



Institute for Information
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Electronics and Applied Mathematics

Robust Spectrum Sensing for Mobile Cognitive Radio

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*For the LORD giveth wisdom: Out of his mouth cometh
knowledge and understanding.*

– Proverbs 2:6 KJV

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Abstract

Cognitive radio (CR) is the solution to the spectrum scarcity issue faced in wireless communications. It allows unlicensed secondary users (SUs) to opportunistically access the spectrum assigned to licensed primary users (PUs), but unused, on a non-interfering basis. Spectrum sensing is the process by which the CR node becomes aware of the spectrum occupancy, in order to decide which frequency bands to use. It is therefore a critical step in CR. The main goal of this thesis is to design robust spectrum sensing algorithms for mobile cognitive radio systems. The contributions are grouped in three parts.

In the first part, a new spectrum sensing algorithm is proposed for mobile CR environments. It exploits the knowledge of the SU's mobility parameters, by using the Bayesian changepoint detection theory. Moreover, it works for practical scenarios where the PU's signal power is unknown to the SU. Simulation results show that the derived algorithm is a good choice when the signal-to-noise ratio (SNR) is unknown and the SU could be required to detect very low SNR signals.

In the second part, the issue of joint transmission and sensing is studied in mobile CR. Considering a scenario with multiple PU appearances and disappearances, a new framework that uses changepoint detection for spectrum sensing is introduced. The optimal system parameters (sensing time, transmission time and detection threshold) that maximize the spectrum utilization are numerically computed and analysed.

In the last part, it is considered that some information about the existing PUs is available in a geolocation database. Then, a new spectrum sensing algorithm, that exploits this geolocation information by using the Bayesian

block-based detection theory, is introduced. An approximate closed-form expression, based on Taylor series expansion, is provided. Numerical results show that the derived algorithm gives the minimum error probability when compared with an algorithm that does not use geolocation information.

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Nomenclature

Acronyms and Abbreviations

5G	Fifth generation of mobile networks (p. 2)
<i>AWGN</i>	Additive White Gaussian Noise (p. 39)
<i>CAF</i>	Cyclic Autocorrelation Function (p. 42)
<i>CFD</i>	Cyclostationary Feature Detection (p. 42)
<i>CR – VANET</i>	Cognitive Radio for Vehicular <i>ad hoc</i> Network (p. 45)
<i>CR</i>	Cognitive Radio (p. 6)
<i>CSD</i>	Cyclic Spectral Density (p. 42)
<i>CUSUM</i>	Cumulative Sum (p. 31)
<i>DSA</i>	Dynamic Spectrum Access (p. 4)
<i>DSO</i>	Digital Switchover (p. 7)
<i>DSRC</i>	Dedicated Short Range Communications (p. 45)
<i>DTT</i>	Digital Terrestrial Television (p. 44)

<i>EBs</i>	Exabytes (p. 2)
<i>ED</i>	Energy Detection or Energy Detector (p. 39)
<i>FCC</i>	Federal Communications Commission (p. 7)
<i>GLR – CUSUM</i>	GLR-based CUSUM (p. 32)
<i>GLRT</i>	Generalized Likelihood Ratio Test (p. 27)
<i>GNSS</i>	Global Navigation Satellite System (p. 18)
<i>GPS</i>	Global Positioning System (p. 20)
<i>GSLRT</i>	Generalized Sequential Likelihood Ratio Test (p. 29)
<i>i.i.d.</i>	independent and identically distributed (p. 28)
<i>IoT</i>	Internet of Things (p. 2)
<i>ITU</i>	International Telecommunication Union (p. 2)
<i>K – L</i>	Kullback-Leibler (p. 36)
<i>LC GLR – Shiryaev</i>	Low-Complexity GLR-Shiryaev (p. 51)
<i>LFD</i>	Least Favourable Distribution (p. 34)
<i>MAP</i>	Maximum a Posteriori probability (p. 27)
<i>MF</i>	Matched Filtering (p. 41)
<i>MIMO</i>	Multiple-Input Multiple-Output (p. 3)
<i>ML</i>	Maximum Likelihood (p. 27)
<i>MLE</i>	Maximum Likelihood Estimate (p. 27)
<i>MSE</i>	Mean Squared Error (p. 75)

<i>NP</i>	Neyman-Pearson (p. 24)
<i>OFCOM</i>	Office of Communications (p. 7)
<i>OSA</i>	Opportunistic Spectrum Access (p. 6)
<i>PDF</i>	Probability Density Function (p. 26)
<i>PSD</i>	Power Spectral Density (p. 5)
<i>PU</i>	Primary User (p. 5)
<i>ROC</i>	Receiver Operating Characteristic (p. 24)
<i>RSU</i>	RoadSide Unit (p. 45)
<i>SFN</i>	Single Frequency Network (p. 44)
<i>SNR</i>	Signal-to-Noise Ratio (p. 2)
<i>SOM</i>	Sensing-Only Mode in Chapter 4 (p. 105)
<i>SPRT</i>	Sequential Probability Ratio Test (p. 28)
<i>SU</i>	Secondary User (p. 5)
<i>TM</i>	Transmission Mode in Chapter 4 (p. 105)
<i>TV</i>	Television (p. 7)
<i>TVWS</i>	TV White Spaces (p. 7)
<i>UHF</i>	Ultra High Frequency (p. 7)
<i>UWB</i>	Ultra Wide Bandwidth (p. 5)
<i>V2I</i>	Vehicle-To-Infrastructure (p. 45)
<i>V2P</i>	Vehicle-To-Person (p. 45)

$V2V$	Vehicle-To-Vehicle (p. 45)
$VANET$	Vehicular <i>ad hoc</i> Network (p. 45)
VHF	Very High Frequency (p. 7)
$WRAN$	Wireless Regional Area Network (p. 37)
$WSPRT$	Weighted SPRT (p. 29)

Symbols and Notations

α	Detection threshold (p. 23)
α_0	Lower stopping threshold for SPRT (p. 29)
α_1	Upper stopping threshold for SPRT (p. 29)
β	Constraint on false alarm probability (p. 25)
I	Identity matrix (p. 39)
w	Noise vector (p. 39)
x	PU signal vector (p. 39)
y	Observed data set (p. 23)
y_1^m	Observed data, from the beginning to the current time sample m (p. 29)
$\chi_{N_s}^2$	Chi-square distribution with N_s degrees of freedom (p. 40)
$\Gamma(.)$	Gamma function (p. 40)
$\Gamma(.,.)$	Incomplete gamma function (p. 140)

γ	Average received power in Chapter 5 (p. 134)
γ_G	Average received power computed with geolocation information in Chapter 5 (p. 135)
$\Lambda(\mathbf{y})$	Decision statistic in block-based detection (p. 23)
$\Lambda(\mathbf{y}_1^m)$	Decision statistic in sequential (hypothesis test or change-point) detection (p. 31)
$\mathbb{E}$	Expectation (p. 32)
$\mathbb{E}_f$	Expectation under the assumption of distribution f (p. 36)
$\mathbb{E}_k$	Expectation under the assumption that the change happens at time sample k (p. 30)
$\mathcal{S}$	Finite set of post-change parameter candidate values in M-Shiryaev algorithm (p. 35)
$\mathcal{CN}(a, b)$	Complex Gaussian distribution with mean a and variance b (p. 53)
$\mathcal{H}_0$	Noise-only hypothesis (changepoint detection) (p. 30)
$\mathcal{H}_1$	Hypothesis of noise-only until τ and signal plus noise from $\tau + 1$ (changepoint detection) (p. 30)
$\mathcal{N}(a, b)$	Gaussian distribution with mean a and variance b (p. 39)
$\mathcal{R}$	Bayes risk (p. 26)
$\Omega(\Lambda(\mathbf{y}), \alpha)$	Detector, given by decision statistic and threshold (p. 24)
$\overline{D}_{D,C}$	Conditional average detection delay (p. 30)
$\overline{D}_{D,WC}$	Worst-case detection delay (p. 31)

$\overline{D}_{DI}$	Redefined average detection delay (p. 61)
$\overline{D}_D$	Average detection delay (p. 32)
$\overline{D}_{FA}$	Average false alarm duration (p. 61)
$\overline{T}_{FA}$	Mean time to false alarm (p. 30)
$\overline{T}_{int}$	Average total duration of interference in Chapter 4 (p. 111)
$\overline{T}_{sow}$	Average total duration of spectrum waste in Chapter 4 (p. 111)
σ_w^2	Variance of noise (p. 39)
σ_x^2	Variance of PU signal (p. 39)
$\sigma_{x,k}^2$	MLE of the signal power that maximizes each term k of the GLR-Shiryaev decision statistic (p. 58)
τ	Time of change, for changepoint detection algorithms (p. 29)
$\tau_i^{(0)}$	The i^{th} PU disappearance time in Chapter 4 (p. 105)
$\tau_i^{(1)}$	The i^{th} PU appearance time in Chapter 4 (p. 105)
$\tilde{\sigma}_x^2$	Estimal PU signal power, for adaptive-threshold methods (p. 72)
ε	Upper margin over the minimum global cost, for unknown-SNR optimal sensing time (Chapter 4) (p. 114)
B_f	Number of frames over which energy detection is performed in Chapter 4 (p. 126)
C_{ij}	Cost of declaring H_i while H_j is true (p. 26)

C_{int}	Cost of interference in Chapter 4 (p. 111)
C_{sow}	Cost of spectrum opportunity waste in Chapter 4 (p. 111)
$D(f_1, f_0)$	Kullback-Leibler divergence of f_1 from f_0 (p. 36)
d	Rate of convergence of the prior distribution for Bayesian changepoint detection (p. 36)
$f(\cdot)$	PDF of a random variable (p. 27)
$f(\mathbf{y} \boldsymbol{\theta})$	Conditional PDF of the observed vector $\mathbf{y}$ (p. 27)
$f_{0,SOM}(z_l)$	Distribution of z_l in sensing-only mode, when only noise is present (Chapter 4) (p. 108)
$f_{0,TM}(z_l)$	Distribution of z_l in transmission mode, when only noise is present (Chapter 4) (p. 108)
$f_0(\mathbf{y})$	PDF of the observed vector $\mathbf{y}$ under hypothesis H_0 (p. 26)
$f_{1,SOM}(z_l)$	Distribution of z_l in sensing-only mode, when PU signal is present (Chapter 4) (p. 108)
$f_{1,TM}(z_l)$	Distribution of z_l in transmission mode, when PU signal is present (Chapter 4) (p. 108)
$f_1(\mathbf{y})$	PDF of the observed vector $\mathbf{y}$ under hypothesis H_1 (p. 26)
$f_G(\cdot)$	PDF computed with geolocation information in Chapter 5 (p. 135)
g'	Complementary ROC function given by $P_{MD} = g'(P_{FA})$ (p. 24)
g	ROC function given by $P_D = g(P_{FA})$ (p. 24)

G_C	Global cost in Chapter 4 (p. 111)
G_P	Global penalty (p. 62)
$h[n]$	Channel gain at time sample n (p. 37)
H_0	Noise-only hypothesis (block-based detection) (p. 23)
H_1	Signal plus noise hypothesis (block-based detection) (p. 23)
k	A probable instant of change (changepoint detection) (p. 33)
l	Frame index in Chapter 4 (p. 106)
L_n	Likelihood ratio of the n^{th} observation (p. 31)
M	Number of post-change parameter candidate values in M-Shiryayev algorithm (p. 35)
m	Current time sample for sequential detection (p. 29)
N	Total communication duration of the SU (p. 60)
n	Time sample (p. 23)
N_f	Number of frames in Chapter 4 (p. 105)
N_s	Observation length or sensing time for block-based detection (p. 23)
N_t	Number of Taylor series terms in Chapter 5 (p. 138)
p	Parameter of the geometric distribution - Probability of change at any given time (p. 33)
P_e	Probability of error (p. 26)

P_D	Probability of (correct) detection (p. 24)
P_{FA}	Probability of false alarm (p. 24)
P_{MD}	Probability of missed detection (p. 24)
$P_G(H_i)$	Prior probability $P(H_i)$ computed with geolocation information in Chapter 5 (p. 135)
P_{int}	Penalty due to interference (p. 62)
P_{sow}	Penalty due to waste of spectral opportunity (p. 62)
$q = P(\tau < 0)$	Probability that the change has occurred before the first observation (p. 33)
$Q_{\chi^2_{N_s}}(.)$	Right-tail probability of a variable following a chi-square distribution with N_s degrees of freedom (p. 40)
$R_y(\rho, \tau_0)$	Cyclic autocorrelation function of the received signal y , where ρ is the cyclic frequency and τ_0 is the the distance between two time instants (weak-sense stationarity) (p. 43)
$S_y(\rho, f_c)$	Cyclic spectral density of the received signal y , where ρ is the cyclic frequency and f_c is the frequency (p. 43)
T_f	Frame size or duration in Chapter 4 (p. 105)
$T_{\cdot}$	Stopping time for sequential (hypothesis test or change-point) detection (p. 28)
T_{β}	Lower bound on the mean time to false alarm (p. 31)
T_G	Guard time in Chapter 4 (p. 106)
T_{se}	Sensing time in Chapter 4 (p. 106)
T_{tr}	Transmission time in Chapter 4 (p. 106)

★ | Nomenclature

$w[n]$	Noise at time sample n (p. 23)
$w_l[n]$	Noise at sensing sample n of frame l in Chapter 4 (p. 106)
$x[n]$	PU signal at time sample n affected by channel (p. 23)
x^*	Complex conjugate of x (p. 41)
$x_l[n]$	PU signal at sensing sample n of frame l in Chapter 4 (p. 106)
$y[n]$	Received (observed) signal at time sample n (p. 23)
$y_l[n]$	Observed signal at sensing sample n of frame l in Chapter 4 (p. 106)
z_l	Combination of sensing samples at frame l in Chapter 4 (p. 108)

1

Introduction

DEFINED as the transmission of information without the use of wires, electrical conductors or any physical medium, wireless communication is one of the most groundbreaking sectors of the communication technology domain, with tremendous progress during these last decades. Information is carried by an electromagnetic wave which is radiated by a transmitting antenna through the air and then recovered by a receiving antenna. If the propagation of electromagnetic waves was mathematically established by James Clerk Maxwell and experimentally confirmed by Heinrich Rudolf Hertz, transmission of information via these waves was demonstrated thereafter. As their properties, behaviours (production, detection, absorption, ...) and applications depend on their central frequency, electromagnetic waves are categorized into several frequency ranges and the overall frequency band is called the electromagnetic spectrum. The radio spectrum (3 Hz to 3000 GHz) is the part of the electromagnetic spectrum used by various communication technologies. In practice, only a small portion (10 MHz to 6 GHz) of the radio spectrum is of interest for wireless communications purposes, because of its good propagation properties, and the lower frequencies are the most suitable for large area services or long-distance communications [1]. As the transmitted information signal contains not a single frequency but a range of frequencies, it should be ensured that the signals of two different communication sys-

tems do not overlap, to avoid interference. The simplest way to achieve this is to allocate a frequency band to each communication system, while including guard bands between them. Hence, it becomes obvious that the constant increase of the number of communication systems will cause the radio spectrum to be crowded. Radio spectrum is therefore a finite and valuable resource [2, 3] that should be efficiently managed by spectrum regulators, in order to ensure fairness among services depending on their bandwidth requirements.

1.1 Context and motivations

1.1.1 Some trends and issues concerning the spectrum utilization

These last years have witnessed a significant increase of the wireless data traffic due to the proliferation of wireless devices and services [4] along with the ever-growing need for multimedia data transmission requiring high data rates, high reliability and low latency [5]. This trend is likely to further increase. The global mobile traffic per month actually reached around 78 Exabytes (EBs) in the third quarter of 2021, as stated in the Ericsson Mobility Report [6], while it is foreseen to grow to 5016 EBs in 2030 according to the *International Telecommunication Union* (ITU) [5, 7]. This fast growth is highly influenced by the *Internet of Things* (IoT) explosion and will be further impacted by the fifth generation of mobile networks (5G) [8]. A direct consequence of this situation is, on the one hand, the increasing need for higher communication channel capacity and, on the other hand, the lack of spectrum to cater for this demand. Therefore, it becomes more than ever a big challenge to make an efficient utilization of the already scarce radio spectrum resource [9].

The capacity C (in bits/s) of a communication channel is given by the Shannon-Hartley theorem [10]

$$C = B \log_2(1 + \text{SNR}) \quad (1.1)$$

and involves the available bandwidth B (in Hertz) and the *signal-to-noise ratio* (SNR). It represents the maximum data rate that can be achieved over a communication channel with an arbitrarily small error probability, by

using an appropriate coding scheme [11, 12]. Therefore, for a practical system transmitting at a rate R (in bits/s), we have

$$R \leq C. \quad (1.2)$$

The spectral efficiency of a communication system is defined as the data rate per unit of bandwidth R/B (in bits/s/Hz). This is an important metric to evaluate how efficiently the spectrum is utilized. Based on (1.1) and (1.2), it is thus bounded as follows

$$R/B \leq \log_2(1 + \text{SNR}). \quad (1.3)$$

The spectral efficiency can be increased by several means such as high order modulations, adaptive modulations and *multiple-input multiple-output* (MIMO) techniques [13, 14, 15, 16]. All these methods come with an increase in hardware and signal processing complexity [16, 17]. As the Shannon bound is being approached by the various existing techniques, it becomes crucial to explore alternative methods for improving the spectrum utilization. This could include new ways of assigning the available radio spectrum by the use of efficient spectrum management and sharing techniques, to allow more room for new wireless services.

1.1.2 Current spectrum management limitations

The traditional spectrum management approach consists in statically allocating a spectrum band on an exclusive basis (through a licence), either to a particular service or a particular operator, which is considered to be the incumbent for this specific band in a specific geographical location. However, several measurement campaigns, carried out in many countries, have shown that the assigned spectrum is usually underutilized [15]. In fact, the spectrum is not fully exploited at all locations and times, and those portions of the spectrum that are not used are generally called *spectrum holes* or *white spaces*, as depicted in Fig. 1.1. Furthermore, the amount of available white spaces can strongly vary from one country to another (either developed or developing) and from one region to another (either urban or rural) in the same country [15, 18, 19]. It also depends on the intended spectrum usage (indoor or outdoor environment). For example, a spectrum occupancy measurement conducted in Singapore showed that

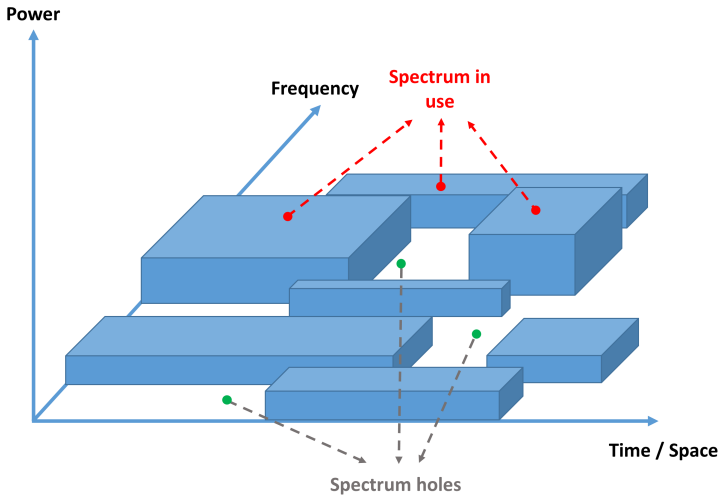


Fig. 1.1 Spectrum holes or white spaces (adapted from [1]).

the average spectrum utilization in the range of 30 MHz to 6 GHz is about 6.5% [20]. Another measurement campaign carried out in an indoor environment in Lagos (Nigeria) showed that the used spectrum in the range of 700 MHz to 2.2 GHz varies from 0.4% to 64.4% depending on the sub-band [21]. Several other measurement results, revealing the large availability of unused spectrum, can be found in the literature [22, 23, 24].

The existence of dynamic and non-uniformly distributed licensed spectrum holes shows that the overall spectrum efficiency is still very low, and this represents an untapped potential for wireless communications. There is clearly a need for an alternative spectrum management to the current static one.

1.1.3 Dynamic spectrum access and cognitive radio

The term *dynamic spectrum access* (DSA) has been given multiple definitions over time, depending on the models or approaches proposed in the research community. In a nutshell, DSA stands for the opposite of the current static spectrum management policy [25]. It is a paradigm in which the spectrum is dynamically shared among services, depending on their needs or requirements and based on predefined rules. DSA is defined in

the IEEE¹ Standard for Definitions and Concepts for Dynamic Spectrum Access as "the real-time adjustment of spectrum utilization in response to changing circumstances and objectives" [26]. It is intended to allow for more flexibility and cleverness in the spectrum usage and to provide an increased spectral efficiency.

Several models have been proposed for DSA in the literature [25, 27, 28], among which the *hierarchical access model*. In the hierarchical access model, two kinds of users are defined. *Primary users* (PUs) are the incumbent users which have been granted the license or right to use specific frequency bands. *Secondary users* (SUs) are users without licence, willing to exploit the spectrum bands assigned to the PUs. They are allowed to use the spectrum as soon as they do not (or only marginally) affect the performance of the PUs. The hierarchical access model is also referred to as unlicensed or license-exempt spectrum access model. This model can be implemented using three approaches [1, 29, 15]:

- In the *underlay* approach, both the PU and the SU simultaneously transmit over the same band. This is achieved by ensuring that the transmit *power spectral density* (PSD) of the SU is so low that the interference on the PU receiver is below a certain threshold. An example of underlay system is the *Ultra Wide Bandwidth* (UWB) communications system, where the transmitted signal is spread all over a wide-band in order to obtain a PSD below the noise floor. Another way is to keep the transmit power of the SU very low, thereby restricting the secondary communication to a short-range communication.
- In the *overlay* approach, the SU still transmits in the same band as the PU, but it chooses appropriate transmission strategies so that it causes a minimum interference on the PU communication. The SU must have knowledge about the data sequence transmitted by the PU and the complete channel state information for both primary and secondary systems, in order to perform interference cancelling techniques. This approach thus clearly needs a strong cooperation between the PU and SU communication systems.
- The idea of the *interweave* approach is to transmit only in the spectrum holes, with the aim of avoiding any interference with the existing PUs. The SU identifies and exploits the unused parts of the

¹IEEE: Institute of Electrical and Electronics Engineers.

spectrum (in frequency, time and space). This approach is also called *opportunistic spectrum access* (OSA) [25] because white spaces are considered as spectrum opportunities that could be used by the SU. To make OSA possible, a SU should be able to accurately (with less error) detect the PU activities in order to transmit in an available spectrum hole or release it as soon as the PU becomes active. This requires constant spectrum monitoring and spectrum agility.

We consider the interweave or opportunistic spectrum access approach in this thesis. Opportunistic spectrum access is often associated to the term *cognitive radio* (CR). Cognitive radio is a technology aiming at enforcing DSA and particularly OSA, by giving wireless devices the capability to detect spectrum holes and use them efficiently to communicate. Coined by Joseph Mitola [30], a cognitive radio is a wireless equipment that is aware of its electromagnetic environment and able to autonomously and automatically adapt or adjust its transmission parameters (carrier frequency, bandwidth, transmission times, transmit power, modulation scheme, ...) accordingly [31, 32]. This intelligence and cognition capability of cognitive radios can be made possible through several steps or functions: the CR has to observe the environment, analyse, decide and adapt itself. The observation and analysis of the environment include the methods used to detect existing spectrum opportunities. A CR can either make the observation itself, or alternatively it can take advantage of an external service to extract information about existing PUs [33]. Three main methods are used in that respect, namely

- the geolocation database method,
- the beacon-based sensing and
- the local spectrum sensing.

In the first two methods, the information about spectrum occupancy is given by an external entity and transmitted to the SU, which is therefore not fully autonomous. But in the last method, the SU directly senses the spectrum band of interest, in order to find spectrum opportunities. These methods are often grouped under the umbrella term *spectrum sensing*, but it is actually a misnomer and only the last one can be called so. We only focus on local spectrum sensing in this thesis, and every mention of spectrum sensing refers to this method. The three methods are further described in

Chapter 2. In brief, spectrum sensing is thus the process by which the CR (or SU) measures the existing PUs activities in a certain frequency band, in order to detect potential spectrum holes. It is the first and most crucial function in CR because all the other functions rely on its outcome. In fact, spectrum sensing directly impacts the amount of interference that will be caused on the existing PUs and the amount of spectrum that will be available for the SUs. It should therefore be carried out accurately.

1.1.4 TV white spaces and CR

In the radio spectrum, the television (TV) band (VHF² and UHF³) is very attractive for long-range wireless communication services, because of its favourable propagation characteristics that allow high coverage and better penetration through obstacles [34]. Many measurement campaigns show that most of the spectrum assigned to TV transmission is not used in many geographical areas [1, 35]. The available TV spectrum portions, called *TV white spaces* (TVWS), have been estimated to vary from a few tens of MHz in dense urban areas, to hundreds of MHz in suburban and rural areas [36, 37]. In developing countries, where there are few TV stations, and especially in sparsely populated rural areas, the amount of TVWS is likely larger. Furthermore, the *digital switchover* (DSO), i.e., the transition to digital TV, has released a significant number of frequency bands, due to the enhanced spectral efficiency of digital TV, thereby increasing the available amount of TVWS.

For all these reasons, TVWS are seen as a significant opportunity to alleviate the spectrum scarcity issue, by offering the possibility to create new and innovative wireless services such as smart grid, indoor localization, public safety and medical applications [38, 39]. In particular, due to their good propagation properties, TVWS are well-suited for delivering broadband access to rural and underserved areas, with minimal infrastructures and at low cost [39, 37]. The decision of the United States *federal communications commission* (FCC) and the United Kingdom *office of communications* (OFCOM) to allow access to TVWS on an unlicensed basis has made them become the first spectrum portions where CR technology is used [38, 40].

²VHF: Very High Frequency (30 MHz to 300 MHz).

³UHF: Ultra High Frequency (300 MHz to 3000 MHz).

1.1.5 Basics of spectrum sensing

Spectrum sensing is based on signal detection theory. In brief, signal detection theory consists in detecting the presence or absence of an event, a pattern or a signal, merged in a background noise. In spectrum sensing, it is the presence or absence of the PU's signal which needs to be detected, by measuring the activity in a specific frequency band. To this end, a decision statistic is computed with the measured data and compared to a detection threshold. A detector is thus composed of both a decision statistic and a detection threshold. A good spectrum sensing algorithm should compute the optimal detector that can accurately detect the presence or absence of any PU.

There are generally two paradigms in terms of optimality:

- the Bayesian paradigm, in which the detector has an *a priori* knowledge about the environment, and
- the non-Bayesian paradigm, in which there is no existing *a priori* knowledge.

In practice, *a priori* knowledge is hard to obtain, but if it is available, it can result in better detection performance. One challenge of spectrum sensing is to have a good trade-off between the amount of required *a priori* knowledge and the performance.

In spectrum sensing, the observation time (or sensing time) before the detection of the PU's activity is an important parameter. On the one hand, it affects the detection accuracy, in that the longer it is, the more accurate the detection will be. But on the other hand, it should not be too long, in order to allow the full use of spectrum opportunities. A detector that gives a good balance between the sensing time and the sensing accuracy is thus desirable.

In the same line, considering the way that the observed signal is processed, there are two families of detectors:

- the block-based detectors which process the signal in block, and
- the sequential detectors which process the signal sample per sample.

Sequential detectors can achieve a lower sensing time than block-based detectors, especially in situations where the SU should quickly detect the PU's activity. But block-based detectors ensure a good detection accuracy. It is important to choose the best approach for spectrum sensing, depending on the CR scenario.

Besides the sensing time, another important aspect is the frequency at which spectrum sensing is performed by the SU in order to vacate the channel when the PU restarts its activity. If spectrum sensing is performed frequently enough, the return of the PU will be quickly detected, but the spectrum might not be fully used due to the time allocated to sensing. In contrast, if it is performed less frequently, the PU's return might be missed and this will affect the performance of the licensed system, due to interference. This parameter highly depends on the PU activity statistics and other environment characteristics, such as the mobility of the SU.

Finally, when a parameter of the signal to detect is not known, there are basically two methods from the detection theory, to deal with that, i.e.,

- the *generalized likelihood ratio test* (GLRT), and
- the mixture-based test.

This is typical in spectrum sensing that the power of the signal to detect is not known. These methods are thus very useful in practical CR scenarios, but they come with additional complexity.

1.2 Objectives and contributions

The main objective of this thesis is to design robust spectrum sensing algorithms for mobile cognitive radio systems. Two main research directions are explored.

- Firstly, considering the Bayesian paradigm, we investigate the possibility of using *a priori* knowledge of the environment characteristics to derive new spectrum sensing algorithms, and we compare their

performance with non-Bayesian algorithms that do not use this *a priori* knowledge. More specifically, in a first scenario, we consider mobility information as *a priori* knowledge and use the sequential detection theory, whereas in the second scenario, we consider geolocation information as *a priori* knowledge and use the block-based detection theory. Moreover, both cases deal with situations where the PU signal power is unknown, by using the GLRT in the first case, and the mixture-based test in the second.

- Secondly, we investigate the question of joint optimization of sensing time and sensing frequency in a mobile cognitive radio scenario, depending on the environment parameters. This is done by using the sequential detection theory.

Some of the research questions we try to answer throughout this thesis are the following:

- Q.1 Is it possible to use environment parameters such as speed, mobility patterns and geolocation information, known by the SU, to take better and/or fast decisions about the spectrum occupancy? How could these parameters be combined with the sensed data, and what would be the resulting performance gain in different scenarios?
- Q.2 How to derive optimal spectrum sensing algorithms and provide simple analytical expressions, when the PU signal power, or equivalently the SNR, is not known? What is the induced loss in performance with respect to situations where the SNR is known? What about the resulting complexity? When some *a priori* knowledge about the SNR is available, is there a trade-off between complexity, performance and accuracy of *a priori* knowledge?
- Q.3 In mobile CR scenarios, what is the most suitable approach for spectrum sensing, between block-based detection and sequential detection? How often should the SU perform spectrum sensing in order to be aware of a situation change? In a joint transmission and sensing system, how to choose the various system parameters (transmission time, sensing time, detection threshold) in order to maximize the spectrum usage?
- Q.4 How could we achieve the best trade-off between interference and spectrum waste in order to maximize the overall spectrum usage, in

a time-limited communication? What is the optimal threshold selection criteria to achieve the maximum spectrum usage? How to choose the optimal threshold in unknown-SNR situations?

Taking these research questions into account, the main contributions of this thesis are listed below.

- C.1 Considering a mobile CR environment, a new spectrum sensing algorithm, called LC GLR-Shiryaev, is proposed to detect a change in the spectrum occupancy when the SU is moving. It exploits the knowledge of the SU's mobility parameters, by using the Bayesian paradigm of sequential changepoint detection. Moreover, LC GLR-Shiryaev is derived from the optimal algorithm in this paradigm, named Shiryaev algorithm, by using the GLRT approach when the PU signal power is unknown to the SU. Simulation results show that LC GLR-Shiryaev is a good choice of algorithm when the SNR is unknown and the SU could be required to detect very low SNR signals. This contribution addresses the research questions Q.1 and Q.2, and it is detailed in Chapter 3.
- C.2 Considering the scenario in contribution C.1, a new performance metric, called the global penalty, is introduced to jointly minimize the interference and spectrum waste durations, thereby maximizing the overall spectrum usage, in a time-limited communication. Moreover, based on the global penalty metric, different methods for setting the optimal threshold in known-SNR and unknown-SNR situations, are suggested. The research question Q.4 is addressed by this contribution, which is presented in Chapter 3.
- C.3 The complexity of LC GLR-Shiryaev is compared with that of a numerical resolution of the GLRT-based Shiryaev algorithm, which assumes some *a priori* knowledge of the SNR. It is shown that the numerical approach has a lower complexity than LC GLR-Shiryaev, but it requires that the *a priori* knowledge be accurate, otherwise its performance worsens. This contribution addresses the research question Q.2, and it is elaborated in Chapter 3.
- C.4 A new joint transmission-sensing framework is proposed for use in mobile CR scenarios, where there may be multiple changes in the spectrum occupancy, depending on the mobility of the SU. The framework uses a changepoint detection algorithm to perform spectrum

sensing sequentially along several frames. Based on the contribution C.2, a new performance metric, called the global cost, is introduced to evaluate the framework performance, by allowing a trade-off between interference and spectrum waste. Using this metric, the optimal transmission time, sensing time and detection threshold are computed and analysed through simulations. Moreover, intuitive rules for choosing the optimal parameters are suggested. Finally, the changepoint detection is replaced in the framework, with a block-based algorithm, called *energy detection* (ED), and a performance comparison is provided. The results show the superiority of changepoint detection. The research question Q.3 is addressed by this contribution, which is presented in Chapter 4.

- C.5 Considering a CR scenario where the PUs are TV broadcasters and assuming that some information about the transmitters is available in a geolocation database, a new spectrum sensing algorithm is introduced. The proposed algorithm uses the available information, combined with the SU's localization, to perform a mixture-based test, as in practice the received PU signal power is not known. It is based on the Bayesian paradigm of block-based detection, because no fast spectrum change is assumed in the considered scenario, and the detection quickness is not a great concern. Moreover, an approximate closed-form expression is provided using Taylor series approximation, and it works well for medium to high average SNR values. The geolocation information is also used to compute the prior probabilities of presence or absence of PUs' activities. Finally, the derived algorithm is compared with ED, that does not use the geolocation information, and it gives the best performance in terms of minimum error probability. This contribution focuses on the research questions Q.1, Q.2, and Q.4, and it is presented in Chapter 5.

1.3 Outline of the thesis

The doctoral dissertation is structured as described below.

Chapter 2 provides a general literature review on existing spectrum sensing techniques. Different methods for detecting primary users activ-

ities in cognitive radio, i.e., geolocation database, beacon-based sensing and spectrum sensing, are first presented. Then, the signal detection theory, which is the basis for spectrum sensing, is introduced, including its various approaches (block-based, sequential, hypothesis testing, changepoint detection) and sub-approaches (Bayesian, non-Bayesian). The existing approaches (GLRT, mixture-based test, etc.) to deal with an unknown parameter (e.g., PU signal or noise power) are also reviewed. Then, some challenges related to signal detection in a CR context (fading, low SNR, sensing time, sensing frequency, detection time, lack of prior knowledge on the PU signal) are highlighted. Moreover, some classical spectrum sensing algorithms are presented and compared in terms of detection performance, sensing time, complexity, and amount of required *a priori* knowledge of the PU. Finally, some use cases scenarios of the spectrum sensing methods developed in the following chapters are underlined.

In **Chapter 3**, the new LC GLR-Shiryaev algorithm, proposed for spectrum sensing in mobile CR is introduced. It is based on the Bayesian changepoint detection approach, reviewed in Chapter 2. First, the mobile CR scenario is presented, and it is postulated that the *a priori* knowledge of the spectrum change could be inferred from the mobility parameters of the SU. Then, applying the GLRT to the optimal Shiryaev algorithm, the new algorithm is derived for practical situations where the PU signal power is unknown. It is compared with many other algorithms of the literature by means of simulation and using the newly proposed global penalty metric. Finally, some optimal threshold setting methods (adaptive and fixed) are suggested when the SNR is unknown, and the computational complexity of the proposed algorithm is discussed.

Chapter 4 introduces the new framework for joint transmission and sensing in mobile CR. The mobile CR scenario of Chapter 3 is reused, but it is stressed that multiple changes in the spectrum occupancy are now considered. Then, the framework and the spectrum sensing method, based on changepoint detection, are presented. Thereafter, the global cost, that serves as a performance metric for optimizing the system, is introduced. The optimal sensing time, transmission time and detection threshold, that minimize the global cost, are computed and analysed using numerical simulations. Furthermore, a discussion is given about the mutual dependence between the optimal parameters. Finally, a performance comparison is given, when energy detection is used for spectrum sensing instead of changepoint detection.

In **Chapter 5**, the Bayesian spectrum sensing algorithm, using *a priori* geolocation information and based on the block-based detection theory, is presented. Then, an approximate closed-form expression is derived. In addition, the prior probabilities of absence and presence are computed based on geolocation information. The proposed algorithm is finally compared with ED algorithm, which does not use the geolocation information.

Lastly, **Chapter 6** gives a general conclusion to the thesis and highlights directions for possible future research.

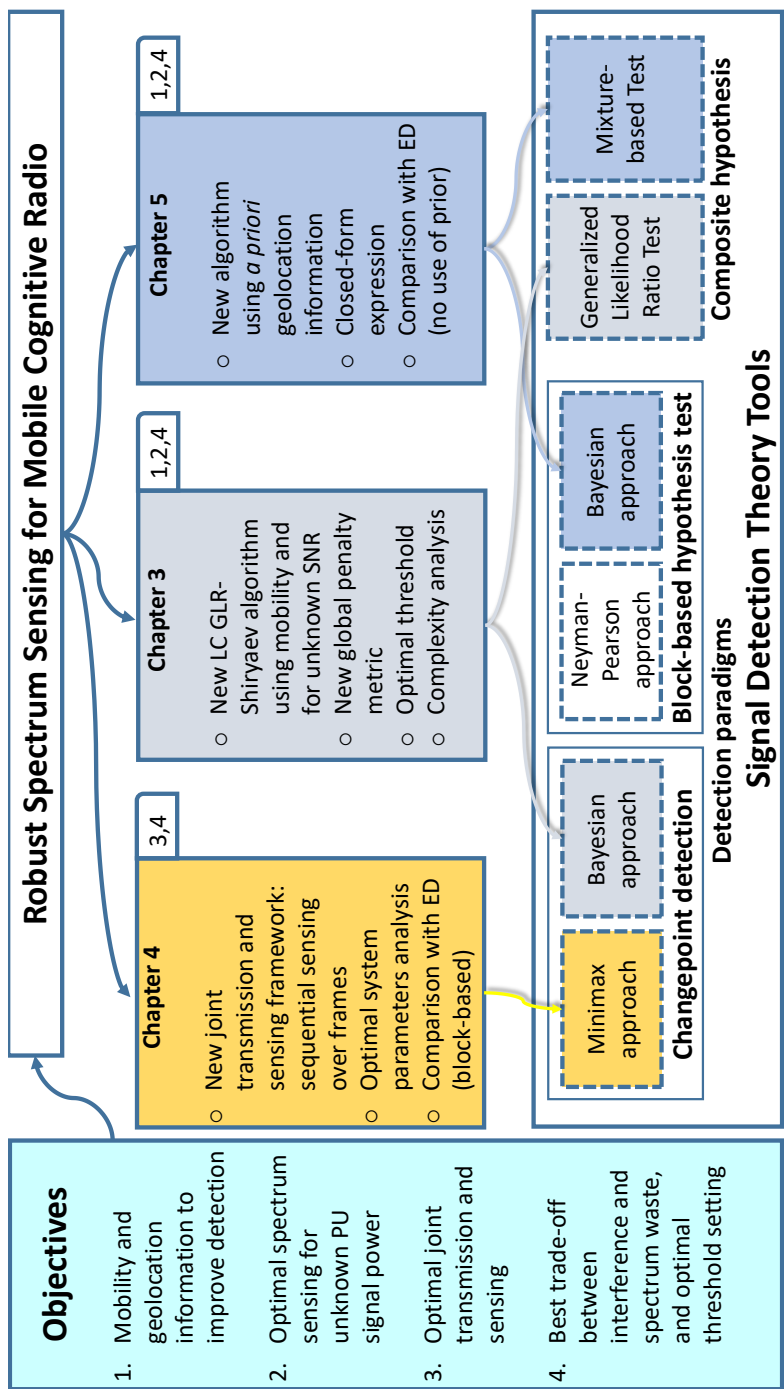


Fig. 1.2 Thesis structure.

1.4 List of publications

Journal papers

- **A. S. V. Chede**, M. Dossou and J. Louveaux, "Geolocation-based Bayesian Spectrum Sensing for Cognitive Radio," Letter under preparation. (**Chapter 5**)

Conference papers

- **A. S. V. Chede**, F. Rottenberg, M. Dossou and J. Louveaux, "Use of Bayesian Changepoint Detection for Spectrum Sensing in Mobile Cognitive Radio," in *IEEE 93rd Vehicular Technology Conference*, Helsinki, Finland, Apr. 2021. (**Chapter 3**)
- **A. S. V. Chede**, F. Rottenberg, M. Dossou and J. Louveaux, "Spectrum Sensing in Mobile Cognitive Radio: A Bayesian Changepoint Detection Approach," in *41st WIC Symposium on Information Theory and Signal Processing in the Benelux*, Online event, May 2021. (**Chapter 3**)
- **A. S. V. Chede**, M. Dossou and J. Louveaux, "Joint Transmission-Sensing Framework Using Changepoint Detection," in *4th International Conference on Advanced Communication Technologies and Networking*, Rabat, Morocco, Dec. 2021. (**Chapter 4**)

2

State of the art

SINCE the introduction of the idea of cognitive radio, the question of how to detect existing primary users activities, or equivalently existing spectrum opportunities, has been the subject of abundant research. Several approaches have been proposed by researchers from academia and industry, each with its advantages, drawbacks and challenges, with the ultimate goal of maximizing the spectrum usage while avoiding deteriorating the performance of exiting PUs. In this chapter, we provide an overview of the various techniques for detecting spectrum opportunities in interweave CR. We further focus on spectrum sensing by introducing and comparing the different approaches of signal detection theory that constitute the basis for spectrum sensing algorithms. Particular attention is given to the specificities of signal detection in the context of spectrum sensing for CR. Then, some major spectrum sensing algorithms are presented and compared. Finally, we present some TVWS parameters and highlight potential applications of the spectrum sensing algorithms proposed in this thesis.

2.1 Different methods for PUs detection

The coexistence of unlicensed SUs and licensed PUs can be realized only at the condition that the SUs do not disturb the primary communication systems. Hence, the SUs need to have an accurate overview of the available spectrum opportunities. In opportunistic spectrum access, PUs activities can be detected in various ways including the *geolocation database method*, the *beacon-based sensing* or *out-of-band sensing*, and the *local spectrum sensing* or *in-band sensing*. As explained in Chapter 1, in the local spectrum sensing method, the decision about the spectrum occupancy is taken by the SU, which directly senses the spectrum band of interest. In the geolocation database method, this decision is taken by an external entity, which is the geolocation database. In the beacon-based sensing, the decision is also external to the SU. It is actually made by the PU transmitter or receiver, which sends the spectrum occupancy information to the SU via a control channel. This is more detailed below.

2.1.1 Geolocation database method

Assuming that all the PUs transmitting in a certain geographical area are perfectly known as well as their transmission parameters and locations, it is possible to estimate the spectrum occupancy in this area. This information can then be used by every SU willing to opportunistically access the spectrum. This is the principle of the geolocation database method.

A geolocation database is a central node, administered by a regulator, that is aware of the spectrum occupancy in a specific region. More precisely, it contains up-to-date information regarding the existing PU transmitters in a certain area. This data includes location, carrier frequency, bandwidth, transmit power, transmission times, antenna parameters (radiation pattern, height), protection requirements, and other parameters [41]. The database inputs this information into a channel model, to estimate the coverage area of the transmitters. With this, a protection contour is defined for each PU, interference-free areas are determined and the spectrum occupancy at each point of the considered geographical area is obtained. A SU that wants to use the spectrum should first retrieve its position through a *global navigation satellite system* (GNSS) and then send it to the database.

The database replies with a list of allowed frequency bands, the associated transmit powers and the allowed duration of operation, depending on the location of the SU. Then, the SU chooses a channel and notifies the database with its transmission parameters, so that the availability of channels is updated. Note that this method needs a cooperation between the SU and the database, on the one hand, and a dynamic update of the database with the PUs broadcasting parameters, on the other hand. This requires an extra communication link, which is typically realized over the Internet. An example of PU detection based on the geolocation database method is shown in Fig. 2.1.

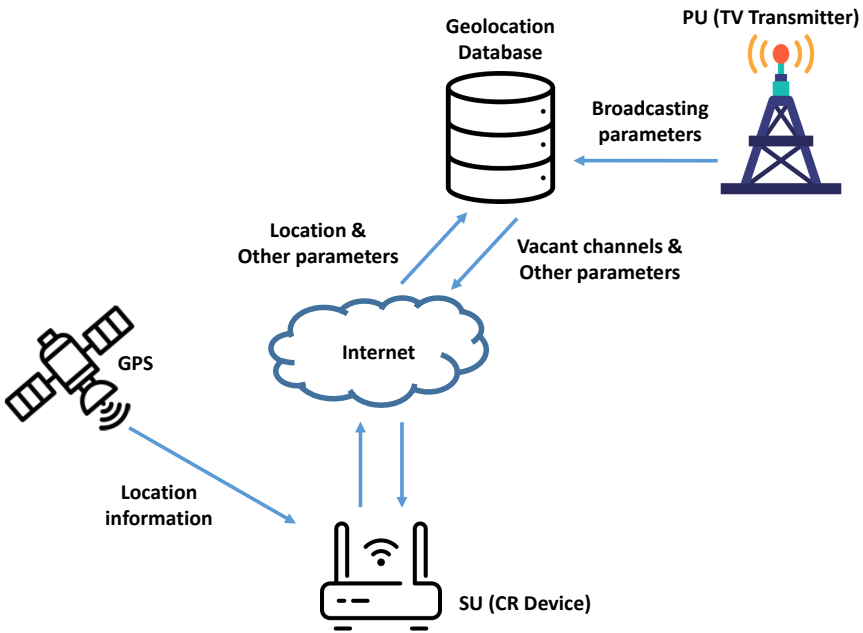


Fig. 2.1 Geolocation database method (adapted from [41])

The detection of PUs activities based on geolocation database depends on several factors. First, this method is not suitable for dynamic primary systems where the network (the locations and activities of the PU transmitters) is quickly changing. It is more appropriate for static primary systems such as TV broadcasting, where all the transmitters are fixed and their operating parameters can be known *a priori* [15]. Actually, this method has

been created to allow the opportunistic use of TVWS by CR. Second, a reliable telecommunication infrastructure is needed to regularly transmit updates between the PUs and the database on the one hand, and between the database and the SUs on the other hand. Third, the database should be regularly updated in order to keep ensuring accurate decisions. The increase of the number of PUs and SUs that should communicate with the database may increase the latency and the implementation complexity [34, 15]. The fourth important factor is the accuracy of the propagation model. It undoubtedly impacts the outcome and thereby the performance of the geolocation-based approach. The requirements for a reliable telecommunication infrastructure, an up-to-date database and an accurate channel model induce a high implementation, maintenance and administrative cost for the geolocation-based spectrum sensing [41]. While these constraints might be easily tackled in some countries, it is not necessarily the case in countries where the Internet backbone infrastructure is unreliable and accurate propagation models does not exist [41]. Finally, the geolocation database method requires the SU to have a geolocation capability with good positioning accuracy. For example, the accuracy recommended by the United States FCC and the United Kingdom OFCOM are respectively 50 m and 100 m [42, 43, 44]. This specification is usually met by existing *Global Positioning System* (GPS) devices [45, 44], and is not a big challenge for the geolocation-based approach.

2.1.2 Beacon-based sensing

In beacon-based or out-of-band sensing, an out-of-band control channel is dedicated for the detection of spectral opportunities. The sensing is actually done by the SU out of the band of interest. PU transmitters or receivers periodically send beacon signals on the control channel, to indicate the presence or absence of PU systems [15]. Therefore, in this method, the SU does not sense the channel of interest, but it senses another channel to know the availability of the desired channel. Besides the spectrum occupancy, other advanced features such as channel quality can be transmitted to the SU [46]. This method can also be used to distinguish different types of PU systems by employing different time slots or orthogonal codes [15].

The main shortcoming of beacon-based sensing is the use of additional spectral resources and power for the transmission of beacon signals. Furthermore, multipath fading and shadowing can cause strong attenuation

to the beacon signals, yielding incorrect detections.

2.1.3 Local spectrum sensing

In local spectrum sensing or in-band sensing, the detection of PUs activities is directly performed by the SUs through measurements in the band of interest. In the literature, local spectrum sensing is often simply referred to as spectrum sensing because, unlike the other methods, it allows the SUs to directly sense the desired spectrum. As the SUs work like detectors, they can just detect *what they see*. For this reason, it is not possible to detect primary receivers, which are passive devices, but only primary transmitters can be detected through their transmissions [1, 15]. This situation can lead to a critical issue depicted in Fig. 2.2. Actually, the SU may have a deep fading channel with the PU transmitter, while having a good channel with the PU receiver. In this situation, it would detect a free channel and start transmitting, thereby causing interference to the PU receivers. This issue is called the *hidden node* problem and cooperative spectrum sensing, performed by several SUs, is seen as a solution for solving it [47, 15, 46].

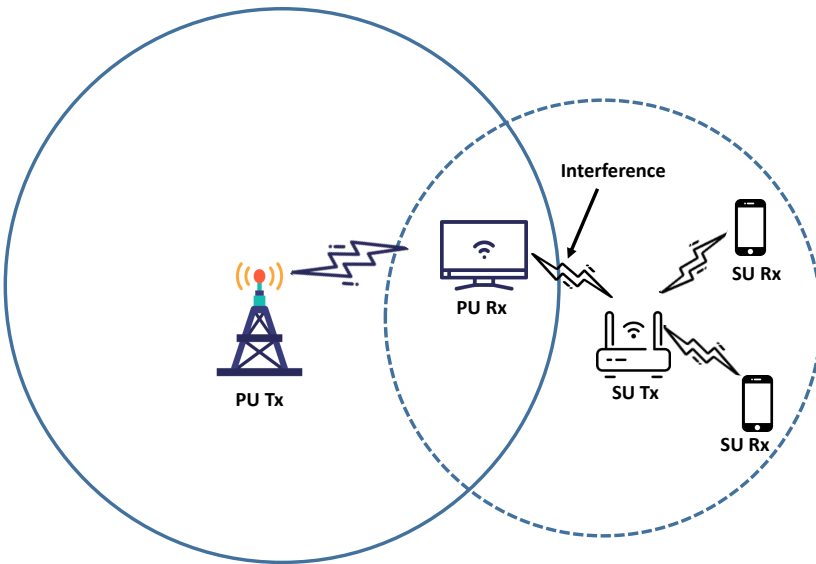


Fig. 2.2 Hidden node problem (adapted from [46]).

Although it has many technical challenges, local spectrum sensing is easier to implement and administer than geolocation-based detection. Moreover, it does not require additional spectrum as beacon-based sensing. It allows SUs to find local spectral opportunities that may not be found by the other methods. For these reasons, we only look into this technique throughout this thesis. In the rest of the document, spectrum sensing is used to designate local spectrum sensing. As spectrum sensing has its roots in the signal detection theory, we introduce the basics of detection theory in the next section.

2.2 Signal detection theory

Signal detection theory is the domain of signal processing that consists in detecting the presence of an event or a pattern merged in noise, based on an observed signal [48]. Two different approaches of detection theory can be distinguished: the *classical detection* approach also called *hypothesis testing* and the *sequential changepoint detection* approach.

In the hypothesis testing, the observed data belongs to a particular distribution among two or multiple hypotheses. The objective is then to distinguish the true hypothesis or, in other words, to find the identity of the true distribution, based on observation [49]. Furthermore, there are two types of hypothesis testing.

- The block-based hypothesis testing processes data in block, i.e., the true hypothesis is decided considering a fixed number of observations, which is the block size. This size is fixed in advance.
- The sequential hypothesis testing processes data sample by sample as they become available, i.e., the total number of observations is not fixed in advance [50].

The changepoint (or quickest) detection approach is slightly different from the hypothesis testing. In this approach, the observed samples are not drawn from the same distribution. It is assumed that, at one moment, there is a change in the distribution of the observed data. The objective of changepoint detection is to detect the change as quickly as possible after its

occurrence [49, 50]. The changepoint detection is thus a sequential process, aiming at achieving a minimum detection delay.

While a block-based processing may be accurate, a sequential processing may take faster decisions about the true hypothesis (hypothesis testing) or the change (changepoint detection). In the next sections, these three detection categories are introduced.

2.2.1 Block-based hypothesis testing

The classical signal detection problem is a hypothesis test, where we want to decide among two possible hypotheses: whether a noisy signal is present or if only noise is present [47]. Assuming that the received signal has been sampled, the hypothesis test is mathematically formulated as

$$\begin{cases} H_0 : & y[n] = w[n], \\ H_1 : & y[n] = x[n] + w[n], \\ & n = 1, 2, \dots, N_s \end{cases} \quad (2.1)$$

where

n is the time sample,

$y[n]$ is the received (observed) signal sample,

$w[n]$ is the noise sample,

$x[n]$ is the sample of the signal to detect,

H_0 means "presence of noise only",

H_1 means "presence of signal and noise" and

N_s is the observation length.

The goal of the detection theory is thus to derive a decision statistic Λ , which is a function of the observed data set ($\mathbf{y} = y[1], \dots, y[N_s]$), and make a decision based on its value [48]. The detection rule is given by

$$\Lambda(\mathbf{y}) \underset{H_1}{\overset{H_0}{\leq}} \alpha \quad (2.2)$$

where α is the detection threshold. It means that if the decision statistic is strictly less than the threshold, then the hypothesis H_0 is decided, otherwise the hypothesis H_1 is decided, and if both are equal, nothing can be decided. A detector is given by a specific expression of $\Lambda(\mathbf{y})$ and a specific

value of α . It is thus defined as

$$\Omega = (\Lambda(\mathbf{y}), \alpha). \quad (2.3)$$

As it can be noticed, this detection rule may result in four different scenarios [47]

- (a) H_1 is decided while H_1 is true ($H_1 | H_1$),
- (b) H_1 is decided while H_0 is true ($H_1 | H_0$),
- (c) H_0 is decided while H_1 is true ($H_0 | H_1$),
- (d) H_0 is decided while H_0 is true ($H_0 | H_0$).

Scenarios (a) and (d) are correct decisions cases, while scenarios (b) and (c) are error cases. Usually in the literature, the error cases are of interest, and they are attributed probabilities which help define the performance metrics of the detector.

- Case (b) is called a *false alarm* and the *probability of false alarm* is defined as $P_{FA} = P(H_1 | H_0)$.
- Case (c) is called a *missed detection* and the *probability of missed detection* is defined as $P_{MD} = P(H_0 | H_1)$.
- As case (a) is complementary to case (c), the *probability of (correct) detection* defined as $P_D = P(H_1 | H_1) = 1 - P_{MD}$ is also used as a performance metric.

A good detector should obtain a low probability of missed detection P_{MD} and a low probability of false alarm P_{FA} . However, both error metrics are related, and a specific detector cannot decrease the value of one metric without increasing the value of the other. Therefore, a good trade-off has to be found between P_{MD} and P_{FA} by choosing an appropriate value of the threshold α . This can be observed in Fig. 2.3. Usually, the function relating the detection probability to the false alarm probability ($P_D = g(P_{FA})$) is called the *receiver operating characteristic* (ROC) and used a performance metric of a detector. By varying the threshold α , an operating point, i.e. a couple (P_D, P_{FA}) , can be chosen along the ROC curve [51]. Similarly, the complementary ROC is defined as $P_{MD} = g'(P_{FA})$. There are various approaches or paradigms for designing an optimal detector, among which the two famous *Neyman-Pearson* (NP) and *Bayesian* approaches [48].

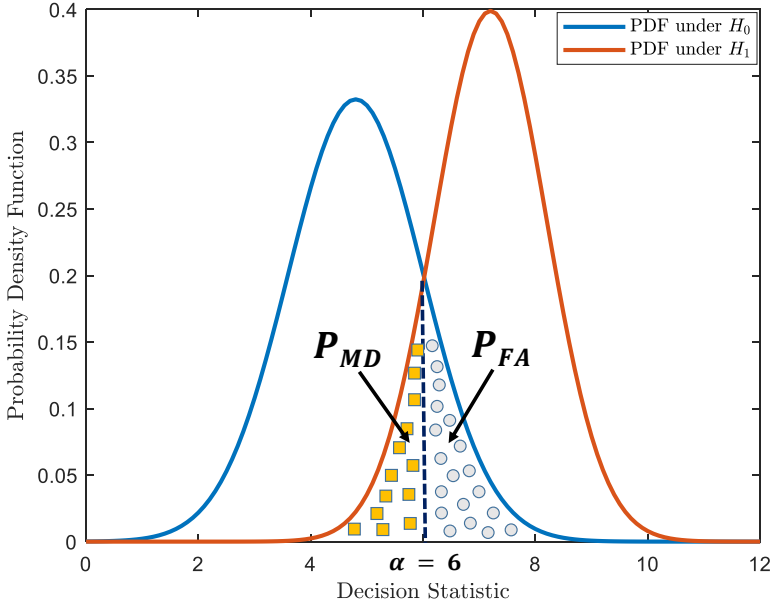


Fig. 2.3 Trade-off between P_{MD} and P_{FA} .

Neyman-Pearson (NP) approach

As both P_{MD} and P_{FA} cannot be simultaneously decreased, a good idea would be to fix one error probability and minimize the other. The NP approach aims to find the detector Ω , that minimizes P_{MD} (or equivalently maximizes P_D) while constraining P_{FA} to a value β

$$\begin{aligned} \arg \min_{\Omega} \quad & P_{MD} = P(H_0 | H_1) \\ \text{s.t.} \quad & P_{FA} = \beta. \end{aligned} \quad (2.4)$$

The optimal detector in the Neyman-Pearson sense is given by [48]

$$\Omega_{opt} = (\Lambda(\mathbf{y}), \alpha) \quad (2.5)$$

where $\Lambda(\mathbf{y}) = \frac{f_1(\mathbf{y})}{f_0(\mathbf{y})}$ and α is derived from $P_{FA} = \beta$.

$f_1(\mathbf{y})$ and $f_0(\mathbf{y})$ are the *probability density functions* (PDFs) of the observed vector $\mathbf{y}$ under the hypotheses H_1 and H_0 . The optimal decision statistic is thus the *likelihood ratio* of the observed data set, and the decision rule is called a *likelihood ratio test*. Note that some variants of the NP detector set the threshold for a fixed value of P_D .

Bayesian approach

In this approach, it is assumed that the *a priori* probabilities of occurrence $P(H_1)$ and $P(H_0)$ of both hypotheses are known. Moreover, a cost is assigned to each of the four possible scenarios. The goal of the Bayesian approach is to minimize the *Bayes risk* defined as [48]

$$\begin{aligned}\mathcal{R} &= \sum_{i=0}^1 \sum_{j=0}^1 C_{ij} P(H_j) P(H_i | H_j) \\ &= C_{00} P(H_0) (1 - P_{FA}) + C_{01} P(H_1) P_{MD} \\ &\quad + C_{10} P(H_0) P_{FA} + C_{11} P(H_1) (1 - P_{MD})\end{aligned}\tag{2.6}$$

where C_{ij} is the cost of declaring H_i while H_j is true.

The optimal detector in the Bayesian sense is given by [48]

$$\Omega_{opt} = (\Lambda(\mathbf{y}), \alpha)\tag{2.7}$$

$$\text{where } \Lambda(\mathbf{y}) = \frac{f_1(\mathbf{y})}{f_0(\mathbf{y})} \text{ and } \alpha = \frac{(C_{10} - C_{00})P(H_0)}{(C_{01} - C_{11})P(H_1)}.$$

The optimal decision statistic is the likelihood ratio of the observed vector, as in the NP approach, but the threshold is set differently. It depends here on the prior probabilities and the costs.

Usually, no cost is assigned to correct decisions and $C_{00} = C_{11} = 0$. Moreover, if the error costs are equal ($C_{01} = C_{10} = 1$), then the Bayes risk boils down to the *probability of error*

$$P_e = P(H_1) P_{MD} + P(H_0) P_{FA},\tag{2.8}$$

and the detection rule is given by

$$\frac{f_1(\mathbf{y})}{f_0(\mathbf{y})} \underset{H_1}{\overset{H_0}{\gtrless}} \frac{P(H_0)}{P(H_1)}.\tag{2.9}$$

Let us denote by $f(\mathbf{y})$, the PDF of the observed data set, regardless of the true hypothesis. Using the Bayes rule

$$P(H_1 | \mathbf{y}) = \frac{f_1(\mathbf{y})P(H_1)}{f(\mathbf{y})},$$

it can be noticed that the minimum P_e detection rule is equivalent to the *maximum a posteriori probability* (MAP) detector

$$P(H_1 | \mathbf{y}) \underset{H_1}{\overset{H_0}{\gtrless}} P(H_0 | \mathbf{y}), \quad (2.10)$$

which, for equal prior probabilities, is equal to the *maximum likelihood* (ML) detector

$$f_1(\mathbf{y}) \underset{H_1}{\overset{H_0}{\gtrless}} f_0(\mathbf{y}). \quad (2.11)$$

Composite hypothesis testing

In some practical scenarios, the PDFs of the signal and noise may have unknown parameters (e.g., signal or noise power) and the likelihood ratio is not fully defined. Assuming that the unknown parameter vectors under H_0 and H_1 are respectively $\boldsymbol{\theta}_0$ and $\boldsymbol{\theta}_1$, the likelihood ratio test is written as follows

$$\Lambda(\mathbf{y} | \boldsymbol{\theta}_0, \boldsymbol{\theta}_1) = \frac{f_1(\mathbf{y} | \boldsymbol{\theta}_1)}{f_0(\mathbf{y} | \boldsymbol{\theta}_0)} \underset{H_1}{\overset{H_0}{\gtrless}} \alpha, \quad (2.12)$$

where $f_1(\mathbf{y} | \boldsymbol{\theta}_1)$ and $f_0(\mathbf{y} | \boldsymbol{\theta}_0)$ are the conditional PDFs of the observed vector. This case is called the *composite hypothesis testing* problem, and two main detection theory tools help deal with that [48, 52].

- The *generalized likelihood ratio test* (GLRT) replace unknown parameters by their ML estimates (MLE) to obtain the decision statistic. The decision rule is thus given by

$$\Lambda_G(\mathbf{y}) = \frac{\sup_{\boldsymbol{\theta}_1} f_1(\mathbf{y} | \boldsymbol{\theta}_1)}{\sup_{\boldsymbol{\theta}_0} f_0(\mathbf{y} | \boldsymbol{\theta}_0)} \underset{H_1}{\overset{H_0}{\gtrless}} \alpha. \quad (2.13)$$

This approach is useful and works quite well, albeit suboptimal [48, 29, 51].

- In the Bayesian approach of composite hypothesis testing (also called the *mixture-based test*), it is assumed that the prior PDFs $f(\theta_0)$ and $f(\theta_1)$ of the unknown parameter vectors are known. The conditional PDFs of the observed data are thus averaged over the unknown vectors. The decision rule is given by

$$\Lambda_B(\mathbf{y}) = \frac{f_1(\mathbf{y})}{f_0(\mathbf{y})} = \frac{\int_{\theta_1} f_1(\mathbf{y} | \theta_1) f(\theta_1) d\theta_1}{\int_{\theta_0} f_0(\mathbf{y} | \theta_0) f(\theta_0) d\theta_0} \underset{H_1}{\overset{H_0}{\leq}} \alpha. \quad (2.14)$$

2.2.2 Sequential hypothesis testing

As stated earlier, unlike the block-based hypothesis testing, the sequential hypothesis testing does not fix the number of observations in advance. The aim of this approach is to identify the true hypothesis with a minimum number of observations [29]. It is based on the fact that as the observations come one by one, the knowledge of the true distribution becomes more precise, and we may decide whether more data are needed to make a final decision or not [50].

Let us assume an independent and identically distributed (i.i.d.) random process $\mathbf{y}$ belonging to a certain distribution whose PDF is either $f_0(\mathbf{y})$ or $f_1(\mathbf{y})$. Always considering the detection of a signal embedded in noise, the sequential hypothesis testing is formulated as

$$\begin{cases} H_0 : & y[n] = w[n], \\ H_1 : & y[n] = x[n] + w[n], \\ & n = 1, 2, \dots \end{cases} \quad (2.15)$$

A decision is thus taken at each time sample, considering the samples observed from the beginning until now.

In sequential tests, there is a stopping time, which is the time when the test takes a final decision about the true hypothesis. It is generally denoted by T , here and in the rest of the document. We want this stopping time to be minimal. The *sequential probability ratio test* (SPRT) developed by Wald [53] is the optimal sequential test that decides between the two hypotheses with the minimum number of observations. Actually it is shown that, for given error probabilities (P_{FA} and P_{MD}), this test minimizes the average sample size under both hypotheses H_0 and H_1 [54]. Denoting the sequence

of observations, from the beginning up to the current time sample m , by $\mathbf{y}_1^m = y[1], \dots, y[m]$, the stopping time of the SPRT is given by [29]

$$T_{\text{SPRT}} = \inf\{m \geq 1 : \Lambda_{\text{SPRT}}(\mathbf{y}_1^m) \leq \alpha_0 \text{ or } \Lambda_{\text{SPRT}}(\mathbf{y}_1^m) \geq \alpha_1\} \quad (2.16)$$

where

$$\Lambda_{\text{SPRT}}(\mathbf{y}_1^m) = \prod_{n=1}^m \frac{f_1(y)}{f_0(y)} \quad (2.17)$$

is the decision statistic and α_0 and α_1 are the lower and upper stopping thresholds. This means that at each time sample, the joint likelihood ratio of all the observations is computed and if $\Lambda_{\text{SPRT}}(\mathbf{y}_1^m) \leq \alpha_0$, H_0 is decided, but if $\Lambda_{\text{SPRT}}(\mathbf{y}_1^m) \geq \alpha_1$, H_1 is decided. If $\alpha_0 < \Lambda_{\text{SPRT}}(\mathbf{y}_1^m) < \alpha_1$, the test continues the observation. The thresholds α_0 and α_1 guarantee that the probabilities of false alarm and missed detection are respectively bounded by [29]

$$P_{\text{FA}} \leq \frac{1}{\alpha_1}$$

and

$$P_{\text{MD}} \leq \alpha_0.$$

As for block-based hypothesis testing, unknown parameters in the PDFs lead to composite hypothesis testing. To solve this issue, Wald suggested the weighted SPRT (WSPRT), which use the same idea as the mixture-based test, and the *generalized sequential likelihood ratio test* (GSLRT), following the GLRT logic [50].

2.2.3 Changepoint detection

As the sequential hypothesis testing, the sequential changepoint detection pertains to the sequential analysis. The difference here lies in the fact that the observed process is not homogeneous. At an unknown time, the distribution changes and the objective is to detect the changepoint (i.e., the time of change) with a minimum detection delay.

Assume that we want to detect a change from a "noise only" state to a "signal plus noise" state. Denoting the changepoint by τ , the model is now

given by [49]

$$\begin{cases} \mathcal{H}_0 : & y[n] = w[n], & n = 1, \dots, m \\ \mathcal{H}_1 : & \exists \tau \in [0, m], \text{ such that} \\ & y[n] = w[n], & n = 1, \dots, \tau \\ & y[n] = x[n] + w[n], & n = \tau + 1, \dots, m \end{cases} \quad (2.18)$$

where we want to distinguish between two hypotheses. The first one, $\mathcal{H}_0$, means that there has not been any change in the received sequence distribution up to now, and the second one, $\mathcal{H}_1$, means there has been a change at time sample τ . Note that this model considers that the first sample of the post-change distribution is $\tau + 1$, as in [50]. As in the sequential hypothesis testing, we use the received sequence, $\mathbf{y}_1^m = y[1], \dots, y[m]$, up to the current time sample m , to detect a possible change in the distribution. There are two classical changepoint detection frameworks in the literature, namely the minimax and Bayesian frameworks.

Minimax framework

The *minimax* approach of changepoint detection assumes that the changepoint is an unknown deterministic variable or a random variable whose distribution is not known [50]. Let us recall that T is the stopping time of the sequential changepoint detection procedure. If $T \leq \tau$, there is a false alarm event and the *mean time to false alarm* is defined as [49]

$$\bar{T}_{\text{FA}} = \mathbb{E}_{\infty}(T), \quad (2.19)$$

where $\mathbb{E}_k$ is the expectation under the assumption that the change happens at time sample k . It is the average number of samples after which a false alarm is made, i.e., the detection procedure stops while there has not been any change. The *conditional average detection delay* is defined as¹ [49]

$$\bar{D}_{\text{D,C}} = \mathbb{E}_{\tau} \{ (T - \tau)^+ \mid \mathbf{y}_1^{\tau} \}. \quad (2.20)$$

It is conditioned on the observed sequence up to the changepoint $\mathbf{y}_1^{\tau}$, and it is a random variable. As the prior distribution of τ is assumed not to be

¹ $x^+ = \max\{0, x\}$.

known, Lorden defined the *worst-case detection delay* as [55]

$$\bar{D}_{D,WC} = \sup_{\tau \geq 0} \text{ess sup} \mathbb{E}_{\tau} \{ (T - \tau)^+ \mid \mathbf{y}_1^{\tau} \}. \quad (2.21)$$

The essential supremum (ess sup) of a random variable W is the smallest number s such that $P(W \leq s) = 1$. Therefore, the metric $\bar{D}_{D,WC}$ chooses the changepoint value that gives the largest possible average detection delay under the worst realization of $\mathbf{y}_1^{\tau}$ [56]. The goal of the minimax approach is to look for the algorithm or procedure Ω that minimizes the worst-case detection delay $\bar{D}_{D,WC}$, subject to a lower bound T_{β} on the mean time to false alarm $\bar{T}_{FA}$. The optimization problem is stated as follows

$$\begin{aligned} \arg \min_{\Omega} \quad & \bar{D}_{D,WC} = \sup_{\tau \geq 0} \text{ess sup} \mathbb{E}_{\tau} \{ (T - \tau)^+ \mid \mathbf{y}_1^{\tau} \} \\ \text{s.t.} \quad & \bar{T}_{FA} \geq T_{\beta}. \end{aligned} \quad (2.22)$$

Cumulative Sum (CUSUM)

Let us assume that the observed process is i.i.d. before the change and after the change (i.i.d. model). The CUSUM algorithm, originally introduced by Page [57], is shown by Lorden [55] and Moustakides [58] to be optimal in the minimax framework. At each time sample $m = n$, CUSUM compares the decision statistic $\Lambda(\mathbf{y}_1^m)$ with the threshold α and an alarm is raised as soon as $\Lambda(\mathbf{y}_1^m)$ exceeds α . The stopping time of CUSUM algorithm is given by [49]

$$T_{CUS} = \inf \{ m \geq 1 : \Lambda_{CUS}(\mathbf{y}_1^m) \geq \alpha \}, \quad (2.23)$$

with²

$$\Lambda_{CUS}(\mathbf{y}_1^m) = \max_{k \leq m} \sum_{n=k}^m \ln(L_n), \quad (2.24)$$

where $L_n = \frac{f_1(\mathbf{y}[n])}{f_0(\mathbf{y}[n])}$ is the likelihood ratio of the n^{th} observation. A recursive version [49] of the CUSUM decision statistic is given by

$$\Lambda_{CUS}(\mathbf{y}_1^m) = \left(\Lambda_{CUS}(\mathbf{y}_1^{m-1}) + \ln(L_m) \right)^+, m \geq 1, \quad (2.25)$$

with $\Lambda_{CUS}(\mathbf{y}_1^0) = \Lambda_{CUS}(\mathbf{y}[0]) = 0$.

² k is the time sample from which, $\ln(L_n)$ has consistently shown positive values, meaning that a change has probably occurred [49].

GLR-CUSUM

When a parameter θ of the likelihood ratio is not known, the GLRT can be applied to the CUSUM algorithm. The new decision statistic is found by replacing the unknown parameter by its MLE, giving

$$\Lambda_{\text{GLR-CUS}}(\mathbf{y}_1^m) = \sup_{\theta} \max_{k \leq m} \sum_{n=k}^m \ln(L_{n|\theta}), \quad (2.26)$$

where $L_{n|\theta}$ is the likelihood ratio conditioned on the unknown parameter θ . Note that the GLR-CUSUM algorithm cannot be written in a recursive manner because, at each time sample m , the estimated value of θ needs to be recomputed considering all the observed data. For this reason, the complexity of GLR-CUSUM grows with the number of available samples.

Bayesian framework

In the alternative *Bayesian* approach, it is assumed that the changepoint τ is a random variable with a known prior distribution. The false alarm risk is measured by the *weighted probability of false alarm*³ [50]

$$\begin{aligned} P_{\text{FA}} &= P(T \leq \tau) \\ &= \sum_{k=1}^{\infty} P(\tau = k)P(T \leq k), \end{aligned} \quad (2.27)$$

where $P(\tau = k)$ is the PDF of τ .

As τ is random, the detection delay $T - \tau$ is random, and the *average detection delay* is defined as

$$\overline{D}_D = \mathbb{E}(T - \tau \mid T > \tau) = \frac{\mathbb{E}\{(T - \tau)^+\}}{P(T > \tau)}. \quad (2.28)$$

The Bayesian approach looks for the optimal algorithm that minimizes the average detection delay $\overline{D}_D$ under a constraint on the false alarm probability P_{FA} , i.e. the optimal procedure Ω which is the solution of the following

³Note that k starts from 1 because $T \geq 1$ so that $P(T \geq 1) = 1$.

optimization problem

$$\begin{aligned} \arg \min_{\Omega} \quad & \bar{D}_D = \frac{\mathbb{E}\{(T-\tau)^+\}}{P(T>\tau)} \\ \text{s.t.} \quad & P_{FA} \leq \beta, \beta \in]0, 1[\end{aligned} \quad (2.29)$$

Shiryaev

Assuming an i.i.d. model and a geometric prior distribution for τ , Shiryaev solved this problem [59, 60, 50]. He assumed a zero-modified geometric distribution

$$P(\tau = k) = (1 - q)p(1 - p)^k, \quad k \geq 0, \quad (2.30)$$

where $q = P(\tau < 0)$ is the probability that the change has occurred before the first observation and $p \in]0, 1]$ is the probability of change at any given time. The *Shiryaev algorithm* is optimal in the Bayesian framework. Its stopping time is given by [50]

$$T_{\text{Shi}} = \inf\{m \geq 1 : \Lambda_{\text{Shi}}(\mathbf{y}_1^m) \geq \alpha\}, \quad (2.31)$$

with

$$\Lambda_{\text{Shi}}(\mathbf{y}_1^m) = \frac{q}{(1 - q)p} \prod_{n=1}^m \frac{L_n}{1 - p} + \sum_{k=1}^m \prod_{n=k}^m \frac{L_n}{1 - p} \quad (2.32)$$

where α is selected⁴ in such a way that $P_{FA} \leq \beta$. When L_n is independent of the changepoint τ , a recursive version of the Shiryaev algorithm can be written as follows [50, 29]

$$\Lambda_{\text{Shi}}(\mathbf{y}_1^m) = \left(1 + \Lambda_{\text{Shi}}(\mathbf{y}_1^{m-1})\right) \frac{L_m}{1 - p}, \quad m \geq 1, \quad (2.33)$$

with $\Lambda_{\text{Shi}}(\mathbf{y}_1^0) = \Lambda_{\text{Shi}}(y[0]) = \frac{q}{(1 - q)p}$.

GLR-Shiryaev

Once again, when some parameters of the distributions are unknown, the Shiryaev algorithm can be extended either using the GLRT approach or the mixture-based test approach. It is worth noting that these two approaches

⁴Authors of [50] showed that a proper way to have $P_{FA} \leq \beta$ is to set $\alpha = \frac{1 - \beta}{p\beta}$.

of composite testing seek to identify a detection procedure that is simultaneously optimal under all possible values of the unknown distributions [61, 62]. In general, it is difficult to find an optimal solution to this problem, and the exact solution can be mathematically intractable. Below, we present other alternatives proposed in the literature to deal with unknown parameters in changepoint detection theory.

The concept of the *least favourable distributions* (LFDs) is introduced in [61], when the pre-change and post-change distributions are not known exactly but belong to known uncertainty classes of distributions. Unlike the GLRT, the objective of this approach is to minimize the maximum detection delay over all possible distributions, while ensuring that the false alarm constraint is satisfied for all possible pre-change distributions. It is demonstrated that if the LFDs can be identified from the uncertainty classes, the detection rule designed for the LFDs is an optimal solution for the problem. It is also shown that the LFDs-based CUSUM and LFDs-based Shiryaev algorithms are suboptimal when compared to CUSUM and Shiryaev algorithms, which have a perfect knowledge of the distributions. Furthermore, because it can be computed recursively, the LFDs-based CUSUM is shown to be simple and less complex to implement than the GLR-based CUSUM. This work does not present a comparison between the LFDs-based Shiryaev and the GLR-based Shiryaev.

Some authors point out that the GLR-based Shiryaev has disadvantages, among which the challenging task of theoretically finding the false alarm probability [50, 62] and its computational complexity due to the complex optimization problems that should be solved at each time sample [62]. For this reason, another approach is introduced in [63, 62], where a parameter of the post-change distribution is unknown but comes from a known finite set S of M values. To solve this problem, two formulations are taken into account. The first formulation considers that there is no prior distribution of S and thus, the problem consists in finding a stopping time that minimizes the average detection delay (over the changepoint) simultaneously for the M candidate values, subject to a worst-case false alarm constraint. The second formulation assumes that S has a prior distribution and therefore, the problem's goal is to minimize the average detection delay (over both the changepoint and the post-change parameter) subject to an average false alarm constraint. To solve both problem formulations, a new multi-chart algorithm named the *M-Shiryaev* procedure is proposed [63, 62]. The algorithm consists in running M parallel Shiryaev

procedures for each post-change parameter candidate value and declaring a change when any one of the procedures stops. M-Shiryaev can be seen as the numerical resolution of the GLR-based Shiryaev over the discrete set $\mathcal{S}$. A *modified M-Shiryaev* algorithm is also proposed, by replacing the summation operation in M-Shiryaev by the maximum operation [62]. Both algorithms are asymptotically (as the false alarm probability vanishes) optimal for post-change parameters within the discrete set $\mathcal{S}$. Moreover, as the M algorithms are computed in a parallel manner, the recursive form of Shiryaev algorithm can be used. Therefore, unlike the GLR-based Shiryaev whose complexity grows with the number of observed samples, the proposed algorithms have a constant complexity with respect to the number of observed samples. Their complexity is linear with respect to M .

In Chapter 3, we derive a new algorithm by applying an approximation of the GLRT approach to the Shiryaev algorithm, when a parameter of the post-change distribution is unknown. The derived algorithm is used for spectrum sensing in a mobile CR scenario and compared with other changepoint detection algorithms. As it is an approximation of the GLR-based Shiryaev, the proposed algorithm is suboptimal but shows good performance. Furthermore, its complexity is compared with that of M-Shiryaev, and the trade-off between complexity, performance and required *a priori* knowledge of the post-change parameter is discussed.

Comparison of CUSUM and Shiryaev algorithms

CUSUM and Shiryaev algorithms have both been proved to be optimal, respectively in the minimax framework and the Bayesian framework, for i.i.d. observations. Moreover, their asymptotic optimalities have been demonstrated in the general non-i.i.d. case. For example, when considering non-i.i.d. models and non-geometric prior distributions for the changepoint, Shiryaev algorithm is asymptotically optimal, i.e., it minimizes the average detection delay as the false alarm probability tends towards zero [64]. CUSUM is also asymptotically optimal for non-i.i.d. observations, i.e., it minimizes the worst-case detection delay as the mean time to false alarm tends towards infinite [65].

As each algorithm is optimal in its own framework, it is important to take a closer look at how both algorithms perform in the same context, i.e., the Bayesian framework. This has been done by few authors in the

literature [64, 66, 50]. Let us introduce two important metrics used in these texts.

Considering a general prior distribution for the changepoint τ , the exponential rate of convergence of the prior distribution is denoted as [66]

$$d = -\lim_{k \rightarrow \infty} \frac{\ln(P(\tau > k))}{k}, \quad d \geq 0. \quad (2.34)$$

If $d > 0$, then the prior distribution has an (asymptotically) exponential tail, but if $d = 0$, the prior distribution has a heavy tail. For the geometric distribution considered above (2.30), we have $d = -\ln(1 - p)$, when $q = 0$. Therefore, the metric d represents the amount of *a priori* knowledge about the time of change [50].

The second metric is the difference between the pre-change and the post-change distributions of the observation, measured as the Kullback-Leibler (K-L) divergence (also called K-L distance). It is defined for i.i.d. models as

$$D(f_1, f_0) = \mathbb{E}_{f_1}(\ln(L_n)) = \int \ln\left(\frac{f_1(y[n])}{f_0(y[n])}\right) f_1(y) dy. \quad (2.35)$$

A large value of $D(f_1, f_0)$ means that when the distribution of the observation changes, this can be highly noticeable in the observed data. Therefore, $D(f_1, f_0)$ represents the amount of information about the time of change available in the observation.

Clearly, these two metrics have an impact on the performance of the changepoint detection algorithm.

Comparing both algorithms from a theoretical point of view, it is shown in [64] that CUSUM loses its asymptotic optimality in the Bayesian framework, when the changepoint has a prior distribution with exponentially decreasing tail ($d > 0$), but it remains asymptotically optimal for heavy-tailed priors ($d = 0$). Moreover, it is shown that for $d > 0$, if $D(f_1, f_0) \gg d$, the observations contain more information about the change than the prior distribution and the performance of Shiryaev algorithm is determined by $D(f_1, f_0)$. But if $D(f_1, f_0) \ll d$, the performance depends more on d [64]. In addition, authors of [66] showed that Shiryaev algorithm performs much better than CUSUM, in terms of average detection delay versus false alarm probability, when $D(f_1, f_0)$ and d have comparable values. This superiority of Shiryaev algorithm is more pronounced for smaller values of false

alarm probability. In contrast, CUSUM's performance gets closer to that of Shiryaev algorithm, when $D(f_1, f_0) \gg d$.

Applying the changepoint detection to a signal plus noise model, the K-L divergence has been related to the SNR [66, 50]. It is important to compare the performance of CUSUM and Shiryaev algorithms in a cognitive radio context. Moreover, in practical situations where the level of the signal to detect (or equivalently the SNR) is not known in advance, it is interesting to see how the modified versions (GLR-based or mixture-based) of both algorithms perform. This is carried out in our first contribution (Chapter 3), where changepoint detection is used for spectrum sensing in a mobile CR scenario.

2.2.4 Challenges of signal detection in CR context

So far, we have reviewed the basics of signal detection theory, on which spectrum sensing is based. However, when it comes to the CR context, signal detection has its own specificities and meanings for the various parameters. Here we discuss the particular aspects of signal detection in CR.

In the CR context, the signal to detect, x , is obviously the PU signal. The effects (path loss, multipath fading, shadowing) of the wireless channel between the PU and the SU can be included by adding a channel gain $h[n]$ to the previous models. If the channel is stationary during the observation length N_s , the time-dependence of h can be removed. The presence of channel effects brings some constraints on spectrum sensing in CR. Deep fading and severe shadowing may cause the received PU signal to be very weak and difficult to distinguish from noise. This may lead to the previously mentioned hidden node problem. Spectrum sensing could thus be required to detect primary signals with very low SNR, as low as -20 dB, as specified in the IEEE 802.22 *wireless regional area network* (WRAN) standard [67], the first CR standard for the opportunistic use of TVWS.

Besides the SNR, the observation length N_s , called *sensing time* or *sensing period* in CR, is also very important. A long observation time is useful so that the detector can make accurate decisions because the more data is available, the easier it is to distinguish the two hypotheses. However, in the CR context, the channel cannot be sensed for an unlimited period of time otherwise the SU may lose some spectral opportunities. For this reason, the sensing time N_s has to be carefully chosen to give a good trade-off

between the *sensing accuracy* and the *waste or lost of spectral opportunities*. This is especially true in fast-fading channels, where the channel quickly changes with time. The sensing time should hence be shorter than the coherence time, to achieve good performance [29]. The choice of N_s is also particularly critical in low-SNR situations. In fact, it is demonstrated that under noise uncertainty, conventional detectors tend to increase the observation length to infinity, in order to achieve good detection accuracy, when the SNR is below a certain threshold. This is called the *SNR wall* problem [68, 69].

It is worth noting that in CR, the interference caused by the SU on the PU is expressed by the probability of missed detection P_{MD} and the waste of spectral opportunities by the SU is expressed by the probability of false alarm P_{FA} . The acceptable thresholds for P_{MD} and P_{FA} are chosen by country regulators and specified in standards. For example, the IEEE 802.22 WRAN standard requires that spectrum sensing be able to achieve $P_{MD} \leq 0.1$ and $P_{FA} \leq 0.1$ for very low receiver sensitivity (-116 dBm) [70, 71].

A previously vacant channel can be reoccupied by the PU. This involves two aspects that should be carefully looked into. First, the *detection time* should be short enough so that the SU could quickly free up the channel to avoid interfering with the PU. Second, spectrum sensing should be realized periodically in order to be able to detect the PU reappearance. Thus, the *sensing frequency* is also an important metric. The FCC and the IEEE WRAN recommend a detection time of no more than 2 seconds [44, 72]. Regarding the sensing frequency, the FCC requires that sensing be performed every 60 second, while the OFCOM considers a tight sensing frequency of 1 second [44].

The lack of prior knowledge on the PU signal structure is typical of practical CR systems because the SU might want to use a spectrum band where multiple technologies coexist [47]. Hence, spectrum sensing should be able to deal with that. In addition, the PU signal distribution (for random signals) is not usually fully defined because one of its parameters (e.g., signal power) is unknown to the SU. In this situation, optimal detectors should be derived using the previously mentioned GLRT and mixture-based test, depending on the available data.

2.3 Spectrum sensing algorithms

Here, we present some classical spectrum sensing algorithms, which are based on the block-based hypothesis testing theory, namely the energy detection, the matched filtering and the cyclostationary spectrum sensing.

2.3.1 Energy detection

Energy detection or *energy detector* (ED) is the most famous spectrum sensing algorithm because of its simplicity of implementation and low computational complexity. Moreover, it does not require *a priori* knowledge of the PU signal and is therefore suitable for detecting random signals. For the latter reason, it is called a blind detector. To detect the presence of a signal, ED computes the received signal energy for a specific time interval and compares it with a threshold.

It is worth highlighting that ED is the optimal NP detector, for a white Gaussian PU signal embedded in *additive white Gaussian noise* (AWGN) [48, 51]. Let us consider the hypothesis test (2.1), and assume that the PU signal vector $\mathbf{x}$ is modelled as a zero mean white Gaussian random process with variance σ_x^2 and the noise vector $\mathbf{w}$ is an AWGN process, independent of the signal, with variance σ_w^2 . We thus have $\mathbf{x} \sim \mathcal{N}(\mathbf{0}, \sigma_x^2 \mathbf{I})$ and $\mathbf{w} \sim \mathcal{N}(\mathbf{0}, \sigma_w^2 \mathbf{I})$, where $\mathbf{I}$ is the identity matrix. We recall that $\mathbf{y} = y[1], \dots, y[N_s]$ is the vector of observed data. The optimal decision rule in the NP sense (2.5) is the likelihood ratio test, given here by

$$\frac{\frac{1}{(2\pi(\sigma_x^2 + \sigma_w^2))^{N_s/2}} \exp\left(-\frac{\sum_{n=0}^{N_s-1} y^2[n]}{2(\sigma_x^2 + \sigma_w^2)}\right)}{\frac{1}{(2\pi\sigma_w^2)^{N_s/2}} \exp\left(-\frac{\sum_{n=0}^{N_s-1} y^2[n]}{2\sigma_w^2}\right)} \underset{H_1}{\overset{H_0}{\leq}} \alpha.$$

Applying a log function to both terms, we obtain

$$\frac{N_s}{2} \ln\left(\frac{\sigma_w^2}{\sigma_x^2 + \sigma_w^2}\right) + \frac{\sigma_x^2}{2\sigma_w^2(\sigma_x^2 + \sigma_w^2)} \sum_{n=0}^{N_s-1} y^2[n] \underset{H_1}{\overset{H_0}{\leq}} \ln(\alpha).$$

The decision rule is finally formulated as

$$\Lambda_{ED}(\mathbf{y}) = \sum_{n=0}^{N_s-1} y^2[n] \underset{H_1}{\overset{H_0}{\leq}} \alpha', \quad (2.36)$$

where α' is found from the P_{FA} constraint. Note that in the literature, some authors use the power as decision statistic, instead of the energy.

The detection probability P_D and the false alarm probability P_{FA} of ED are well documented [48, 73] and follow from the fact that the decision statistic is the sum of the squares of N_s Gaussian variables, which hence follows a chi-square distribution with N_s degrees of freedom ($\chi_{N_s}^2$). From [48], we have

$$\begin{aligned} P_{FA} &= P(H_1 | H_0) \\ &= P(\Lambda(\mathbf{y}) > \alpha' | H_0) \\ &= Q_{\chi_{N_s}^2} \left(\frac{\alpha'}{\sigma_w^2} \right) \end{aligned} \quad (2.37)$$

and

$$\begin{aligned} P_D &= P(H_1 | H_1) \\ &= P(\Lambda(\mathbf{y}) > \alpha' | H_1) \\ &= Q_{\chi_{N_s}^2} \left(\frac{\alpha'}{\sigma_x^2 + \sigma_w^2} \right), \end{aligned} \quad (2.38)$$

where $Q_{\chi_{N_s}^2}(u) = \int_u^\infty f(t)dt$ is the right-tail probability of a $\chi_{N_s}^2$ variable, the PDF⁵ being $f(u) = \frac{1}{2^{N_s/2}\Gamma(N_s/2)} u^{N_s/2-1} \exp(-u/2), u \geq 0$. For large values of N_s , the chi-square distribution can be approached by a Gaussian distribution, thanks to the central limit theorem, which is useful to simplify the performance metrics derivations but obviously does not give accurate results. Remark that the false alarm probability P_{FA} depends only on the noise power σ_w^2 , while the detection probability P_D depends on both the noise power σ_w^2 and the signal power σ_x^2 . As the threshold α' is found from the constraint on P_{FA} , ED does not require knowing the signal power σ_x^2 . This is where its name of blind detector comes from.

While ED is a simple algorithm, it has some drawbacks. One of them is its bad performance at low SNR. It can be seen from (2.38) that

$$P_D = Q_{\chi_{N_s}^2} \left(\frac{\alpha' / \sigma_w^2}{\sigma_x^2 / \sigma_w^2 + 1} \right)$$

and that when $\text{SNR} = \sigma_x^2 / \sigma_w^2$ decreases, the argument of the right-tail probability increases and hence, P_D decreases. Moreover, the threshold α' is cho-

⁵The chi-square PDF is defined assuming that u is the sum of the squares of N_s real Gaussian variables $\sim \mathcal{N}(0, 1)$.

sen assuming perfect knowledge of noise power, but in practical scenarios this knowledge might not exist. If there is uncertainty in the noise power, the performance will decrease, giving rise to the SNR wall phenomenon, mentioned in Section 2.2.4.

2.3.2 Matched filtering

In certain scenarios, we may have some knowledge of the PU signal structure and this can be exploited to achieve better detection performance. This prior knowledge of the PU signal consists in features such as modulation format, pulse shapes, phase, data rate, statistical properties, or others [29, 15]. Assuming the limiting case where the PU signal is fully known, its presence can be detected by coherently demodulating the received signal. This is the *matched filtering* (MF) algorithm, also called *coherent sensing*. This spectrum sensing algorithm requires the PU to send preambles or periodic pilots in order to allow timing and carrier synchronization [29, 15, 74, 46]. A less strong assumption is to consider that PU signal is not known, but its pilot signal is deterministic and known by the SU. The MF thus focuses on the pilot detection to detect the presence of the PU [68].

The MF is derived using the NP approach and assuming a deterministic PU signal. Its decision rule is given by [48]

$$\Lambda_{MF}(\mathbf{y}) = \sum_{n=0}^{N_s-1} y[n]x^*[n] \underset{H_1}{\overset{H_0}{\leq}} \alpha', \quad (2.39)$$

where $x^*[n]$ is the complex conjugate of $x[n]$ and α' is found from the P_{FA} constraint.

The MF correlates the known transmit signal x with the received signal y . It is the optimal detector which maximizes the SNR over an AWGN channel. It, therefore, performs well at low SNR and is more robust to the SNR wall problem than ED [47, 68]. Moreover, it can achieve good detection performance with lower sensing time than ED [15, 74].

Coming to practical CR aspects, the MF has some limitations. When the SU wants to use a spectrum band where several distinct PU systems are operating, it should be aware of all the possible pilots and PU transmit signals, and it should provide a dedicated matched filter for each of them. This is impractical, because as the number of different PU systems

increases, the SU receiver complexity also increases [74, 47, 15]. The MF is thus more suitable in spectrum bands where all the PU systems use the same communication technology.

2.3.3 Cyclostationary Feature Detection

Obtaining *a priori* knowledge of the PU signal structure or its pilot signal may be difficult, as we have already seen, especially in environments with different PU systems. Instead, one may wonder if it is possible to use known features, common to most PU signals, to detect their presence. This is what *cyclostationary feature detection* (CFD) does. Due to modulations, carriers, cyclic prefixes, codes, hopping sequences, pilots, and other factors, most communication signals exhibit periodic statistics [47, 15]. More specifically, their mean and autocorrelation function are periodic functions of time, and these signals are said to be *cyclostationary* [29]. At the opposite, noise is a wide-sense stationary process and does not show periodic features. The presence of cyclostationarity can thus help differentiate PU signals from noise. CFD looks for cyclostationarity by analysing the *cyclic autocorrelation function* (CAF) of the received signal. The CAF of the received signal y is defined as [51]

$$R_y(\rho, \tau_0) = \frac{1}{N_s} \sum_{n=0}^{N_s-1} r_y[n, \tau_0] e^{-j2\pi\rho n}, \quad (2.40)$$

where ρ is the cyclic frequency and $r_y[n, \tau_0] = \mathbb{E}(y[n]y^*[n + \tau_0])$ is the autocorrelation function. The *cyclic spectral density* (CSD) of y is defined as the Fourier transform of the CAF and is given by [51]

$$S_y(\rho, f_c) = \sum_{\tau_0} R_y(\rho, \tau_0) e^{-j2\pi f_c \tau_0}. \quad (2.41)$$

The received signal is cyclostationary if a non-zero value of ρ exists such that $R_y(\rho, \tau_0) > 0$. Equivalently, $S_y(\rho, f_c)$ has peaks at cyclic frequencies that are multiples of the fundamental frequency of the received signal [29]. CFD can thus be performed both in the time domain and the frequency domain [51]. This spectrum sensing algorithm works well in low SNR conditions compared to the previous ones, and it is more robust to noise uncertainty. Note that CFD can also be used to distinguish between different types of PU systems that have different periods of cyclostationarity

[46, 29]. The major drawbacks of this algorithm are the high complexity, the requirement for high sampling rate and large amount of samples when the received signal length increases, which results in long sensing time [15, 34].

2.3.4 Comparison of spectrum sensing algorithms

In summary, energy detection is the simplest spectrum sensing algorithm in terms of implementation, with the lowest computational complexity, and it does not require any *a priori* knowledge of the PU signal. Nevertheless, its performance drops at low SNR and under noise uncertainty. The matched filtering is the most accurate algorithm in terms of detecting the presence of a specific PU signal with minimum sensing time and under low SNR conditions. However, it requires a precise knowledge of all the signals to detect or at least their pilot signals, and should be able to perform coherent demodulation for all of them, which does not come without an increase in the complexity. The cyclostationary feature detection might appear to be a good compromise between the performance and the requirements for *a priori* knowledge of the PU signal. It is robust against noise uncertainties and works well at low SNR. But the long sensing time of this algorithm is a real disadvantage. Moreover, it is shown that cyclostationary features may be lost due to severe channel fading, making CFD less efficient in these conditions. This comparison is summarized in Table 2.1 below.

Table 2.1 Comparison of ED, CFD and MF.

Algorithm	Advantages	Drawbacks
ED	- Simple implementation - Low complexity - No need for a priori knowledge	- Bad performance at low SNR - Sensitive to noise uncertainty
CFD	- No need for a priori knowledge (use known features) - Robust against noise uncertainty - Works well at low SNR	- High complexity - High sampling rate - Long sensing time
MF	- Best performance - Good at low SNR - Minimum sensing time	- Requires precise knowledge of signal or pilot - Increased complexity

2.4 TVWS characteristics and some use case scenarios

As mentioned in Chapter 1, TVWS are the first spectrum portions where CR has been used in many parts of the world. In this section, we present some of their characteristics, as they are the bands of interest for the different algorithms designed in this thesis. Furthermore, we present some potential applications of our work.

2.4.1 TVWS characteristics

TVWS bands are comprised between 470 MHz and 790 MHz. Depending on the regulations in each country, some portions of the spectrum have been allocated to licensed services. For example, in Benin, the upper band (703 MHz – 790 MHz) is already allocated to cellular networks. In the remaining portions (470 MHz – 703 MHz), only 4 channels of 8 MHz are allocated to digital TV. The *digital terrestrial television* (DTT) network consists of 29 licensed users (broadcasting sites) grouped into 4 *single frequency networks* (SFNs) distributed over the whole territory, as shown in Table 2.2. Each SFN covers specific geographical areas and all the transmitters within one SFN use a single channel.

Table 2.2 TV bands utilization by DTT in Benin.

Network	No of sites	Channel	Frequency band
SFN1	12	33	(566 – 574) MHz
SFN2	4	24	(494 – 502) MHz
SFN3	8	35	(582 – 590) MHz
SFN4	5	29	(534 – 542) MHz

Considering the 470 MHz – 703 MHz band, which represents around 29 sub-bands of 8 MHz, spectrum sensing can be performed in the whole band in two ways.

- *Narrowband spectrum sensing*: Channels of 8 MHz are sensed one by one. This will need a low sampling rate but will result in a long sensing time for the whole band.
- *Wideband spectrum sensing*: The whole band is sensed at a time in

order to detect the available channels. This will need a high sampling rate but will result in a short sensing time.

Throughout this thesis, we consider the narrowband spectrum sensing approach and always assume looking at a single 8 MHz channel at a time.

2.4.2 Application scenarios

The various spectrum sensing algorithms developed in this thesis are general, and they can be used in different mobile CR scenarios. Here, we introduce some use case scenarios where they are applicable, i.e., vehicular networks and wireless *ad hoc* networks.

Vehicular networks or *vehicular ad hoc networks* (VANETs) have emerged as an innovative technology with promises of new and appealing services that will improve road safety, drivers' comfort and ubiquitous wireless communications [75, 76]. Common applications in vehicular environments can be classified in two categories, namely the safety applications (collision warning, traffic monitoring, ...) and the non-safety applications (Internet access, entertainments, ...). In the United States, the FCC has allocated 75 MHz in the 5.9 GHz *dedicated short range communications* (DSRC) for vehicular communications. However, several studies show that the DSRC band is not enough to cater for the increasing requirements of emerging vehicular services [77]. Moreover, in presence of traffic congestion, the spectrum scarcity problem becomes more serious in this band, and this has a huge impact on time-critical safety applications [78, 77, 79]. For this reason, it has been proposed to opportunistically exploit the TVWS in addition to the DSRC band, in order to increase the available bandwidth to support more services and users. It thus becomes crucial to add cognitive capabilities to vehicular networks. The *cognitive radio for vehicular ad hoc networks* (CR-VANETs) technology constitutes an interesting and a fast-emerging application domain of CR, where the spectrum sensing algorithms proposed in this thesis can be implemented.

In vehicular networks, several topologies can be implemented, namely the *vehicle-to-infrastructure* (V2I) communication, the *vehicle-to-vehicle* (V2V) communication and the *vehicle-to-person* (V2P) communication. The first one is a centralized topology, in which the communication happens between a vehicle and an infrastructure usually called a *roadside unit* (RSU).

In the latter two topologies, the information is directly exchanged between two vehicles or one vehicle and a pedestrian. Based on these architectures, spectrum sensing can be performed in several ways in CR-VANETs.

- Firstly, the spectrum sensing task can be assigned to the RSU, which is also in charge of channel allocation to the vehicles. The result is then sent to the vehicles via a control channel. This is the centralized sensing (Fig. 2.4).

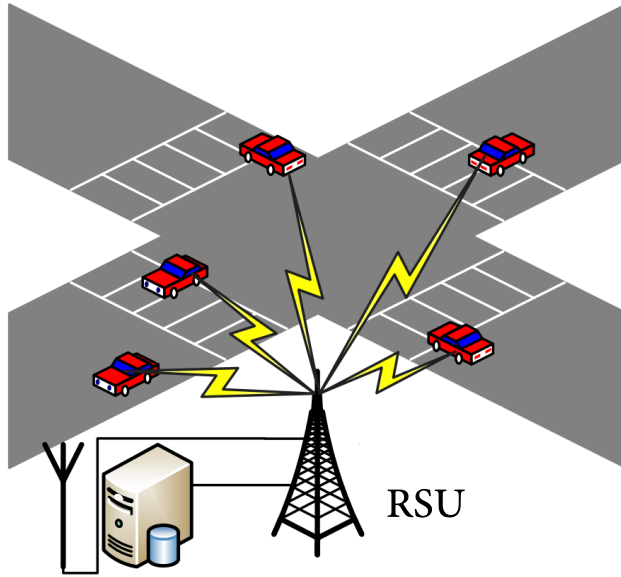


Fig. 2.4 Centralized sensing [80].

- Secondly, the vehicles can perform spectrum sensing themselves and share the results among them via a control channel. This approach is the distributed cooperative sensing (Fig. 2.5).
- Another approach is to perform a centralized cooperative spectrum sensing, where each vehicle performs spectrum sensing and then sends the result to the RSU which aggregates all the results and takes a final decision (Fig. 2.6).

In Chapters 3 and 4 of this thesis, we do not envisage the case where spectrum sensing is done by the RSU, but we consider a mobile CR scenario where spectrum sensing is done by the moving vehicles themselves.

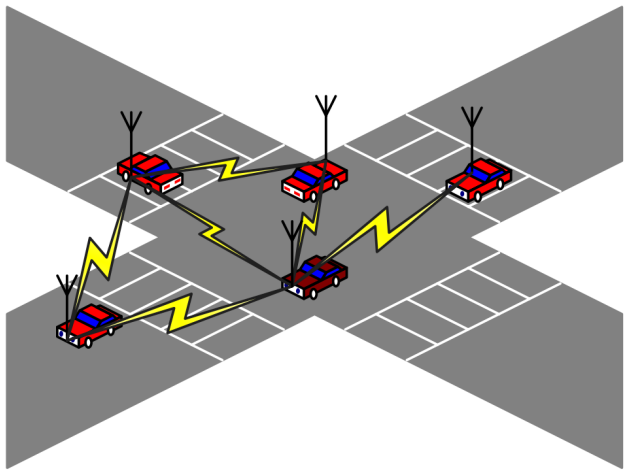


Fig. 2.5 Distributed cooperative sensing [80].

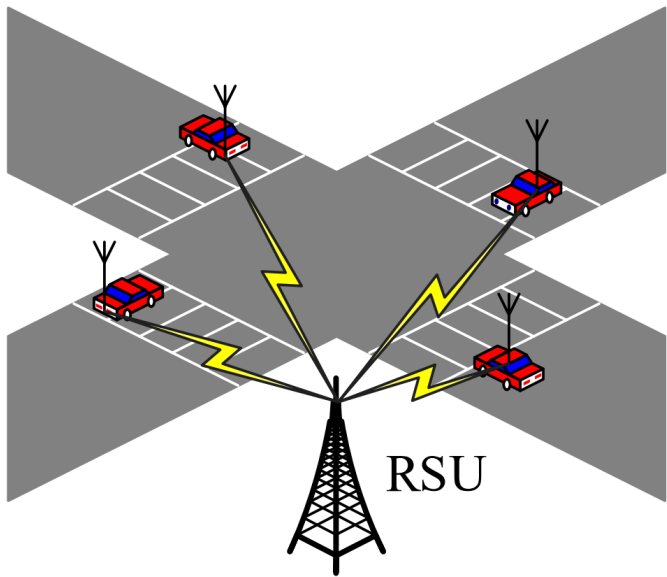


Fig. 2.6 Centralized cooperative sensing [80].

This relates to the distributed cooperative sensing and the centralized cooperative sensing approaches (Fig. 2.5 and 2.6). However, we do not perform cooperative sensing in the proposed algorithms. We only consider

the observations made by a single SU vehicle to take decisions about the spectrum occupancy. Yet, our work could be extended to a cooperative framework, where the sensing results of several vehicles are used to take faster and more accurate decisions.

The geolocation-based spectrum sensing algorithm proposed in Chapter 5 can be used in many scenarios where a user, knowing its localization and having access to geolocation information of existing PUs, wants to set up a new opportunistic communication. A typical example of such scenarios is wireless *ad hoc* networks for disaster management or military operations.

3

Spectrum Sensing Using Bayesian Changepoint Detection

3.1 Introduction

Spectrum sensing is more challenging in mobile environments. In fact, when the SU is moving, the spectrum occupancy may become more dynamic and the spectrum sensing algorithm should deal with fast situation changes. Mobility raises some issues among which the best choice of sensing frequency, the most suitable approach between hypothesis testing and changepoint detection, and the possibility to use mobility parameters (speed, direction, ...) to make better decisions. The issue of sensing frequency is more relevant to the joint transmission-sensing problem, because it involves the question of trade-off between the sensing time and the time allocated for SU transmission. This is addressed in Chapter 4. In this chapter, we only consider spectrum sensing and look at the last two issues. As already discussed in Chapter 2, spectrum sensing algorithms can use either the hypothesis testing approach or the changepoint detec-

tion approach. Hypothesis testing gives the current channel state, i.e., the PU is absent or present, while changepoint detection detects a change in the channel state, i.e., the PU has appeared or disappeared. Changepoint detection approach can be viewed as a method for tracking the channel state. In mobility scenarios, it is preferable to track the spectrum occupancy when the SU is moving, and to quickly detect the appearance or disappearance of a PU. The goal is to limit the time during which a SU would interfere after the PU appearance and the time it would waste after the PU has vacated the spectrum. For this reason, we think that changepoint detection is a better alternative to hypothesis testing for spectrum sensing in mobility scenarios, and we use this approach in this chapter.

Many works in the literature, apply changepoint detection to cognitive radio and wireless sensor network using both minimax and Bayesian approaches. Following the minimax approach, authors of [49] developed a changepoint detection framework using CUSUM (when both noise and signal power are known), GLR-based CUSUM (when only noise power is known), and a nonparametric test (when both are unknown). The quickest change detection over multiple independent channels is considered in [81], using a Bayesian formulation, with an application to a spectrum sensing scenario where a SU searches for idle channels in a wideband spectrum. Another quickest wideband spectrum sensing is introduced in [82], where the channels are considered to be correlated. Performance of CUSUM is studied in a cognitive radio context, over various fading channels for one [83] and multiple antennas [84]. Changepoint detection is also applied to collaborative spectrum sensing, where CUSUM is used either at the fusion center [85], or at the local nodes [86] or at both sides [87]. Furthermore, Bayesian and minimax changepoint detection schemes are applied to wireless sensor networks (for single and multiple sensors, in centralized and decentralized frameworks) with an additional constraint on the energy consumed by the sensing algorithm [88, 89, 90, 91, 92].

In this chapter, we consider a mobile cognitive radio scenario, and try to evaluate whether some knowledge about the environment and the mobility parameters of the SU can help in improving the detection of changes in the spectrum occupancy. To do so, we assume that the mobility parameters of the SU can be summarized in some *a priori* knowledge on the average time of spectrum change, or in other words, the prior distribution of the changepoint. We therefore use Bayesian changepoint detection methods in which this *a priori* knowledge can be included. Given that the SU

does not usually know the signal power of the PUs or other SUs using the spectrum, we apply the GLRT approach to the optimal Bayesian Shiryaev algorithm. As the GLR-based Shiryaev involves a complex non-convex optimization problem, we use an approximation of the GLRT which results in a low-complexity algorithm termed as *low-complexity GLR-Shiryaev* (LC GLR-Shiryaev). The derived algorithm is compared with other existing minimax (CUSUM, GLR-CUSUM) and Bayesian (Shiryaev, M-Shiryaev) algorithms for realistic CR scenarios. In addition, unlike previous works in the literature, we consider that both interference and spectrum waste durations are important to evaluate the performance of detection algorithms in a CR context. Hence, we introduce a new performance metric, called the *global penalty*, which jointly evaluates the costs of interference and spectrum waste induced by the changepoint detection algorithms, in a time-limited communication context. Furthermore, correctly setting the detection threshold is an important issue in spectrum sensing. We thus suggest a new way of setting the optimal threshold based on the global penalty metric, and this optimal threshold shows to depend on the SNR value. It is therefore only useful when the SNR is perfectly known by the SU. When the SNR is unknown, setting an optimal threshold is more difficult. We propose some adaptive-threshold methods which try to estimate the SNR and a fixed-threshold method that works in average for a certain SNR distribution. Finally, we perform a complexity analysis of the proposed LC GLR-Shiryaev algorithm and the M-Shiryaev algorithm, which was presented in Section 2.2.3 as a numerical GLRT Bayesian approach over a finite set of values of the unknown parameter. The goal is to find the trade-off between performance and complexity when choosing among both algorithms.

The simulation results show that, albeit suboptimal because it is an approximation of the GLR-based Shiryaev, the LC GLR-Shiryaev algorithm has a good performance. Moreover, it outperforms its non-Bayesian equivalent (i.e., GLR-CUSUM) at low SNR. Hence, it is a good choice for a practical mobile CR, when the SU does not have any *a priori* knowledge about the SNR and could be required to detect very low SNR signals. The simulations also show that the optimal threshold does not depend on the SU's communication length, and that the adaptive and fixed methods give almost similar performance under certain conditions. The complexity analysis shows that the recursive version of M-Shiryaev has a lower complexity than LC GLR-Shiryaev. However, it requires an accurate *a priori* knowledge

of the SNR, otherwise its performance decreases. Even if it adds some complexity, LC GLR-Shiryaev has the advantage of not requiring any *a priori* knowledge of the SNR.

To summarize, the main contributions of this chapter are:

- Derivation of the new LC GLR-Shiryaev algorithm,
- Introduction of the new global penalty metric that jointly considers interference and spectrum waste,
- Performance comparison of LC GLR-Shiryaev and other changepoint detection algorithms,
- Optimal threshold setting in known-SNR and unknown-SNR cases,
- Complexity analysis of LC GLR-Shiryaev and M-Shiryaev.

The rest of this chapter is organized as follows. The system model is introduced in Section 3.2. In Section 3.3, the LC GLR-Shiryaev algorithm is derived. Then, numerical results are given in Section 3.4, to evaluate and compare the performance of the considered algorithms, under different conditions. The computation of optimal thresholds in unknown-SNR situations is presented in Section 3.5. In addition, Section 3.6 shows the complexity analysis of LC GLR-Shiryaev and M-Shiryaev. Finally, Section 3.7 concludes the chapter and Section 3.A gives some proofs.

3.2 System model

3.2.1 Scenario

We consider the mobile CR scenario depicted in Fig. 3.1. Some fixed PUs are transmitting in the same frequency band, each having its coverage area. They could be, for example, TV transmitters in a TV broadcasting network or base stations in a mobile network. A mobile SU (vehicle SU1) is looking for opportunities to use the same spectrum band in order to set up a secondary vehicle-to-vehicle (V2V) communication with the vehicle SU2 or a secondary vehicle-to-infrastructure (V2I) communication with the roadside unit (RSU). The presence of another secondary system already using

the spectrum is possible, but, for simplicity, a PU and another SU (from a distinct secondary system) are called PU below, without distinction. It is assumed that, initially, a first spectrum sensing has been performed and the frequency band of interest is free. Hence, the SU tunes to the available band and starts using it. As it is moving, the spectrum occupancy may change, due to getting in the range of a PU. In order to detect a potential change in the given band, the SU is continuously measuring the activity in this band.

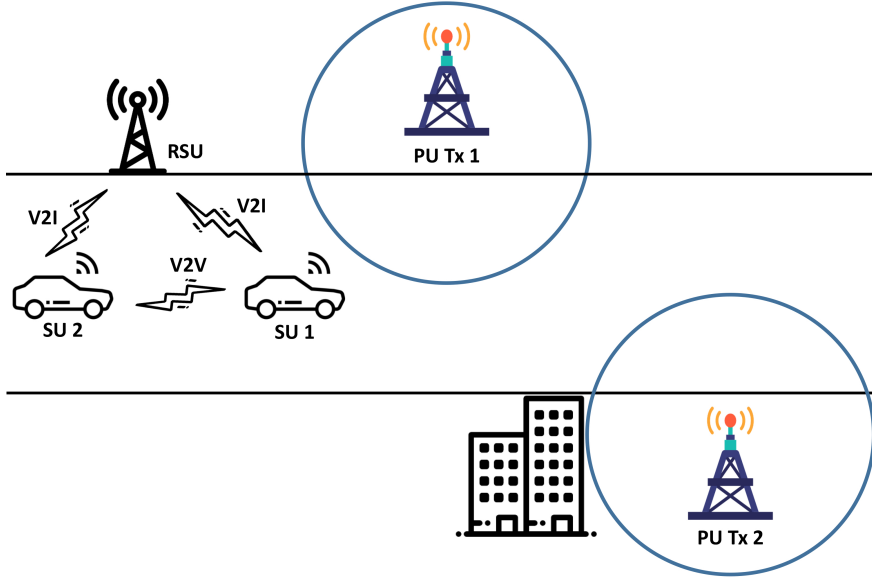


Fig. 3.1 Mobile CR scenario.

3.2.2 Signal model

The observed signal sample at time n is denoted by $y[n]$. When there is no PU activity, it is given by $y[n] = w[n]$, where $w[n]$ is the Additive White Gaussian Noise (AWGN) sample. When the PU transmits a signal, the SU observes $y[n] = x[n] + w[n]$, $x[n]$ being the transmitted signal affected by fading. It is assumed that $x[n]$ and $w[n]$ are independent circularly symmetric complex Gaussian, $x[n] \sim \mathcal{CN}(0, \sigma_x^2)$, $w[n] \sim \mathcal{CN}(0, \sigma_w^2)$, and independent of each other.

As said previously, we assume that, initially, the SU is outside of the ranges of PUs. Thus, the observed samples y follow a complex Gaussian distribution with variance σ_w^2 , whose PDF is denoted by $f_0(y)$. At an unknown time sample τ , a PU appears while the SU is moving, and the distribution changes to a complex Gaussian distribution with variance $\sigma_x^2 + \sigma_w^2$, whose PDF is denoted by $f_1(y)$. At each time sample m , the SU tries to detect the possible change in the distribution, based on the received sequence $y_1^m = y[1], \dots, y[m]$ up to the current time sample. The SU should thus distinguish between the two hypotheses of the changepoint detection model (2.18), that we recall below

$$\begin{cases} \mathcal{H}_0 : & y[n] = w[n], & n = 1, \dots, m \\ \mathcal{H}_1 : & \exists \tau \in [0, m], \text{ such that} \\ & y[n] = w[n], & n = 1, \dots, \tau \\ & y[n] = x[n] + w[n], & n = \tau + 1, \dots, m. \end{cases}$$

Note that in our model, we consider the detection of a PU appearance, but the detection of a PU disappearance can be performed following the same logic.

To include mobility in the model, we make the assumption that the probability of spectrum change is linked with the mobility parameters of the SU. For this reason, we consider the Bayesian framework, in which it is assumed that the time sample of PU appearance, τ , is a random variable with known prior distribution. Like in the literature, we consider the geometric prior distribution (2.30). This choice is motivated by the fact that the experience of detecting the PU appearance can be viewed as a sequence of trials with a certain number of failures (no appearance) before the first success (appearance). Considering the probability that the PU has appeared before the first observation, to be null ($q = P(\tau < 0) = 0$), the prior distribution is written as follows

$$P(\tau = k) = p(1 - p)^k, \quad k \geq 0. \quad (3.1)$$

It thus depends on only one parameter $p \in]0, 1]$, which is the probability that a PU appears at each time sample or, in other words, the average number of PU appearances per unit of time (sample). We postulate that if the speed of the SU and other characteristics of its environment are known, p can be known with a certain degree of accuracy. In the next section, we give more details about this.

3.2.3 Link between the parameter p and the mobility of the SU

Intuitively, a high speed of the SU may correspond to a high value of p because the spectrum situation may change more frequently. It should also be mentioned that p depends on the sampling frequency of the SU. In fact, the average number of time samples before the PU appearance, represented by $1/p$, does not have the same value depending on how fast the SU takes and processes samples. If the sampling rate is high, the SU will observe several samples before the PU appearance and the value of p will be small. But a low sampling rate will result in few samples observed before the PU appearance and the value of p will be higher. Clearly, all this will impact the detection performance of the system. Processing a lot of samples will give a high resolution for observing the time of spectrum change, but will induce a high complexity. Processing few samples will give a low resolution but will induce a low complexity.

In a practical mobile CR scenario, the SU is expected to be able to transmit for some reasonable amount of time, so the value of p is rather small. As highlighted in Section 2.4, we consider in this thesis that the PU is transmitting on a 8 MHz TV channel and that we are performing narrowband spectrum sensing on a single channel. With a sampling frequency of 8 MHz, it follows that the sampling period is equal to 125 ns. An average time of change of 10 s thus corresponds to observing $8 \cdot 10^7$ samples in average before the PU appearance, and results in $p = 125 \cdot 10^{-10}$. Actually this value of p is very low. As said above, in a practical system, processing too many samples with a sequential spectrum sensing algorithm may result in a high complexity. Moreover, a computer simulation with such value of p will take too much time to run. For this reason, we assume that the samples used for sensing are collected at a lower frequency than the sampling frequency. Assuming for example that they are collected every 50 ms, an average time of change of 10 s corresponds to observing 200 samples in average before the PU appearance, and results in $p = 0.005$. Considering a vehicle moving at the speed of 30 km/h, according to the scenario of Fig. 3.1, these parameter values imply that a spectrum change occurs in average after a distance 83 m. These example values are summarized in Table 3.1.

Table 3.1 Example of parameters values

Parameter	Value
Bandwidth	8 MHz
Sampling period	125 ns
Sensing sampling period	50 ms
Average time before change	10 s
p	0.005
Speed	30 km/h
Average distance before change	83 m

3.3 Low-complexity GLR-Shiryaev algorithm

Let us assume now that the PU's signal power σ_x^2 is unknown to the SU. This is actually the case for practical CR scenarios. In this section, we proceed to the derivation of the LC GLR-Shiryaev algorithm to solve this issue. It is obtained by applying an approximation of the GLRT approach to the optimal Bayesian Shiryaev algorithm.

3.3.1 GLR-Shiryaev

Let us remind the Shiryaev algorithm whose stopping time (2.31) is given by

$$T_{\text{Shi}} = \inf\{m \geq 1 : \Lambda_{\text{Shi}}(\mathbf{y}_1^m) \geq \alpha\} \quad (3.2)$$

with the decision statistic, where q is set to zero in (2.32),

$$\Lambda_{\text{Shi}}(\mathbf{y}_1^m) = \sum_{k=1}^m \prod_{n=k}^m \frac{L_n}{1-p} \quad (3.3)$$

L_n being the likelihood ratio of the n^{th} observation, given here by

$$L_n = \frac{f_1(y[n])}{f_0(y[n])} = \frac{\sigma_w^2}{\sigma_x^2 + \sigma_w^2} \exp\left(\frac{|y[n]|^2 \sigma_x^2}{\sigma_w^2 (\sigma_x^2 + \sigma_w^2)}\right). \quad (3.4)$$

As σ_x^2 is not known, the likelihood ratio is not defined, but it is con-

ditional to σ_x^2 . It is denoted now as $L_{n|\sigma_x^2}$. The Shiryaev statistic can be rewritten as a function of σ_x^2 , as follows

$$\begin{aligned}\Lambda_{\text{Shi}}(\mathbf{y}_1^m, \sigma_x^2) &= \sum_{k=1}^m \prod_{n=k}^m \frac{L_{n|\sigma_x^2}}{1-p} \\ &= \sum_{k=1}^m \prod_{n=k}^m \left(\frac{1}{1-p} \right) \frac{\sigma_w^2}{\sigma_x^2 + \sigma_w^2} \exp \left(\frac{|y[n]|^2 \sigma_x^2}{\sigma_w^2 (\sigma_x^2 + \sigma_w^2)} \right).\end{aligned}$$

The GLRT approach consists here in maximizing the statistic $\Lambda_{\text{Shi}}(\mathbf{y}_1^m, \sigma_x^2)$ over the set of possible values of σ_x^2 . The GLR-Shiryaev statistic is thus given by

$$\begin{aligned}\Lambda_{\text{GLR-Shi}}(\mathbf{y}_1^m) &= \sup_{\sigma_x^2} \Lambda_{\text{Shi}}(\mathbf{y}_1^m, \sigma_x^2) \\ &= \sup_{\sigma_x^2} \sum_{k=1}^m \prod_{n=k}^m \left(\frac{1}{1-p} \right) \frac{\sigma_w^2}{\sigma_x^2 + \sigma_w^2} \exp \left(\frac{|y[n]|^2 \sigma_x^2}{\sigma_w^2 (\sigma_x^2 + \sigma_w^2)} \right) \\ &= \sup_{\sigma_x^2} \sum_{k=1}^m \frac{1}{(1-p)^{m-k+1}} \left(\left(\frac{\sigma_w^2}{\sigma_x^2 + \sigma_w^2} \right)^{m-k+1} \right. \\ &\quad \left. \exp \left(\frac{\sigma_x^2}{\sigma_w^2 (\sigma_x^2 + \sigma_w^2)} \sum_{n=k}^m |y[n]|^2 \right) \right).\end{aligned}\tag{3.5}$$

We thus have to solve the following optimization problem

$$\begin{aligned}\max_{\sigma_x^2 \geq 0} \quad \Lambda_{\text{Shi}}(\mathbf{y}_1^m, \sigma_x^2) &= \sum_{k=1}^m \frac{1}{(1-p)^{m-k+1}} \left(\left(\frac{\sigma_w^2}{\sigma_x^2 + \sigma_w^2} \right)^{m-k+1} \right. \\ &\quad \left. \exp \left(\frac{\sigma_x^2 \sum_{n=k}^m |y[n]|^2}{\sigma_w^2 (\sigma_x^2 + \sigma_w^2)} \right) \right),\end{aligned}\tag{3.6}$$

The objective function $\Lambda_{\text{Shi}}(\mathbf{y}_1^m, \sigma_x^2)$ can be shown to be a non-concave function and its derivative is a nonlinear function whose zeros are hard to find analytically (proof is given in Section 3.A.1). To relax the optimization problem, we make an approximation in the following section.

3.3.2 Approximation of GLR-Shiryaev

Instead of directly maximizing the sum $\Lambda_{\text{Shi}}(\mathbf{y}_1^m, \sigma_x^2)$, each term of the sum is maximized independently, making the problem easier to solve. In other words, rather than computing the σ_x^2 that maximizes $\Lambda_{\text{Shi}}(\mathbf{y}_1^m, \sigma_x^2)$, we compute for each term of the sum, the $\sigma_{x,k}^2$ that maximizes this term. The new decision statistic is thus

$$\Lambda_{\text{GLR-Shi}}(\mathbf{y}_1^m) \approx \sum_{k=1}^m \frac{1}{(1-p)^{m-k+1}} \sup_{\sigma_{x,k}^2} \left(\left(\frac{\sigma_w^2}{\sigma_{x,k}^2 + \sigma_w^2} \right)^{m-k+1} \exp \left(\frac{\sigma_{x,k}^2}{\sigma_w^2 (\sigma_{x,k}^2 + \sigma_w^2)} \sum_{n=k}^m |y[n]|^2 \right) \right). \quad (3.7)$$

Looking at one term at a time, the logarithm of the k^{th} term is redefined as $h_k(\sigma_{x,k}^2)$, and the optimization problem that needs to be solved is the following

$$\begin{aligned} \max_{\sigma_{x,k}^2 \geq 0} \quad h_k(\sigma_{x,k}^2) &= (m-k+1) \ln \left(\frac{\sigma_w^2}{\sigma_{x,k}^2 + \sigma_w^2} \right) \\ &+ \frac{\sigma_{x,k}^2}{\sigma_w^2 (\sigma_{x,k}^2 + \sigma_w^2)} \sum_{n=k}^m |y[n]|^2. \end{aligned} \quad (3.8)$$

This problem can be easily solved using the Lagrangian method. The Lagrangian is defined as

$$\begin{aligned} \mathcal{L}(\sigma_{x,k}^2, \lambda) &= h_k(\sigma_{x,k}^2) - \lambda(-\sigma_{x,k}^2) \\ &= (m-k+1) \ln \left(\frac{\sigma_w^2}{\sigma_{x,k}^2 + \sigma_w^2} \right) + \frac{\sigma_{x,k}^2}{\sigma_w^2 (\sigma_{x,k}^2 + \sigma_w^2)} \sum_{n=k}^m |y[n]|^2 + \lambda \sigma_{x,k}^2 \end{aligned} \quad (3.9)$$

where λ is the Lagrange multiplier. Its first derivative is given by

$$\begin{aligned} \frac{\partial \mathcal{L}(\sigma_{x,k}^2, \lambda)}{\partial \sigma_{x,k}^2} &= (m-k+1) \frac{\frac{-\sigma_w^2}{(\sigma_{x,k}^2 + \sigma_w^2)^2}}{\frac{\sigma_w^2}{\sigma_{x,k}^2 + \sigma_w^2}} + \frac{\sigma_{x,k}^2 + \sigma_w^2 - \sigma_{x,k}^2}{(\sigma_{x,k}^2 + \sigma_w^2)^2} \frac{\sum_{n=k}^m |y[n]|^2}{\sigma_w^2} + \lambda \\ &= \frac{\sum_{n=k}^m |y[n]|^2}{(\sigma_{x,k}^2 + \sigma_w^2)^2} - \frac{m-k+1}{\sigma_{x,k}^2 + \sigma_w^2} + \lambda. \end{aligned}$$

The Karush-Kuhn-Tucker (KKT) conditions for this problem are given by

$$\begin{cases} \frac{\partial \mathcal{L}(\sigma_{x,k}^2, \lambda)}{\partial \sigma_{x,k}^2} = 0 & \iff \frac{\sum_{n=k}^m |y[n]|^2}{(\sigma_{x,k}^2 + \sigma_w^2)^2} - \frac{m-k+1}{\sigma_{x,k}^2 + \sigma_w^2} + \lambda = 0 \\ \frac{\partial \mathcal{L}(\sigma_{x,k}^2, \lambda)}{\partial \lambda} \geq 0 & \iff \sigma_{x,k}^2 \geq 0 \\ \lambda \geq 0 & \iff \lambda \geq 0 \\ \lambda \sigma_{x,k}^2 = 0 & \iff \lambda = 0 \text{ or } \sigma_{x,k}^2 = 0. \end{cases} \quad (3.10)$$

The solutions can be split in two cases:

- **Case 1:** $\sigma_{x,k}^2 > 0 \implies \lambda = 0, \sigma_{x,k}^2 = \frac{\sum_{n=k}^m |y[n]|^2}{m-k+1} - \sigma_w^2$,
which is obtained if $\sum_{n=k}^m |y[n]|^2 > (m-k+1)\sigma_w^2$.
- **Case 2:** $\sigma_{x,k}^2 = 0 \implies \lambda = \frac{(m-k+1)\sigma_w^2 - \sum_{n=k}^m |y[n]|^2}{\sigma_w^4}$,
which is obtained for the reversed inequality, and thus satisfies $\lambda \geq 0$.

To verify if $\tilde{\sigma}_{x,k}^2 = \frac{\sum_{n=k}^m |y[n]|^2}{m-k+1} - \sigma_w^2$ is a maximum of $h_k(\sigma_{x,k}^2)$, we compute the second-order derivative at this value. It is given by

$$\frac{\partial^2 h_k}{\partial (\sigma_{x,k}^2)^2}(\tilde{\sigma}_{x,k}^2) = \frac{-(m-k+1)^3}{\left(\sum_{n=k}^m |y[n]|^2\right)^2} \leq 0.$$

Consequently, the solution to the optimization problem (3.8) can be compactly written as

$$\tilde{\sigma}_{x,k}^2 = \left(\frac{\sum_{n=k}^m |y[n]|^2}{m-k+1} - \sigma_w^2 \right)^+. \quad (3.11)$$

Inserting it in (3.7), we obtain the approximation of the GLR-Shiryaev decision statistic. As the approximation has a lower complexity than the original GLR-Shiryaev, we call it *low-complexity GLR-Shiryaev* (LC GLR-Shiryaev) algorithm and its decision statistic is

$$\Lambda_{\text{LC GLR-Shi}}(\mathbf{y}_1^m) = \sum_{k=1}^m \frac{1}{(1-p)^{m-k+1}} \left(\left(\frac{\sigma_w^2}{\tilde{\sigma}_{x,k}^2 + \sigma_w^2} \right)^{m-k+1} \exp \left(\frac{\tilde{\sigma}_{x,k}^2}{\sigma_w^2 (\tilde{\sigma}_{x,k}^2 + \sigma_w^2)} \sum_{n=k}^m |y[n]|^2 \right) \right). \quad (3.12)$$

In summary, at each time sample m , the SU just has to compute the decision

statistic $\Lambda_{\text{LC GLR-Shi}}(\mathbf{y}_1^m)$ in (3.12) and compare it with the threshold α , to decide of the PU appearance.

3.4 Numerical results

In this section, we perform some simulations in order to assess the performance of LC GLR-Shiryaev in comparison with other changepoint detection algorithms. More specifically, the comparison is made with the optimal Bayesian Shiryaev algorithm which has a perfect knowledge of the PU's signal power and will serve as benchmark to evaluate the loss due to this lack of knowledge of LC GLR-Shiryaev. Then, in order to evaluate the precision of the approximation (3.7), done in deriving LC GLR-Shiryaev, a comparison can be made with a numerical solution using a grid search over a finite set of values for σ_x^2 . This is equivalent to the M-Shiryaev algorithm and therefore, M-Shiryaev is added to the comparison. We also make the comparison with the non-Bayesian algorithms which do not use the knowledge about the PU appearance, i.e., the mobile environment parameters. We simulate the performance of the optimal minimax CUSUM and the GLR-CUSUM, which is the non-Bayesian equivalent of LC GLR-Shiryaev, in order to evaluate the gain of knowing the mobility parameters in practical CR scenarios.

3.4.1 Simulation setup and performance metric

Simulation setup

It is assumed that the SU has a limited amount of time, defined by the number N of measurement samples, for secondary communication and the time sample of PU appearance τ is geometrically generated with parameter p . The PU may not appear until the end of the SU communication. We are interested in the performance of each algorithm during the limited time frame N .

Global penalty performance metric

In CR, it is desirable to reduce the interference with the PU as much as possible, but also to maximize the spectrum usage. For this reason, we define a new criterion that tries to take both effects into account. The criterion also has to take into account for how long the interference or spectrum waste is present with respect to the duration of the communication. For this reason, the average detection delay, representing the interference duration, is first redefined as

$$\overline{D}_{DI} = \mathbb{E} \left\{ \left(\min\{T, N\} - \min\{\tau, N\} \right)^+ \right\}. \quad (3.13)$$

This varies from the usual average detection delay definition of the literature (2.28) in two ways. First, it is no longer conditional to the correct detection event $\{T > \tau\}$, but the average is done over all cases (correct detection and false alarm) since the cases with $\{T < \tau\}$ must be counted as creating no interference penalty. Moreover, it also handles the cases where there is no detection until the end of the frame $\{T = N\}$, and the cases where the PU does not appear during the frame $\{\tau > N\}$. Similarly, the *average false alarm duration* is defined as the average time between a false alarm decision and the changepoint, and it is written as follows

$$\overline{D}_{FA} = \mathbb{E} \left\{ \left(\min\{\tau, N\} - \min\{T, N\} \right)^+ \right\}. \quad (3.14)$$

This metric represents the average time of spectrum waste during the time frame of communication. Depending on the specific CR application, one may want to specify a cost or a penalty for the interference induced by the SU, and a different one for spectrum waste. For example, the PU can be privileged by giving a high penalty to interference or conversely, the quality of service of the SU can be prioritized by giving a high penalty to spectrum waste. The penalties induced by interference and waste of spectral opportunity are respectively denoted by P_{int} and P_{sow} . Finally, the overall *global penalty* induced by the detection algorithm during the time frame N can be defined as

$$G_P = P_{int} \overline{D}_{DI} + P_{sow} \overline{D}_{FA}. \quad (3.15)$$

This new performance metric allows to define the relative importance of interference time and spectrum waste time, and thereby, to obtain the de-

sired trade-off between them.

Penalties computation

To compute the penalties values in this work, we first make a simple assumption of a multi-band transmission with uniform power allocation. We assume that the SU wants to opportunistically use the whole band, but it performs spectrum sensing independently in each sub-band. Then, we compute the loss in the total channel rate (in bits/sample) when the SU fails in detecting the PU activity on a sub-band, which represents the interference penalty P_{int} . And finally, we compute the loss in the total channel rate (in bits/sample) when the SU releases a sub-band before the PU appearance, which represents the spectrum waste penalty P_{sow} . As the penalties are expressed in bits/sample, the global penalty stands for the average total number of bits lost during the time frame N . This is more elaborated in Section 3.A.2.

3.4.2 Simulations results & Discussion

The simulation parameters are given the following values: $p \in \{0.005, 0.05\}$, received SNR $= \frac{\sigma_s^2}{\sigma_w^2} \in \{-10, -5, 0, 5\}$ dB when the PU is present, SU communication duration $N = 1000$ samples, penalties $P_{int} = 0.5$ bits/sample and $P_{sow} = 0.14$ bits/sample.

In order to concurrently visualize the interference and the spectral opportunity waste induced by each algorithm, the $\overline{D}_{DI}$ vs $\overline{D}_{FA}$ curve, obtained by varying the threshold, is represented in Fig. 3.2 to 3.5. It can be viewed as a kind of complementary receiver operating characteristics (ROC) curve for changepoint spectrum sensing algorithms. The curve is plotted for $p = 0.005$ and different SNR values. It can be noticed that Shiryaev algorithm has the best performance at every SNR value, i.e., for a given average false alarm duration, it has the smallest average detection delay. Its non-Bayesian equivalent, CUSUM algorithm, has a comparable performance at high SNR values, but performs worse for low SNR values. Concerning the GLR-based algorithms, GLR-CUSUM has better performance than LC GLR-Shiryaev and M-Shiryaev at high SNR, whereas it is outperformed by both algorithms at low SNR. It is also interesting to note that at very low SNR (-10 dB), LC GLR-Shiryaev and M-Shiryaev, which do not know

the SNR, perform better than CUSUM, which perfectly knows the SNR. Intuitively, we can say that Bayesian algorithms outperform non-Bayesian ones at low SNR, because they use the prior information about the change, which is more useful at low SNR than at high SNR.

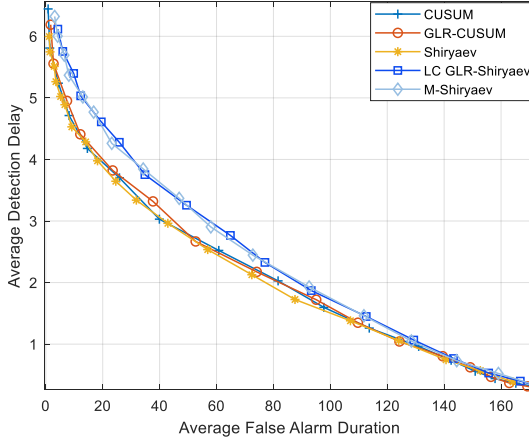


Fig. 3.2 $\bar{D}_{DI}$ vs $\bar{D}_{FA}$ for $p = 0.005$ and SNR = 5 dB.

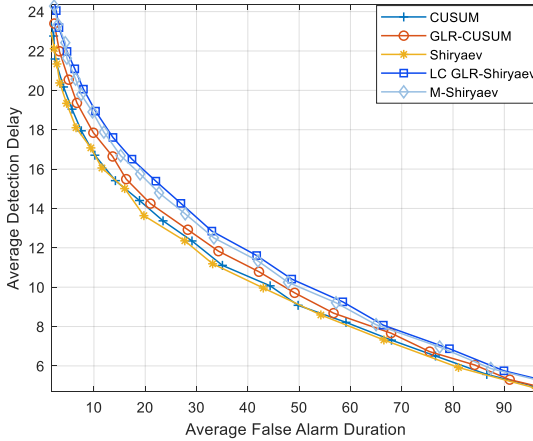


Fig. 3.3 $\bar{D}_{DI}$ vs $\bar{D}_{FA}$ for $p = 0.005$ and SNR = 0 dB.

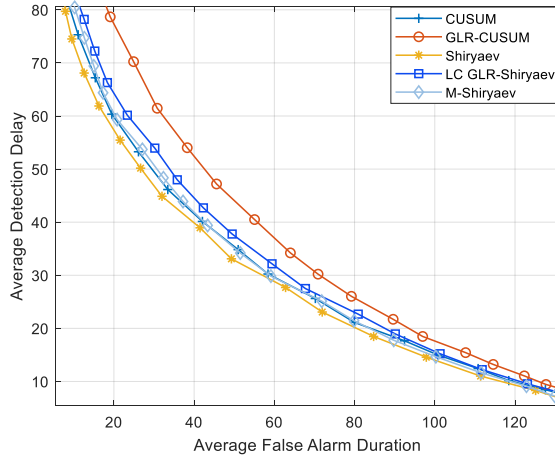


Fig. 3.4 $\bar{D}_{DI}$ vs $\bar{D}_{FA}$ for $p = 0.005$ and SNR = -5 dB.

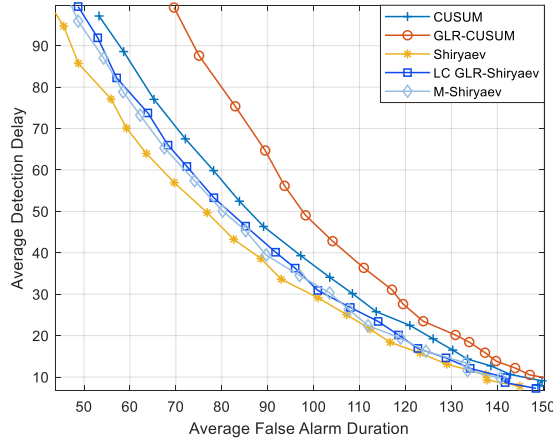


Fig. 3.5 $\bar{D}_{DI}$ vs $\bar{D}_{FA}$ for $p = 0.005$ and SNR = -10 dB.

It is worth noting that for all the algorithms, the average detection delay $\bar{D}_{DI}$, obtained for a constant average false alarm duration $\bar{D}_{FA}$, significantly increases when the SNR decreases. The explanation for that is simple. Actually, false alarms do not depend on the presence of a PU and

thus, a threshold can be set to obtain a constant $\overline{D}_{FA}$ at every SNR. This is shown in Fig. 3.6, where we also see that $\overline{D}_{FA}$ is a decreasing function of the threshold. For a constant threshold value, a low-SNR signal is obviously detected more slowly than a high-SNR signal, and yields a larger $\overline{D}_{DI}$. Note also that the performance gap between GLR-CUSUM and LC GLR-Shiryaev is higher in low-SNR regions, as presented in Fig. 3.7.

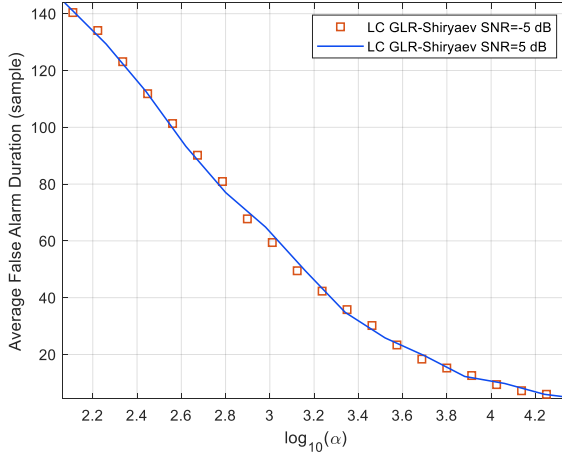


Fig. 3.6 $\overline{D}_{FA}$ vs $\log_{10}(\alpha)$ for $p = 0.005$ and $\text{SNR} = \{-5, 5\}$ dB.

As it can be seen in the previous figures, LC GLR-Shiryaev and M-Shiryaev have comparable performance in every case, showing that LC GLR-Shiryaev is a good approximation of GLR-Shiryaev. The performance of M-Shiryaev is impacted by the choice of the range of possible values of σ_x^2 . It is shown with further simulations that when a narrower range of possible values of σ_x^2 is chosen around the true value, the M-Shiryaev algorithm approaches the optimal Shiryaev algorithm. For example, the performance of M-Shiryaev is shown in Fig. 3.8 and 3.9 when the true SNR value is 0.5 dB and the considered ranges of possible SNR values are respectively $\{-5, -4, \dots, 5\}$ dB and $\{-2, -1, \dots, 2\}$ dB. A performance gain can be noticed in the last case. Hence, obtaining a good performance for M-Shiryaev requires an accurate *a priori* knowledge of the range, which is usually not available. In practice, a wider range will be chosen with multiple possible values of σ_x^2 , in order to avoid missing the true value and to keep reasonable performance. But this will come at a cost of increased com-

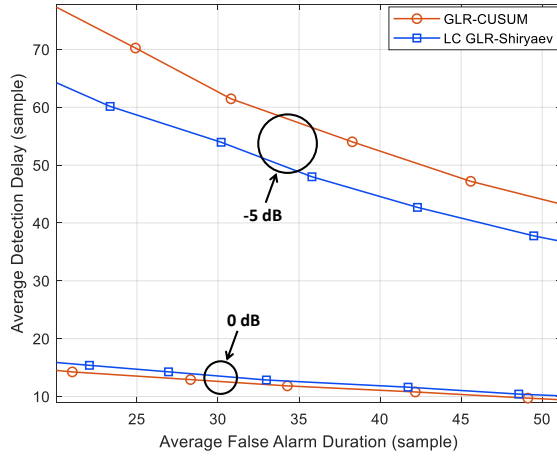


Fig. 3.7 $\bar{D}_{DI}$ vs $\bar{D}_{FA}$ for $p = 0.005$ and $SNR = \{-5, 0\}$ dB.

plexity, as it will be shown in Section 3.6. The derived LC GLR-Shiryaev, for its part, has the advantage of not requiring any *a priori* knowledge as it uses an analytical form. It also does not require to compute multiple parallel procedures.

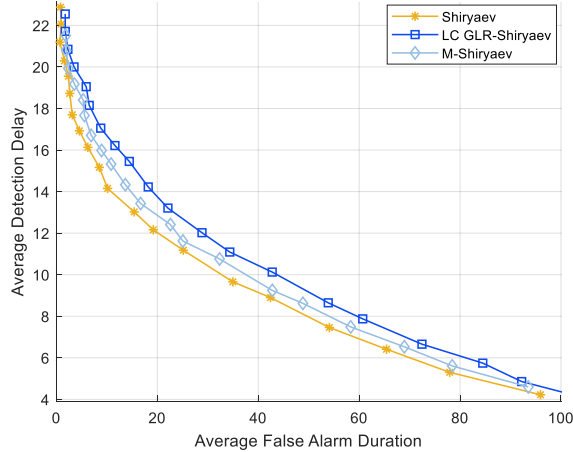


Fig. 3.8 M-Shiryaev performance for $p = 0.005$, true $SNR = 0.5$ dB and possible $SNR \in \{-5, -4, \dots, 5\}$ dB.

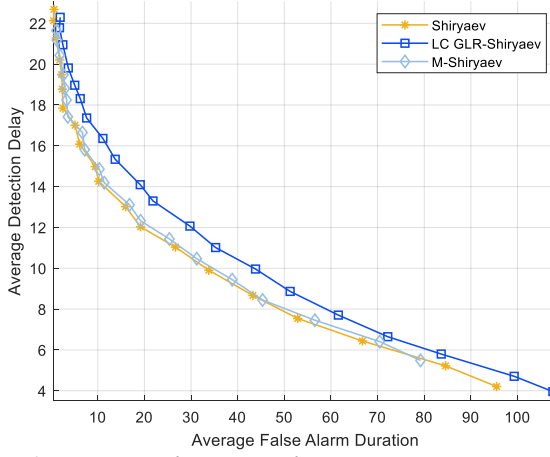


Fig. 3.9 M-Shiryaev performance for $p = 0.005$, true SNR = 0.5 dB and possible SNR $\in \{-2, -1, \dots, 2\}$ dB.

The choice of the threshold α is an important issue, and choosing a specific value will put the spectrum sensing algorithm at a specific operating point on the $\bar{D}_{DI}$ vs $\bar{D}_{FA}$ curve. As it is not possible to simultaneously minimize $\bar{D}_{DI}$ and $\bar{D}_{FA}$, it is desirable to choose a threshold that gives a good compromise between them. We suggest to fix this threshold with the help of the G_P vs $\log_{10}(\alpha)$ curves plotted in Fig. 3.10 – 3.13.

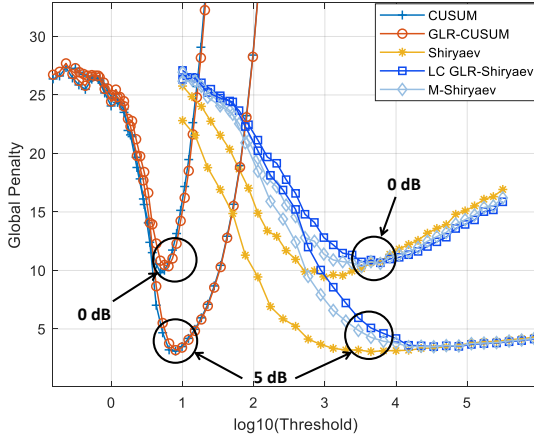


Fig. 3.10 G_P vs $\log_{10}(\alpha)$ for $p = 0.005$ and SNR = 5 dB & 0 dB.

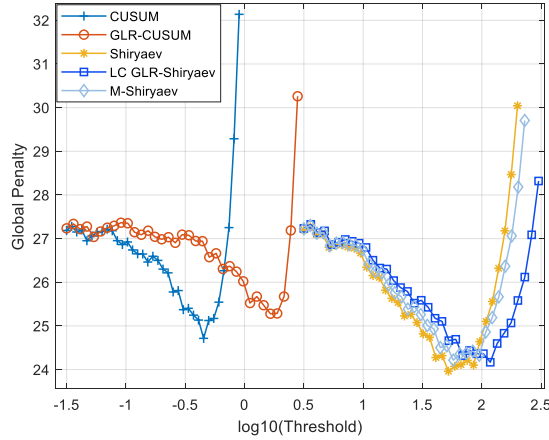


Fig. 3.11 G_P vs $\log_{10}(\alpha)$ for $p = 0.005$ and $\text{SNR} = -10$ dB.

First, it can be seen that, for every algorithm, a minimum global penalty exists, and the corresponding threshold depends on the SNR and the value of p . By comparing Fig. 3.10 and Fig. 3.11, the same tendency as before can be noticed: LC GLR-Shiryaev and M-Shiryaev yield lower minimum global penalties than CUSUM and GLR-CUSUM at low SNR, for $p = 0.005$.

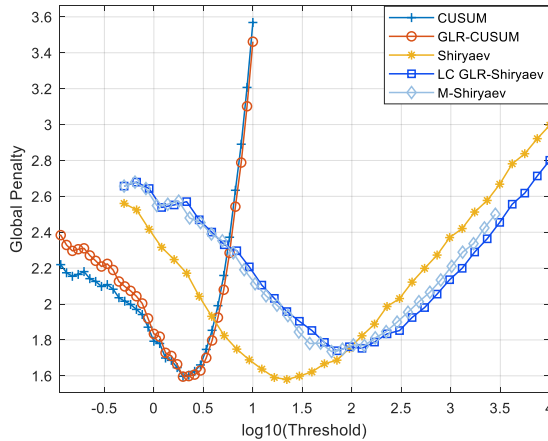


Fig. 3.12 G_P vs $\log_{10}(\alpha)$ for $p = 0.05$ and $\text{SNR} = 5$ dB.

The same trend is observed when p is increased tenfold ($p = 0.05$) in Fig. 3.12 and Fig. 3.13. Moreover, looking at Fig. 3.10 and Fig. 3.11, it can be noticed that CUSUM and GLR-CUSUM are more sensitive to the threshold variation, i.e., small variations of the threshold around its optimal value can result in high variations of the global penalty

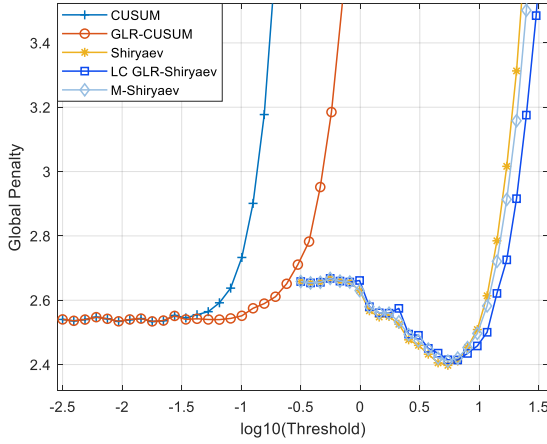


Fig. 3.13 G_p vs $\log_{10}(\alpha)$ for $p = 0.05$ and $\text{SNR} = -10$ dB.

These results show that Shiryayev-based algorithms work better than CUSUM-based algorithms at low SNR. This can be explained by the fact that at low SNR, the PU's signal becomes more difficult to distinguish from noise and the prior knowledge of the PU appearance, obtained through the mobility parameters, can help increase the detection performance. These results are consistent with the literature on changepoint detection theory, since a low value of the Kullback-Leibler divergence (2.35) corresponds to a low value of the SNR in a CR context [66, 50]. This advantage of Shiryayev-based algorithms over CUSUM-based algorithms in low SNR ranges is valuable in practical mobile CR. In fact, multipath fading and shadowing can happen very frequently and the SU could be required to detect primary signals with very low SNR (low as -20 dB, according to IEEE 802.22 WRAN standard) [67]. Moreover, when the SU does not know the SNR, the derived LC GLR-Shiryayev is a good spectrum sensing algorithm candidate.

3.4.3 Optimal threshold setting

In the literature, the optimal threshold is classically chosen so that the false alarm probability is below or equal to a given value. In the current work, this would be equivalent to define a threshold for a constant average false alarm duration, as mentioned in the previous paragraph. But instead, we suggest choosing the optimal threshold as the one that minimizes the global penalty. It is already underlined that the minimum global penalty, and therefore the optimal threshold, depends on p and the SNR. Furthermore, it can be noticed that it is approximately independent of the SU time frame duration N . This can be seen in Fig. 3.14, where the G_P vs $\log_{10}(\alpha)$ curves are plotted for two different SU communication lengths. If the SNR

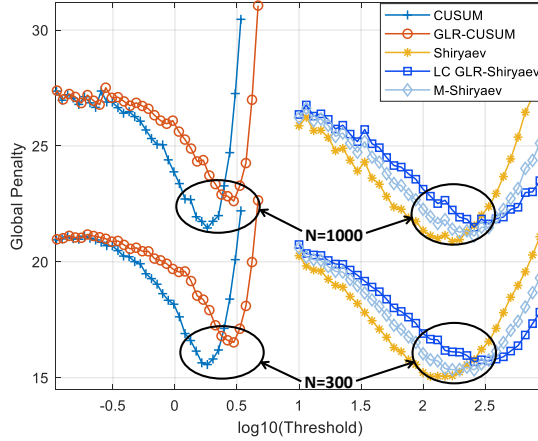


Fig. 3.14 G_P vs $\log_{10}(\alpha)$ for $p = 0.005$, $\text{SNR} = -5$ dB, $N \in \{300, 1000\}$ samples.

is known, the optimal thresholds can be pre-computed offline and recovered from a table during the actual sensing. Based on simulations, the optimal thresholds have been computed for a set of given SNR values. The results are presented in Table 3.2 below. It can be noticed for each algorithm that when the SNR increases, the optimal threshold increases too. To better understand how the optimal threshold is chosen, we also plot the average detection delay, the average false alarm duration and the global penalty of LC GLR-Shiryayev at the optimal thresholds, with respect to the

Table 3.2 Optimal thresholds computed through simulations.

SNR (dB) \ Algorithm	-10	-5	0	5
p=0.005				
CUSUM	0.4516	1.8248	4.8819	7.9771
GLR-CUSUM	1.6521	2.9753	6.3437	7.9771
Shiryaev	52.2811	171.9072	966.7053	4.4367e+03
LC GLR-Shiryaev	117.8190	258.0862	6.0174e+03	3.3839e+04
M-Shiryaev	57.4652	258.0862	3.2712e+03	3.3839e+04
p=0.05				
CUSUM	0.0179	0.1317	0.5987	1.9684
GLR-CUSUM	0.0179	0.1838	0.6471	1.9684
Shiryaev	5.4419	6.0174	6.7289	22.0537
LC GLR-Shiryaev	6.5786	9.3193	10.3830	70.7107
M-Shiryaev	5.4419	5.3150	9.2552	62.3929

SNR, in Fig. 3.15. We can see that $\bar{D}_{FA}$ and G_P are monotonically decreas-

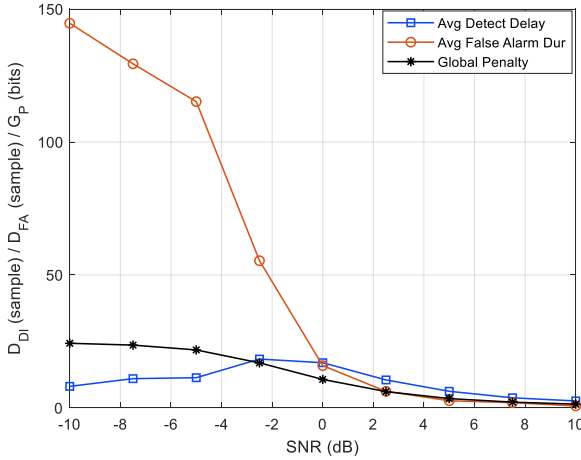


Fig. 3.15 $\bar{D}_{DI}, \bar{D}_{FA}, G_P$ vs SNR at optimal thresholds, for LC GLR-Shiryaev ($p = 0.005$).

ing functions of the SNR, while $\bar{D}_{DI}$ initially increases and then decreases. This way of setting the threshold ensures that the average total number of bits lost during the communication is always kept to the minimum. This

may require to increase a bit the interference in order to substantially decrease the waste of spectral opportunity, as it can be seen for SNR values ranging from -10 dB to 0 dB. Note that at high SNR (from 0 dB to 10 dB), both can be decreased and this results in a considerably decreased global penalty. The monotonic decrease of G_P means that we can make the most effective use of the spectrum when the PU signal power is the highest.

As the GLR-based algorithms do not know the SNR, it is problematic to set the optimal threshold (with minimum global penalty) for them. To deal with that, the SNR could be estimated first or a fixed threshold, independent of the SNR, could be used. This is more detailed in the next section.

3.5 Optimal threshold setting in unknown-SNR situations

Here, we suggest different methods to compute the optimal threshold for changepoint detection-based spectrum sensing algorithms which do not have the knowledge of the SNR (GLR-CUSUM and LC GLR-Shyriaev). We suggest two (02) ways to do that: an *adaptive-threshold* method and a *fixed-threshold* method.

3.5.1 Adaptive threshold

The first method tries to estimate the SNR, or equivalently the PU signal power σ_x^2 , at each time sample m , based on the observations. Then, the optimal threshold corresponding to the estimated value is drawn from Table 3.2 and used at this time sample. As the optimal threshold can be pre-computed only for a finite number of σ_x^2 values, we choose the one corresponding to the σ_x^2 value that is the closest to the estimated value. Let us designate by $\tilde{\sigma}_x^2$, the estimated PU signal power. This method is called *adaptive*, as $\tilde{\sigma}_x^2$ is sequentially refined and the threshold is updated accordingly. To compute $\tilde{\sigma}_x^2$, we suggest four (04) different approaches.

First approach

In the first approach, we suggest reusing the value of $\tilde{\sigma}_x^2$, used to compute the decision statistic.

Let us first consider the GLR-CUSUM decision statistic (2.26) which, according to our signal model, is given by

$$\begin{aligned}\Lambda_{\text{GLR-CUS}}(\mathbf{y}_1^m) &= \sup_{\sigma_x^2} \max_{k \leq m} \sum_{n=k}^m \left(\frac{|y[n]|^2 \sigma_x^2}{\sigma_w^2 (\sigma_x^2 + \sigma_w^2)} + \ln \frac{\sigma_w^2}{\sigma_x^2 + \sigma_w^2} \right) \\ &= \max_{k \leq m} \sum_{n=k}^m \left(\frac{|y[n]|^2 \tilde{\sigma}_x^2}{\sigma_w^2 (\tilde{\sigma}_x^2 + \sigma_w^2)} + \ln \frac{\sigma_w^2}{\tilde{\sigma}_x^2 + \sigma_w^2} \right).\end{aligned}\quad (3.16)$$

Here, $\tilde{\sigma}_x^2$ is the value that maximizes the decision statistic. In other words, considering the observation vector $\mathbf{y}_1^m$, the likelihood ratios of the m sub-vectors $\mathbf{y}_k^m$, for $k = 1, \dots, m$, is computed, and the signal power value that gives the maximum likelihood ratio is chosen as $\tilde{\sigma}_x^2$.

Let us now consider the LC GLR-Shiryaev statistic (3.12)

$$\begin{aligned}\Lambda_{\text{LC GLR-Shi}}(\mathbf{y}_1^m) &= \sum_{k=1}^m \frac{1}{(1-p)^{m-k+1}} \left(\left(\frac{\sigma_w^2}{\tilde{\sigma}_{x,k}^2 + \sigma_w^2} \right)^{m-k+1} \right. \\ &\quad \left. \exp \left(\frac{\tilde{\sigma}_{x,k}^2}{\sigma_w^2 (\tilde{\sigma}_{x,k}^2 + \sigma_w^2)} \sum_{n=k}^m |y[n]|^2 \right) \right),\end{aligned}$$

where we compute, for $k = 1, \dots, m$, the $\tilde{\sigma}_{x,k}^2$ that maximizes each term (the terms contain also the likelihood ratios of observation sub-vectors). Here, the statistic uses all the m values of $\tilde{\sigma}_{x,k}^2$ for the summation, but we are interested in one value to compute the threshold. Following the same idea as the GLR-CUSUM statistic, we suggest using the $\tilde{\sigma}_{x,k}^2$ that gives the highest term, as the most reliable value of $\tilde{\sigma}_x^2$. Note that the only difference with GLR-CUSUM is the $\frac{1}{(1-p)^{m-k+1}}$ expression that multiplies each likelihood ratio.

Second approach

Furthermore, we suggest a second approach with the aim of improving the $\tilde{\sigma}_x^2$ computed in the first approach. It simply consists in averaging the $\tilde{\sigma}_x^2$

computed at the current time sample, with the ones computed at previous time samples, and use the average for the threshold computation.

Third approach

In the third approach, we suggest computing $\tilde{\sigma}_x^2$, by maximizing the likelihood ratio, but considering only the last observed sample ($k = m$). The $\tilde{\sigma}_x^2$ obtained in this approach is similar for GLR-CUSUM and LC GLR-Shiryaev.

Fourth approach

Similarly to the second approach, the fourth approach performs an average over time of the $\tilde{\sigma}_x^2$ computed in the third approach. In other words, the maximum likelihood estimate of σ_x^2 , using only the last observation is averaged with the ones previously computed.

3.5.2 Fixed threshold

Secondly, we suggest a *fixed-threshold* method, where the threshold is not adaptively computed, but it is set to give the optimal performance in average. More specifically, we assume a certain SNR distribution. Then, by means of simulations, we compute the optimal threshold that minimizes the average global penalty G_p for this distribution. The fixed threshold is used for spectrum sensing, independently of the true SNR. Note that the obtained optimal threshold and the corresponding performance will clearly depend on the distribution parameters.

3.5.3 Simulation results and discussion

Here, we evaluate the performance of the algorithms, when the unknown-SNR thresholds are used, compared to the optimal known-SNR thresholds. Using Monte Carlo simulations, the global penalty is computed at various SNR values. The following parameters values are used: $p = 0.005$, $\text{SNR} \in [-10, 10]$ dB, $N = 1000$ samples, $P_{int} = 0.5$ bits/sample and $P_{sow} = 0.14$ bits/sample.

Adaptive threshold

The performance of the four adaptive-threshold approaches is conditioned by the accuracy of the PU signal power estimation performed in each approach. For this reason, we first plot the *mean squared error* (MSE) of the estimation with respect to the time sample, at various SNR values, for each approach. For $\tau = 200$ samples, $\text{SNR} \in \{-10, 0, 10\}$ dB, we obtain the results shown in Fig. 3.16 – 3.23, for the four approaches. We can notice in Fig. 3.16 and 3.17 that the first and second approaches give slightly similar performance for GLR-CUSUM and LC GLR-Shiryaev. As already mentioned, the third and fourth approaches result in the same estimates for both algorithms. Thereafter, we compare the different approaches irrespective of the algorithm.

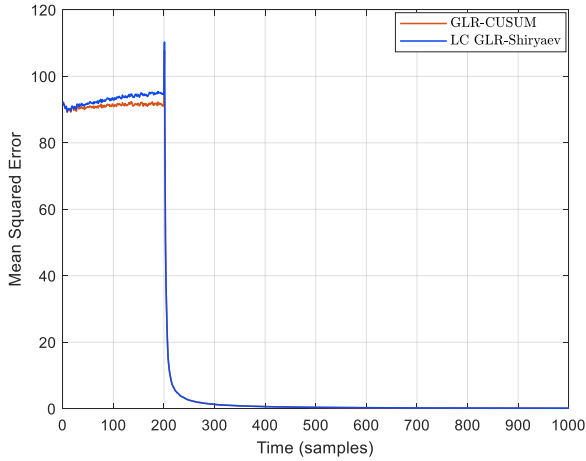


Fig. 3.16 MSE of the first adaptive approach, for $\tau = 200$ samples and $\text{SNR} = 10$ dB.

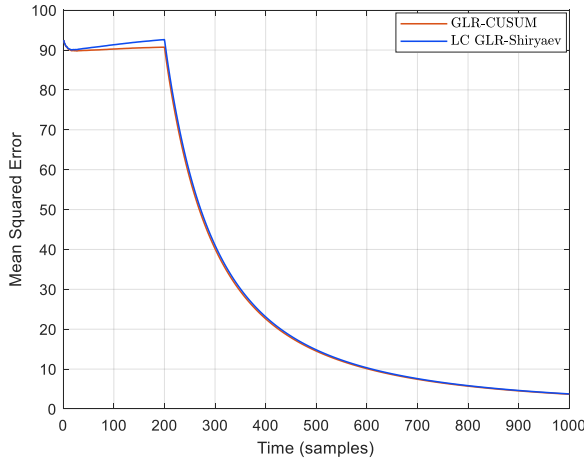


Fig. 3.17 MSE of the second adaptive approach, for $\tau = 200$ samples and SNR = 10 dB.

For SNR = -10 dB, the performance of the four approaches are shown in Fig. 3.18 and 3.19. The performance improves slowly after τ , in the first and second approaches. Instead, the performance deteriorates after τ , in the third approach. The second and fourth approaches which average over time have almost the same and the best performance.

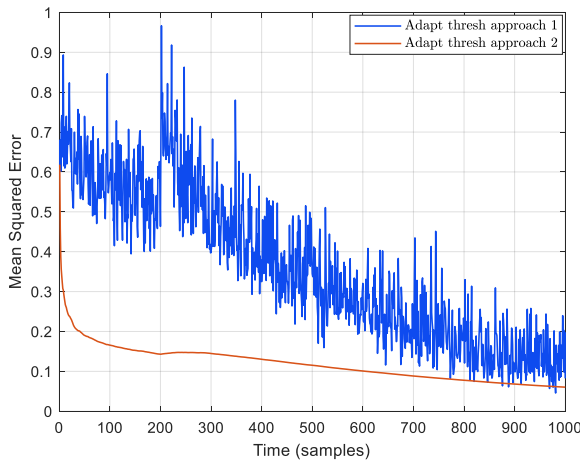


Fig. 3.18 MSE of the first and second adaptive approaches, for $\tau = 200$ samples and SNR = -10 dB.

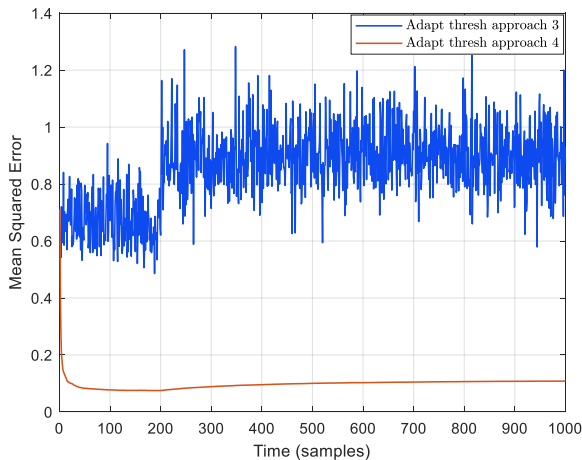


Fig. 3.19 MSE of the third and fourth adaptive approaches, for $\tau = 200$ samples and SNR = -10 dB.

For SNR = 0 dB, the performance of the four approaches are shown in Fig. 3.20 and 3.21. Now we observe a fast improvement of the estimation after τ , in the first, second and fourth approaches. The third approach always has the worst performance.

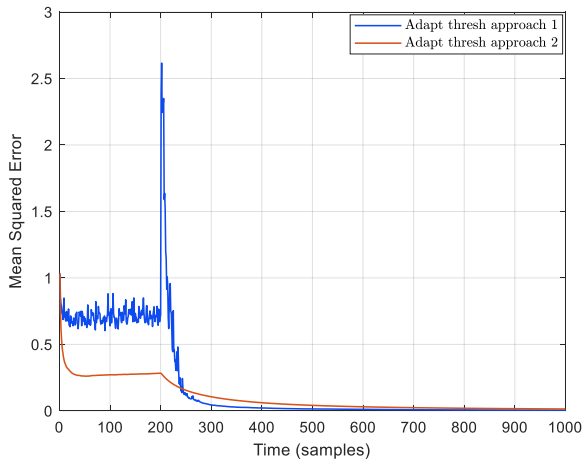


Fig. 3.20 MSE of the first and second adaptive approaches, for $\tau = 200$ samples and SNR = 0 dB.

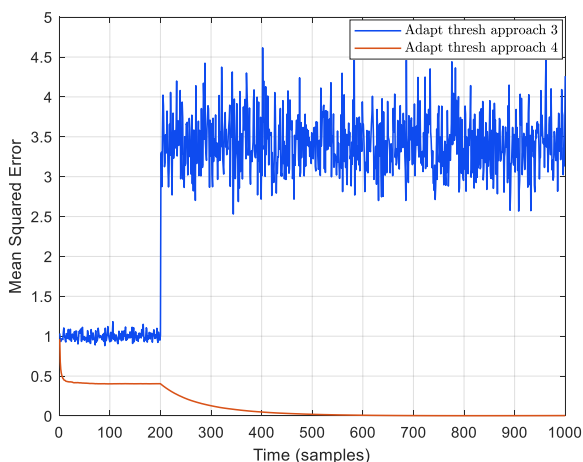


Fig. 3.21 MSE of the third and fourth adaptive approaches, for $\tau = 200$ samples and SNR = 0 dB.

For SNR = 10 dB, the performance of the four approaches are shown in Fig. 3.22 and 3.23. While the second and fourth approaches still have the same performance and a slow improvement after τ , the first approach is the best that gives good estimation few samples after τ .

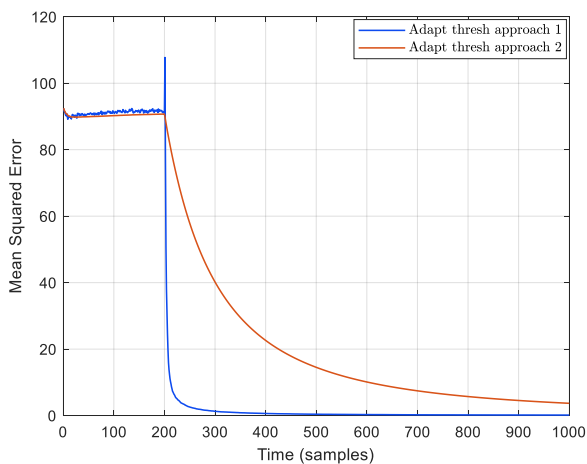


Fig. 3.22 MSE of the first and second adaptive approaches, for $\tau = 200$ samples and SNR = 10 dB.

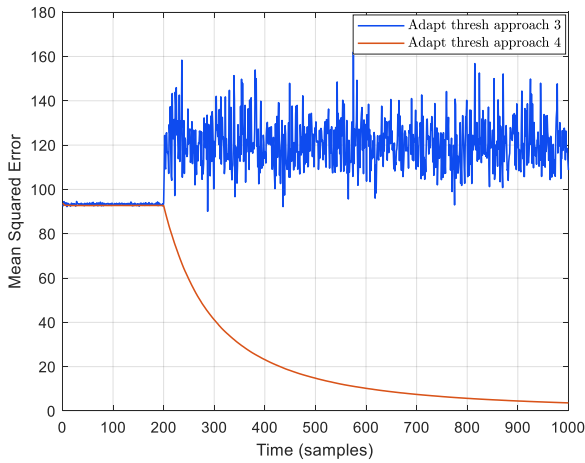


Fig. 3.23 MSE of the third and fourth adaptive approaches, for $\tau = 200$ samples and SNR = 10 dB.

Based on the MSE curves, we can conclude that for middle to high SNR values, the first approach can give a good estimation of the PU signal power, few samples after the PU appearance. The second and fourth approaches, which perform an average over time, improve the estimation only when the SNR is low. The third approach has the worst performance certainly because it only considers one sample, which is not enough to get an accurate estimation.

We then plot the global penalty curves obtained with the adaptive thresholds for GLR-CUSUM and LC GLR-Shiryaev. The performance of GLR-CUSUM is shown in Fig. 3.24. It can be seen that the first approach gives good results at high SNR, but very poor results at low SNR. This is consistent with the MSE curves analyses. The bad performance at low SNR is due to the fact that the estimation is very inaccurate, even many samples after the PU appearance. We plot the average detection delay and average false alarm duration corresponding to this approach in Fig. 3.25 and 3.26. We can observe that the high global penalty at low SNR is mostly due to large detection delays. The average false alarm duration is constant at every SNR because, before the PU appearance, the estimation always gives the same result (in average), and thus the same threshold is used, independently of the true SNR.

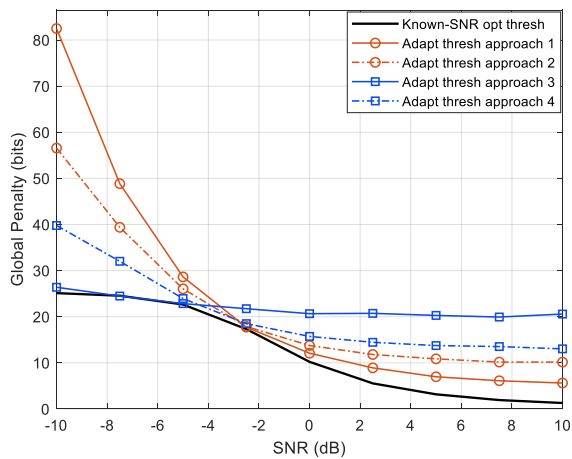


Fig. 3.24 Comparison of unknown-SNR adaptive thresholds and optimal known-SNR threshold for GLR-CUSUM.

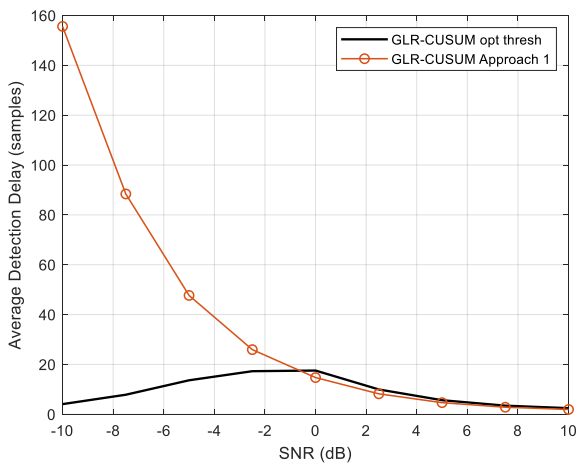


Fig. 3.25 $\bar{D}_{DI}$ vs SNR for adaptive threshold approach 1 and optimal known-SNR threshold (GLR-CUSUM).

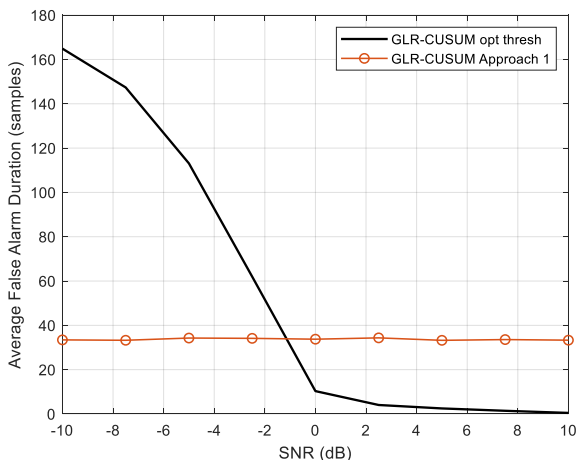


Fig. 3.26 $\bar{D}_{FA}$ vs SNR for adaptive threshold approach 1 and optimal known-SNR threshold (GLR-CUSUM).

The second approach, which averages the estimated $\hat{\sigma}_x^2$ obtained in the first approach, improves the global penalty at low SNR, but slightly degrades it at high SNR (Fig. 3.24). In fact, averaging over time decreases the detection delays at every SNR value, while it increases the false alarm duration. This is shown in Fig. 3.27 and 3.28.

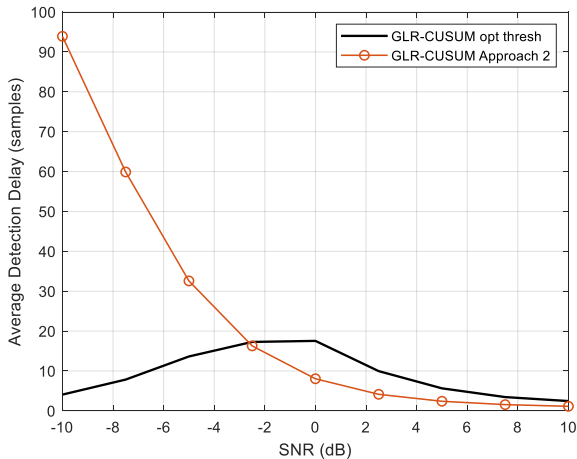


Fig. 3.27 $\bar{D}_{DI}$ vs SNR for adaptive threshold approach 2 and optimal known-SNR threshold (GLR-CUSUM).

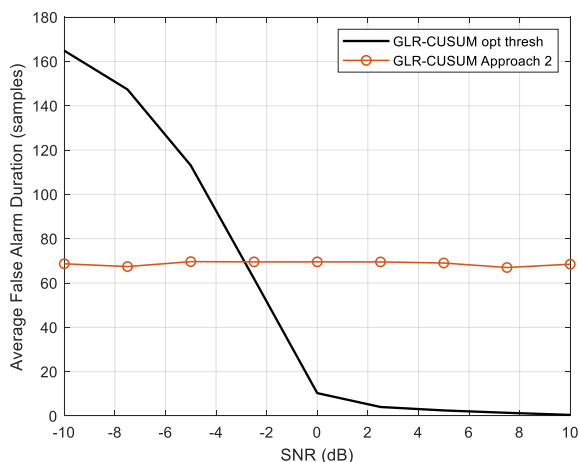


Fig. 3.28 $\bar{D}_{FA}$ vs SNR for adaptive threshold approach 2 and optimal known-SNR threshold (GLR-CUSUM).

The third approach, unsurprisingly, gives the worst global penalty performance at high SNR, but it seems to perform well at low SNR (Fig. 3.24). This bad performance at high SNR is mostly due to large false alarm durations, as shown in Fig. 3.29.

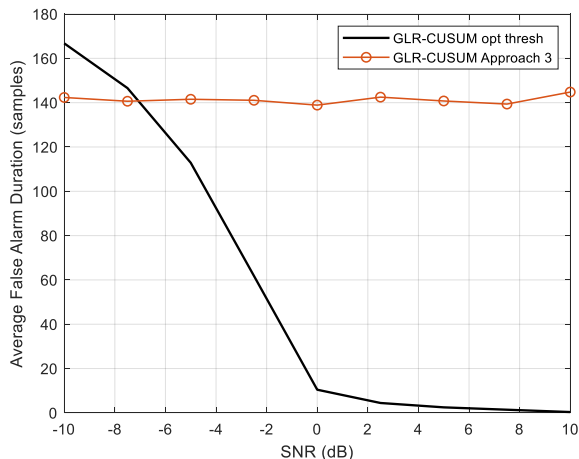


Fig. 3.29 $\bar{D}_{FA}$ vs SNR for adaptive threshold approach 3 and optimal known-SNR threshold (GLR-CUSUM).

The detection delays are relatively low, as depicted in Fig. 3.30. This suggests the idea that in this approach, the PU signal power is almost always underestimated after the PU appearance. The seemingly good result at low SNR is not due to accurate estimation, but to the fact that the balance between detection delays and false alarm durations results in global penalties closer to the optimal ones.

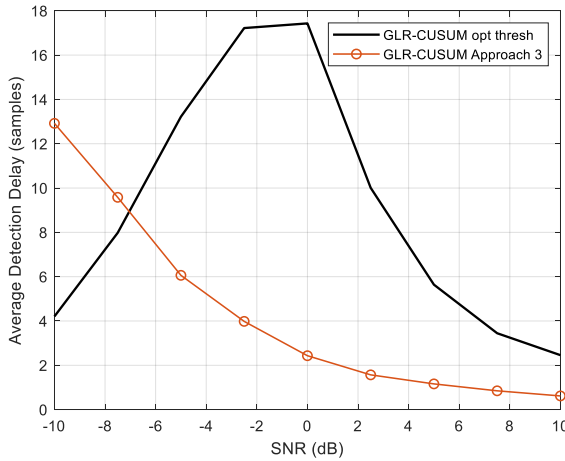


Fig. 3.30 $\bar{D}_{\text{DI}}$ vs SNR for adaptive threshold approach 3 and optimal known-SNR threshold (GLR-CUSUM).

The fourth approach, which averages the estimated $\tilde{\sigma}_x^2$ obtained in the third approach, slightly improves the global penalty at high SNR, but degrades it at low SNR (Fig. 3.24). We can see that it is closer to the second approach.

The performance of LC GLR-Shiryayev, when using the four adaptive approaches, is shown in Fig. 3.31. At low SNR, the first approach gives a performance closer to the optimal one, but at high SNR the global penalty does not decrease enough. This is in contrast with the GLR-CUSUM case, whereas the estimation in this approach gives almost similar results for both algorithms. Looking at the average detection delay and average false alarm duration, depicted in Fig. 3.32 and 3.33, we see that the bad performance (with respect to the optimal one) observed at high SNR is mostly due to the high false alarm duration, which is much lower for GLR-CUSUM. This large false alarm duration of LC GLR-Shiryayev is explained by the

fact that its decision statistic grows faster than the GLR-CUSUM decision statistic. This can be noticed in Fig. 3.34, where the average ratio of the decision statistic over the threshold is plotted in the first approach for both algorithms.

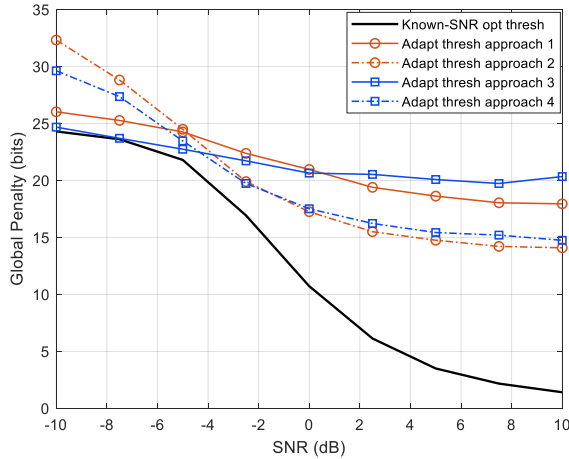


Fig. 3.31 Comparison of unknown-SNR adaptive thresholds and optimal known-SNR threshold for LC GLR-Shiryaev.

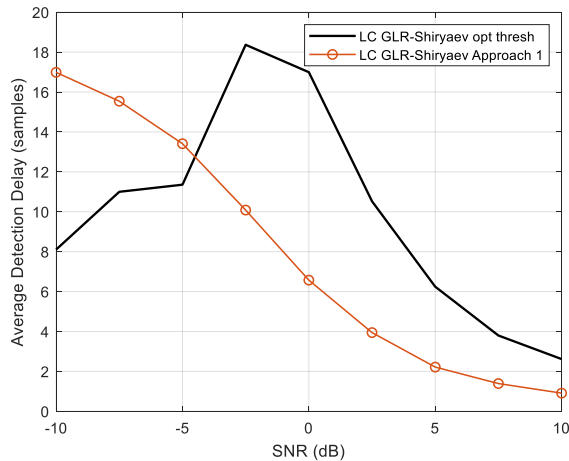


Fig. 3.32 $\bar{D}_{DI}$ vs SNR for adaptive threshold approach 1 and optimal known-SNR threshold (LC GLR-Shiryaev).

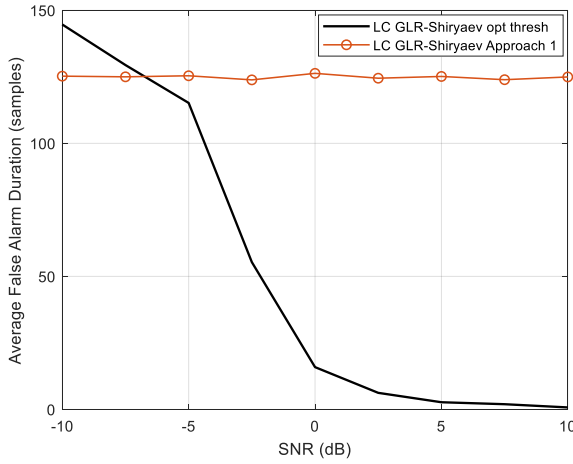


Fig. 3.33 $\bar{D}_{FA}$ vs SNR for adaptive threshold approach 1 and optimal known-SNR threshold (LC GLR-Shiryayev).

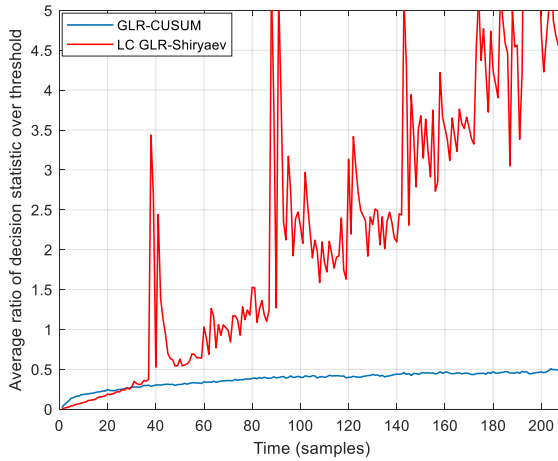


Fig. 3.34 Average ratio of decision statistic over threshold (approach 1) for GLR-CUSUM and LC GLR-Shiryayev.

The second approach, which averages over time the $\hat{\sigma}_x^2$ of the first one, improves the global penalty at high SNR but degrades it at low SNR.

For the third and fourth approach, the same trend is observed as in the GLR-CUSUM case.

The performance obtained with the first and second approaches, for both algorithms, is shown in Fig. 3.35. The best performance at low SNR

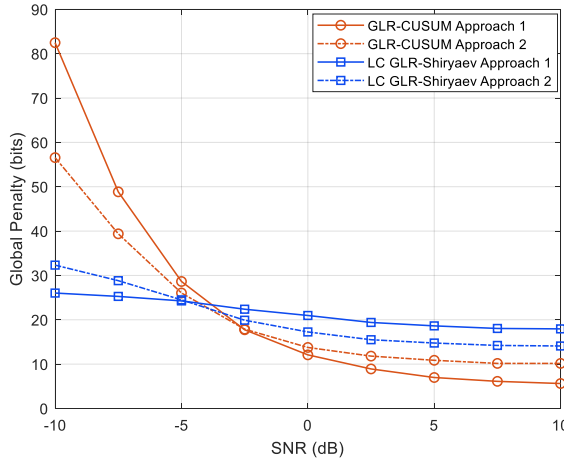


Fig. 3.35 Comparison of unknown-SNR adaptive thresholds (approach 1 & 2) for GLR-CUSUM and LC GLR-Shiryaev.

is obtained with LC GLR-Shiryaev, while at high SNR, it is obtained with GLR-CUSUM. The very poor performance of GLR-CUSUM at low SNR can be explained by its high sensitivity to the threshold, that has been previously mentioned. In fact, this sensitivity is more pronounced at low SNR and an error on the optimal threshold (resulting from an estimation error) can considerably degrade the global penalty.

In summary, it is challenging to obtain a very good performance with adaptive-threshold methods that try to estimate the SNR while detecting. In fact, a reliable estimate of the SNR cannot be obtained before the PU appearance time τ and this will necessarily add a delay to the detection. Moreover, even if at high SNR, the estimation becomes accurate few samples after τ (approach 1), the estimated values before τ may result in very large false alarm durations. At low SNR values, a good estimation needs more samples and results in larger detection delays. Averaging the estimated values over time is a good option but does not profit both low SNR and high SNR regions at the same time.

Fixed threshold

We compute the optimal fixed threshold in this section, by making the assumption that the PU signal amplitude is Rayleigh distributed and thus that the received power, or the SNR, is exponentially distributed. When we respectively choose the values -5 dB, 0 dB and 5 dB as the mean SNR, the computed optimal fixed thresholds for GLR-CUSUM and LC GLR-Shiryaev are shown in Table 3.3 below.

Table 3.3 Unknown-SNR fixed thresholds computed through simulations for $p = 0.005$.

SNR (dB) Algorithm	-5	0	5
GLR-CUSUM	2.2758	3.0599	4.4773
LC GLR-Shiryaev	193.0698	449.8433	2.8118e+03

The obtained performance in comparison with the optimal thresholds is plotted in Fig. 3.36 and 3.37. As might be expected, the fixed thresholds

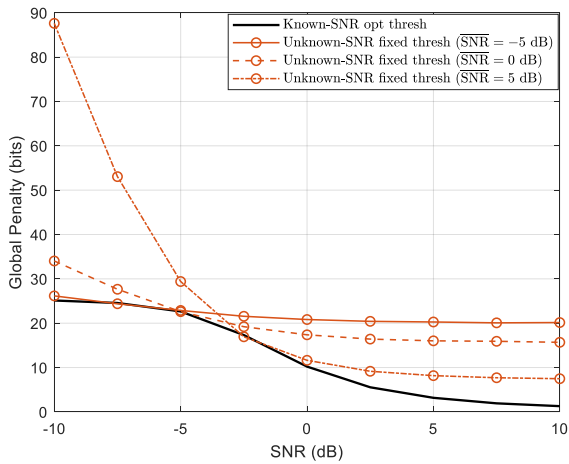


Fig. 3.36 Comparison of optimal threshold with unknown-SNR fixed thresholds (mean SNR = $\{-5, 0, 5\}$ dB) for GLR-CUSUM.

cannot work well at every SNR. Depending on the average SNR chosen for the computation, they might perform better at high SNR and worse at low SNR, or vice versa.

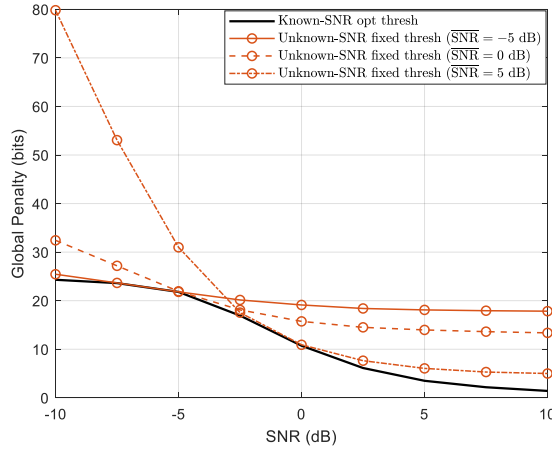


Fig. 3.37 Comparison of optimal threshold with unknown-SNR fixed thresholds (mean SNR = $\{-5, 0, 5\}$ dB) for LC GLR-Shiryaev.

To compare them for both GLR-CUSUM and LC GLR-Shiryaev, we plot the curves shown in Fig. 3.38. In contrast with the adaptive-threshold approaches where the best performance is obtained at low SNR with LC GLR-Shiryaev and at high SNR with GLR-CUSUM, we can see here that LC GLR-Shiryaev has the best performance at almost every SNR value, when the same distribution parameters are used.

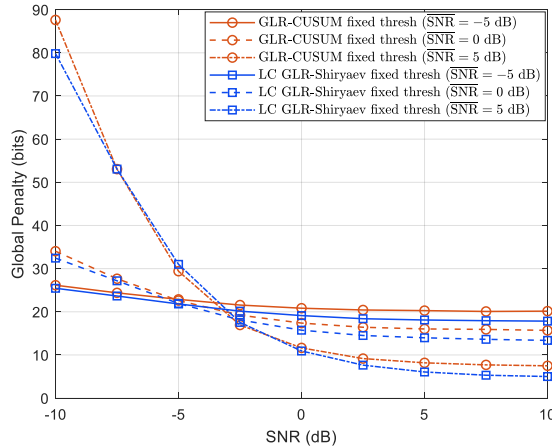


Fig. 3.38 Comparison of unknown-SNR fixed thresholds (mean SNR = $\{-5, 0, 5\}$ dB) for GLR-CUSUM and LC GLR-Shiryaev.

In brief, a fixed threshold can work well depending on the available *a priori* knowledge about the SNR, translated into the distribution parameters used for the threshold computation.

Adaptive versus fixed threshold

Comparing the fixed-threshold method with the first and second adaptive-threshold approaches, we obtain the results presented in Fig. 3.39 and 3.40. Let us first consider the GLR-CUSUM case (Fig. 3.39). The fixed threshold computed with a mean SNR of 5 dB performs closely to the first adaptive-threshold approach, even if it is slightly worse for $\text{SNR} \in [-10, -5]$ dB and $\text{SNR} \in [5, 10]$ dB. The fixed threshold computed with a mean SNR of 0 dB, and constraining the SNR to be within the range $[-10, 10]$ dB, performs closely to the second adaptive-threshold approach. It is better than the adaptive method at low SNR, but slightly worse at high SNR. Let us now consider the LC GLR-Shiryayev case (Fig. 3.40). The fixed threshold computed with a mean SNR of -5 dB performs better than the first adaptive-threshold approach at every SNR. The same is observed for the fixed threshold computed with a mean SNR of 0 dB with respect to the second adaptive-threshold approach.

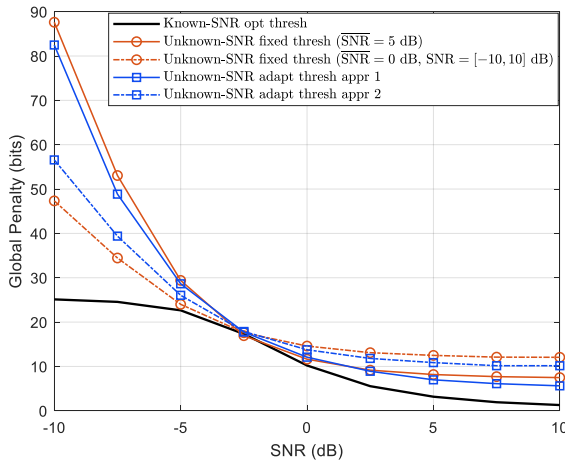


Fig. 3.39 Comparison of optimal threshold, unknown-SNR adaptive thresholds and unknown-SNR fixed thresholds (mean $\text{SNR} = \{0, 5\}$ dB) for GLR-CUSUM.

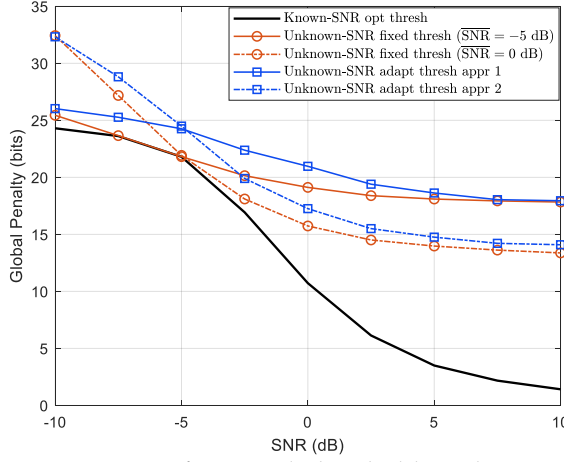


Fig. 3.40 Comparison of optimal threshold, unknown-SNR adaptive thresholds and unknown-SNR fixed thresholds (mean SNR = $\{-5, 0\}$ dB) for LC GLR-Shiryaev.

In summary, a simple fixed threshold can perform closely to the proposed adaptive-threshold methods and even perform better in certain cases, if the distribution parameters, assumed for the threshold computation, are appropriately chosen. Obviously, if the SNR is *a priori* known to belong to a smaller range, this could help increase the performance of both threshold-setting approaches in this range.

3.6

Complexity analysis of LC GLR-Shiryaev and M-Shiryaev

In this section, we evaluate the computational complexity of the derived LC GLR-Shiryaev and the M-Shiryaev algorithm, for hardware implementation. As already noticed, LC GLR-Shiryaev is an analytical solution to the scenarios where the SU does not know the PU signal power σ_x^2 . It does not require any *a priori* knowledge of σ_x^2 , but at each time sample m , it has to compute $m \tilde{\sigma}_{x,k}^2$ values, to obtain the decision statistic (3.12). Regarding the M-Shiryaev algorithm, it is rather a numerical solution. It assumes that the SU knows M possible values of σ_x^2 (termed here as $\sigma_{x,i}^2$ for $i \in [1, M]$), and consists in running M parallel Shiryaev algorithms for each of these

values. The PU appearance is declared when any one of the parallel algorithms exceeds the threshold. So, M-Shiryaev has to compute M Shiryaev decision statistics for known values of the PU signal power. Considering our signal model, we can write them as follows

$$\Lambda_{\text{Shi},i}(\mathbf{y}_1^m) = \sum_{k=1}^m \frac{1}{(1-p)^{m-k+1}} \left(\left(\frac{\sigma_w^2}{\sigma_{x,i}^2 + \sigma_w^2} \right)^{m-k+1} \exp \left(\frac{\sigma_{x,i}^2}{\sigma_w^2 (\sigma_{x,i}^2 + \sigma_w^2)} \sum_{n=k}^m |y[n]|^2 \right) \right) \text{ for } i \in [1, M]. \quad (3.17)$$

Using the recursive implementation of Shiryaev, the statistics become

$$\Lambda_{\text{Shi},i}(\mathbf{y}_1^m) = (1 + \Lambda_{\text{Shi},i}(\mathbf{y}_1^{m-1}) - 1) \left(\frac{1}{1-p} \right) \frac{\sigma_w^2}{\sigma_{x,i}^2 + \sigma_w^2} \exp \left(\frac{|y(m)|^2 \sigma_{x,i}^2}{\sigma_w^2 (\sigma_{x,i}^2 + \sigma_w^2)} \right), \quad \text{for } i \in [1, M]. \quad (3.18)$$

We thus want to compare the complexities in computing the decision statistics (3.12, 3.17, 3.18) at each time sample m , for both algorithms.

3.6.1 Computational complexity calculation

We express the complexity in terms of the number of elementary operations needed to run the algorithms at each time sample m . The most used elementary operations are *real summation* (RS), *real multiplication* (RM), *real division* (RD), *real exponential function* (RE). A real exponentiation a^b can be expressed in terms of RM, assuming that fast exponentiation algorithms use at most $2 \log_2(b)$ RM.

LC GLR-Shiryaev's complexity

The complexity of LC GLR-Shiryaev depends on both the computations of the $\tilde{\sigma}_{x,k}^2$ and $\Lambda_{\text{LC GLR-Shi}}(\mathbf{y}_1^m)$ (3.12).

To compute each $\tilde{\sigma}_{x,k}^2$, we need $(2m - 2k + 2)$ RM, $(2m - 2k + 4)$ RS and 1 RD. Thus, the computation of the m $\tilde{\sigma}_{x,k}^2$ needs $\sum_{k=1}^m (2m - 2k + 2) = (m^2 + m)$ RM, $\sum_{k=1}^m (2m - 2k + 4) = (m^2 + 3m)$ RS and m RD.

3 | Spectrum Sensing Using Bayesian Changepoint Detection

As the term $\sum_{n=k}^m |y[n]|^2$ is already computed in $\hat{\sigma}_{x,k}^2$, it can be reused in the computation of the decision statistic. Thus, the k^{th} term of $\Lambda_{LC\ GLR-Shi}(\mathbf{y}_1^m)$ needs $(4 + 4\log_2(m - k + 1))$ RM, 7 RS, 3 RD and 1 RE. To compute and add the m terms, we need

$\sum_{k=1}^m (4 + 4\log_2(m - k + 1)) = (4m + 2m\log_2(m))$ RM, $(7m + m - 1)$ RS, $3m$ RD and m RE.

In total, at each time sample m , we need $(m^2 + 5m + 2m\log_2(m))$ RM, $(m^2 + 11m - 1)$ RS, $4m$ RD and m RE, to run LC GLR-Shiryaev.

M-Shiryaev's complexity

Following the same reasoning, the complexity of both implementations of M-Shiryaev is computed. At each time sample m , M statistics are computed.

In the direct implementation (3.17), the terms $\sum_{n=k}^m |y[n]|^2$ and $\frac{1}{(1-p)^{m-k+1}}$ can be computed once and reused in the $M - 1$ other statistics. In total, it needs $(m^2 + (4M + 1)m + (M + 1)m\log_2(m))$ RM, $(m^2 + (4M + 3)m)$ RS, $((2M + 1)m)$ RD and (mM) RE.

The recursive implementation (3.18) needs $(5M + 2)$ RM, $(3M + 2)$ RS, $(2M + 1)$ RD and M RE.

The complexity analysis can be summarized in Table 3.4, where the LC GLR-Shiryaev, the direct M-Shiryaev and the recursive M-Shiryaev algorithms are respectively designated by their decision statistics equations numbers (3.12, 3.17, 3.18).

Table 3.4 Complexity analysis of LC GLR-Shiryaev and M-Shiryaev.

	No. of RM	No. of RS	No. of RD	No. of RE
(3.12)	$m^2 + 5m$ $+ 2m\log_2(m)$	m^2 $+ 11m - 1$	$4m$	m
(3.17)	$m^2 + (4M + 1)m$ $+ (M + 1)m\log_2(m)$	m^2 $+ (4M + 3)m$	$(2M + 1)m$	mM
(3.18)	$5M + 2$	$3M + 2$	$2M + 1$	M

3.6.2 Numerical results of the complexity

Numerical comparisons are shown in this section by analysing the behaviour of the complexities for different values of m and M . For a fixed value of M , we plot the complexity with respect to the current number of time samples m . We just focus on the number of real multiplications, as the other operations have almost the same trend. As it can be seen in Fig. 3.41, the complexities of LC GLR-Shiryaev and the direct M-Shiryaev grow quadratically ($O(m^2)$) when the number of samples increases, but the recursive M-Shiryaev has a constant complexity ($O(M)$). The direct M-Shiryaev is worse than LC GLR-Shiryaev when M is high, but has a closer complexity when M is very low. As shown in Fig. 3.42, the only case when LC GLR-Shiryaev and the recursive M-Shiryaev have comparable complexity is when M is high and the number of observed samples is low.

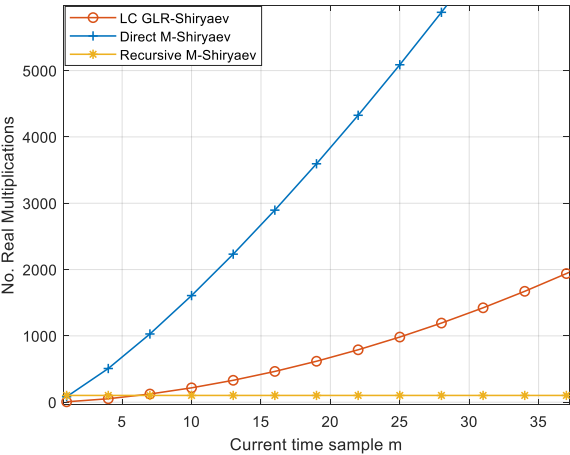


Fig. 3.41 Complexity in terms of No. of Real Multiplications for $M = 20$.

To conclude, the recursive M-Shiryaev has the lowest complexity for relatively moderate values of M . However, this requires an accurate *a priori* knowledge of the SNR, otherwise the performance will degrade. LC GLR-Shiryaev has the advantage of not requiring any *a priori* knowledge, but it adds some complexity, especially when the number of observed samples is large. Therefore, to make a choice between both algorithms, we should

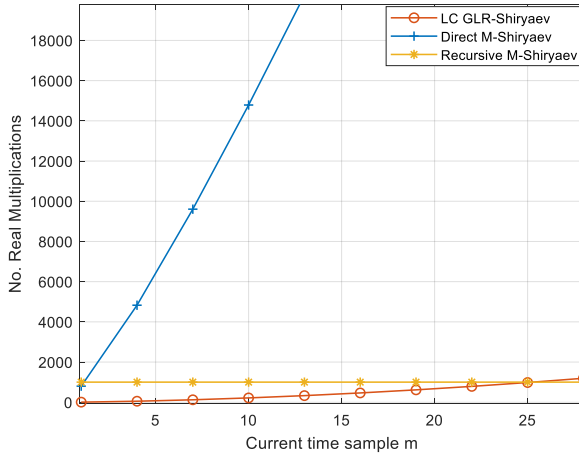


Fig. 3.42 Complexity in terms of No. of Real Multiplications for $M = 200$.

consider the accuracy of existing knowledge about the SNR, the performance and the complexity induced.

3.7 Conclusion

In this chapter, Bayesian changepoint detection is used for spectrum sensing in mobile cognitive radio. Assuming that the mobility parameters of the SU can be translated into an *a priori* probability of PU appearance, a low-complexity algorithm, called LC GLR-Shiryaev, is introduced. Based on the Shiryaev algorithm, it uses an approximation of the GLRT approach for situations where the PU's signal power is unknown to the SU. Even if it is suboptimal, LC GLR-Shiryaev has a good performance and shows a reduced complexity, when compared with the GLR-based Shiryaev, which needs to solve a complex non-convex optimization problem at each time sample. Moreover, thanks to the prior knowledge of the PU appearance, it gives a better performance than the GLR-based CUSUM in low SNR situations, which are very typical in CR.

In addition, through the introduction of the new global penalty metric,

a new way of setting the optimal threshold is introduced. It minimizes the average loss in the overall channel usage during a time-limited communication of the SU, by appropriately balancing between the interference time and the spectrum waste time. As the optimal threshold depends on the SNR, some adaptive and fixed threshold methods are suggested in unknown-SNR situations. Under certain parameters values assumptions for the fixed threshold, both methods give almost the same performance.

Finally, a complexity analysis of LC GLR-Shiryaev and M-Shiryaev algorithms is performed. While being less complex than the proposed LC GLR-Shiryaev, the recursive implementation of M-Shiryaev needs an accurate *a priori* knowledge of the SNR, which is usually not available, to give a good performance. In contrast, albeit more complex, LC GLR-Shiryaev does not require any *a priori* knowledge of the SNR and performs quite well.

3.A Appendix

3.A.1 Proof of non-concavity of the GLR-Shiryaev statistic

Let us recall the optimization problem (3.6)

$$\max_{\sigma_x^2 \geq 0} \Lambda_{\text{Shi}}(\mathbf{y}_1^m, \sigma_x^2) = \sum_{k=1}^m \frac{1}{(1-p)^{m-k+1}} \left(\left(\frac{\sigma_w^2}{\sigma_x^2 + \sigma_w^2} \right)^{m-k+1} \exp \left(\frac{\sigma_x^2 \sum_{n=k}^m |y[n]|^2}{\sigma_w^2 (\sigma_x^2 + \sigma_w^2)} \right) \right),$$

that needs to be solved for the GLR-Shiryaev algorithm derivation. We can write

$$\Lambda_{\text{Shi}}(\mathbf{y}_1^m, \sigma_x^2) = \frac{1}{(1-p)^{m+1}} \sum_{k=1}^m (1-p)^k \left(\left(\frac{\sigma_w^2}{\sigma_x^2 + \sigma_w^2} \right)^{m-k+1} \exp \left(\frac{\sigma_x^2 \sum_{n=k}^m |y[n]|^2}{\sigma_w^2 (\sigma_x^2 + \sigma_w^2)} \right) \right),$$

and, as $\frac{1}{(1-p)^{m+1}}$ is a positive constant, the optimization problem can be equivalently written as follows

$$\min_{\sigma_x^2 \geq 0} \Delta(\sigma_x^2) = - \sum_{k=1}^m (1-p)^k \left(\left(\frac{\sigma_w^2}{\sigma_x^2 + \sigma_w^2} \right)^{m-k+1} \exp \left(\frac{\sigma_x^2 \sum_{n=k}^m |y[n]|^2}{\sigma_w^2 (\sigma_x^2 + \sigma_w^2)} \right) \right). \quad (3.19)$$

The function $\Lambda_{\text{Shi}}(\mathbf{y}_1^m, \sigma_x^2)$ is concave in σ_x^2 if $\Delta(\sigma_x^2)$ is convex. We can write

$$\Delta(\sigma_x^2) = \sum_{k=1}^m q_k g_k(\sigma_x^2),$$

with $q_k = (1-p)^k > 0$ and $g_k(\sigma_x^2) = - \left(\frac{\sigma_w^2}{\sigma_x^2 + \sigma_w^2} \right)^{m-k+1} \exp \left(\frac{\sigma_x^2 \sum_{n=k}^m |y[n]|^2}{\sigma_w^2 (\sigma_x^2 + \sigma_w^2)} \right)$.

If each $g_k(\sigma_x^2)$ is convex, then $\Delta(\sigma_x^2)$ is convex. The second order derivative

of $g_k(\sigma_x^2)$ is given by

$$\frac{\partial^2 g_k(\sigma_x^2)}{\partial (\sigma_x^2)^2} = \frac{g_k(\sigma_x^2)}{(\sigma_x^2 + \sigma_w^2)^4} \left((m - k + 1) - \sum_{n=k}^m |y[n]|^2 \right) \left((m - k + 1) - \sum_{n=k}^m |y[n]|^2 + 2(\sigma_x^2 + \sigma_w^2) \right).$$

As $\forall \sigma_x^2 \geq 0$, $\frac{g_k(\sigma_x^2)}{(\sigma_x^2 + \sigma_w^2)^4} < 0$, $g_k(\sigma_x^2)$ is convex if for every possible realization of the observation $\mathbf{y}_1^m$, we have

$$\left((m - k + 1) - \sum_{n=k}^m |y[n]|^2 \right) \left((m - k + 1) - \sum_{n=k}^m |y[n]|^2 + 2(\sigma_x^2 + \sigma_w^2) \right) \leq 0, \\ \forall \sigma_x^2 \geq 0.$$

By means of simulations, we can observe that for several realizations of $\mathbf{y}_1^m$ and different parameters values, there are several values of k , for which this is not true. Therefore, $g_k(\sigma_x^2)$ is a non-convex function and accordingly, the GLR-Shiryaev statistic $\Lambda_{\text{Shi}}(\mathbf{y}_1^m, \sigma_x^2)$ is non-concave.

Another way to prove the non-concavity of the GLR-Shiryaev statistic could be to plot the statistic opposite, $-\Lambda_{\text{Shi}}(\mathbf{y}_1^m, \sigma_x^2)$, for several observation realizations and parameters values, to check for non-convexity cases. An example of that is shown in Fig. 3.43, where the curve $-\Lambda_{\text{Shi}}(\mathbf{y}_1^m, \sigma_x^2)$ vs σ_x^2 is plotted for the following parameters values $p = 0.005$, $\tau = 123$, $m = 125$, and $\mathbf{y}_1^m$ is generated for $\sigma_x^2 = 5$ and $\sigma_w^2 = 1$. The existence of a local minimum can be observed, showing that the function $-\Lambda_{\text{Shi}}(\mathbf{y}_1^m, \sigma_x^2)$ is non-convex.

3.A.2 Penalties computation

Let us assume a multi-band transmission for the secondary user. We denote the total available number of sub-bands and power for secondary transmission respectively by N_b and σ_S^2 . The noise power in each sub-band at the SU receiver is denoted by σ_N^2 . Assuming a uniform power allocation and a constant SNR at every sub-band, the channel rate at each sub-band can be obtained from the Shannon capacity in AWGN channel [11] and is

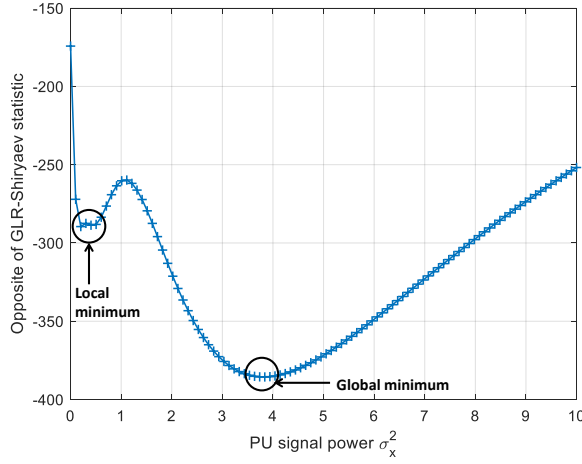


Fig. 3.43 Typical realization of the GLR-Shyriaev statistic opposite $-\Lambda_{\text{Shi}}(\mathbf{y}_1^m, \sigma_x^2)$ with respect to the PU signal power σ_x^2 .

given by

$$R_{sb} = \frac{1}{2} \log_2 \left(1 + \frac{\sigma_S^2}{N_b \sigma_N^2} \right) \text{ bits/sample}, \quad (3.20)$$

where $\frac{\sigma_S^2}{N_b \sigma_N^2}$ is the SNR at each sub-band. The total channel rate is given by

$$R = \frac{N_b}{2} \log_2 \left(1 + \frac{\sigma_S^2}{N_b \sigma_N^2} \right) \text{ bits/sample}. \quad (3.21)$$

Assume that a PU is transmitting in one sub-band and the SU did not detect this activity. This will result in an interference and the data transmitted in this sub-band will be lost. The total channel rate will become

$$R_{int} = \frac{N_b - 1}{2} \log_2 \left(1 + \frac{\sigma_S^2}{N_b \sigma_N^2} \right) \text{ bits/sample}. \quad (3.22)$$

The interference penalty is the loss in the total channel rate, caused by the interference. It is given by

$$\begin{aligned} P_{int} &= R - R_{int} \\ &= \frac{1}{2} \log_2 \left(1 + \frac{\sigma_S^2}{N_b \sigma_N^2} \right) \text{ bits/sample}. \end{aligned} \quad (3.23)$$

If the SU vacates a sub-band, thinking it is occupied by a PU, while it is not (false alarm), this will result in spectrum waste. The available power σ_S^2 will be redistributed among the $N_b - 1$ other sub-bands and the total channel rate will now become

$$R_{sow} = \frac{N_b - 1}{2} \log_2 \left(1 + \frac{\sigma_S^2}{(N_b - 1) \sigma_N^2} \right) \text{ bits/sample.} \quad (3.24)$$

The penalty due to spectral opportunity waste is the loss in the total channel rate given by

$$\begin{aligned} P_{sow} &= R - R_{sow} \\ &= \frac{N_b}{2} \log_2 \left(1 + \frac{\sigma_S^2}{N_b \sigma_N^2} \right) - \frac{N_b - 1}{2} \log_2 \left(1 + \frac{\sigma_S^2}{(N_b - 1) \sigma_N^2} \right) \text{ bits/sample.} \end{aligned} \quad (3.25)$$

Using as numerical values $N_b = 29$ sub-bands (taken as example from Section 2.4) and $\text{SNR} = \frac{\sigma_S^2}{N_b \sigma_N^2} = 0$ dB, we obtain the values $P_{int} = 0.5$ and $P_{sow} = 0.14$, which are used in the simulations.

4

Joint Transmission and Sensing Framework

4.1 Introduction

A cognitive radio network is reliable by the ability of the secondary user to maximize the spectrum usage while minimizing the interference on the primary user. This strongly depends on how sensing and transmission are scheduled by the SU. It is impossible for a SU hardware that does not use full duplex techniques to simultaneously transmit data and sense the spectrum. For this reason, the current CR hardwares use an approach based on periodic sensing [93]. Time is split into blocks or frames, where an amount of time is allocated to spectrum sensing and another to data transmission. Obviously, the longer the sensing time, the more accurate the spectrum sensing outcome will be. However, there will be too much spectrum waste if the sensing time is too long. In contrast, the shorter the sensing time, the more time will be available for transmission, but a too short sensing time will induce too much interference because of sensing inaccuracy. Therefore, there should be a trade-off between the sensing duration and the transmission duration [94].

This topic is abundantly covered in the literature. Most of the existing works assume a sense-then-transmit strategy [94]. The CR frame begins with the sensing period, and if the PU is detected to be absent, the transmission period is used for transmitting. If the PU is detected to be present, there is no transmission until the next frame, where spectrum sensing is performed again. Energy detection (ED) is often used as spectrum sensing method because of its simplicity. While some authors derive the optimal sensing time for a fixed frame duration [95], others jointly optimize the sensing and transmission durations [96, 97]. For example, the optimal sensing time that maximizes the SU throughput, with a constraint on the interference to the PUs, is derived in [95]. Authors of [96] propose a periodic sensing scheduling scheme and found the optimal sensing frequency and sensing time that minimize the sensing overhead and average detection time. In [97], a joint optimization of sensing and transmission times is performed to maximize the energy efficiency, subject to spectrum sensing accuracy and limited interference. Unlike the previous works that only use a block-based sensing (ED), authors of [94] investigate the throughput-sensing trade-off for CR networks when quickest sensing (or changepoint detection) is used. Always assuming a sense-then-transmit strategy, they compute the optimal frame length that maximizes the throughput and conclude that quickest sensing could achieve higher throughput than block-based sensing, while requiring shorter frame length and sensing time. Note that an independent quickest sensing is carried out in each frame.

In this chapter, we investigate the issue of joint transmission and sensing in a mobile CR scenario. We consider the same scenario as in Chapter 3, but here we assume that multiple changes in the spectrum occupancy may occur when the SU is moving. We always assume, as in Chapter 3, that the SU has a limited amount of time for communication, and that the *a priori* probabilities of PU appearance and disappearance can be deduced from the SU's mobility parameters. Taking all this into consideration, a new joint transmission-sensing framework is introduced. Unlike previous works in the literature, that perform a new spectrum sensing at the beginning of each frame to detect the current channel state, we suggest sensing the spectrum sequentially along several frames, in order to detect changes in the channel state. For this, we use a changepoint detection algorithm over successive frames. This choice is motivated by the fact that if the spectrum occupancy does not change too quickly, we can combine observations

at successive frames to rapidly detect a change. This will result in less time allocated for sensing per frame than in the case of independent sensing at each frame. To achieve this, not only the duration of observation in each frame, but also the frequency of observation, should be carefully chosen. Our goal is thus to find the optimal parameters (sensing time, transmission time, frame size) for the joint transmission-sensing framework. To compute the optimal sensing and transmission times, we introduce a new performance metric, called the global cost, that gives a trade-off between interference and spectrum waste times during a time-limited communication of the SU. This metric is inspired from the global penalty presented in [98] and Chapter 3. In most of the existing works, the detection threshold is not optimized together with sensing time and transmission time, but it is fixed for a desired detection performance or to meet a certain interference constraint. In this work, we suggest to jointly optimize the detection threshold with sensing time and transmission time, to satisfy an overall minimum global cost. By means of simulations, the optimal parameters values are studied in detail, with respect to the SNR and the SU mobility parameters, translated into the *a priori* probabilities of PU appearance and disappearance. Our main contributions in this chapter are the following:

- We introduce a new joint transmission-sensing framework, based on sequential changepoint detection.
- We also introduce the global cost metric that jointly evaluates the costs of interference and spectrum waste induced by the framework.
- Then, we use the global cost metric to compute the optimal parameters.
 - Firstly, we optimize the parameters one by one, by fixing the others. Hence, for fixed frame size and threshold, we compute the optimal sensing time for given SNR values, and we suggest a simple method to derive a range of optimal sensing times when the SNR is unknown but lies within a certain interval. Then, for fixed sensing time and threshold, we compute the optimal transmission time (or equivalently the optimal frame size).
 - Secondly, always with a fixed threshold, we perform a joint optimization of the sensing time and the transmission time by using exhaustive search, and we suggest a simple algorithm that is more time-efficient than exhaustive search for the optimization.

- Thirdly, we jointly optimize the detection threshold with the sensing and transmission times, using exhaustive search.

The numerical results show that all the parameters (frame size, sensing time, transmission time and threshold) are interrelated and by jointly choosing optimal values, we can achieve a better global cost than when only some parameters are optimized. Furthermore, the optimal parameters distinctively depend on the SNR and the SU mobility parameters. It is actually noticed that the optimal sensing time strongly depends on the SNR and slightly on the threshold, while the optimal transmission time strongly depends on the mobility parameters and the threshold. The optimal threshold, for its parts, depends on both the SNR and the SU mobility parameters. Therefore, based on these results, we give intuitive rules to choose the optimal parameters.

- Finally, for comparison purposes, we replace the changepoint detection algorithm with the classical ED, and perform spectrum sensing firstly frame by frame and secondly over a block of frames. The superiority of changepoint detection is shown.

The rest of the chapter is organized as follows. The system model of the joint transmission-sensing framework, including the scenario description and the spectrum sensing method, is introduced in Section 4.2. Section 4.3 presents some performance metrics and formulates the optimization problem. Then, in Section 4.4, the numerical results for the joint optimization of sensing time, transmission time and threshold, are given. The comparison with ED algorithm is presented in Section 4.5. Finally, Section 4.6 concludes the chapter.

4.2 System model

4.2.1 System description

As stated in the introduction, the scenario considered in this chapter is similar to the one of Chapter 3 (Fig. 3.1), except some differences. We assume a CR system with multiple geographically-separated fixed PUs transmitting

in a frequency band, and a mobile SU, which tries to access the same band opportunistically. As it moves, the SU may enter or leave the coverage area of different PUs, and the spectrum occupancy will change. We consider that the SU is communicating for a certain amount of time N , during which there may be several PU appearances and disappearances. Let us denote by $\tau_i^{(1)}$ the i^{th} PU appearance time and $\tau_i^{(0)}$ the i^{th} PU disappearance time, which are both assumed to be *a priori* geometrically distributed with parameter p . The parameter p thus represents the average number of PU appearances or disappearances per unit of time sample. As in Chapter 3, it is assumed that p can be inferred from the mobility parameters of the SU. The SU should not only quickly detect each spectrum release in order to start the secondary communication, but it should also quickly detect each PU appearance to avoid interference. For this reason, it should be continuously measuring the activity in the band, to maximize the use of spectrum opportunities and minimize interference on PUs. Without loss of generality, we assume that the SU looks at only one band, but the same reasoning applies for a multi-band system.

We consider the joint transmission-sensing scheme shown in Fig. 4.1, where the communication time N is split into N_f frames of duration T_f ($N = N_f T_f$). Unlike the existing works that sense the spectrum on a frame by frame basis (one frame at a time, irrespective of the other frames), we suggest to sequentially perform spectrum sensing over successive frames. This means that we do not take an independent decision at each frame, but we sequentially combine observations at several frames, to detect a change in the spectrum. The SU has two operation modes. The *sensing-only mode* (SOM) is turned on when the spectrum has been detected to be busy. In this mode, the whole frame is used for spectrum sensing until the next change detection. When the spectrum is detected to be free, the SU switches to the *transmission mode* (TM). In the transmission mode, each frame is composed of a transmission block of time T_{tr} and a sensing block of time T_{se} . They are separated by guard blocks of times T_{Ga} and T_{Gb} , to allow switching between both operations, as only one half-duplex radio frequency chain is used. A frame duration is thus given by

$$T_f = T_{tr} + T_{se} + T_{Ga} + T_{Gb}.$$

It is supposed that initially, the SU is not covered by any PU signal, and it turns on the transmission mode until it detects a PU appearance and

switches to the sensing-only mode.

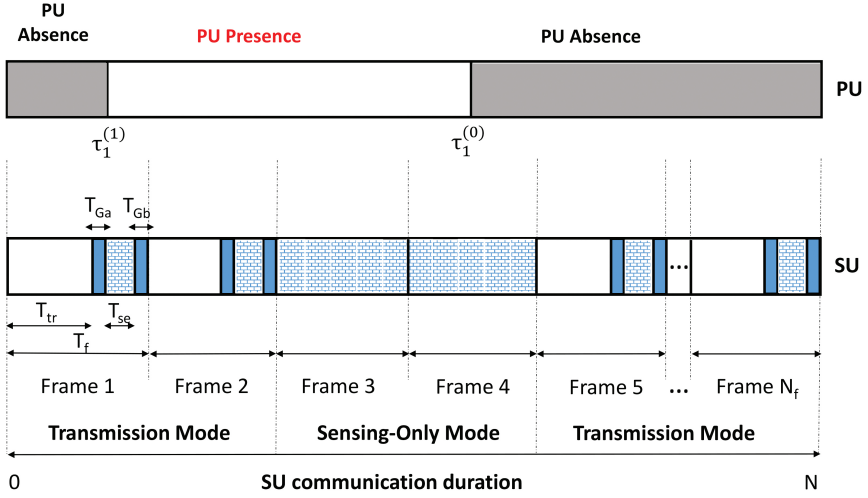


Fig. 4.1 Joint transmission-sensing system model.

4.2.2 Spectrum sensing

The signal observed by the SU is denoted by $y_l[n]$, where $l = 1, 2, \dots, N_f$ is the frame index, and n is the sensing sample index within the frame. When the SU is out of the PUs ranges, $y_l[n] = w_l[n]$, where $w_l[n]$ is the Additive White Gaussian Noise (AWGN) sample at the frame l . When it is within a PU range, $y_l[n] = x_l[n] + w_l[n]$, where $x_l[n]$ is the transmitted signal affected by the channel. It is assumed that $x_l[n]$ and $w_l[n]$ are independent circularly symmetric complex Gaussian variables, $x_l[n] \sim \mathcal{CN}(0, \sigma_x^2)$, $w_l[n] \sim \mathcal{CN}(0, \sigma_w^2)$, and independent of each other.

It is worth mentioning that our goal in this work is to find the optimal performance that can be achieved by our framework in case of perfect knowledge of the SU's mobile environment and the PU signal. For this reason, we decide, as a first step, to keep the model simple by assuming that the received signal power σ_x^2 is constant for all the PUs and that it is perfectly known by the SU. Thus, the spectrum sensing algorithm assumes this knowledge. Moreover, we do not use a Bayesian detection approach, that includes the SU mobility parameters, as in Chapter 3. But any change-

point detection algorithm could be used in the framework, and Bayesian algorithms could be considered thereafter, to observe the effect of using the SU mobility parameters on the framework performance.

For spectrum sensing, we use the CUSUM changepoint detection algorithm [57, 49] over several frames. As a reminder, a changepoint detection algorithm observes a data sequence and has to decide when there is a change in the distribution of the observed samples. A decision statistic is computed at each time sample, based on the previous samples, and is compared with a threshold. In this work, we use the CUSUM algorithm as presented in Chapter 2, but with a slight modification. Instead of computing a decision statistic at the sample scale, we do it at the frame scale. We do this to avoid the complexity of taking multiple decisions per frame and to take advantage of the combination of several samples within one frame. Thus, depending on the current mode, the sensing samples of each frame l ($y_l[1], \dots, y_l[T_{se}]$ in the TM and $y_l[1], \dots, y_l[T_f]$ in the SOM) are combined to form one single sample z_l , which is then input to the CUSUM algorithm. We compute z_l by doing the average power of the sensing samples of the frame l . This is a simple combining scheme that helps reduce the noise power. Note that, either for PU appearance detection or PU disappearance detection, the frames involved in spectrum sensing are the frames of the current mode. Hence, spectrum sensing is restarted when the SU switches to another mode. The spectrum sensing scheme is depicted in Fig. 4.2. At

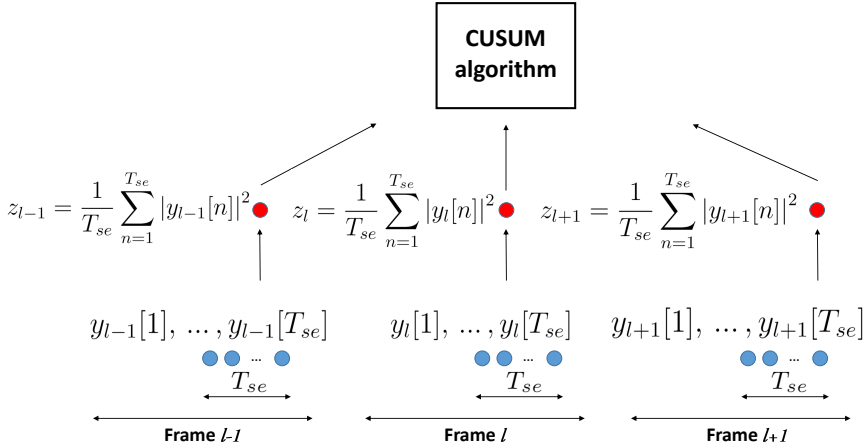


Fig. 4.2 Spectrum sensing scheme in the transmission mode.

4 | Joint Transmission and Sensing Framework

the end of each frame, the CUSUM algorithm computes a decision statistic C_j , where j refers to the current frame, and compares it with a threshold α . The detection time is given by [49]

$$T_{\text{CUS}} = \inf\{j \geq 1 : C_j \geq \alpha\}, \text{ with } C_j = \max_{k \leq j} \sum_{l=k}^j \ln L(z_l), \quad (4.1)$$

where $L(z_l)$ is the likelihood ratio¹ of z_l .

In the transmission mode, $z_l = \frac{1}{T_{se}} \sum_{n=1}^{T_{se}} |y_l[n]|^2$ follows a central chi-square distribution with $2T_{se}$ degrees of freedom, because it is the sum of the squares of $2T_{se}$ real Gaussian variables. Each of these variables has a zero mean and a variance of $\frac{\sigma_w^2}{2T_{se}}$ in case of PU absence and $\frac{\sigma_x^2 + \sigma_w^2}{2T_{se}}$ in case of PU presence. The PDFs of z_l in cases of PU absence and presence are thus respectively given by (from equation (2.32) in [99])

$$f_{0, \text{TM}}(z_l) = \frac{z_l^{T_{se}-1}}{\left(\frac{\sigma_w^2}{T_{se}}\right)^{T_{se}} \Gamma(T_{se})} \exp\left(-\frac{z_l}{\frac{\sigma_w^2}{T_{se}}}\right) \quad (4.2)$$

and

$$f_{1, \text{TM}}(z_l) = \frac{z_l^{T_{se}-1}}{\left(\frac{\sigma_x^2 + \sigma_w^2}{T_{se}}\right)^{T_{se}} \Gamma(T_{se})} \exp\left(-\frac{z_l}{\frac{\sigma_x^2 + \sigma_w^2}{T_{se}}}\right), \quad (4.3)$$

where $\Gamma(T_{se})$ is the gamma function² of T_{se} [99]. In the sensing-only mode, $z_l = \frac{1}{T_f} \sum_{n=1}^{T_f} |y_l[n]|^2$ follows a central chi-square distribution with $2T_f$ degrees of freedom. The PDFs $f_{0, \text{SOM}}(z_l)$ and $f_{1, \text{SOM}}(z_l)$ are the same as in the TM, except that T_{se} is replaced with T_f . As in the TM, we try to detect the PU appearance, and in the SOM, we try to detect the PU disappearance, the likelihood ratios in both modes are differently defined. They are given by

$$L(z_l) = \begin{cases} \frac{f_{1, \text{TM}}(z_l)}{f_{0, \text{TM}}(z_l)} = \left(\frac{\sigma_w^2}{\sigma_x^2 + \sigma_w^2}\right)^{T_{se}} \exp\left(\frac{z_l T_{se} \sigma_x^2}{\sigma_w^2 (\sigma_x^2 + \sigma_w^2)}\right), & \text{if TM} \\ \frac{f_{0, \text{SOM}}(z_l)}{f_{1, \text{SOM}}(z_l)} = \left(\frac{\sigma_x^2 + \sigma_w^2}{\sigma_w^2}\right)^{T_f} \exp\left(\frac{-z_l T_f \sigma_x^2}{\sigma_w^2 (\sigma_x^2 + \sigma_w^2)}\right), & \text{if SOM.} \end{cases} \quad (4.4)$$

¹ k is the frame sample after which, $\ln L(z_l)$ has consistently shown positive values, meaning that a change has probably occurred [49].

² $\Gamma(T_{se}) = (T_{se} - 1)!$.

4.3

4.3.1

In this section, we introduce some metrics in order to evaluate the performance of the joint transmission-sensing framework. It has been established so far that in CR, the amount of interference caused on the PUs and the amount of spectrum opportunities used by the SU are two important criteria to assess how robust the sensing is. It is profitable to maximize the SU spectrum usage while minimizing the interference on the PUs. Because both situations are antagonist, as already stated, a good compromise should be found in the context of joint transmission-sensing.

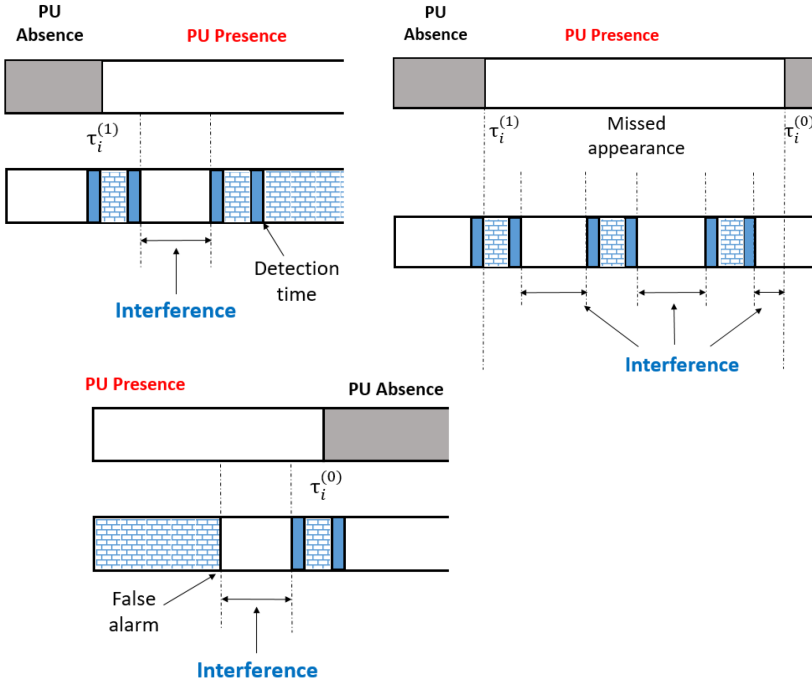


Fig. 4.3 Interference scenarios.

As multiple changes in the spectrum occupancy may occur, and they may not be perfectly detected, the detection errors (on PU appearance and disappearance) will generate interference and spectrum waste. On

one side, interference can be created by three kinds of errors. When a PU appearance is detected with a delay or completely missed until the next disappearance, this will induce interference with the PU during the transmission time of each frame of the SU. Furthermore, when there is a false alarm on a PU disappearance, this will result in interference, due to an earlier switching to the transmission mode while the PU is still present. These three scenarios are depicted in Fig. 4.3.

On the other side, spectrum waste can be created by several scenarios. When a PU disappearance is detected with a delay, or it is missed, the SU remains in the sensing-only mode while it could transmit data, and the entire portion of the frames, since the disappearance time, is wasted. Moreover, a false alarm on a PU appearance will cause the SU to release the spectrum and switch to the sensing-only mode, while it could continue transmitting. Finally, the time blocks allocated for sensing and guard times when the PU is absent, should also account for spectrum waste because they could be used for transmission. These four scenarios are shown in Fig. 4.4.

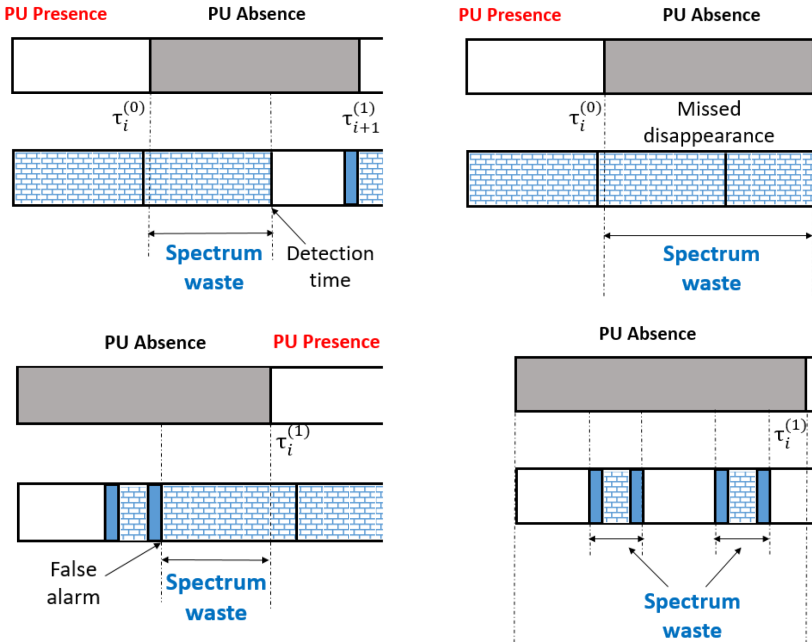


Fig. 4.4 Spectrum waste scenarios.

All this considered, we define the total interference time during the SU communication time N , taking into account the various types of interference, and total spectrum waste time similarly. Let us denote the average total duration of interference and spectrum waste by $\bar{T}_{int}$ and $\bar{T}_{sow}$. Aiming to simultaneously minimize the total interference on the PU and maximize the spectrum opportunity, we suggest a new performance metric, called the *global cost*. Similarly to the global penalty introduced in [98] and (3.15), it is obtained by assigning a cost C_{int} to $\bar{T}_{int}$, and another cost C_{sow} to $\bar{T}_{sow}$. The global cost G_C is thus given by

$$G_C = C_{int} \bar{T}_{int} + C_{sow} \bar{T}_{sow}. \quad (4.5)$$

While the global penalty metric considers only one PU appearance detection, the global cost considers the detection of multiple PU appearances and disappearances. Moreover, it accounts for the joint impact of sensing time and transmission time to compute the interference and spectrum waste times. This metric will help derive the optimal parameters that give a good balance between interference and spectrum waste.

4.3.2 Optimal transmission-sensing

We define the optimal transmission-sensing scheme as the one that minimizes the global cost induced by the framework. It is given by the following optimization problem

$$\arg \min_{T_{se}, T_{tr}, \alpha} G_C(T_{se}, T_{tr}, \alpha). \quad (4.6)$$

We thus want to find the optimal transmission duration T_{tr} , sensing duration T_{se} and detection threshold α , that minimize the global cost G_C . Obviously, the frame duration T_f is indirectly optimized when T_{se} and T_{tr} are optimized.

4.4 Simulations & numerical results

In this section, we compute the optimal parameters through numerical simulations. Foremost, the frame size is fixed and the optimal sensing time (and indirectly the optimal transmission time) is studied for various values of p and SNR. Then, for a fixed sensing time, the optimal frame size (and indirectly the optimal transmission time) is studied. Thereafter, the optimal couple of transmission and sensing times is computed and analysed. Finally, the three parameters, i.e., sensing time, transmission time and detection threshold, are jointly optimized. A small discussion about the mutual dependence between the optimal parameters concludes the section.

4.4.1 Optimal sensing time and transmission time for fixed frame size and threshold

Initially, we assume that the frame size T_f and the threshold are fixed, and we try to find the optimal sensing time T_{se} (and thus the optimal transmission time T_{tr}) that gives the minimum global cost. We consider the following parameters values for the simulations³: $N = 2500$ samples, $T_f = 50$ samples, $N_f = 50$ frames, $T_{Ga} = T_{Gb} = 1$ sample, $\alpha = 3.5$, SNR = 0 dB, $C_{int} = 0.5$ bits/sample, $C_{sow} = 0.14$ bits/sample and $p = 0.003$. We compute the averages of the performance metrics by using Monte Carlo simulations with 10000 realizations. We first plot the average total interference and spectrum waste metrics with respect to the sensing time. On one side, the average total duration of interference is a monotonically decreasing function of the sensing time (Fig. 4.5). This is due to two reasons. First, when more samples are used for sensing, the PU appearances are quickly and well detected so that the interference time is reduced. Second, increasing T_{se} leads to a decrease of the transmission time T_{tr} and consequently a decrease of the potential time of interference with PUs. On the other side, the average total duration of spectrum waste is an increasing function of T_{se} (Fig. 4.6), because using more samples for sensing decreases the time allocated for transmission and inevitably increases the amount of time wasted while the spectrum was free.

³The costs are defined similarly as the penalties in [98] and Chapter 3.

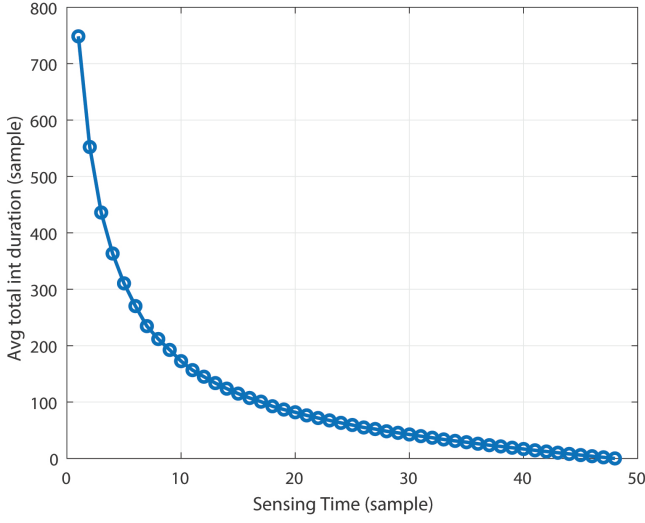


Fig. 4.5 Average interference time $\bar{T}_{int}$ versus sensing time T_{se} .

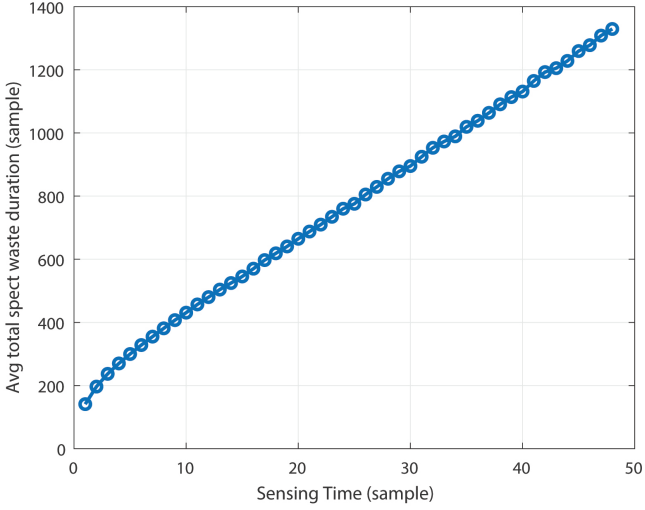


Fig. 4.6 Average spectrum waste time $\bar{T}_{sow}$ versus sensing time T_{se} .

The optimal sensing time is the one that gives a good compromise between interference time and spectrum waste time. To obtain it, we plot the G_C vs T_{se} curves for a fixed frame size. The optimal sensing time varies

with respect to the received SNR, as it can be noticed in Fig. 4.7, where the simulation is done for $\text{SNR} \in [0, 15]$ dB. When the SNR increases, the optimal T_{se} decreases, meaning that the higher the SNR is, the fewer samples are needed for reliable spectrum sensing and the more time is available for interference-free transmission.

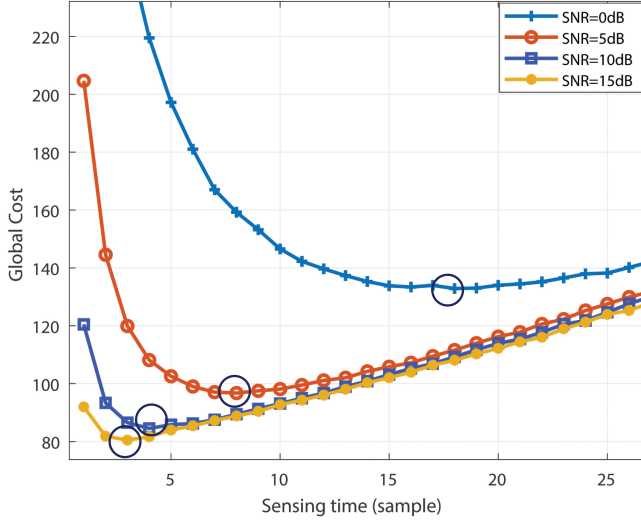


Fig. 4.7 Global cost (G_C) versus sensing time (T_{se}).

As assumed in Section 4.2, the SNR is perfectly known by the SU. The optimal sensing times for different SNR values can thus be pre-computed and stored in a table, and then recovered during the joint transmission-sensing operation. However, in more practical situations where the SNR is unknown by the SU, it can be problematic to choose the optimal sensing time. To solve this issue, we propose a simple method when the SNR is unknown but belongs to a certain interval. First, we suggest deriving a range of optimal sensing times that keep the global cost to the minimum, with a certain margin ε (Fig. 4.8). Then, the intersection of the derived ranges for different SNR values is computed. The resulting range is the optimal range of sensing times that always keep the global cost to the minimum, with a margin ε , when the SNR lies between the considered values. Considering an upper margin $\varepsilon = 10$ bits over the minimum global cost, and a SNR ranging from 0 to 15 dB, we obtain the optimal ranges plotted in Fig. 4.9. We can observe that if $\text{SNR} \in [3, 15]$ dB, the optimal T_{se} can be chosen in

the set {7,8,9}.

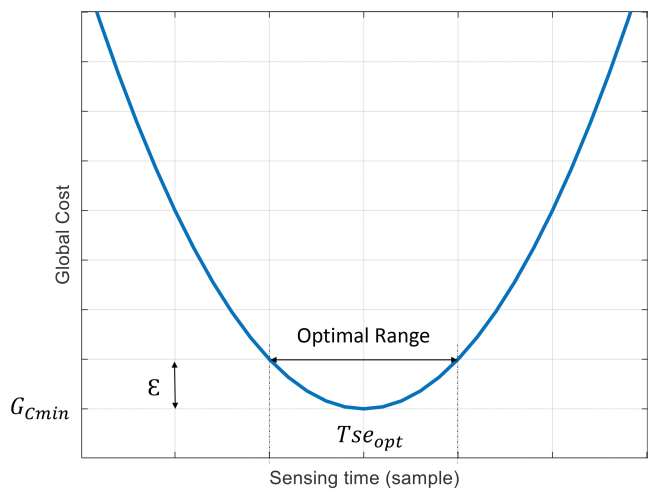


Fig. 4.8 Computation of optimal range of sensing times.

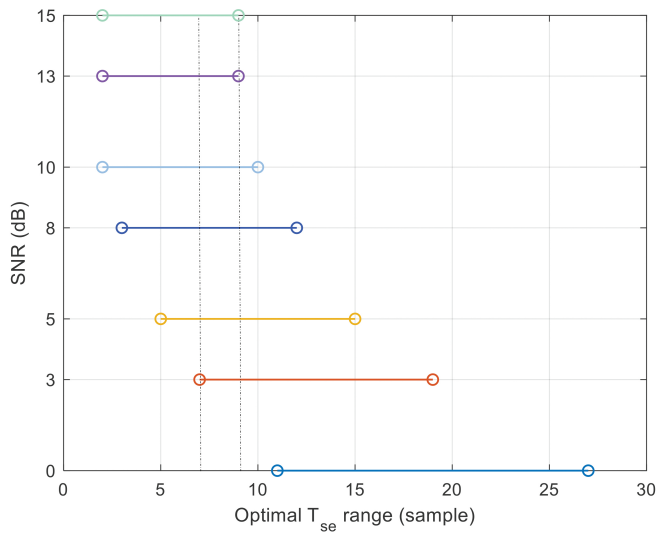


Fig. 4.9 Optimal ranges for various SNR.

4.4.2 Optimal frame size for fixed sensing time and threshold

Beside the optimal time allocated for sensing, it is important to consider the sensing frequency (or the frame size) which also has an impact on the performance of the framework. For this reason, we investigate the optimal frame size T_f for fixed T_{se} and α . We perform some simulations for fixed values of sensing time T_{se} and total communication time N , while varying the frame size T_f . For $T_{se} = 5$ samples, $N = 2500$ samples, $\alpha = 3.5$ and $p = 0.003$, we plot the G_C vs T_f curves for various SNR values. As we can see in Fig. 4.10, the optimal frame size T_f varies slightly with respect to the SNR value. We then plot the same curves for a fixed SNR value (5 dB) but

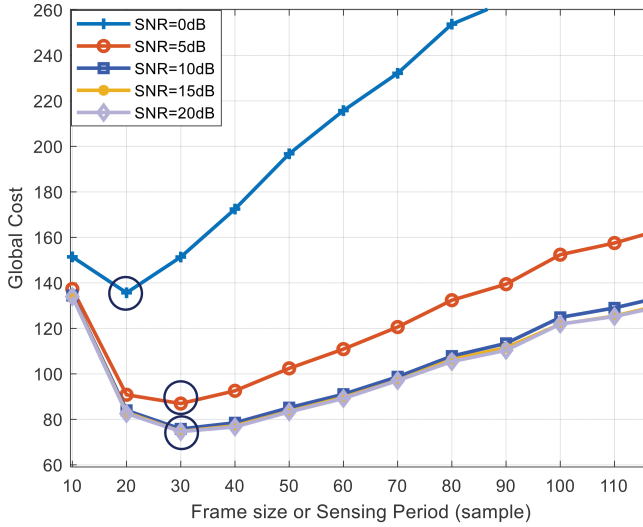


Fig. 4.10 G_C vs T_f for different SNR values with $p = 0.003$.

varying p ($p \in [0.001, 0.01]$). It can be noticed in Fig. 4.11 that the optimal frame size T_f decreases when the probability of PU appearance and disappearance p increases. This means that the more frequent it is expected to get PU appearances and disappearances, due to mobility, the more frequent the SU should perform spectrum sensing and the shorter should be the frame size T_f . To summarize, the optimal frame size depends more on the SU mobility parameters than the SNR value. In the next section, we jointly compute the optimal frame size and sensing time.

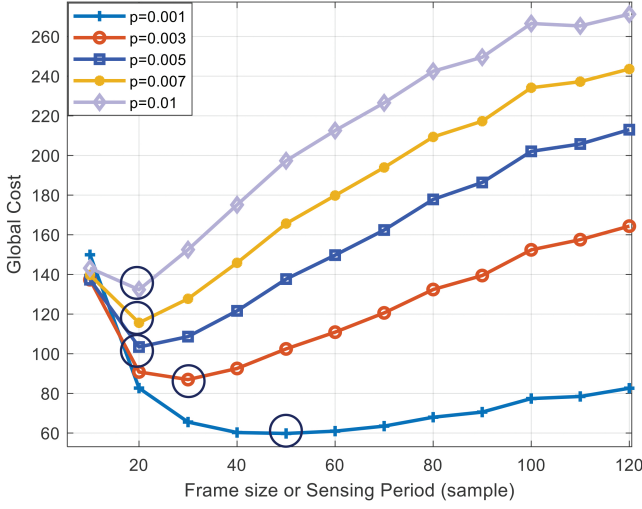


Fig. 4.11 G_C vs T_f for different p values with SNR = 5 dB.

4.4.3 Joint computation of optimal transmission time and sensing time for fixed threshold

In the two previous sections, we compute the optimal sensing time by fixing the frame size and vice versa. Here, we jointly optimize both parameters using numerical simulations. It is also equivalent to the joint optimization of sensing and transmission times.

Exhaustive search

We use exhaustive search to find the optimal (T_{se}, T_{tr}) couple. The three-dimensional curve of the global cost G_C versus T_{se} and T_{tr} is shown in Fig. 4.12. We can see that when T_{tr} is increased too much with a small T_{se} , the global cost is increased, obviously, due to interference. The same is observed when T_{se} is increased too much with a small T_{tr} , due, this time, to spectrum waste. It is also noticed that there is a minimum global cost, and it is obtained for $(T_{se} = 4$ samples, $T_{tr} = 17$ samples), when $p = 0.003$ and SNR = 5 dB.

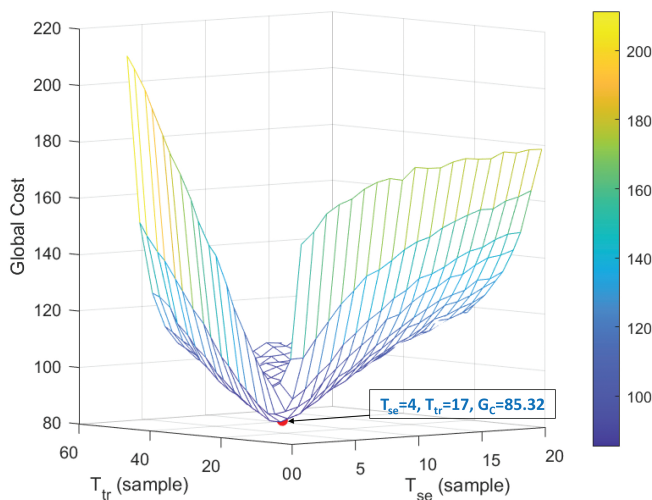


Fig. 4.12 G_C vs (T_{se}, T_{tr}) for $p = 0.003$ and SNR = 5 dB.

With further simulations, we can observe how the optimal (T_{se}, T_{tr}) couple varies with respect to SNR and p values. On the one hand, the SNR is varied from 0 dB to 20 dB for $p = 0.003$, and the optimal parameters are shown in Table 4.1. It can be seen that the optimal T_{se} varies a lot (from 12 to 1 samples), while the optimal T_{tr} slightly varies (from 17 to 14 samples).

Table 4.1 Optimal sensing and transmission times for $p = 0.003$.

Parameter \ SNR (dB)	SNR (dB)			
	0	5	10	20
T_{se} (sample)	12	4	2	1
T_{tr} (sample)	17	17	14	14
G_C (bit)	129.2	85.32	67.64	56.54

On the other hand, when p is varied from 0.001 to 0.01 for SNR = 5 dB, the optimal T_{tr} varies a lot (from 35 to 9 samples), while the optimal T_{se} has a constant value (4 samples), as shown in Table 4.2.

Table 4.2 Optimal sensing and transmission times for SNR= 5 dB.

Parameter \ p	0.001	0.003	0.005	0.007	0.01
T_{se} (sample)	4	4	4	4	4
T_{tr} (sample)	35	17	12	9	9
G_C (bit)	57.78	85.32	101.7	113.96	127.7

This result confirms what was already observed in the previous section and leads us to conclude that the optimal sensing time depends more on the SNR value, while the optimal transmission time (or equivalently the optimal frame size) depends more on the probability of appearance and disappearance of the PU, inferred from the SU mobility parameters. A similar result is found in [100], where energy detection is used in a context of joint transmission-sensing. It is highlighted that the optimal sensing time is much less sensitive than the optimal transmission time when the primary activity parameter changes.

Simple algorithm

Here, we propose a simple strategy to find the joint optimal parameters values, because of the time complexity of exhaustive search. We suggest the following steps:

- Initialize the optimal sensing time at $T_{se} = 1$ sample.
- Then, for the initial T_{se} value, find the optimal transmission time T_{tr} , using Monte Carlo simulations.
- For the optimal T_{tr} computed, find the corresponding optimal sensing time T_{se} , using Monte Carlo simulations.
- Repeat the process as long as the global cost decreases.
- Use the last (T_{se}, T_{tr}) couple as the optimal one.

This method is more time-efficient than the exhaustive search (about one third of the simulation time), and shows comparable performance. It is depicted in Fig. 4.13, that for $p = 0.003$ and SNR = 5 dB, the exhaustive search gives ($T_{se} = 4$ samples, $T_{tr} = 17$ samples, $G_C = 85.32$ bits) and the algorithm gives ($T_{se} = 3$ samples, $T_{tr} = 14$ samples, $G_C = 85.43$ bits).

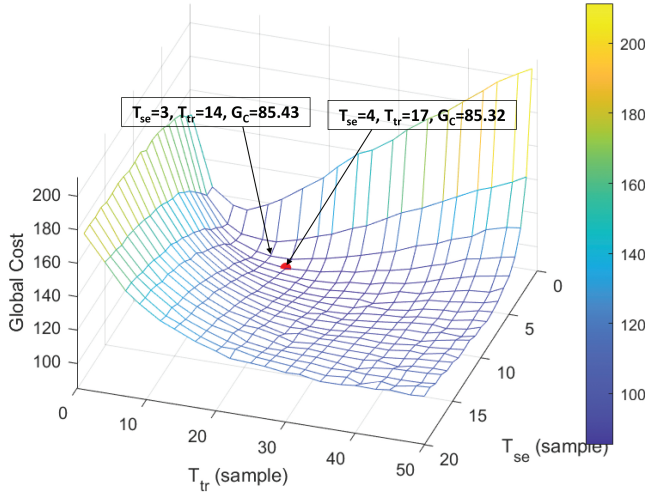


Fig. 4.13 Optimal (T_{se}, T_{tr}) for $p = 0.003$ and $\text{SNR} = 5$ dB, with both methods.

4.4.4 Joint optimization of sensing time, transmission time and detection threshold

In the previous sections, we assumed a fixed threshold to simplify the analysis. However, beside the sensing and transmission times, the detection threshold also has a significant impact on the performance of the joint transmission-sensing framework. For this reason, it is important to verify if we can further decrease the global cost by appropriately choosing an optimal threshold. In this section, we jointly optimize the threshold α with the sensing time T_{se} and the transmission time T_{tr} , using exhaustive search.

Firstly, we compute the optimal (T_{se}, T_{tr}, α) triplet for $p = 0.003$ and $\text{SNR} \in \{0, 5, 10, 15\}$ dB. The minimum global cost obtained at each threshold value and the corresponding optimal (T_{se}, T_{tr}) couple are shown in Fig. 4.14 – 4.17. The optimal threshold α and the corresponding (T_{se}, T_{tr}) are indicated with green color. At first glance, we can notice that the minimum achievable global cost can vary a lot from one threshold value to another. Therefore, the performance of the joint transmission-sensing framework is clearly sensitive to the threshold variation. Note that this sensitivity is higher at low SNR than at high SNR. For $\text{SNR} = 0$ dB (Fig. 4.14), the

optimal (T_{se}, T_{tr}) couple changes a lot (from $(T_{se} = 10$ samples, $T_{tr} = 32$ samples) to $(T_{se} = 11$ samples, $T_{tr} = 20$ samples)) by varying the threshold α . But for $\text{SNR} \in \{5, 10, 15\}$ dB (Fig. 4.14 and 4.15), the optimal (T_{se}, T_{tr}) couple remains almost unchanged in the considered range of thresholds.

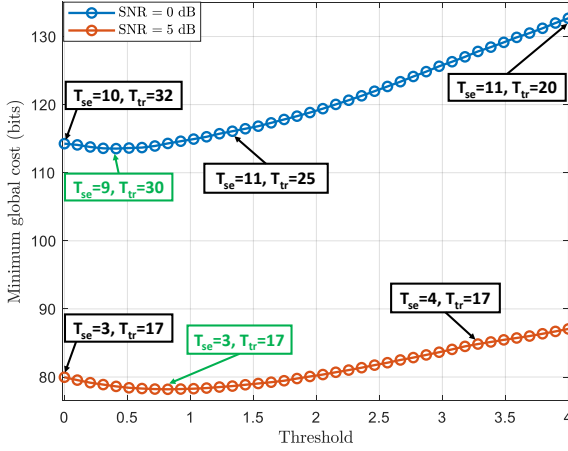


Fig. 4.14 Minimum G_C vs α and corresponding (T_{se}, T_{tr}) for $p = 0.003$ and $\text{SNR} \in \{0, 5\}$ dB.

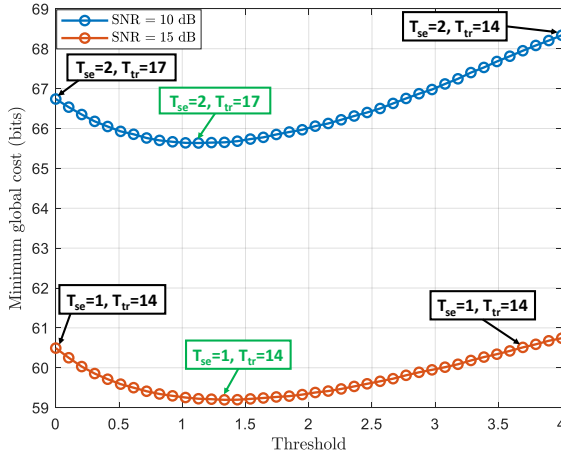


Fig. 4.15 Minimum G_C vs α and corresponding (T_{se}, T_{tr}) for $p = 0.003$ and $\text{SNR} \in \{10, 15\}$ dB.

The optimal (T_{se}, T_{tr}, α) triplets and corresponding G_C for the considered SNR values, are shown in Table 4.3. It can be seen that the optimal threshold α increases with the SNR. It is also observed that the optimal sensing time T_{se} decreases a lot with the SNR (from 9 to 1 samples) similarly to the previous results. However, the optimal transmission time T_{tr} also decreases a lot with the SNR (from 30 to 14 samples) unlike the case of fixed threshold (Table 4.1), where T_{tr} is almost constant for a fixed value of p . In fact, this change in T_{tr} value is more noticed at low SNR. This is due to the following reasons. First, to be able to detect a low SNR signal, we obviously need to increase T_{se} . But if the frame size T_f is kept constant, this may induce too much spectrum waste and thus an increase of the global cost G_C . Thus, T_f , and consequently T_{tr} , needs to be increased. This situation is more perceived when the threshold is optimized, simply because by varying the threshold, we gain more degrees of freedom. For example, if T_{se} and T_{tr} are increased due to low SNR, the threshold α can also be decreased to keep a good balance between quick detections and false alarms, resulting in a lower G_C . But when α is constrained to be constant, T_{se} is further increased while T_{tr} is slightly increased and G_C has a larger value. This can be seen by comparing Table 4.1 and Table 4.3, for SNR = 0 dB. The second reason for an increase of T_{tr} at low SNR is related to the detection of PU disappearances, which is also taken into account by G_C . In fact, the frame size T_f is also the sensing duration for PU disappearance detection in our model. And as at low SNR, it is more profitable to increase the sensing time, the system tends to increase the optimal T_f and thus the optimal T_{tr} .

Table 4.3 Optimal (T_{se}, T_{tr}, α) triplets for $p = 0.003$.

SNR (dB) \ Parameter	0	5	10	15
T_{se} (sample)	9	3	2	1
T_{tr} (sample)	30	17	17	14
α	0.41	0.82	1.12	1.33
G_C (bit)	113.53	78.19	65.63	59.19

Secondly, we compute the optimal (T_{se}, T_{tr}, α) triplet for SNR = 5 dB and $p \in \{0.001, 0.003, 0.007, 0.01\}$. The minimum global cost obtained are shown in Fig. 4.16 and 4.17. We can first notice that the minimum G_C is more sensitive to the threshold variation for higher values of the proba-

bility of PU appearance and disappearance p . Furthermore, the optimal (T_{se}, T_{tr}) couple slightly changes for lower values of p (from $(T_{se} = 3$ samples, $T_{tr} = 32$ samples) to $(T_{se} = 4$ samples, $T_{tr} = 38$ samples), for $p = 0.001$), while it remains almost unchanged for higher values of p .

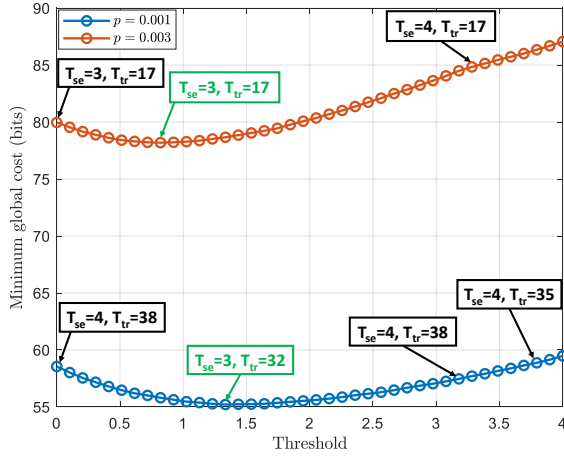


Fig. 4.16 Minimum G_C vs α and corresponding (T_{se}, T_{tr}) for SNR = 5 dB and $p \in \{0.001, 0.003\}$.

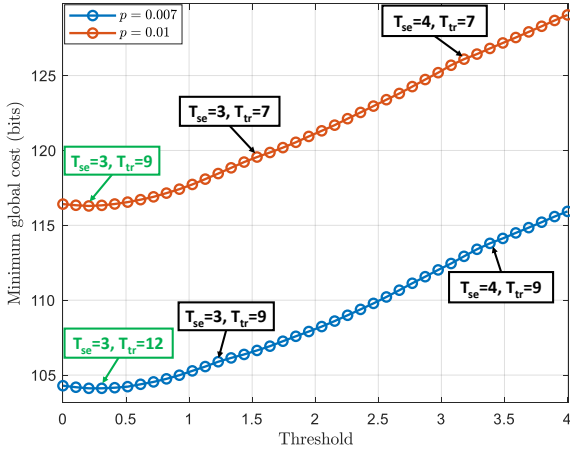


Fig. 4.17 Minimum G_C vs α and corresponding (T_{se}, T_{tr}) for SNR = 5 dB and $p \in \{0.007, 0.01\}$.

The optimal (T_{se}, T_{tr}, α) triplets and corresponding G_C for the considered values of p , are shown in Table 4.4. It can be seen that the optimal threshold α decreases with p , which intuitively means that with fast PU appearances and disappearances, the detection threshold should be decreased to get fast detections. Similarly to the case of fixed α (Table 4.2), we can notice that the optimal T_{se} is constant for every value of p and the optimal T_{tr} decreases a lot when p increases.

Table 4.4 Optimal (T_{se}, T_{tr}, α) triplets for SNR = 5 dB.

Parameter \ p	0.001	0.003	0.007	0.01
T_{se} (sample)	3	3	3	3
T_{tr} (sample)	32	17	12	9
α	1.33	0.82	0.30	0.20
G_C (bit)	55.21	78.19	104.11	116.29

To recap, the joint optimization of the threshold with the sensing and transmission times allows us to conclude the following:

- The optimal sensing time T_{se} depends more on the SNR. It slightly varies with the threshold α for a fixed SNR value, but when it is jointly chosen with the optimal α , it is always constant at fixed SNR. Therefore, in a practical system, we suggest choosing T_{se} according to the SNR.
- The optimal transmission time T_{tr} is clearly dependent on the probability of PU appearance and disappearance p . But also, at low SNR, it becomes very sensitive to the threshold variation. Both parameters (T_{tr} and α) should be carefully and jointly chosen to avoid degrading the performance because G_C is more sensitive to the threshold at low SNR (Fig. 4.14). Moreover, when p is low, the optimal T_{tr} also depends on α , even if this time, a small variation of (T_{tr}, α) will not be that harmful to the performance because G_C is less sensitive to the threshold (Fig. 4.16). For higher values of p , T_{tr} is slightly dependent on the threshold α , and G_C is very sensitive to α . To sum up, we suggest to jointly choose T_{tr} and α according to p and the SNR.

4.4.5 Discussion on the optimal parameters

In sum, we see that by playing with detection accuracy, detection quickness and sensing frequency, which are obtained for a given (T_{se}, T_{tr}, α) triplet, we can achieve an overall optimization of the spectrum usage. Furthermore, all the parameters, T_f, T_{se}, T_{tr} and α , are interrelated, which makes the analysis difficult. We have just seen in the previous section that the optimal T_{tr} is very dependent on α while the optimal T_{se} is slightly dependent on α . In addition, we have seen that at low SNR, not only T_{se} is increased, but also T_{tr} is increased (because of the sensing frequency T_f) in order to avoid too much spectrum waste. This mutual dependence is accentuated by the fact that our model accounts for both PU appearances and disappearances. In fact, the frame size, or sensing frequency T_f for appearance detection, is also the sensing time for disappearance detection, as the SU switches to the sensing-only mode. And it is obviously linked to the detection threshold, which is the same threshold for both appearance and disappearance detection. It might be easier to independently study appearances and disappearances, but in a real scenario, they are interconnected because an error on the detection of a PU appearance will certainly impact the detection of the next PU disappearance and so on. Our model has the advantage of considering all this.

In order to further improve the joint transmission-sensing model, we may increase the number of degrees of freedom by:

- Using distinct thresholds for appearance and disappearance detections,
- Performing spectrum sensing sample per sample, instead of frame per frame, for the sensing-only mode (disappearance detection). This would first avoid the dependence on the frame size T_f and may result in faster detection of PU disappearances.

4.5 Comparison with energy detection

4.5.1 Methodology

In this section, the performance of the proposed joint transmission-sensing framework is evaluated, when energy detection algorithm is used against CUSUM algorithm. Let us note some differences that come up in the framework when using ED. First, ED is an hypothesis testing algorithm, and it cannot detect changes in the channel state (PU appearance or disappearance), but it can detect the current channel state (PU presence or absence). So, unlike the CUSUM case where the algorithm applied depends on the current mode (TM or SOM), both hypotheses (PU presence or absence) are periodically tested for ED. Second, ED is a block-based spectrum sensing algorithm. Thus, the total sensing duration (sum of sensing times per frame) is fixed in advance, unlike for CUSUM where we do not know in advance after how many frames the algorithm will stop. Moreover, even if the current mode does not change, a new sensing is performed periodically.

According to this, we suggest two approaches to insert ED in the framework:

- Firstly, we perform ED independently frame by frame. The total sensing duration in this case is thus T_{se} .
- Secondly, we define a block as the set of frames after which a new spectrum sensing is performed, and B_f is the number of frames inside each block. ED is performed independently block by block. The total sensing duration here is $B_f T_{se}$. The first approach is thus a special case of the second, when $B_f = 1$.

4.5.2 Simulation results

For comparison with CUSUM, we perform the simulations using the optimal (T_{se}, T_{tr}) couple in CUSUM case when $p = 0.003$ and $\text{SNR} = 5$ dB (Table 4.4), and compare the global cost obtained for different threshold values. The results obtained with both approaches are presented in Fig. 4.18. As it can be noticed, the minimum global cost obtained with frame by

frame ED ($B_f = 1$) is closer to that of CUSUM. But when ED is performed over a block of two frames ($B_f = 2$), the obtained performance is worse than CUSUM performance. This is not surprising. In fact, even though we perform spectrum sensing sequentially over frames with CUSUM, the algorithm is somewhat constrained to take decisions only at the end of each frame. Despite the fact that the initial objective was to combine data per frame, this may not result in the fastest detection. This is the reason why a simple ED, that also takes decisions at the end of each frame, performs closer even without combining data over several frames. We see that when ED is done over a longer period of time (just a block of two frames), the performance worsens.

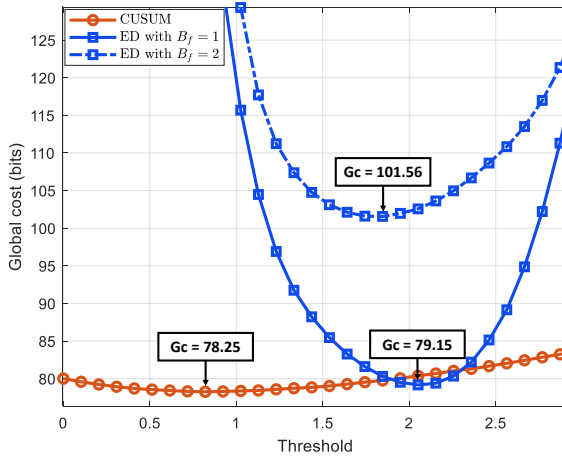


Fig. 4.18 G_C vs α curves of CUSUM, frame by frame ED and block by block ED, for ($T_{se} = 3$ samples, $T_{tr} = 17$ samples), SNR = 5 dB and $p = 0.003$.

This result reinforces our idea that performing the CUSUM algorithm sample per sample for the sensing-only mode (PU disappearance detection) could result in faster detections and improve the performance. This implies that the frames would have no more a constant size in the SOM.

4.6 Conclusion

In this chapter, a new joint transmission-sensing framework is introduced for mobile cognitive radio systems where the SU is moving for a certain period of time, and multiple PU appearances and disappearances may occur. Using the CUSUM changepoint detection algorithm, spectrum sensing is performed sequentially over successive frames, instead of independent frame by frame sensing suggested in the literature so far. Using the idea of the global penalty metric proposed in the previous chapter, a new performance metric, named the global cost, is suggested to evaluate the total loss in the channel usage due to the joint effect of interference and spectrum waste caused by the framework.

Then, the optimal system parameters, namely the sensing time, the transmission time and the detection threshold, that minimize the global cost are computed and analysed by means of simulations. The optimal parameters depend on the received SNR and the mobility parameters of the SU. The optimal sensing time shows to strongly depend on the SNR, while the optimal transmission time is very sensitive to the mobility parameters, and the optimal threshold depends on both. Moreover, the threshold should be jointly chosen with the transmission time, which is also the sensing frequency.

Finally, the performance of ED is compared with that of CUSUM in the context of the proposed framework. It is found that frame by frame ED performs closer to CUSUM because in the framework, CUSUM is, in some ways, forced to take decisions at end of frames. On its side, block by block ED has the worst performance due to the fact that spectrum sensing is forced over a longer time period.

5

Geolocation-based Bayesian Spectrum Sensing

5.1 Introduction

In heterogeneous cognitive radio networks, it is difficult to have an *a priori* knowledge of the existing primary systems. For this reason, blind spectrum sensing algorithms, that does not need any prior information about the PUs, are highly desirable. However, the performance of blind detection algorithms is limited, and they do not make the most use of the available spectrum. As it has been seen throughout this thesis, the maximum utilization of the spectrum can be achieved only when the PUs' parameters are perfectly known. This is noticed in Chapter 3, where the optimal threshold depends on the received SNR, and in Chapter 4, where both the optimal threshold and the optimal sensing time per frame are sensitive to the SNR. Therefore, if some *a priori* knowledge of the PUs is available to the SU, it can be used to increase the performance of spectrum sensing algorithms. As discussed in Chapter 2, a good detector should make a good compromise between the amount of required prior knowledge and the achieved performance.

Due to the challenges of spectrum sensing to meet their very tight requirements, many regulators around the world consider that this technology alone is not yet reliable enough to guarantee the protection of channel incumbents [44, 101, 102]. For this reason, geolocation database is considered as the most feasible method [103, 102]. However, the geolocation database implementations are too conservative because a large range of protection contour is defined to avoid interference with the PUs [104, 102]. This results in a huge waste of spectrum opportunities. Instead of using geolocation database alone, another approach is to combine it with spectrum sensing. This could be done in two ways [102]. Firstly, spectrum sensing results could help in adjusting the protection contour and fine-tuning the various parameters used in the geolocation database method. In view of this, a technique to build and improve geolocation database from sensed data, and using statistical algorithms, is presented in [105]. It helps to calculate the emission limits of SUs. Secondly, geolocation database information could be used to adjust the sensitivity of spectrum sensing, in order to improve the detection performance. Considering a wideband sensing of the TV band, the authors in [106] and [107] use simple methods that consist in performing spectrum sensing only in the TV channels considered available by the geolocation database method. This speeds up the sensing process, thereby reducing the sensing time for the whole band, and also relaxes the sensitivity and processing capabilities required for SUs. Furthermore, authors in [108, 109, 110] use a geolocation-assisted compressive spectrum sensing approach to reduce the number of required samples in a wideband spectrum sensing context. In fact, the prior information obtained from the geolocation database is used to infer the sparse nature of the wideband spectrum, and by using this sparsity, spectrum sensing is performed at lower sampling rates than Nyquist rates. This has the effect of improving the detection accuracy and lowering the computational complexity. Finally, assuming a heterogeneous CR network with one PU and multiple existing SUs, a non-coherent spectrum sensing scheme, based on energy detection, is suggested to simultaneously and distinctively detect the PU signal and the SUs signals in [111]. Using the available geolocation information about the PU and the SUs, the adjacent channel interference and the co-channel interference induced by the SUs are distinguished from the received PU signal power and the noise power. Then the noise power is used to compute the detection threshold, and the signal powers of the PU and the SUs are used for detection, resulting in an improved performance.

In this chapter, we consider a CR scenario where the primary system is composed of multiple TV transmitters. Assuming that some information about the transmitters is available in a geolocation database, we use this information as *a priori* knowledge to derive a new spectrum sensing algorithm. Unlike the previous chapters where sequential changepoint detection theory is used, we consider the block-based detection theory in this chapter. The reason is that we do not assume fast changes in the spectrum occupancy. We consider the Bayesian approach of block-based detection, where the optimal decision rule (2.9) compares the likelihood ratio of observation to the ratio of both hypotheses prior probabilities. In a real-world scenario, the received signal power is not deterministic, but it is distributed according to fading statistics, and the likelihood ratio should be computed, either using the GLRT or the mixture-based test if the fading distribution is known. Assuming a Rayleigh fading, we compute the average received power based on the geolocation information and use this to obtain the prior distribution of the received power. Then, using this geolocation-based prior distribution, we perform a mixture-based test to derive a new detection rule. As it is difficult to directly solve the integral involved in the detection rule, an approximate closed-form expression is derived using Taylor series approximation. With few terms of the Taylor series, the approximation is accurate for medium to high average SNR values, when a long enough sensing time is chosen. The derived algorithm is compared with energy detection, which does not use the available *a priori* geolocation information, and it gives the best performance in terms of minimum error probability. Additionally, we suggest computing the prior probabilities of presence and absence, and thus the threshold, by using the geolocation information. By doing so, the detection and false alarm probabilities of the derived algorithm increase quickly with the SNR, but it always keeps the minimum error probability. All these results show that by using *a priori* geolocation information about the PUs, we are able to achieve a better compromise between false alarms and good detections than without this information, even if the received power signal is not perfectly known.

The main contributions of this chapter are:

- Introduction of a Bayesian spectrum sensing algorithm based on geolocation information,
- Derivation of a closed-form approximation that is accurate for medium to high average SNR values,

- Computation of prior probabilities based on geolocation information,
- Performance comparison of the derived algorithm with energy detection.

The remainder of the chapter is structured as follows. The system model, including the considered scenario and the signal model, is introduced in Section 5.2. Then, the Bayesian detection rule, based on prior geolocation information, is derived in Section 5.3. The approximate closed-form expression of the detection rule is also given. A numerical comparison between the derived algorithm and energy detection algorithm is done in Section 5.4. Finally, a conclusion is given in Section 5.5 and some proofs are presented in Section 5.A.

5.2 System model

5.2.1 Scenario

We consider a CR scenario with multiple fixed PUs transmitting in different bands and a SU that wants to opportunistically use the available bands (Fig. 5.1). It is assumed that the PUs are TV transmitters of a digital terrestrial television (DTT) network. Their respective broadcasting parameters, i.e., location, carrier frequency, bandwidth, transmit power, antenna height, minimum admissible power by a PU receiver, etc., are assumed to be available in a geolocation database. The SU has a geolocation capability, and it is assumed that it can retrieve the PUs' parameters from the geolocation database. To do this, the SU first sends its location to the database which replies with the geolocation information G of each existing PU. Let $G = (P_T, f_c, D, P_{adm})$, where P_T is the PU transmit power, f_c is the carrier frequency, D is the distance between the PU transmitter and the SU (computed using the locations), and P_{adm} is the minimum power admissible by a PU receiver. Our goal is to derive a Bayesian spectrum sensing algorithm that uses the geolocation data G as *a priori* information to improve the detection.

Due to the fading channel between the PUs transmitters and the SU, the received signal power is unknown and random. But we assume that it has

a certain prior distribution, and we propose to compute this distribution using the available geolocation information G . Then, the prior distribution is used to derive the Bayesian detector. The signal model is presented in the next section.

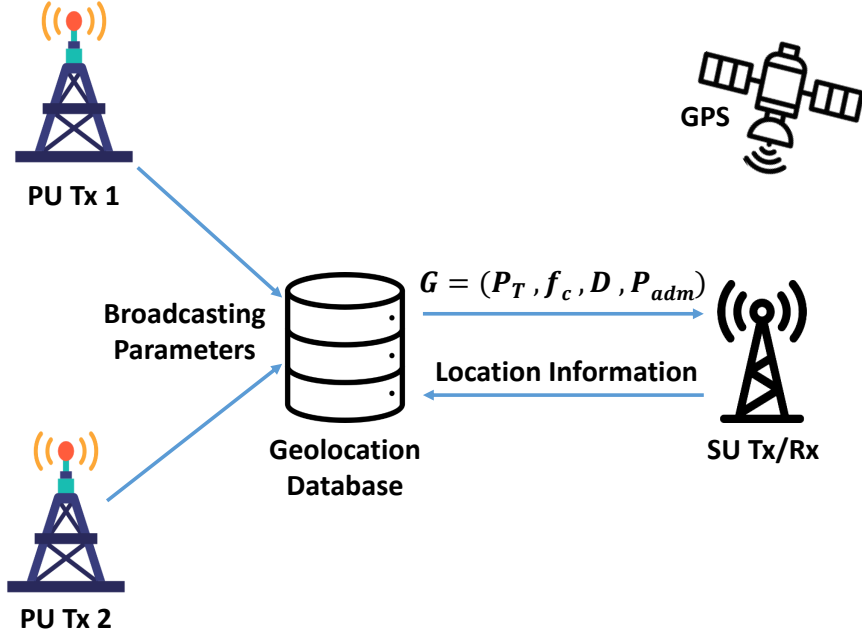


Fig. 5.1 Geolocation-based spectrum sensing scenario

5.2.2 Signal model

We assume that spectrum sensing is performed in one band at a time, but this could be done simultaneously in multiple bands, as already stated in the previous chapters. Looking at one band, let us recall the block-based signal detection problem (2.1) given by

$$\begin{cases} H_0 : y[n] = w[n], \\ H_1 : y[n] = x[n] + w[n], \\ n = 1, 2, \dots, N_s \end{cases} \quad (5.1)$$

where

n is the time sample,

$y[n]$ is the received (observed) signal sample,

$w[n]$ is the noise sample,

$x[n]$ is the PU transmitted signal sample affected by fading,

H_0 means "presence of noise only",

H_1 means "presence of PU signal and noise" and

N_s is the sensing time.

As in the previous chapters, we assume that $x[n]$ and $w[n]$ are independent circularly symmetric complex Gaussian, $x[n] \sim \mathcal{CN}(0, \sigma_x^2)$, $w[n] \sim \mathcal{CN}(0, \sigma_w^2)$, and independent of each other. The PDFs of the observed vector¹ $\mathbf{y}$ under the hypotheses H_1 and H_0 are respectively given by

$$f_1(\mathbf{y}) = \frac{1}{(\pi(\sigma_x^2 + \sigma_w^2))^{N_s}} \exp\left(-\frac{\sum_{n=1}^{N_s} |y[n]|^2}{\sigma_x^2 + \sigma_w^2}\right) \quad (5.2)$$

and

$$f_0(\mathbf{y}) = \frac{1}{(\pi\sigma_w^2)^{N_s}} \exp\left(-\frac{\sum_{n=1}^{N_s} |y[n]|^2}{\sigma_w^2}\right). \quad (5.3)$$

The noise power σ_w^2 is assumed to be perfectly known. Moreover, we assume that the PU signal is affected by a Rayleigh fading channel and thus the received power σ_x^2 is exponentially distributed, with the following PDF

$$f(\sigma_x^2) = \frac{1}{\gamma} \exp\left(-\frac{\sigma_x^2}{\gamma}\right), \quad (5.4)$$

where γ is the average received power. The distribution of the received power depends on only one parameter, which is the average received power γ . We suggest computing this average received power, by using the geolocation information G . In practical scenarios, γ is obtained either from empirical or deterministic channel models. Here, we use a simple free-space path loss model, that takes G as input, to compute γ . Obviously, another more robust channel model could be used. The average received power computed based on geolocation information G is given by

$$\gamma_G = P_T \left(\frac{c}{4\pi f_c D} \right)^2 \quad (5.5)$$

¹ $\mathbf{y} = y[1], \dots, y[N_s]$.

where c is the speed of light. The PDF of the received power can thus be written as

$$f_G(\sigma_x^2) = \frac{1}{\gamma_G} \exp\left(-\frac{\sigma_x^2}{\gamma_G}\right). \quad (5.6)$$

Now, assuming the Bayesian approach of classical hypothesis testing, the two hypotheses can be assigned *a priori* probabilities $P(H_0)$ and $P(H_1)$. We think that it is realistic to infer the *a priori* probabilities of PU signal absence and presence by combining geolocation information with channel models. For this reason, we suggest computing them using $f_G(\sigma_x^2)$ and the minimum power admissible by a PU receiver P_{adm} . The geolocation-based prior probabilities are defined as

$$\begin{aligned} P_G(H_1) &= P(\sigma_x^2 > P_{adm}) \\ &= \exp\left(-\frac{P_{adm}}{\gamma_G}\right) \end{aligned} \quad (5.7)$$

and

$$P_G(H_0) = 1 - \exp\left(-\frac{P_{adm}}{\gamma_G}\right). \quad (5.8)$$

Basically, $P(H_0)$ is the value of the cumulative distribution function of the received power at P_{adm} , and $P(H_1)$ is the complementary value.

5.3 Geolocation-based Bayesian detector

In this section, we derive a Bayesian detection rule that uses the geolocation information G , with the aim of improving the spectrum sensing accuracy.

5.3.1 Problem formulation

We know from (2.8) and (2.9) in Chapter 2, that the optimal detector in the Bayesian sense, minimizes the error probability, which is given here by

$$P_e = P_G(H_1)P_{MD} + P_G(H_0)P_{FA}, \quad (5.9)$$

and the detection rule is given by

$$\frac{f_1(\mathbf{y})}{f_0(\mathbf{y})} \underset{H_1}{\overset{H_0}{\gtrless}} \frac{P_G(H_0)}{P_G(H_1)}. \quad (5.10)$$

Obviously, the optimal detector requires to perfectly know the distribution of the received signal and that of noise. However, due to the fading channel between the PU and the SU, the received signal power σ_x^2 is not constant and hence the decision statistic of the Bayesian detector (the likelihood ratio) cannot be defined. It is now conditioned on σ_x^2 and can be written as

$$\frac{f_1(\mathbf{y} | \sigma_x^2)}{f_0(\mathbf{y})}. \quad (5.11)$$

As already known, this is the composite hypothesis testing problem, which can be solved either by using a GLRT or a mixture-based test (Bayesian approach to composite hypothesis testing). The GLRT does not need any *a priori* knowledge of the unknown signal power σ_x^2 , but it tries to estimate it based on the observed data. In the current work, we adopt the mixture-based test that uses the geolocation information G to compute the prior distribution of the unknown signal power σ_x^2 . The geolocation-based Bayesian detection rule can thus be written as follows

$$\int_0^\infty \frac{f_1(\mathbf{y} | \sigma_x^2)}{f_0(\mathbf{y})} f_G(\sigma_x^2) d\sigma_x^2 \underset{H_1}{\overset{H_0}{\gtrless}} \frac{P_G(H_0)}{P_G(H_1)}. \quad (5.12)$$

In other words, we suggest the geolocation-based Bayesian detector given by

$$\Omega = (\Lambda_{GB}(\mathbf{y}), \alpha) \quad (5.13)$$

where

$$\Lambda_{GB}(\mathbf{y}) = \frac{P_G(H_1)}{P_G(H_0)} \int_0^\infty \frac{f_1(\mathbf{y} | \sigma_x^2)}{f_0(\mathbf{y})} f_G(\sigma_x^2) d\sigma_x^2 \quad (5.14)$$

and

$$\alpha = 1. \quad (5.15)$$

5.3.2 Derivation of a closed-form expression

In this section, we derive a closed-form expression for the proposed detector, by solving the integral involved in the computation of the decision

statistic $\Lambda_{GB}(\mathbf{y})$. By replacing the PDFs in (5.14), we obtain

$$\begin{aligned}\Lambda_{GB}(\mathbf{y}) &= \frac{P_G(H_1)}{P_G(H_0)} \int_0^\infty \frac{\frac{1}{(\pi(\sigma_x^2 + \sigma_w^2))^{N_s}} \exp(-\frac{\sum_{n=1}^{N_s} |y[n]|^2}{\sigma_x^2 + \sigma_w^2})}{\frac{1}{(\pi\sigma_w^2)^{N_s}} \exp(-\frac{\sum_{n=1}^{N_s} |y[n]|^2}{\sigma_w^2})} \frac{1}{\gamma_G} \exp(-\frac{\sigma_x^2}{\gamma_G}) d\sigma_x^2 \\ &= \frac{P_G(H_1)}{\gamma_G P_G(H_0)} \int_0^\infty \left(\frac{\sigma_w^2}{\sigma_x^2 + \sigma_w^2} \right)^{N_s} \\ &\quad \exp \left(-\frac{\sum_{n=1}^{N_s} |y[n]|^2}{\sigma_x^2 + \sigma_w^2} + \frac{\sum_{n=1}^{N_s} |y[n]|^2}{\sigma_w^2} - \frac{\sigma_x^2}{\gamma_G} \right) d\sigma_x^2.\end{aligned}$$

Moreover,

$$\begin{aligned}\Lambda_{GB}(\mathbf{y}) &= \frac{P_G(H_1)}{\gamma_G P_G(H_0)} \int_0^\infty \left(\frac{1}{\sigma_x^2/\sigma_w^2 + 1} \right)^{N_s} \\ &\quad \exp \left(\frac{\sigma_x^2/\sigma_w^2}{\sigma_w^2(\sigma_x^2/\sigma_w^2 + 1)} \sum_{n=1}^{N_s} |y[n]|^2 - \frac{\sigma_x^2/\sigma_w^2}{\gamma_G/\sigma_w^2} \right) d\sigma_x^2 \\ &= \frac{\sigma_w^2 P_G(H_1)}{\gamma_G P_G(H_0)} \int_0^\infty \left(\frac{1}{\sigma_x^2/\sigma_w^2 + 1} \right)^{N_s} \\ &\quad \exp \left(\frac{\sigma_x^2/\sigma_w^2}{(\sigma_x^2/\sigma_w^2 + 1)} \frac{\sum_{n=1}^{N_s} |y[n]|^2}{\sigma_w^2} - \frac{\sigma_x^2/\sigma_w^2}{\gamma_G/\sigma_w^2} \right) d\left(\frac{\sigma_x^2}{\sigma_w^2}\right).\end{aligned}$$

By performing the change of variable $v = \frac{\sigma_x^2}{\sigma_w^2} + 1$, we have

$$\Lambda_{GB}(\mathbf{y}) = \frac{\sigma_w^2 P_G(H_1)}{\gamma_G P_G(H_0)} \int_1^\infty \left(\frac{1}{v} \right)^{N_s} \exp \left(\frac{v-1}{v} \frac{\sum_{n=1}^{N_s} |y[n]|^2}{\sigma_w^2} - \frac{v-1}{\gamma_G/\sigma_w^2} \right) dv.$$

Furthermore, we can write

$$\Lambda_{GB}(\mathbf{y}) = \frac{\sigma_w^2 P_G(H_1)}{\gamma_G P_G(H_0)} \exp \left(\frac{\sum_{n=1}^{N_s} |y[n]|^2}{\sigma_w^2} + \frac{\sigma_w^2}{\gamma_G} \right) I \quad (5.16)$$

with

$$I = \int_1^\infty v^{-N_s} \exp \left(-\frac{\sum_{n=1}^{N_s} |y[n]|^2}{\sigma_w^2} \frac{1}{v} - \frac{\sigma_w^2}{\gamma_G} v \right) dv. \quad (5.17)$$

The integral I can be rewritten as

$$I = \int_1^\infty \psi(v) dv \quad (5.18)$$

where

$$\psi(v) = v^{-N_s} \exp(-\delta/v) \exp(-\kappa v) \quad (5.19)$$

with $\delta = \frac{\sum_{n=1}^{N_s} |y[n]|^2}{\sigma_w^2}$ and $\kappa = \frac{\sigma_w^2}{\gamma_G}$. Note that κ is the inverse of the average SNR at reception and δ includes the energy of the observed signal.

A closed-form solution does not actually exist for I and we need to find an approximate solution. The function $\psi(v)$ is the product of three terms, and we can try to approximate one of them, using Taylor series expansion around a carefully chosen point. For example, if we are able to prove that the term $\exp(-\kappa v)$ does not change too quickly around the maximum v_m of the function $\psi(v)$, in the interval $[1, \infty[$, this term can be approached with few terms of its Taylor series expansion around v_m . We can numerically prove that under certain conditions, this approximation can be done. We make the following propositions.

Proposition 5.1. *The maximum of $\psi(v)$ in the interval $[1, \infty[$, is given by*

$$v_m = \max \left(1, \frac{\sqrt{N_s^2 + 4\kappa\delta} - N_s}{2\kappa} \right). \quad (5.20)$$

Proof. See Section 5.A.1. □

Proposition 5.2. *For $1/\kappa \geq 0.3162$, and a large enough value of N_s , we can obtain an accurate approximation of the function $\psi(v)$ by replacing $\exp(-\kappa v)$ with its first N_t Taylor series terms ($N_t \geq 4$) around the maximum v_m of $\psi(v)$. This range of values for κ corresponds to average received SNR values greater than or equal to -5 dB. The minimum value of N_s , for which the approximation works, decreases with the average received SNR.*

Proof. See Section 5.A.2. □

The Taylor series expansion of $\exp(-\kappa v)$ around v_m is given by

$$\exp(-\kappa v) = \exp(-\kappa v_m) \left(1 + \sum_{r=1}^{\infty} \frac{(v - v_m)^r}{r!} (-\kappa)^r \right). \quad (5.21)$$

Therefore, for $\frac{\gamma_G}{\sigma_w^2} \geq 0.3162$, or average received SNR greater than or equal

to -5 dB, we can write

$$\begin{aligned}
 I &= \int_1^\infty v^{-N_s} \exp(-\delta/v) \exp(-\kappa v) dv \\
 &\approx \int_1^\infty v^{-N_s} \exp(-\delta/v) \exp(-\kappa v_m) \left(1 + \sum_{r=1}^{N_t} \frac{(v - v_m)^r}{r!} (-\kappa)^r\right) dv \\
 &\approx \exp(-\kappa v_m) \int_1^\infty v^{-N_s} \exp(-\delta/v) \left(1 + \sum_{r=1}^{N_t} \frac{(v - v_m)^r}{r!} (-\kappa)^r\right) dv \\
 &\approx \exp(-\kappa v_m) \left\{ \left(\int_1^\infty v^{-N_s} \exp(-\delta/v) dv \right. \right. \\
 &\quad \left. \left. + \sum_{r=1}^{N_t} \frac{(-\kappa)^r}{r!} \int_1^\infty v^{-N_s} (v - v_m)^r \exp(-\delta/v) dv \right) \right\}.
 \end{aligned}$$

Using the Binomial expansion, we can write

$$(v - v_m)^r = \sum_{\dot{r}=0}^r \binom{r}{\dot{r}} v^{\dot{r}} (-v_m)^{r-\dot{r}}, \quad (5.22)$$

where

$$\binom{r}{\dot{r}} = \frac{r!}{\dot{r}! (r - \dot{r})!}.$$

By inserting (5.22) into I , we have

$$\begin{aligned}
 I &\approx \exp(-\kappa v_m) \left\{ \left(\int_1^\infty v^{-N_s} \exp(-\delta/v) dv \right. \right. \\
 &\quad \left. \left. + \sum_{r=1}^{N_t} \frac{(-\kappa)^r}{r!} \int_1^\infty v^{-N_s} \sum_{\dot{r}=0}^r \binom{r}{\dot{r}} v^{\dot{r}} (-v_m)^{r-\dot{r}} \exp(-\delta/v) dv \right) \right\} \\
 &\approx \exp(-\kappa v_m) \left\{ \left(\int_1^\infty v^{-N_s} \exp(-\delta/v) dv \right. \right. \\
 &\quad \left. \left. + \sum_{r=1}^{N_t} \frac{(-\kappa)^r}{r!} \sum_{\dot{r}=0}^r \binom{r}{\dot{r}} (-v_m)^{r-\dot{r}} \int_1^\infty v^{-N_s} v^{\dot{r}} \exp(-\delta/v) dv \right) \right\} \\
 I &\approx \exp(-\kappa v_m) \left\{ \left(I_1 + \sum_{r=1}^{N_t} \sum_{\dot{r}=0}^r \frac{(-\kappa)^r}{\dot{r}! (r - \dot{r})!} (-v_m)^{r-\dot{r}} I_2 \right) \right\}
 \end{aligned}$$

with

$$I_1 = \int_1^\infty v^{-N_s} \exp(-\delta/v) dv, \quad (5.23)$$

and

$$I_2 = \int_1^\infty v^{\dot{r}-N_s} \exp(-\delta/v) dv. \quad (5.24)$$

The integrals I_1 and I_2 can be easily solved using closed-form solutions (2.33.10) or (2.33.11) in [112]. The solutions are given as follows

$$I_1 = \frac{\Gamma(N_s - 1, 0) - \Gamma(N_s - 1, \delta)}{\delta^{N_s-1}} \quad (5.25)$$

or

$$I_1 = \frac{(N_s - 2)!}{\delta^{N_s-1}} \left(1 - \exp(-\delta) \left(\sum_{\dot{s}=0}^{N_s-2} \frac{\delta^{\dot{s}}}{\dot{s}!} \right) \right), \quad (5.26)$$

and

$$I_2 = \frac{\Gamma(N_s - \dot{r} - 1, 0) - \Gamma(N_s - \dot{r} - 1, \delta)}{\delta^{N_s-\dot{r}-1}} \quad (5.27)$$

or

$$I_2 = \frac{(N_s - \dot{r} - 2)!}{\delta^{N_s-\dot{r}-1}} \left(1 - \exp(-\delta) \left(\sum_{\dot{s}=0}^{N_s-\dot{r}-2} \frac{\delta^{\dot{s}}}{\dot{s}!} \right) \right), \quad (5.28)$$

where $\Gamma(\cdot, \cdot)$ is the incomplete gamma function given by

$$\Gamma(\eta, u) = \int_u^\infty \exp(-t) t^{\eta-1} dt.$$

Finally, an approximate closed-form expression of the geolocation-based Bayesian decision statistic is given by

$$\Lambda_{GB}(\mathbf{y}) \approx \frac{\sigma_w^2 P_G(H_1)}{\gamma_G P_G(H_0)} \exp \left(\frac{\sum_{n=1}^{N_s} |y[n]|^2}{\sigma_w^2} + \frac{\sigma_w^2}{\gamma_G} (1 - v_m) \right) \left\{ \left(I_1 + \sum_{r=1}^{N_t} \sum_{\dot{r}=0}^r \frac{\left(-\frac{\sigma_w^2}{\gamma_G} \right)^r}{\dot{r}! (r - \dot{r})!} (-v_m)^{r-\dot{r}} I_2 \right) \right\} \quad (5.29)$$

where

I_1 and I_2 are obtained from (5.25 – 5.28), with $\delta = \frac{\sum_{n=1}^{N_s} |y[n]|^2}{\sigma_w^2}$,

$N_t \geq 4$ is the number of Taylor series terms considered for the approximation,

and v_m is obtained from (5.20), with $\kappa = \frac{\sigma_w^2}{\gamma_G}$.

5.4 Numerical results

In this section, we numerically evaluate the performance of the geolocation-based Bayesian detector introduced in the previous section. In the first instance, we verify the precision of the derived approximate closed-form expression, when it is used for spectrum sensing. In the second instance, we compare the proposed detector to the classical energy detection algorithm.

5.4.1 Precision of the closed-form expression

Here, we check the correctness of the decision statistic closed-form expression (5.29), by comparing it with the numerical resolution of (5.16). To do this, we plot the detection and false alarm probabilities versus the average received SNR, obtained through simulations. For each average SNR value, the PU signal power is exponentially generated, and the performance is computed using Monte Carlo simulations. Using fixed prior probabilities $P_G(H_0) = P_G(H_1) = 0.5$ and sensing time $N_s = 100$ samples, the curves are plotted for the numerical resolution and the approximate closed-form expressions obtained respectively with $N_t = 4$ and $N_t = 12$. As it can be seen in Fig. 5.2 and 5.3, with $N_t = 4$, the closed-form expression is a very good approximation of the detector when the average SNR is greater than -5 dB. With an increased number of Taylor series terms ($N_t = 12$), the approximation can be accurate down to -10 dB. For lower SNR values, the approximation does not give accurate performance. It can thus be concluded that the derived closed-form expression is a good approximation of the geolocation-based Bayesian detector, for medium to high average SNR values, when the sensing time N_s is long enough.

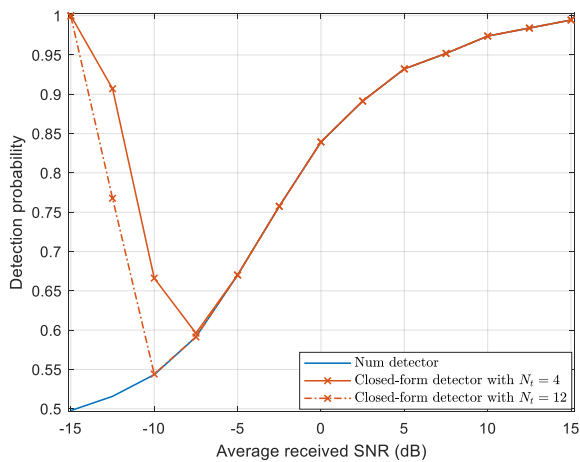


Fig. 5.2 Detection probability vs average received SNR of both numerical and analytical detector.

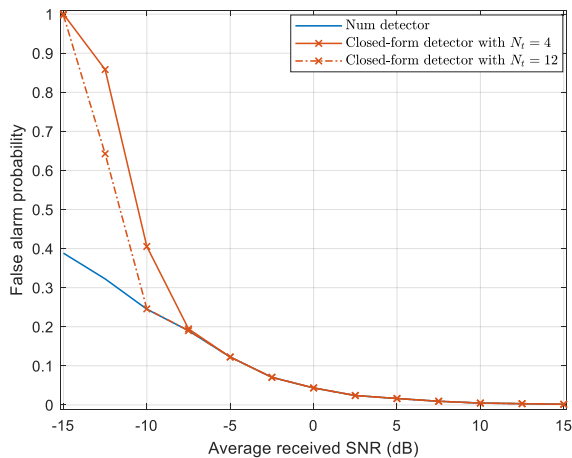


Fig. 5.3 False alarm probability vs average received SNR of both numerical and analytical detector.

5.4.2 Comparison with energy detection

To evaluate the gain of combining geolocation information with spectrum sensing, we compare our algorithm with energy detection (ED) that does not use any *a priori* knowledge of the PU signal power, and therefore makes no use of G , but ensures the maximum detection probability (P_D) for a fixed false alarm probability (P_{FA}).

As a first step, we make the comparison by fixing the prior probabilities $P_G(H_0) = P_G(H_1) = 0.5$ and setting the sensing time to $N_s = 100$ samples. ED is computed with target $P_{FA} = 0.1$. The probabilities of detection, false alarm and error obtained through numerical simulations are respectively presented in Fig. 5.4 – 5.6. The geolocation-based algorithm has the best detection probability at low SNR, but gets slightly worse at high SNR. This trend is reversed for the false alarm probability. However, the geolocation-based algorithm always keeps the error probability to the minimum.

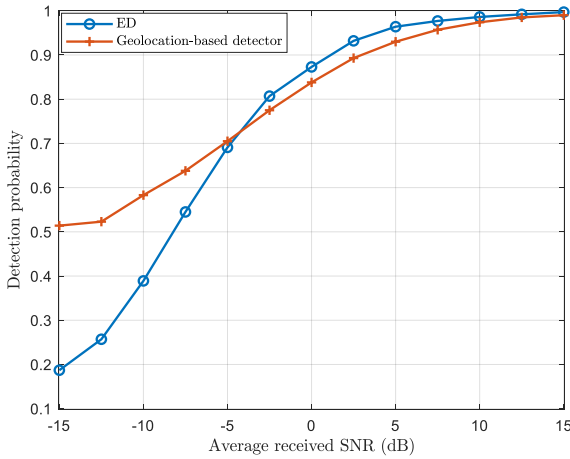


Fig. 5.4 Detection probability vs average received SNR for the compared algorithms, when target $P_{FA} = 0.1$ for ED.

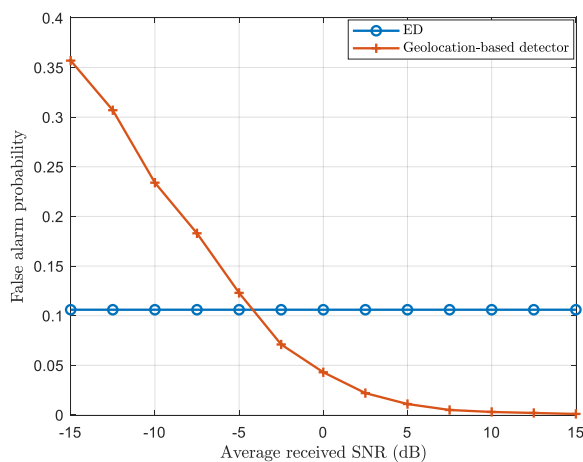


Fig. 5.5 False alarm probability vs average received SNR for the compared algorithms, when target $P_{FA} = 0.1$.

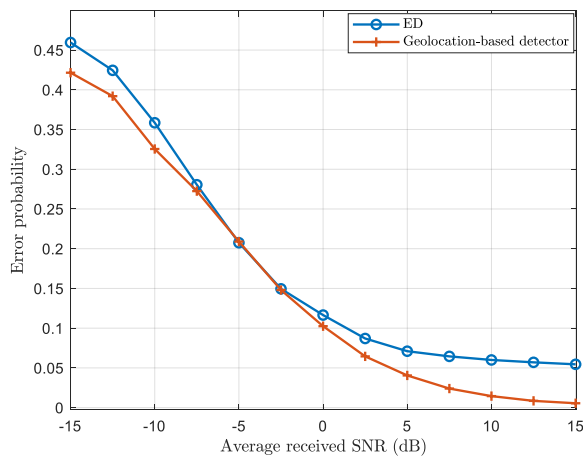


Fig. 5.6 Error probability vs average received SNR for the compared algorithms, when target $P_{FA} = 0.1$.

When the target P_{FA} of ED is increased (Fig. 5.8), it shows a higher detection probability at every SNR value (Fig. 5.7), but still has a poor total error probability (Fig. 5.9).

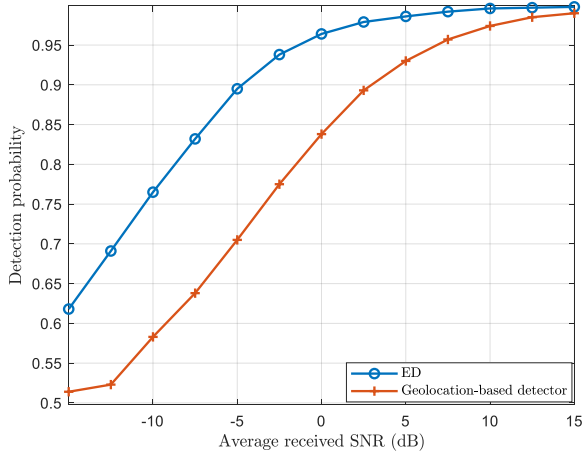


Fig. 5.7 Detection probability vs average received SNR for the compared algorithms, when target $P_{FA} = 0.475$ for ED.

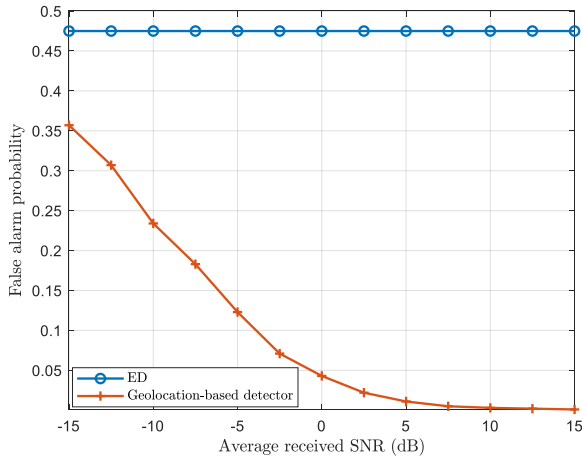


Fig. 5.8 False alarm probability vs average received SNR for the compared algorithms, when target $P_{FA} = 0.475$.

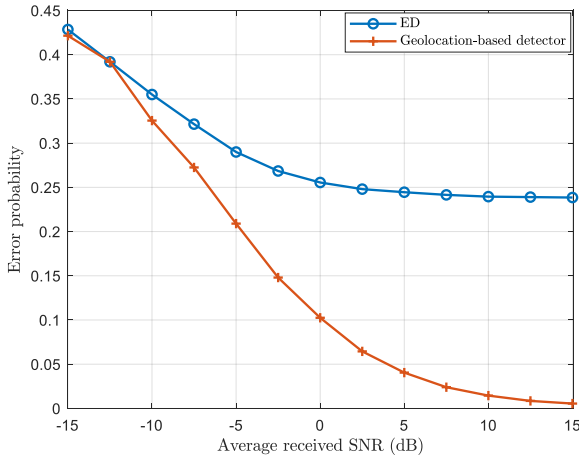


Fig. 5.9 Error probability vs average received SNR for the compared algorithms, when target $P_{FA} = 0.475$.

As a second step in the performance comparison, we compute the prior probabilities $P_G(H_1)$ and $P_G(H_0)$, using the suggested expressions (5.7) and (5.8). Note that the prior probabilities are computed with the average received power and the minimum admissible power. For the simulations, in order to give a sense to the computed $P_G(H_1)$ and $P_G(H_0)$, the hypothesis about the presence H_1 or the absence H_0 is conditioned on the actual received power σ_x^2 . In other words, the cases where $\sigma_x^2 < P_{adm}$, are considered as H_0 and the reversed cases are considered as H_1 . The sensing time is always $N_s = 100$ samples and the minimum SNR admissible by a PU receiver (P_{adm}/σ_w^2) is set to -15 dB. The results are shown in Fig. 5.10 – 5.12. As it can be seen, the detection probability of the geolocation-based algorithm quickly increases with the SNR, and it is better than that of ED. However, the false alarm probability also increases very fast, while ED keeps a constant false alarm because its threshold does not change. The reason for this is that when the average received power γ_G exceeds a bit the minimum admissible power P_{adm} , the probability of presence $P_G(H_1)$ quickly increases and the detection threshold decreases. This results in a fast increase of P_D and P_{FA} . To avoid this, a solution may be to add some costs to appropriately counterbalance $P_G(H_1)$ and $P_G(H_0)$. Nonetheless, the geolocation-based algorithm always keeps the minimum error probability.

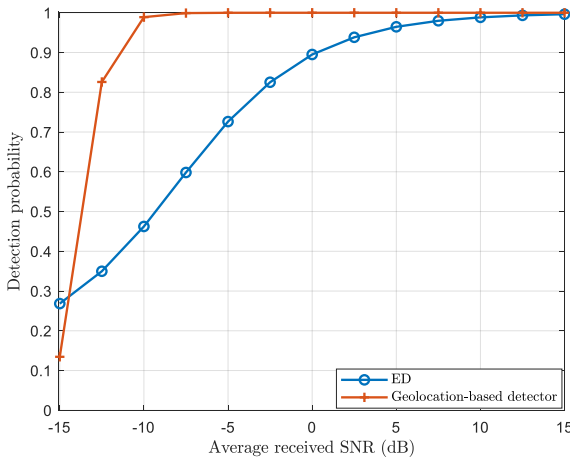


Fig. 5.10 Detection probability vs average received SNR when $P_G(H_1)$ and $P_G(H_0)$ are computed using the geolocation information.

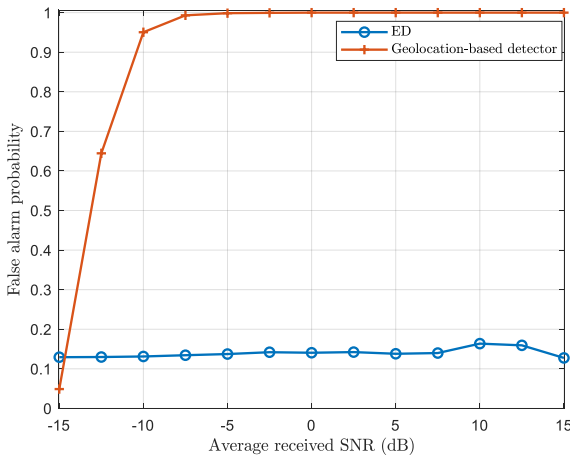


Fig. 5.11 False alarm probability vs average received SNR when $P_G(H_1)$ and $P_G(H_0)$ are computed using the geolocation information.

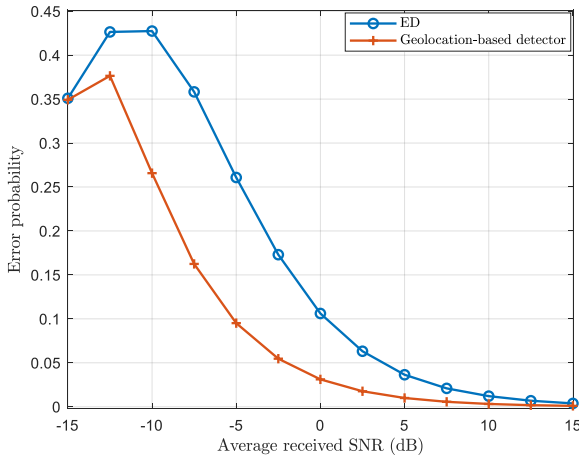


Fig. 5.12 Error probability vs average received SNR when $P_G(H_1)$ and $P_G(H_0)$ are computed using the geolocation information.

5.5 Conclusion

In this chapter, we consider the block-based hypothesis testing framework and introduce a new spectrum sensing algorithm that makes the use of prior knowledge about the PUs to improve the detection. The information concerning the PUs is retrieved from a geolocation database and used to compute the prior fading distribution of the received power, depending on the location of the SU. Then, using the Bayesian approach of composite hypothesis testing, a new detector, that does not require the perfect knowledge of the actual PU signal power, is derived. Moreover, an approximate closed-form expression is derived for the detection algorithm, and it shows to be an accurate approximation for medium to high average SNR values. In addition, a new way of computing the prior probabilities of presence and absence is suggested.

When compared with the energy detection algorithm, the derived algorithm gives the minimum error probability. Therefore, having an *a priori* geolocation information about the PUs, a good trade-off between false alarms and good detections can be achieved, by using the Bayesian detection theory.

5.A Proofs

5.A.1 Proof of proposition 5.1

Let us recall the expression (5.18) of I as follows

$$I = \int_1^{\infty} \psi(v) dv$$

where

$$\psi(v) = v^{-N_s} \exp(-\delta/v) \exp(-\kappa v).$$

We want to find the maximum of $\psi(v)$ in the interval $[1, \infty[$, which is equivalent to solve the following

$$\max_{v \geq 1} \zeta(v) = -N_s \ln(v) - \frac{\delta}{v} - \kappa v \quad (5.30)$$

with $\zeta(v) = \ln(\psi(v))$.

This can be easily done using the Lagrangian method. We define the Lagrangian for this problem as

$$\begin{aligned} \mathcal{L}(v, \lambda) &= \zeta(v) - \lambda(-v + 1) \\ &= -N_s \ln(v) - \frac{\delta}{v} - \kappa v + \lambda v - \lambda \end{aligned} \quad (5.31)$$

where λ is the Lagrange multiplier. Its first derivative with respect to v is given by

$$\frac{\partial \mathcal{L}(v, \lambda)}{\partial v} = \frac{-N_s}{v} + \frac{\delta}{v^2} - \kappa + \lambda.$$

Let us find the zeros of this derivative. We have

$$\begin{aligned} \frac{\partial \mathcal{L}(v, \lambda)}{\partial v} = 0 &\iff \frac{-N_s}{v} + \frac{\delta}{v^2} - \kappa + \lambda = 0 \\ &\iff \frac{(\lambda - \kappa)v^2 - N_s v + \delta}{v^2} = 0 \\ &\iff (\lambda - \kappa)v^2 - N_s v + \delta = 0. \end{aligned}$$

We obtain two zeros as follows

$$v_1 = \frac{N_s - \sqrt{N_s^2 - 4(\lambda - \kappa)\delta}}{2(\lambda - \kappa)} \text{ and } v_2 = \frac{N_s + \sqrt{N_s^2 - 4(\lambda - \kappa)\delta}}{2(\lambda - \kappa)}.$$

The Karush-Kuhn-Tucker (KKT) conditions for the optimization problem (5.30) are given by

$$\begin{cases} \frac{\partial \mathcal{L}(v, \lambda)}{\partial v} = 0 & \iff v = \frac{N_s - \sqrt{N_s^2 - 4(\lambda - \kappa)\delta}}{2(\lambda - \kappa)} \text{ or } v = \frac{N_s + \sqrt{N_s^2 - 4(\lambda - \kappa)\delta}}{2(\lambda - \kappa)} \\ \frac{\partial \mathcal{L}(v, \lambda)}{\partial \lambda} \geq 0 & \iff v - 1 \geq 0 \\ \lambda \geq 0 & \iff \lambda \geq 0 \\ \lambda(-v + 1) = 0 & \iff \lambda = 0 \text{ or } v = 1. \end{cases} \quad (5.32)$$

The solutions can be split in two cases:

- **Case 1:** $v > 1 \implies \lambda = 0, v = \frac{\sqrt{N_s^2 + 4\kappa\delta} - N_s}{2\kappa} \text{ or } v = -\frac{N_s + \sqrt{N_s^2 + 4\kappa\delta}}{2\kappa}$
The second zero is discarded because it gives a negative value which contradicts $v > 1$. We keep $v = \frac{\sqrt{N_s^2 + 4\kappa\delta} - N_s}{2\kappa}$ which is obtained for $\sqrt{N_s^2 + 4\kappa\delta} > 2\kappa + N_s$ or simply $N_s + \kappa - \delta < 0$.
- **Case 2:** $v = 1 \implies N_s - \sqrt{N_s^2 - 4(\lambda - \kappa)\delta} = 2(\lambda - \kappa)$, considering only the first zero. By developing the equation, we obtain $\lambda = \kappa$ or $\lambda = N_s + \kappa - \delta$. As κ is always a positive quantity, this case is thus obtained for $\lambda = N_s + \kappa - \delta \geq 0$.

To verify if $v_m = \frac{\sqrt{N_s^2 + 4\kappa\delta} - N_s}{2\kappa}$ is a maximum of $\zeta(v)$, we compute the second-order derivative at this value. The second order derivative of $\zeta(v)$ is given by

$$\frac{\partial^2 \zeta(v)}{\partial v^2} = \frac{Nv - 2\delta}{v^3}.$$

As the denominator is always positive in the interval $[1, \infty[$, we just check

the sign of the numerator. By inserting v_m in the numerator, we obtain

$$\begin{aligned}
 N_s v_m - 2\delta &= N_s \left(\frac{\sqrt{N_s^2 + 4\kappa\delta} - N_s}{2\kappa} \right) - 2\delta \\
 &= \frac{N_s}{2\kappa} \left(\sqrt{N_s^2 + 4\kappa\delta} - N_s - \frac{4\kappa\delta}{N_s} \right) \\
 &= \frac{N_s}{2\kappa} \left(\sqrt{N_s \left(N_s + \frac{4\kappa\delta}{N_s} \right)} - \left(N_s + \frac{4\kappa\delta}{N_s} \right) \right) \\
 &= \frac{N_s}{2\kappa} \left(\left(\sqrt{N_s} \right) \sqrt{\left(N_s + \frac{4\kappa\delta}{N_s} \right)} - \sqrt{\left(N_s + \frac{4\kappa\delta}{N_s} \right)} \sqrt{\left(N_s + \frac{4\kappa\delta}{N_s} \right)} \right)
 \end{aligned}$$

$$N_s v_m - 2\delta \leq 0,$$

because $N_s \leq N_s + \frac{4\kappa\delta}{N_s}$.

Consequently, the solution to the optimization problem (5.30) can be compactly written as

$$v_m = \max \left(1, \frac{\sqrt{N_s^2 + 4\kappa\delta} - N_s}{2\kappa} \right). \quad (5.33)$$

5.A.2 Proof of proposition 5.2

We remind below the expression (5.18) of I

$$I = \int_1^\infty \psi(v) dv$$

where

$$\psi(v) = v^{-N_s} \exp(-\delta/v) \exp(-\kappa v).$$

Here, we numerically prove that for $1/\kappa \geq 0.3162$, and a large enough value of N_s , we can obtain an accurate approximation of the function $\psi(v)$ by replacing $\exp(-\kappa v)$ with its first N_t Taylor series terms ($N_t \geq 4$) around the maximum v_m of $\psi(v)$.

We recall that N_s is the observation length, $\kappa = \frac{\sigma_w^2}{\gamma_G}$ is the inverse of the average received SNR, and $\delta = \frac{\sum_{n=1}^{N_s} |y[n]|^2}{\sigma_w^2}$. Typical values of N_s ranges

from 1 to 100 samples or higher. Considering an average SNR range from -20 dB to 20 dB, κ ranges between 0.01 and 100 in reverse order. Finally, δ depends on the observation and thus is a random variable, but we can have an idea of its average value. Let us denote by $\mathbb{E}_{f_0}$ and $\mathbb{E}_{f_1}$ the expectations under the hypotheses H_0 and H_1 . We have

$$\begin{aligned}\mathbb{E}_{f_0}(\delta) &= \mathbb{E}\left(\frac{\sum_{n=1}^{N_s} |w[n]|^2}{\sigma_w^2}\right) \\ &= \frac{\sum_{n=1}^{N_s} \mathbb{E}\left(|w[n]|^2\right)}{\sigma_w^2} \\ &= N_s\end{aligned}\tag{5.34}$$

and

$$\begin{aligned}\mathbb{E}_{f_1}(\delta) &= \mathbb{E}\left(\frac{\sum_{n=1}^{N_s} |x[n] + w[n]|^2}{\sigma_w^2}\right) \\ &= \frac{\sum_{n=1}^{N_s} \mathbb{E}\left(|x[n] + w[n]|^2\right)}{\sigma_w^2} \\ &\leq \frac{\sum_{n=1}^{N_s} \mathbb{E}\left(|x[n]|^2 + |w[n]|^2\right)}{\sigma_w^2} \\ &\leq \frac{N_s (\sigma_x^2 + \sigma_w^2)}{\sigma_w^2}.\end{aligned}\tag{5.35}$$

$\mathbb{E}_{f_1}(\delta)$ is random, and we have

$$\mathbb{E}(\mathbb{E}_{f_1}(\delta)) \leq N_s \left(\frac{\gamma_G}{\sigma_w^2} + 1 \right) = N_s (1/\kappa + 1).\tag{5.36}$$

According to (5.34) and (5.36), the minimum value of δ in average is equal to N_s , which is obtained when there is only noise, and its maximum value in average is $N_s(1/\kappa + 1)$, which is obtained when the PU signal is present. At very low SNR, even in case of PU presence, the value of δ is closer to the noise-only case. To summarize, the minimum value of δ is typically between 1 and 100, or higher depending on N_s . Its maximum value is obtained for the minimum value of κ (high average SNR) and is in average equal to $101 N_s$, i.e., between 101 and 10100.

In the interval of interest $[1, \infty[$, $\exp(-\kappa v)$ is an exponentially decreases-

ing function. The decay rate is higher for high values of κ and lower for low values of κ . This is shown in Fig. 5.13, where the $\exp(-\kappa v)$ curve is plotted for $\kappa \in \{0.03, 0.3\}$ or average received SNR $\in \{15.2, 5.2\}$ dB. For very low decay rate ($\kappa \rightarrow 0$), this function tends to be a straight line. Note that $\exp(-\kappa v)$ can be more accurately approximated with few of its first Taylor series expansion terms if the decay rate is low, i.e., for low values of κ (high average SNR).

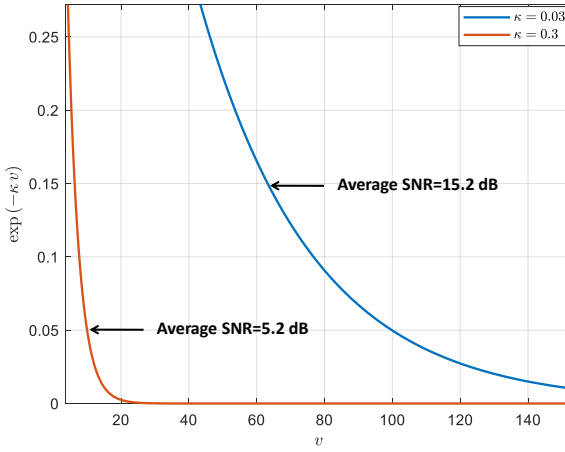


Fig. 5.13 $\exp(-\kappa v)$ vs v for $\kappa \in \{0.03, 0.3\}$ or average SNR $\in \{15.2, 5.2\}$ dB.

In the same interval, the power function v^{-N_s} is a decreasing function which decreases at a higher rate for high values of N_s and always has the value 1 at $v = 1$. Knowing that in block-based spectrum sensing, the higher N_s is, the better the detection performance is, the decay rate of v^{-N_s} may often be high.

The function $\exp(-\delta/v)$ exhibits two parts, as depicted in Fig. 5.14. It first shows a rapid (exponential) increase for $v \in [1, \delta/2]$ and then a slow (logarithmic) increase for $v \in [\delta/2, \infty]$, with an asymptotic increase towards 1 when $v \rightarrow \infty$. Note that there is a change of concavity at $v = \delta/2$ (the zero of the second order derivative). Moreover, if δ is low, the first part of the curve is short, the growth rate is high and $\exp(-\delta/v)$ converges more quickly towards the value 1. But if δ is high, the first part of the curve is long and the growth rate is low. For very low values (around 0.02) of

δ , $\exp(-\delta/v)$ can be assimilated to the asymptote in the interval of $[1, \infty[$, but in practice δ will not have such lower values as already shown. As the typical values of δ considered in our model are large, the first part will be rather long (low growth rate). Nevertheless, the change of concavity can make the approximation difficult by requiring a lot of terms.

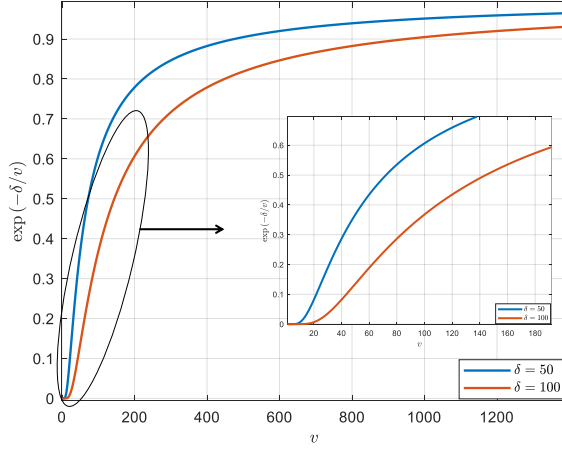


Fig. 5.14 $\exp(-\delta/v)$ vs v for $\delta \in \{50, 100\}$.

Based on the above analysis, $\exp(-\kappa v)$ is the function that can be easily approximated because it does not vary too quickly in the interval $[1, \infty[$, but only for low values of κ , which corresponds to high values of the average SNR. For high values of κ (low SNR), the approximation of $\exp(-\kappa v)$ will become inaccurate, and it will require too many terms. The worst is that by increasing the number of terms, the approximation will quickly diverge after a certain interval and lead to large errors and inaccuracy. Moreover, this inaccuracy will be amplified if the maximum v_m of $\psi(v)$, around which the approximation is done, has a low value (i.e., is around the fast decay area of $\exp(-\kappa v)$). This is actually the case, as shown in Fig. 5.15, where it can be seen that v_m has a low value, for high values of κ (it approaches 1 as κ grows). Hence, the approximation of $\psi(v)$ may be more difficult for low SNR values. For this reason, we need to know the parameters (κ , N_s and N_t) values for which the function $\psi(v)$ can be accurately approximated by approximating $\exp(-\kappa v)$. In what follows, we show some numerical results to check this. The Taylor series expansion of $\exp(-\kappa v)$ around v_m is given by

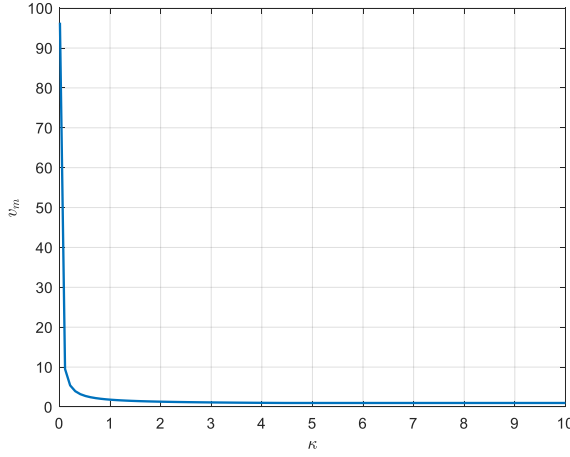


Fig. 5.15 Maximum v_m of $\psi(v)$ vs average SNR inverse κ for $N_s = 20$ and δ computed with (5.36).

$$\exp(-\kappa v) = \exp(-\kappa v_m) \left(1 + \sum_{r=1}^{\infty} \frac{(v - v_m)^r}{r!} (-\kappa)^r \right). \quad (5.37)$$

The approximations of $\exp(-\kappa v)$ and $v^{-N_s} \exp(-\delta/v) \exp(-\kappa v)$, for $\kappa = 0.1$ (average SNR = 10 dB), $N_s = 10$ and $N_t \in \{2, 4\}$ are shown in Fig. 5.16 and 5.17.

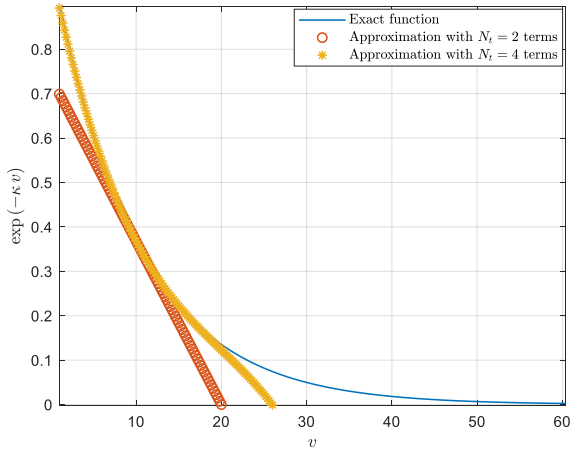


Fig. 5.16 Approximation of $\exp(-\kappa v)$ for $\kappa = 0.1$ (average SNR = 10 dB), $N_s = 10$ and $N_t \in \{2, 4\}$.

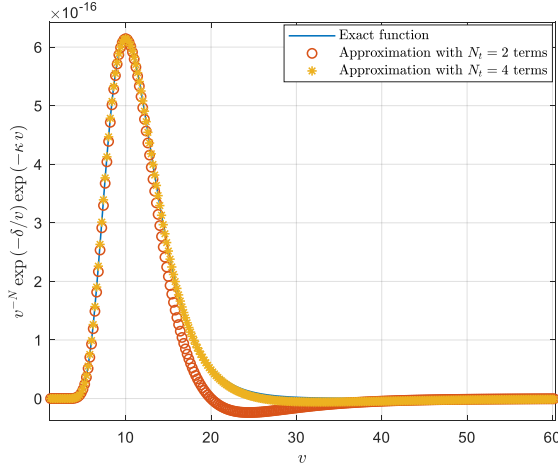


Fig. 5.17 Approximation of $v^{-N_s} \exp(-\delta/v) \exp(-\kappa v)$ for $\kappa = 0.1$ (average SNR = 10 dB), $N_s = 10$ and $N_t \in \{2, 4\}$.

We can observe that with 4 terms of the Taylor series expansion, we obtain a very good approximation of $\psi(v)$. For the same observation length N_s and number of terms, we plot the approximations for $\kappa = 1$ (average SNR = 0 dB) in Fig. 5.18 and 5.19.

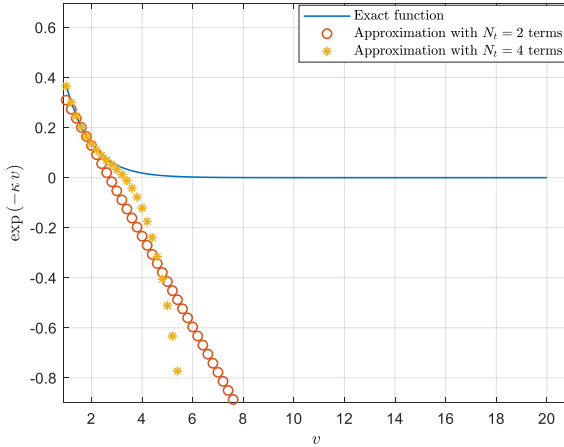


Fig. 5.18 Approximation of $\exp(-\kappa v)$ for $\kappa = 1$ (average SNR = 0 dB), $N_s = 10$ and $N_t \in \{2, 4\}$.

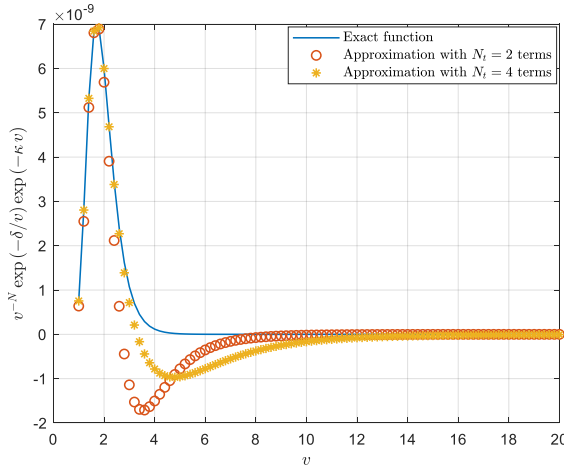


Fig. 5.19 Approximation of $v^{-N_s} \exp(-\delta/v) \exp(-\kappa v)$ for $\kappa = 1$ (average SNR = 0 dB), $N_s = 10$ and $N_t \in \{2, 4\}$.

We see that the approximation becomes less accurate because of the high decay rate of $\exp(-\kappa v)$ around the maximum v_m of $\psi(v)$. By increasing the number of terms ($N_t \in \{6, 8\}$), we can see in Fig. 5.21 that even if the approximation of $\exp(-\kappa v)$ becomes more accurate around v_m , after a certain interval, there is a large divergence which has a great impact on the accuracy of $\psi(v)$. But when we keep $N_t \in \{2, 4\}$, and increase the observation length to $N_s = 25$, we obtain a much better accuracy than previously. This can be seen in Fig. 5.20. Thus, the approximation seems to work better for high values of N_s , when the average SNR decreases. Furthermore, always for average SNR = 0 dB, we plot the approximation for $N_s = 15$ and $N_t \in \{6, 8\}$, and we can observe in Fig. 5.22 that we obtain a much better result than in Fig. 5.21, just by increasing a bit N_s . This suggests the idea that the approximation at a certain average SNR can become accurate when increasing the number of terms N_t , only for a minimum observation length N_s . Actually, this is due to the fact that for high values of N_s , the function $\psi(v)$ quickly vanishes when v increases (because of the v^{-N_s} term) and even if higher order approximations of $\exp(-\kappa v)$ quickly diverge from a certain value of v , the effect on $\psi(v)$ is nullified.

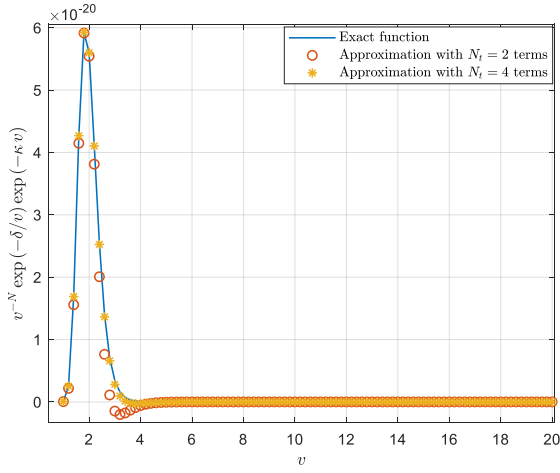


Fig. 5.20 Approximation of $v^{-N_s} \exp(-\delta/v) \exp(-\kappa v)$ for $\kappa = 1$ (average SNR = 0 dB), $N_s = 25$ and $N_t \in \{2, 4\}$.

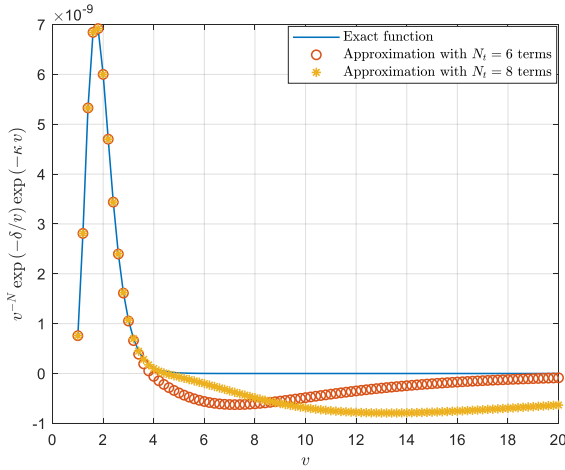


Fig. 5.21 Approximation of $v^{-N_s} \exp(-\delta/v) \exp(-\kappa v)$ for $\kappa = 1$ (average SNR = 0 dB), $N_s = 10$ and $N_t \in \{6, 8\}$.

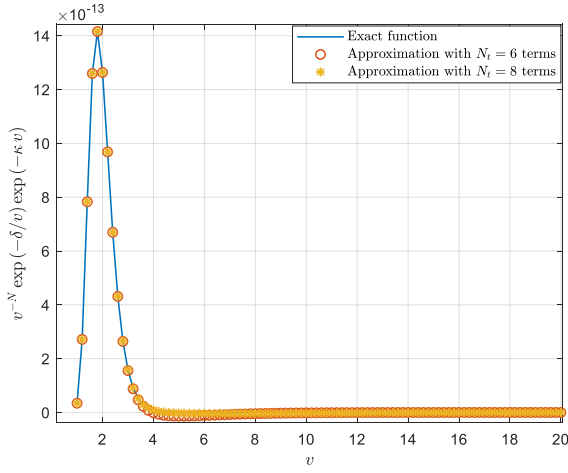


Fig. 5.22 Approximation of $v^{-N_s} \exp(-\delta/v) \exp(-\kappa v)$ for $\kappa = 1$ (average SNR = 0 dB), $N_s = 15$ and $N_t \in \{6, 8\}$.

With further simulations, we notice that for $\kappa = 10$ (average SNR = -10 dB), a good accuracy is obtained for $N_s = 100$ and $N_t = 12$. At lower average SNR, it becomes more difficult to obtain a good approximation even when N_s and N_t are increased. At this point, we can conclude that the suggested approximation works quite well for medium to high average SNR values (average SNR ≥ -5 dB or $\kappa \leq 3.162$), by using $N_t = 4$ terms and $N_s \geq 55$ samples. Note that $N_s = 55$ is the minimum observation length for average SNR = -5 dB. It can be decreased for higher SNR values.

6

Conclusion

In this chapter, we summarize the main contributions of this thesis and give some perspectives for future works.

6.1 Summary

The goal of this thesis was the design of new spectrum sensing algorithms for cognitive radio access to unlicensed TV white spaces. Our first objective was to use some *a priori* information about the environment characteristics, such as mobility and geolocation information, to propose new algorithms that would perform better. Secondly, these algorithms should work when the PU signal power is unknown to the SU, or when a few *a priori* knowledge was available. We also looked into how a good compromise could be obtained between interference and spectrum waste, in a context where the SU has a limited time for communication. Finally, we focused on the best system parameters (sensing time, transmission time, threshold), to maximize the spectrum usage. In a nutshell, our contributions are the following.

In Chapter 3, we investigated the use of changepoint detection theory for spectrum sensing in mobile CR environments. Unlike the hypothesis

testing approach, where the goal is to detect the current channel state assuming that it does not change during the observation, the goal of changepoint detection approach is to detect, with a minimum detection delay, a change in the observation. Our motivation for using changepoint detection was that in a mobile environment, there may be fast spectrum changes. It is thus preferable to continuously track the spectrum occupancy when the SU is moving, in order to detect the appearance or disappearance of the PU. Assuming that the mobility parameters of the SU could be translated to some *a priori* knowledge on the average time of spectrum change, we considered the Bayesian framework of changepoint detection, which includes this *a priori* knowledge. Based on this, we derived a new algorithm (LC GLR-Shiryaev) for situations where the PU signal power is not known, by applying an approximation of the GLRT approach to the optimal Bayesian changepoint detection algorithm. The derived LC GLR-Shiryaev algorithm shows better performance at low SNR, when compared with its non-Bayesian equivalent, which does not use the mobility parameters. LC GLR-Shiryaev is thus a good candidate for mobile CR systems, where the SU does not know the PU signal power, and it could be required to detect very low SNR signals. This is, indeed, the case in practical mobile CR scenarios, due to the presence of multipath fading and shadowing effects. Moreover, to better evaluate the performance of spectrum sensing algorithms in a time-limited communication context, we introduced a new performance metric, called the global penalty, that allows a trade-off between both interference time and spectrum waste time. This metric helped derive the optimal detection threshold, which depends on the SNR and the prior knowledge of the time of spectrum change. As LC GLR-Shiryaev works without the SNR knowledge, we suggested two approaches to provide the optimal threshold in unknown-SNR situations. The first approach consists in adaptive-threshold methods which estimate the SNR, and the second approach consists in a fixed-threshold method that is optimal in average, considering a certain SNR distribution. It was shown that the fixed-threshold method is almost equivalent to the adaptive-threshold methods under certain conditions. Finally, we performed a complexity analysis of the LC GLR-Shiryaev algorithm in comparison with the M-Shiryaev algorithm, which is a numerical resolution of the GLRT approach to the optimal Bayesian changepoint detection algorithm, and assumes some *a priori* knowledge of the PU signal power. It was noticed that M-Shiryaev had the lowest complexity, but required an accurate *a priori* knowledge, which might be difficult to obtain in practice.

In Chapter 4, we investigated the problem of joint transmission and sensing in a mobile CR scenario with multiple PU appearances and disappearances. We considered a system where the communication time is split into several frames, with a time period allocated to sensing and another to transmission. Our motivation was to find the best combination of sensing time, transmission time and detection threshold that should be chosen for a maximum spectrum usage, and to analyse how they depend on the environment (mobility parameters and SNR). Stemming from the idea of Chapter 3 that changepoint detection is more appropriate in mobile environments, we proposed a new framework that uses the CUSUM changepoint detection algorithm to perform spectrum sensing. The novelty of our framework is that in addition to use a changepoint detection algorithm, spectrum sensing is performed sequentially by combining observations frame by frame and then inputting them into the CUSUM algorithm over successive frames. In previous works, spectrum sensing was done independently frame by frame. To compute the optimal system parameters, we introduced a new performance metric, called the global cost, that gives a trade-off between the interference and spectrum waste durations during a time-limited communication of the SU. This metric is almost identical to the global penalty of Chapter 3, but it considers the joint effect of sensing time and transmission time, and also accounts for the detection of multiple PU appearances and disappearances. The optimal system parameters, i.e., sensing time, transmission time and threshold, that minimize the global cost, were jointly computed and analysed through numerical simulations. It was found that the optimal sensing time is strongly dependent on the SNR and slightly on the threshold, while the optimal transmission time was strongly sensitive to the mobility parameters and to the threshold. Concerning the optimal threshold, it depends both on the SNR and the mobility parameters. Based on these results, intuitive rules were given to choose these optimal system parameters. Lastly, we compared CUSUM and ED algorithms, in the context of our framework. It was found that the superiority of CUSUM over ED is more pronounced when ED is performed over more than one frame. This suggested the idea that performing CUSUM sample per sample over successive frames would yield a better system performance than performing it frame by frame, as it is currently done in our framework. In fact, this would allow CUSUM to take detection decisions as quickly as possible, especially for PU disappearances detections.

In Chapter 5, we investigated the use of available *a priori* information from a geolocation database, to derive a new spectrum sensing algorithm. Assuming that the PUs are TV transmitters, and that the environment is not subject to fast spectrum changes, we considered the Bayesian paradigm of the block-based hypothesis testing, unlike in previous chapters. As the received PU signal power is not deterministic in a real-world scenario, it is difficult to get an optimal decision statistic. To solve this, we computed the distribution of the PU signal power, by using the *a priori* geolocation information. Then, using the mixture-based test, we suggested a new Bayesian detection rule that includes the geolocation information. As there was no existing closed-form expression for the detector, we provided an approximate closed-form expression, by using Taylor series approximation. The obtained approximation is quite accurate with few terms of the Taylor series, for average received SNR greater than or equal to -5 dB, when the sensing time is long enough. By increasing the number of terms, it is possible to get a good accuracy down to -10 dB. Additionally, we suggested computing the prior probabilities of PU presence and absence, by using the geolocation information. The comparison of our algorithm with energy detection, that does not use the *a priori* geolocation information, showed that by using the geolocation information about the PUs, we can achieve the minimum error probability, even if the received power signal is not perfectly known.

6.2 Future works

The various research directions of this thesis have paved the way for novel avenues and research problems. Some of them are listed below.

- In Chapter 3 and 4, we assumed that the *a priori* probability of spectrum change could be inferred from the mobility parameters of the SU. However, we did not consider any specific model relating them in this thesis. Such a model could be derived either mathematically, using stochastic geometry, or experimentally, by means of field measurements. This might be an interesting subject to investigate, in order to allow the use of LC GLR-Shiryaev in real-world CR systems. Moreover, it is important to study the effects of a misestimation of

this *a priori* probability over the system performance.

- Another potential direction concerning the LC GLR-Shiryaev algorithm, derived in Chapter 3, is to reduce its computational complexity, which grows quadratically with the number of samples (Table 3.4). This could be done in two ways. First, a window-based GLRT could be proposed. It consists in performing the GLRT only over a limited-number of observation samples and not all the observed samples. Obviously, the choice of optimal window size would depend on the SNR and the SU's mobility, and this will have an impact on the system performance. Adaptive likelihood ratios are also a very attractive alternative to GLRT, in order to reduce the complexity.
- Instead of using the GLRT proposed in Chapter 3, the mixture-based approach of Chapter 5 could be used in a Bayesian changepoint detection model, by using a fading distribution.
- As discussed in Chapter 4, Bayesian changepoint detection could be considered in the joint transmission-sensing framework. It would thus be interesting to evaluate the framework performance when using for example the LC GLR-Shiryaev algorithm. Moreover, as it was highlighted, the framework performance could be improved if spectrum sensing was performed sample per sample, for disappearances detections. This should be verified. The effect of different probabilities of appearance and disappearance should also be investigated.
- The approximate closed-form of the Bayesian spectrum sensing algorithm proposed in Chapter 5 is not accurate for very low average SNR values (less than -10 dB). It would be interesting to provide another closed-form for this SNR region. Another direction would be to use a more general fading model than the Rayleigh fading, to derive the algorithm. This would result in a robust spectrum sensing algorithm for any kind of fading channel. For more practical scenarios, the interference between the PUs could also be considered to derive a new algorithm. Finally, the computation of the prior probabilities of presence and absence could be improved to avoid obtaining such a high false alarm probability.

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