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ABSTRACT: This paper presents a probabilistic model for the service life assessment of bridge deteriorating truss. Damage evolution of each structural member is dealt as a transition process through different states of performance. The transition process is modeled as a Markovian renewal process which formulation allows to consider eventual improvements of the structural performance. In this way, the model is used to set up selective maintenance and/or rehabilitation interventions. Then the preventive maintenance scenarios selected are economically compared.

1 INTRODUCTION

Over time structural systems in aggressive environments are subjected to a deterioration of their mechanical properties. Since the system is weak from the degradation process, it is less able to withstand the applied actions; therefore the system tends to decrease its structural reliability. When the reliability level reaches a lower value than the acceptable threshold, the system is no longer able to satisfy the requirements for which it was conceived and, unless some maintenance or rehabilitation interventions are performed, the end of system's service life occurs.

In order to assess the service life of a structural system interacting with an aggressive environment, and to eventually set up a strategy of essential and/or preventive maintenance, some basic knowledge about actual development of the deterioration process, as well as uncertainty involved in its modeling, is necessary. Usually a valuable set of useful data can be gathered from monitoring activities; but such activities tend to be greatly limited in space and time and only a few observed data are usually available due to economic reasons.

Since the structural analysis of a deteriorated system is affected by uncertainty, it cannot follow the classic deterministic approach, but rather a probabilistic one. A probabilistic approach requires a huge amount of "historic" information on deterioration evolution as well as on performance development of the structure over time. Nevertheless more often than not a lack of monitoring data compromises the probabilistic approach efficiency.

Thence technics of simulation can be carried out in order to bypass this gap. Recently Monte Carlo

Simulation technic is used to simulate the behaviour of deteriorated structures. It requires a basic knowledge about the evolution of some parameters defining structural behaviour.

Using Monte Carlo Simulation, the paper proposes a probabilistic approach to analyze and model deterioration as a transition process through different states of performance. The deterioration process leads structure's member up to a loss of performance. When the deterioration reaches a dangerous level, the member moves into a lower performance state. When the performance decrease of damaged members sets up a mechanism or unacceptable displacements, the failure of the whole system occurs. Hence the transition process is modeled as a Markovian Renewal Process (MRP).

Usually MRP is used to define the failure time of a system or to define the transition probabilities state by state for the whole system considered. Moreover this paper deals with another employment of MRP that is the study of deterioration development of each member of the system and the evaluation of times of transition between states. MRP is also used as a tool to plan selective maintenance member by member and for different instant.

The proposed procedure is applied to predict the service life of a bridge steel truss and to investigate suitable selective maintenance scenarios. Referring to economic point of view the scenarios outlined will be finally compared each other.

2 DETERIORATION ANALYSIS OF A BRIDGE STEEL TRUSS

This paper points out the deterioration process of a bridge steel truss. Since the whole bridge's structure as well as its fundamental elements are usually stuck into aggressive environment, they suffer a decrease of both cross-sectional area A and material strength $\bar{\sigma}$. At first we assume that the same damage process $\delta=\delta(t)$ weighs the structure properties down and that the damage rates are related with the aggressiveness of the environment.

The approach is applied to a statically indeterminate truss system with members' area $A=2500\text{mm}^2$ and total volume $V=0.0723\text{m}^3$. The allowable material strength is $\bar{\sigma}=140\text{MPa}$. Moreover we assume that buckling failures are to be avoided. The structure is subjected to a set of forces $F=25\text{kN}$ as shown in Figure 1. The initial values of member's cross-sectional area A and material strength $\bar{\sigma}$, as well as the force F , are assumed as deterministic.

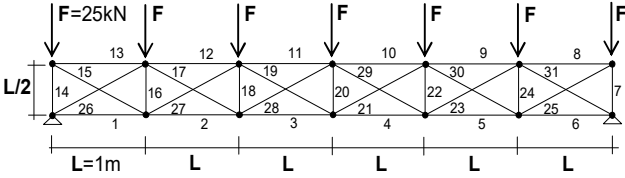


Fig. 1 Statically indeterminate truss system with members area $A=2500\text{mm}^2$ and total volume $V=0.0723\text{m}^3$.

Since deterioration process of building materials is affected by uncertainty, a probabilistic approach seems to be appropriated for the system analysis. However, the simulation and modeling of the damage development needs for a huge monitoring-based dataset. Usually monitoring actions are limited in space and time, compromising the right modeling of the phenomenon investigated. To overcome the problem this paper proposes a probabilistic approach associated with a Monte Carlo simulation. This combined method allows us to simulate the deterioration evolution for several structures with same typology. In Monte Carlo simulation an appropriate deterioration model must be implemented. Hereby the deterioration model proposed by Biondini *et al.* (2008) has been implemented into our computer code for structural analysis and Monte Carlo simulation.

2.1 Deterioration law

Following Biondini *et al.* (2008), by denoting Γ as a generic material property, the material deterioration over time t can be evaluated as following:

$$\Theta(t) = \Theta_0[1 - \delta(t)] \quad (1)$$

where “0” denotes the initial undamaged state, and the deterioration over time t is measured by a time-

variant damage index $\delta=\delta(t)\in[0;1]$. The following damage model is assumed (Biondini *et al.* 2008):

$$\delta(\tau) = \begin{cases} \omega^{1-\rho} \tau^\rho & , \tau \leq \omega \\ 1 - (1 - \omega)^{1-\rho} (1 - \tau)^\rho & , \omega < \tau < 1 \\ 1 & , \tau \geq 1 \end{cases} \quad (2)$$

where $\tau = t/T_C$, T_C is the normalized time instant when the failure threshold $\delta=1$ is reached and ω and ρ are shape parameters of the damage function.

The damage parameters ρ and ω must be chosen according to the actual damage process development. Damage rates are assumed associated with the aggressiveness of the environment, as well as with the level of acting stress.

The following linear relationship is assumed (Biondini *et al.*, 2008):

$$\rho = \rho_a + (\rho_b - \rho_a) \xi \quad (3)$$

$$\omega = \omega_a + (\omega_b - \omega_a) \xi \quad (4)$$

where the subscript “a” refers to damage associated with environmental aggression, and the subscript “b” refers to damage associated with loading effects and ξ refers to ratio between the level of acting stress and the limit state value.

Thereby the proposed damage law is able to represent damage mechanisms induced by environmental deterioration, like carbonation of concrete, corrosion of steel or material fatigue. These mechanisms are usually present and interacting. Thence basing on experimental observations and/or laboratory accelerated test data, a proper calibration of the damage parameters is required.

By experimental evidence the deterioration process can be considered as a loss of performance affecting a material system. Whenever deterioration reaches a given level, the system suffers a “failure” and a loss of performance. This phenomenon represents the transition of the *system* through different service states characterized by different levels of performance. Thus, the deterioration process (failure process) is defined as a *transition process* (Cox, 1962). It depends on a huge number of random variables (r.v.) and it is right to dial it as a stochastic process. A suitable r.v. able to describe this process is *service lifetime* τ_i , defined as: “the waiting time spent by the material in the performance state i before a transition” (Garavaglia *et al.* 2004).

If the deterioration process is assumed to be a state transition Biondini & Garavaglia (2005) have shown that the Markov Renewal Process (Howard, 1971, Limnios & Oprisan, 2001) seems to be suitable to describe such a dynamic process. The MRP can represent the development of the system's life among different service states with different waiting times; it is also able to take in account the age t_0 of

the system, i.e. the time already spent by the system into the current service state before the prevision is carried out. This is one of the main aspect in reliability analysis when maintenance must be planned.

2.2 Deterioration modeling

A MRP is completely determined if we know its initial laws (Limnios & Oprisan, 2001):

- The state i of the system at the initial time $t = 0$, and the time $t_i = t_0$ spent in the initial state (i.e. the initial conditions).
- The probability density function $f_{ik}(t)$ of the waiting time t_{ik} spent by the system into the service state i provided that the next state after transition is k , or $f_{ik}(t) = P\{t < t_{ik} < t+dt\}$.
- $\Pr(\text{next state } j, \tau_{ij} \leq t \mid \text{previous state } i) = p_{ij} F_{ij}(t)$
for every $t \in (0, +\infty)$ and $i, j \in E$. (5)

Assuming i as present state, the (5) describes the probability that transition into the next state j , occurs by time t ; p_{ij} are the transition probabilities of the Markov chain and $F_{ij}(t)$ are the distribution functions associated to waiting times in state i before moving to state j .

The marginal distribution function of the waiting times τ_i :

$$\Pr(\tau_1 \leq t_1, \dots, \tau_i \leq t_i \mid \text{current state } (i-1)) = \prod_{i=1}^n F_{(i-1),i}(t_i) \quad \forall n \geq 0 \quad \text{and} \quad t_i \geq 0 \quad (6)$$

describes the probability of transition into each state i by time t_i .

Assuming i as present state and t_0 as the time already spent by the system into i -state, if the distribution functions $F_{ij}(t)$ are defined and $\tau_{ij} = t_0$, the probability to move into state j , Δt in the next can be defined as follow:

$$P_{\Delta t|t_0}^{(ij)} = \frac{[F_{ij}(t_0 + \Delta t) - F_{ij}(t_0)] p_{ij}}{\sum_{k=1}^S [1 - F_{ik}(t_0)] p_{ik}}. \quad (7)$$

The probability p_{ij} can be obtained by experimental observations through the ratio:

$$p_{ij} = \frac{\text{observed transitions from } i \text{ to } j}{\text{observed transitions from } i}. \quad (8)$$

Probability (7) can be evaluated both for the system and for each member composes the system.

In this paper the MR approach is applied to evaluate the transition process of each structural member of a simple case study. The aim is the schedule of maintenance actions on single or multiple members based on the probable instant of transition between states evaluated with the MRP. Therefore, (7) is evaluated for each member of the bridge truss in Figure 1. Then the probability $P_{\Delta t|t_0}^{(m,ij)}$ where m is the

number identifying the structural member is analyzed.

Since the deterioration leads to decrease both cross-sectional area A and material strength $\bar{\sigma}$ of each structural member and the failure of a member as well as of the whole system is related to the loss of performance, anytime these parameters can reach a dangerous level. Assuming the stress value σ recorded/evaluated at every monitoring as a risk parameter and considering it as the ratio between the internal forces and the deteriorated area of the elements' cross sections, using a computer code for structural analysis the performance decrease can be quantified. Hereafter this parameter will be assumed as random variable in the deterioration process. In order to model the time evolution of such variable within a Markov renewal modeling, the following assumptions are introduced (Biondini & Garavaglia, 2005):

- The structure as well as each component member are undamaged at the initial time $t_0 = 0$.
- At each instant t the members are considered to occupy a given state i , with $i > 0$, when $\sigma_i \leq \sigma \leq \sigma_{(i+1)}$, where σ_i and $\sigma_{(i+1)}$ are the lower and upper thresholds respectively, which characterize the state i .
- The member performance moves from a state i , $i > 0$, to another state j , with $j > i$, characterized by a lower level of performance, σ_i , $\sigma_i < \sigma_j$, during a time interval τ_{ij} . Of course, condition $j < i$, with $\sigma_i > \sigma_j$, is also possible if some maintenance is operated.

In MRP each waiting time τ_{ij} must be modeled by choosing an appropriate PDF. This is a hard choice and it can be “not unique”. It should be made on the basis of physical knowledge of phenomenon and on the basis of the characteristic distributions in their tails where, usually, no much data are recorded.

The physical knowledge, based on experimental evidence, suggests that the deterioration of building materials increases following a characteristic time-dependent law due to aggressive environment. Therefore, considering the increasing hazard rate functions $\lambda(t)$ over the time, the deteriorated material's behavior can be modeled by using distributions satisfying this tendency. Distributions obeying to this law are, for example, the Weibull distributions where, if $t \rightarrow \infty$, $\lambda(t)$ tends to an infinite value too, and Gamma distributions where, if $t \rightarrow \infty$, $\lambda(t)$ tends to an asymptotic value. In this paper, starting from the previous considerations Weibull distributions are chosen in order to model structural members deterioration. The parameters involved in Weibull distributions are estimated by the maximum likelihood criterion using a computer code where the routine ROSE of IMSL Fortran Library, involved the Rosenbrock's optimization method, was implemented.

3 APPLICATION OF MRP TO A CASE STUDY

3.1 Monte Carlo simulation as support of MRP

Every probabilistic approach requires several experimental data usually difficult to gather. In this study, the samples were obtained using a Monte Carlo simulation. In this simulation process the damage parameters ρ_a , ρ_b , ω_a , ω_b , and T_C , are modeled as random variables with prescribed probability distribution. Each distribution is chosen on the basis of physical knowledge of phenomena investigated; starting from mean and standard deviation (Ciampoli, 1999, Gupta & Lawsirirat, 2005) the model is developed.

During its life the structure as well as its single members, move into different service state; the transitions can occur state by state, otherwise can involve two or more states, which means that a transition can involve two or more states for each step. Thence the choice of the state size is a tricky issue that can compromise the modeling. The following three different states are assumed: *State 1* relates to a low damage, *State 2* relates to moderate damage and *State 3* relates to heavy damage. The states' boundaries are chosen on the basis of expert judgment and are collected in Table 1.

Table 1 State boundaries

State	$\sigma_{\min}$ (MPa)	$\sigma_{\max}$ (MPa)	Damage level
State 1	>0.00	≤ 1100	Low (undamaged)
State 2	>1100	≤ 1250	Moderate
State 3		>1250	Heavy

For each simulation cycle, the time-variant structural analysis is carried out by updating step-by-step the stiffness matrix of the deteriorated structural members. Referring to the limit state i , $|\sigma| \leq \sigma_{\max}^i$, the member's transition happens when the member reaches the upper bound $\sigma_{\max}^i$ of the state i . At every simulation, the transition time for each member is recorded and it is defined as the waiting time τ_{ij} spent by the member into state i before moving into a successive state j .

Considering the limit state $(|\sigma| - \bar{\sigma}) = 0$, the system failure is reached when the number of failed members is able to activate a certain mechanism. The time in which the failure occurs is called *failure time*. While whenever a member comes to a given damage threshold, the time related to the reaching of this damage state is called *crossing time*.

The approach here presented is applied to probabilistically investigate the next transition of each member of the system in Figure 1, from the current state i since t_0 , with $t_0 > 0$ to the state j in the next interval Δt . Therefore using a population of 5000 samples of the same structural system a MC simulation has been built. Each sample was affected by random

deterioration process following the damage law (2); the members' crossing times connected with each damage threshold and the failure time of the whole system have been recorded and modeled with Weibull distributions.

3.2 Life- service evaluation

The development of a computer code for structural analysis, implemented with the MRP and then applied to each member of the structure, state by state, allows the evaluation of the crossing times. Using Eq. (7) the probability $P_{\Delta t | t_0}^{(m,ij)}$ for each m members of the system ($m=31$) is evaluated. Considering new buildings, when $P_{\Delta t | t_0}^{(m,ij)}$, with $i \neq j$, reaches a value major or equal to $r \cdot 10^{-4}$ (r =positive number) the risk of transition from i to j is assumed upcoming; therefore maintenance could be planned in instants close to which the probability is recorded (Fig. 2).

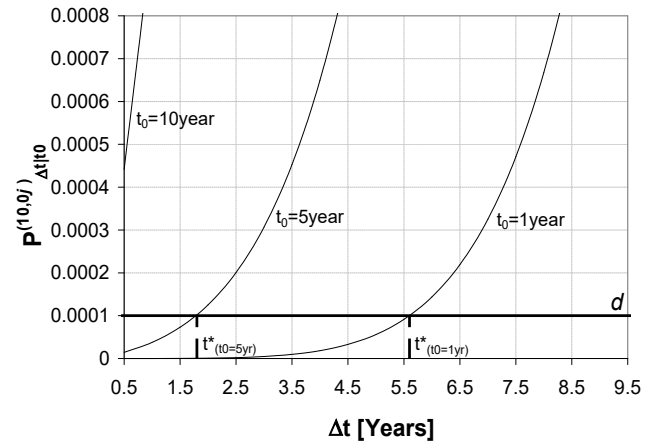


Fig. 2. Member 10: Probability of transition $P_{\Delta t | t_0}^{(m,ij)}$ vs Δt ($i=0$, vs. $\forall j$) evaluated at different t_0 . Damage $d=1 \cdot 10^{-4}$ is the damage threshold; instants t^* are the instants when the member exceeds d if the time t_0 spent into state $i=0$ is 5 years or 10 years respectively.

4 MAINTENANCE PLANNING BASED ON MRP EVIDENCE

The maintenance of existing building can be approached as follows:

- Replacing* all damaged members: the initial system reliability is wholly restored and future maintenance can be applied at constant intervals.
- Repairing* all damaged members: the initial system reliability is almost restored, future maintenance could be required at every closer interval.
- Replacing/repairing* several damaged members: the initial system reliability is just partially restored.

The application of the MRP on each member of a system can lead the decision on *when* and *where* to operate maintenance, therefore, it's interesting to investigate possible selective maintenance scenarios on the basis of MRP results and the economic implications.

4.1 Selective maintenance scenarios

This paper presents the results of two selective maintenance actions approached with the MRP and focused on the replacement of the deteriorated members only.

The Markovian approach applied on 5000 simulation obtained by MC technic has pointed out the possibility of moving through contiguous states or directly in states with higher damage also for the same member of a structure (Fig. 3); this happens because the 5000 systems are subjected to a random deterioration always different and the behavior of the members changes time by time.

Figure 3 shows the tendency of each member of the system to move into one of the possible future damage states. Therefore, evaluating the waiting times spent by the system into the initial state 0 before the next state transition and assuming as dangerous the threshold $d=1*10^{-4}$, Figure 3 suggests to make a first selective maintenance in advance on several members, and afterwards a second one on the whole system.

On the bases of results shown in Figure 3, different selective maintenance scenarios can be proposed (Fig. 4). Furthermore each scenario must be economically evaluated too. The economical investigation of different maintenance scenario will be the future step of this research.

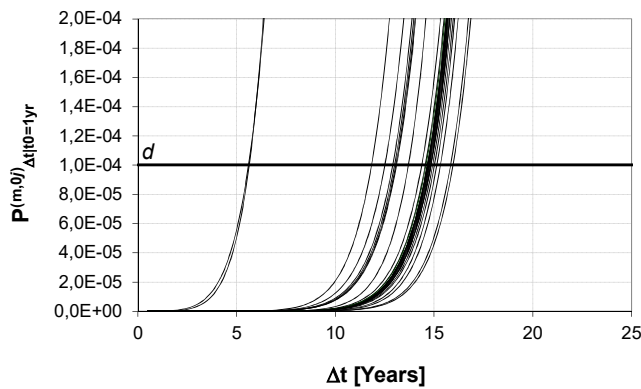


Fig. 6 Probability of transition $P_{\Delta t|t_0}^{(m,j)}$ vs Δt evaluated on the 31 members of the system with $t_0=1$ year and transition 0j from initial state 0 to state j ($j=1,2$). This figure describes the probability of transition from initial state versus one of the future damage states.

Within the two selective maintenance scenarios investigated, maintenance actions are scheduled assuming $t_0=1$ year: considering a one-year-old sys-

tem, just the replacement of deteriorated elements is assumed to carry out.

Figure 4 shows two possible selective maintenance scenarios: scenario (1) maintenance is renewed each $\Delta t=8.5$ years with two different actions: $\Delta t=8.5$ years replacing of 10 members (3, 4, 5, 10, 11, 19, 21, 23, 28 and 29), $\Delta t=17$ years overall replacing of system's members; scenario (2) maintenance is renewed each $\Delta t=7.5$ years with two different actions: $\Delta t=7.5$ years replacing of two members (10 and 11), $\Delta t=15$ years overall replacing of system's members.

Comparing the two scenarios, scenario (1) presents the replacement of more members respect to scenario (2); the decision was made on the basis of the low but increasing $P_{\Delta t|t_0}^{(m,j)}$ after the maintenance instant chosen, which suggests an in advance replacement. The two scenarios will be now compared to evaluate the different costs of an in advanced maintenance on several members of the system and 2 overall maintenances, as in (1), and, vice versa, 3 overall maintenances and the in advanced maintenance just on two members of the system, as in (2).

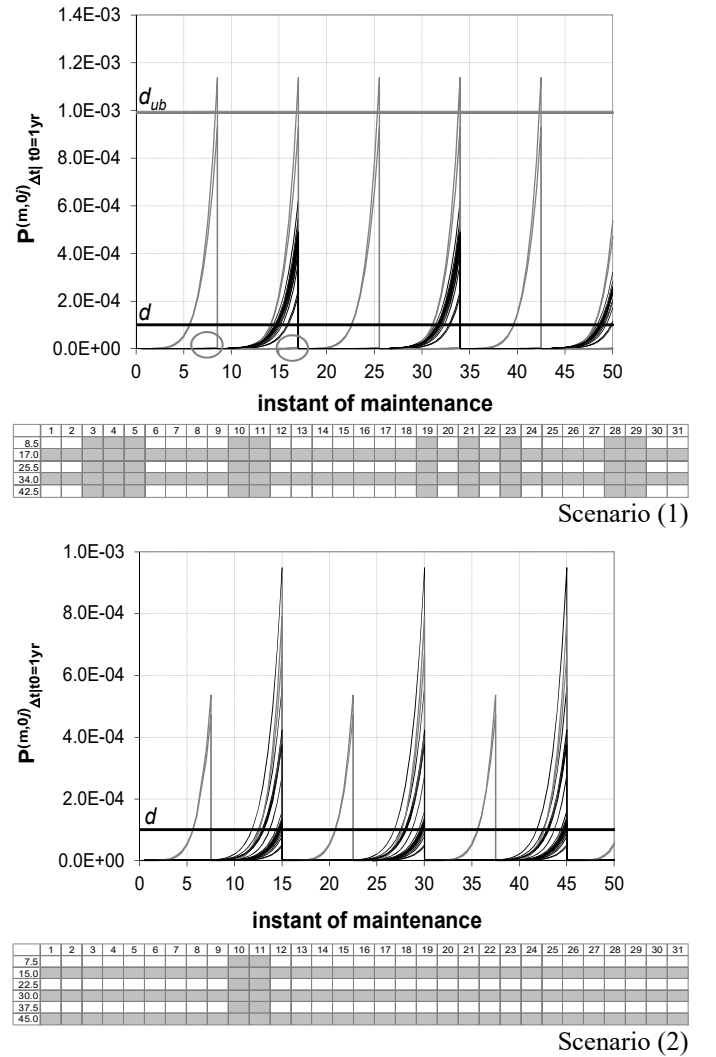


Fig. 4 Two possible selective maintenance scenarios. Considering maintenance instants, the figures show the system's leaning towards initial level of performance when the replacement of damaged elements is carried out. Tables show which members are replaced. Circles identify members with a low probability P

to cross d before $\Delta t=8.5$ years and $\Delta t=17$ years respectively. The sudden increasing of P after Δt suggests an in advance replacement.

4.2 Maintenance costs: comparison between scenarios

Since any maintenance scenario must be associated to relative maintenance cost, the cost effectiveness of the choice must be mainly taken into account. For instance, considering a prescribed maintenance scenario, the total cost of maintenance C_m can be evaluated by summing the costs C_q of the individual interventions (Kong and Frangopol, 2003),

$$C_m = \sum_{q=1}^n \frac{C_q}{(1+\nu)^{t_q}} = \sum_{q=1}^n C_{0q} \quad (9)$$

where the cost C_q of the q^{th} rehabilitation has been referred to the initial time of construction by taking into account a proper discount rate of money ν (Kong and Frangopol, 2003). Here, the cost C_q of the individual intervention is assumed as follows:

$$C_q = C_f + \sum_{k=1}^m \delta_k \cdot V_k \cdot c_{qk} = C_f + C_{(1,m)} \quad (10)$$

where $C_f = \alpha C_0$ is a fixed cost computed as a percentage α of the initial cost C_0 , δ_k is the damage index of element k of the structural system, V_k is the volume of element k , and c_{qk} is the volume unit cost for restoring the element k . Hereafter $C_{(1,m)}$ will identify the sum shows in (10). Basing on this cost model, different maintenance scenarios can be compared and optimal maintenance strategies can be evaluated (Biondini et al. 2008, Biondini & Frangopol, 2008). The scenarios previously presented are now economically compared.

In Figure 5 scenario (1) is compared with scenario (2); the total cost C_2 , computed as the sum of initial cost C_0 and replacement cost $C_{(1,m)}$, of the scenario (2) normalized to the cost C_1 of the scenario (1), $c_2 = C_2/C_1$, versus the discount rate ν , for different values α of the fixed cost of maintenance $C_f = \alpha C_0$.

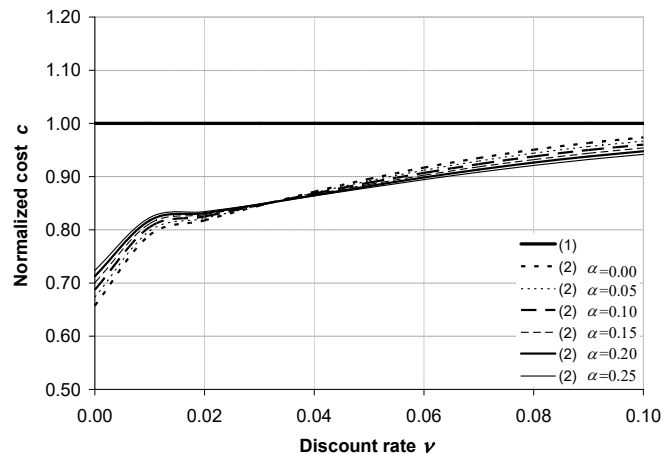


Fig. 5 Total cost $C_m = C_2$, computed by (9) where C_q is the sum of initial cost C_0 and replacement cost $C_{(1,m)}$, of the scenario (2)

normalized to the cost C_1 of the scenario (1), or $c_2 = C_2/C_1$, versus the discount rate ν , for different values α of the fixed cost of maintenance $C_f = \alpha C_0$.

From the cost analysis and comparison stands out the economic primacy of scenario (2) over scenario (1) for all the discount rate ν and for all the percentage α investigated. Thence an in advanced maintenance on the whole system seems to be preferred instead of a much more selective maintenance.

If we consider that replacement costs have a strong incidence on the maintenance cost C_m , i.e. the double $(C_{(1,m)} * 2)$, the situation changes and scenario (2) becomes convenient only for small values of the discount rate ν (Fig. 7).

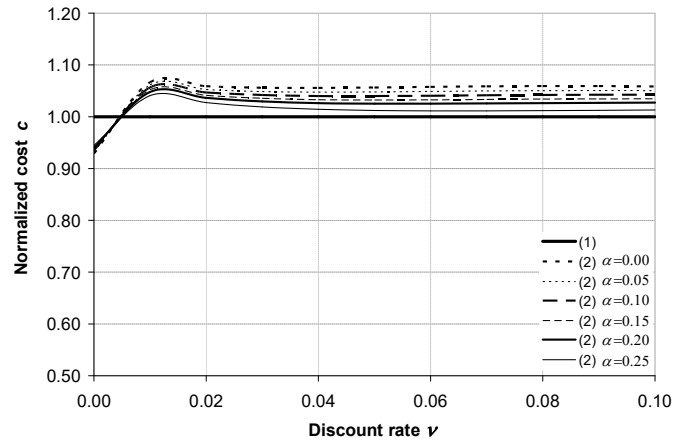


Fig. 5 Total cost C_m , computed following (9) where C_q is the sum of initial cost C_0 and replacement cost $C_{(1,m)}$, where $C_{(1,m)}$, is weighted with a coefficient 2. Scenario (2) is normalized to the cost C_1 of the scenario (1), and evaluated for different discount rate ν , and different values α of the fixed cost of maintenance $C_f = \alpha C_0$.

In this application the choice to replace the deteriorated members, without analyzing the system after the first replacement, is the mainly critical issue within the initial application phase. The new replacement might extend the waiting time of the next maintenance with cost effectiveness. However comparing with results obtained by an analysis where replacements of new members are taken into account, even though the results here shown surely ensure structural safety, they probably provide less economic benefit. Therefore, integrating the maintenance cost assessment with the evaluation of new members' replacement will be the next step of the research.

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