Full Revelation under Optional Verification

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Abstract

We analyse a Sender-Receiver game in which the Sender can choose between a costless cheap-talk message and a costly verifiable message. In a discrete setting we state conditions under which the Receiver always learns the true state while the Sender uses different message types in different states. On the contrary we show that such a fully revealing equilibrium does not exist if the state and action space are continuous.

Keywords: cheap-talk, communication, costly disclosure, full revelation, Sender-Receiver game, verifiable information.
JEL: C72, D82

1. Introduction

We study a Sender-Receiver game in which only the Sender knows about the true state of the world. She can communicate with the Receiver who then chooses an action. The utility of both players depends on the state and the action. The Sender can choose between costless cheap-talk messages as introduced by Crawford and Sobel [1] and a costly verifiable message that reveals the true state. The literature on verifiable messages goes back to Grossman [2] and Milgrom [3]. In their models the Sender can decide how much information she likes to reveal about an item she wants to sell. In our model the Sender only has one verifiable message that completely reveals the state. We argue that if the Sender could verify partially the unraveling argument would hold and in equilibrium the Sender will reveal the complete information.

Our analysis focuses on fully revealing equilibria. In such an equilibrium the Receiver always learns about the true state. In a setting with only verifiable messages the existence of such equilibria has been shown, see for example Jovanovic [4]. As our model combines two different message types, we focus our attention on fully revealing equilibria in which the Sender uses both message types. If the preferences of Sender and Receiver are similar there might exist a fully revealing equilibrium in which the Sender uses only cheap-talk, but then this equilibrium would also exist without the availability of the verifiable message.

We state conditions for the existence of a fully revealing equilibrium in which the Sender uses both message types when the action and state space are discrete. On the other hand we show that no such equilibrium can exist in a continuous setting.

Our model can be applied to several classical examples. In a job interview the Sender is looking for employment and has to convince the employer (Receiver) of her qualification. To

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do so she can either just give a list of certain skills or she can prove that she has those skills by certificates and reports. As these certificates are costly, e.g. the TOEFL test or the Cambridge certificate for English language certification, the Sender might only be interested to pay if her skill level is sufficiently high.

In the used-car market a potential buyer (Receiver) often relies on the details given by the seller (Sender). These details are often cheap-talk, but the seller could verify the quality of her car by an independent and costly consultant.

Combinations of these two different message types have been done, but with different assumptions and a different focus. Eső and Galambos [5] start with the cheap-talk model and add a possibility for the Sender to verify her information. Compared to our paper, they make stronger assumptions about the utility functions. They prove that in equilibrium the state space is split into different intervals, where the Sender uses the same message type within an interval. This confirms the result we derive in the continuous setting. Even under their additional assumptions there is no fully revealing equilibrium in which the Sender uses different types of messages in different states of the world. Bertomeu and Cianciaruso [6] extend the verifiable disclosure setting by proposing a model with several verifiable messages which differ in their precision and a single cheap-talk message. They create a general model to unify different models of verifiable disclosure. The authors show that the existence of the cheap-talk messages is beneficial for the Sender while mandatory disclosure decreases the Sender's utility. Bertomeu and Marinovic [7] assume that a firm reports about one verifiable and one unverifiable component and they derive the optimal behavior under certain regulations. They assume that cheap-talk messages can never be completely reliable so in their model cheap-talk and verifiable messages are not perfect substitutes. This implies that there can be no full revelation through cheap-talk.

2. The model

A state of the world ω is randomly drawn from a state space Ω. The Sender learns about ω and sends a message to the Receiver. We assume that the set of possible cheap-talk messages M corresponds to the state space Ω and that the verifiable message v is unique in each state of the world. So, the Sender chooses a message from $M \cup \{v\}$, i.e. she either sends a cheap-talk message or the verifiable message v. There is no possibility for partial disclosure. While sending any cheap-talk message is costless, the Sender has to pay a cost $c(\omega) > 0$ if she sends the verifiable message. An economic explanation for this cost can be either the payment for a certificate or the investment into effort.

After reading the message, the Receiver chooses an action from the action set A. The utility function of the Sender is $u_S(a, \omega)$ for $m \in M$ (cheap-talk) and $u_S(a, \omega) - c(\omega)$ for $m = v$ (verifiable message). For the Receiver the utility function is not dependent on the type of the message, but is given by $u_R(a, \omega)$. Both players have complete knowledge of the utility functions. By $a_R^*(\omega)$ we denote the action the Receiver prefers in the state ω. We assume this action is unique for all states.

3. Full revelation

We focus our attention on fully revealing Perfect Bayesian equilibria.

Definition 1 (Fully revealing equilibrium)**.** *A Perfect Bayesian equilibrium is fully revealing if the Receiver knows the true state of the world after receiving the message of the Sender.*

This setting combines the possibilities of cheap-talk and verifiable messages and even though there might exist fully revealing equilibria in which the Sender uses only cheap-talk messages or only the verifiable message, we focus on equilibria in which the Sender uses different message types in different states.

Proposition 1 (Discrete setting)**.**

Assume that Ω and A are discrete. There exists a fully revealing equilibrium with both message types used if and only if there exists $\hat{\Omega} \subsetneq \Omega$ with $\hat{\Omega} \neq \emptyset$ such that

$$1.\ \forall \omega_i \notin \hat{\Omega} : u_S(a_R^*(\omega_i), \omega_i) - c(\omega_i) > u_S(a_R^*(\omega_j), \omega_i)\ \forall \omega_j \in \hat{\Omega}$$
$$2.\ \forall \omega_i \in \hat{\Omega} : u_S(a_R^*(\omega_i), \omega_i) > u_S(a_R^*(\omega_j), \omega_i)\ \forall \omega_j \in \hat{\Omega}, \omega_j \neq \omega_i$$

This Proposition states that in some states ($\hat{\Omega}$) the Receiver trusts the Sender's cheap-talk messages while in all other states he enforces the use of the verifiable message. This requires that for states in $\Omega \backslash \hat{\Omega}$ the Sender dislikes the optimal actions that the Receiver chooses in the states in $\hat{\Omega}$ so much that she prefers to pay for the verifiable message (Condition 1). In addition the Sender is not allowed to have an incentive to deviate to another cheap-talk message if the true state is an element of $\hat{\Omega}$ (Condition 2).

It is to point out that even if there exists a fully revealing equilibrium, other equilibria might exist as well. Even several fully revealing equilibria might exist at the same time, because there may be several subsets $\hat{\Omega}$ that satisfy Condition 1 and 2 of Proposition 1. A fully revealing equilibrium is the best possible equilibrium for the Receiver while the Sender might be better off in a different equilibrium[2].

Allowing the Sender to choose between cheap-talk or the verifiable message can yield to the existence of a fully revealing equilibrium that would not exist if the Sender can only send messages of one type. A simple example is as follows: $\Omega = \{\omega_1, \omega_2\}$, $A = \{a_1, a_2\}$, $a_R^*(\omega_i) = a_i$ and the Sender always prefers a_1 over a_2. There is no fully revealing equilibrium with only one message type used, but there can be full revelation in our model if $c(\omega_1) < u_S(a_1, \omega_1) - u_S(a_2, \omega_1)$ holds. Then, the Receiver can threaten to reply to all cheap-talk messages with the action a_2 and the Sender will use cheap-talk in ω_2 and the verifiable message in ω_1.

Note that in the discrete setting a fully revealing equilibrium not always has to exist. Modifying the previous example such that the Sender prefers a_1 in ω_2 and a_2 in ω_1 gives one example where full revelation is impossible.

In a continuous setting we show that there never can be a fully revealing equilibrium where the Sender uses different message types in different states.

Proposition 2 (Continuous setting)**.**

Assume that Ω and A are continuous and that $a_R^*(\omega) : \Omega \to A$ is continuous and that $u_S(a, \omega) : A \times \Omega \to \mathbb{R}$ is continuous in both arguments. Then there exists no fully revealing equilibrium in which the Sender uses different message types in different states.

Unlike in the discrete setting, a fully revealing equilibrium in which the Sender uses both message types is not possible as long as the utility functions of both players are continuous. The

[2]Equilibrium selection in games with verifiable messages have been studied for example by Hart et al. [8] and Rappoport [9].

only fully revealing equilibria which might exist are those in which the Sender uses the same message type in all states.

In a continuous state and action space with continuous utilities for Sender and Receiver, there exists no action $\hat{a}$ that the Receiver can use as a threat point so that the Sender uses verifiable messages in some states, but cheap-talk in the state in which the Receiver's most preferred action is $\hat{a}$. The reasoning is as follows: Let $\hat{\omega}$ denote the state of the world in which the Receiver wants the action $\hat{a}$, i.e. $a_R^*(\hat{\omega}) = \hat{a}$. At a state $\hat{\omega} + \epsilon_1$ close to $\hat{\omega}$ the Receiver prefers another action, but because of the continuity it is close to $\hat{a}$, i.e. $\hat{a} + \epsilon_2$. If the Receiver uses $\hat{a}$ to reply to cheap-talk, the Sender uses cheap-talk not only in $\hat{\omega}$, but also for states close to $\hat{\omega}$. In that case her utility is $u_S(\hat{a}, \hat{\omega} + \epsilon_1)$ which is larger than $u_S(\hat{a} + \epsilon_2, \hat{\omega} + \epsilon_1) - c(\hat{\omega} + \epsilon_1)$ because of the continuity in the utility function. The Sender can only have an incentive to pay for the verifiable message in states close to $\hat{\omega}$ if there is some discontinuity in the utility function of the Sender or if the cost for verification does not exist in some states.

Example: Quadratic loss utility functions.

For concave utility functions, especially for quadratic loss utilities the implications of Proposition 2 are strong. Assume $A = \Omega = [0, 1]$ and that the utility functions are $u_R = -(a - \omega)^2$ and $u_S = -(a - \omega - b(\omega))^2$, where $b(\omega) \in \mathbb{R}$ is the continuous state dependent bias function of the Sender. For positive values of b, the Sender wants to have a higher action than state. This is similar to the example of Crawford and Sobel [1], but we allow the bias function to be state-dependent.

As an immediate conclusion from Proposition 2 we see that there can be no fully revealing equilibrium where the Sender uses cheap-talk and verifiable messages. Furthermore, as long as the Sender is biased in some states, she will not always tell the truth by cheap-talk. In addition, it is impossible to have full revelation where the Sender uses only the verifiable messages. The Sender would only use the verifiable message in all states if the Receiver replies to all cheap-talk messages with an action $\hat{a}$ the Sender dislikes in all states. Given the utility function of the Receiver, such a threat point $\hat{a}$ is the Receiver's best reply in one state, i.e. there exists $\hat{\omega}$ such that $a_R^*(\hat{\omega}) = \hat{a}$. In this state $\hat{\omega}$ the Sender will never use the verifiable message, because independent of the message type the Receiver picks the action $\hat{a}$ and the Sender can save the cost of the verifiable message $c(\hat{\omega}) > 0$ by using cheap-talk. So there exists no fully revealing equilibrium in which the Sender only uses the verifiable message. Combined with Proposition 2 we get that in this case no fully revealing equilibrium exists at all.

Corollary 1.

For $A = \Omega = [0, 1]$ and quadratic loss utility functions for the players (with $b(w) \not\equiv 0$), no fully revealing equilibrium exists.

4. Concluding Remarks

For a discrete setting we have stated conditions for a fully revealing equilibrium in which the Sender uses both message types and we have shown that such a equilibrium does not exist in a continuous model. Our results focus on the existence, but other questions are left unanswered for further research: How can we modify the model such that a fully revealing equilibrium (with both message types) also exists in the continuous model? How close can we get to a fully revealing equilibrium in the continuous setting otherwise? Which are then the messages the Sender chooses in the different states? Bertomeu and Cianciaruso [6] provide conditions for a

unique equilibrium in their setting and it might be interesting to see if such conditions can be adapted to our model.

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Appendix

Proof. Proposition 1

Only if: The equilibrium is as follows: For $\omega \in \hat{\Omega}$ the Receiver trusts the cheap-talk and in all other states the Sender uses the verifiable message.

If: Proof by contradiction.

Step 1. Let us assume that Condition 1 does not hold. This implies that there exist $\omega_i \notin \hat{\Omega}$ and $\omega_j \in \hat{\Omega}$ such that $u_S(a_R^*(\omega_j), \omega_i) > u_S(a_R^*(\omega_i), \omega_i) - c(\omega_i)$ holds. So the Sender prefers cheap-talk (and action $a_R^*(\omega_j)$) over the verifiable message (and action $a_R^*(\omega_i)$) and there will be no full revelation, because $a_R^*(\omega_i) \neq a_R^*(\omega_j)$.

Step 2. We assume that Condition 2 does not hold. Let us assume that there exists ω_k such that $u_S(a_R^*(\omega_k), \omega_k) \not> u_S(a_R^*(\omega_j), \omega_k)\ \forall \omega_j \neq \omega_k$. This implies that there exists ω_j such that $u_S(a_R^*(\omega_k), \omega_k) < u_S(a_R^*(\omega_j), \omega_k)$ holds. Then the Receiver has an incentive to lie in ω_k and send the cheap-talk message ω_j, so there is no full revelation. □

Proof. Proposition 2

We show that there is no fully-revealing equilibrium in which the Sender uses both message types. This then implies that if there exists a fully revealing equilibrium the Sender uses only one message type.

Let us assume that there is a fully revealing equilibrium in which the Sender sends a cheap-talk message in $\hat{w}$ and uses the verifiable message in all other states. This means that the Receiver replies to all cheap-talk messages with the action $a_R^*(\hat{\omega})$. The Sender has an incentive to use the verifiable message in state ω if the utility she gets from the action $a_R^*(\omega)$ minus the cost for the verifiable message is higher than her utility from the action that is the Receiver's reply to cheap-talk $a_R^*(\hat{\omega})$. This requires for all states $\omega \neq \hat{\omega}$:

$$u_S(a_R^*(\omega), \omega) - c(\omega) \geq u_S(a_R^*(\hat{\omega}), \omega).$$

So for the states close to $\hat{\omega}$:

$$u_S(a_R^*(\hat{\omega} + \epsilon), \hat{\omega} + \epsilon) - c(\hat{\omega} + \epsilon) \geq u_S(a_R^*(\hat{\omega}), \hat{\omega} + \epsilon)$$

For $|\epsilon| \to 0$, using the continuity of u_S and a_R^* this yields to:

$$u_S(a_R^*(\hat{\omega}), \hat{\omega}) - c(\hat{\omega}) \geq u_S(a_R^*(\hat{\omega}), \hat{\omega})$$

This requires $c(\hat{\omega}) \leq 0$, which is a contradiction and shows that no fully revealing equilibrium can exist in which the Sender uses a cheap-talk message in one state and the verifiable message in all other states.

The same contradiction arises if we assume that the Sender uses cheap-talk in several states or an interval of states. □

References

[1] V. P. Crawford, J. Sobel, Strategic Information Transmission, Econometrica 50 (1982) 1431–51. URL: https://ideas.repec.org/a/ecm/emetrp/v50y1982i6p1431-51.html. doi:10.2307/1913390.

[2] S. J. Grossman, The Informational Role of Warranties and Private Disclosure about Product Quality, Journal of Law and Economics 24 (1981) 461–83. URL: https://ideas.repec.org/a/ucp/jlawec/v24y1981i3p461-83.html. doi:10.1086/466995.

[3] P. R. Milgrom, Good News and Bad News: Representation Theorems and Applications, Bell Journal of Economics 12 (1981) 380–391. URL: https://ideas.repec.org/a/rje/bellje/v12y1981iautumnp380-391.html. doi:10.2307/3003562.

[4] B. Jovanovic, Truthful disclosure of information, The Bell Journal of Economics 13 (1982) 36–44. URL: http://www.jstor.org/stable/3003428. doi:10.2307/3003428.

[5] P. Eső, A. Galambos, Disagreement and Evidence Production in Strategic Information Transmission, International Journal of Game Theory 42 (2013) 263–282. URL: https://ideas.repec.org/a/spr/jogath/v42y2013i1p263-282.html. doi:10.1007/s00182-012-0344-8.

[6] J. Bertomeu, D. Cianciaruso, Verifiable disclosure, Economic Theory 65 (2018) 1011–1044. URL: https://doi.org/10.1007/s00199-017-1048-x. doi:10.1007/s00199-017-1048-x.

[7] J. Bertomeu, I. Marinovic, A theory of hard and soft information, The Accounting Review 91 (2016) 1–20. URL: https://doi.org/10.2308/accr-51102. doi:10.2308/accr-51102. arXiv:https://doi.org/10.2308/accr-51102.

[8] S. Hart, I. Kremer, M. Perry, Evidence games: Truth and commitment, American Economic Review 107 (2017) 690–713. URL: http://www.aeaweb.org/articles?id=10.1257/aer.20150913. doi:10.1257/aer.20150913.

[9] D. Rappoport, Evidence and skepticism in verifiable disclosure games (2017). doi:10.2139/ssrn.2978288.