter

This study confirms the need of a new model for sea ice dynamics. Thanks to the collaboration with the Laboratoire de Glaciologie et de Géophysique de l'Environnement (LGGE), we developed a new rheology based on the elasto-brittle framework. We tested the model in realistic simulations of the Arctic sea ice deformation over short periods of time (Girard et al., 2011). The simulated ice deformation exhibits power law tail distributions and spatial scaling. The next chapter presents this completely new model and the adaptations made to simulate sea ice deformation on longer periods.

CHAPTER 4

THE ELASTO-BRITTLE MODEL

This Chapter is based on the following paper:

S. Bouillon, J. Weiss, L. Girard, T. Fichefet, V. Legat, and D. Amitrano. A further step towards the integration of the elasto-brittle rheology in sea ice models. *to be submitted to Journal of Geophysical Research*, 2013.

Abstract

In this chapter, the elasto-brittle (EB) rheology is adapted for long simulations and implemented in a finite element sea ice model. This rheology includes a progressive damage model and reproduces the heterogeneity of ice pack deformation. The simulated deformation of the Arctic ice pack exhibits statistical properties similar to observations. Two parameterisations of damage recovery (healing) are proposed and compared. We show that healing has a strong impact on the persistency of ice deformation patterns and that could explain part of the strong seasonality of the deformation activity. This study is a further step towards the integration of the EB rheology in large-scale sea ice models, leading to a much better representation of the complex behaviour of the Arctic ice pack.

4.1 Introduction

Large-scale sea ice models relatively well represent the average motion of the ice pack, but poorly resolve the deformation fields at small spatial and temporal scales (Kwok et al., 2008). Girard et al. (2011) proposed a new approach with the development of the elasto-brittle (EB) model and performed realistic simulations of the Arctic sea ice deformation at high resolution (10 km), on a temporal scale of 3 days. Statistical analyses of the results highlight the spatial heterogeneity of the simulated ice deformation. Shear and divergence rate distributions produced with the EB model compare much better to observations than those obtained with viscous-plastic (VP) model in the same configuration.

The objective of this study is to prepare the integration of the EB model in coupled ice-ocean models by studying its behaviour on long time scales and with various spatial resolutions. We also propose two parameterisations of the healing process and we study their impact on the persistency of high ice deformation features.

Other models are being developed to improve the representation of sea ice dynamics. Hutchings et al. (2005) showed that isotropic models can produce oriented failure zones if the initial ice strength is randomly distributed and if weakening is allowed. Another approach consists in developing anisotropic models (Wilchinsky and Feltham, 2012; Schreyer et al., 2006). Several approaches are also developed in order to validate the simulated deformation fields. Coon et al. (2007) defined a metric based on the correlation of the location and orientation of discontinuities between simulated and observed deformation fields. However, these models and metrics do not take into account the scaling properties of the ice deformation. We think that models should be validated in regard of the ice pack statistical properties, as done in Girard et al. (2009).

In section 4.2, the different components of the EB model are detailed as well as the adaptations made for running simulations on longer periods than a few days. In section 4.3, we analyse results from a simulation of the Arctic ice pack deformation at high resolution. These results confirm the ability of EB to produce heterogeneous ice deformation fields and power-law tailed distributions. The impact of two healing parameterisations on the persistency and activity of sea ice deformation are investigated in one-month simulations at a lower resolution.

4.2 Model description

The first application of the EB model made by Girard et al. (2011) showed its ability to reproduce heterogeneous sea ice deformation fields in a series of 3-days simulations of the Arctic ice pack deformation. However, an adaptation of the model

Symbol	Meaning	Values and units
h	Ice thickness	m
A	Ice concentration	
E	Effective elastic modulus	Nm^{-1}
Y	Young modulus	$5 \times 10^8 Nm^{-2}$
c^*	Ice strength constant	20
ν	Poisson coefficient	0.3
μ	Internal friction coefficient	0.7
C	Cohesion	10000 to 20000 Nm^{-1}
σ_0	Tensile strength	20000 Nm^{-1}
d_0	Damage factor	0.85

Table 4.1: Principal variables and parameters used in the model

is required for integration on longer periods of time and to study the evolution of deformation patterns. In the following section, we present the EB model, as well as the modifications made for this study. Variables and model parameters are listed in Table 4.1.

4.2.1 Progressive damage models

The EB model is inspired from progressive damage models used in rock mechanics. These models reproduce some characteristics of brittle failure of disordered materials, such as the localization of the deformation and power-law distributions of dissipated energy and avalanche sizes (Amitrano et al., 1999; Girard et al., 2010).

One of the simplest damage models for disordered materials is the random fuse model (RFM) that emulates the fracturing process by simulating the behaviour of a large network of fuses with random tolerance (Nukala et al., 2005). The voltage applied at both ends of the network is slowly increased and generates an electric current through the elementary components. When the tolerance is reached in one fuse, it breaks and the current is instantaneously redistributed through the fuse network. Other components may reach the limit and blow up, leading to large avalanches of breaking events. The avalanche size, defined here as the number of breaking events for each increment of voltage, has the same statistical properties as the one observed during the fracturing of geo-materials and the RFM model is a powerful tool to study damage localization and power-law distribution of avalanche sizes (Alava et al., 2006). Many variants exist, as the random spring model (RSM) where fuses and currents are replaced by elastic springs and forces (Nukala et al., 2005), but the key point is the emergence of a complex behaviour from interactions

between a large number of elementary components. Even if these models are very simplified, their behaviour has many similarities with fracturing in ice.

The EB model is somewhat similar to RFM and RSM models. In the present study, the elementary components are finite-element triangles connected by their vertices and covering all the ice pack. Each of them responds as a linear elastic material with its own effective stiffness and tolerance. If the internal stress in an element reaches a given threshold, its stiffness is reduced, implying an elastic redistribution of the stress. As in the RFM and RSM models, the complexity of the results does not come from a complex material law or from the external forcing but emerges from interactions between ice elements. An advantage of EB compared to more simplified models is that shear and divergence strains can be computed, and compared to observations.

Another advantage of the EB model, is that, even if elements are treated as being distinct, they remain connected and cover all the ice pack with no gap between them. This approach contrasts with the discrete model of Hopkins and Thorndike (2006), where thousands of independent ice pieces float on the ocean surface and interact with each other by collisions or frozen joints. Here, the continuity of the medium is preserved in some way. There is no explicit representation of individual floes or leads.

The EB model also differs from fracture models used to predict the evolution of a single crack in an homogeneous medium by taking into account the energy release of the advancing crack tip. In our meso-scale approach, micro-scale processes are indirectly taken into account by the relationship between the damage variable and the effective elastic stiffness. However, this model is not exactly continuous as each element has its own material properties (cohesion and damage) which are not derived from an underlying continuous field. In particular, the damage variable may vary dramatically between elements and from one time step to the next. In consequence, the results of the model depend on the spatial and temporal resolutions and are not supposed to converge with increasing resolution.

4.2.2 The constitutive equation

Hooke's law under plane-stress relates the internal stress tensor $\boldsymbol{\sigma}$ to the deformation tensor, given by $\boldsymbol{\epsilon} = \frac{1}{2}(\nabla \mathbf{U} + (\nabla \mathbf{U})^T)$, where $\mathbf{U}$ is the displacement at the nodes from a stress-free configuration:

$$\sigma_{ij} = E D \epsilon_{ij}, \quad \text{with} \quad \sigma_{ij} = \begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{bmatrix} \quad \text{and} \quad \epsilon_{ij} = \begin{bmatrix} \epsilon_{11} \\ \epsilon_{22} \\ \epsilon_{12} \end{bmatrix}, \quad (4.1)$$

where E is the effective elastic stiffness and $\mathbf{D}$ is the unit elasticity tensor defined as:

$$\mathbf{D} = \frac{1}{(1-\nu^2)} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix}, \quad (4.2)$$

where ν is Poisson's ratio.

For each element, a scalar damage variable d is defined, equal to 1 for an undamaged element and to 0 for a completely damaged element. The value of d influences the mechanical properties through the effective elastic stiffness E defined as:

$$E = E_0 d, \quad (4.3)$$

where E_0 is the undamaged elastic stiffness, which is parameterised as:

$$E_0 = Y h e^{-c^*(1-A)}, \quad (4.4)$$

where Y is Young's modulus, h is the mean ice thickness, c^* is a non-dimensional parameter and A is the ice concentration. This parameterisation mimics the one proposed by Hibler (1979) for sea ice strength in the VP modelling framework. The linear relationship with the ice thickness is obvious and the exponential relationship with the ice concentration allows a rapid decrease of the internal stress when ice cover is not compact.

The EB model is then isotropic at the element scale, as the damage variable only modifies the magnitude of the elastic stiffness. However, the combination of damaging and long-range elastic interactions spontaneously leads to anisotropic strain patterns, as shown by Girard (2010), and below.

4.2.3 The collective behaviour

Damage occurs when the stress state falls outside a failure envelope defined as a combination of a Mohr-Coulomb criterion,

$$\sigma_s \leq -\mu \sigma_{kk} + 2C, \quad (4.5)$$

and a maximum tensile stress criterion,

$$\sigma_{kk} \leq 2\sigma_0. \quad (4.6)$$

where σ_s , the shear stress, is given by $\sigma_s = \sqrt{(\sigma_{11} - \sigma_{22})^2 + 4\sigma_{12}^2}$ and σ_{kk} , the compressive stress (also called normal stress), is given by the trace of the stress tensor. The three positive parameters, C , μ and σ_0 , are the cohesion, the internal friction

coefficient and the tensile strength, respectively. This failure envelope well corresponds to in-situ stress measurements (Weiss et al., 2007). The friction coefficients μ is set to 0.7, which is a common value for geo-materials (Amitrano et al., 1999). This value also coincides with results from laboratory tests (Schulson et al., 2006) and seems to be scale-independent (Weiss and Schulson, 2009).

The cohesion C is defined as the maximum shear stress in pure shear deformation and depends on the spatial scale. In-situ measurements give values around $40 \times 10^3 Nm^{-2}$ (Weiss et al., 2007). Much higher values, about $5 \times 10^6 Nm^{-2}$, are observed with specimens of ice of a few centimetres (Schulson et al., 2006). This dependence on the scale is explained by the higher probability of having a larger defect when increasing the length scale, resulting in lower resistance to shear and traction. As the internal stress in the model is supposed to be vertically integrated, the value of the cohesion should linearly depends on the ice thickness but we do not use this relationship and we rather randomly draw its value from a uniform distribution between $C_{min} = 10 \times 10^3$ and $C_{max} = 20 \times 10^3 Nm^{-1}$, as in Girard et al. (2011). Similarly, the tensile stress, σ_0 , is set to $20 \times 10^3 Nm^{-1}$ with no dependence on the ice thickness. A limit could also be defined for the compressive stress to represent the crushing of the ice under high confinement, but this is less frequent and not taken into account here.

When the stress state falls outside the failure envelope, the local value of d is multiplied by a constant damage factor d_0 , smaller than 1. The value of d_0 is not critical (Amitrano et al., 1999) and is usually taken between 0.85 and 0.95 (Girard et al., 2011). After n damage events, the local effective elastic stiffness is equal to:

$$E = E_0 d_0^n. \quad (4.7)$$

The damage variable can be related to the density of micro-cracks within an element. Kachanov (1992) related the elastic stiffness of a fractured material to the crack density ρ by the following relationship:

$$E = E_0 (1 + c\rho)^{-1}, \quad (4.8)$$

where c is a constant and where the crack density is defined by

$$\rho = \frac{1}{L^2} \sum l^2, \quad (4.9)$$

where l is the length of each crack and L^2 is the area of the region of interest. For a two-dimensional material with non-interacting, randomly oriented cracks, c is found equal to π . If we assume that one damage event corresponds to one crack, the two expressions (4.7) and (4.8) are actually almost equivalent. The two expressions give the same value when there is no crack/damage ($\rho = 0$) and asymptotically

tend to zero when the number of cracks/damages increases. If we further assume that all cracks have the same length, the crack density is given by $\rho = \frac{l^2}{L^2} n$. Equation (4.7) is then well approximated by Equation (4.8), with the same derivative for $n = 0$, if we choose $c = -\log(d_0) \frac{L^2}{l^2}$. The damage may then be related to n by the following expression:

$$d = (1 - \log(d_0)n)^{-1}, \quad (4.10)$$

instead of $d = d_0^n$. This alternative formulation of the damage variable has not been tested in our study but allows the interpretation of the damage variable in terms of the crack density. With the assumption that all cracks have the same length and if $c = \pi$, we find an approximate value for the crack length $l = \sqrt{\frac{-\log(d_0)}{c}} L \approx \frac{L}{4}$.

The redistribution of the stress is made by the quasi-static equation

$$\nabla \cdot \boldsymbol{\sigma} + \mathbf{F} + \mathbf{G}(\mathbf{u}) = 0, \quad (4.11)$$

where the first term depends on ice deformation, the second term groups the external stresses independent on ice deformation and velocity, and the third term corresponds to the velocity-dependent terms. The displacement is fixed at zero on the coast and a zero stress condition is applied at the ice edge.

The external stress $\mathbf{F}$ is progressively applied thanks to a loading factor f that slowly increases from 0 to 1 by a small increment (10^{-3}). This increment is of the same order of magnitude than the ratio between the time step (20 seconds) and the spin-up duration (24 hours) used in Girard et al. (2011). After each increment of the external stress, damage is applied on the elements that violate the failure criterion. Interactions between elements are allowed by cycling through the redistribution of the stress and the application of damage. The external stress is upgraded only when no more element exceeds the failure criterion.

The quasi-static approximation implies that a damage in one element instantaneously affects all the other elements. This hypothesis is justified by the high speed of elastic waves compared to the crack velocity. Marsan et al. (2011) identified bursts of low-frequency waves propagating at a speed of about 440 m s^{-1} . The associated seismic activity correlates with ice-cover deformation at the regional scale (400 km) and exhibits a strong intermittency. Besides, the crack velocity of a single crack in sea ice is evaluated at about 10 m s^{-1} (Petrenko and Gluschenkov, 1996; Dempsey, 2000). This is high enough to split one element of 10 km in ~ 1000 seconds. Since the crack velocity is much lower than the elastic wave speed, we assume that no elasto-dynamic equation is needed and that the propagation of the stress is instantaneous compared to damage increase.

Contrary to classical laboratory mechanical tests, the post-failure dynamics has to be considered in our case and the simulation has to continue even after the damage localization. In this study, we simply rely on the other two dominant terms of the momentum equation, $\boldsymbol{\tau}_a$, the atmospheric stress and $\boldsymbol{\tau}_w$, the oceanic drag term, to produce a reasonable ice velocity field. The horizontal atmospheric drag term depends on the wind velocity $\mathbf{u}_a$ by a quadratic relationship:

$$\boldsymbol{\tau}_a = A\rho_a c_a |\mathbf{u}_a| \mathbf{u}_a, \quad (4.12)$$

where the air density $\rho_a = 1.3 \text{ kg m}^{-3}$ and the drag coefficient $c_a = 0.0012$. $\boldsymbol{\tau}_w$ is given by a linear parameterisation $\boldsymbol{\tau}_w = A\rho_o c_w (\mathbf{u}_o - \mathbf{u})$, where $c_w = 0.66 \times 10^{-3}$ is the water drag coefficient, $\rho_o = 1025 \text{ kg m}^{-3}$ is the reference density of seawater and $\mathbf{u}_o$ is the oceanic surface velocity which is set to zero in our study (ocean at rest). The other terms related to Coriolis effect, the slope of the sea surface elevation and ice acceleration are less important for compact Arctic sea ice (Leppäranta, 2005) and are neglected in this study for simplicity. All these terms may be included in $\mathbf{F}$ and $\mathbf{G}(\mathbf{u})$.

The behaviour of the simulated ice pack is very different before and after damage localization. Before damage localization, the ice pack resists deformation as an elastic material. After damage localization, the ice pack behaves as an ensemble of individual plates of various dimensions, delimited by highly damaged linear features. In the first case, the deformation is recoverable while it is permanent in the second case. To distinguish the two cases, we analyse the magnitude of the deformation when a piece of ice is entirely surrounded by damaged elements or not. The wind stress magnitude is given by $\tau_a = \rho_a c_a u_a^2$ and is equal to $\tau_a = 0.15 \text{ Nm}^{-2}$, for wind speed $u_a = 10 \text{ m/s}$. The ice-ocean drag magnitude is given by $\tau_w = \rho_w c_w u$. In the first case, a portion l of the boundary is still undamaged and resists deformation with an effective elastic modulus $E = 500 \times 10^6 \text{ Nm}^{-1}$ (from Equation (4.4) for 1-metre thick concentrated ice). If the ice-ocean drag term is neglected, the magnitude of the deformation is given by $\epsilon = (\int_S \tau_a dS) / (\oint_{\partial S} E dl) = \frac{L^2 \tau_a}{lE}$. This deformation is elastic (recoverable) and would be about $\epsilon = 3 \times 10^{-4}$, if $l = L$. In the second case, sea ice is entirely damaged on the boundary of the considered area S . The velocity of this piece of ice is then given by $u = (\int_S \tau_a dS) / (\int_S \rho_w c_w dS) = 20 \text{ km day}^{-1}$. Deformation concentrates at the boundary of S , as it is disconnected from its surroundings. If the surroundings are not moving, the magnitude of the deformation rate at a spatial scale of 10 km is about 2 day^{-1} . This deformation is permanent (not linked to the elastic term) and is much larger than in the first case. On a daily time scale, permanent deformation is four orders of magnitude larger than elastic deformation. The clear separation of scales between recoverable and permanent deformations allows to neglect the elastic part of the deformation and to consider all the deformation as permanent at the end of the loading process.

At this moment, it is important to note that there is no time scale associated with the quasi-static Equation 4.11, nor with the damage processes. An arbitrary time scale has to be chosen to define deformation rate and velocity. As we consider that all the deformation is permanent, ice velocity can directly be defined by $\mathbf{u} = \frac{\mathbf{U}}{\Delta t}$ where Δt is an arbitrary time scale chosen equal to 6 hours. The strain rate tensor is then computed by $\dot{\boldsymbol{\epsilon}} = \frac{1}{2} (\nabla \mathbf{u} + (\nabla \mathbf{u})^T)$ and its invariants by

$$\dot{\epsilon}_{kk} = \dot{\epsilon}_{11} + \dot{\epsilon}_{22}, \quad (4.13)$$

$$\dot{\epsilon}_s = \sqrt{(\dot{\epsilon}_{11} - \dot{\epsilon}_{22})^2 + 4\dot{\epsilon}_{12}^2}. \quad (4.14)$$

Our interpretation of EB in this study can be summarized as follows. The damage process and the instantaneous elastic stress redistribution allow to localize damage along linear features delimiting distinct plates. The velocity of each plate is given by the equilibrium between the oceanic and atmospheric drag terms. The separation of scale between elastic and permanent deformations allows to consider all the deformation as permanent at the end of each loading step. The velocity is then directly given by the displacement divided by an arbitrary time scale Δt .

4.2.4 Healing parameterisations

As explained in Girard et al. (2010), once damage localization has occurred, most of the deformation is accommodated by persistent features. Introducing damage recovery allows to heal the ice and to simulate evolving deformation patterns. Healing parameterisations could be based on the refreezing in leads and ridges but the link with the damage variable is not very clear. Equation (4.10) gives a possible relationship between the level of damage d and the number of cracks, but these cracks could correspond either to leads or ridges and the model gives no information about their size. In the followings, we present two healing parameterisations that will be validated regarding their effects on the deformation field. The parallel with the refreezing in leads is only used to evaluate the order of magnitude of the healing time scale.

Several studies show a rapid recovery of the ice thickness in narrow leads. Petrich et al. (2007) observed in leads of $\sim 10\text{cm}$ width that only 100 to 200 hours were needed to recover 1 meter of ice at the surface air temperature $T_a = -15^\circ\text{C}$. This time scale is much larger than the damage time scale and both processes are considered decoupled in this study. The healing only acts on damage every global time step.

We have tested two simple healing parameterisations based on a unique parameter t_h , which is the time needed to heal a cell from the completely damaged state $d = 0$ to the undamaged state $d = 1$. The first parameterisation assumes a constant

healing rate equal to $\frac{1}{t_h}$ leading to

$$d(t) = \frac{t}{t_h} + d(0). \quad (4.15)$$

The second parameterisation proposed by Girard (2010) assumes that the rate of healing $d'(t)$ is inversely proportional to the damage variable d :

$$d'(t) = \frac{1}{2t_h d(t)}. \quad (4.16)$$

By solving the differential equation

$$d' d = \frac{1}{2t_h}, \quad (4.17)$$

one obtains:

$$d(t) = \sqrt{\frac{t}{t_h} + d(0)^2}. \quad (4.18)$$

This parameterisation is inspired from the vertical growth of the mean ice thickness (Maykut, 1986), but actually nothing indicates that the healing rate depends on the level of damage, especially when it is interpreted as a measure of the crack density.

4.3 Results and discussion

Several simulations have been performed with the model described in the previous section. In a first step, the spatial scaling of the simulated ice deformation is analysed from a simulation at high resolution (10 km) on a period of 5 days and without healing. In a second step, the effects of healing parameterisations on sea ice dynamics are investigated from results obtained at a resolution of 50 km and on a period of one month. Apart from the resolution, all simulations share the same setup which is described hereafter.

The mesh is made of triangular elements of uniform size and covers all the oceanic surface located north of $50^\circ N$ (see Figure 4.1 for the mesh with a resolution of 50 km). The coasts are defined at the prescribed resolution by a B-spline interpolation of a coarsened set of the Global Self-consistent, Hierarchical, High-resolution Shoreline Database (GSHHS) following the method explained by Lambrechts et al. (2008). Sea ice concentration is interpolated and the mesh is adapted to force a 10 times lower resolution where the ice concentration is lower than 0.1. All these operations are performed with the Gmsh mesh generator¹ and the MadLib library². For the high resolution simulation, the Canadian Arctic Archipelago is not taken into account to limit the computational cost.

¹<http://www.geuz.org/gmsh>

²<http://sites.uclouvain.be/madlib>

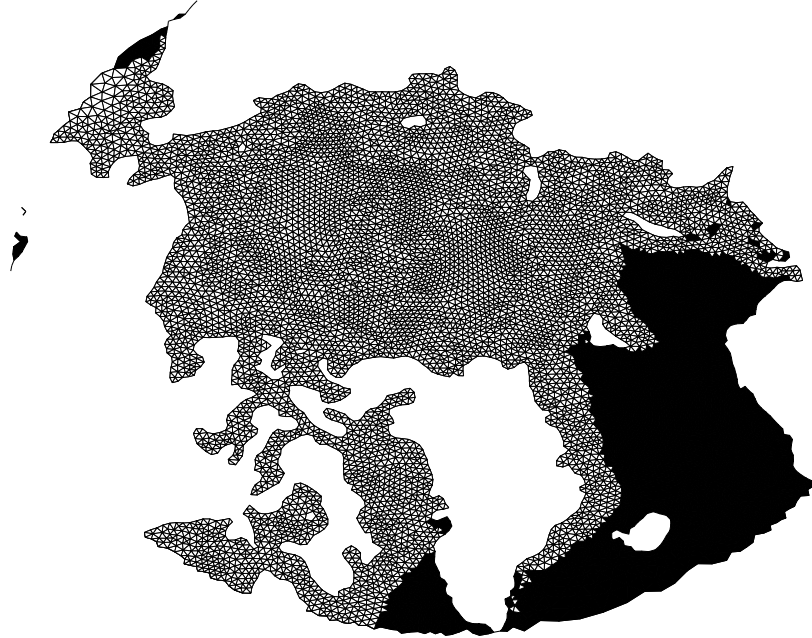


Figure 4.1: Initial mesh of the ice pack with a mean resolution of 50 km. The rest of the oceanic surface is also meshed with a resolution varying from 50 to 500 km but a mask is applied to do not compute ice motion where there is no ice.

In each element, the sea ice volume, concentration and damage are considered constant. The velocity field is continuous with nodal values defined at triangle vertices. Thanks to the continuous approach, we can use classical finite element methods to compute the ice velocity field. Equations are solved on the sphere with the method developed by Comblen et al. (2009) for the resolution of partial differential equations on curved surfaces. This method is based on both local and global coordinate systems, and avoids the so-called pole singularity issue.

For all simulations, ice thicknesses and concentrations are initialized as in Girard et al. (2011) from the state of the ice pack coming from a simulation conducted with the coupled ice-ocean model NEMO-LIM2 with the VP rheology in the ORCA025-G70 configuration (Bernard et al., 2006; Barnier et al., 2007). As the EB framework is intended to describe the mechanical behavior of dense pack ice (Girard et al., 2011), results will only be analysed in a region of interest, hereafter called the "central Arctic". It is represented by elements with visible borders in Figure 4.2. In this region, the ice concentration is higher than 0.95 and the mean thickness varies from 1 to 9 m. We verified that the inclusion or exclusion of elements near the coasts does

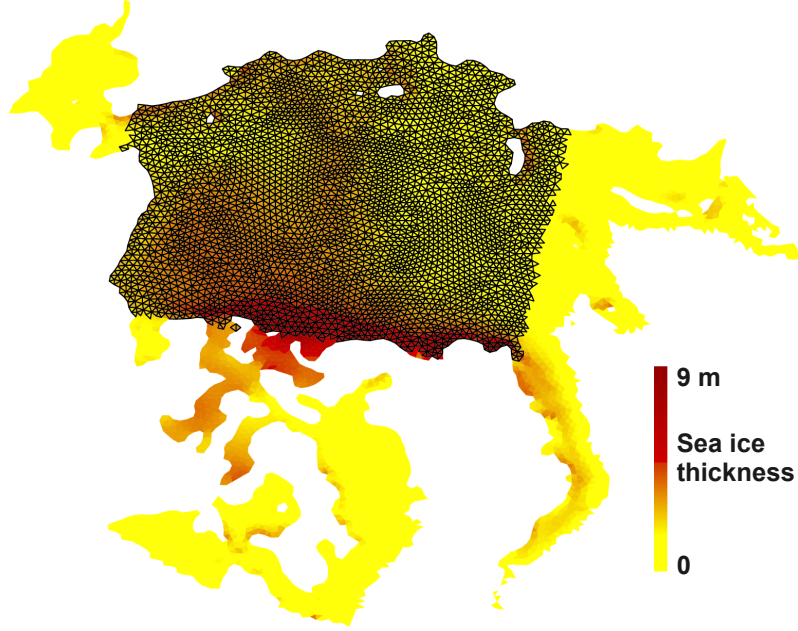


Figure 4.2: Initial ice thicknesses [m] on the whole computational domain. Elements with concentration below 0.15 are not drawn. Elements with visible border form our "central Arctic" region used for the analysis.

not influence the analysis. The initial elastic stiffness field computed from Equation (3.56) is shown in Figure 4.3. The initial value for damage is 1 (no damage). The values of the parameters are listed in Table 4.1. The daily 10 meter winds come from the NCEP-NCAR reanalysis which has a mean resolution of about 200 km in the Arctic (Kalnay et al., 1996). All the simulations start the first January 2007. The oceanic drag term is given by a linear drag formula:

$$\boldsymbol{\tau}_w = A\rho_o c_w (\mathbf{u}_o - \mathbf{u}), \quad (4.19)$$

where c_w is the water drag coefficient, ρ_o is the reference density of seawater and $\mathbf{u}_o \equiv (u_o, v_o)$ is the oceanic surface velocity which is set to zero in our study (ocean at rest). This drag term may be thought as the linearization of the classical quadratic formulation, given by $A\rho_o c_D |u|u$ in one-dimensional case. The value of c_w is equal to 0.66×10^{-3} and corresponds to a linearization of the quadratic drag around $u = 0.06 \text{ m s}^{-1}$ with the quadratic water drag coefficient c_D equal to 0.0055.

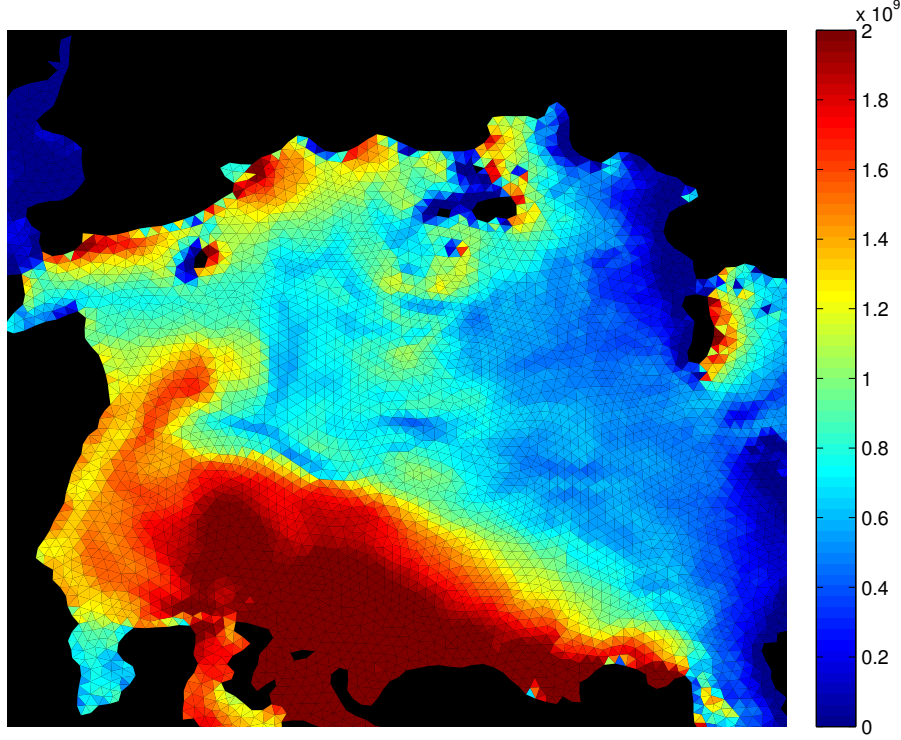


Figure 4.3: Initial elastic parameter E [N/m] computed from the initial ice thicknesses and concentrations.

The atmospheric drag term depends on the wind velocity $\mathbf{u}_a$ by a quadratic relationship:

$$\boldsymbol{\tau}_a = A\rho_a c_a |\mathbf{u}_a| \mathbf{u}_a, \quad (4.20)$$

where $\rho_a = 1.3 \text{ kg m}^{-3}$ and $c_a = 0.0012$. Both stress terms depend on the ice concentration as suggested by Connolley et al. (2004) and Hunke and Dukowicz (2003).

4.3.1 Spatial scaling analysis of a simulation at high resolution

A first simulation of 5 days (January 1-5, 2007) is performed at a resolution of 10 km without healing. Figures 4.4 and 4.5 show snapshots of the shear and divergence rates, respectively. They are computed after 5 days of simulation starting without damage. The temporal and spatial scales are given by the time step (6 hours) and the resolution of the grid (15 km), respectively. Linear features of high ice deformation are observed in both fields representing active faults. These one-element

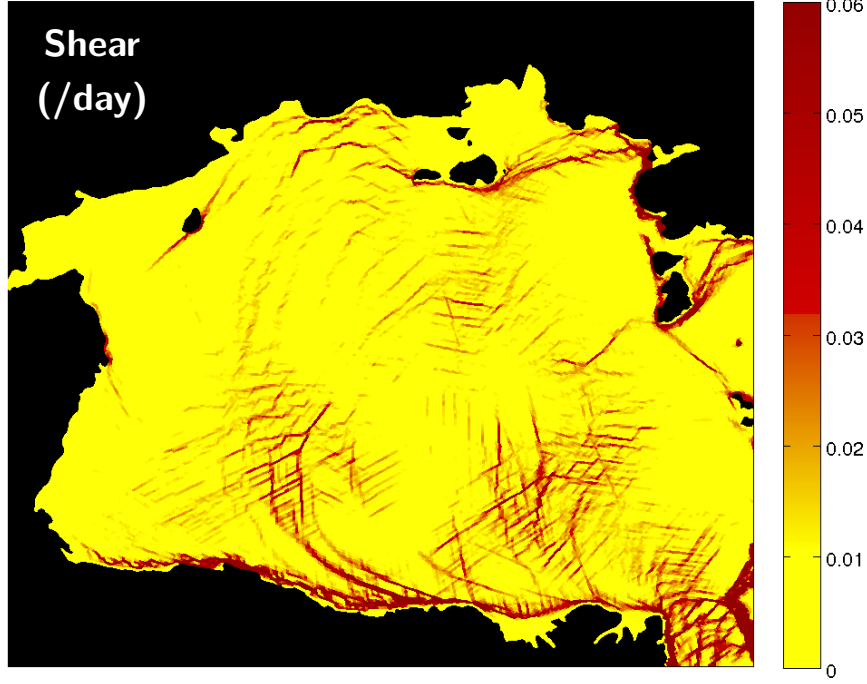


Figure 4.4: Shear rates at the last time step of the 5 days of simulation on a mesh with a mean resolution of 15 km corresponding to a spatial scale of 10 km. The temporal scale is 6 hours.

wide features delimit large areas with almost no deformation and qualitatively reproduce the characteristic patterns of the ice deformation computed from RGPS (Kwok, 2006). The clear anisotropy in the deformation fields does not come from initial conditions but emerges from local damages and elastic interactions between elements. As in the previous application of the EB model by Girard et al. (2011), the shear and divergence fields exhibit strong spatial clustering similar to what is observed from RGPS data (Weiss and Schulson, 2009).

The probability density functions (PDF) of the simulated absolute divergence and shear rates, $p(\dot{\epsilon})$, exhibits a power-law decay over 3 orders of magnitude, where $p(\dot{\epsilon}) \sim \dot{\epsilon}^{-\eta}$, with $\eta = 2.6$ for divergence (not shown) and $\eta = 2.7$ for the shear (Figure 4.6). Such power-law distributions with $\eta < 3$ are consistent with RGPS observations (Girard et al., 2009) and indicate that the deformation is dominated by extreme values. With such distributions, the standard deviation is, analytically, in-

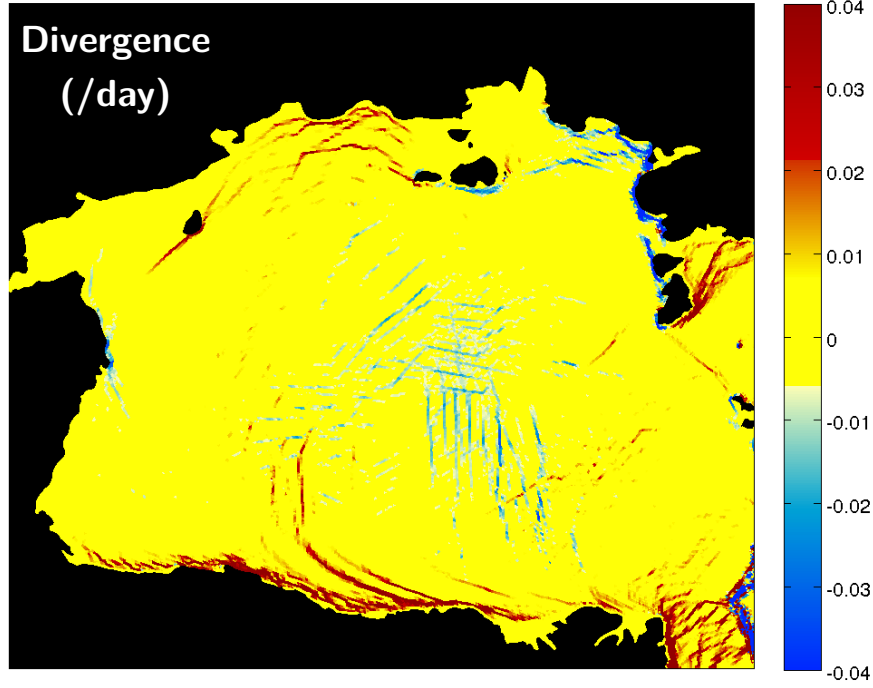


Figure 4.5: Divergence rates at the last time step of the 5 days of simulation on a mesh with a mean resolution of 15 km corresponding to a spatial scale of 10 km. The temporal scale is 6 hours.

finity (Marsan et al., 2004). These PDFs are obtained for the last 3 days of the simulation from the central Arctic region and take into account all the outputs given every 6 hours. Classical models cannot reproduce such fat tail distributions, even at high resolution (Girard et al., 2009).

A second characteristic of sea ice dynamics is the spatial correlation of the deformation field. It is evaluated here by computing the deformation field at several spatial scales (10 to 1000 km) by a coarse graining procedure (Marsan et al., 2004; Girard et al., 2011). The procedure is illustrated in Figure 4.7, which displays the shear rates at length scales going from 20 to 640 km. The deformation rate at a certain length scale is computed on a regular array of points separated by a distance $l/2$. For each point, we take into account all triangles whose centre is within a square of length l centred on that point and we compute the average of velocity

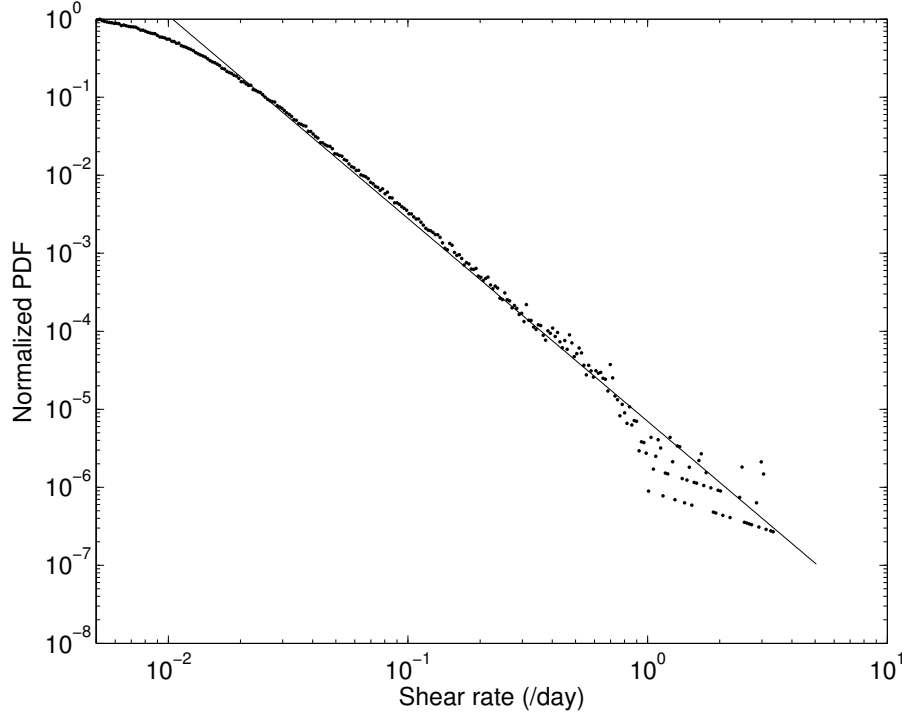


Figure 4.6: Normalized PDFs of shear rates (/day) at the spatial scale of 10 km and temporal scale of 6 hours. The distribution has a power-law tail with exponent equal to -2.7.

derivatives weighted by the surface. Strain rate invariants are registered and the spatial scale L is given by the square root of the surface covered by the considered triangles.

The localization of the deformation is very similar to observations, where most of the ice deformation is accommodated by a small portion of the ice cover. For smaller scales (10 and 20 km), the largest 15% of the deformation is accommodated by less than 1% of the considered area, with very high deformation rates on the order of 1 day^{-1} . As in Stern and Lindsay (2009), we first determine the minimum number of elements whose sum of deformation rates is larger than 15% of the sum of all deformation rates and we then compute the ratio of the area covered by these elements to the total area considered. For larger scales, extreme values of the deformation are lower, but the distribution is still with a power-law tail and the localization remains important. For example, at scales of 160 and 320 km, the area fraction

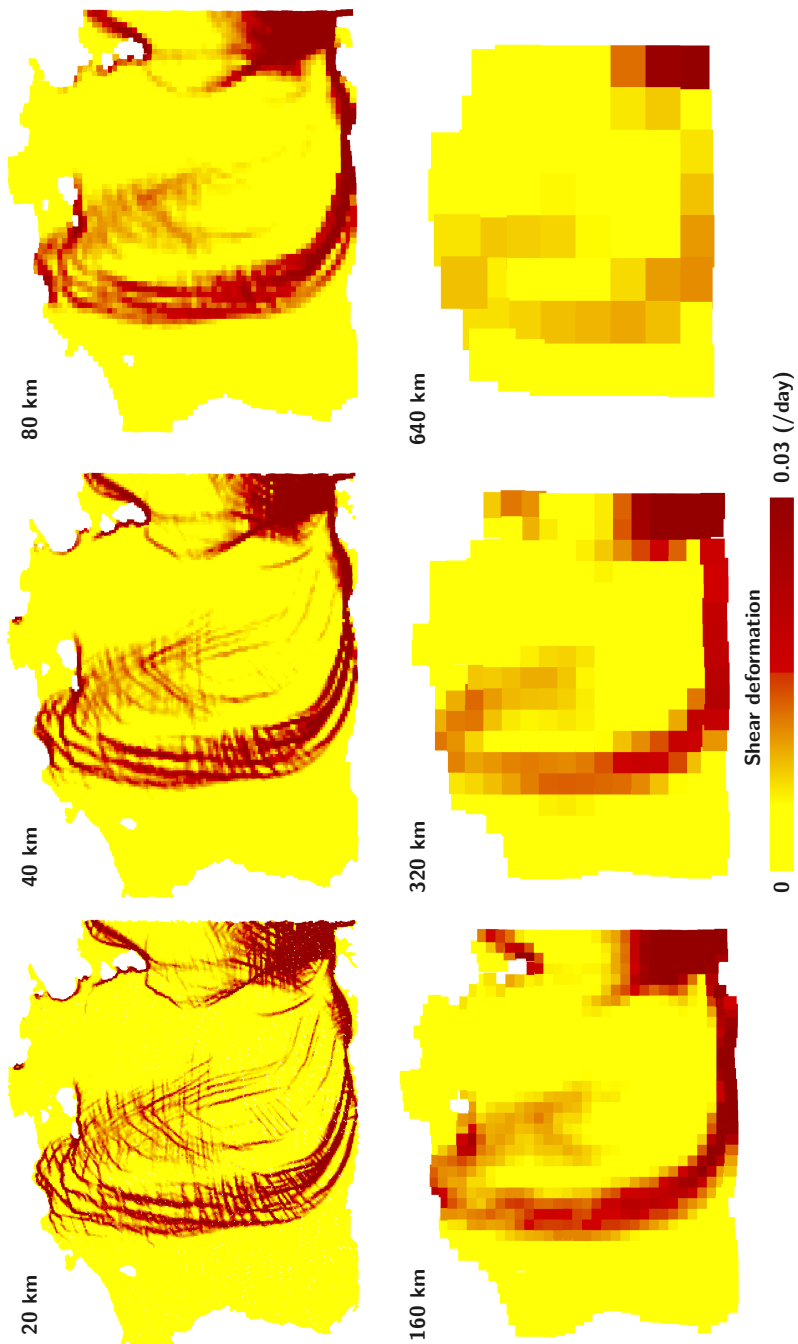


Figure 4.7: Shear deformation rates computed at the spatial scales of 20, 40, 80, 160, 320 and 640 km. Each square corresponds to one bin of data and is drawn at half its actual size for illustration. Those squares overlap in reality.

accommodating the largest 15% of the deformation evaluated for all snapshots of 6 hours is equal to $2.6 \pm 1.1\%$ and $4.2 \pm 1.6\%$, respectively, which is consistent with the values found by Stern and Lindsay (2009) and Marsan et al. (2004) from RGPS data. As it can be seen from Figure 4.7, shear rates higher than 0.03 day^{-1} are still observed at scales larger than 160 km.

Strong localization of the ice deformation rate is observed for all snapshots after two days of simulations. Figure 4.8 illustrates the localization of the deformation calculated at the first time step of day 3. The localization of the deformation rate is not only diagnosed visually but can be detected through the effect on its moments $\dot{\epsilon}_L^q$. The average value of the moments of the ice deformation is computed for each length scale L defined by the coarsening procedure. The power-law relationship $\langle \dot{\epsilon}_L^q \rangle \sim L^{\beta(q)}$ holds from 10 to 320 km. The last two bins (640 and 1320 km) diverge from the power-law because of the finite size of the domain. The fact that the slope is higher for higher moments indicates that large deformations are more localized towards small scales than lower deformations.

The fact that $\beta(q)$ increases quadratically with q , as illustrated in Figure 4.9, indicates the multifractality of the deformation rate. In this example (first time step of day 3), $\beta(q)$ can be interpolated by a quadratic function $aq^2 + bq$, with $a = 0.16$ and $b = -0.1$. Similar values are found for all snapshots. The EB model is then found able to produce multifractal heterogeneous deformation fields.

The values of $\beta(1)$ vary between 0.06 and 0.02 and are lower than those obtained from RGPS data (~ 0.2) (Marsan et al., 2004) and than the values obtained with the EB model (~ 0.1) (Girard et al., 2011) with a model resolution of 10 km and 6-hourly wind forcing having a spatial resolution of about 80 km. The lower values in our case may be due to the smoother wind field used in our study which is only given at a resolution of 200 km. As explained by Stern and Lindsay (2009), if the wind stress gradient is larger, the exponent is more negative.

4.3.2 Temporal scaling and influence of healing

Several 30-days simulations (January 1-30, 2007) are performed at the resolution of 50 km to study spatial and temporal scaling and the effects of healing on sea ice dynamics. First of all, we estimate the variability of the $\beta(1)$ exponent computed with the same coarsening procedure as in the high resolution case. The same kind of analysis is performed in the temporal dimension to look at the intermittency of the simulated ice deformation. Eventually, the effects of healing on the evolution of the ice deformation are analysed and linked to the observed seasonality of the activity of the Arctic sea ice cover.

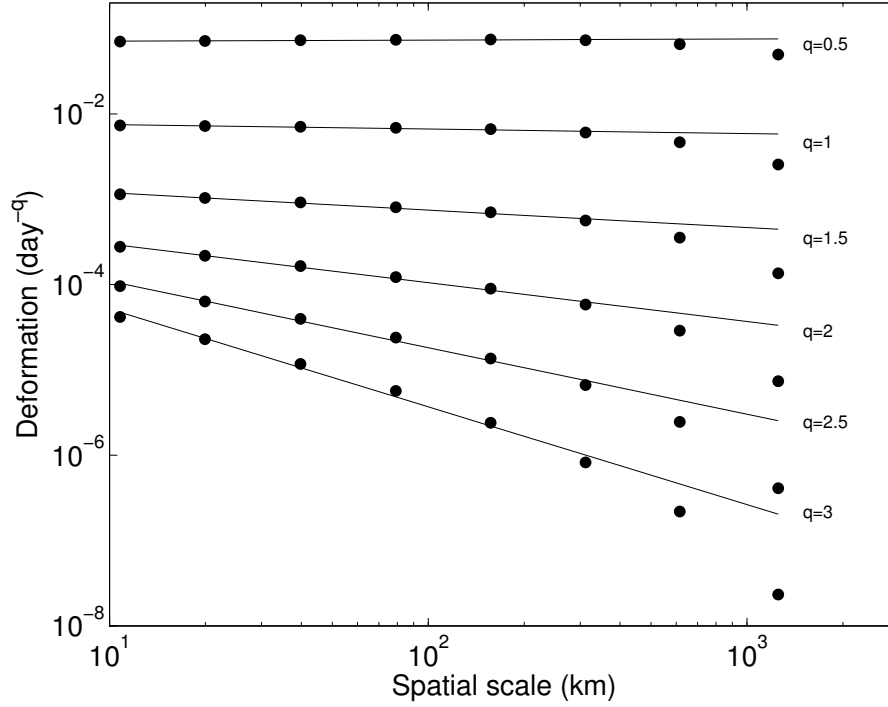


Figure 4.8: Moments of the deformation as a function of the spatial scale. Each dot corresponds to the domain average value of the q -moment of the shear deformation computed at the length scale L . The lines are the best linear fit in the log-log space for length scales from 10 to 320 km.

The simulations are conducted with the same initial conditions, forcing and parameters as the simulation at high resolution. At the end of each time step (6 hours), the outputs are computed and the external wind is updated. For the analysis, the first two days of simulation are not taken into account, as in the simulation at high resolution.

On periods larger than a few days, the advection of sea ice is not negligible but its implementation leads to several problems. With Eulerian methods, the numerical diffusion of the damage variable is too important because this field strongly varies between elements. With these methods, the localization of the deformation is not preserved from one time step to the other. To avoid this problem, we implemented a Lagrangian method with a moving grid (Lietaer, 2011). No numerical diffusion arises during the advection of discontinuous fields and the conservation of sea ice

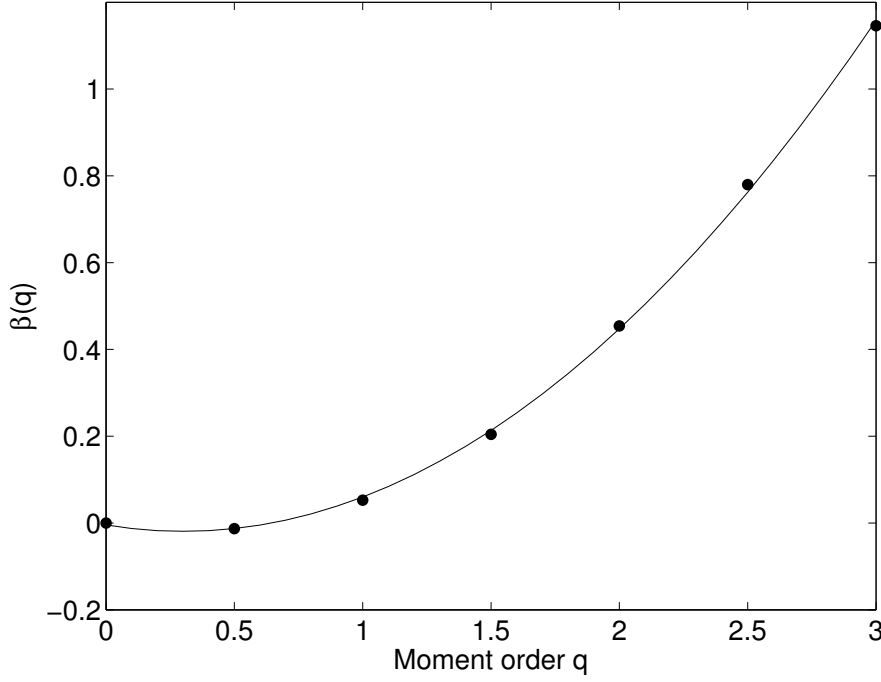


Figure 4.9: Functions $\beta(q)$ versus moment order q (circles) and the best-fit quadratic curve.

volume and concentration is naturally handled. As the grid deforms, elements may become highly distorted and generate numerical instabilities. The needed adaptation of the mesh complicates the analysis of statistical properties and generates numerical diffusion in the elements that are adapted. We have tested the influence of neglecting the advection by comparing ice deformation distributions obtained with and without advection. As no difference is detected, we do not consider advection for this study which focus on statistical and scaling properties. As the thermodynamics is also not considered, sea ice thickness and concentration remain constant thorough all the simulated period.

The heterogeneity of the deformation is estimated at spatial scales ranging from 40 to 1280 km and from time scales ranging from 6 hours to 16 days. A power-law relationship is observed up to 320 km, as in the high resolution case. However, the power-law exponent $\beta(1)$ has a large variability as shown in Figure 4.10 for a simulation with the first healing parameterisation (constant healing rate) and $t_h = 4$ years. The mean value of $\beta(1)$ is about 0.1 at a temporal scale of 6 hours, which

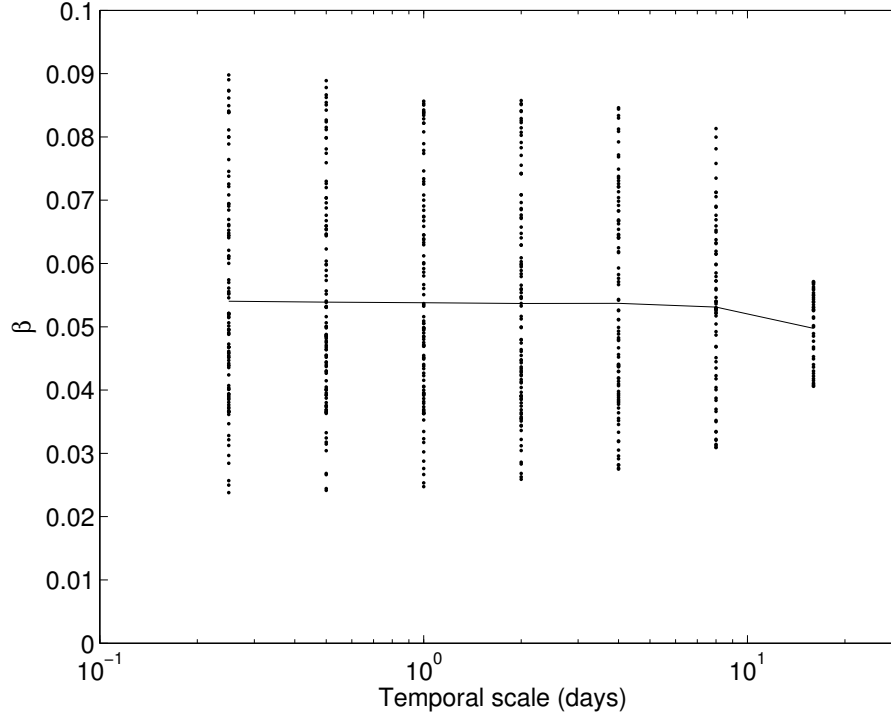


Figure 4.10: Values of the $\beta(1)$ exponent computed at different temporal scales from a 1-month of simulation with the first healing parameterisation (constant healing rate) and $t_h = 4$ years. The line connects the mean values for each temporal scale.

is higher than in the high resolution simulation. The mean value does not change when increasing temporal scale from 6 hours to 3 days. This is in contradiction with the logarithmic decrease of $\beta(1)$ when increasing the temporal scale observed in Hutchings et al. (2011) and Rampal et al. (2008). The absence of effect of the time scale indicates that our implementation of EB does not produce temporal scaling, as shown below. The standard deviation is quite large (about 0.07), but similar to the large variability of the $\beta(1)$ exponent observed by Stern and Lindsay (2009). The mean value of $\beta(1)$ is not affected by healing, meaning that this process does not influence the localization of the deformation. This is consistent with the fact that instantaneous deformation fields are not smoother with healing than without. Narrow bands of high deformation can be seen even with $t_h = 0.75$ month (not shown).

The problem of temporal scaling is confirmed by the absence of any significant power-law relationship between the mean absolute deformation rate $\langle \dot{\epsilon}_\tau \rangle$ and the temporal scale τ considered to compute the deformation. This inability to reproduce the intermittency of the deformation may come from the fact that we consider each time step as an isolated loading experiment.

However, our implementation reproduces some interesting characteristics of the evolution of the ice pack deformation. Faults position are recorded thanks to the damage variable. Due to the changing external forcing, faults can be reactivated several times similarly to what is observed from in-situ and satellite data. The healing module slowly clears the memory of previous damages and makes the ice recover its initial resistance to deformation. We perform a series of simulations to compare both parameterisations and to evaluate the influence of the healing time parameter t_h , with t_h ranging from 4 years to 0.75 month. An additional simulation without healing is used for comparison.

First of all, let us compare the two parameterisations in a simple example by computing the damage update in one time step for a cell that has been damaged 50 times. With the first parameterisation (constant healing rate), the healing in one time step is equal to 1.7×10^{-4} with $t_h = 4$ years and corresponds to cancel 3 damage events. The second parameterisation (healing rate inversely proportional to damage) exhibits a strong healing rate even for large values of t_h . In this example, with $t_h = 4$ years, the damage update is equal to 1.3×10^{-2} and corresponds to cancel 23 damage events. Another difference between the two parameterisations is the influence of t_h , which is much weaker in the second one. The ratio of the healing rate between $t_h = 0.75$ month and $t_h = 4$ years is about 64 for the first parameterisation and about 8 for the second one.

An interesting diagnostic to show the persistence of linear kinematic features on periods of several weeks or months is the activity. Coon et al. (2007) and Kwok (2006) defined the activity as the percentage of time for which the deformation rate is higher than an arbitrary limit. In our study, a cell is considered active if the shear rate is greater than 0.01 day^{-1} or if the divergence rate is greater than 0.0066 day^{-1} . These limits are chosen to correspond to the minimum values for which the PDFs of the deformation rate start to exhibit a power-law tail. These limits are 3 times lower than in Coon et al. (2007) and Kwok (2006), but give a similar mean activity.

The activity over one month is drawn in Figure 4.11 for simulations without healing, with $t_h = 4$ years and with $t_h = 0.75$ month for the first parameterisation. Without healing, some patterns stay active for more than 15 days (activity higher than 50%). The activity decreases with $t_h = 4$ years and becomes very low for the simulation with $t_h = 0.75$ month, except in regions where the undamaged elastic modulus

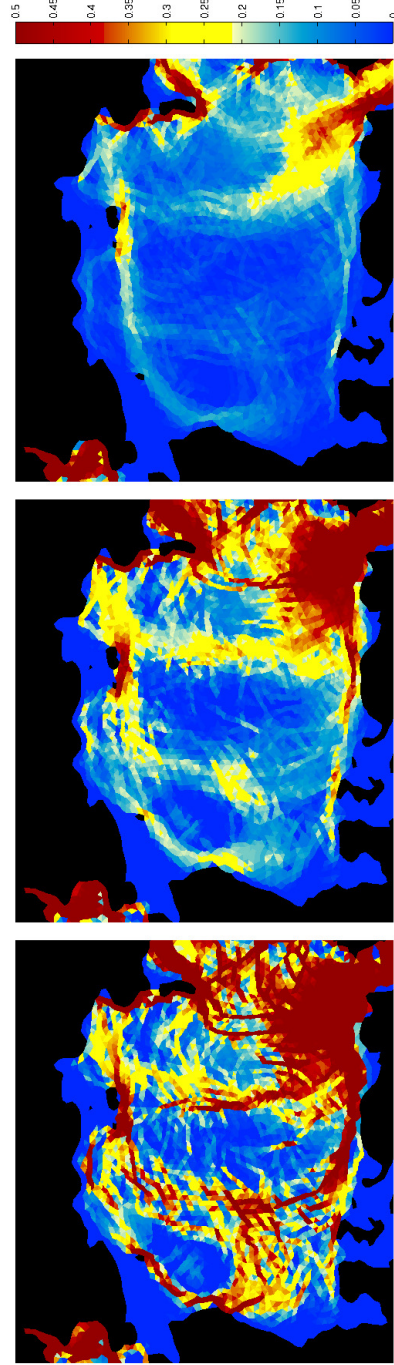


Figure 4.11: Activity defined as the percentage of time steps for which the shear rate in a cell is greater than 0.01 day^{-1} or for which the divergence rate is greater than 0.0066 day^{-1} . This percentage is computed for the whole 30-days simulation. The results on the left hand side are obtained without healing, the results in the middle are for $t_h = 4$ years and the results on the right hand side are obtained with $t_h = 0.75$ month and the first parameterisation, for the three cases.

is lower than 0.6×10^8 (Figure 4.3). The three monthly mean activity fields shown in Figure 4.11 may be compared with the activity fields for periods November-December, January-February and March-April presented in Figure 6 of Kwok (2006). For the first period (November-December), large parts of the domain experience an activity higher than 50% as in the case without healing. For the second period (January-February), linear features are still observed, with an activity around 25%. For the third period (March-April), the activity is about 10% in linear features as in our case with $t_h = 0.75$ month, and activity higher than 50% is only observed in first year ice regions. This comparison is relatively limited but indicates that the magnitude of the activity is comparable in simulations and observations and that the value of the healing parameter should vary as a function of the season. t_h could then be related to the air temperature or to the heat budget in leads.

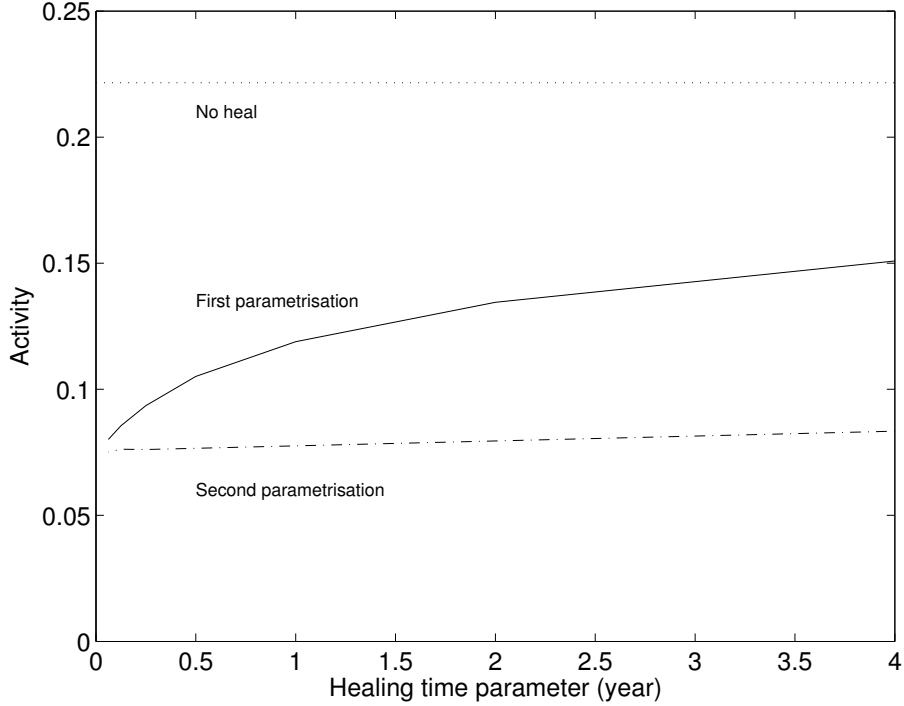


Figure 4.12: Mean activity in the central Arctic as a function of the healing time parameter [*years*]. The dotted line indicates the level of activity for the simulation without healing. The full line is for the constant healing rate parameterisation and the dashed-dotted line for the healing rate inversely proportional to $t_h d$

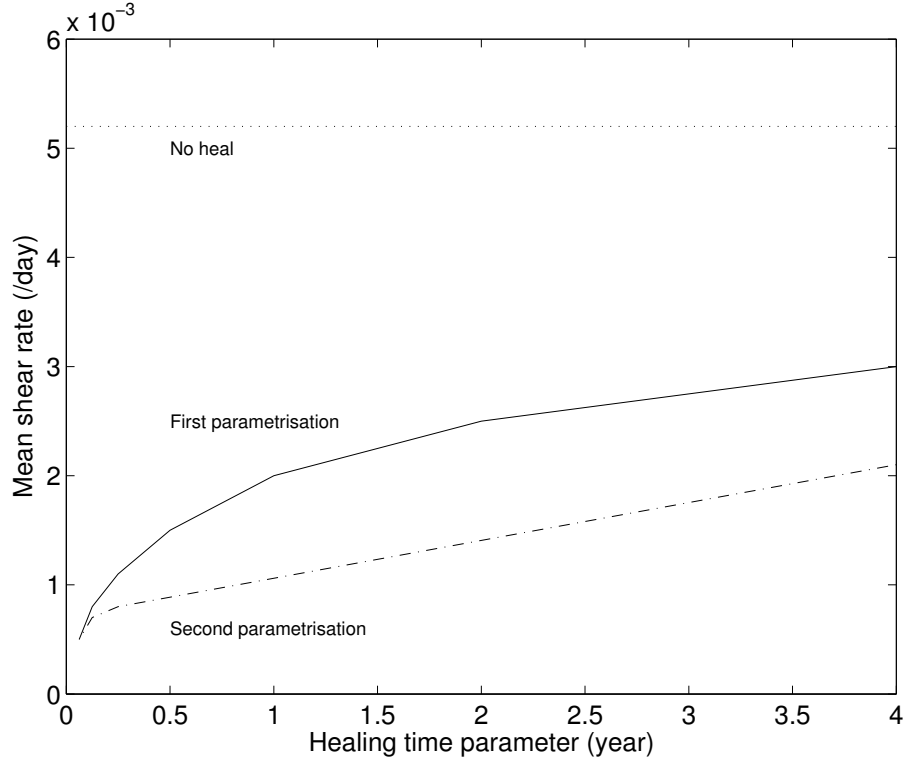


Figure 4.13: Mean shear rate [day^{-1}] in the central Arctic as a function of the healing time parameter [$years$]. The dotted line indicates shear rate for the simulation without healing. The full line is for the constant healing rate parameterisation and the dashed-dotted line for the healing rate inversely proportional to $t_h d$

To better evaluate the possible link between the strong seasonality of the deformation fields and the healing, we look at the average value of the mean activity and shear rate. These diagnostics are also used to compare the two healing parameterisations and to analyse the impact of the value of t_h . Figures 4.12 and 4.13 show the effect of the healing time parameter on the monthly mean activity and shear rate in the central Arctic region. For the first parameterisation, both the mean activity and shear rate increase with the healing time parameter, following a square root function (full curve in Figures 4.12 and 4.13). The activity is about 8% for $t_h = 0.75$ month and rises to 15% for $t_h = 4$ years. The dependence on t_h is also well pronounced for the mean shear rate that increases by a factor 6 from $t_h = 0.75$ month to $t_h = 4$ years. For the second parameterisation, the healing time parameter t_h

has almost no impact on the mean activity, that remains around 8% (dash-dotted curve in Figure 4.12). The high healing rate with the second parameterisation explains the low value of the activity that corresponds to almost no memory of previous damages. The parameter t_h has however an impact on the mean shear rate but to a lesser degree than with the first parameterisation (dash-dotted curve in Figure 4.13).

There is a strong difference between the situations with and without healing. With $t_h = 4$ years and the first parameterisation, the activity is reduced by 32% and the mean shear rate is decreased by 42% when compared to the values without healing (dotted curve in Figures 4.12 and 4.13). Such discrepancy is explained by the strong effect of the healing on highly damaged elements even for large values of t_h . For example, with $t_h = 4$ years and the constant healing rate parameterisation, if the damage variable is equal to $d = d_0^{50}$, it will increase by 60% in one time step. For the same forcing, the deformation rate will then rapidly decrease compared to the situation without healing.

The mean activity and shear rate computed from RGPS data by Kwok (2006) exhibit a strong seasonality. The observed activity is the highest for the late fall (from 10 to 30%) and is minimum in midwinter (< 10%). This has been explained by the increasing thickness in winter and by the seasonal structure of the atmospheric boundary layer, but the effect of healing on the activity can also explain part of this strong seasonality.

As an illustration of the effect of healing on the simulated sea ice dynamics, Figure 4.14 shows the evolution of the shear rate for day 5 to day 8 in the simulation with $t_h = 4$ years on the left and in the simulation with $t_h = 0.75$ month on the right. Both are with the first parameterisation of healing and the shear rate is computed over the last 6-hours time step of each day. The situation at day 5 is almost the same in both simulations (first line of Figure 4.14). There is almost no activity in the central Arctic region. At day 6, a large feature appears, spanning all the Arctic and being quite similar in the two simulations. In the simulation with $t_h = 4$ years, this feature remains active for several days, while it disappears after 2 days in the simulation with $t_h = 0.75$ month.

4.4 Conclusions

In our interpretation of the elasto-brittle model (EB), we consider each global time step as a distinct loading experiment starting from a stress-free initial state. In addition, we consider all the deformation as permanent at the end of each global time step. As there is no time-scale associated with the elastic stress redistribution nor with the damage processes, we arbitrary define a time-scale, $\Delta t = 6 h$, to have a

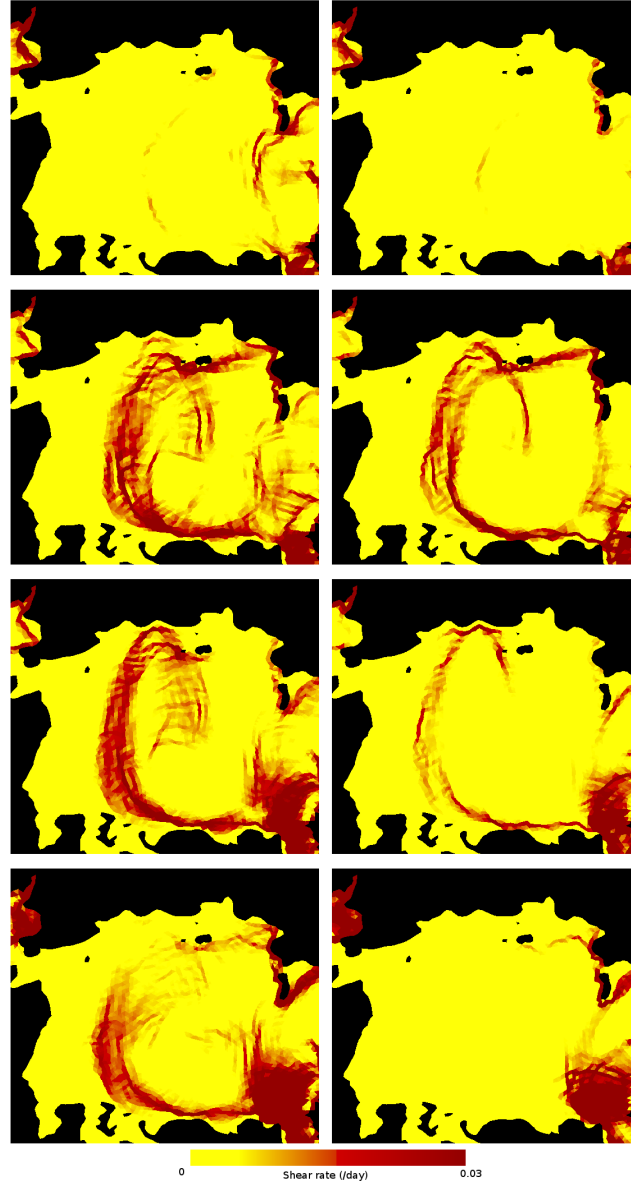


Figure 4.14: Evolution of the shear rate from day 5 to day 8 (from top to bottom) with the first healing parameterisation. The color scale goes from 0.0 (yellow) to 0.03 day^{-1} (red). The simulation on the left is with a healing time parameter $t_h = 4$ years and the simulation on the right with a healing time parameter $t_h = 0.75$ month. The shear rate is computed over the last time step of each day.

link between displacement and velocity. A drawback of this implementation of the EB model is the absence of intermittency in the deformation fields, coming from the fact that we isolate global time steps from each other.

Results obtained with the EB model have been analysed in terms of their statistical properties. Sea ice shear and divergence rate distributions are consistent with observations, with similar values for the slope of the power-law tails. The heterogeneity of the deformation is generated by EB even with a wind forcing at low spatial and temporal resolutions and no ocean currents. The coarsening procedure confirms the strong spatial localization and the multifractality of the simulated deformation fields.

Healing does not have an impact on the spatial scaling properties, but has a strong impact on the persistence of the deformation pattern. In consequence, it has a large impact on both the mean deformation rate and activity computed on a monthly time scale. A simple parameterisation with a constant healing rate allows to control the mean activity through the value of the healing time parameter. The effect of healing could partly explain the strong seasonality of the mean shear rate and activity observed from RGPS data. A second parameterisation with a healing rate inversely proportional to the level of damage produces a much weaker variability of the activity which is strongly reduced. No persistence of the deformation is observed with the second parameterisation. This sensitivity analysis is in favour of the constant healing rate parameterisation for its ability to reproduce some persistency of the ice deformation and to control the deformation activity.

CHAPTER 5

CONCLUSIONS

Current sea ice models are far from simulating the complex behaviour of the Arctic ice pack. When used at high resolution, they do not provide realistic deformation fields that are responsible for sea ice ridging and leads opening. The impacts of having a better representation of sea ice dynamics may then be crucial for many applications related to sea ice and in particular for the accuracy of climate change predictions, not only in the Arctic but also at a much larger scale.

This thesis contributes to a better representation of the ice pack mechanical behaviour. In the following, the main achievements are summarized and some perspectives for sea ice modelling are proposed.

Achievements

In this thesis, we adopted a two-fold strategy. On the one hand, we started from a widely used sea ice model and we improved the accuracy and performance of the numerical methods used to solve sea ice dynamics. On the other hand, we adopted a completely new approach with the implementation of the elasto-brittle rheology that produces more realistic results. We adapted it for a better integration in classical sea ice models and we confirmed its ability to reproduce some aspects of the complex behaviour of the Arctic ice pack.

The elastic-viscous-plastic model in LIM

In **Chapter 2**, we present the implementation of the elastic-viscous-plastic (EVP) sea ice rheology formulation of Hunke and Dukowicz (2002) in the Louvain-la-Neuve sea Ice Model, version 2 (LIM2), which has until now used a viscous-plastic (VP) sea ice dynamics (Hibler, 1979). Two centred difference discretizations of the EVP rheology have also been introduced, one defined on a B-grid and the second defined on a C-grid. These centred difference formulations help reducing computational costs for sea ice dynamics by about a factor of two compared to the more sophisticated approach of Hunke and Dukowicz (2002).

The conclusion of this chapter is that the different implementations of EVP lead to fairly similar results that compare well with observations and with previous LIM simulations. The C-grid version, however, offers an easier coupling with the ocean model NEMO and the possibility of representing ice transport across one-cell-wide straits and passages. This formulation on a C-grid is now the standard method used in the state-of-the-art NEMO-LIM model (Vancoppenolle et al., 2009).

This implementation of EVP has also been used to compare sea ice velocities and deformations produced by NEMO-LIM at high resolution to observations (Girard et al., 2009). We find that VP and EVP produce ice drift similar to observations in terms of large-scale velocity field and distributions of velocity fluctuations. However neither VP nor EVP produces deformation with power-law tailed distribution and spatial and temporal scaling that characterize the heterogeneity and intermittency of sea ice deformation.

In **Chapter 3**, we reinterpret the EVP model as an iterative solver. This allows to define a clear distinction between the numerical and physical parameters and to remove the constraint on the number of iterations. The interpretation of results and the choice of parameters are simplified and clear criteria are defined to ensure stability of the method and to avoid oscillations during the iterative solving process. The new parameters and diagnostics have also proven useful in comparing simulations from other configurations to distinguish the real cause of numerical problems.

By analysing the stress states during the iterative process, we find that the convergence of the compressive stress component is four times lower than the convergence of the shear stress components, with the classical EVP formulation proposed by Hunke and Dukowicz (1997). We propose a new regularization which corrects this discrepancy and allows faster convergence.

A major result of this chapter is the evidence of the strong link between the generation of linear bands of high ice deformation and numerical problems. Their ap-

pearance precludes the convergence of the iterative method and correlates with the persistence of significant errors in large parts of the domain even after many iterations. The new regularization completely filters out these features and allows the solution to converge everywhere. In previous studies, similar features were observed and compared to linear kinematic features as defined by Kwok (2006). Wang and Wang (2009); Maslowski and Lipscomb (2003) concluded that viscous-plastic models were successful in modelling sea ice dynamics at small scales. However, our analysis indicates that such features could arise from numerical artefacts and correlate with a strong lack of convergence towards the solution of the non linear VP equations.

This study, along with the analysis we made with VP in the finite element model SLIM (Girard et al., 2011), confirms the difficulty of the plastic-like rheologies to produce realistic deformation fields, and motivates the implementation of an alternative model.

The elasto-brittle model in SLIM

In chapter 4, we present an implementation of the elasto-brittle (EB) model in the finite element sea ice model SLIM. The EB model is inspired from progressive damage model. The complexity emerges from interactions between a large number of elements. The behaviour of each element is simple. It deforms as a linear elastic material until a certain limit, beyond which its elastic modulus is reduced.

In this study, we propose the following method to integrate EB in sea ice models. First of all, we consider each global time step of the sea ice model as an isolated loading experiment. The external forcing is increased by small steps, up to its nominal value. At the end of the loading experiment, all the deformation is considered as permanent and the ice velocities is computed as the displacement divided by the time step. The next time step starts in an unloading state with no memory of previous deformation.

This implementation allows to simulate ice deformation fields with power-law tail distribution and the right spatial scaling properties. For the first time, we also show the multifractality of the simulated deformation fields. However, this implementation of EB does not reproduce the intermittency of the ice deformation which is a major characteristic of sea ice dynamics.

In this study, we also decouple healing from damaging and we propose two parameterisations for healing. However, the link between the damage variable and physical quantities, like the number of leads or ridges, is not clear enough to propose a comprehensive parameterisation based on the refreezing in leads and ridges.

Instead, we discriminate the two parameterisations by looking at their effects on sea ice deformation activity. The sensitivity analysis we made is in favour of the constant healing rate parameterisation for its ability to control deformation activity. The effect of healing could partly explain the observed strong seasonality of the mean shear rate and deformation activity.

Perspectives

The recently identified characteristics of the ice cover open a broad range of perspectives for modellers. Many sub-grid scale parameterisations should be reconsidered in the light of sea ice scaling properties. For example, using power-law distributions of lead widths allows to estimate that more than 70% of the upward heat fluxes during winter occur through leads that only correspond to 1 or 2% of the ice cover (Marcq and Weiss, 2012). A similar approach could be applied to all air-ocean fluxes through leads (e.g., Steiner et al., 2013). To rethink the ice thickness redistribution scheme which is now based on large-scale deformation (Thorndike et al., 1975) could be done by considering deformation distributions at small spatial and temporal scale (Stern and Rothrock, 1995).

These characteristics are also very useful to validate models of sea ice dynamics. First of all, simulated deformation distributions should be systematically compared to those obtained from observations. If the power-law tail is missing, the mechanical thickness redistribution (ridging/leads opening) will be severely underestimated with non-negligible implications in terms of melting rate in summer and sea ice formation in winter. Secondly, diagnostics related to the scaling properties of the ice deformation should be generalized in high resolution models, in order to verify if the spatial and temporal clustering of deformation is reproduced. Finally, simulated sea ice velocity fields should also be validated in terms of their diffusion and dispersion properties, by using the approach presented in Rampal et al. (2009) and Rampal et al. (2008) to analyse trajectories of drifting buoys fixed on the sea ice cover.

Perspectives for plastic rheologies

As most of the sea ice models solve the non-linear VP equations with iterative algorithm, either from a simplistic linearization of the VP implicit equations (outer-loop, small time steps compared to the forcing scale, pseudo-time stepping, ...) or from more complex algorithms (EVP, JFNK), our first recommendation is the implementation of diagnostics to evaluate the errors in the approximate solution of the non-linear equations. This helps in better interpreting results and in making a clear distinction between the real properties of the model and numerical artefacts. Diagnostic as simple as the difference in velocity between successive approximate solutions during the iterative process already gives a good idea of the confidence in

results. Note that the mean error is not sufficient since the solution could be very bad in some limited areas of the domain.

The new regularization ensuring the same convergence for all stress components should be implemented in other sea ice models as it improves the stability and convergence of the EVP method. However EVP still exhibits slow rate of convergence towards the solution of the non-linear VP equations. New numerical methods could be implemented, e.g. the Jacobian-free Newton–Krylov method (Lemieux et al., 2012), but as the main assumptions of VP and its ability to produce realistic deformation fields are now strongly questioned, such developments may not be the top priority.

Perspectives for the elasto-brittle rheology

The development of alternative sea ice dynamics models is getting a lot of attention, for example, with the anisotropic rheology (Tsamados et al., 2013), or the elastic-decohesive rheology (Schreyer et al., 2006). However, only the EB rheology has proven its ability to reproduce some of the statistical properties of the ice pack deformation.

The present implementation of the EB rheology produces deformation fields having power-law tailed distributions and spatial scaling properties. Further statistical validation could be done against Lagrangian buoys trajectories to look at the spatial and temporal scaling properties.

The link between the damage variable and a measurable quantity, like the density of leads and ridges, should be developed by using lead/ridge statistics that can be derived from satellites images (Röhrs and Kaleschke, 2012).

The post-failure dynamics should also be better represented. In the present studies, once the damage has localized, the ice pack is divided into several plates moving independently. Friction and collision between these plates should be taken into account. The development of this post-failure dynamics should be done in parallel with the validation of the simulated ice drift.

In our study, damage recovery is treated with simple parameterisations which are not directly linked to the external conditions. However, the sensitivity study of the monthly mean activity highlights the strong impact of the healing time parameter t_h which could be linked to air temperature to reproduce the seasonality of the observed activity. A more complex parameterisation could allow healing only when the top ocean layer is at its freezing point and is losing heat. The relation between t_h and the open water heat flux could be tuned by comparing the simulated activity

to the observed one. The melting of sea ice already impacts the dynamics through the decrease of the ice thickness and concentration, and its effect on the level of damage could be neglected.

EB in a coupled sea ice–ocean models

In addition to the achievements presented in the main text, we also obtained preliminary results with the EB rheology in NEMO-LIM. This was one of the tasks of the European EMBRACE¹ project which aims at increasing the realism of existing European Earth system models. High northern latitudes are one of the key regions of this project because physical processes in this area strongly affect the transport of water, heat, salinity, nutrients and carbon in the ocean and influence the response of the Earth system to the increase of carbon dioxide and heat.

The EB rheology has been implemented in NEMO-LIM using the finite difference method. The momentum equation is solved by the successive relaxation method used in Chapter 2 for the implicit formulation of the VP model on a B-grid. The algorithm for the damage has the same structure as the implementation of EB in SLIM presented in Chapter 4. When activated, the EB module replaces the VP or EVP module. The rest of the model remains untouched. The effects of the new rheology on lead opening, ice thickness redistribution and transport naturally comes from the use of velocity and deformation rate computed by the EB module.

A simulation has been performed with the same grid (ORCA05), initial conditions and forcing as in Chapter 3 with the LIM2 version of the sea ice model. The sea ice divergence and shear rate fields for the first time step are shown in Figure 5.1. This figure can be compared with the results obtained with the EVP model under the same conditions (Figure 3.9). With the EB rheology, high deformation bands are not especially aligned with the grid but organize themselves into patterns similar to those observed in the Arctic. These results are promising for the integration of the EB rheology in classical sea ice–ocean models and will be used to study the impact of the localization of the deformation on the redistribution of the ice thickness. Comparisons with results obtained with the VP rheology will be used to assess the impact of using the new rheology.

¹<http://www.embrace-project.eu>

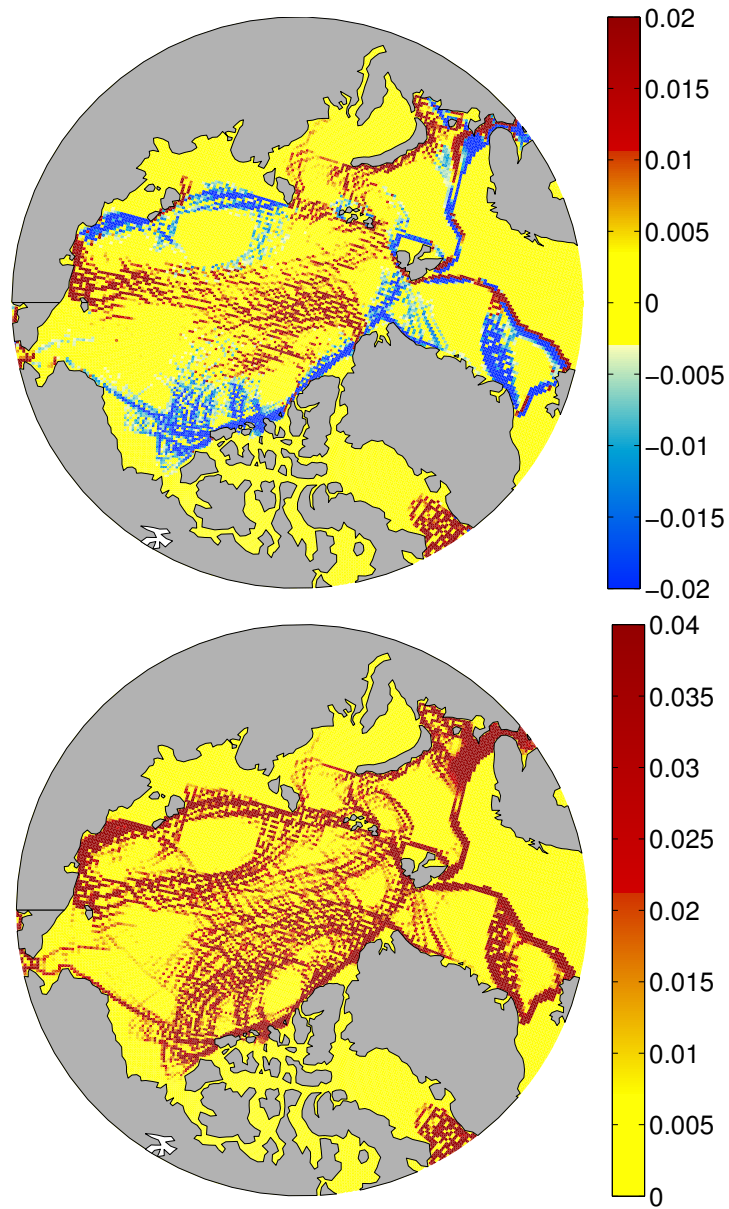


Figure 5.1: Divergence (top) and shear (bottom) rates after one time step of 4 hours with the elasto-brittle rheology in the NEMO-LIM model. Initial conditions and wind forcing are the same as in the simulations of Chapter 3. This result can be compared to those obtained with the EVP rheology and shown in Figure 3.9.

REFERENCES

- K. A. Aagaard and E. C. Carmack. The role of sea ice and other fresh water in the arctic circulation. *J. Geophys. Res.*, 94:14485–14498, 1989. 1
- M. J. Alava, P. K. Nukala, and S. Zapperi. Statistical models of fracture. *Advances in Physics*, 55(3-4):349–476, 2006. 69
- D. Amitrano, J. Grasso, and D. Hantz. From diffuse to localised damage through elastic interaction. *Geophysical research letters*, 26(14):2109–2112, 1999. 10, 69, 72
- B. Barnier, L. Brodeau, J. Le Sommer, J. Molines, T. Penduff, S. Theetten, A. Treguier, G. Madec, A. Biastoch, C. Böning, et al. Eddy-permitting ocean circulation hindcasts of past decades. *Clivar Exchanges*, pages 8–10, 2007. 77
- B. Bernard, G. Madec, T. Penduff, J. Molines, A. Treguier, J. Le Sommer, A. Beckmann, A. Biastoch, C. Böning, J. Dengg, et al. Impact of partial steps and momentum advection schemes in a global ocean circulation model at eddy-permitting resolution. *Ocean Dynamics*, 56(5):543–567, 2006. 77
- S. Blaise, R. Comblen, V. Legat, J.-F. Remacle, E. Deleersnijder, and J. Lambrechts. A discontinuous finite element baroclinic marine model on unstructured prismatic meshes. Part I: space discretization. *Ocean Dynamics*, 60:1371–1393, 2010. 12
- S. Bouillon, M. A. M. Maqueda, V. Legat, and T. Fichefet. An elastic-viscous-plastic sea ice model formulated on Arakawa B and C grids. *Ocean Modelling*, 27:174–184, 2009. 42, 45, 53, 59
- R. H. Bourke and R. P. Garrett. Sea ice thickness distribution in the arctic ocean. *Cold Regions Science and Technology*, 13:259–280, 1987. 26
- T. Boyer, S. Levitus, J. Antonov, M. Conkright, T. O’Brien, and C. Stephens. World ocean database 1998. vol. 1: Temperature of the atlantic ocean. Technical report, Technical report, NOAA Atlas NESDIS 27, US Government Printing Office, Washington, DC 166 pp, 1998. 54

- L. Brodeau, B. Barnier, A. Treguier, T. Penduff, and S. Gulev. An era40-based atmospheric forcing for global ocean circulation models. *Ocean Modelling*, 31(3): 88–104, 2010. 54
- A. Chmel and V. S. Smirnov. Correlated energy exchange in drifting sea ice. *International Journal of Oceanography*, 2011, 2011. 9
- A. Chmel, V. Smirnov, and M. Astakhov. The arctic sea-ice cover: Fractal space-time domain. *Physica A*, 357:556, 2005. 9
- R. Colony and A. Thorndike. Sea ice motion as a drunkard's walk. *Journal of Geophysical Research*, 90(C1):965–974, 1985. 8
- R. Comblen, S. Legrand, E. Deleersnijder, and V. Legat. A finite element method for solving the shallow water equations on the sphere. *Ocean Modelling*, 28:12–23, 2009. 77
- R. Comblen, J. Lambrechts, J.-F. Remacle, and V. Legat. Practical evaluation of five part-discontinuous finite element pairs for the non-conservative shallow water equations. *International Journal for Numerical Methods in Fluids*, 73:701–724, 2010. 12
- J. Comiso, C. L. Parkinson, R. Gersten, and L. Stock. Accelerated decline in the arctic sea ice cover. *Geophys. Res. Lett.*, 35:L01703, 2008. 1
- W. M. Connolley, J. M. Gregory, E. Hunke, and A. J. McLaren. On the consistent scaling of terms in the sea-ice dynamics equation. *J. Phys. Oceanogr.*, 34:1776–1780, 2004. 15, 40, 79
- M. Coon, R. Kwok, G. Levy, M. Pruis, H. Schreyer, and D. Sulsky. Arctic ice dynamics joint experiment (aidjex) assumptions revisited and found inadequate. *Journal of Geophysical Research*, 112:C11S90, 2007. 9, 35, 68, 88
- B. de Brye, A. de Brauwere, O. Gourgue, T. Kärnä, J. Lambrechts, R. Comblen, and E. Deleersnijder. A finite-element, multi-scale model of the scheldt tributaries, river, estuary and rofi. *Coastal Engineering*, 57(9):850–863, 2010. 12
- J. Dempsey. Research trends in ice mechanics. *International journal of solids and structures*, 37(1):131–153, 2000. 73
- E. Driesschaert, T. Fichefet, H. Goosse, P. Huybrechts, I. Janssens, A. Mouchet, G. Munhoven, V. Brovkin, and S. Weber. Modeling the influence of greenland ice sheet melting on the atlantic meridional overturning circulation. *Geophysical Research Letters*, 34:L10707, 2007. 11
- E. E. Ebert and J. A. Curry. An intermediate one-dimensional thermodynamic sea ice model for investigating ice–atmosphere interactions. *J. Geophys. Res.*, 98: 10085–10109, 1993. 1

- R. Ezraty and J. Piollé. Sea-ice drift in the central arctic combining quikscat and ssm/i sea ice drift data—user's manual (v1. 0, april 2004), 2004. 26
- T. Fichefet and M. A. M. Maqueda. Sensitivity of a global sea ice model to the treatment of ice thermodynamics and dynamics. *Journal of Geophysical Research*, 102:12609–12646, 1997. 11, 14, 15, 29, 54
- T. Fichefet and M. A. M. Maqueda. Modelling the influence of snow accumulation and snow-ice formation on the seasonal cycle of the antarctic sea-ice cover. *Climate Dynamics*, 15:251–268, 1999. 11, 15
- T. Fichefet, H. Goosse, and M. A. M. Maqueda. *Ocean Modeling and Parameterization*, volume C516 of *NATO ASI Series*, chapter On the large-scale modeling of sea ice–ocean interactions, pages 399–422. Kluwer Academic Publisher, Dordrecht, 1998. 39
- G. M. Flato and W. D. Hibler. Modeling pack ice as a cavitating fluid. *Journal of Physical Oceanography*, 22:626–651, 1992. 24
- L. Girard. *Vers un nouveau cadre de modélisation rhéologique de la banquise*. PhD thesis, Université Joseph-Fourier-Grenoble I, 2010. 71, 76
- L. Girard, J. Weiss, J. Molines, B. Barnier, and S. Bouillon. Evaluation of high-resolution sea ice models on the basis of statistical and scaling properties of arctic sea ice drift and deformation. *Journal of Geophysical Research*, 114(C8), 2009. 9, 35, 68, 80, 81, 96
- L. Girard, D. Amitrano, and J. Weiss. Failure as a critical phenomenon in a progressive damage model. *Journal of Statistical Mechanics: Theory and Experiment*, 2010(01):P01013, 2010. 10, 69, 75
- L. Girard, S. Bouillon, J. Weiss, D. Amitrano, T. Fichefet, and V. Legat. A new modelling framework for sea ice mechanics based on elasto-brittle rheology. *Annals of Glaciology*, 52(57):123–132, 2011. 10, 35, 65, 68, 72, 73, 77, 80, 81, 84, 97
- H. Goosse. *Modelling the large-scale behaviour of the coupled ocean-sea ice system*. PhD thesis, Fac. des Sci. Appl. Univ. Cath. de Louvain, Louvain-la-Neuve, Belgium, 1997. 15
- H. Goosse and T. Fichefet. Importance of ice-ocean interactions for the global ocean circulation: A model study. *Journal of Geophysical Research*, 104:23337–23355, 1999. 11
- J. M. N. T. Gray. Loss of hyperbolicity and ill-posedness of the viscous-plastic sea ice rheology in uniaxial divergent flow. *Journal of Physical Oceanography*, 29(11): 2920–2929, 1999. 24

- C. Haas, A. Pfaffling, S. Hendricks, L. Rabenstein, J.-L. Etienne, and I. Rigor. Reduced ice thickness in arctic transpolar drift favors rapid ice retreat. *Geophys. Res. Lett.*, 35:L17501, 2008. 1
- W. D. Hibler. A dynamic thermodynamic sea ice model. *Journal of Physical Oceanography*, 9:817–846, 1979. 10, 14, 15, 16, 17, 34, 39, 40, 41, 71, 96
- M. Hofmann and M. A. M. Maqueda. Performance of a second-order moment advection scheme in an ocean general circulation model. *Journal of Geophysical Research*, 111:C05006, 2006. 11
- M. M. Holland and C. M. Bitz. Polar amplification of climate change in coupled models. *Clim. Dynam.*, 21:221–232, 2003. 1
- M. A. Hopkins and A. S. Thorndike. Floe formation in arctic sea ice. *Journal of Geophysical Research*, 111:C11S23, 2006. 70
- E. Hunke and J. Dukowicz. The sea ice momentum equation in the free drift regime. Technical report, Technical Report LA-UR-03-2219, Los Alamos National Laboratory, 2003. 40, 79
- E. C. Hunke. Viscous-plastic sea ice dynamics with the evp model: Linearization issues. *J. Comput. Phys.*, 170:18–38, 2001. 38, 42, 47, 49, 50, 51, 52, 57, 59
- E. C. Hunke and J. K. Dukowicz. The elastic-viscous-plastic sea ice dynamics model in general orthogonal curvilinear coordinates on a sphere-incorporation of metric terms. *Monthly Weather Review*, 130:1848–1865, 2002. 14, 15, 17, 18, 29, 32, 34, 35, 38, 42, 59, 96
- E. C. Hunke and J. K. Dukowicz. An elastic-viscous-plastic model for sea ice dynamics. *Journal of Physical Oceanography*, 27:1849–1867, 1997. 10, 14, 15, 17, 18, 38, 39, 46, 96
- E. C. Hunke and W. H. Lipscomb. Cice: the los alamos sea ice model documentation and software user’s manual. Technical report, T-3 Fluid Dynamics Group, Los Alamos National Laboratory, Los Alamos NM 87545, 2008. 15, 18, 20, 38
- E. C. Hunke and Y. Zhang. A comparison of sea ice dynamics models at high resolution. *Monthly Weather Review*, 127:396–408, 1999. 18, 46
- J. Hutchings, P. Heil, and W. Hibler. Modeling linear kinematic features in sea ice. *Monthly weather review*, 133(12):3481–3497, 2005. 68
- J. Hutchings, A. Roberts, C. Geiger, and J. Richter-Menge. Spatial and temporal characterization of sea-ice deformation. *Annals of Glaciology*, 52(57):360–368, 2011. 6, 8, 9, 87

- J. Hutchings, P. Heil, A. Steer, and W. Hibler III. Subsynoptic scale spatial variability of sea ice deformation in the western weddell sea during early summer. *Journal of Geophysical Research*, 117(C1):C01002, 2012. 6
- R. Jaiser, K. Dethloff, D. Handorf, A. Rinke, and J. Cohen. Impact of sea ice cover changes on the northern hemisphere atmospheric winter circulation. *Tellus A*, 64(0), 2012. 2
- M. Kachanov. Effective elastic properties of cracked solids. *Appl Mech Rev*, 45(8), 1992. 72
- E. Kalnay, M. Kanamitsu, R. Kistler, W. Collins, D. Deaven, L. Gandin, M. Iredell, S. Saha, G. White, J. Woollen, Y. Zhu, A. Leetmaa, B. Reynolds, M. Chelliah, W. Ebisuzaki, W. Higgins, J. Janowiak, K.C. Mo, C. Ropelewski, J. Wang, R. Jenne, and D. Joseph. The ncep/ncar 40-year reanalysis project. *Bull. Amer. Meteor. Soc.*, 77:437–470, 1996. 78
- L. Kantha and C. Clayson. *Numerical models of oceans and oceanic processes*, volume 66. Academic Press, 2000. 21
- R. Kwok. Deformation of the arctic ocean sea ice cover between november 1996 and april 1997: a qualitative survey. *Solid Mechanics And Its Applications*, 94:315–322, 2001. 3
- R. Kwok. Contrasts in sea ice deformation and production in the arctic seasonal and perennial ice zones. *Journal of geophysical research*, 111(C11):C11S22, 2006. 3, 80, 88, 90, 92, 97
- R. Kwok and D. A. Rothrock. Decline in arctic sea ice thickness from submarine and icesat records: 1958 – 2008. *Geophys. Res. Lett.*, 36:L15501, 2009. 1
- R. Kwok, E. C. Hunke, W. Maslowski, D. Menemenlis, and J. Zhang. Variability of sea ice simulations assessed with rgps kinematics. *J. Geophys. Res*, 113:C11012, 2008. 35, 68
- J. Lambrechts, R. Comblen, V. Legat, C. Geuzaine, and J. Remacle. Multiscale mesh generation on the sphere. *Ocean dynamics*, 58(5):461–473, 2008. 76
- W. Large and S. Yeager. Diurnal to decadal global forcing for ocean and sea-ice models: The data sets and flux climatologies. *NCAR Tech. Note TN-460+ STR*, 2004. 54
- S. W. Laxon, K. A. Giles, A. L. Ridout, D. J. Wingham, R. Willatt, R. Cullen, R. Kwok, A. Schweiger, J. Zhang, C. Haas, et al. Cryosat-2 estimates of arctic sea ice thickness and volume. *Geophysical Research Letters*, 2013. 1
- J.-F. Lemieux and B. Tremblay. Numerical convergence of viscous-plastic sea ice models. *J. Geophys. Res*, 114:C05009, 2009. 38, 46

- J.-F. Lemieux, B. Tremblay, J. Sedlacek, P. Tupper, S. Thomas, H. D., and J.-P. Auclair. Improving the numerical convergence of viscous-plastic sea ice models with the jacobian-free newton-krylov method. *J. Comp. Phys.*, 229:2840–2852, 2010. 38, 46
- J.-F. Lemieux, D. Knoll, B. Tremblay, D. Holland, and M. Losch. A comparison of the jacobian-free newton-krylov method and the evp model for solving the sea ice momentum equation with a viscous-plastic formulation: A serial algorithm study. *Journal of Computational Physics*, 2012. 38, 42, 48, 50, 51, 52, 59, 63, 99
- M. Leppäranta. *The Drift of Sea Ice*. Springer, Helsinki, 2005. 15, 74
- S. Levitus. Climatological atlas of the world ocean. *Eos, Transactions American Geophysical Union*, 64(49):962–963, 1983. 26
- O. Lietaer. *Finite element methods for sea ice modeling*. PhD thesis, Université catholique de Louvain, 2011. 11, 12, 85
- O. Lietaer, T. Fichefet, and V. Legat. The effects of resolving the Canadian Arctic Archipelago in a finite element sea ice model. *Ocean Modelling*, 24:140–152, 2008. 12
- O. Lietaer, E. Deleersnijder, T. Fichefet, M. Vancoppenolle, R. Comblen, S. Bouillon, and V. Legat. The vertical age profile in sea ice: Theory and numerical results. *Ocean Modelling*, 2011. 12
- R. Lindsay, J. Zhang, A. Schweiger, M. Steele, and H. Stern. Arctic sea ice retreat in 2007 follows thinning trend. *Journal of Climate*, 22(1):165–176, 2009. 1
- W. Lipscomb, E. C. Hunke, W. Maslowski, and J. Jakacki. Ridging, strength, and stability in high-resolution sea ice models. *J. Geophys. Res.*, 112:C03S91, 2007. 38
- J. Liu, J. A. Curry, H. Wang, M. Song, and R. M. Horton. Impact of declining arctic sea ice on winter snowfall. *Proceedings of the National Academy of Sciences*, 109(11):4074–4079, 2012. 2
- M. Losch and S. Danilov. On solving the momentum equations of dynamic sea ice models with implicit solvers and the elastic-viscous-plastic technique. *Ocean Modelling*, 2011. 38
- M. Losch, D. Menemenlis, J.-M. Campin, P. Heimbach, and C. Hill. On the formulation of sea-ice models. part 1: Effects of different solver implementations and parameterizations. *Ocean Modelling*, 33, 1-2:129–144, 2010. ISSN 1463-5003. 38, 42
- G. Madec. *NEMO Ocean engine*. Pôle de modélisation de l’Institut Pierre-Simon Laplace, May 2008. 53

- G. Madec, P. Delecluse, M. Imbard, and C. Lévy. OPA 8.1 Ocean General Circulation Model reference manual. Note du Pôle de modélisation, p.91. Technical Report XX, Institut Pierre Simon Laplace (IPSL), 1998. 11, 54
- S. Manabe and R. J. Stouffer. Sensitivity of a global climate model to an increase of CO_2 concentration in the atmosphere. *J. Geophys. Res.*, 85:5529–5554, 1980. 1
- S. Marcq and J. Weiss. Influence of sea ice lead-width distribution on turbulent heat transfer between the ocean and the atmosphere. *The Cryosphere, Volume 6, Issue 1, 2012, pp. 143-156*, 6:143–156, 2012. 2, 98
- D. Marsan and J. Weiss. Space/time coupling in brittle deformation at geophysical scales. *Earth and Planetary Science Letters*, 296(3-4):353 – 359, 2010. 3, 9
- D. Marsan, H. Stern, R. Lindsay, and J. Weiss. Scale dependence and localization of the deformation of arctic sea ice. *Physical review letters*, 93(17):178501, 2004. 3, 4, 5, 6, 7, 8, 81, 84
- D. Marsan, J. Weiss, J.-P. Metaxian, J. Grangeon, P.-F. Roux, and J. Haapala. Low-frequency bursts of horizontally polarized waves in the arctic sea-ice cover. *Journal of Glaciology*, 57(202):231–237, 2011. 73
- O. Marti, P. Braconnot, J. Dufresne, J. Bellier, R. Benshila, S. Bony, P. Brockmann, P. Cadule, A. Caubel, F. Codron, et al. Key features of the ipsl ocean atmosphere model and its sensitivity to atmospheric resolution. *Climate dynamics*, 34(1):1–26, 2010. 11
- T. Martin and R. Gerdes. Sea ice drift variability in arctic ocean model intercomparison project models and observations. *Journal of Geophysical Research*, 112: C04S10, 2007. 27
- W. Maslowski and W. Lipscomb. High resolution simulations of arctic sea ice, 1979–1993. *Polar Research*, 22(1):67–74, 2003. 9, 36, 64, 97
- F. Massonnet, T. Fichefet, H. Goosse, C. M. Bitz, G. Philippon-Berthier, M. M. Holland, and P.-Y. Barriat. Constraining projections of summer arctic sea ice. *The Cryosphere*, 6(6):1383–1394, 2012. 2
- G. Maykut. *The surface heat and mass balance*. University of Washington, 1986. 76
- M. G. McPhee. Turbulent heat fluxes in the upper ocean under sea ice. *Journal of Geophysical Research*, 97:5365–5379, 1992. 15
- M. Montoya, A. Griesel, A. Levermann, J. Mignot, M. Hofmann, A. Ganopolski, and S. Rahmstorf. The earth system model of intermediate complexity climber-3 α . part i: description and performance for present-day conditions. *Climate Dynamics*, 26:237–263, 2005. 11, 14

- P. Nukala, S. Zapperi, and S. Simunovic. Statistical properties of fracture in a random spring model. *Physical Review E*, 71(6):066106, 2005. 69
- J. Nye. Meaning of two-dimensional strain-rate in a floating ice cover. *Aidjex Bull.:(United States)*, 6(21), 1973. 3
- D. K. Perovich, W. B. Tucker III, and K. A. Ligett. Aerial observations of the evolution of ice surface conditions during summer. *Journal of Geophysical Research*, 107, 2002. 1
- V. Petrenko and O. Gluschenkov. Crack velocities in freshwater and saline ice. *Journal of geophysical research*, 101(B5):11541–11, 1996. 73
- C. Petrich, P. Langhorne, and T. Haskell. Formation and structure of refrozen cracks in land-fast first-year sea ice. *Journal of geophysical research*, 112(C4):C04006, 2007. 75
- D. F. Porter, J. J. Cassano, and M. C. Serreze. Local and large-scale atmospheric responses to reduced arctic sea ice and ocean warming in the wrf model. *Journal of Geophysical Research*, 117(D11):D11115, 2012. 1, 2
- P. Rampal, J. Weiss, D. Marsan, R. Lindsay, and H. Stern. Scaling properties of sea ice deformation from buoy dispersion analysis. *Journal of Geophysical Research*, 113(C3):C03002, 2008. 3, 6, 8, 9, 87, 98
- P. Rampal, J. Weiss, D. Marsan, and M. Bourgoïn. Arctic sea ice velocity field: General circulation and turbulent-like fluctuations. *Journal of Geophysical Research*, 114(C10):C10014, 2009. 8, 98
- P. Rampal, J. Weiss, C. Dubois, and J. Campin. Ipcc climate models do not capture arctic sea ice drift acceleration: Consequences in terms of projected sea ice thinning and decline. *J. Geophys. Res*, 116:C00D07, 2011. 2
- D. A. Randall. Geostrophic adjustment and the finite-difference shallow-water equations. *Monthly Weather Review*, 122:1371–1377, 1994. 23, 24
- D. A. Randall, R. Wood, S. Bony, R. Colman, T. Fichefet, J. Fyfe, V. Kattsov, A. Pitman, J. Shukla, J. Srinivasan, R. Stouffer, A. Sumi, and K. Taylor. *Climate Models and Their Evaluation. In: Climate Change 2007: The Physical Science Basis. Contribution of Working Group I to the Fourth Assessment Report of the Intergovernmental Panel on Climate Change [Solomon, S. and D. Qin and M. Manning and Z. Chen and M. Marquis and K.B. Averyt and M. Tignor and H.L. Miller (eds.)]*. Cambridge University Press, Cambridge, United Kingdom and New York, NY, USA, 2007. 14
- J. Röhrs and L. Kaleschke. An algorithm to detect sea ice leads by using amsr-e passive microwave imagery. *The Cryosphere*, 6:343–352, 2012. 99

- D. Rothrock, Y. Yu, and G. Maykut. Thinning of the arctic sea-ice cover. *Geophys. Res. Lett.*, 26(23):3469–3472, 1999. 1
- C. Schmitt, C. Kottmeier, S. Wassermann, and M. Drinkwater. Atlas of antarctic sea ice drift, 2004. 28
- H. L. Schreyer, D. L. Sulsky, L. B. Munday, M. D. Coon, and R. Kwok. Elastic-decohesive constitutive model for sea ice. *Journal of Geophysical Research*, 111: C11S26, 2006. 68, 99
- E. M. Schulson, A. L. Fortt, D. Iliescu, and C. E. Renshaw. Failure envelope of first-year arctic sea ice: The role of friction in compressive fracture. *Journal of Geophysical Research*, 111:C11S25, 2006. 72
- M. C. Serreze, M. M. Holland, and J. Stroeve. Perspectives on the arctic’s shrinking sea-ice cover. *Science*, 315:1533–1536, 2007. 1
- N. Steiner, W. Lee, and J. Christian. Enhanced gas fluxes in small sea ice leads and cracks-effects on co2 exchange and ocean acidification. *Journal of Geophysical Research: Oceans*, 2013. 2, 98
- H. Stern and R. Lindsay. Spatial scaling of arctic sea ice deformation. *Journal of Geophysical Research*, 114(C10):C10017, 2009. 4, 6, 82, 84, 87
- H. L. Stern and D. A. Rothrock. Open water production in arctic sea ice: Satellite measurements and model parameterizations. *Journal of Geophysical Research*, 100:20601–20612, 1995. 98
- J. Stroeve, V. Kattsov, A. Barrett, M. Serreze, T. Pavlova, M. Holland, and W. Meier. Trends in arctic sea ice extent from cmip5, cmip3 and observations. *Geophysical Research Letters*, 39(16):L16502, 2012a. 2
- J. C. Stroeve, M. C. Serreze, M. M. Holland, J. E. Kay, J. Malanik, and A. P. Barrett. The arctic’s rapidly shrinking sea ice cover: a research synthesis. *Climatic change*, 110(3):1005–1027, 2012b. 1
- A. Thorndike. Diffusion of sea ice. *Journal of Geophysical Research*, 91(C6):7691–7696, 1986. 8
- A. S. Thorndike and R. Colony. Estimating the deformation of sea ice. *AIDJEX Bulletin*, 37:25–36, 1977. 2
- A. S. Thorndike, D. A. Rothrock, G. A. Maykut, and R. Colony. The thickness distribution of sea ice. *Journal of Geophysical Research*, 80:4501–4513, 1975. 98
- R. Timmermann, H. Goosse, G. Madec, T. Fichefet, C. Etche, , and V. Dulière. On the representation of high latitude processes in the orca-lim global coupled sea ice-ocean model. *Ocean Modelling*, 8:175–201, 2005. 11, 15, 25, 26

- M. Tsamados, D. Feltham, and A. Wilchinsky. Impact of a new anisotropic rheology on simulations of arctic sea ice. *Journal of Geophysical Research: Oceans*, 2013. 99
- M. Vancoppenolle, T. Fichefet, H. Goosse, S. Bouillon, G. Madec, and M. A. M. Maqueda. Simulating the mass balance and salinity of arctic and antarctic sea ice. 1. model description and validation. *Ocean Modelling*, 27(1):33–53, 2009. 11, 96
- P. Wadhams and N. Davis. Further evidence of ice thinning in the arctic ocean. *Geophys. Res. Lett.*, 27(24):3973–3975, 2000. 1
- K. Wang and C. Wang. Modeling linear kinematic features in pack ice. *Journal of Geophysical Research*, 114(C12):C12011, 2009. 35, 61, 62, 97
- M. Wang and J. Overland. A sea ice free summer arctic within 30 years: An update from cmip5 models. *Geophysical Research Letters*, 39(18):L18501, 2012. 1
- J. Weiss. Scaling of fracture and faulting of ice on earth. *Surveys in Geophysics*, 24: 185–227, 2003. 9
- J. Weiss. Intermittency of principal stress directions within arctic sea ice. *Physical Review E*, 77(5):056106, 2008. 9
- J. Weiss. *Sea ice drift, deformation and fracturing - A perspective across scales*, volume in press. Springer, 2013. 8
- J. Weiss and D. Marsan. Scale properties of sea ice deformation and fracturing. *Comptes Rendus Physique*, 5(7):735–751, 2004. 8, 9
- J. Weiss and E. Schulson. Coulombic faulting from the grain scale to the geophysical scale: lessons from ice. *Journal of Physics D: Applied Physics*, 42(21):214017, 2009. 72, 80
- J. Weiss, E. M. Schulson, and H. L. Stern. Sea ice rheology from in-situ, satellite and laboratory observations: fracture and friction. *Earth and Planetary Science Letters*, 255:1–8, 2007. 9, 72
- J. Weiss, D. Marsan, and P. Rampal. Space and time scaling laws induced by the multiscale fracturing of the arctic sea ice cover. In *IUTAM Symposium on Scaling in Solid Mechanics*, pages 101–109. Springer, 2009. 9
- A. Wilchinsky and D. Feltham. Rheology of discrete failure regimes of anisotropic sea ice. *Journal of Physical Oceanography*, 42(7):1065–1082, 2012. 68
- J. Zhang and W. D. Hibler. On an efficient numerical method for modelling sea ice dynamics. *Journal of Geophysical Research*, 102:8691–8702, 1997. 10, 14, 31, 38, 46

-
- J. Zhang and D. Rothrock. Modeling arctic sea ice with an efficient plastic solution. *Journal of geophysical research*, 105(C2):3325–3338, 2000. 14, 31, 38, 39
- J. Zhang and D. A. Rothrock. Effect of sea ice rheology in numerical envestigations of climate. *Journal of Geophysical Research*, 110:C08014, 2005. 42