Riemannian algorithms and estimation bounds for synchronization of rotations

We estimate unknown rotation matrices R_i in SO(n) from a set of measurements of relative rotations $R_i R_j^T$. Each measurement is either slightly noisy, or an outlier bearing no information. We study the case where most measurements are outliers. We propose a Maximum Likelihood Estimator (MLE) approach, explicitly acknowledging outliers in the noise model.

The MLE maximizes the log-likelihood function over the parameter space. That space is a product of rotation groups, possibly quotiented to account for invariance under a common rotation of the estimators.

To compute the MLE, we use Riemannian trust-region methods to maximize the log-likelihood function over the parameter space. That space is a matrix manifold, hence tools and analyses from (Absil et al., *Optimization Algorithms on Matrix Manifolds*, PUP, 2008) apply gracefully.

We derive Riemannian Cramer-Rao bounds for synchronization, valid for a broad class of problem dimensions and noise distributions. These bounds admit a simple expression in terms of an information-weighted Laplacian of the measurement graph. Numerical tests suggest the MLE is asymptotically efficient in many cases.

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