Degrees of Freedom of 2-Tier Networks without Channel State Information at the Transmitter

Máximo Morales-Céspedes, Luc Vandendorpe, Ana García Armada

***Abstract*—We characterize the degrees of freedom (DoF) of the 2-tier heterogeneous networks without channel state information at the transmitters (CSIT) or cooperation among transmitters. We consider conventional transmitters while the users are equipped with a multi-mode receiver. In contrast to previous works focused on 2-tier networks, the derived DoF consider the management of the intracell, the intercell and inter-tier interference. The derived bounds define the DoF region for any 2-tier network. Furthermore, these bounds are achievable by combining blind interference alignment (BIA) schemes properly.**

***Index Terms*—Degrees of Freedom, 2-tier networks, Blind Interference Alignment, multi-mode receivers.**

I. Introduction

Determining the capacity is still an open problem for most of the channel configurations. Several interference alignment (IA) schemes have been proposed to obtain the maximum Degrees of Freedom (DoF), as a first order approximation of the capacity, for interference networks [1]. However, implementing these schemes is subject to providing channel state information at the transmitter (CSIT), which results in a huge overhead that makes the implementation of IA a challenge [2].

In [3], a scheme referred to as Blind Interference Alignment (BIA) is proposed to achieve the optimal DoF for the broadcast channel (BC) without CSIT [4]. BIA is based on exploiting the channel correlation of the users over a coherence period [5]. In [6], the use of multi-mode receivers is proposed for this matter. Recently, determining the DoF region and its achievability have been considered for caching [7] and information retrieval [8]. The DoF in the interference channel (IC) is analyzed in [9], [10]. The DoF without CSIT for homogenous cellular networks and its achievability through BIA are presented in [11]. It is shown that achieving the DoF for cellular networks requires to differentiate the users depending on their connectivity, i.e., the number of transmitters from which each user receives a useful signal. For 2-tier networks a cognitive BIA scheme is proposed in [12]. However, this approach is subject to treating the interference among the transmitters of the lower tier as noise, which limits the usefulness of this approach. To the best of our knowledge, the achievable DoF without CSIT for generic 2-tier networks, i.e., accounting for the intracell, intercell and inter-tier interference in any tier, has not been determined yet.

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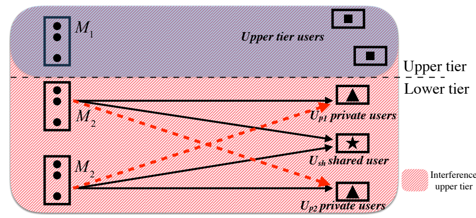

Fig. 1. 2-tier scenario. All the users in the lower tier are subject to interference because of transmission in the upper tier.

In this work we present the DoF without CSIT in a generic 2-tier network. It is interesting to remark that the results obtained in this work are not restricted to cellular systems and are applicable to any 2-tier network. Moreover, we propose a BIA strategy for achieving the derived bounds.

II. System model

We consider a 2-tier network where an upper tier interferes to a lower tier as is shown in Fig. 1. Each transmitter in the upper and lower tier is composed of M_1 and M_2 sources of information, e.g., antennas, respectively. The users are equipped with a multi-mode receiver, so that their channel can vary among a set of preset modes providing a linearly independent channel response each. For instance, by using reconfigurable antennas [6] or reconfigurable photodetectors [13]. Similarly as [12], we consider a strategy to maximize the usage of the lower tier so that each user is connected to the upper tier only if the interference from the lower tier can be treated optimally as noise. Therefore, the users belonging to the upper tier are not interfered by the transmitters of the lower tier

Focussing on the lower tier, we consider B, $b = \{1, \dots, B\}$, transmitters where the receiving users can be divided between private and shared users depending on their connectivity. The transmitted signal in the lower tier at time i can be written as

$$\mathbf{x}[i] = \begin{bmatrix} \mathbf{x}_1[i] & \dots & \mathbf{x}_B[i] \end{bmatrix} \in \mathbb{C}^{M_{\Sigma_2} \times 1}, \tag{1}$$

where $M_{\Sigma_2} = BM_2$ and $\mathbf{x}_b \in \mathbb{C}^{M_2 \times 1}$ is the signal of the transmitter b. At the lower tier, the private users receive a strong signal from a single transmitter while the signal from all other transmitters of the lower tier is treated as noise. The threshold to categorize a user as private depends on the network design. A signal-to-interference ratio greater than 3 dB is usually considered to treat the interference as noise [14]. Thus, the signal received by the private user k connected to the transmitter b of the lower tier for the mode l is

$$y_l^{[p_{k,b}]}[i] \approx \tag{2}$$
$$\mathbf{h}_b^{[p_{k,b}]}\left(l^{[p_{k,b}]}[i]\right)\mathbf{x}_b[i] + \mathbf{g}^{[p_{k,b}]}\left(l^{[p_{k,b}]}[i]\right)\mathbf{x}_{\Sigma_1}[i] + z^{[p_{k,b}]}[i],$$

where $\mathbf{h}_b^{[p_{k,b}]} \in \mathbb{C}^{M_2 \times 1}$ is the channel between the transmitter b and the private user $p_{k,b}$, $\mathbf{g}^{[p_{k,b}]} \in \mathbb{C}^{M_1 \times 1}$ is the channel between the upper tier and the private user $p_{k,b}$, $\mathbf{x}_{\Sigma_1} \in \mathbb{C}^{M_1 \times 1}$ is the signal transmitted in the upper tier and $z^{[p_{k,b}]}$ is additive white Gaussian noise (AWGN).

The signal received by the shared user $sh_{k'}$, which receives a useful signal from the B transmitters, for the mode l is

$$y_l^{[sh_{k'}]}[i] = \mathbf{h}^{[sh_{k'}]}\left(l^{[sh_{k'}]}[i]\right)\mathbf{x}[i] + \mathbf{g}^{[sh_{k'}]}\left(l^{[sh_{k'}]}[i]\right)\mathbf{x}_{\Sigma_1}[i] + z^{[sh_{k'}]}[i], \tag{3}$$

where $\mathbf{h}^{[sh_{k'}]} \in \mathbb{C}^{M_{\Sigma_2} \times 1}$ is the channel between the set of B transmitters and the user $sh_{k'}$, $\mathbf{g}^{[sh_{k'}]}$ contains the channel between the upper tier and the user $sh_{k'}$ and $z^{[sh_{k'}]}$ is AWGN.

The transmitted signals are subject to an average power constraint $\mathbb{E}[\|\mathbf{x}_{\Sigma_1}\|^2] \leq P_1$ and $\mathbb{E}[\|\mathbf{x}\|^2] \leq P_2$. Each private and shared user can vary among M_2 and M_{Σ_2} modes, respectively. We assume that the transmitters do not have any CSIT or cooperation among them. The channel coefficients are drawn from a continuous distribution and stay constant for a sufficiently large number of symbol extensions.

III. Converse proof

Theorem 1. *For a 2-tier network as described in Section II, the sum-DoF outer bound in the lower tier where the transmitter b serves to U_{p_b} private users and U_{sh} shared users are served simultaneously by the B transmitters is given by*

$$\begin{aligned} &\text{maximize} \quad \sum_{b=1}^{B} d_{\Sigma_{p_b}} + d_{\Sigma_{sh}} \\ &\text{subject to} \\ &\nu U_{p_b}^{*}\left(M_2-1\right)\sum_{b=1}^{B} d_{\Sigma_{p_b}} \leq \sum_{b}^{B} U_{p_b}\left(\nu M_{\Sigma_2} - \nu d_{\Sigma_2} - d_{\Sigma_1}\right) \\ &\nu\left(M_{\Sigma_2}-1\right)d_{\Sigma_{sh}} \leq U_{sh}\left(\nu M_{\Sigma_2} - \nu d_{\Sigma_2} - d_{\Sigma_1}\right) \end{aligned} \tag{4}$$

where $\nu = \lceil \frac{M_1}{M_{\Sigma_2}} \rceil$, $U_{p_b}^{*} = \sum_{b'=1, b' \neq b}^{B} U_{p_{b'}}$, $d_{\Sigma_{p_b}}$ and $d_{\Sigma_{sh}}$ are the sum-DoF by the private users at the transmitter b and the shared users, respectively. Moreover, in (4) d_{Σ_2} and d_{Σ_1} denote the sum-DoF in the lower and upper tier, respectively.

Proof. For the sake of simplicity, we consider a lower tier composed of $B = 2$ transmitters. The message intended to the private user $p_{k,b}$ is denoted as $W^{[p_{k,b}]}$, which results in the user rate $R^{[p_{k,b}]}$, and the whole set of messages for the $U_{p,b}$ users at the transmitter b is given by $\mathcal{W}_{\Sigma}^{[p_b]}$. Similarly, the message intended to the shared user $sh_{k'}$ is $W^{[sh_{k'}]}$ resulting in the user rate $R^{[sh_{k'}]}$. The set of messages to the U_{sh} shared users is given by $\mathcal{W}_{\Sigma}^{[sh]}$. In the following, we denote the regular entropy as $H(\cdot)$ and the differential entropy as $h(\cdot)$.

A. Case $M_1 \leq M_{\Sigma_2}$

1) Private users: Without loss of generality, let us focus on the private user $p_{1,1}$. Assume that M_2 modes are available at the considered user, which provide $l = \{1, \ldots, M_2\}$ random observations each corresponding to a different channel. Notice that the received signal only changes with the selected mode l. Moreover, since there is no CSIT, each realization of the user should have probability of error approaching zero to achieve reliable decoding. Thus, applying the Fano's inequality to codebooks spanning n channel realizations,

$$\begin{aligned} nR^{[p_{1,1}]} &\leq I\left(W^{[p_{1,1}]}; (y_l^{[p_{1,1}]})^n\right) \\ &\leq n\left(\log(P_2) + o\left(\log(P_2)\right)\right) - h\left((y_l^{[p_{1,1}]})^n | W^{[p_{1,1}]}\right) + o(n). \end{aligned} \tag{5}$$

Due to the space limitations, the term $o\left(\log(P_2)\right) + o(n)$ is omitted from now on. Let us define $\mathbf{y}^{[p_{1,1}]} = (y_1^{[p_{1,1}]}, \ldots, y_{M_2}^{[p_{1,1}]})^n$. Adding up M_2 inequalities as (5) for $l = \{1, \ldots, M_2\}$ and using $h(A, B) \leq h(A) + h(B)$

$$\begin{aligned} nM_2 R^{[p_{1,1}]} &\leq nM_2 \log(P_2) - h\left(\mathbf{y}^{[p_{1,1}]} | W^{[p_{1,1}]}\right) \\ &\overset{(b)}{\leq} nM_2 \log(P_2) - I\left(\mathcal{W}_{\Sigma*}^{[p_1]}; \mathbf{y}^{[p_{1,1}]} | W^{[p_{1,1}]}\right) \\ &\quad - h\left(\mathbf{y}^{[p_{1,1}]} | \mathcal{W}_{\Sigma}^{[p_1]}\right) \\ &\overset{(c)}{\leq} nM_2 \log(P_2) - H\left(\mathcal{W}_{\Sigma*}^{[p_1]} | W^{[p_{1,1}]}\right) \\ &\quad + H\left(\mathcal{W}_{\Sigma*}^{[p_1]} | \mathbf{y}^{[p_{1,1}]}, W^{[p_{1,1}]}\right) - h\left(\mathbf{y}^{[p_{1,1}]} | \mathcal{W}_{\Sigma}^{[p_1]}\right) \\ &\overset{(d)}{\leq} nM_2 \log(P_2) - \sum_{k=2}^{U_{p_1}} nR^{[p_{k,1}]} - h\left(\mathbf{y}^{[p_{1,1}]} | \mathcal{W}_{\Sigma}^{[p_1]}. \mathcal{W}_{\Sigma}^{[p_2]}\right). \end{aligned} \tag{6}$$

In (6) the step (b) is given by $I(X;Y|Z) = h(X|Z) - h(X|Y,Z)$ defining $\mathcal{W}_{*}^{[p_1]} = \{W^{[p_{2,1}]}, \ldots, W^{[p_{U_{p_1},1}]}\}$ while the step (c) uses the definition of mutual information and the fact that $\{W^{[p_{1,1}]}, \mathcal{W}_{\Sigma*}^{[p_1]}\} = \mathcal{W}_{\Sigma}^{[p_1]}$. To justify the step (d) in (6) note that $\mathbf{y}^{[p_{1,1}]}$ contains all the possible outputs for all the private users served by the transmitter $b = 1$. Therefore, from $\mathbf{y}^{[p_{1,1}]}$ it must be possible to decode all the messages $\mathcal{W}_{\Sigma}^{[p_1]}$. Thus, the remaining uncertainty is just due to noise, i.e., $H(\mathcal{W}_{\Sigma*}^{[p_1]} | \mathbf{y}^{[p_{1,1}]}, W^{[p_{1,1}]}) \leq o\left(n\right)$. Moreover, in (d) we use the independence between messages and the fact that conditioning cannot increase the entropy.

Following the same procedure for the private user $p_{1,2}$ we obtain a similar inequality as (6). Thus, adding up both inequalities for $b = \{1, 2\}$ and using $h(A) + h(B) \geq h(A, B)$

$$\begin{aligned} nM_2(R^{[p_{1,1}]} + R^{[p_{1,2}]}) &\leq 2nM_2 \log(P_2) - \sum_{k=2}^{U_{p_1}} nR^{[p_{k,1}]} \\ &\quad - \sum_{k=2}^{U_{p_2}} nR^{[p_{k,2}]} - h\left(\mathbf{y}^{[p_{1,1}]}, \mathbf{y}^{[p_{1,2}]} | \mathcal{W}_{\Sigma}^{[p_1]}, \mathcal{W}_{\Sigma}^{[p_2]}\right). \end{aligned} \tag{7}$$

From $\mathbf{y}^{[p_{1,1}]}, \mathbf{y}^{[p_{1,2}]}$ we have $M_{\Sigma_2} = 2M_2$ generic linear equations, which can therefore be solved to recover M_{Σ_2} input symbols from both transmitters, subject to noise distortion. Thus, we can recover the remaining messages of the lower tier, which provide $R_{\Sigma_2}^{[sh]}$, within an $no(\log(P))$ term (not shown due to space limitations). Thus,

$$\begin{aligned} nM_2(R^{[p_{1,1}]} + R^{[p_{1,2}]}) &\leq 2nM_2 \log(P_2) \\ &\quad - n\left(R_{\Sigma_2} - R^{[p_{1,1}]} - R^{[p_{1,2}]}\right) - h\left(\mathbf{y}^{[p_{1,1}]}, \mathbf{y}^{[p_{1,2}]} | \mathcal{W}_{\Sigma_2}\right), \end{aligned} \tag{8}$$

where $\sum_{k=2}^{U_{p_1}} nR^{[p_{k,1}]} + \sum_{k=2}^{U_{p_2}} nR^{[p_{k,2}]} + nR_{\Sigma_2}^{[sh]} = n(R_{\Sigma_2} - R^{[p_{1,1}]} - R^{[p_{1,2}]})$. Similarly to the step (b) in (6), we use $I(X;Y|Z) = h(X|Z) - h(X|Y,Z)$ given by the messages transmitted in the tier 1, i.e., $\mathcal{W}_{\Sigma_1}$, in (8)

$$
\begin{aligned}
nM_2(R^{[p_{1,1}]} + R^{[p_{1,2}]}) &\leq 2nM_2 \log(P_2) \\
&\quad - n\left(R_{\Sigma_2} - R^{[p_{1,1}]} - R^{[p_{1,2}]}\right) \\
&\quad - I\left(\mathcal{W}_{\Sigma_1}; \mathbf{y}^{[p_{1,1}]}, \mathbf{y}^{[p_{1,2}]} | \mathcal{W}_{\Sigma_2}\right) \\
&\quad - h\left(\mathbf{y}^{[p_{1,1}]}, \mathbf{y}^{[p_{1,2}]} | \mathcal{W}_{\Sigma_1}, \mathcal{W}_{\Sigma_2}\right) \qquad (9) \\
&\overset{(a)}{\leq} 2nM_2 \log(P_2) - n\left(R_{\Sigma_2} - R^{[p_{1,1}]} - R^{[p_{1,2}]}\right) - nR_{\Sigma_1},
\end{aligned}
$$

notice that given the messages from both tiers, $\mathcal{W}_{\Sigma_1}$ and $\mathcal{W}_{\Sigma_2}$, the outputs contain uncertainty only due to noise. The step (a) uses the definition of mutual information. Thus, the sum-rate in the first tier is given by considering the independence between messages. Moreover, given all the messages transmitted in the tier 2, we can subtract the symbols $\mathcal{W}_{\Sigma_2}$ from the signal received at the private users of the tier 2. Since the condition $M_{\Sigma_2} \geq M_1$, we can invert the channel to the upper tier to resolve the set of messages $\mathcal{W}_{\Sigma_1}$ within noise distortion, i.e., $H(\mathcal{W}_{\Sigma_1}|\mathbf{y}^{[p_{1,1}]}, \mathbf{y}^{[p_{1,1}]}, \mathcal{W}_{\Sigma_2})$ carries 0 DoF.

2) Shared users: For the shared user sh_1, without loss of generality, who wants the message $W^{[sh_1]}$, for any realization l, $l = \{1, \ldots, M_{\Sigma_2}\}$, applying the Fano's inequality

$$
nR^{[sh_1]} \leq n\log(P_2) - h\left(\left(y_l^{[sh_1]}\right)^n | W^{[sh_1]}\right). \qquad (10)
$$

Adding up these bounds for all M_{Σ_2} and following a similar procedure as (6) and (8),

$$
\begin{aligned}
nM_{\Sigma_2} R^{[sh_1]} \leq & nM_{\Sigma_2}\log(P_2) - \left(nR_{\Sigma_2} - R^{[sh_1]}\right) \\
& - h\left((y_1^{[sh_1]}, \ldots, y_{M_{\Sigma_2}}^{[sh_1]})^n | \mathcal{W}_{\Sigma_2}\right). \qquad (11)
\end{aligned}
$$

Let us define $\mathbf{y}^{[sh_1]} = (y_1^{[sh_1]}, \ldots, y_{M_{\Sigma_2}}^{[sh_1]})^n$, which contains $M_{\Sigma_2} = BM_2$ realizations of $sh_{k'}$. Similarly to step (9),

$$
\begin{aligned}
nM_{\Sigma_2} R^{[sh_1]} &\leq nM_{\Sigma_2}\log(P_2) - n\left(R_{\Sigma_2} - R^{[sh_1]}\right) \\
&\quad - I\left(\mathcal{W}_{\Sigma_1}; \mathbf{y}^{[sh_1]} | \mathcal{W}_{\Sigma_2}\right) - h\left(\mathbf{y}^{[sh_1]} | \mathcal{W}_{\Sigma_1}, \mathcal{W}_{\Sigma_2}\right) \\
&\overset{(a)}{\leq} nM_{\Sigma_2}\log(P_2) - n\left(R_{\Sigma_2} - R^{[sh_1]}\right) - nR_{\Sigma_1}, \qquad (12)
\end{aligned}
$$

where given $\mathcal{W}_{\Sigma_1}$ and $\mathcal{W}_{\Sigma_2}$ the outputs contain uncertainty only due to noise. The step (a) in (12) is given by the definition of mutual information, which provides the rate R_{Σ_1} employing the independence between messages and an entropy term $H(\mathcal{W}_{\Sigma_1}|\mathbf{y}^{[sh_1]}, \mathcal{W}_{\Sigma_2})$. This term carries 0 DoF since we can invert the channel of the tier 1 from the $M_{\Sigma_2} \geq M_1$ realizations of $\mathbf{y}^{[sh_1]}$ and the knowledge of $\mathcal{W}_{\Sigma_2}$.

B. Case $M_1 > M_{\Sigma_2}$

1) Private users: For simplicity, let us consider the case for $M_1 = 2M_{\Sigma_2} = 4M_2$. The proof extends easily to $M_1 > M_{\Sigma_2}$ as wells for arbitrary values of M_1. For each private user in the lower tier let us generate an auxiliary receiver who wants the same message and has the same channel statistics. Without loss of generality, consider $\mathbf{y}_a^{[p_{1,b}]} = \left(y_{1,a}^{[p_{1,b}]}, \ldots, y_{M_2,a}^{[p_{1,b}]}\right)^n$ as the a-th independent set of output signals. For the sake of simplicity, we relabel $a = 1$ as the outputs of the inequality (8) and $a = 2$ for an auxiliary user. Adding up both inequalities for $a = \{1, 2\}$ and using $h(A) + h(B) \geq h(A, B)$ we obtain

$$
\begin{aligned}
2nM_2(R^{[p_{1,1}]} + R^{[p_{1,2}]}) &\leq 4nM_2\log(P_2) \\
&\quad - 2n\left(R_{\Sigma_2} - R^{[p_{1,1}]} - R^{[p_{1,2}]}\right) \\
&\quad - h\left(\mathbf{y}_1^{[p_{1,1}]}, \mathbf{y}_1^{[p_{1,2}]}, \mathbf{y}_2^{[p_{1,1}]}, \mathbf{y}_2^{[p_{1,2}]} | \mathcal{W}_{\Sigma_2}\right), \qquad (13)
\end{aligned}
$$

Following a similar procedure as in (9) we obtain

$$
\begin{aligned}
2nM_2(R^{[p_{1,1}]} + R^{[p_{1,2}]}) &\leq 4nM_2\log(P_2) \qquad (14) \\
&\quad - 2n\left(R_{\Sigma_2} - R^{[p_{1,1}]} - R^{[p_{1,2}]}\right) - nR_{\Sigma_1}.
\end{aligned}
$$

In this case notice that $\mathbf{y}_1^{[p_{1,1}]}, \mathbf{y}_1^{[p_{1,2}]}, \mathbf{y}_2^{[p_{1,1}]}, \mathbf{y}_2^{[p_{1,2}]}$ contain $M_{\Sigma_2} = 4M_2 = M_1$ independent realizations so that it is possible to resolve the messages $\mathcal{W}_{\Sigma_1}$ within noise distortion, i.e., $H\left(\mathcal{W}_{\Sigma_1}|\mathbf{y}_1^{[p_{1,1}]}, \mathbf{y}_1^{[p_{1,2}]}, \mathbf{y}_2^{[p_{1,1}]}, \mathbf{y}_2^{[p_{1,2}]}, \mathcal{W}_{\Sigma_2}\right)$ carries 0 DoF.

2) Shared users: Similarly, we create an auxiliary user for the shared user sh_1. Denoting $\mathbf{y}_a^{[sh_1]} = \left(y_{1,a}^{[sh_1]}, \ldots, y_{M_{\Sigma_2},a}^{[sh_1]}\right)^n$ we obtain $a = \{1, 2\}$ inequalities as (11). Following a similar procedure as for the private users

$$
\begin{aligned}
2nM_{\Sigma_2} R^{[sh_1]} &\leq \qquad (15) \\
& 2nM_{\Sigma_2}\log(P_2) - 2n\left(R_{\Sigma_2} - R^{[sh_1]}\right) - nR_{\Sigma_1}.
\end{aligned}
$$

This procedure can be carried out for an arbitrary value of $M_1 > M_{\Sigma_2}$ creating $\nu = \left\lceil \frac{M_1}{M_{\Sigma_2}} \right\rceil$ auxiliary users and following the same steps as described above.

C. Achievable Degrees of Freedom

Let us focus on the inequalities of the private users for both cases (see (9) and (14)). Replacing $p_{1,1}$ and $p_{1,2}$ with any private user denoted as $p_{k,1}$ and $p_{j,2}$, respectively, we obtain $U_{p1} + U_{p_2}$ inequalities for $k = \{1, \ldots, U_{p_1}\}$ and $j = \{1, \ldots, U_{p_2}\}$. Dividing these inequalities by $n\log(P_2)$ and considering the limit $n \to \infty$ and $P_2 \to \infty$ after some rearrangement the DoF bound is

$$
\nu\left(M_2 - 1\right)\left(d^{[p_{k,1}]} + d^{[p_{j,2}]}\right) \leq \nu M_{\Sigma_2} - \nu d_{\Sigma_2} - d_{\Sigma_1}. \qquad (16)
$$

Adding up the U_{p_1} and U_{p_2} inequalities we obtain

$$
\begin{aligned}
\nu U_{p_2}\left(M_2 - 1\right) d_{\Sigma_{p_1}} + & \nu U_{p_1}\left(M_2 - 1\right) d_{\Sigma_{p_2}} \leq \qquad (17) \\
& U_{p_1} U_{p_2}\left(\nu M_{\Sigma_2} - \nu d_{\Sigma_2} - d_{\Sigma_1}\right).
\end{aligned}
$$

Similarly, considering (12) and (15) for a generic shared user $sh_{k'}$, $k' = \{1, \ldots, U_{sh}\}$ and taking the limits $n \to \infty$ and $P_2 \to \infty$ the DoF bound after some rearrangement is

$$
\nu\left(M_{\Sigma_2} - 1\right) d^{[sh_1]} \leq \nu M_{\Sigma_2} - \nu d_{\Sigma_2} - d_{\Sigma_1}. \qquad (18)
$$

Adding up the U_{sh} inequalities

$$
\nu\left(M_{\Sigma_2} - 1\right) d_{\Sigma_{sh}} \leq U_{sh}\left(\nu M_{\Sigma_2} - \nu d_{\Sigma_2} - d_{\Sigma_1}\right). \qquad (19)
$$

The extension to B transmitters results straightforward considering the variable $U_{p_b}^*$ defined in the Theorem 1. $\square$

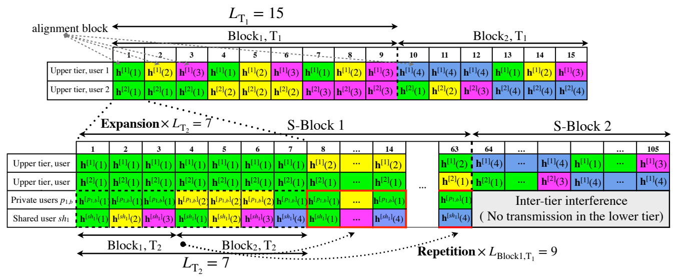

Fig. 2. Supersymbol construction for $B = 2$, $M_2 = 2$, $U_{p_1} = U_{p_2} = U_{sh} = 1$. The upper tier is given by $M_1 = 4$ and 2 users.

Corollary 1. *The sum-DoF in the lower tier for a symmetric 2-tier network corresponds to the close-form expression*

$$d_{\Sigma_2} \leq \frac{M_{\Sigma_2}\left(U_p + U_{sh}\left(\frac{M_2-1}{M_{\Sigma_2}-1}\right)\right)}{(M_2-1)+U_p+U_{sh}\left(\frac{M_2-1}{M_{\Sigma_2}-1}\right)}\left(1-\frac{d_{\Sigma_1}}{\nu M_{\Sigma_2}}\right). \tag{20}$$

Proof. Easily obtained using $U_p = U_{p_1} = U_{p_2}$ in (4). □

IV. Achievability through BIA

In this section we propose a BIA scheme that achieves the DoF outer-bound derived in the previous section. The proposed scheme, referred to as 2-tier BIA (tBIA), results from the combination of BIA schemes that are DoF-optimal at each tier without considering the inter-tier interference such as standard BIA (sBIA) [6], network BIA (nBIA) [11] or cognitive BIA (cogBIA) [12]. These schemes are based on generating a pattern of receiving modes denoted as *supersymbol* composed of two differentiated parts; Block 1 where simultaneous transmission occurs and Block 2 during which the signal space is transmitted in orthogonal fashion. Transmission is based on forming *alignment blocks* over which the multi-mode receiver of each user varies among the same number of modes as transmitters is connected while all other users maintain a constant mode as in shown in Fig. 2.

The proposed tBIA scheme considers a BIA scheme at the upper tier given by a supersymbol that comprises L_{T_1} symbol extensions divided into L_{Block_1,T_1} and L_{Block_2,T_1}. Similarly, the lower tier considers a BIA scheme that achieves the optimal DoF without considering the inter-tier interference, which comprises L_{T_2} symbol extensions divided into L_{Block_1,T_2} and L_{Block_2,T_2}. Obviously, implementing these BIA schemes independently is subject to inter-tier interference at the lower tier. Moreover, the signal space given by the supersymbol of the upper tier does not allow to align the inter-tier interference.

In order to generate a signal space sufficiently large to align the inter-tier interference, the supersymbol of the upper tier is expanded L_{T_2} times. Therefore, the resulting supersymbol comprises $L_{T_1} \times L_{T_2}$ symbol extensions. For illustrative purposes we show this procedure in Fig. 2 for a specific case where sBIA and nBIA are considered for the upper and lower tier, respectively. We can differentiate 2 sections in the resulting supersymbol corresponding to the expansion of the Block 1 and Block 2 of the upper tier, which are referred to as S-Block 1 and S-Block 2. Since the resulting signal space of the upper tier is transmitted in orthogonal fashion during S-Block 2, the transmitters of the lower tier must remain in silence during this section in order to measure the inter-tier interference and subtract it afterwards. Therefore, transmission in the lower tier occurs during the S-Block 1, which comprises $L_{T_2} \times L_{\text{Block}_1,T_1}$ symbol extensions. Notice that the inter-tier interference can now be aligned because of the expansion of the signal space. Moreover, the supersymbol of the lower tier without considering the inter-tier interference can be repeated L_{Block_1,T_1} times while satisfying the alignment between tiers.

The proposed tBIA scheme simply expands the BIA structure at the upper tier, and therefore, the achievable DoF are equal to the achieved by the considered BIA scheme, e.g., sBIA or nBIA. However, transmission in the lower tier only occurs during S-Block 1. Thus, according the proposed supersymbol structure, the achievable sum-DoF per symbol extension is

$$d_{\Sigma_2} = \frac{d^*_{\Sigma_2} \times L_{\text{Block}1,T_1}}{L_{T_1}}, \tag{21}$$

where $d^*_{\Sigma_2}$ denotes the achievable DoF at the lower tier without considering the inter-tier interference. It can be easily checked that the proposed tBIA scheme achieves the DoF outer-bound derived in Section III. For the considered case (see Fig. 2), $\text{DoF}_2 = \frac{16}{7}$ (see [11]), so that $\text{DoF}_{T_2} = \frac{16}{7} \times \frac{9}{15} \approx 1.37$, which correponds to the outer bound for the considered case, while the upper tier achieves a sum-DoF $d_{\Sigma_1} = \frac{8}{5}$ [6].

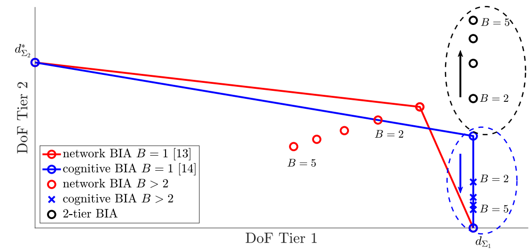

Fig. 3. Sum-DoF region for 2-tier networks. Solid lines correspond to the sum-DoF region for $B = 1$. For $B > 2$ the points represents the vertex of the sum-DoF region considering the sum-DoFs per transmitter.

V. DoF Region for 2-tier Networks

The DoF region for a 2-tier network without CSIT is depicted in Fig. 3. Considering a single transmitter at the lower tier, it can be seen that the nBIA scheme obtains a sum-DoF greater than cogBIA at the cost of cooperation between both tiers. Notice that neither the upper nor the lower tiers achieve the optimal sum-DoF for nBIA. In contrast, the cogBIA scheme achieves the optimal sum-DoF at the upper tier while the lower tier obtains a non-zero sum-DoF. However, as the number of transmitters at lower tier that interfere among them increases the sum-DoF decreases for both schemes. It can be seen that the proposed tBIA scheme solves this issue.

VI. Conclusions

In this letter, the bounds that define the DoF region without CSIT nor cooperation for the generic 2-tier network are characterized. These bounds can be reached by combining distinct BIA schemes in a cognitive fashion. The scheme proposed in this letter can be used for deriving the DoF of the K-tier networks in further works.

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