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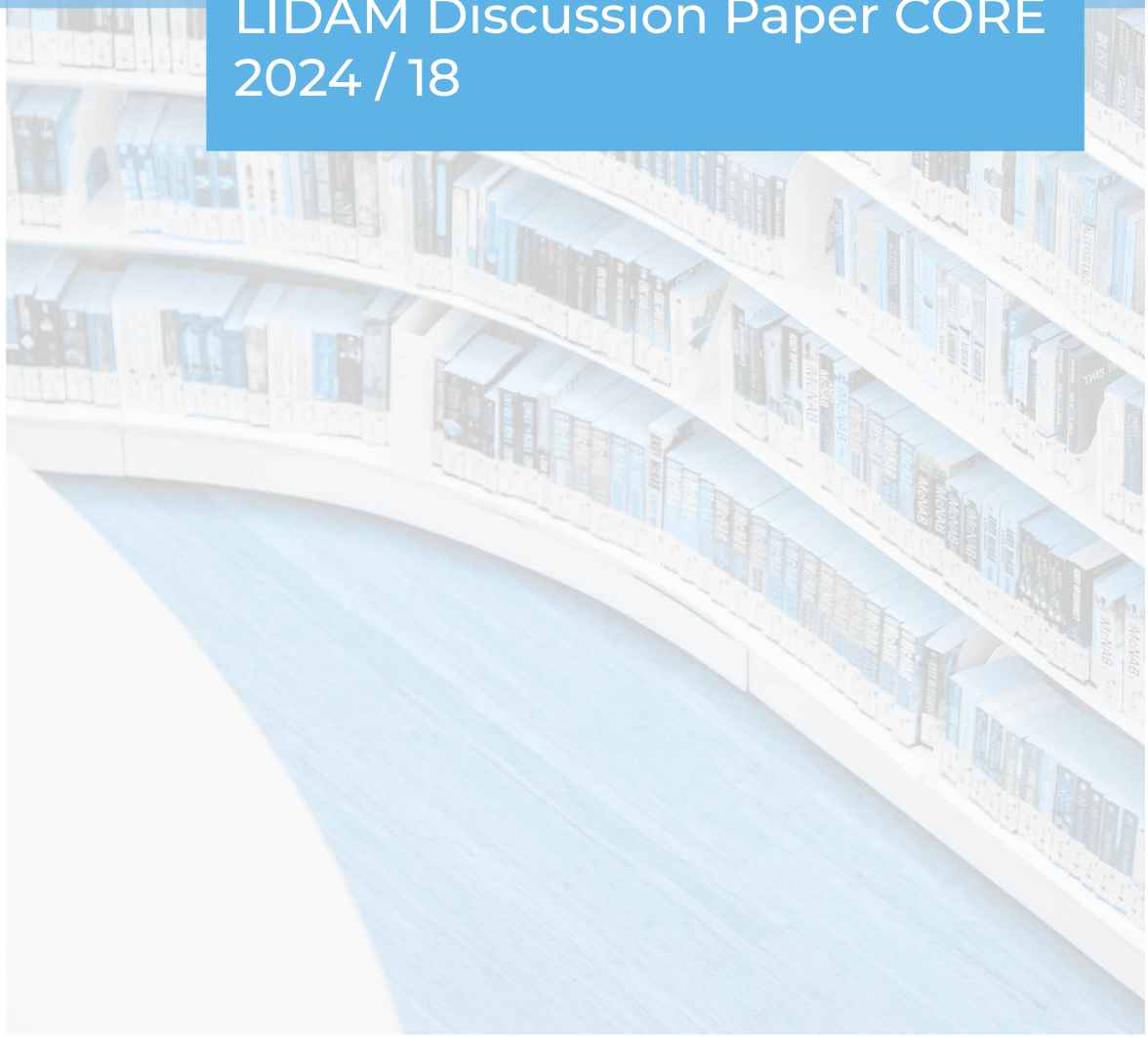

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The Intriguing Relation Between Parenting Styles and Eldercare

Simon Fan Yu Pang Pierre Pestieau*

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Abstract

This paper develops an overlapping generations model analyzing parenting choices from a life cycle perspective. Young parents educate their children to foster their human capital development. Strict discipline requires minimal time from parents yet but can strain intergenerational relations. Pedagogical practice preserves familial bonds but demands significant time and effort, adversely affecting parental income. As parents age, they desire caregiving support from their adult children, who may bring earlier conflicts with their parents into the care environment. We suggest that the prevalence of strict discipline declines when the probability of living into old adulthood increases. We then incorporate health investments into the model to endogenize longevity and investigate the transitional dynamics of life expectancy, parenting styles, and human capital stock. Moreover, we examine how the interaction between parenting styles and monetary transfers induces children's provision of eldercare in a bargaining framework. We characterize multiple stationary Markov perfect equilibria, shedding light on the observed diversity in parenting across different cultures.

JEL Classifications: I19, I21, J14

Keywords: parenting; longevity; old-age support; human capital; health investment

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1 Introduction

The term "tiger mother," popularized by Yale Law professor Amy Chua's memoir *Battle Hymn of the Tiger Mother*, refers to strict parenting that relentlessly drives children to excel academically and professionally with little regard for their happiness. While Chua believes that her approach of setting high expectations and sternly disciplining her two daughters is justified by their remarkable achievements, strict parenting has sparked controversies in the developmental psychology literature (Chao 1994). Doepke and Zilibotti (2017) draw attention to the role of parenting styles in economic research. Their landmark work explores the effects of socioeconomic environment, such as income inequality, division of labor, and technological advancement, on child-rearing practices, attributing the long-term decline of authoritarian parenting to rising returns to independence.

This paper contributes to the emerging economic theories of parenting styles from a life course perspective. Family dynamics entail reciprocal processes, where interactions between parents and children generate a lasting influence on each other's emotions and behaviors. Parents educate their children in early life to equip them with knowledge and skills for better life opportunities that they aspire for their children to achieve (de la Croix and Michel 2002; Francesconi and Heckman 2016). Among various educational approaches that facilitate the development of children's human capital, strict discipline may cause children's hostility toward their parents, and pedagogical methods that prioritize responsiveness avoids damaging the parent-child closeness.[1] When parents reach old age, they usually value the caregiving support provided by their adult children (Canta and Cremer 2019; Pestieau 2022). But the inclination of children to assume the responsibilities of family caregiving depends on how they were treated by their parents during childhood (Vangen and Herlofson 2023). The expectation that children may carry the relationship histories with their parents into the care environment plays a crucial role in shaping parenting styles.

Given the potential merit of pedagogical practices in preserving family bonds, the next question arises: Why do some parents find an incentive to choose harsh disciplinary measures? We propose two underlying reasons. First of all, parenting style is often conceptualized as the degree of parents'

[1]There is a vast body of empirical literature investigating the impact of parenting on children's human capital. For example, Fiorini and Keane (2014) show that children's time spent in educational activities together with parents fosters their cognitive skill development. Zhang, Qin, and Zhou (2020) find that both demandingness and responsiveness of parents have positive effects on children's academic performance in China.

patience in response to children's misbehavior (e.g., Burton, Phipps, and Curtis 2002); in particular, harsh discipline emerges as an effortless and expedient solution. Imagine that a mother attempts to motivate her nine-year-old son to put in more effort at school. Implementing pedagogical practices, such as communicating the value of effort, setting clear rules, and teaching self-control, involves a substantial and continuous investment of time (Doepke and Zilibotti 2019; Kearney, Levine, and Pardue 2022). Yet expressing anger through yelling at the boy, a behavior that can be accomplished within a minute, will swiftly convey the message of his lack of effort being unacceptable. In short, while both approaches teach a lesson to the child, they differ in terms of the time commitment by the parent. Parents who prioritize their careers and income may resort to harsh discipline because it enables them to dedicate more time toward increasing their earnings.

The other potential reason for the frequent adoption of harsh discipline is low life expectancy. In societies with a short average lifespan, the role of children as future companions and sources of caregiving support holds less significance. Due to the anticipation of a low probability of surviving into old age, young parents tend to disregard the costs associated with harsh parenting and may not fully recognize the long-term benefits associated with responsive parenting. This rationale helps to explain the prevalence of physical punishment within families in ancient times when most people die young. Over the last century, life expectancy at birth has nearly doubled in many countries (e.g., Hazan 2009; Haberkern 2023). Greater longevity heightens parental concern for intergenerational care, thereby discouraging harsh discipline that can hinder the quality of lifelong familial bonds. In Section 2, we perform an empirical test of the negative relationship between life expectancy and harsh parenting in the US and China, which motivates our positive theory.

This paper analyzes parenting styles, human capital formation, and eldercare in a unified framework through the lens of intergenerational conflicts and reciprocity. We build an overlapping generations model formalizing the transition from being a child to assuming the obligation of a caregiver for aging parents. Individuals live for childhood, young adulthood, and middle adulthood and may live into old adulthood with a probability. Children receive education from their parents, taking the home environment (i.e., parental human capital and parenting styles) as given. Young adults work for a wage and educate their children using two approaches. One is pedagogical practice, which is time-demanding (and therefore adversely affects parental income) but does not put strain on the parent-child relations. The other is harsh discipline, which does not exhaust parental resources but

leads to family estrangement. Both approaches promote children's human capital acquisition, and their impacts on children's attention toward parents persist into later life. Middle-aged adults live on savings and allocate time between leisure and eldercare (if their aging parents are alive). Their attitudes and behaviors toward their parents in the care environment hinges on their own childhood experiences. Old adults may undergo physical and cognitive changes that disable them to live independently, and thus, they desire their children's attention and support.

We lay out the benchmark model in Section 3, where individuals are assumed to face an exogenous probability of living into old adulthood and spend a constant fraction of time taking care of their parents during middle adulthood in compliance with a social norm. We show that parents employ harsh discipline less frequently when they are more likely to live into old adulthood. As a result, children enjoy greater childhood happiness, and parents receive better attention from their children as they become in need of help. Contrary to conventional wisdom, our analysis suggests that a higher wage rate does not necessarily result in an increase in work hours. This is because individuals, who are concerned by their children's future earnings potential, may choose to allocate time away from work to engage in their children's education. This section complements the literature that examines the impact of longevity on economic development (e.g., Cervellati and Sunde 2005; Hazan and Zoabi 2006; de la Croix and Licandro 2013).

In Section 4, we drop the assumption of an exogenous life expectancy and instead consider an endogenous health investment that enhance the likelihood of individuals living into old adulthood. Children often develop enduring negative sentiments toward parents who frequently employ harsh disciplinary measures. Consequently, these parents experience limited happiness in the prospect of spending their later years with their children; they are inclined to invest less in health, ultimately compromising their longevity and well-being. When forward-looking individuals make a life plan including health investments, they tend to mitigate the potential damage on their connections with children by refraining from treating children poorly. In other words, the increase in lifespan resulting from endogenous health investments steers parenting styles toward a less tough manner.

Klimaviciute and Pestieau (2023) posit that the provision of eldercare by family members can be motivated by social norms, interfamily exchange, and a wish to help. In Sections 3 and 4, we focus on the first motive: middle-aged adults spend a specified portion of their time accompanying their parents out of social obligation although they do not enjoy doing so. In Section 5, we consider

the exchange motive by formulating the provision of eldercare to be a Nash bargaining process, in which parents adopt both emotional and pecuniary incentives to shape their children's choices. We extend the model by allowing for flexible time allocation for eldercare during middle adulthood, and this extension carries significance when social norms mandate only minimal caring obligations. Parents consider the long-term detrimental impacts of strictness on their children's attention toward them when choosing educational approaches during young adulthood, and employ wealth transfers as incentives to induce greater engagement in eldercare from their children. We demonstrate that strictness will strain the parent-child relationship, leading children to avoid spending time together with their parents and discouraging their caregiving support to their parents. In the steady state, we characterize the stationary Markov perfect equilibrium. With the help of a simulation, we find the coexistence of multiple equilibria given the same parameter constellation, which implies that cultural backgrounds may lead to the adoption of distinct parenting styles.

Our extended model is closely related to the exchange bequest literature, which considers children choosing a level of attention to provide to their parents and parents remunerating them in the prospect of bequests (See the review by Masson and Pestieau 1997). Bernheim, Shleifer, and Summers (1985) find that the parent-child interactions, measured by children's visits or phone calls to parents, are more frequent in households where parents possess more bequeathable wealth. Canta and Cremer (2019) develop a model in which parents commit to a rule specifying a transfer conditional on the level of informal care provided by their children. We move one step further, arguing that adult children's attention to their parents depends on their childhood experience in addition to inter vivos wealth transfers.

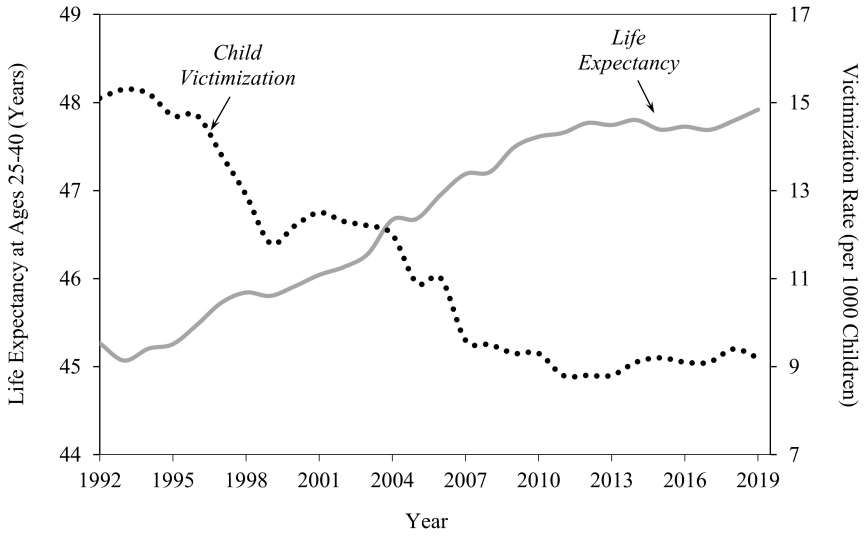
2 Motivating Evidence

In this section, we aim to outline some empirical observations that our positive theory of parenting can account for. We first provide a visual impression of the variations of life expectancy and harsh parenting over time and across states in the US and then perform a simple panel regression to test their correlation. We also undertake a robustness check by adopting the family-level survey data collected from the rural China.

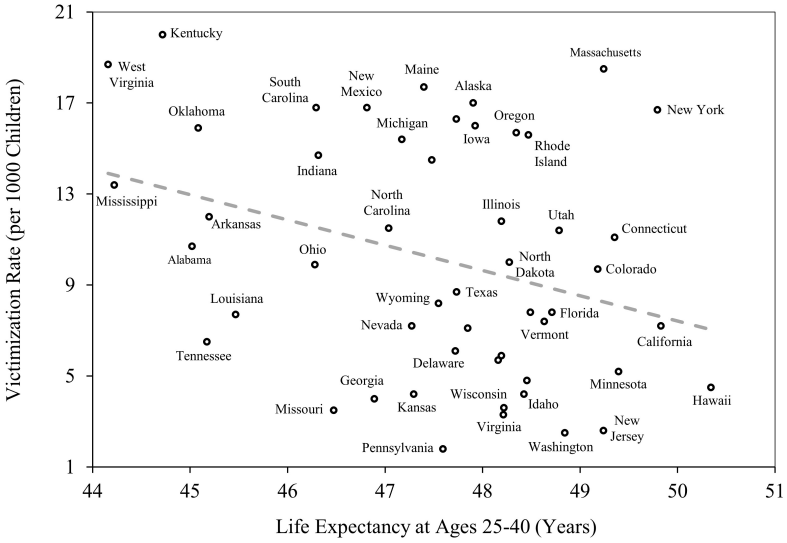
Figure 1 illustrates the child victimization rates and the average remaining life expectancies of

Figure 1: Life Expectancy versus Child Victimization in the US

(a) National: 1992–2019



(b) US states: Year 2019



young adults in the US.[2] Plot (a) shows their trends at the national level between 1992 and 2019. In 1992, Americans between the ages of 25 and 40 expected additional 45.3 years of life on average; however, as of 2019, the same age group have gained about 2.7 years in their projected lifespans, signifying an extension of one year per decade. Simultaneously, the rate of child victimization has

[2]Because most child victims are abused by their mothers, fathers, or a combination of both, we use the child victimation rate to measure the prevalence of harsh parenting. Data on child victims are obtained from the US Children's Bureau. Data on life expectancy at a given age are found in the US Mortality Database. We calculate the simple mean of the remaining life expectancy of Americans aged between 25 and 40 in a given year.

Table 1: Life Expectancy and Harsh Parenting: Based on the US Panel Data

Dependant Variable: Child Victimization Rate (per 1000 children)						
	(1)	(2)	(3)	(4)	(5)	(6)
Life Exp 25-40	-1.761^{**}	-1.652^{***}	-1.114	-1.901^{**}		
	(0.663)	(0.597)	(1.024)	(0.765)		
Life Exp 20-45					-1.769^{**}	-1.934^{**}
					(0.674)	(0.774)
Controls	No	No	Yes	Yes	No	Yes
Observations	1,173	1,071	1,173	1,071	1,173	1,071

Notes: Odd columns report fixed-effects estimations, while even columns present difference GMM estimations. Two alternative measures for parents' life expectancy of each state in a given year are used: Life Exp 25-40 (the average remaining years of life of people aged between 25 and 40) and Life Exp 20-45 (the average remaining years of life of people aged between 20 and 45). Controls refer to the inclusion of real GDP per capita and ethnic composition. Robust standard errors are in parentheses. ** and *** indicate statistical significance at the 5% and 1% level, respectively.

experienced a substantial decline from 15.1 cases per 1000 children to 9.2 per 1000 children.

In plot (b), we draw a scatterplot that presents the correlation between the child victimization rates and the period life expectancies in 2019 across the states. People in Hawaii, California, and Minnesota have the country's longest life expectancy when they reach young adulthood (aged 25–40), yet their children are least affected by maltreatment. In contrast, the period life expectancies in West Virginia, Kentucky, and Oklahoma stand out among the lowest, whereas these states exhibit alarmingly high child maltreatment rates. The straight line that best fits the spots has a significantly negative slope: the Pearson coefficient is -0.318, with p-value being 0.023.

We proceed to test the relationship between child victimization rates and life expectancies by applying panel data regression analyses. We adopt the data coming from the US states (the federal district included) between 1998 and 2020 (Data on the state-level child victims remain unavailable until 1998). Table 1 summarizes our estimation results. In Column (1), we present the regressions based on the fixed-effects estimations without including any control variables. It reveals that an additional year of parental lifespan is associated with a statistically significant fall of 1.761 victims per 1000 children on average. Column (2) differs from Column (1) in that difference Generalized Method of Moments (GMM) estimation methods are employed to deal with the endogeneity bias. The significant negative correlation seems robust, although the focal coefficient is adjusted slightly downward to 1.652.

Columns (3) and (4) control for the real income per capita as well as the racial composition.[3] These control variables are included because earlier empirical research finds that parenting styles tend to be influenced by income and culture (e.g., Weinberg 2001, Doepke, Sorrenti, and Zilibotti 2019). The coefficient for life expectancy may have minor variations, but it consistently maintains a significant and negative sign in the GMM estimation. In Columns (5) and (6), we use an alternative measurement for parents' life expectancy, namely the average remaining years of life for adults at ages 20–45. The correlation continues to exhibit a similar pattern.

Let us turn to show additional evidence about the impact of life expectancy on parenting styles using data from a different country, China. We rely on the China Family Panel Studies (CFPS) that collects individual- and family-level information of the Chinese population. We focus our attention on the CFPS 2020 survey on a total of 791 rural children aged between 9 and 15 and their parents. The parenting variables in the survey are constructed from two questions asking children (a) *How often do your parents talk to you in an amicable tone?* and (b) *how often do your parents criticize you?* To answer both questions, children select their responses from the following options: "Never" (1 point), "Occasionally" (2 points), "Sometimes" (3 points), "Often" (4 points), and "Always" (5 points). To construct the variable of parents' life expectancies, we first estimate the remaining life expectancies of rural men and women by age group and province based on the China Population Census Yearbook (CPCY) 2020 and then match the estimates with the geographic identifiers and age groups of the parents of each child participating in the CFPS 2020.[4]

Table 2 displays the results of the ordered probit regressions based on the Chinese survey data. In the left panel, the dependent variable is the child's response to the question how frequently their parents use an amicable tone of voice. We first investigate a baseline specification without control variables and then control for child characteristics (i.e., gender and educational stage) and parental education. Both coefficients are significantly positive: parents who have a shorter life expectancy tend to communicate with their children in a less amicable manner. In the right panel, we employ the alternative dependent variable, namely the child's response to the question as for the frequency

[3]To construct the real income per capita by state and year, we divide real GDP (in millions of chained 2012 dollars) by population size, with data being obtained from the US Bureau of Economic Analysis. We obtain percentage shares of each race by state and year from the National Center for Health Statistics, where populations are classified into four racial categories: White, Black or African American, Asian or Pacific Islander, and American Indian or Alaska Native.

[4]See the CPCY 2020 from the National Bureau of Statistics of China and the calculation methods of life expectancy at the template provided by the South East Public Health Observatory of the UK. Parents' life expectancies in 2020 are measured as the mean of the remaining years of life of the father and the mother.

Table 2: Life Expectancy and Harsh Parenting: Based on the Chinese Cross-Sectional Data

	Frequency of Using an Amicable Tone		*Frequency of Using Criticism*	
	(1)	(2)	(3)	(4)
Life Exp	0.331^{***}	0.246^{***}	-0.220^{***}	-0.120^{***}
	(0.011)	(0.044)	(0.009)	(0.032)
Controls	No	Yes	No	Yes
Observations	791	791	791	791

Notes: Controls mean the inclusion of the child's gender, educational stage, and type of accommodation place, child-parents age gap, and parental education. All the specifications are ordered probit models with province fixed effects; standard errors in parentheses are clustered at the province level. *** indicates statistical significance at 1% level.

with which their parents criticize them. Both coefficients are negative and statistically significant, which implicates that parents with a longer life expectancy are more likely to criticize their children less frequently than their peers.

Overall, we show that parents' lifespan extension is significantly associated with a reduction of harsh parenting practices in both the US and China. Moreover, there is an extensive body of empirical literature on family reciprocity showing that harsh parenting toward underage children has a long-term negative consequence on later intergenerational help and care (Paris 2001; Silverstein et al. 2002; Kim et al. 2013; Baumbach, Hughes, Derain, and Liu 2023). These empirical findings serve as a motivation for the development of our model in the next section.

3 The Basic Model

Consider an overlapping generations economy with a continuum of individuals, who live up to four periods and are endowed with one unit of time in each period. In childhood, they accumulate human capital and do not make decisions. In young adulthood, they allocate time between bearing and educating a child (parenting) and working for a wage.[5] Besides, they choose their consumption and savings subject to their income constraints. In middle adulthood, they retire, live on savings, and spend time on leisure or family eldercare. The probability of living into old adulthood, denoted as $\pi \in (0, 1)$, is constant. Old adults desire private consumption and their children's attention.

[5]For simplicity, our model assumes the fertility rate to be one and abstracts from the relationship between fertility and parenting. See Doepke, Sorrenti, and Zilibotti (2019) for a discussion about this relationship.

Denote individuals who were born and spend childhood in period t as "generation t." In period $t+1$, a representative (young) member of generation t has a human capital stock of H_t and chooses her parenting. She spends a fraction $p_t \in (0,1)$ of time on pedagogical practices and adopts harsh discipline at the frequency of $d_t \geqslant 0$. We assume that d_t is an attitude (e.g., shout at the child, let the child stand on the wall), which does not consume any time from the young parent. With the wage rate per efficiency unit of labor being normalized to one, the young adult earns a total income of $(1-p_t)H_t$. Let c_t^y and s_t denote her consumption and saving, respectively. The young adult's budget constraint amounts to

$$c_t^y + s_t = (1-p_t)H_t, \tag{1}$$

and the young-age utility is denoted by $v(c_t^y)$, where $v' > 0$.

When the member of generation t enters middle adulthood in period $t+2$, she chooses to spend a fraction $e_t \in [0,1]$ of time on family eldercare and the remaining time on leisure; in particular, she enjoys leisure throughout this period (i.e., $e_t = 0$) if her parent dies at the beginning of his old adulthood. Furthermore, in exchange for φ units of her potential old-age consumption, the middle-aged adult purchases an actuarially fair insurance by paying a premium of $\pi\varphi$, where φ is a positive parameter. Assume that the infratemporal interest rate is zero.[6] The middle-age consumption, c_t^m, equals saving net of the insurance premium:

$$c_t^m = s_t - \pi\varphi. \tag{2}$$

The middle-aged adult obtains utility from both consumption and leisure, and her utility is denoted by $u(c_t^m, e_t)$, where u increases with c_t^m and decreases with e_t.

If the member of generation t lives into old adulthood (i.e., period $t+3$), she obtains a utility of $w(\varphi)$ from consumption, which is normalized to zero. She also receives an attention of a_t from her child, which takes the following simple form:

$$a_t = e_{t+1} l[-z(d_t)]. \tag{3}$$

Equation (3) shows that the child's attention on the parent has two dimensions: the length of time

[6]The assumption of zero interest rate can be derived from a constant-return-to-scale aggregate production function where labor is the only factor of production.

that the child accompanies the parent (e_{t+1}) and the child's love/affection toward the parent ($l(.)$). Denote $-z(d_t)$ as the disutility suffered by the child of the member of generation t, where $z' > 0$ and $z(0) = 0$. The more severe the childhood disutility caused by harsh parenting, the less affection the child expresses to the parent in times of need, namely $l' > 0$.[7] In sum, the *expected* old-age utility can be written as $\pi[w(\varphi) + a_t] + (1 - \pi)0 = \pi a_t$.

Taken together, the lifetime utility of a representative member of generation t is given by

$$U_t = -z(d_{t-1}) + v(c_t^y) + u(e_t, c_t^m) + \pi a_t + g(H_{t+1}), \tag{4}$$

where $g(.)$ captures the parental warm glow derived from the quality of the child (as measured by the child's human capital stock) and $g' > 0$. Based on earlier empirical findings (Black, Devereux, and Salvanes 2005, Doepke, Sorrenti, and Zilibotti 2019), we consider that human capital development is governed by both intergenerational transmissions and educational methods. Specifically, $H_{t+1} = h(H_t, d_t, p_t)$ increases with parental human capital (H_t), frequency of harsh discipline (d_t), and time spent on pedagogical practice (p_t). Each input exhibits diminishing returns, namely

$$\frac{\partial h}{\partial H_t} > 0 \qquad \frac{\partial h}{\partial d_t} > 0 \qquad \frac{\partial h}{\partial p_t} > 0 \qquad \frac{\partial^2 h}{\partial H_t^2} < 0 \qquad \frac{\partial^2 h}{\partial d_t^2} < 0 \qquad \frac{\partial^2 h}{\partial p_t^2} < 0.$$

For simplicity, we assume that $e_t = e > 0$ is exogenous and is determined by a social norm in this section and the next.[8] (In Appendix A, we derive that a social norm regarding e can be endogenously determined as a dynamic Nash equilibrium in an intergenerational supergame). Under this assumption, we plug (1) – (3) into (4) to write the lifetime utility as

$$U_t = -z(d_{t-1}) + v[(1 - p_t)H_t - s_t] + u\left(e, s_t - \pi\varphi\right) + \pi el[-z(d_t)] + g[h(H_t, p_t, d_t)]. \tag{5}$$

Each individual optimally chooses her parenting (d_t, p_t) and saving (s_t) during young adulthood. The next proposition summarizes the comparative statics of d_t in equilibrium.

[7] To save algebra and highlight essentials, we abstract from the impact of p_t on $l(.)$ despite that children may love their parents more when receiving more responsive parenting. We would get the qualitatively same results if we instead assume that l depends on both p_t and d_t.

[8] For instance, in many southern and central European countries, adult children are normatively or legally obligated to support their aging parents in need (Haberkern and Szydlik 2010). Barigozzia, Cremer, and Roeder (2020) develop a model in which women conform to the norm of long term care and feel guilty if they provide less care to old parents than the average level.

Proposition 1 *(i)* d_t *decreases with* π*;*

(ii) d_t *increases with* g' *and decreases with* (e, l', z')*.*

Proof. See Appendix B. ■

Proposition 1(i) predicts a negative relationship between parental lifespan and harsh parenting. High expectations of living into old adulthood make parents concerned about their future eldercare. As parents anticipate their aging and potential need for assistance, they may become more aware of the role that their adult children will play in the caregiving dynamics. Recognizing that maintaining a harmonious relationship with their children enhances the likelihood of attaining attention and care in old age, the rational forward-looking parents tend to adjust their child-rearing practices in young age by reducing aggression and conflict.

Proposition 1(i) offers a potential reason behind the prevalence of corporal punishment inflicted by parents in ancient era when life expectancy was generally lower. For instance, some Bible verses may be interpreted as an endorsement of harsh parenting.[9] Parents' rights to control their children and use physical punishment at home are undisputed in ancient China (Paris 2001). This result is also broadly consistent with our empirical findings presented in Section 2.

Proposition 1(ii) states that parents resort to strict approaches driven by a belief that such methods will ultimately enhance the quality of their children. They may not push their children to excel if they have low aspirations for their children (small g'). The marginal attention gain of preventing harsh discipline is measured by the term $el'z'$ based on equation (3). Social norms that require adult children to provide their aging parents with intensive support (large e) tend to refrain parents from adopting harsh disciplinary methods during young adulthood. In their quest for more attention from their children, parents are inclined to avoid behaviors that harm their relationships with their children when such behaviors significantly hurt the children (large $-z'$) and/or translates into children's strong negative feelings toward their parents (large l').

The choice of the frequency of harsh parenting has profound effects on both parents and children. On the one hand, a smaller d_t translates into a larger $l[-z(d_t)]$ and hence a larger a_t. Interpreted literally, reduced strictness not only increases children's happiness but also allows parents to attain better attention from their children during old adulthood. On the other hand, a smaller d_t re-

[9] "Folly is bound up in the heart of a child, but the rod of discipline will drive it far from him." (Proverbs 22:15); "Do not withhold discipline from a child; if you punish them with the rod, they will not die. Punish them with the rod and save them from death." (Proverbs 23: 13-14).

sults in a smaller H_{t+1}. This explains why Tiger moms like Amy Chua, who emphasize children's high achievements, advocate for a highly rigorous parenting style.

To unveil the equilibrium choices of (p_t, s_t), we resort to specific functions. Each member of generation t receives a childhood disutility caused by harsh parenting in the following form:

$$-z(d_{t-1}) = -d_{t-1} \leqslant 0. \tag{6}$$

Her young-age utility is increasing and concave in c_t^y:

$$v(c_t^y) = \ln c_t^y. \tag{7}$$

During middle adulthood, the member of generation t obtains a utility of $1 - e_t^2$ from leisure if her parent is alive and a utility of 1 otherwise. It follows that her expected utility is a weighted sum of $\pi(1 - e_t^2) + (1 - \pi) = 1 - \pi e_t^2$. Her middle-age utility consists of two additively separable terms attached to leisure and consumption, as follows:

$$u(e_t, c_t^m) = 1 - \pi e_t^2 + \ln c_t^m. \tag{8}$$

During old adulthood, she receives her child's affection/love at the level of

$$l[-z(d_t)] = \mu - d_t, \tag{9}$$

where the parameter $\mu > 0$ measures the degree of the child's resilience against harsh discipline. Moreover, the parental utility derived from the child's human capital stock is specified by

$$g(H_{t+1}) = \delta \ln H_{t+1}, \tag{10}$$

where the parameter $\delta > 0$ indicates the parent's preference over the quality of the child. Finally, the development of human capital follows the Cobb-Douglas functional form:

$$H_{t+1} = (H_t)^\alpha (1 + d_t)^\beta (p_t)^\gamma, \tag{11}$$

where $\alpha, \beta, \gamma \in (0,1)$ reflect diminishing returns to different types of parental inputs.

Based on equations (6) – (11) and given $e_t = e$, the lifetime utility function can be rewritten as

$$U_t = -d_{t-1} + \ln[(1-p_t)H_t - s_t] + 1 - \pi e^2 + \ln(s_t - \pi\varphi) + \pi e(\mu - d_t) + \delta \ln[(H_t)^\alpha (1+d_t)^\beta (p_t)^\gamma].$$

Taking the first order conditions of U_t with respect to p_t and s_t and rearranging solves

$$p_t = \frac{\gamma\delta}{2+\gamma\delta}\left(1 - \frac{\pi\varphi}{H_t}\right), \qquad s_t = \frac{H_t + (1+\gamma\delta)\pi\varphi}{2+\gamma\delta}. \tag{12}$$

Based on equation (12), the following proposition discusses the changes in p_t and s_t in response to the changes in various parameters.

Proposition 2 *(i) an increase in (π, φ) leads to an increase in s_t and a fall in p_t;*
(ii) an increase in (δ, γ) leads to a fall in s_t and an increase in p_t;
(iii) an increase in H_t leads to an increase in (s_t, p_t).

Proof. See Appendix B. ■

Proposition 2(i) demonstrates the comparative statics of (s_t, p_t) with respect to (π, φ). As the life-cycle hypothesis predicts, longer-lived individuals (larger π) tend to invest a larger proportion of their incomes in financial savings during the prime earning years (larger s_t) to smooth consumption. A similar effect will arise when the consumption level required in the old age is higher (larger φ). Besides, either larger π or larger φ incentivizes parents to raise their earnings, persuading them to devote less time to children's education (smaller p_t).

According to Proposition 2(ii), the stronger the parents' concern about their children's human capital acquisition (larger δ), the greater the effort they will make in educating their children (larger p_t). This in turn requires them to cut work hours, which negatively affects their incomes and thus savings (smaller s_t). Besides, the improved efficiency of pedagogical practices (larger γ) prompts parents to allocate more time toward children's education (larger p_t), and consequently, the income effect induces them to save less (smaller s_t).

Note that H_t is a state variable for generation t. The positive effect of H_t on s_t, as presented in Proposition 2(iii), seems intuitive. All else being equal, a higher wage per unit of time (larger H_t) provides the young with an increased disposable income, allowing them to save more (larger s_t).

The impact of H_t on p_t has a less straightforward intuition. On the one hand, young adults tend to work more hours to take advantage of a higher wage ("*labor supply effect*"). On the other hand, since parental human capital plays a complementary role to pedagogical practices in accumulating children's human capital that concerns parents, a larger stock of parental human capital enhances the efficiency of pedagogical methods, thereby inducing parental time inputs ("*education efficiency effect*"). Proposition 2(iii) shows that the latter effect outweighs the former effect, leading to a net positive response of the time spent on children's education to income growth.

The positive relationship between parental human capital and responsive parenting has found empirical audience. Guryan, Hurst, and Kearney (2008) find that mothers with a college education or higher spend 4.5 hours more every week in childcare than mothers with a high school diploma or less. Ramey and Ramey (2010) report that between the 1980s and the mid 1990s, college-educated mothers in the US have increased their time spent with children by over nine hours per week (or equivalently 70%), despite a drastic increase in the average pay. Agostinelli, Doepke, Sorrenti, and Zilibotti (2022) show that wealthier parents spend more time on childcare than less affluent ones.

Given that individual choices of (p_t, s_t) depend on the state variable H_t, we proceed to explore the steady state of the economy. In accordance with equation (11) that specifies the transitional dynamics of H_t for all t, we define the steady state as follows:

Definition 1 *The economy reaches the steady state when $H_t = H$ for any t.*

Based on Definition 1, we establish the following proposition that characterizes the steady state:

Proposition 3 *The steady-state human capital stock is determined by*

$$\left(1+\frac{1-\alpha}{\gamma}\right)\ln H-\ln(H-\pi\varphi)=\frac{\beta}{\gamma}\ln\frac{\beta\delta}{\pi e}+\ln\frac{\gamma\delta}{2+\gamma\delta}, \tag{13}$$

where H decreases with π and e if and only if $\frac{H}{\pi\varphi}>1+\frac{\gamma}{1-\alpha}$.

Proof. See Appendix B. ■

Proposition 3 states that the steady-state stock of human capital depends on seven parameters, namely $(\alpha, \beta, \gamma, \delta, \pi, \varphi, e)$. Individuals with greater longevity (larger π) carry out harsh discipline less frequently as they have their children's future eldercare in mind (Proposition 1). Meanwhile, greater longevity stimulates the forward-looking young adults to increase their earnings and devote

less to the time-demanding education of their children (Proposition 2(i)), which in turn depresses the formation of children's human capital. Both forces will contribute to a decrease in the steady-state human capital stock (smaller H). Furthermore, when the middle-aged children are required to allocate a great proportion of time to look after their aging parents (larger e), their parents will hesitate to discipline them harshly during childhood (smaller d), which translates into a lower stock of human capital in the steady state.

4 Endogenous Lifespan

Our analysis so far rests on the assumption that an individual's probability of surviving to old adulthood is constant and exogenous. This assumption is empirically relevant based on substantial scientific finding that genes are a major determinant of human lifespan and aging (Browner et al. 2004). Yet it is worth noting that other elements, such as lifestyle choices and access to health care, also play an important role in affecting how long an individual lives. In this section, we follow the literature regarding the endogenous lifetime (Chakraborty 2004; de La Croix and Ponthière 2010) to consider that one can produce health stock and extend his expected lifespan by purposeful health investments. In real life, such private investments may include undertaking aerobic exercises at the gym, having regular health checkups, and taking dietary supplements.

The extended model shares the same game structure as in Section 3 except that each member of generation t makes a private investment of m_t in health during middle adulthood. In other words, each individual chooses (d_t, p_t, s_t) in period $t+1$ and m_t in period $t+2$. Health investments help to increase her probability of living into old adulthood, as expressed by

$$k_t = \pi + k(m_t), \tag{14}$$

where $k_t \in [\pi, 1)$. Health investments exhibit diminishing returns (i.e., $k' > 0$ and $k'' < 0$); the survival probability equals π as in Section 3 if no investment is made ($k(0) = 0$). Note that health investments increase the insurance premium to φk_t, which decreases the middle-age consumption level to $c_t^m = s_t - \varphi k_t - m_t$.

Given the specific functions in (6) – (11), taking into account her parent's survival probability

k_{t-1} and her health investment m_t, a member of generation t expects her lifetime utility to be

$$\begin{aligned} U_t = &-d_{t-1} + \ln[(1-p_t)H_t - s_t] + \ln(s_t - \varphi k_t - m_t) + 1 - k_{t-1}e^2 \\ &+ k_t(\mu - d_t)e + \delta \ln\left[(H_t)^{\alpha}(1+d_t)^{\beta}(p_t)^{\gamma}\right]. \end{aligned} \tag{15}$$

The rational forward-looking individual aims to maximize U_t by making a life plan. Specifically, she optimally chooses (d_t, p_t, s_t) during young adulthood and m_t during middle adulthood. Her optimal choices are presented in the following proposition:

Proposition 4 *Given (14),* (d_t, p_t, s_t, m_t) *in equilibrium are determined by four equations:*

$$\frac{2+\gamma\delta}{H_t - \varphi[\pi + k(m_t)] - m_t}\left(\varphi + \frac{1}{k'}\right) + \frac{\beta\delta}{\pi + k(m_t)} = (1+\mu)e, \tag{16a}$$

$$d_t = \frac{\beta\delta}{[\pi + k(m_t)]e} - 1, \tag{16b}$$

$$p_t = \frac{\gamma\delta}{2+\gamma\delta}\left[1 - \frac{\varphi\pi + \varphi k(m_t) + m_t}{H_t}\right], \tag{16c}$$

$$s_t = \frac{H_t + (1+\gamma\delta)[\varphi\pi + \varphi k(m_t) + m_t]}{2+\gamma\delta}. \tag{16d}$$

Proof. See Appendix B. ■

Equation (16a) shows the equilibrium solution to m_t given the state vector $(H_t; \beta, \gamma, \delta, \varphi, \mu, \pi, e)$ in period t. Equations (16b) suggests that holding (β, δ, π, e) constant, d_t and m_t are inversely associated. As individuals expect investments in health to extend lifespan, they are more concerned for cherishing quality moments with their children. To address this concern, they nurture enduring familial bonds by mitigating harsh behaviors toward their children in young adulthood.

Equations (16c) and (16d) indicate that (p_t, s_t) can be written as a function of m_t; in a special case of $m_t = 0$, they can be simplified as in equation (12). Given $(\gamma, \delta, \varphi, H_t)$, there is a negative correlation between p_t and m_t. Intuitively, a stronger tendency to make future health investments prompts young adults to extend work hours to earn money. Subject to time constraint, they reduce effort in the time-demanding pedagogical practices. Moreover, the positive relationship between s_t and m_t can be interpreted as the tendency to prioritize saving during young adulthood in response to the anticipation of larger health expenses during middle adulthood. This is largely in accordance with the motive of consumption smoothing.

Table 3: Benchmark Values for Key Parameters

Notation	Description	Value
α	Efficiency of intergenerational transmission of human capital	0.6
β	Efficiency of harsh discipline	0.4
γ	Efficiency of pedagogical practice	0.4
δ	Preference over the quality of the child	5
μ	Children's resilience against harsh discipline	10
φ	Old-age consumption level	0.2
π	Survival probability (without health investments)	0.6
θ	Efficiency of health investment	0.3
e	Time spent on eldercare	0.45

To further explore the features of the transitional dynamics and the steady state, we resort to a numerical exercise. We assume that equation (14) takes the following specific functional form:

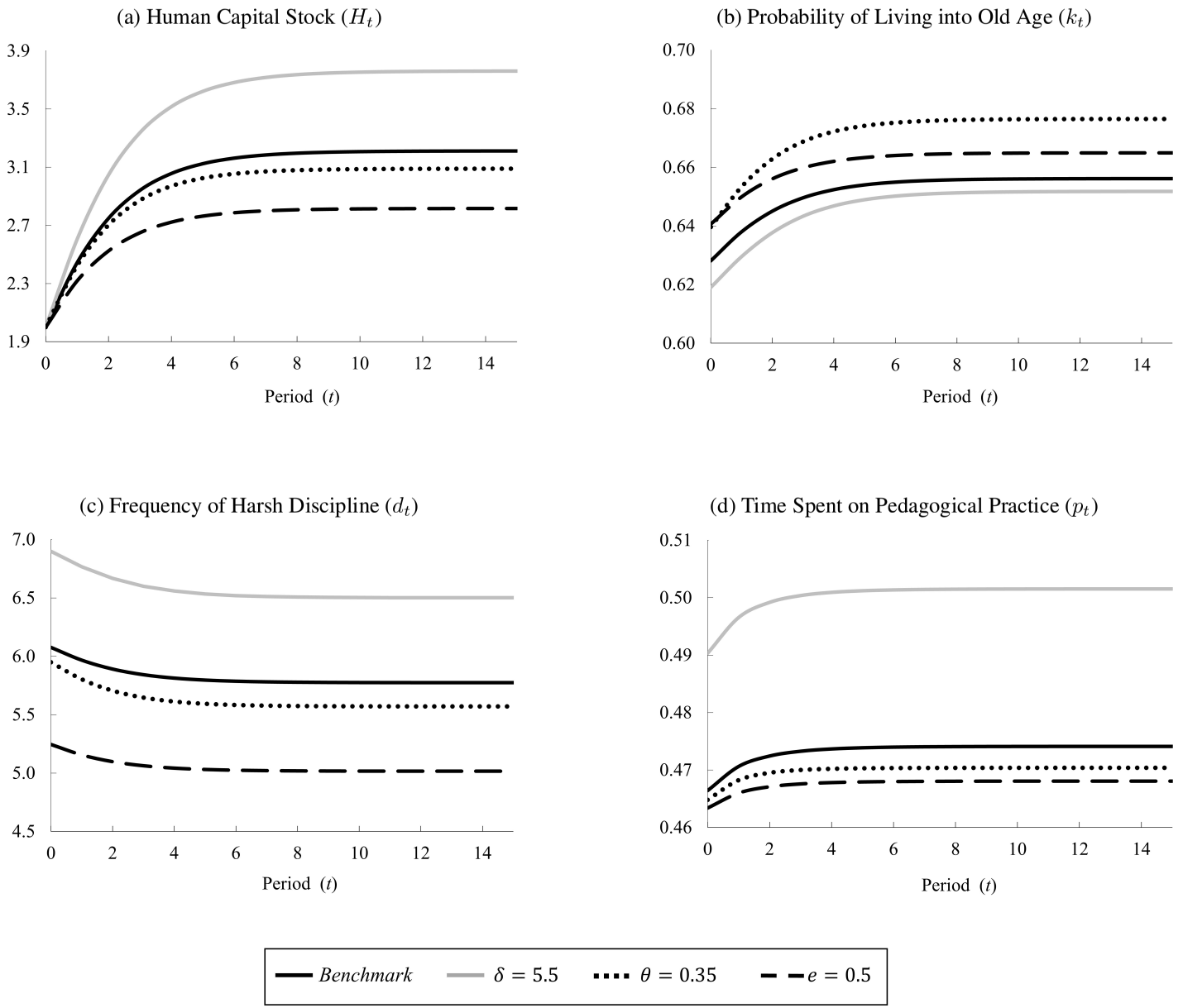
$$k_t = \pi + \theta\sqrt{m_t}, \quad \text{where } \theta > 0.$$

In Figure 2, we draw dark solid curves to demonstrate the numerical equilibrium outcomes in every period based on the configuration given in Table 3. Starting from the human capital stock of $H_0 = 2$ in plot (a), we derive that $v_0 = 0.0087$ by equation (16a) and then $\{d_0 = 6.0765, p_0 = 0.4664\}$ by equations (16b) and (16c). It follows directly from equation (11) that $H_1 = 2.4437$, and so forth. Over time, the human capital stock will progress toward the steady state.

The increase in human capital (or, equivalently, wage per unit of time) facilitates an increase in health investments that promote longevity. As shown in plot (b), the survival probability in old adulthood gradually evolves to a higher level. The concurrent growth of H_t and k_t aligns with the historical observations. For example, American men's average years of schooling rise from 8.71 in the 1850s to 15.87 in the 1970s, while their life expectancy at age five has experienced a sustained increase from 52.5 to 70.73 years during the same period (Hazan 2009).

The growing probability of living into old age, in turn, gravitates a change in parenting styles. Harsh disciplinary measures toward children gradually diminish: the dark curve of d_t in plot (c) follows a downtrend until the steady state is attained. Meanwhile, pedagogical practices become

Figure 2: Transitional Dynamics and Steady State When Health Investments Promote Longevity



more common over time. In plot (d), the dark curve of p_t displays an upward trend till the steady state. Our simulation outcome that H_t and d_t exhibit contrary trajectories is in line with Weinberg's (2001) finding that lower earners have limited capability to mold their children's behaviors through monetary incentives, leading to their increased reliance on the non-pecuniary corporal punishment.

The other three curves in each plot of Figure 2 give a visual presentation of comparative static analyses. The grey curves are drawn based on the values in Table 3 except that δ increases from 5 to 5.5. All else being equal, as parents develop a heightened concern for their children's human capital accumulation, they tend to engage more in children's education, as manifested by a higher frequency of strict discipline coupled with increased time spent on pedagogical practices (d_t and p_t shift upward). Being better educated, children obtain higher earnings (H_t moves upward). Despite the potential income effect that can motivate individuals to invest in health, the increased frequency of harsh discipline may hinder such investments, mainly due to its harmful consequences for the old-age well-being. In plot (b), the grey curve lies below the dark curve, which implicates that engaging in harsh parenting has a net detrimental effect on parents' desire to thrive into old adulthood during which they need their children's caregiving.

Next, we draw the dotted curves by using $\theta = 0.35$, *ceteris paribus*. The improved efficiency of health investments promotes healthy aging and longevity (i.e., k_t shifts upward). The prospect of needing assistance in old age prompts young parents to reduce harsh practices against children (i.e., d_t shifts downward). Yet a consequence of reduced parental involvement is the hindrance of children's human capital accumulation, which further undermines the future generations' human capital development (i.e., H_t shifts downward). As parental human capital declines, the marginal contribution of pedagogical practices decreases, resulting in a fall in such efforts made by parents in every period (i.e., p_t moves downward) .

Finally, the dashed curves differ from the dark solid curves in that e increases from 0.45 to 0.5. If the society has a higher expectation about adult children's care for aging parents by default, then parents tend to cherish the quality moments spent together with their children during old adulthood and respond by holding back demanding tactics during young adulthood (i.e., d_t shifts downward). By a similar logic, individuals are more willing to live longer by investing in health in each period (i.e., k_t shifts upward), whereas both the use of pedagogical practices and the human capital stock hit a lower level (i.e., H_t and p_t move downward).

5 Intergenerational Transfer and Eldercare

In the basic model of Section 3, individuals are assumed to follow the social norm to spend a specified fraction of time on eldercare (i.e., $e_t = e$ for all t). This section aims to investigate the interaction between parenting styles and wealth transfers induces children's caregiving. We focus on the scenario where the social norm is so weak that children provide no eldercare services unless they receive a transfer from their parents (In Appendix A, we discuss the endogenous determination of the social norm regarding e in an intergenerational supergame and suggest that $e = 0$ can be one potential Nash equilibrium).

We formulate the provision of family eldercare to be determined during a bargaining process between aging parents and their middle-aged children. Our modelling strategy is close in spirit to the strategic bequest literature (e.g., Bernheim, Shleifer, and Summers 1985), which suggests that parents withhold intergenerational transfers until death as a means of maintaining a bargaining chip that can be played to obtain assistance and support from children. The exchange of "time for money" is common, with children typically dedicating time to support their parents and attaining pecuniary benefits from them in return (Haberkern 2023). Our analysis departs from this line of literature by emphasizing inter vivo transfers rather than bequests.

As in Section 3, generation t choose (d_t, p_t, s_t) in young adulthood. In addition, they choose (e_t, f_t) during middle adulthood, where f_t represents a transfer to children. In period $t + 2$, each (middle-aged) member of generation t and his aging parent enter a Nash bargaining to determine the provision of her eldercare. To make the bargaining model tractable, we rely on specific functional forms in equations (6) – (11).

We discuss three possible scenarios. First, if the parent dies at the beginning of old adulthood, the child takes his parent's bequest f_{t-1} for granted without offering any eldercare ($e_t = 0$). Second, if the parent survives to old adulthood and receives caregiving support from her child ($e_t > 0$), then the child receives a transfer of f_{t-1}, while the parent obtains an attention of $e_t(\mu - d_{t-1})$. Given his parent's transfer f_{t-1} and his future transfer to his own child f_t, the child's current utility can be written as

$$\widetilde{u}_t = 1 - \pi e_t^2 + \ln(s_t + f_{t-1} - f_t - \pi\varphi).$$

Third, if the child refuses to provide support to his parent ($e_t = 0$), his refusal leads to a whole

period of leisure and a renunciation of his parent's transfer. In this case, his current utility is

$$\hat{u}_t = 1 + \ln(s_t - f_t - \pi\varphi).$$

In response, the old parent gives the money to a young adult (i.e., a health worker) in exchange for nursing services at the time length of f_{t-1}/H_{t+1}, given that the wage of young adults is H_{t+1} per unit of time in period $t+2$. This outside option enables her to gain an attention of $\lambda f_{t-1}/H_t$. We assume that λ is a small positive parameter so that the parent places a higher value on her child's caregiving than the care provided by a nursing house.

Without loss of generality, we assume a symmetric bargaining power of the child vis-à-vis the parent. The Nash bargaining equation can be written as

$$\max_{e_t} \underbrace{\left[e_t(\mu - d_{t-1}) - \lambda f_{t-1}/H_{t+1}\right]}_{\text{parent's surplus}} \underbrace{\left(\widetilde{u}_t - \hat{u}_t\right)}_{\text{child's surplus}}.$$

The maximization of the above match surplus is to set e_t through the intergenerational negotiation, as follows:

$$\begin{aligned}
& (\mu - d_{t-1})\left(\widetilde{u}_t - \hat{u}_t\right) + \left[e_t(\mu - d_{t-1}) - \frac{\lambda f_{t-1}}{H_{t+1}}\right]\frac{\partial \widetilde{u}_t}{\partial e_t} = 0 \\
\Leftrightarrow \quad & \ln\frac{s_t + f_{t-1} - f_t - \pi\varphi}{s_t - f_t - \pi\varphi} - \pi e_t^2 = -2\pi e_t\left[\frac{\lambda f_{t-1}}{(\mu - d_{t-1})H_{t+1}} - e_t\right] \\
\Leftrightarrow \quad & \pi e_t\left[\frac{2\lambda f_{t-1}}{(\mu - d_{t-1})H_{t+1}} - 3e_t\right] = \ln\frac{s_t - f_t - \pi\varphi}{s_t + f_{t-1} - f_t - \pi\varphi},
\end{aligned} \tag{17}$$

which indicates that the child's optimal choice of e_t is a function of $(d_{t-1}, f_{t-1}, f_t, s_t, H_{t+1})$.

We proceed to analyze the child's optimal choices of another four variables, namely (d_t, p_t, s_t, f_t). His lifetime utility function can be written as

$$\begin{aligned}
U_t = & -d_{t-1} + \ln[(1 - p_t)H_t - s_t] + \ln(s_t + f_{t-1} - f_t - \pi\varphi) + 1 - \pi e_t^2 \\
& + \pi e_{t+1}(\mu - d_t) + \delta \ln\left[(H_t)^\alpha (1 + d_t)^\beta (p_t)^\gamma\right].
\end{aligned} \tag{18}$$

In particular, the next proposition discusses the child's response to harsh parenting:

Proposition 5 *Given* (β, δ, π)*,* e_{t+1} *decreases with* d_t*.*

Proof. See Appendix B. ■

Proposition 5 unveils the intriguing correlation between the frequency of harsh discipline experienced by a child in early life and the subsequent decline in his willingness to spend time with his parent. Frequent strictness gives rise to a strained parent-child relationship: the child may perceive the parent as authoritarian or associate his parent with fear and restrictions, which can erode the emotional bonds between them. Over the life course, the strain will manifest as a decreased sense of connection and a diminished inclination to spend time together. The lack of children's attention adversely affects the well-being of their aged parents. Conversely, implementing reduced strictness serves as an long-term insurance that will benefit the parents, contingent on their survival.

The implication of Proposition 5 is in line with earlier empirical findings. For example, Paris (2001) finds that individuals who report less controlling mothering indicate greater willingness to provide eldercare in the US. Silverstein et al. (2002) suggest that the motivation of adult children to provide support to their aging parents is rooted in earlier family experiences based on the US survey data. Kim et al. (2013) find that a tiger parenting profile is associated with a greater sense of alienation and less of a sense of family obligation in the Chinese American families. Baumbach, Hughes, Derain, and Liu (2023) conduct a survey to show that childhood experiences may affect the quality of care provision by adult children to their aging parents.

To better capture the bargaining outcome, we draw on the literature on infinitely-repeated intergenerational interactions (e.g., Maskin and Tirole 2001) to focus on the stationary Markov perfect equilibrium, as defined below:

Definition 2 *The sequence $\{x_t\}_0^\infty$ is a stationary Markov perfect equilibrium if (i) for any t, x_t is the optimal strategy for a member of generation t who believes that future generations will play the strategy $\{x_k\}_{t+1}^\infty$, (ii) an individual's strategies are functions of the payoff-relevant state variable(s) only, and (iii) individuals facing the same state variable(s) play the same strategy, $x_t = x$ for all t.*

By Definition 2, individuals play the same Markov strategy in the same state in any subgame. Let's consider the steady state where the human capital stock remains time-invariant ($H_t = H$). In a stationary Markov perfect equilibrium, each generation makes the same choices, namely $d_t = d$, $p_t = p$, $s_t = s$, $e_t = e$, and $f_t = f$. We characterize the stationary Markov perfect equilibrium in the following proposition.

Proposition 6 *In the steady state, the stationary Markov perfect equilibrium is governed by equation (13) and the following five equations:*

$$d = \frac{\beta\delta}{\pi e} - 1, \tag{19a}$$

$$p = \frac{\gamma\delta}{2+\gamma\delta}\left(1 - \frac{\pi\varphi}{H}\right), \tag{19b}$$

$$s = \frac{H + (1+\gamma\delta)\pi\varphi}{2+\gamma\delta}, \tag{19c}$$

$$\frac{2\lambda\pi f}{[(1+\mu)\pi e - \beta\delta]H} - \frac{1}{\pi e^2}\ln\left[1 - \frac{(2+\gamma\delta)f}{H - \pi\varphi}\right] = 3, \tag{19d}$$

$$\frac{\pi\lambda}{H}\left[\frac{f}{(1+\mu)\pi e - \beta\delta} + \frac{H - \pi\varphi}{2+\gamma\delta}\right] - \frac{(2+\gamma\delta)f}{H - \pi\varphi - (2+\gamma\delta)f} + \frac{(1+\mu)\pi e - \beta\delta}{2\pi e^2} = 3. \tag{19e}$$

Proof. See Appendix B. ■

Proposition 6 presents the implicit solutions to (d, p, s, e, f, H) in the stationary Markov perfect equilibrium. Equation (13), which pins down the steady state, suggests that e can be written as a function of H. Because d is a decreasing function of e as in equation (19a), d can be derived as depending on H only. Likewise, substituting $e(H)$ into equation (19d) helps to solve $f = f(H)$, while plugging $e(H)$ and $f(H)$ into equation (19e) obtains the solution to H. Finally, equations (19b) and (19c) describe both p and s as a function of H. The system of six non-linear equations gives rise to the possibility of multiple equilibria. We rely on a simulation to examine more features of equilibrium solutions.

We perform the simulations based on the parameter values in Table 3 except for θ (which is irrelevant in this section) and e (which is endogenous in this section). Also, we set $\lambda = 0.5$. There are two sets of numerical solutions to the set of equations in Proposition 6, as presented below:

Solution I:	$d = 3.632$	$p = 0.473$	$e = 0.720$	$s = 0.637$	$f = 0.312$	$H = 2.189$
Solution II:	$d = 7.475$	$p = 0.485$	$e = 0.393$	$s = 1.118$	$f = 0.239$	$H = 4.114$

Solution I presents a steady-state equilibrium with low parental involvement. Young parents tend not to dispense harsh tactics frequently toward their children and allocate 47.3% of time to pedagogical practices. Low parental inputs leads the steady-state human capital to end up with a low stock. Old parents attain intensive eldercare from their children ($a = e(\mu - d) = 4.583$) and reward

them with a sizeable transfer ($f = 0.312$). Solution II reflects a more demanding parenting style. Parents employ both harsh and pedagogical practices intensively during young adulthood to foster their children's human capital acquisition. When they become old, they provide their children with a modest transfer ($f = 0.239$) and anticipate a reciprocation of limited attention from their children ($a = 0.993$).

The presence of multiple equilibria within a given set of parameters sheds some light on the real-life observations on different parenting styles. These differences may be attributed to factors that remain unaddressed in our model, such as cultural diversity. For instance, in many East Asian economies highly influenced by Confucian philosophy, parents emphasize discipline and follow a more stringent and structured upbringing. Black parents are observed to use physical punishment more frequently than Hispanic and white parents, in part due to the historical influence of colonial practices in Africa. Immigrant parents, who often encounter substantial challenges and sacrifices to provide better life opportunities for their children, are inclined to invest more in their children's education to ensure their future success.

6 Conclusion

Population aging has emerged as a prominent global demographic trend marked by the increase in life expectancy, which creates a pressing demand for eldercare in many societies (Pestieau 2022). Longer life spans increases the shared lifetime of parents and children in adulthood. In this paper, we emphasize that the manner parents treat their children in early life may generate a lasting impact on the quality of their relationship. Children who experience responsive parenting maintain healthy interactions with their parents after they transition into adulthood. A fond attachment to parents provides the basis for them to willingly reciprocate when their parents age. Conversely, children who vividly remember being harshly treated by parents during childhood are more likely to develop resentment toward parents. Repairing a damaged parent-child relationship is challenging, leading adult children to pay less attention to their aging parents. This paper takes a life cycle perspective and addresses the dynamic reciprocity between parents and children.

We develop an overlapping generations model, wherein parenting matters to children's human capital stock, familial bond, and parental income. Educating the child in a pedagogical way is time

demanding and thus reduces the parent's work hours and incomes; however, it does not undermine the child's motivation of looking after the parent in later life. Harsh parenting is the easy way out, but it creates a more authoritarian atmosphere within the family and reduces the child's propensity to assume the role of a future family caregiver. Our model formalizes such intergenerational conflicts associated with the choices of parenting styles. With greater life expectancy, forward-looking parents choose to engage less in harsh behaviors that can undermine their lifelong connections with their children. That is, greater longevity serves as a catalyst for deterring harsh parenting. We find empirical evidence for this key implication of our model.

We extend our basic model by allowing for the endogenous survival probability resulting from individual health investments during middle adulthood. To the extent that parents frequently employ tough discipline during their children's early years, they will anticipate limited joy in spending time with their children in later years. Consequently, this hinders their inclination to invest in health, thereby reducing their welfare. When making a life plan, the forward-looking parents will refrain from practicing harsh discipline to avoid creating family estrangement, with the expectation that they will receive better caregiving support from their children in times of need.

We also propose a Nash bargaining model, wherein aging parents use transfers as a means to garner their children's attention. The availability of alternative (professional) care services enables adult children to have flexible choices about their use of time, and their propensity to accompany their aging parents depends on how they were treated by their parents during childhood. We show that adult children will allocate more time to family eldercare if their parents use harsh disciplinary measures less frequently toward them in early life. By characterizing the stationary Markov perfect equilibrium in the steady state, our model allows for the emergence of diverse parenting styles within a given set of parameters.

This paper aims to develop a positive theory that connects parenting styles to human capital development and eldercare. The interconnections between these factors underscores the significance of societal progress in safeguarding the well-being of future generations. While we delve into this issue in absence of any public policies, it is recognized that the normative question of determining the optimal public policies regarding parenting and eldercare is equally important. We will turn to address the policy concerns in another segment of our research.

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Appendix A: The Credible Social Norms of Eldercare

Eldercare is often driven by family norms, which encompass explicit and implicit guidelines within a culture, imposing invisible constraints on the behaviors of family members (e.g., Barigozzia, Cremer, and Roeder 2020; Klimaviciute and Pestieau 2023). In this appendix, we aim to examine the condition under which the filial norm (i.e., individuals spend a proportion e of time taking care of aging parents) can be sustained in a Nash equilibrium. In line with the intergenerational trade literature (e.g., Ehrlich and Lui 1991; Becker 1993), we consider that generation $t+1$ will conform to the norm to set $e_{t+1} = e$ if generation t are observed to spend a proportion $e_t = e$ of time taking care of (the living) generation $t-1$, and will leave their aging parents uncared otherwise. To make our analysis tractable, we proceed with the specific functions as in (6) – (11).

Rewrite the lifetime utility of a representative member of generation t in (5) as

$$U_t = -d_{t-1} + v[(1-p_t)H_t - s_t] + 1 - \pi e_t^2 + \ln(s_t - \pi\varphi) + \pi e_{t+1}(\mu - d_t) + \delta \ln[(H_t)^\alpha (p_t)^\gamma] + \beta\delta \ln(1+d_t). \tag{A1}$$

An individual conforming to the norm to choose $e_t = e$ in period $t+2$ expects her child to choose $e_{t+1} = e$ in period $t+3$. In other words, if she provides caregiving support to her parent, she will benefit such support from her child during old adulthood. Her optimal choice of harsh discipline on her child satisfies

$$d_t = \frac{\beta\delta}{\pi e_{t+1}} - 1. \tag{A2}$$

Moreover, the optimal choices of (p_t, s_t) are independent of (d_t, e_{t+1}, e_t). Substituting (A2) and $e_t = e_{t+1} = e$ into (A1) and rearranging obtains

$$U_t^C = 1 - d_{t-1} + v[(1-p_t)H_t - s_t] + \ln(s_t - \pi\varphi) + \delta \ln[(H_t)^\alpha (p_t)^\gamma] + \pi e(\mu + 1 - e) + \beta\delta\left(\ln\frac{\beta\delta}{\pi e} - 1\right), \tag{A3}$$

where the superscript C denotes conformity.

An individual who decides to deviate from the filial norm in period $t+2$ will optimally choose $e_t = 0$, which leads her child to set $e_{t+1} = 0$, too. In this case, her utility function in (A1) can be

rewritten as

$$U_t^D = 1 - d_{t-1} + v[(1-p_t)H_t - s_t] + \ln(s_t - \pi\varphi) + \delta \ln[(H_t)^\alpha (p_t)^\gamma] + \beta\delta \ln(1+d_t),$$

where the superscript D denotes deviation. A larger d_t always increases U_t^D so that the individual will set d_t to the maximum level, as denoted by $\bar{d}$. By (A2), she will always choose $d_t = \bar{d}$ when $e \leqslant \frac{\beta\delta}{\pi(1+\bar{d})}$. To save algebra, we limit our attention to the case $e > \frac{\beta\delta}{\pi(1+\bar{d})}$.

Clearly, the individual will find that an deviation from the social norm makes herself worse off if and only if $U_t^C \geqslant U_t^D$. In other words, the social norm is credible if and only if

$$e + \frac{\beta\delta}{\pi e}\left[\ln(1+\bar{d}) - \ln\frac{\beta\delta}{\pi e} + 1\right] < 1 + \mu. \tag{A4}$$

Any e that meets condition (A4) can be sustained in an intergenerational Nash equilibrium. It is straightforward to infer that such an effort should be neither too high nor too low. For any given value of e, each generation's optimal choice of harsh discipline satisfies (A2). We therefore obtains two immediate inferences. First, there can be multiple Nash equilibria representing different social norms regarding eldercare, which in turn influences parenting styles; our analysis in Section 3 can be regarded as one of these equilibria given a specific value of e. Second, the diligent provision of eldercare within families maintained by the social norm contributes to deterring harsh parenting.

Appendix B: Mathematical Proofs

Proof of Proposition 1

Given d_{t-1} and H_t, the first order condition of U_t in (5) with respect to d_t yields

$$\frac{\partial U_t}{\partial d_t} = \pi e l' z' + g' \frac{\partial h}{\partial d_t} = 0 \qquad \Leftrightarrow \qquad \frac{\partial h}{\partial d_t} = \frac{\pi e l'(-z')}{g'}. \tag{B1}$$

Holding other things unchanged, $\frac{\partial h}{\partial d_t}$ increases with $(\pi, e, l', -z')$ and decreases with g'. Because $\frac{\partial^2 h}{\partial d_t^2} < 0$, d_t decreases with $(\pi, e, l', -z')$ and increases with g'.

Proof of Proposition 2

(i) It is straightforward to see from equation (12) that p_t decreases with φ and π, while s_t increases with φ and π.

(ii) Differentiating p_t and s_t in (12) with respect to δ and γ yields

$$\frac{dp_t}{d\delta} = \frac{2\gamma}{(2+\gamma\delta)^2}\left(1 - \frac{\pi\varphi}{H_t}\right) = \frac{2p_t}{\delta(2+\gamma\delta)} > 0, \tag{B2}$$

$$\frac{dp_t}{d\gamma} = \frac{2\delta}{(2+\gamma\delta)^2}\left(1 - \frac{\pi\varphi}{H_t}\right) = \frac{2p_t}{\gamma(2+\gamma\delta)} > 0, \tag{B3}$$

$$\frac{ds_t}{d\delta} = \frac{-\gamma(H_t - \pi\varphi)}{(2+\gamma\delta)^2} = -\frac{p_t H_t}{\delta(2+\gamma\delta)} < 0, \tag{B4}$$

$$\frac{ds_t}{d\gamma} = \frac{-\delta(H_t - \pi\varphi)}{(2+\gamma\delta)^2} = -\frac{p_t H_t}{\gamma(2+\gamma\delta)} < 0. \tag{B5}$$

(iii) It is straightforward to see from equation (12) that both s_t and p_t increase with H_t.

Proof of Proposition 3

By Definition 1 and (11), the steady state can be characterized by $H^{1-\alpha} = (1+d)^{\beta} p^{\gamma}$. By inserting (B1) and (12), we rewrite this expression as

$$H^{1-\alpha} = \left(1 + \frac{\beta\delta}{\pi e} - 1\right)^{\beta}\left[\frac{\gamma\delta}{2+\gamma\delta}\left(1 - \frac{\pi\varphi}{H}\right)\right]^{\gamma} \qquad \Leftrightarrow \qquad \frac{H^{\frac{1-\alpha+\gamma}{\gamma}}}{H - \pi\varphi} = \left(\frac{\beta\delta}{\pi e}\right)^{\frac{\beta}{\gamma}} \frac{\gamma\delta}{2+\gamma\delta}, \tag{B6}$$

which can be rearranged as (13). Totally differentiating (13) with respect to H and π yields

$$\begin{aligned} & \left(\frac{1-\alpha}{\gamma}+1\right)\frac{dH}{H}-\frac{dH-\varphi d\pi}{H-\pi\varphi}+\frac{\beta}{\gamma\pi}d\pi=0 \\ \Leftrightarrow \quad & \frac{dH}{d\pi}=-\frac{H}{(1-\alpha)(H-\pi\varphi)-\gamma\pi\varphi}\left[\varphi\gamma+\frac{\beta}{\pi}(H-\pi\varphi)\right], \end{aligned} \tag{B7}$$

which is negative if and only if $(1-\alpha)(H-\pi\varphi)>\gamma\pi\varphi$, namely $\frac{H}{\pi\varphi}>1+\frac{\gamma}{1-\alpha}$.

Totally differentiating (13) with respect to H and e derives

$$\begin{aligned} & \left(\frac{1-\alpha}{\gamma}+1\right)\frac{dH}{H}-\frac{dH}{H-\pi\varphi}+\frac{\beta}{\gamma}de=0 \\ \Leftrightarrow \quad & \frac{dH}{de}=-\frac{\beta H}{(1-\alpha)(H-\pi\varphi)-\gamma\pi\varphi}. \end{aligned} \tag{B8}$$

which is negative if and only if $\frac{H}{\pi\varphi}>1+\frac{\gamma}{1-\alpha}$.

Proof of Proposition 4

The first order conditions of U_t in (15) with respect to (d_t, p_t, s_t) obtain

$$\frac{\partial U_t}{\partial d_t}=-k_t e+\frac{\beta\delta}{1+d_t}=0 \qquad \Leftrightarrow \qquad d_t=\frac{\beta\delta}{k_t e}-1, \tag{B9}$$

$$\frac{\partial U_t}{\partial p_t}=-\frac{H_t}{(1-p_t)H_t-s_t}+\frac{\gamma\delta}{p_t}=0 \qquad \Leftrightarrow \qquad p_t=\frac{\gamma\delta}{1+\gamma\delta}\left(1-\frac{s_t}{H_t}\right), \tag{B10}$$

$$\frac{\partial U_t}{\partial s_t}=-\frac{1}{(1-p_t)H_t-s_t}+\frac{1}{s_t-\varphi k_t-m_t}=0 \quad \Leftrightarrow \quad s_t=\frac{(1-p_t)H_t+\varphi k_t+m_t}{2}. \tag{B11}$$

Combining (B10) and (B11) derives the expression for s_t as

$$s_t=\frac{H_t+(1+\gamma\delta)(\varphi k_t+m_t)}{2+\gamma\delta} \tag{B12}$$

Moreover, the first order condition of U_t in (15) with respect to m_t yields

$$\frac{\partial U_t}{\partial m_t}=\frac{-\varphi k'-1}{s_t-\varphi k_t-m_t}+k'(\mu-d_t)e=0 \qquad \Leftrightarrow \qquad \frac{1}{s_t-\varphi k_t-m_t}\left(\varphi+\frac{1}{k'}\right)=(\mu-d_t)e.$$

Inserting (B9) and (B12) into the above expression and rearranging yields equation (16a).

Proof of Proposition 5

Differentiating (18) with respect to d_t and setting it to zero derives the interior solution:

$$d_t = \frac{\beta\delta}{\pi e_{t+1}} - 1, \tag{B13}$$

which shows that e_{t+1} decreases with d_t given (β, δ, π).

Proof of Proposition 6

Taking the first order condition of (18) with respect to (p_t, s_t) and rearranging obtains

$$p_t = \frac{\gamma\delta}{2+\gamma\delta}\left(1 - \frac{\pi\varphi + f_t - f_{t-1}}{H_t}\right), \tag{B14}$$

$$s_t = \frac{H_t + (1+\gamma\delta)(\pi\varphi + f_t - f_{t-1})}{2+\gamma\delta}. \tag{B15}$$

Taking the first order condition of (18) with respect to f_t and rearranging pins down the optimal intergenerational transfer in the following equation:

$$\frac{1}{s_t + f_{t-1} - f_t - \pi\varphi} + 2\pi e_t \frac{\partial e_t}{\partial f_t} = \pi(\mu - d_t)\frac{\partial e_{t+1}}{\partial f_t}. \tag{B16}$$

Note that $\dfrac{\partial e_{t+1}}{\partial f_t}$ and $\dfrac{\partial e_t}{\partial f_t}$ can be derived from total differentiations of equation (17), as follows:

$$\frac{\partial e_{t+1}}{\partial f_t} = \frac{(\mu - d_t)H_{t+2} + 2\pi\lambda e_{t+1}(s_{t+1} + f_t - f_{t+1} - \pi\varphi)}{2\pi(\mu - d_t)(s_{t+1} + f_t - f_{t+1} - \pi\varphi)H_{t+2}\left[3e_{t+1} - \frac{\lambda f_t}{(\mu - d_t)H_{t+2}}\right]} > 0, \tag{B17}$$

$$\frac{\partial e_t}{\partial f_t} = \frac{f_{t-1}}{2\pi(s_t + f_{t-1} - f_t - \pi\varphi)(s_t - f_t - \pi\varphi)\left[3e_t - \frac{\lambda f_{t-1}}{(\mu - d_{t-1})H_{t+1}}\right]} > 0. \tag{B18}$$

By Definition 1, we have $H_t = H$ in the steady state. In the stationary Markov perfect equilibrium, each generation play the same strategies, namely $d_t = d$, $p_t = p$, $s_t = s$, $e_t = e$, and $f_t = f$. Therefore, we can rewrite (B13), (B14), and (B15) as (19a) – (19c). Moreover, equation (17) can

be rewritten as

$$\begin{aligned} & \pi e\left[\frac{2\lambda f}{(\mu-d)H}-3e\right]=\ln\frac{s-f-\pi\varphi}{s-\pi\varphi} \\ \Leftrightarrow \quad & \frac{2\lambda f}{(\mu-d)eH}=3+\frac{1}{\pi e^2}\ln\left(1-\frac{f}{s-\pi\varphi}\right). \end{aligned} \tag{B19}$$

Inserting (19a) and (19c) into the above expression and rearranging obtains (19d).

Furthermore, substituting (B17) and (B18) into (B16) derives

$$\begin{aligned} & \frac{1}{s_t+f_{t-1}-f_t-\pi\varphi}+\frac{f_{t-1}e_t}{(s_t-f_t-\pi\varphi)(s_t+f_{t-1}-f_t-\pi\varphi)\left[3e_t-\frac{\lambda f_{t-1}}{(\mu-d_{t-1})H_{t+1}}\right]} \\ & =\frac{(\mu-d_t)H_{t+2}+2\pi\lambda e_{t+1}(s_{t+1}+f_t-f_{t+1}-\pi\varphi)}{2H_{t+2}(s_{t+1}+f_t-f_{t+1}-\pi\varphi)\left[3e_{t+1}-\frac{\lambda f_t}{(\mu-d_t)H_{t+2}}\right]}. \end{aligned} \tag{B20}$$

In the steady state and the stationary Markov equilibrium, equation (B20) can be rewritten as

$$\begin{aligned} & \frac{1}{s-\pi\varphi}+\frac{fe}{(s-f-\pi\varphi)(s-\pi\varphi)\left[3e-\frac{\lambda f}{(\mu-d)H}\right]}=\frac{(\mu-d)H+2\pi\lambda e(s-\pi\varphi)}{2H(s-\pi\varphi)\left[3e-\frac{\lambda f}{(\mu-d)H}\right]} \\ \Leftrightarrow \quad & 3e-\frac{\lambda f}{(\mu-d)H}+\frac{fe}{s-f-\pi\varphi}=\frac{\mu-d}{2}+\frac{\pi\lambda e(s-\pi\varphi)}{H}. \end{aligned} \tag{B21}$$

Inserting (19a) and (19c) into (B21) obtains

$$\begin{aligned} & 3e-\frac{\lambda f}{[1+\mu-\beta\delta/(\pi e)]H}+\frac{fe}{\frac{H-\pi\varphi}{2+\gamma\delta}-f}=\frac{1+\mu-\beta\delta/(\pi e)}{2}+\frac{\pi\lambda e}{H}\cdot\frac{H-\pi\varphi}{2+\gamma\delta} \\ \Leftrightarrow \quad & 3+\frac{(2+\gamma\delta)f}{H-\pi\varphi-(2+\gamma\delta)f}-\frac{1}{2e}\left(1+\mu-\frac{\beta\delta}{\pi e}\right)=\frac{\pi\lambda}{H}\left(\frac{H-\pi\varphi}{2+\gamma\delta}\right)+\frac{(\lambda f)/(eH)}{1+\mu-\beta\delta/(\pi e)} \\ \Leftrightarrow \quad & \frac{\pi\lambda}{H}\left[\frac{H-\pi\varphi}{2+\gamma\delta}+\frac{f}{(1+\mu)\pi e-\beta\delta}\right]=3+\frac{(2+\gamma\delta)f}{H-\pi\varphi-(2+\gamma\delta)f}-\frac{(1+\mu)\pi e-\beta\delta}{2\pi e^2}, \end{aligned}$$

which can be rearranged as (19e).