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A note on the "size" of determinacy and indeterminacy★

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Summary. It is shown in this note that in an incomplete markets economy with uncountably many states of the world there may be uncountably many isolated equilibria as well as uncountably many non-isolated equilibria. Moreover, both subsets can be simultaneously of second category. Therefore, none of the subsets can be considered negligible with respect to the other, neither from a cardinality point of view nor from a topological one. Unfortunately, this fact prevents from claiming that these economies may have "typically" determinate equilibria – even though uncountably many of them – as would have been desirable for comparative statics exercises.

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1 Introduction

This note addresses a question left open in [5] relative to the local uniqueness or not of the uncountably many equilibria found in the incomplete markets economies with uncountably many states of the world considered in that article. Namely, are all of these equilibria isolated? If not, are indeterminate equilibria much "fewer", in some sense, than determinate ones? In what follows I will try to shed some light on these questions. These are important questions since, as pointed in [5], "*a careful study of* [*comparative statics analysis*] *must deal not only with the cardinality but also the topological properties of the equilibrium set*". After stating that his analysis would deal only with the cardinality issue, Mas-Colell points in [5] that "*it is certainly true that for comparative statics purposes local uniqueness is the key property. Thus it could be asked if local uniqueness* (*relative to the* L_∞ *topology*) *may still*

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in some sense be a generic property. This is a difficult question and we have no answer to offer". My goal in this note is to provide a contribution to answering this question.

The indeterminacy issue in general equilibrium theory refers to situations in which, roughly speaking, there are either "too many" equilibria or they are not sufficiently apart from each other, or both. Thus, it makes reference to a combination of cardinal and topological properties of the set of equilibria which are differently related between them in different frameworks. Specifically, an economy with a set of equilibria with the cardinality of the continuum is considered indeterminate whenever this implies that equilibria are not locally unique. This is so in the simplest situations, *i.e.* whenever the topological space of equilibria is separable[1]. But uncountable equilibrium sets need not consist of non-isolated equilibria in some other natural non-separable topologies.

As pointed above, for comparative statics purposes, the important property is the local uniqueness of equilibria, independently of the cardinality of the set of all equilibria. Thus it is worthwhile to inspect the topological structure of the set of equilibria of economies with uncountably many of them, in order to check if comparative statics can still be done.

When there is a finite number of states, it has been shown that local uniqueness of equilibria is the typical situation in the real assets case (see [3]), whereas models with nominal assets are typically indeterminate (see [2]). As for the case in which there are uncountably many states of the world and a finite number of real assets, it is shown in [5] that, for anyone of an open set of economies with incomplete markets, the set of equilibria has at least the cardinality of the continuum. However, what it is not claimed in [5] is that such a set of equilibria does not consist of isolated points.

In this note I shall provide a simple example of such an economy, showing that this set has isolated as well as non-isolated equilibria (this economy will by no means be pathological; actually, any other economy close enough to this one in the topology of uniform convergence on compact sets[2] would satisfy the same properties). Given this fact, it would be desirable for comparative statics purposes to be able to show that the set of non-isolated equilibria is in some sense negligible, meager or small with respect to the set of isolated ones. This would allow to claim that actual equilibria are isolated and, therefore, comparative statics can be done upon them. Nevertheless both subsets of isolated and non-isolated equilibria turn out to have the cardinality of the continuum in this economy. This means that none of this sets can be claimed to be "smaller" than the other on the grounds of their cardinality. Notwithstanding, the complete characterization of non-isolated equilibria provided by the corollary to Proposition 1 seems to

[1] *i.e.* it contains a countable dense subset.

[2] This is the topology considered in [5]. Notice that there is no natural topology with which to endow the space of economies. For a study on how to define the notion of genericity with infinitely many parameters see [1].

convey the idea that these should still be, in some sense, fewer than the isolated ones.

Thus, as an alternative, one could conjecture that the set of non-isolated equilibria is topologically small compared to the set of isolated ones. Specifically, it may well be that the set of non-isolated equilibria is a topological space of the first category[3] (roughly speaking, it is constituted by countably many "negligible" pieces) while the set of isolated equilibria is one of the second category (that is to say, not of first category)[4]. Nevertheless, it will be proved in Proposition 3 that this is not so and that, indeed, both sets are of the second category. That is to say, none of them is "negligible" with respect to the other, neither with respect to their cardinality, nor with respect to their category. This is not good news for any approach trying to fix the indeterminacy issue and to extend comparative statics arguments to economies with uncountably many equilibria.

2 The example

Consider an incomplete markets economy with two periods in which trade on real assets takes place in the first period and trade on commodities and consumption takes place in the second period. An equilibrium of such an economy will determine an asset trade, the payoffs of the assets will redistribute the endowments among agents at the beginning of the second period contingently to the state of the world, and therefore associated to the equilibrium there will be a correspondence from states of the world to spot equilibrium prices for the commodities in the second period. Thus, the equilibrium prices for the commodities is a selection of this correspondence. Typically, for another equilibrium with a different asset trade, the correspondence of spot equilibria itself will be different. Nevertheless, if we consider the case in which there are no assets at all, then the set of all the equilibrium prices for the commodities in the second period is the set of selections from a unique correspondence of spot equilibria.

Let us be more specific about the economy. Consider an economy which consists of two agents in a world with two periods and uncountably many states of the world identified with the reals in $[0, 1]$. They trade and consume two commodities in the second period and nothing in the first period. In the first period, the state of the world is not known, while in the second period the state is realized. Actually, in the first period agents cannot insure against the uncertainty since there is not any asset in this economy. In the second period, the agents receive their endowments contingent to the state of the

[3] *i.e.* it is the union of countably many nowhere dense subsets.

[4] With a topological space of first category we can sequentially remove "negligible" pieces of it and, doing so, eventually wipe it out; it can be thought of as "small" or "thin" in this sense. On the contrary, no similar process can make disappear a second category topological space, there will always remain "big" pieces of it to be removed, what makes them, in this sense, "big" or "thick" spaces.

world. Let us assume, for the sake of simplicity, that there is no aggregate risk in this economy: the total amount of each of the two commodities is normalized to be 1 for both commodities, independently of the state of the world realized. Thus we can think of the economy as a collection of uncountably many identical square Edgeworth boxes as in figure 1. The only uncertainty in the economy comes from the way in which the partition of the total endowments among the two agents depends on the state of the world: their respective endowment will be determined by the point on the segment joining A and B in the Edgeworth box of figure 1 at a distance x from A whenever the state $x \in [0, 1]$ is realized, taking as unit of measure the segment AB itself. Moreover, the agents have (state independent Von Neumann-Morgenstern) utility functions such that their indifference curves follow the pattern shown in figure 1, where the resulting contract curve is also depicted[5]. These preferences are continuous, differentiable, monotone and convex. In the appendix I provide an example of utility functions generating such indifference curves and contract curve. They imply no loss of generality, as it will be seen below.

The absence of any kind of asset prevents the agents to insure against the uncertainty of their endowments in the second period. In that period the agents receive their endowment according to the state of the world realized and trade in the Edgeworth box to attain a Walrasian (spot) equilibrium allocation. Note that for any initial endowments in the second period lying in the shadowed region in figure 1 there are three spot equilibria, two on its boundary and one outside it. The appearance of such a cusp in the projection on the Edgeworth box of the graph of the correspondence from endowments to spot equilibrium relative prices between commodities is the only important characteristic for what follows, and it is robust to perturbations in the preferences. Specifically, for this economy such correspondence is (see figure 2)

$$\Gamma(x) = \begin{cases} \{2\} & \text{if } 0 \leq x < \frac{1}{3} \\ \{\frac{1}{2}, \frac{9}{2}x - 1, 2\} & \text{if } \frac{1}{3} \leq x \leq \frac{2}{3} \\ \{\frac{1}{2}\} & \text{if } \frac{2}{3} < x \leq 1 \end{cases} \tag{1}$$

For the sake of simplicity, let us assume that, instead of the correspondence (1), it is the following correspondence (2) which is the actual spot equilibria correspondence of the economy. There is no loss of generality in doing this and all the reasoning henceforth could be redone for the correspondence (1) up to a rescaling of ordinates and a translation of the origin. The advantage of reasoning on the following correspondence is that the notation is much less cumbersome and the heart of the argument appears more clean. Thus, let $\Gamma : \mathbb{R} \longrightarrow \mathbb{R}$ be the correspondence such that

[5] Discard the portions of indifference curves out of the consumption set of each agent, R^2_+.

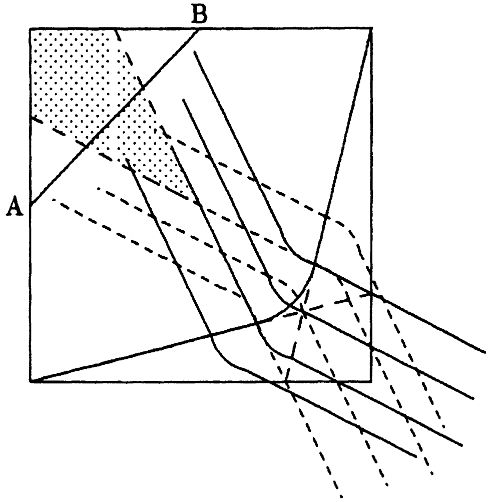

Figure 1

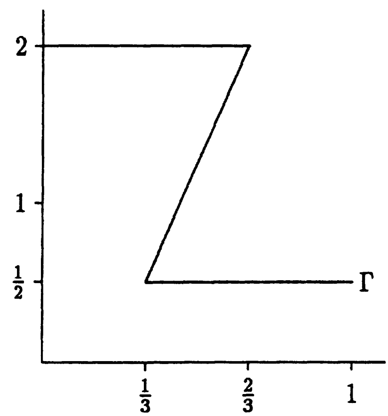

Figure 2

$$\text{(2)} \qquad \Gamma(x) = \begin{cases} \{1\} & \text{if } x < -1 \\ \{-1, x, 1\} & \text{if } -1 \leq x \leq 1 \\ \{-1\} & \text{if } x > 1 \end{cases}$$

Think of the ordinates as the possible spot equilibrium relative prices of commodity 1 in terms of commodity 2 at each state of the world x in abscissae. The fact that the graph of this correspondence has two kinks and takes negative values is not relevant for what follows. The whole reasoning would work the same with a smooth S-shaped and upwards shifted correspondence.

As it was discussed above, we can identify the set of equilibria if this economy with the set $\mathcal{S}$ of selections of Γ, *i.e.* the set of real-valued functions

f on $\mathbb{R}$ such that $f(x) \in \Gamma(x)$, $\forall x \in \mathbb{R}$. Clearly there is uncountably many selections and, hence, of equilibria. As notion of distance between selections (equilibria), I shall take the metric d induced by the norm of the essential supremum with respect to the Lebesgue measure, *i.e.* the set of selections (equilibria) is a metric subspace of $\mathscr{L}_\infty$. According to this metric, two selections f and g are considered to be the same one, or to be at a distance zero, whenever $f(x) = g(x)$ for every real x but for those in a set of null measure, what will be denoted as $\forall_{a.e.} x \in \mathbb{R}$ henceforth[6]. The next proposition characterizes completely the isolated equilibria of the economy.

Proposition 1. *$f \in \mathscr{S}$ is an isolated point of the metric space $(\mathscr{S}, d)$ if, and only if,*

(1) $\exists \varepsilon > 0 | \forall_{a.e.} x \in (-1, -1+\varepsilon),\ f(x) = 1$ and $\forall_{a.e.} x \in (1-\varepsilon, 1),\ f(x) = -1$

Proof. Let $f \in \mathscr{S}$ be such that (1) holds. Then, necessarily, $\varepsilon < 1$, otherwise (1) cannot be satisfied in $(1-\varepsilon, -1+\varepsilon)$. Let $B_\varepsilon(f)$ be a ball centred at f with radius ε. Let g be a selection of Γ in $B_\varepsilon(f)$. Then $f(x) - \varepsilon < g(x) < f(x) + \varepsilon$, $\forall_{a.e.} x \in \mathbb{R}$. That is to say, $\forall_{a.e.} x \in (-1, -1+\varepsilon)$, $f(x) = 1$ and hence $1 - \varepsilon < g(x) < 1 + \varepsilon$; but $0 < 1 - \varepsilon$ and thus $g(x) = 1$ too necessarily (a symmetric reasoning leads to conclude that $\forall_{a.e.} x \in (1-\varepsilon, 1)$, $f(x) = -1$ and $g(x) = -1$ too). Besides, $\forall_{a.e.} x \in [-1+\varepsilon, 1-\varepsilon]$, if $f(x) = -1$, then $-1-\varepsilon < g(x) < -1 + \varepsilon$, but $-1+\varepsilon \leq x$ and thus $g(x) < x < 1$ implying $g(x) = -1$ too; if $f(x) = 1$, then $1 - \varepsilon < g(x) < 1 + \varepsilon$, but $x \leq 1 - \varepsilon$ and thus $-1 < x < g(x)$ implying $g(x) = 1$ too; finally, if $f(x) = x$, then $x - \varepsilon < g(x) < x + \varepsilon$; but $-1 + \varepsilon \leq x \leq 1 - \varepsilon$ and thus $-1 \leq x - \varepsilon < g(x) < x + \varepsilon \leq 1$ implying $g(x) = x$ too. Obviously, $f(x) = g(x)$ also if $x < -1$ or $x > 1$. Therefore, if $g \in B_\varepsilon(f)$, then $g = f$.

Conversely, if f is isolated, then $\exists \eta > 0$ such that $B_\eta(f) = \{f\}$. Now, assume that (1) were not true, *i.e.* assume that $\forall \varepsilon > 0$, either $\exists A \subset (-1, -1+\varepsilon)$, A of non-null measure, such that $\forall x \in A$, $f(x) \in \{-1, x\}$ (and thus there is a non-null measure subset A' of A such that either $f(x) = -1$, $\forall x \in A'$ or $f(x) = x$, $\forall x \in A'$) or $\exists B \subset (1-\varepsilon, 1)$, B of non-null measure, such that $\forall x \in B$, $f(x) \in \{x, 1\}$ (and thus there is a non-null measure subset B' of B such that either $f(x) = x$, $\forall x \in B'$ or $f(x) = 1$, $\forall x \in B'$). Now assume $\varepsilon < \eta$. If $\exists A' \subset A \subset (-1, -1+\varepsilon)$, A' of non-null measure, such that $\forall x \in A'$, $f(x) = -1$, then $\forall x \in A'$, $-1 - \eta < -1 < x < -1 + \varepsilon < -1 + \eta$, *i.e.* $f(x) - \eta < x < f(x) + \eta$. Therefore, if g is a selection such that $g(x) = f(x)$, $\forall x \notin A'$ and $g(x) = x$, $\forall x \in A'$, then $g \in B_\eta(f)$ and $g \neq f$. If, on the contrary, $\exists A' \subset A \subset (-1, -1+\varepsilon)$, A' of non-null measure, such that $\forall x \in A'$, $f(x) = x$, then $\forall x \in A'$, $x - \eta < x - \varepsilon < -1 < x + \varepsilon < x + \eta$, *i.e.* $f(x) - \eta < -1 < f(x) + \eta$. Therefore, if g is a selection such that $g(x) = f(x)$, $\forall x \notin A'$ and $g(x) = -1$, $\forall x \in A'$, then $g \in B_\eta(f)$ and $g \neq f$. Thus for any

[6] As a matter of fact, the points of the metric space of selections with this metric are not the selections themselves but their equivalence classes induced by the relation "being equal but for a null measure set".

$\varepsilon \in (0, \eta)$, $f(x) = 1$, $\forall_{a.e.} x \in (-1, -1+\varepsilon)$. It can be shown similarly that for any $\varepsilon \in (0, \eta)$, $f(x) = -1$, $\forall_{a.e.} x \in (1, 1-\varepsilon)$. □

Proposition 1 provides an implicit characterization of the non-isolated equilibria.

Corollary. *$f \in \mathscr{S}$ is non-isolated if, and only if, $\forall \varepsilon > 0$, either $\exists A \subset (-1, -1+\varepsilon)$, A of non-null measure, such that $\forall x \in A$, $f(x) \in \{-1, x\}$ or $\exists B \subset (1-\varepsilon, 1)$, B of non-null measure, such that $\forall x \in B$, $f(x) \in \{x, 1\}$*

It is immediate that there are uncountably many both isolated and non-isolated equilibria. The next proposition shows that the set of non-isolated equilibria of this economy is open. In general, the subset of non-isolated points (limit points) of a metric space is not necessarily open, as can be easily seen in a simple counter-example[7], and the fact that in our metric space it is open indeed, will be important for establishing its category afterwards.

Proposition 2. *The subset of non-isolated selections of Γ is open and non-empty.*

Proof. The non-emptyness of the set of non-isolated selections of Γ is immediate; for instance, $f(x) = 1$, $\forall x < 1$ and $f(x) = -1$, $\forall x \geq 1$ is non-isolated. Thus, let f be a non-isolated selection of Γ, and $B_\varepsilon(f)$ a ball centred at f with radius $0 < \varepsilon < 1$. Let g be any selection in $B_\varepsilon(f) \setminus \{f\}$. Such a g exists since f is non-isolated. Then, according to the previous corollary, for any $\delta > 0$, either $\exists A \subset (-1, -1+\delta)$, A of non-null measure, such that $\forall x \in A$, $f(x) \in \{-1, x\}$, or $\exists B \subset (1-\delta, 1)$, B of non-null measure, such that $\forall x \in B$, $f(x) \in \{x, 1\}$. If $\delta < \varepsilon$ and there exists such an A, then any $x \in A$ would satisfy $-1-\varepsilon < x-\varepsilon < -1 < x < -1+\varepsilon < x+\varepsilon$, since $\delta < \varepsilon$ and $-1 < x < -1+\delta$. And similarly, if there exists such a B, then any $x \in B$ would satisfy $x - \varepsilon < 1-\varepsilon < x < 1 < x+\varepsilon < 1+\varepsilon$, since $\delta < \varepsilon$ and $1-\delta < x < 1$. That is to say, if $\delta < \varepsilon$ and there is such an A, then $\forall x \in A \subset (-1, -1+\delta)$, $\Gamma(x) \cap B_\varepsilon(f(x)) = \{-1, x\}$; and if, alternatively, there is such a B, then $\forall x \in B \subset (1-\delta, 1)$, $\Gamma(x) \cap B_\varepsilon(f(x)) = \{x, 1\}$ (recall: $\delta < \varepsilon < 1$ and hence $x < -1+\delta < -1+\varepsilon < 0$ and therefore $x+\varepsilon < 1$ if $x \in X_0$, or $0 < 1-\varepsilon < 1-\delta < x$ and therefore $-1 < x-\varepsilon$ if $x \in X_1$). Now, since for almost every x, $g(x)$ must be in $\Gamma(x) \cap B_\varepsilon(f(x))$, then it is necessarily verified that $\forall \delta \in (0, \varepsilon)$, if such an A exists, then $\exists A' \subset A \subset (-1, -1+\delta)$, A' of non-null measure, such that $\forall x \in A'$, $g(x) \in \{-1, x\}$; and if, alternatively, such a B exists, then $\exists B' \subset B \subset (1-\delta, 1)$, B' of non-null measure, such that $\forall x \in B'$, $g(x_1) \in \{x, 1\}$. According to the previous corollary of Proposition 1, this means that g is non-isolated. □

The fact that the set of non-isolated equilibria is open has a strong implication on the category of this set. This is what is stated in the following.

[7] Consider the metric space formed by the reals $\{0, 1, \frac{1}{2}, \frac{1}{3}, \ldots\}$ with the Euclidean metric. The subset of limit points does not coincide with its interior (the first is nonempty while the second is empty).

Proposition 3. *The subset of non-isolated equilibria and the subset of isolated equilibria of this economy are both of second category in the set of all equilibria.*

Proof. In effect, the set of equilibria of this economy is a closed subset of the set of essentially bounded real functions on $\mathbb{R}$, $\mathscr{L}_\infty$. This is a consequence of the fact that the correspondence is closed. But $\mathscr{L}_\infty$ is complete (see [4]) and since any closed subset of a complete metric space is itself complete, then the set of equilibria of this economy is a complete metric space. This guarantees, by the Baire Category Theorem, that it is a Baire space. (see [6]). Now, any open set of a Baire space is of second category. Therefore, the set of non-isolated equilibria of this economy is of second category. On the other hand, the set of isolated equilibria is a union of open sets (each isolated equilibrium being an open ball of sufficiently small radius) and thus it is an open itself[8]. This implies that it is of second category too. □

3 Conclusion

This note shows by means of a robust example that the set of non-isolated equilibria of an incomplete markets economy without assets and uncountably many states of the world may not be negligible in a cardinal or topological sense. The same can be expected to hold when assets are introduced as far as the structure remains incomplete. This result means a drawback in the expectations of finding some way to fix the indeterminacy problem when continua of equilibria appear.

Appendix

In order to define the utility function $u : \mathbb{R}^2_+ \longrightarrow \mathbb{R}$ of the agent in the southwest corner of the Edgeworth box consider its partition in Figure A1. Thus, let us define the utility function u as follows:

(1) if $(x, y) \in \mathbb{R}^2_+$ are such that

(13) $$y = 1/4x \quad \text{and} \quad 0 \leq x \leq 4/5 \ ,$$

or

(14) $$y = 4x - 3 \ \text{and} \ 4/5 < x \ ,$$

then $u(x, y) = x + y$;

(2) if $(x, y) \in \mathbb{R}^2_+$ are such that

[8] Notice that the fact that the set of equilibria can be partitioned in two non-empty open sets is a consequence of the fact that it is not connected.

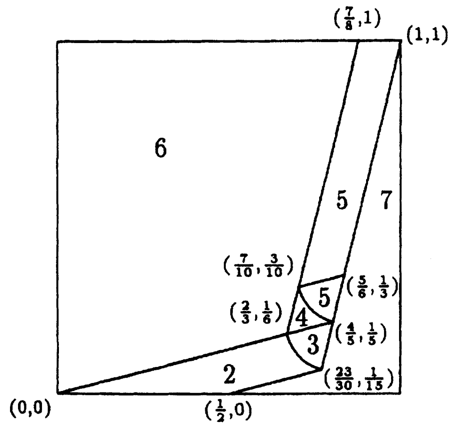

Figure A1

(15) $$\frac{1}{4}x - \frac{1}{8} \leq y < \frac{1}{4}x \ \text{ and } \ \sqrt{\left(x - \frac{4}{5}\right)^2 + \left(y - \frac{1}{5}\right)^2} > \frac{\sqrt{17}}{30} \ ,$$

and $(x', y') \in \mathbb{R}^2_+$ are (the only values) such that

(16) $$y' = \frac{1}{4}x', \ \sqrt{(x - x')^2 + (y - y')^2} = \frac{\sqrt{17}}{30} \ \text{ and } \ x < x', y < y' \ ,$$

then $u(x, y) = u(x'', y'')$, where x'', y'' are (the only values) such that

(17) $$y'' = \frac{1}{4}x'', \ \sqrt{(x'' - x')^2 + (y'' - y')^2} = \frac{\sqrt{17}}{30} \ \text{ and } \ x'' < x, y < y'' \ ;$$

(3) if $(x, y) \in \mathbb{R}^2_+$ are such that

(18) $$4x - 3 \leq y < \frac{1}{4}x \ \text{ and } \ \sqrt{\left(x - \frac{4}{5}\right)^2 + \left(y - \frac{1}{5}\right)^2} \leq \frac{\sqrt{17}}{30} \ ,$$

and $(x', y') \in \mathbb{R}^2_+$ are (the only values) such that

(19) $$y' = 4x' - 3, \ \sqrt{(x - x')^2 + (y - y')^2} = \frac{\sqrt{17}}{30} \ \text{ and } \ x < x', y < y' \ ,$$

then $u(x, y) = u(x'', y'')$, where x'', y'' are (the only values) such that

(20) $$y'' = \frac{1}{4}x'', \ \sqrt{(x'' - x')^2 + (y'' - y')^2} = \frac{\sqrt{17}}{30} \ \text{ and } \ x'' < x, y < y'' \ ;$$

(4) if $(x, y) \in \mathbb{R}^2_+$ are such that

(21) $$\frac{1}{4}x < y \leq 4x - \frac{5}{2} \ \text{ and } \ \sqrt{\left(x - \frac{4}{5}\right)^2 + \left(y - \frac{1}{5}\right)^2} \geq \frac{\sqrt{17}}{30} \ ,$$

and $(x', y') \in \mathbb{R}^2_+$ are (the only values) such that

$$(22)\quad y' = 4x' - 3,\ \sqrt{(x-x')^2 + (y-y')^2} = \frac{\sqrt{17}}{30} \quad \text{and} \quad x < x', y < y' \ ,$$

then $u(x,y) = u(x'',y'')$, where x'', y'' are (the only values) such that

$$(23)\quad y'' = \frac{1}{4}x'',\ \sqrt{(x''-x')^2 + (y''-y')^2} = \frac{\sqrt{17}}{30} \quad \text{and} \quad x < x'', y'' < y \ ;$$

(5) if $(x,y) \in \mathbb{R}^2_+$ are such that either

$$(24)\qquad 4x - 3 < y < \frac{1}{4}x + \frac{1}{8} \quad \text{and} \quad \sqrt{\left(x - \frac{5}{6}\right)^2 + \left(y - \frac{2}{6}\right)^2} < \frac{\sqrt{17}}{30} \ ,$$

or

$$(25)\qquad 1/4x + 1/8 \le y \quad \text{and} \quad 4x - 3 < y \le 4x + 5/2$$

and $(x',y') \in \mathbb{R}^2_+$ are (the only values) such that

$$(26)\quad y' = 4x' - 3,\ \sqrt{(x-x')^2 + (y-y')^2} = \frac{\sqrt{17}}{30} \quad \text{and} \quad < x', y < y' \ ,$$

then $u(x,y) = u(x'',y'')$, where x'', y'' are (the only values) such that

$$(27)\quad y'' = 4x'' - 3,\ \sqrt{(x''-x')^2 + (y''-y')^2} = \frac{\sqrt{17}}{30} \quad \text{and} \quad x < x'', y'' < y;$$

(6) if $(x,y) \in \mathbb{R}^2_+$ are such that

$$(28)\qquad 1/4x < y \quad \text{and} \quad 4x - 5/2 < y$$

and $(x',y') \in \mathbb{R}^2_+$ are (the only values) such that, either

$$(29)\quad y' = 1/4x' \quad \text{and} \quad x' \le 2/3, \quad \text{or} \quad y' = 4x' - 5/2 \quad \text{and} \quad x' > 2/3 \ ,$$

and

$$(30)\qquad 2x + y = 2x' + y' \ ,$$

then $u(x,y) = u(x',y')$;

(7) if $(x,y) \in \mathbb{R}^2_+$ are such that

$$(31)\qquad y < 1/4x - 1/8 \quad \text{or} \quad y < 4x - 3$$

and $(x',y') \in \mathbb{R}^2_+$ are (the only values) such that, either

$$(32)\quad y' = 1/4x' - 23/30 \quad \text{and} \quad x' \le 4/5, \ \text{or} \ y' = 4x' - 3 \quad \text{and} \quad x' > 23/30 \ ,$$

and

$$(33)\qquad 1/2x + y = 1/2x' + y' \ ,$$

then $u(x,y) = u(x',y')$.

Finally, the utility function of the agent in the northeast corner of the Edgeworth box is defined symmetrically with respect to the diagonal of $\mathbb{R}^2_+$.

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