Price-quantity competition with varying toughness

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Abstract

In order to formalize the variety of oligopolistic competition regimes, we adopt an approach (pioneered by Shubik, 1959) where firms behave strategically both in price and quantity. The corresponding concept of oligopolistic equilibrium allows for a parameterized continuum of regimes with varying competitive toughness. In particular, the Cournot and the competitive outcomes coincide, respectively, with the softest and the toughest oligopolistic equilibrium outcome. The set of all equilibrium outcomes are compared with those obtained in three alternative approaches, respectively based on pricing-schemes, conjectural variations and supply functions. Finally, we explore the possibility of endogenizing strategically the choice of competitve toughness by the firms.

1 Introduction

A fundamental problem of oligopoly theory is equilibrium indeterminacy. This indeterminacy is not only of the kind associated with a given equilibrium concept and the possibility of multiple solutions. It is mainly related to the choice of the equilibrium concept itself. This is most simply demonstrated in static models by the choice between quantity competition (usually assimilated to the approach of Cournot, 1838) and price competition (linked to Bertrand's 1883 critique of Cournot). However, this second kind of indeterminacy only reflects the variety of observed competition regimes varying in their degree of toughness.

In order to formalize this variety of oligopoly regimes, we shall adopt an approach that was pioneered by Shubik (1959), where firms do not privilege one strategic variable but behave strategically in price-quantity pairs. This approach leads to a comprehensive concept of oligopolistic equilibrium providing a unified formulation of the whole spectrum of enforceable noncooperative equilibria. The proposed concept remains Cournotian and considers oligopolistic competition as a generalization of monopoly. Two types of competition are at stake: the struggle of the whole industry for market size, and the struggle of each individual firm for its market share. The first type of competition is represented in the spirit of Cournot by a constraint involving aggregate demand. The second type of competition, linked to a Bertrand-like price war, will however be regulated through a device that can be interpreted as a "meeting competition clause" introduced in the sales contracts (Salop, 1985) and then formalized as a second constraint. The set of solutions (corresponding to different competition regimes) can be parameterized according to the relative values of the Lagrange multipliers associated with these two constraints. This parameterization is interpreted in terms of competitive toughness. Thus, at one extreme, we find the Cournot solution as the softest oligopolistic equilibrium, and, at the other extreme, we find the competitive equilibrium when competitive toughness is maximal. As mentioned by Shubik (1959), the price equilibrium (corresponding to Bertrand competition) is a particular price-quantity equilibrium, which coincides with the competitive equilibrium when all firms are producing at equilibrium. All other enforceable outcomes are intermediate to these two extremes.

These outcomes will be compared with those obtained in three alternative approaches, respectively based on pricing-schemes, conjectural variations and supply functions.[1] As far as the first approach is concerned, the present concept of oligopolistic equilibrium simply coincides with the one of "min-pricing scheme equilibrium" which, however, does not offer a natural parameterization. Considering the other two approaches, they do offer such a parameterization, but the set of outcomes obtained both through conjectural variations and as supply function equilibria is larger than the set of oligopolistic equilibrium out-

[1] For the first approach, see d'Aspremont, Dos Santos Ferreira and Gérard-Varet (1991). For a synthetic presentation of the other two, see Vives (1999). Martin (2002) also refers to the "coefficient of cooperation" approach (Cyert and De Groot, 1973), inspired by the Edgeworth "coefficient of effective sympathy", and establishes its equivalence with the conjectural variations approach.

comes. The former set even includes the collusive solution.

In section 2 we define and characterize the concept of oligopolistic equilibrium for an industry producing a single homogeneous good. In section 3, we compare it with the alternative concepts. In section 4, we develop two simple examples and examine the possibility of endogenizing (strategically) the choice of competitive toughness by the firms. We briefly conclude in section 5.

2 Oligopolistic equilibrium: definition and characterization

We take an industry consisting in n firms ($n > 1$), each firm i producing the same good with a technology described by an increasing cost function C_i, which is continuously differentiable on $(0, \infty)$ and such that $C_i(0) = 0$. The demand D for the good is a function of market price P, with a finite negative continuous derivative over all the domain where it is positive.

In our approach, following Shubik (1959), neither the price nor the quantity will be privileged as a strategic variable. Each firm i chooses, simultaneously with its competitors, a pair (p_i, q_i) stating the quantity q_i it proposes to sell at price p_i. It is supposed to adopt a best-price policy, by introducing a *meeting competition clause* in its sales contracts, guaranteeing its customers that they are not paying more than elsewhere. It is also supposed to take into account the collective constraint that total supply cannot exceed total demand at the best price (equivalently, each firm is constrained by its residual demand). Formally, given the strategic choices $(p_{-i}, q_{-i}) \in \mathbb{R}_+^{2(n-1)}$ of others and referring to the min-pricing scheme $\underline{P}(p_i, p_{-i}) \equiv \min_j \{p_j\}$, each firm profit is given by

$$\Pi_i(p_i, p_{-i}, q_i, q_{-i}) = \underline{P}(p_i, p_{-i})\, q_i - C_i(q_i), \tag{1}$$

and submiitted to the constraint that the total quantity supplied $\overline{Q}(q_i, q_{-i}) \equiv \sum_j q_j$ be upper bounded by the aggregate demand $D(\underline{P}(p_i, p_{-i}))$ at the best price. More precisely, an *oligopolistic game* is defined by having each firm strategies in $\mathbb{R}_+^2$ and payoffs Π_i specified as follows:

$$\begin{aligned} \Pi_i(p, q) &= \underline{P}(p_i, p_{-i})\, q_i - C_i(q_i), \\ \text{if } \overline{Q}(q_i, q_{-i}) &\leq D(\underline{P}(p_i, p_{-i})), \\ \Pi_i(p, q) &= -C_i(q_i), \text{ otherwise.} \end{aligned} \tag{2}$$

An *oligopolistic equilibrium* is then a $2n$-tuple (p^*, q^*) which is a *Nash equilibrium* of the oligopolistic game and which satisfies the *additional restriction*

$$\overline{Q}(q^*) = D(\underline{P}(p^*)). \tag{3}$$

This last requirement is meant to exclude rationing of consumers at equilibrium, and accordingly to eliminate irrelevant equilibria.

This definition of an oligopolistic equlibrium coincides with the one of a $P^{\min}$-equilibrium as defined in d'Aspremont, Dos Santos Ferreira and Gérard-Varet (1991)[2] with $P^{\min} \equiv \underline{P}$.

In the following lemma we give an alternative formulation of the concept of oligopolistic equilibrium that will lead to a meaningful parameterization of the set of equilibria.

Lemma 1 *A $2n$-tuple (p^*, q^*) is an oligopolistic equilibrium (i.e. a $P^{\min}$-equilibrium) if and only if, for each firm i, (p_i^*, q_i^*) is solution to the following program:*

$$\max_{(p_i,q_i)\in\mathbb{R}_+^2} \left\{ p_i q_i - C_i(q_i) : p_i \le \min_{j\neq i}\left\{p_j^*\right\} \text{ and } p_i \le D^{-1}\left(\overline{Q}\left(q_i, q_{-i}^*\right)\right)\right\}. \quad (4)$$

Proof. To prove sufficiency suppose that, for each i, (p_i^*, q_i^*) is solution to (4), but that, for some i, and some $(p_i, q_i) \in \mathbb{R}_+^2$, $\underline{P}\left(p_i, p_{-i}^*\right) q_i - C_i(q_i) > \underline{P}(p^*)\, q_i^* - C_i(q_i^*) \ge 0$ (recall that $C_i(0) = 0$), while $\overline{Q}\left(q_i, q_{-i}^*\right) \le D\left(\underline{P}\left(p_i, p_{-i}^*\right)\right)$. Letting $p_i' = \underline{P}\left(p_i, p_{-i}^*\right)$, the pair (p_i', q_i) satisfies the two constraints of (4) and gives higher profit to firm i, which is a contradiction.

To prove necessity suppose (p^*, q^*) is an oligopolistic equilibrium, but that, for some i, and some $(p_i, q_i) \in \mathbb{R}_+^2$, $p_i q_i - C_i(q_i) > p_i^* q_i^* - C_i(q_i^*) \ge 0$, $p_i \le \min_{j\neq i}\left\{p_j^*\right\}$ and $p_i \le D^{-1}\left(\overline{Q}\left(q_i, q_{-i}^*\right)\right)$. Then $\underline{P}\left(p_i, p_{-i}^*\right) = p_i$, and so (p_i, q_i) is a profitable deviation for firm i, again a contradiction. ■

Notice that the equilibrium is restricted to have all prices positive so that at least one firm produces and the consumer is not rationed. However, when deriving first order conditions to the program (4) of firm i, we have to distinguish the case where, at the solution, firm i is active ($q_i^* > 0$) and the case where it is not[3] ($q_i^* = 0$). Introducing Kuhn and Tucker multipliers $(\lambda_i, \nu_i) \in \mathbb{R}_+^2 \setminus \{0\}$ associated with the first and second constraints in (4), respectively, general first order conditions require, by the nonnegativity of (p_i, q_i), that $q_i^* - \lambda_i - \nu_i \le 0$ and $(q_i^* - \lambda_i - \nu_i)\, p_i^* = 0$, as well as $p_i^* - C_i'(q_i^*) + \nu_i / D'(\underline{P}(p^*)) \le 0$ and $(p_i^* - C_i'(q_i^*) + \nu_i / D'(\underline{P}(p^*)))\, q_i^* = 0$. Therefore, if firm i is active, we get

$$\begin{bmatrix} q_i^* \\ p_i^* - C_i'(q_i^*) \end{bmatrix} = \lambda_i \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \nu_i \begin{bmatrix} 1 \\ -1/D'(\underline{P}(p^*)) \end{bmatrix}, \quad (5)$$

whereas, if firm i is inactive, the equality $(-\lambda_i - \nu_i)\, p_i^* = 0$ implies $\lambda_i = \nu_i = 0$.

The multiplier λ_i is associated with the min-pricing constraint and the multiplier ν_i with the market size constraint. They can be interpreted as the shadow costs firm i would accept to bear in order to ease the pressure coming from its

[2] The paper stresses the importance of the manipulabilty of the chosen pricing scheme P. Generally one assumes full range ($P\left(\mathbb{R}_+^n\right) = \mathbb{R}_+$) which only ensures collective manipulability. The min-pricing scheme $\underline{P}$ is individually manipulable downwards. As shown in the paper, if the pricing scheme P is also individualy manipulable upwards then the corresponding P-equilibrium reduces to the Cournot solution.

[3] Shubik (1959) suggests to call such a firm a "firm-in-being" (by analogy to the term "fleet in being").

competitors inside and outside the industry, respectively. For an active firm, the normalized parameter $\theta_i \equiv \lambda_i / (\lambda_i + \nu_i) \in [0, 1]$ can accordingly be interpreted as the (relative) *competitive toughness* of firm i within the industry. For an inactive firm i, we conventionally set $\theta_i = 1$.

We can express the degree of monopoly of each active firm i at equilibrium (p^*, q^*), with $p_i^* = \underline{P}(p^*)$ and $q_i^* > 0$, as a function of θ_i:

$$\frac{p_i^* - C_i'(q_i^*)}{p_i^*} = (1 - \theta_i) \frac{q_i^* / \overline{Q}(q^*)}{-\epsilon D(\underline{P}(p^*))} \equiv \mu_i(\theta_i, \underline{P}(p^*), q^*) . \tag{6}$$

Oligopolistic equilibria may hence be characterized by first order conditions (6), together with equality (3), giving a system of $n+1$ equations:

$$\mu_i(\theta_i, P^*, q^*) = \frac{P^* - C_i'(q_i^*)}{P^*} \; (i = 1, ..., n) \text{ and } \overline{Q}(q^*) = D(P^*) \tag{7}$$

in $n+1$ unknowns (P^*, q^*), parameterized by $\theta = (\theta_1, ..., \theta_n)$ in the set $\Theta^* \subset [0, 1]^n$ of the parameter values entailing a solution of the system (7) and ensuring sufficiency of the first order conditions. We thus obtain endogenously a parameterization by θ of the set of oligopolistic equilibria.

We shall not examine here sufficiency of first order conditions. Local curvature requirements should be somewhat softened by the presence of a kink in the boundary of the admissible set. In any case, sufficient global conditions are necessarily weaker than those ensuring existence of standard solutions, such as Cournot or competitive equilibria, since these solutions, as we shall now see, are enforceable as particular instances of oligopolistic equilibria. The following proposition, the first statement of which does not depend upon differentiability of the cost and demand functions, characterizes the set of potential oligopolistic equilibria: this set includes the Cournot solution at one extreme, when competitive toughness is minimal ($\theta \equiv 0$), the competitive equilibrium at the other extreme, when competitive toughness is maximal ($\theta \equiv 1$), and all outcomes corresponding to intermediate values of θ, including the outcome corresponding to Bertrand competition.

We let $\left(P^C, q^C\right)$ denote the Cournot outcome, with the quantity vector q^C satisfying:

$$q_i^C \in \arg \max_{q_i \in [0,\infty)} \left\{ D^{-1}\left(\overline{Q}\left(q_i, q_{-i}^C\right)\right) q_i - C_i(q_i) \right\}, \; i = 1, ..., n, \tag{8}$$

and $P^C = D^{-1}\left(\overline{Q}\left(q^C\right)\right)$. The competitive (Walrasian) outcome is denoted $\left(P^W, q^W\right)$ with

$$q_i^W \in \arg \max_{q_i \in [0,\infty)} \left\{ P^W q_i - C_i(q_i) \right\}, i = 1, ..., n, \tag{9}$$

and P^W such that $\overline{Q}\left(q^W\right) = D\left(P^W\right)$. The Bertrand outcome $\left(P^B, q^B\right)$ with

$P^B = \min_i \left\{p_i^B\right\}$ is characterized by

$$\begin{aligned} p_i^B &\in \arg\max_{p_i \in [0,\infty)} \left\{ \begin{array}{c} p_i \min\left\{q_i(p_i), d_i\left(p_i, p_{-i}^B\right)\right\} \\ -C_i\left(\min\left\{q_i(p_i), d_i\left(p_i, p_{-i}^B\right)\right\}\right) \end{array} \right\} &(10) \\ \text{with } q_i(p_i) &= \arg\max_{q_i \in [0,\infty)} \left\{p_i q_i - C_i(q_i)\right\} \\ d_i\left(p_i, p_{-i}^B\right) &= \frac{D\left(\min\left\{p_i, p_{-i}^B\right\}\right)}{\# \arg\min\left\{p_i, p_{-i}^B\right\}}, \text{ if } p_i = \min\left\{p_i, p_{-i}^B\right\}, \\ &= 0, \text{ otherwise.} &(11) \end{aligned}$$

Finally, the collusive (monopoly) outcome (P^m, q^m) corresponds to

$$(P^m, q^m) \in \arg\max_{(P,q) \in \mathbb{R}_+^{n+1}} \left\{ P \sum_i q_i - \sum_i C_i(q_i) : \sum_i q_i \leq D(P) \right\}. \tag{12}$$

We can now state the following

Proposition 2 *The outcomes $\left(P^C, q^C\right)$ and $\left(P^W, q^W\right)$ are oligopolistic equilibrium outcomes, as well as any outcome $\left(P^B, q^B\right)$ such that $P^B = D^{-1}\left(\overline{Q}\left(q^B\right)\right)$, but (P^m, q^m) is not (unless it is also a Cournot outcome).*[4] *Moreover, assuming that $\theta = (0, ..., 0)$ (resp. $\theta = (1, ..., 1)$) belongs to Θ^*, and that, for any i, $D^{-1}\left(q_i + \sum_{j \neq i} q_j\right) q_i - C_i(q_i)$ is quasi-concave (resp. $C_i(q_i)$ is convex) in q_i, any oligopolistic equilibrium corresponding to $\theta = (0, ..., 0)$ (resp. $\theta = (1, ..., 1)$) leads to a Cournot (resp. a competitive) outcome.*

Proof. If $\left(P^C, q^C\right)$ were not an oligopolistic equilibrium outcome, some i would be able, through some choice $(p_i, q_i) \in \mathbb{R}_+^2$, to get a profit $\min\left\{p_i, P^C\right\} q_i - C_i(q_i) > P^C q_i^C - C_i\left(q_i^C\right)$. But the constraint in (2) requires: $q_i + \sum_{j \neq i} q_j^C \leq D\left(\min\left\{p_i, P^C\right\}\right)$. Hence,

$$\begin{aligned} D^{-1}\left(q_i + \sum_{j \neq i} q_j^C\right) q_i - C_i(q_i) &\geq \\ \min\left\{p_i, P^C\right\} q_i - C_i(q_i) &> D^{-1}\left(\sum_i q_i^C\right) q_i^C - C_i\left(q_i^C\right), \end{aligned}$$

contradicting the assumption that q^C is a Cournot equilibrium. Similarly, if (P^r, q^r) were not an oligopolistic equilibrium outcome (with $r = W, B$ and

[4] Coincidence of Cournot and collusive outcomes, impossible under differentiability of the cost and demand functions, cannot otherwise be excluded. If, for instance, demand is non-differentiable, one may have (under cost symmetry) the first order conditions:

$$\begin{aligned} 1/n\left[-\epsilon^- D(P^*)\right] < 1/\left[-\epsilon^- D(P^*)\right] &\leq C'\left(D(P^*)/n\right) \\ &\leq 1/n\left[-\epsilon^+ D(P^*)\right] < 1/\left[-\epsilon^+ D(P^*)\right] \end{aligned}$$

with $P^* = P^C = P^m$, and with $\epsilon^- D(P^*)$ and $\epsilon^+ D(P^*)$ the left hand and right hand demand elasticities at P^*, respectively.

$P^B = D^{-1}\left(\overline{Q}\left(q^B\right)\right)$), some firm i would be able, through some choice $(p_i, q_i) \in \mathbb{R}_+^2$, such that $q_i + \sum_{j \neq i} q_j^r \leq D\left(\min\left\{p_i, P^r\right\}\right)$ to get a profit

$$\begin{aligned} \min\left\{p_i, P^r\right\} q_i - C_i\left(q_i\right) &> P^r q_i^r - C_i\left(q_i^r\right), \text{ so that} \\ P^r q_i - C_i\left(q_i\right) &> P^r q_i^r - C_i\left(q_i^r\right). \end{aligned}$$

If $r = W$, we contradict the assumption that $\left(P^W, q^W\right)$ is a competitive outcome. If $r = B$, the same argument holds if $\min\left\{p_i, P^B\right\} = P^B$. Otherwise, with $p_i < P^B$, a deviation would a fortiori be possible in the Bertrand game (where i would be constrained by the aggregate instead of the residual demand). Finally, if (P^m, q^m) is a collusive but not a Cournot outcome, it must be such that for some i, some $q_i \in \mathbb{R}_+$ and $P = D^{-1}\left(q_i + \sum_{j \neq i} q_j^m\right)$,

$$\begin{aligned} & P q_i - C_i\left(q_i\right) + P \sum_{j \neq i} q_j^m - \sum_{j \neq i} C_j\left(q_j^m\right) \\ \leq\ & P^m \sum_j q_j^m - \sum_j C_j\left(q_j^m\right) < P q_i - C_i\left(q_i\right) + P^m \sum_{j \neq i} q_j^m - \sum_{j \neq i} C_j\left(q_j^m\right) \end{aligned}$$

implying $P < P^m$. Therefore, (P, q_i) is an admissible deviation for firm i in the oligopoly game, which entails profit $P q_i - C_i\left(q_i\right)$, so that (P^m, q^m) cannot be an oligopolistic equilibrium outcome.

To prove the last statement, recall that in the case of Cournot equilibrium the first order conditions (which are sufficient by assumption) are:

$$\frac{p_i^* - C_i'\left(q_i^*\right)}{p_i^*} = \frac{q_i^* / \overline{Q}\left(q^*\right)}{-\epsilon D\left(\underline{P}\left(p^*\right)\right)} \equiv \mu_i\left(0, \underline{P}\left(p^*\right), q^*\right) \text{ for all } i.$$

Thus, by (6), the first order conditions for an oligopolistic equilibrium with $\theta = (0, ..., 0) \in \Theta^*$ correspond to the first order conditions for a Cournot equilibrium. With $\theta = (1, ..., 1) \in \Theta^*$, we see from (6) that each firm equalizes marginal cost to price, so that we get the condition (assumed to be sufficient) for a competitive equilibrium. The result follows. ■

To conclude, an oligopolistic equilibrium with minimal admissible competitive toughness ($\theta_i = 0$) for any firm i corresponds to a Cournot equilibrium (when it exists). At the other extreme, of maximal competitive toughness for all firms, an oligopolistic equilibrium corresponds to a competitive equilibrium (when it exists). The latter remark applies to the Bertrand solution under the assumption of identical linear cost functions ($C_i\left(q_i\right) = c q_i$ for any i), since in that case it coincides with the competitive outcome.

3 Relationship with conjectural variations and supply function equilibria

We have already mentioned that the definition of an oligopolistic equlibrium coincides with the one of a $P^{\min}$-equilibrium. The added value of the present approach is to deliver a natural and interpretable parameterization. We now want to establish a connection between the parameterized notion of oligopolistic equilibrium and other parameterized concepts. The first, in chronological order, is the well-known *conjectural variations* approach (Bowley, 1924): each firm i when choosing its quantity is supposed to make a specified type of conjecture concerning the reaction of the other firms to any of its deviations. But these conjectures are introduced directly into the first order conditions. They are not part of the description of the oligopoly game.

Following the presentation in Dixit (1986), a sufficient specification[5] consists in introducing conjectural variation parameters $r_i = \sum_{j \neq i} \partial q_j / \partial q_i$ for each i. The corresponding first order conditions are:

$$\frac{P^* - C_i'(q_i^*)}{P^*} = (1 + r_i) \frac{q_i^* / \overline{Q}(q^*)}{-\epsilon D(P^*)}, \tag{13}$$

giving the same characterization as (6) with $r_i = -\theta_i$. In other words, comparing first order conditions, the set of oligopolistic equilibrium outcomes appears as the selected subset of outcomes obtained by conjectural variations restricted to be compensating (non-collusive), i.e. r_i to be in the interval[6] $[-1, 0]$, for every i. The concept of oligopolistic equilibrium thus provides some game-theoretic foundation to the concept of conjectural variations, since the conjectural variation terms (within the relevant class) can be identified to the parameterization of the equilibria of a fully specified game.

Another well-known, but more recent, approach assumes that firms strategies are *supply functions* (Grossman, 1981 and Hart, 1982).[7] A supply function equilibrium is a Nash equilibrium of a game where each firm i strategies are functions S_i associating with every price p_i in $[0, \infty)$ a quantity $q_i = S_i(p_i)$. The functions S_i may be restricted to some admissible set $\mathbf{S}$. For any n-tuple S of supply functions in $\mathbf{S}^n$, the price $\hat{P}(S)$ is said to be well-defined if it is nonnegative and uniquely solves the equation

$$\sum_{j=1}^{n} S_j(P) = D(P). \tag{14}$$

The corresponding payoffs are defined by:

$$\begin{aligned} \Pi_i^{\mathbf{s}}(S_i, S_{-i}) &= \hat{P}(S_i, S_{-i}) S_i\left(\hat{P}(S_i, S_{-i})\right) - C_i\left(S_i\left(\hat{P}(S_i, S_{-i})\right)\right), \quad (15) \\ &\quad \text{if } \hat{P} \text{ is well-defined} \\ \Pi_i^{\mathbf{s}}(S_i, S_{-i}) &= 0, \text{ otherwise.} \end{aligned}$$

Observe that at an equilibrium S^*, for any firm i, maximizing $\Pi_i^{\mathbf{s}}\left(S_i, S_{-i}^*\right)$ amounts to select P^* in

$$\arg\max_{P \in \mathbb{R}_+} \left\{ P D_i^*\left(P, S_{-i}^*\right) - C_i\left(D_i^*\left(P, S_{-i}^*\right)\right) \right\}, \tag{16}$$

[5] Dixit considers the more general case where r_i is a function of both q_i and $\sum_{j \neq i} q_j$.

[6] Matching variations ($r_i > 0$) are excluded, and in particular those leading to the collusive solution.

[7] See also Singh and Vives (1984).

with $D_i^*\left(P, S_{-i}^*\right) = \max\left\{D(P) - \sum_{j\neq i} S_j^*(P), 0\right\}$ the residual demand function. Indeed, firm i could as well choose any supply function S_i for which $S_i(P) = D_i^*\left(P, S_{-i}^*\right)$ has the unique solution P^*.

The multiplicity of supply function equilibria is well-known. In order to compare this concept with our own, we shall restrict strategies to the set $\mathbf{S}_+$ of *non-decreasing supply functions*.[8] We then have the following characterization:

Proposition 3 *For any supply function equilibrium $S^* \in \mathbf{S}_+^n$, there is an oligopolistic equilibrium (p^*, q^*) such that $q^* = S^*\left(\hat{P}(S^*)\right)$ and, for any j, $p_j^* = \hat{P}(S^*)$. Conversely, for any oligopolistic equilibrium (p^*, q^*), there is a supply function equilibrium $S^* \in \mathbf{S}_+^n$ such that $S^*(\underline{P}(p^*)) = q^*$.*

Proof. Let $S^* \in \mathbf{S}_+^n$ be a supply function equilibrium. We show that (p^*, q^*) with $q^* = S^*\left(\hat{P}(S^*)\right)$ and, for any j, $p_j^* = \hat{P}(S^*) = \underline{P}(p^*)$, is an oligopolistic equilibrium. Using, for any i, the fact that the residual demand $D_i^*\left(P, S_{-i}^*\right)$ is decreasing in P and that the profit $p_i q_i - C_i(q_i)$ is increasing in p_i we get, by (16), that (p_i^*, q_i^*) maximizes $p_i q_i - C_i(q_i)$ on

$$\Delta_i \equiv \left\{(p_i, q_i) \in \mathbb{R}_+^2 \mid q_i \leq D_i^*\left(p_i, S_{-i}^*\right)\right\}.$$

For (p^*, q^*) to be an oligopolistic equilibrium, (p_i^*, q_i^*) should maximize $p_i q_i - C_i(q_i)$ on

$$\hat{\Delta}_i \equiv \left\{(p_i, q_i) \in \mathbb{R}_+^2 \mid p_i \leq \underline{P}(p^*), q_i \leq \max\left\{D(p_i) - \sum_{j\neq i} q_j^*, 0\right\}\right\},$$

for every i. Since, by construction, $(p^*, q^*) \in \hat{\Delta}_i$ and $\hat{\Delta}_i \subset \Delta_i$, this is clearly true.

To prove the converse, let us suppose that (p^*, q^*) is an oligopolistic equilibrium. We may construct an associated supply function equilibrium by using simple supply functions, namely by letting, for every i, $S_i^*(P) = q_i^*$ if $P \leq p_i^*$, and $S_i^*(P) = \infty$ otherwise. Clearly, the solution to (16) cannot be larger than $\underline{P}(p^*)$, hence any profitable deviation by some firm i from S_i^* must involve a price below $\underline{P}(p^*)$ and thus constitute a deviation with respect to the oligopolistic equilibrium. The result follows. ■

We see that, with the restrictions imposed on the admissible class of supply functions, the outcomes corresponding to the two sets of equilibrium outcomes coincide, including the Cournot and the competitive outcomes.

To establish more clearly the relation between the two concepts, we may consider the first order condition to firm i program (16) at equilibrium, while restricting to differentiable supply functions in $\mathbf{S}_+$. Denoting $q_i^* = D_i^*\left(P^*, S_{-i}^*\right)$ and $\overline{Q}(q^*) = \sum_{i=1}^n q_i^*$, we get:

$$q_i^* + \left(D'(P^*) - \sum_{j\neq i} S_j^{*\prime}(P^*)\right)(P^* - C_i'(q_i^*)) = 0, \tag{17}$$

[8] As Delgado and Moreno (2000) do,. However they assume in addition that firms are identical.

which is equivalent to

$$\frac{P^* - C_i'(q_i^*)}{P^*} = \frac{q_i^*/\overline{Q}(q^*)}{\sum_{j\neq i}\left(q_j^*/\overline{Q}(q^*)\right)\epsilon S_j^*(P^*) + [-\epsilon D(P^*)]} = \mu_i(\theta_i, P^*, q^*), \tag{18}$$

with μ_i as defined in (6), taking

$$\theta_i = 1 - \frac{[-\epsilon D(P^*)]}{\sum_{j\neq i}\left(q_j^*/\overline{Q}(q^*)\right)\epsilon S_j^*(P^*) + [-\epsilon D(P^*)]}. \tag{19}$$

Not surprisingly, the Cournot solution corresponds to an elasticity $\epsilon S_j^*(P^*)$ of the supply functions equal to 0, for all j, and the competitive solution to $\epsilon S_j^*(P^*) = \infty$, for at least two j's. Notice also that varying the elasticities of the supply functions in the relevant class allows us to cover the whole range of admissible values of the θ_i. The competitive toughness of firm i, as measured by θ_i at the oligopolistic equilibrium, is seen to correspond positively to a measure of the "reactivity of the other firms" (with respect to prices) as anticipated by firm i at the supply function equilibrium. Although the elasticity of a supply function chosen by any firm is indifferent from the point of view of the firm itself (for which only the price-quantity pair actually implemented matters), it is crucial in shaping the anticipations of its competitors.

4 The strategic choice of competitive toughness: two standard cases

In order to illustrate the concept of oligopolistic equilibrium, and visualize the set of equilibrium outcomes as parameterized by $\theta \in \Theta^*$, we refer to two standard cases of homogeneous duopoly, the cases of linear and isoelastic demand with linear cost functions. Since technological lead is an important feature in many applied models (for example those analysing the relationship between innovation and competition), we will admit asymmetric costs and assume that each firm i has a constant marginal cost c_i (by convention, $c_1 \leq c_2$).

4.1 Linear demand

In the first example, we suppose demand $D(P)$ to be linear, equal to $a - P$ (with $a > c_2$), so that Marshallian elasticity is $P/D(P)$. The case where only the efficient firm is active at equilibrium is simple. The set of equilibria is then described by all prices between the competitive price $P^W = c_1$ and the Bertrand price $P^B = c_2$. When both firms are active, from condition (6), equilibrium (p^*, q^*) with $p_1^* = p_2^* = P^*$ is seen to verify for each firm i

$$\frac{P^* - c_i}{P^*} = (1-\theta_i)\frac{q_i^*}{q_1^* + q_2^*}\frac{D(P^*)}{P^*} = (1-\theta_i)\frac{q_i^*}{P^*}, \tag{20}$$

giving a market share for firm 1

$$\frac{q_1^*}{q_1^* + q_2^*} = \frac{1}{1-\theta_1}\frac{P^* - c_1}{a - P^*} = 1 - \frac{1}{1-\theta_2}\frac{P^* - c_2}{a - P^*}, \tag{21}$$

with

$$P^* = \frac{(1-\theta_1)\, c_2 + (1-\theta_2)\, c_1 + (1-\theta_1)(1-\theta_2)\, a}{(1-\theta_1) + (1-\theta_2) + (1-\theta_1)(1-\theta_2)}. \tag{22}$$

The equilibrium price P^* is lower bounded by the Bertrand price $P^B = c_2$, and upper bounded by the Cournot price $P^C = (a + c_1 + c_2)/3$. Also, these expressions are valid only for values of θ in $[0, 1 - (c_2 - c_1)/(a - c_2)] \times [0, 1]$. Indeed, for values of θ_1 in $[1 - (c_2 - c_1)/(a - c_2), 1]$, firm 2 becomes inactive and we are back to the first case.

To illustrate, we can take the values for the costs $c_1 = 2/3$ and $c_2 = 1$ and, choosing $a = 10/3$, so that $P^* \in [1, 5/3]$, we obtain the following representation of the set of equilibrium outcomes with the two firms active, in the market price-market share space $(P^*, q_1^*/(q_1^* + q_2^*))$. This set is given by the region bounded by the vertical axis and the two thick curves, the lower one corresponding to $\theta_1 = 0$, the upper one to $\theta_2 = 0$. The increasing solid curves (including the thick one) are three elements of the family of curves stemming from firm 1 first order condition (with $\theta_1 = 1/3$, $1/6$, 0, respectively, from left to right). Similarly, the decreasing solid curves are three elements, corresponding to the same values of θ_2, of the family of curves stemming from firm 2 first order condition.

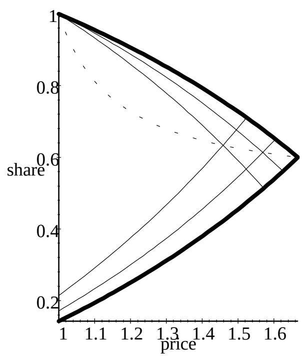

Figure 1

The dotted curve in Figure 1 corresponds to the uniform case $\theta_1 = \theta_2 = \bar{\theta}$. It links the Cournot outcome (determined at the point of intersection of the two thick curves, with $\bar{\theta} = 0$) to the Bertrand outcome (associated with the highest possible market share for the efficient firm and the highest value of $\bar{\theta}$, c_1/c_2). We observe that, as $\bar{\theta}$ increases, the market expands and splits more and more asymmetrically in favor of the efficient firm.

4.2 Isoelastic demand

The demand function is now taken to be $D(P) = P^{-\varepsilon}$, with $\varepsilon > 1/2$. From condition (6) we see that, at equilibrium (p^*, q^*) with $p_1^* = p_2^* = P^*$, the degree of monopoly of firm i is

$$\frac{P^* - c_i}{P^*} = \frac{1-\theta_i}{\varepsilon} \frac{q_i^*}{q_1^* + q_2^*}, \tag{23}$$

giving the market share for firm 1

$$\frac{q_1^*}{q_1^* + q_2^*} = \frac{\varepsilon}{1-\theta_1} \frac{P^* - c_1}{P^*} = 1 - \frac{\varepsilon}{1-\theta_2} \frac{P^* - c_2}{P^*}, \tag{24}$$

with

$$P^* = \frac{(1-\theta_1)\, c_2 + (1-\theta_2)\, c_1}{(1-\theta_1) + (1-\theta_2) - (1-\theta_1)(1-\theta_2)/\varepsilon}, \tag{25}$$

The equilibrium price P^* is lower bounded by the Bertrand price $P^B = c_2$, and upper bounded by the Cournot price $P^C = (c_1 + c_2)/(2 - 1/\varepsilon)$. It is decreasing in each θ_i. These expressions are valid only for values of θ in $[0, 1 - \varepsilon(1 - c_1/c_2)] \times [0, 1]$. For values of θ_1 in $[1 - \varepsilon(1 - c_1/c_2), 1]$, firm 2 becomes inactive.

Figure 2 shows, for parameter values $\varepsilon = 1$, $c_1 = 2/3$ and $c_2 = 1$, the set of equilibrium outcomes with both firms active, in the market price-market share space $(P^*, q_1^*/(q_1^* + q_2^*))$ as the region bounded by the vertical axis and the two thick curves. The solid curves are associated with constant values of competitive toughness, corresponding to the same values of θ_i as above: 1/3, 1/6, 0, respectively, from left to right. These curves are increasing for firm 1 and decreasing for firm 2. The dotted curve is associated with varying equal degrees of competitive toughnes for both firms. Figure 2 is strikingly similar to Figure 1, and leads to the same comments.

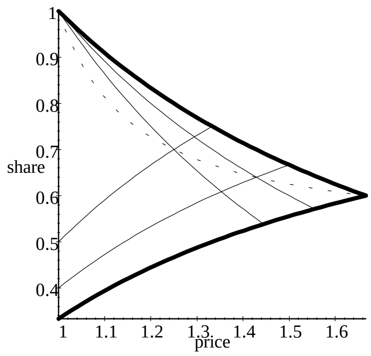

Figure 2

Some significant properties concerning the effects of competitive toughness on profits can be deduced from both examples. In the uniform case ($\theta_1 = \theta_2 = \bar{\theta}$):[9]

- the inefficient firm loses as competitive toughness increases (both its market share and the price decrease);
- if the cost advantage of the efficient firm is large enough, its profit increases as competitive toughness increases (a higher market share more than compensates a lower price);
- if costs are equal for both firms (implying equal market shares), profits decrease as competitive toughness increases.

In the case where competitive toughness is asymmetric, effects are more intricate. If the value of θ_i is kept unchanged and θ_j, $j \neq i$, increases, the price as well as firm i market share fall, implying a lower profit for firm i. However, an increase in the value of θ_i while keeping θ_j, $j \neq i$, constant, is associated with a larger market share for firm i but with a lower price, so that an increase in its profit is not guaranteed.

4.3 Choosing θ strategically

An important feature of the parameterization that we have introduced is that it allows for different levels of competitive toughness for different firms. In other words, competitive toughness is an individual characteristic of firm behavior, rather than a characteristic of the industry, as it would result from restricting the analysis to the uniform case. A natural question is to consider the situation where firms would choose their level of competitive toughness strategically. We shall not investigate extensively this question here, but we may at least show what answer we get in the two previous examples.

With linear costs, the profit of firm i is equal to the product of the degree of monopoly, the market share and the aggregate expenditure level. Hence, when increasing its competitive toughness, a firm increases its market share and may or may not increase the aggregate expenditure, but anyway must endure a loss in its degree of monopoly. In the linear demand case, using (20) and (21), the equilibrium profit of firm i combines these three components as follows:

$$\Pi_i \left(P^*, P^*, q_1^*, q_2^*\right) = (1 - \theta_i) \left(\frac{q_i^*}{q_1^* + q_2^*}\right)^2 (a - P^*)^2 = \frac{(P^* - c_i)^2}{1 - \theta_i}. \qquad (26)$$

The equilibrium profit of firm i is thus an increasing function of θ_i directly, and a decreasing function of θ_i through the price P^*. From (22), it can be computed

[9] These properties correspond to properties (b), (c) and (d) in Definition 2.2 of Boone (2001), where the "intensity of competition" parameter is supposed to be the same for all firms. Cournot and Bertrand regimes, but not the intermediate cases, are considered in an example. In Boone (2000), the parameter measuring "competitive pressure" is differentiated but his examples concentrate on comparative statics when the fundamentals vary.

to be

$$\Pi_i^*(\theta) = (1-\theta_i)\left(\frac{(c_j - c_i) + (1-\theta_j)(a - c_i)}{(1-\theta_i) + (1-\theta_j) + (1-\theta_i)(1-\theta_j)}\right)^2. \tag{27}$$

In order to determine the best reply of firm i in θ_i, it is useful to derive the sign of the partial derivative of $\Pi_i^*(\theta_i, \theta_j)$ with respect to θ_i:

$$\text{sign}\{\partial_i \Pi_i^*(\theta)\} = \text{sign}\{1 - (2-\theta_j)\theta_i\}. \tag{28}$$

From this formula, the quasi-concavity of $\Pi_i^*(\theta_i, \theta_j)$ in θ_i is straightforwardly established since the sign can only switch from positive to negative. If the two firms have equal unit costs, then at equilibrium the sign given by (28) is equal to zero for both firms, which is obtained for $\theta_i = \theta_j = 1$. Otherwise, the efficient firm 1 will choose the lowest competitive toughness that eliminates its rival that is $\theta_1 = 1 - (c_2 - c_1)/(a - c_2)$ entailing the equiibrium price $P^* = c_2$. Therefore the only oligopolistic equilibrium outcome in the linear demand case is the Bertrand outcome.

In the isoelastic demand case, combining the degree of monopoly (23), the market share (24) and the aggregate expenditure $P^{*1-\varepsilon}$ gives the following expression for the profit function at equilibrium:

$$\Pi_i(P^*, P^*, q_1^*, q_2^*) = \frac{1-\theta_i}{\varepsilon}\left(\frac{q_i^*}{q_1^* + q_2^*}\right)^2 P^{*1-\varepsilon} = \frac{\varepsilon}{1-\theta_i}\left(\frac{P^* - c_i}{P^*}\right)^2 P^{*1-\varepsilon}. \tag{29}$$

From (25), the equilibrium profit of firm i can be written as a function of θ:

$$\begin{aligned}\Pi_i^*(\theta) &= \varepsilon(1-\theta_i)\left(\frac{c_j - c_i + (1-\theta_j)c_i/\varepsilon}{(1-\theta_i)c_j + (1-\theta_j)c_i}\right)^2 \\ &\quad \times \left(\frac{(1-\theta_i)c_j + (1-\theta_j)c_i}{(1-\theta_i) + (1-\theta_j) - (1-\theta_i)(1-\theta_j)/\varepsilon}\right)^{1-\varepsilon}.\end{aligned}$$

As above, in order to determine the best reply of firm i in θ_i, it is useful to derive the sign of the partial derivative of $\Pi_i^*(\theta_i, \theta_j)$ with respect to θ_i, sign$\{\partial_i \Pi_i^*(\theta)\}$, is equal to the sign of the following expression

$$\begin{aligned}&(\varepsilon - 1)[c_j - (1 - (1-\theta_j)/\varepsilon)c_i](1-\theta_i)(1-\theta_j) + \\ &[(1-\theta_i) + (1-\theta_j) - (1-\theta_i)(1-\theta_j)/\varepsilon][(1-\theta_i)c_j - (1-\theta_j)c_i].\end{aligned} \tag{30}$$

If $\varepsilon = 1$, an equilibrium, with both firms active clearly exists for any $\theta \in [0, c_1/c_2) \times [0,1]$ such that $(1-\theta_1)/(1-\theta_2) = c_1/c_2$, requiring that $c_1/c_2 > 1/2$. If $c_1/c_2 \leq 1/2$, then we get the Bertrand equilibrium with only the efficient firm active, choosing $\theta_1 = c_1/c_2$ (with, by convention, $\theta_2 = 1$). If $\varepsilon \neq 1$, from the two first order conditions for an interior maximum $\partial_1 \Pi_1^*(\theta) = \partial_2 \Pi_2^*(\theta) = 0$, we get

$$c_j - (1 - (1-\theta_j)/\varepsilon)c_i = -c_i + (1 - (1-\theta_i)/\varepsilon)c_j,$$

leading to

$$(1-\theta_j)\, c_i = -\,(1-\theta_i)\, c_j,$$

which is impossible for interior values of θ. Therefore, we may again consider the Bertrand equilibrium with $\theta_1 = 1-\varepsilon\,(1-c_1/c_2)$ and $\theta_2 = 1$, and verify that

$$\text{sign}\,\{\partial_1 \Pi_1^*\,(1-\varepsilon\,(1-c_1/c_2)\,,1)\} = \text{sign}\left\{\varepsilon^2\,(1-c_1/c_2)^2\, c_2\right\} = 1.$$

So $\theta^* = (1-\varepsilon\,(1-c_1/c_2)\,,1)$ together with the oligopolistic equilibrium corresponding to Bertrand (with only one firm active when $c_1 < c_2$) is always a subgame perfect equilibrium of the two-stage game. However, as we now show, we may in addition obtain the Cournot outcome as a subgame perfect equilibrium (with $\theta^* = (0,0)$) for all values of ε in $(1/2,1]$. We need to verify that $\text{sign}\{\partial_i \Pi_i^*\,(0,0)\} \leq 0$, for $i = 1,2$. This is ensured by the condition $c_i/c_j \geq (1+\varepsilon-1/\varepsilon)\,/\varepsilon$, for $i,j = 1,2$, $i \neq j$, and $\varepsilon \in (1/2,1)$. This condition is satsfied for any cost configuration if $1+\varepsilon-1/\varepsilon \leq 0$ (*i.e.* $1/2 < \varepsilon \leq \left(\sqrt{5}-1\right)/2 = .618$). Otherwise, the condition forbids a too large efficiency gap between the two firms, and the more so the closer ε becomes to one. The existence of a subgame perfect equilibrium leading to the Cournot outcome in the complementary case ($\varepsilon < 1$) is easily explained by the fact that, in that case, among the three terms in the firm payoff – the market share, the degree of monopoly and the aggregate expenditure level – the last two are now decreasing in the firm competitive toughness.

5 Conclusion

The concept of oligopolistic equilibrium proposed here provides a parameterization of regimes with varying competitive toughness that goes well beyond the simple Cournot-Bertrand dichotomy. It offers an alternative to the conjectural variations approach, with better foundations, and to the supply function equilibrium approach, using simpler strategies (price-quantity pairs). Moreover, it leads to a natural narrowing down of the plethoric set of outcomes obtained in both these approaches and can be readily extended to the composite good case.[10]

To formalize, in a unified and synthetic form, the parameterized set of competition regimes that are enforceable as non-cooperative equilibria is also meant to provide a convenient tool to analyze some relevant issues for competition and economic policy. A first attempt to use this concept and the associated parameterization for that purpose is proposed in a companion paper studying the relationship between competitive toughness and the incentives to R&D investment and growth.[11]

Of course, many issues remain to be treated. The existence problem would deserve more development, in particular to explore the role of the kink in the

[10] See d'Aspremont, Dos Santos Ferreira and Gérard-Varet (2003).

[11] d'Aspremont, Dos Santos Ferreira and Gérard-Varet (2004).

producer's feasibility frontier in relieving curvature conditions. However, for existence, there is a change in focus, since now the main question is the determination of the set of enforceable competition regimes. A complementary and vast issue remains of course the one of equilibrium selection, of which the strategic choice of competitive toughness by the firms in a preliminary stage (as briefly sketched out in the last section) is only one instance.

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