Chapter 1

Introduction

Development of innovative materials has always been one of the most important engineering tasks over the years. In this field, composite materials (i.e. an heterogeneous material made of two or more different constituents) always had an important position and have a much longer history than what we can firstly think of. Wood as such can be considered as a composite material made of very long fibers of cellulose held together by a much weaker substance. The very first composite produced by man was cob: a plastic matrix made of earth is reinforced by plant fibers. More than 4000 years ago, Mongols already produced a sophisticated composite material: a bow made of a wooden frame assembled with horn and tendons. In more recent times, interest for composite materials was permanently growing up. The main interest is to obtain thermo-mechanical properties which are impossible to get with a single homogeneous material. A good example is concrete. The mix of gravel and cement gives a good compressive strength but is much less satisfying in tension. This can be improved by adding metal rods in order to obtain reinforced concrete.

There are two main ways for assembling the different constituents. Several layers made of different materials can be superposed in order to produce a laminated composite. Another possibility is to consider a matrix in which particles are added in order to get a particle-reinforced composite. This work will be limited to the latter type. However, this include a wide range of materials, from fiber-reinforced composites to nanocomposites, from random to perfectly arranged microstructures. An homogeneous material containing holes can also be considered as this type of composite. Even if material properties of all the constituents are well established, predicting the effective response by taking into account geometrical, mechanical and thermal properties is certainly not an easy task. Logically, this strongly depends on the constitutive behavior of each constituent so that different subdivisions can be considered: linear (thermo-)elastic composites, linear viscoelastic composites, elasto-plastic composites, elasto-viscoplastic composites,... In this work, we will concentrate our efforts to predict the behavior of materials made of at least one plastic phase. Main applications of such materials include:

- Polymer matrix reinforced with ceramic, glass or Kevlar fibers in order to improve stiffness and strength. Some applications include boat hulls, aircraft wings, sport equipments, car parts,...
- Polymer matrix with low modulus rubber particles in order to improve toughness and impact resistance (e.g.: car bumper).
- Metal matrix with ceramic particles or short fibers mainly for high temperature applications.
- Concrete matrix reinforced by elasto-plastic metallic fibers (for better strength in tension or bending) or polymeric fibers (for better ductility, lower density).

Furthermore, rate-dependent plastic effects occur in several types of matrix (e.g.: metallic (aluminum alloy, titanium,...), polymer), especially at high temperatures where such effects cannot be neglected. Typical applications include aeronautics (e.g.: aeropropulsion systems). Such composites can be reinforced by elastic inclusions or even elasto-viscoplastic ones.

The range of materials concerned by this study is thus really wide and we are concerned with the prediction of their effective behavior. In order to shorten a long and costly experimental campaign, numerical simulations are of great help. Numerical predictions of such materials represent an extraordinary tool in order to speed up the development of new materials. The main goal of this thesis is to develop efficient numerical methods based on robust algorithms for the prediction of the behavior of elasto-(visco)plastic composites by taking into account geometrical and mechanical aspects. More especially, two approaches will be used throughout this work: finite element simulations and mean-field homogenization schemes. No damage will be considered and all the developments are made in the context of small perturbations.

The structure of the thesis does not rigorously follow the historical developments of the research. This is done in order to get a more logical and readable structure of the text. The first section deals with the scale transition problem from a macroscopic structure to the definition of a representative volume element (RVE) of the heterogeneous microstructure. A boundary value problem is defined over this RVE and several solution methods are suggested. One of these methods, the finite element (FE) one, is presented in chapter 3. Since this technique gives the most accurate predictions, it will be used later as validation tool of other solution methods. This chapter has been the subject of the following publication:

- Pierard, O., González, C., Segurado, J., LLorca, J. and Doghri, I. Micromechanics of elasto-plastic materials reinforced with ellipsoidal inclusions. *Mechanics of Materials*, Submitted for publication, 2006.

Subsequent chapters are devoted to the development of mean-field homogenization schemes for a wide range of material constitutive behaviors. This technique relies on simplifying hypotheses so that an approximate solution is found at a much lower computational cost than a corresponding FE simulation. These are introduced in chapter 4 for linear behaviors (isothermal elasticity, thermo-elasticity, viscoelasticity) where analytical solutions are available in most cases. This chapter has been the subject of the following publication:

- Pierard, O., Friebel, C. and Doghri, I. Mean-field homogenization of multi-phase thermo-elastic composites: a general framework and its validation. *Composites Science and Technology*, 64 (2004), 1587-1603.

Mean-field homogenization schemes are later extended to elasto-plastic behaviors where a linearization of the local constitutive laws is required in order to apply homogenization schemes valid in linear elasticity. More precisely, an incremental formulation is introduced in chapter 5 which links at each state of deformation the strain rate to the stress rate. This chapter has been the subject of the following publications:

- Pierard, O. and Doghri, I. A study of various estimates of the macroscopic tangent operator in the incremental homogenization of elasto-plastic composites. *International Journal for Multiscale Computational Engineering*, Accepted for publication, 2006.

- Pierard, O., González, C., Segurado, J., LLorca, J. and Doghri, I. Micromechanics of elasto-plastic materials reinforced with ellipsoidal inclusions. *Mechanics of Materials*, Submitted for publication, 2006.

Finally, one of the main goals of this work is to predict the behavior of heterogeneous elasto-viscoplastic materials, which is introduced in chapter 6. For this, an affine type formulation of the constitutive laws is used to reduce the problem to a linear thermo-viscoelastic one. This chapter has been the subject of the following publications:

- Pierard, O. and Doghri, I. An enhanced affine formulation and the corresponding numerical algorithms for the mean-field homogenization of elasto-viscoplastic composites. *International Journal of Plasticity*, 22 (2006), 131-157.

- Pierard, O., Segurado, J., LLorca, J. and Doghri, I. Micromechanics of particle-reinforced elasto-viscoplastic composites: finite element simulations versus affine homogenization. *International Journal of Plasticity*, Submitted for publication, 2006.

In each chapter, an important part is devoted to the validation of the developed methods.