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Procedia Engineering 66 (2013) 598 – 607

Procedia Engineering

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5th Fatigue Design Conference, Fatigue Design 2013

Two-scale time homogenization for isotropic viscoelastic-viscoplastic homogeneous solids under large numbers of cycles

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Abstract

A two-scale time homogenization approach for coupled viscoelastic-viscoplatic (VE-VP) homogeneous solids and structures subjected to large numbers of cycles, is proposed. The main aim is to give a description of the long time behaviour, by calculating the evolution of internal variables within the structure, while reducing the computational overhead. This method consists in decomposing the original VE-VP initial-boundary problem into coupled micro-chronological (fast time scale) and macro-chronological (slow time-scale) problems. The proposed methodology was implemented and studied for J_2 VP coupled with VE using fully implicit time integration and a return-mapping algorithm. An illustration of the time homogenization on a simple case is presented and a good agreement with the reference solution is observed.

Keywords: Time homogenization, two-scale modeling, viscoelasticity (VE), viscoplasticity (VP), numerical algorithm.

1. Introduction

Fatigue life prediction poses a great challenge to mechanics and materials science communities. Actually, fatigue of structures is a multiscale phenomenon in space (due to the presence of heterogeneities in the microstructure of the

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doi:10.1016/j.proeng.2013.12.111

material) and time (because the load period could be in the order of seconds whereas the component life may span years). Fatigue experiments are generally limited to small structural components, therefore the prediction of long-term behaviour of these structures involves some sort of modeling, and it requires significant computational resources, in particular, for nonlinear solids subjected to high cycle fatigue. In this work we will consider only the computation of the mechanical response of homogeneous materials subjected to large numbers of cycles. The proposed methodology is called two-scale time homogenization, which is an extension of the asymptotic spatial homogenization.

2. Coupled viscoelastic-viscoplastic (VE-VP) constitutive model

The observed behaviour of many materials like polymers is generally time dependent (Zhang and Moore, [3]); that is the stress response always depends on the rate of loading. This section presents the multidimensional theory of a viscoelastic-viscoplastic model within the context of von Mises plasticity under isothermal conditions.
The constitutive model is based on the assumption that the total strain is subdivided into viscoelastic (VE) and viscoplastic (VP) portions.

$$\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}^{ve} + \boldsymbol{\varepsilon}^{vp}, \tag{1}$$

where ε is the total infinitesimal strain tensor, $\boldsymbol{\varepsilon}^{ve}$ and $\boldsymbol{\varepsilon}^{vp}$ are its VE and VP parts, respectively.
Concerning the VE component, linear VE constitutive relations are adapted as (Christensen, [9]):

$$\boldsymbol{\sigma}(t) = \int_{-\infty}^{t} \mathbf{E}(t-\xi) : \frac{\partial \boldsymbol{\varepsilon}^{ve}}{\partial \xi} d\xi, \tag{2}$$

where $\boldsymbol{\sigma}$ is the Cauchy stress tensor, **E** the fourth-rank relaxation tensor and ξ is the integral argument.

For the VP part, a $\mathbf{J_2}$ constitutive model (Lemaitre and Chaboche, [6]) is used for the simulations presented in this work. It considers an isotropic, pressure-independent, VP response. In this case the yield function f ($\boldsymbol{\sigma}$, R) is defined by:

$$f(\boldsymbol{\sigma}, R) \equiv \sigma_{eq} - \sigma_y(\dot{\boldsymbol{\varepsilon}}) - R(p), \tag{3}$$

where σ_y is the initial yield stress, which may depend on the strain rate, and σ_{eq} the von Mises equivalent stress defined by:

$$\sigma_{eq} \equiv \left(\frac{3}{2} \mathbf{s}:\mathbf{s} \right)^{\frac{1}{2}}. \tag{4}$$

Here, s is the deviatoric part of the stress tensor.
The only hardening function R(p) considered in the subsequent simulations is a power-law defined as:

$$R(p) = kp^n, \text{ if } p > 0; \; R(p) = 0, \text{ if } p = 0, \tag{5}$$

where the two parameters k and n represent the hardening modulus and exponent respectively.

When the transformation is VE, then f ($\boldsymbol{\sigma}$, R) ≤ 0, but if there is a VP deformation, then f ($\boldsymbol{\sigma}$, R) may be positive. The variable p is called accumulated plastic strain, because it measures the length of the plastic strain path, and is linked to the VP law:

$$p(t) = \int_0^t \dot{p}(\xi) d\xi, \text{ and } \dot{p} = \sqrt{\frac{2}{3} \dot{\boldsymbol{\varepsilon}}^{vp} : \dot{\boldsymbol{\varepsilon}}^{vp}}. \tag{6}$$

In the framework of associated VE-VP, the VP dissipation potential defines the yield surface and the VP flow direction, for more information, see (Lemaitre and Chaboche, [6]). This VP dissipative potential corresponds to the well-known von Mises yield surface f. The VP strain rate is obtained by the normality rules of the generalized standard materials and is governed by a flow rule represented by the VP operator **B**:

$$\dot{\boldsymbol{\varepsilon}}^{vp} = \mathbf{B}(\bar{x}, \boldsymbol{\sigma}, p) = \dot{p} \frac{\partial f}{\partial \boldsymbol{\sigma}}. \tag{7}$$

The precise sign of the plastic multiplier $\dot{p}$ is determined by the following conditions:

$$\dot{p} = C\left(\vec{x}, \boldsymbol{\sigma}, p\right),\ \text{if}\ f > 0;\ \dot{p} = 0,\ \text{if}\ f \le 0, \tag{8}$$

where C is a VP operator. In this work we will use Norton's power law:

$$C\left(\vec{x}, \boldsymbol{\sigma}, p\right) = g_v(\sigma_{eq}, p) = \frac{\sigma_y}{\eta}\left(\frac{f}{\sigma_y}\right)^m, \tag{9}$$

where the two parameters η and m represent the VP modulus and exponent respectively.

For a generic time interval [t_n, t_{n+1}], and using the appropriate numerical integration of equation (2), the VE-VP constitutive model can be defined incrementally (Miled, [2]). Equations (1) and (2) can then be written as follows:

$$\Delta\boldsymbol{\varepsilon}\left(\vec{u}\right) = \Delta\boldsymbol{\varepsilon}^{vp} + \hat{\mathbf{E}}^{-1}(\Delta t) : \mathbf{F}\left(\vec{x}, \boldsymbol{\sigma}, \boldsymbol{\varepsilon}^{ve}\right), \tag{10}$$

where Δt represents the time increment, $\hat{\mathbf{E}}$ is the incremental relaxation modulus and $\mathbf{F}$ is a second order tensor which depends on the stress at the start and the end of the time step and the viscoelastic strain at the start of the time step. The second term of the sum in equation (10) represents the viscoelastic increment.

3. Time homogenization of coupled viscoelastic-viscoplastic solids subjected to periodic loadings

The Goal of the time homogenization method is to predict the response of nonlinear materials under a large number of cycles while simulating a much smaller number. This approach is an extension of the asymptotic spatial homogenization method and was developed by Guennouni [10], for elasto-viscoplastic homogeneous materials. This technique was also used in the case of cyclic loadings by Yu & Fish [7,8] and Aubry & Puel [4]. In this work, we develop the theory for the coupled VE-VP model of Section 2.

3.1. Definition of two time scales

Consider a spatial domain Ω subjected to body forces $\vec{f}$. Γ_u and Γ_f correspond to its boundary portions where displacements $\vec{u}_b^T$ and tractions $\vec{g}^T$ are prescribed, respectively. We denote by $\vec{n}$ the outer unit vector normal to the boundary and by $\vec{u}^I$ and σ^I the initial displacement and stress respectively.

The framework typically considers cyclic loadings of the following form:

$$g^T(\vec{x}, t) = g^*(\vec{x})\lambda(t)\ \text{and}\ u_b^T(\vec{x}, t) = u_b^*(\vec{x})\beta(t), \tag{11}$$

where λ(t) and β (t) are the sum of a rapid harmonic oscillation and a slow loading and they are of the following form:

$$a\left(t\right) + b\left(t\right)\left(\sin\left(\frac{2\pi t}{T}\right) + c\right), \tag{12}$$

Here, T is a small positive scaling parameter that represents the load period and it is supposed to be very small compared to the observation time T_M, so that $T/T_M \ll 1$.

Two time scales are defined. The first corresponds to the macro time scale $\bar{t} \in [0, T_M]$ that is characteristic of the long-term behaviour of the solution. The second is a fast time-scale τ associated with the varying behaviour of the evolving variables resulting from the periodic loading.

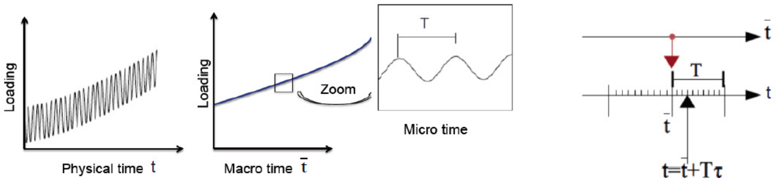


Fig.1. Macro and micro time coordinates.

Figure 1 explains the relation between the two time scales: if we look at the loading in the macro scale we will see a continuous line but if we zoom in we will see periodic fluctuations of period T. The macro time $\bar{t}$ corresponds to the peaks of the load, and it is a multiplier of T. The physical time t can then be defined as the sum of the macro time $\bar{t}$ and a fraction τ of the period T, $\tau \in [0, 1]$:

$$t = \bar{t} + T\tau, \tag{13}$$

Loadings can then be written as a function of the fast time variable τ and the slow time variable $\bar{t}$:

$$g^T(\vec{x},t) = g(\vec{x},\bar{t},\tau) \text{ and } u_b^T(\vec{x},t) = u_b(\vec{x},\bar{t},\tau). \tag{14}$$

All the response fields in the body $\Phi^T(\vec{x},t)$ at a given spatial location $\vec{x}$ are assumed to depend simultaneously on both time-scales and are then considered as a function of the two-time variables $\bar{t}$ and τ, and are expressed as:

$$\Phi^T(\vec{x},t) = \Phi(\vec{x},\bar{t},\tau). \tag{15}$$

Using the chain rule and the relation between the time scales, the time derivation in the two time scales are given as:

$$\dot{\Phi} = \frac{d\Phi(\vec{x},\bar{t},\tau)}{dt} = \frac{\partial\Phi(\vec{x},\bar{t},\tau)}{\partial\bar{t}} + \frac{1}{T}\frac{\partial\Phi(\vec{x},\bar{t},\tau)}{\partial\tau}. \tag{16}$$

The average value $\bar{\Phi}(\vec{x},\bar{t})$ with respect to τ of the function $\Phi(\vec{x},\bar{t},\tau)$ is given by:

$$\bar{\Phi}(\vec{x},\bar{t}) = \int_0^1 \Phi(\vec{x},\bar{t},\tau)\, d\tau. \tag{17}$$

A general decomposition of all variables $\Phi(\vec{x},\bar{t},\tau)$ is proposed in the form:

$$\Phi(\vec{x},\bar{t},\tau) = \bar{\Phi}(\vec{x},\bar{t}) + \tilde{\Phi}(\vec{x},\bar{t},\tau), \tag{18}$$

where $\tilde{\Phi}(\vec{x},\bar{t},\tau)$ represents the oscillatory portion of $\Phi(\vec{x},\bar{t},\tau)$.

The initial-boundary VE-VP problem is then expressed using the two time scales:

Equilibrium equation:

$$\nabla.\boldsymbol{\sigma}\left(\vec{x},\bar{t},\tau\right) + \vec{f}\left(\vec{x},\bar{t},\tau\right) = \vec{0}, \quad \text{on } \Omega\times\left[0,T_M\right]\times\left[0,1\right] \tag{19}$$

Constitutive equations:

$$\boldsymbol{\varepsilon}\left(\vec{u}\right) = \boldsymbol{\varepsilon}^{ve}\left(\vec{x},\bar{t},\tau\right) + \boldsymbol{\varepsilon}^{vp}\left(\vec{x},\bar{t},\tau\right), \quad \text{on } \Omega\times\left[0,T_M\right]\times\left[0,1\right] \tag{20}$$

$$\Delta\boldsymbol{\varepsilon}\left(\vec{u}\right) = \Delta\boldsymbol{\varepsilon}^{vp}\left(\vec{x},\bar{t},\tau\right) + \hat{\mathbf{E}}^{-1}(\Delta\bar{t}) : \mathbf{F}\left(\boldsymbol{\sigma},\boldsymbol{\varepsilon}^{ve}\right), \quad \text{on } \Omega\times\left[0,T_M\right]\times\left[0,1\right] \tag{21}$$

$$\dot{\boldsymbol{\varepsilon}}^{vp} = \mathbf{B}\left(\vec{x},\boldsymbol{\sigma},p\right), \quad \text{on } \Omega\times\left[0,T_M\right]\times\left[0,1\right] \tag{22}$$

$$\dot{p} = C\left(\vec{x},\boldsymbol{\sigma},p\right), \quad \text{on } \Omega\times\left[0,T_M\right]\times\left[0,1\right] \tag{23}$$

Kinematic equation:

$$\boldsymbol{\varepsilon}\left(\vec{u}\right) = \frac{1}{2}\left(\nabla\vec{u} + {}^t\,\nabla\vec{u}\right), \quad \text{on } \Omega\times\left[0,T_M\right]\times\left[0,1\right] \tag{24}$$

Boundary conditions:

$$\vec{u}\left(\vec{x},\bar{t},\tau\right) = \vec{u}_b\left(\vec{x},\bar{t},\tau\right), \quad \text{on } \Gamma_u\times\left[0,T_M\right]\times\left[0,1\right] \tag{25}$$

$$\boldsymbol{\sigma}\cdot\vec{n}=\vec{g}\left(\bar{x},\bar{t},\tau\right),\quad \text{on } \Gamma_f\times\left[0,T_M\right]\times\left[0,1\right] \tag{26}$$

Initial conditions:

$$\vec{u}\left(\bar{x},\bar{t}=0,\tau=0\right)=\vec{u}^I\left(\bar{x}\right) \text{ and } \boldsymbol{\sigma}\left(\bar{x},\bar{t}=0,\tau=0\right)=\boldsymbol{\sigma}^I\left(\bar{x}\right),\quad \text{on } \Omega \tag{27}$$

Where $\nabla.$ and ∇ denote the divergence and gradient operators, respectively.

3.2. Asymptotic expansion

Each mechanical variable $\Phi(\vec{x},\bar{t},\tau)$ is supposed to be regular enough so that it can be expanded into an asymptotic series of powers of T (Bensoussan, [1]). Given that T/ T_M <<<1, we can assume that:

$$\Phi(\bar{x},\bar{t},\tau)=\sum_{i=0}^{\infty}T^i\Phi_i(\bar{x},\bar{t},\tau), \tag{28}$$

where $\Phi_i(\vec{x},\bar{t},\tau)$ are $\boldsymbol{\tau}$-periodic functions and i denotes the order of the terms in the expansion.

As T is small, expression (28) can be regarded as consisting of a leading term $\Phi_0(\vec{x},\bar{t},\tau)$, plus a series of rapidly decreasing amplitude. The asymptotic expansions are injected into equations (19) to (27) of the problem. Using the chain rule (16) and gathering terms of equal order, the problem can then be rewritten in terms of the powers of T. For example the 1/T-terms of the viscoplastic flow rules (22) and (23) are:

$$\frac{\partial\boldsymbol{\varepsilon}_0^{vp}\left(\bar{x},\bar{t},\tau\right)}{\partial\tau}=0,\quad \text{on } \Omega\times\left[0,T_M\right]\times\left[0,1\right] \tag{29}$$

$$\frac{\partial p_0\left(\bar{x},\bar{t},\tau\right)}{\partial\tau}=0,\quad \text{on } \Omega\times\left[0,T_M\right]\times\left[0,1\right] \tag{30}$$

And the T^0-terms lead to the following equations:

$$\frac{\partial\boldsymbol{\varepsilon}_0^{vp}}{\partial\bar{t}}+\frac{\partial\boldsymbol{\varepsilon}_1^{vp}}{\partial\tau}=\mathbf{B}\left(\bar{x},\boldsymbol{\sigma}_0,p_0\right),\quad \text{on } \Omega\times\left[0,T_M\right]\times\left[0,1\right] \tag{31}$$

$$\frac{\partial p_0}{\partial\bar{t}}+\frac{\partial p_1}{\partial\tau}=C\left(\bar{x},\boldsymbol{\sigma}_0,p_0\right),\quad \text{on } \Omega\times\left[0,T_M\right]\times\left[0,1\right] \tag{32}$$

Equations (29) and (30) show that the zeroth-order of the VP strain and that of the accumulated plastic strain are a function of the slow time variable only, hence at the zero-order the rapid evolution of VP deformation is blocked:

$$\boldsymbol{\varepsilon}_0^{vp}\left(\bar{x},\bar{t},\tau\right)=\boldsymbol{\varepsilon}_0^{vp}\left(\bar{x},\bar{t}\right) \text{ and } p_0\left(\bar{x},\bar{t},\tau\right)=p_0\left(\bar{x},\bar{t}\right). \tag{33}$$

In the same way, we can derive the zeroth-order equations coming from the other equations of the mechanical problem.

3.3. Resolution of the problem

To solve the various order initial boundary problems, we follow the decomposition defined in (18). Taking the average of all zeroth-order equations we obtain a decomposition of the source problem into micro- and macro-chronological problems:

– Macro-chronological problem:

$$\nabla.\bar{\boldsymbol{\sigma}}_0\left(\bar{x},\bar{t}\right)+\bar{\vec{f}}\left(\bar{x},\bar{t}\right)=\vec{0},\quad \text{on } \Omega\times\left[0,T_M\right] \tag{34}$$

$$\boldsymbol{\varepsilon}\left(\bar{\vec{u}}_0\right)=\bar{\boldsymbol{\varepsilon}}_0^{ve}\left(\bar{x},\bar{t}\right)+\bar{\boldsymbol{\varepsilon}}_0^{vp}\left(\bar{x},\bar{t}\right),\quad \text{on } \Omega\times\left[0,T_M\right] \tag{35}$$

$$\Delta\boldsymbol{\varepsilon}\left(\bar{\bar{u}}_0\right)=\Delta\bar{\boldsymbol{\varepsilon}}_0^{vp}\left(\vec{x},\bar{t}\right)+\hat{\mathbf{E}}^{-1}(\Delta\bar{t}):\bar{\mathbf{F}}\left(\boldsymbol{\sigma}_0,\boldsymbol{\varepsilon}_0^{ve}\right),\quad \text{on } \Omega\times\left[0,T_M\right] \tag{36}$$

$$\frac{d\boldsymbol{\varepsilon}_0^{vp}}{d\bar{t}}=\bar{\mathbf{B}}\left(\vec{x},\boldsymbol{\sigma}_0,p_0\right),\quad \text{on } \Omega\times\left[0,T_M\right] \tag{37}$$

$$\frac{dp_0}{d\bar{t}}=\bar{C}\left(\vec{x},\boldsymbol{\sigma}_0,p_0\right),\quad \text{on } \Omega\times\left[0,T_M\right] \tag{38}$$

$$\boldsymbol{\varepsilon}\left(\bar{\bar{u}}_0\right)=\frac{1}{2}\left(\nabla\bar{\bar{u}}_0+{}^t\nabla\bar{\bar{u}}_0\right),\quad \text{on } \Omega\times\left[0,T_M\right] \tag{39}$$

$$\bar{\bar{u}}_0\left(\vec{x},\bar{t}\right)=\bar{\bar{u}}_b\left(\vec{x},\bar{t}\right),\quad \text{on } \Gamma_u\times\left[0,T_M\right] \tag{40}$$

$$\bar{\boldsymbol{\sigma}}_0\cdot\vec{n}=\bar{\vec{g}}\left(\vec{x},\bar{t}\right),\quad \text{on } \Gamma_f\times\left[0,T_M\right] \tag{41}$$

The macro-chronological problem is then completely defined taking into account the following initial conditions:

$$\bar{\bar{u}}_0\left(\vec{x},\bar{t}=0\right)+\tilde{\bar{u}}_0\left(\vec{x},\bar{t}=0,\tau=0\right)=\vec{u}^I\left(\vec{x}\right)\ \text{ and }\ \bar{\boldsymbol{\sigma}}_0\left(\vec{x},\bar{t}=0\right)+\tilde{\boldsymbol{\sigma}}_0\left(\vec{x},\bar{t}=0,\tau=0\right)=\boldsymbol{\sigma}^I\left(\vec{x}\right),\quad \text{on } \Omega \tag{42}$$

– Micro-chronological problem:

$$\nabla\tilde{\boldsymbol{\sigma}}_0\left(\vec{x},\bar{t},\tau\right)+\tilde{\vec{f}}\left(\vec{x},\bar{t},\tau\right)=\vec{0},\quad \text{on } \Omega\times\left[0,T_M\right]\times\left[0,1\right] \tag{43}$$

$$\boldsymbol{\varepsilon}\left(\tilde{\bar{u}}_0\right)=\tilde{\boldsymbol{\varepsilon}}_0^{ve}\left(\vec{x},\bar{t},\tau\right)+\tilde{\boldsymbol{\varepsilon}}_0^{vp}\left(\vec{x},\bar{t},\tau\right),\quad \text{on } \Omega\times\left[0,T_M\right]\times\left[0,1\right] \tag{44}$$

$$\Delta\boldsymbol{\varepsilon}\left(\tilde{\bar{u}}_0\right)=\hat{\mathbf{E}}^{-1}(\Delta\bar{t}):\tilde{\mathbf{F}}\left(\boldsymbol{\sigma}_0,\boldsymbol{\varepsilon}_0^{ve}\right),\quad \text{on } \Omega\times\left[0,T_M\right]\times\left[0,1\right] \tag{45}$$

$$\tilde{\bar{u}}_0\left(\vec{x},\bar{t},\tau\right)=\tilde{\bar{u}}_b\left(\vec{x},\bar{t},\tau\right),\quad \text{on } \Gamma_u\times\left[0,T_M\right]\times\left[0,1\right] \tag{46}$$

$$\tilde{\boldsymbol{\sigma}}_0\cdot\vec{n}=\tilde{\vec{g}}\left(\vec{x},\bar{t},\tau\right),\quad \text{on } \Gamma_f\times\left[0,T_M\right]\times\left[0,1\right] \tag{47}$$

Equations (43) to (47) correspond to the resolution of a VE problem only, since the VP flow rule does not depend on the fast time variable explicitly.

4. Illustration on a simple case

Consider a one-dimensional cylindrical bar of length L clamped at one end (x = 0) and submitted at the other end **(**x = L**)** to a sinusoidal displacement with a period T=0.1s and amplitude U which is taken as U/L=0.07:

$$u_b^T\left(x=L,t\right)=U\left(0.45\sin\left(\frac{2\pi}{T}t\right)+0.55\right). \tag{48}$$

The body forces $\vec{f}$ are neglected and the time observation T_M=1000 s. The nonlinear material behaviour of the bar consists in J2 VE-VP model in its uniaxial expression. The fitted material parameters are listed in Table 1.
The prescribed displacement expressed in terms of the fast time coordinate is given as:

$$u_b\left(x=L,\bar{t},\tau\right)=U\left(0.45\sin\left(\frac{2\pi}{T}\left(\bar{t}+T\tau\right)\right)+0.55\right). \tag{49}$$

The numerical 1D model is solved using a return-mapping algorithm (Doghri, [5]). The mico-chronological problem (43)-(47) is solved only once (no iterations) at each macro time increment, whereas the macro-chronological problem (34)-(41) is nonlinear and solved iteratively at each macro time step using the Newton Raphson method. Comparisons between the reference calculation and the homogenized one are given in figures (2)-(5). Numerical solution for the source problem is obtained using a very fine time step Δt=0.005 s, which represents one twentieth of the load period. Whereas the time-homogenized computation was started by a full calculation until the time t=0.075s which represents the three-quarters of the first cycle and then the calculations are done using a time step of 0.1s.

Table 1. Constitutive model parameters for polyamide (PA) at 40°C identified from experimental measures of Baquet (2007) [2].

Viscoelastic parameters			
Initial shear modulus	G_0=1074 MPa		
Initial bulk modulus	K_0=3222 MPa		
G_i (MPa)	g_i(s)	K_i (MPa)	k_i(s)
158	0.021	472	0.007
80	0.378	242	0.126
37	0.648	111	0.216
Viscoplastic parameters			
Hardening function	k=79 MPa	n=0.15	
Viscoplastic function	η=305 MPA.s	m=4.02	
Yield stress	σ_y=40 MPa		

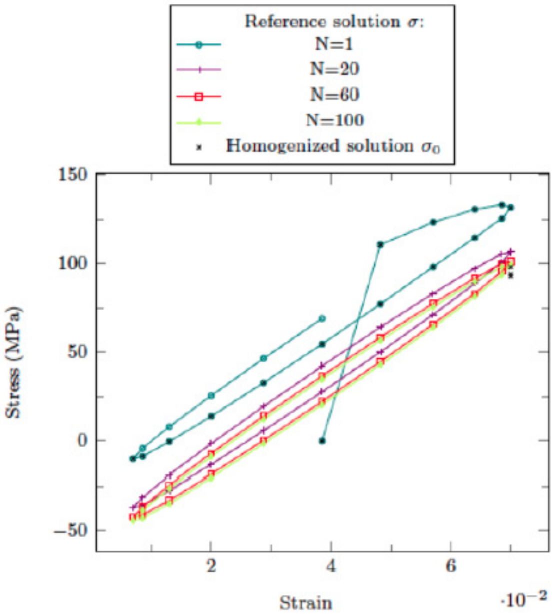


Fig.2. Hysteresis loops: First hundred cycles.

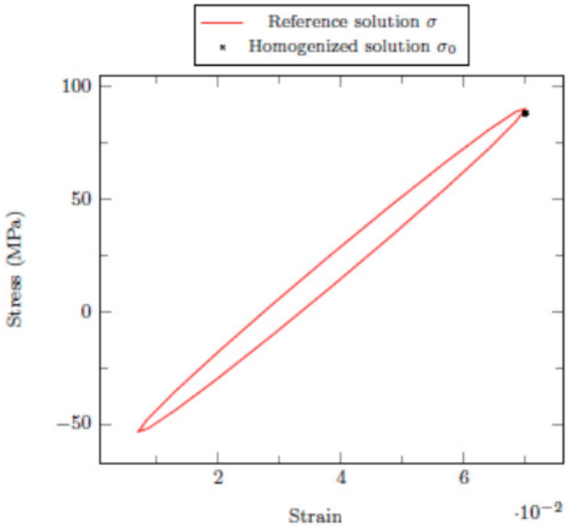
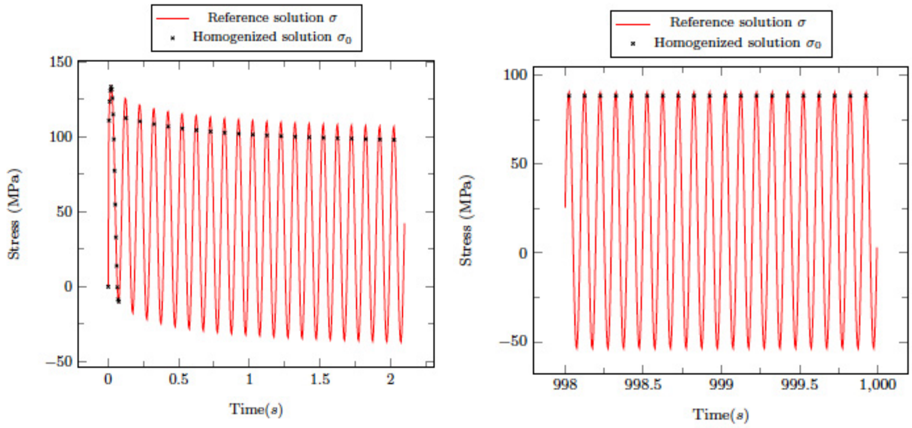


Fig.3. Hysteresis loops: Last twenty cycles.

In figure 2 are represented the hysteresis loops of the stress-strain data. Some selected cycles (N=1,20,60,100) are reported. The marked lines show the results obtained by reference calculation. The black crosses show the results obtained by homogenized calculation. The appearance of loops on the stress-strain curve is directly related to the presence of permanent plastic deformation. It is noted that even if the applied strain is small, its repeated action can cause a softening of the material. The shape of the stress-strain loop rapidly changed, shrinking with increasing number of cycles. For the first hundred cycles it is seen that the black crosses don't coincide with the peaks of constraints, however, for the last twenty cycles shown in figure 3, the black crosses are nearly perfectly positioned at the peaks of constraints. This can also be noticed in figure 4 where a comparison between the evolution of the zeroth-order homogenized solution σ_0 and the reference solution σ is shown. The red continuous line represents the stress obtained with full calculation, whereas the black crosses represents the stress obtained with the time homogenization method. Figure 4 (a) gives the oscillatory stress field at the beginning of the loading at [0, 2s] which corresponds to the first twenty cycles. It is noticed that the homogenized solution does not really coincide with the reference solution. This is because only the zero-order in the asymptotic expansion is considered. In addition, for the first twenty cycles the load period is not sufficiently small compared to the observation time which is equal in this case to 2 s. Unlike what is shown in figure 4 (b) for the last twenty cycles where it is observed that the homogenized solution is almost superposed with the reference solution since the period T is now considered small enough compared to the time observation.

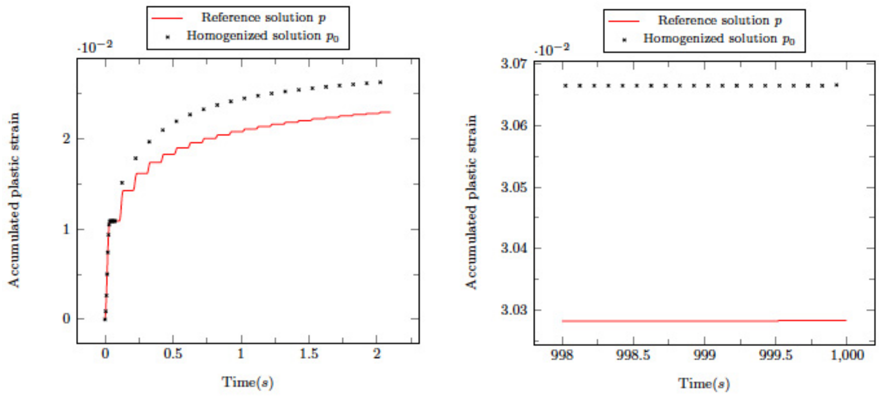
Figure 5 presents the accumulated plastic strain at the beginning (figure 5 (a)) and the end (figure 5 (b)) of the loading. The relative error between the zeroth-order homogenized solution p_0 and the reference solution p in the beginning of loading is about 12.8%, nevertheless at the end of the simulation it is about 1.2%.

For the reference calculation the CPU time is 55.15 s, while with the time homogenization method the CPU time is 1.31 s. The relative gain in computation time, between the reference and the time-homogenized computation, is about 97%, in this simple example, which gives a hint about the potential of the time homogenization method.



(a) (b)

Fig. 4. Evolution of the stress: homogenized solution in comparison with reference solution. (a) The first twenty cycles. (b) The last twenty cycles.



(a) (b)

Fig. 5. Evolution of the accumulated plastic strain: homogenized solution in comparison with reference solution. (a) The first twenty cycles. (b) The last twenty cycles.

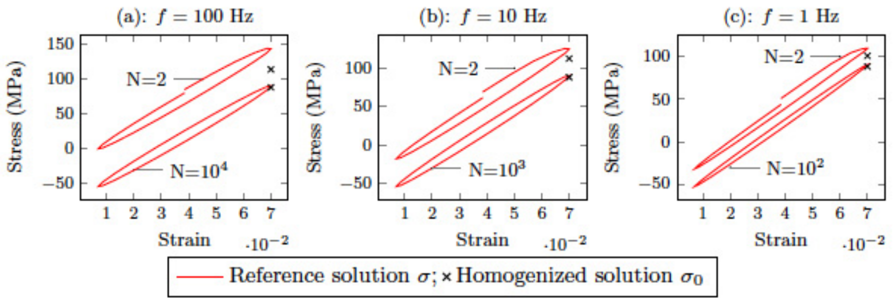


Fig. 6. Hysteresis loops for different frequencies f: homogenized solution in comparison with reference solution. Fitted material parameters are listed in Table 1.

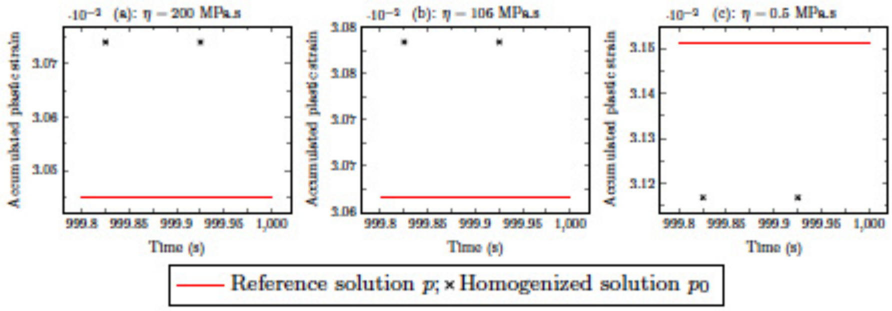


Fig. 7. Evolution of the accumulated plastic strain for the same frequency f=10 Hz and different values of viscous parameter η: (a) η =200 MPa.s(b) η =106 MPa.s(, (c) η =0.5 MPa.s. (Fitted material parameters are listed in Table 1

A comparative study of the complete method and temporal homogenization in terms of loading frequency and viscous parameter is done. Figure 6 presents the cyclic response of polyamide (PA) at 40°C, for different loading frequencies 100 Hz (figure 6 (a)), 10 Hz (figure 6 (b)) and 1 Hz (figure 6 (c)). The time observation T_M is fixed to 1000 s. As the number of loading cycles (N) increased the loop becomes smaller. It can be observed that the material is rate-dependent. The flow stress increases nonlinearly with an increase of the loading rate. As mentioned before the homogenized solution does not really coincide with the reference solution for the first cycles, whereas it is almost superposed with the reference solution for the last cycle.

The effect of the viscous parameter η on the evolution of the accumulated plastic strain is illustrated in figure 7. Results are shown only for the two last cycles. The time observation T_M is fixed to 1000 s, the frequency f is fixed to 10 Hz and the viscous parameter η is varied. It is noticed that the accumulated plastic strain increases when η decreases. Temporal homogenization approach has been found to be in good agreement with the reference solution for different values of η. Several simulations for $\eta \in [0.5, 400]$ MPa.s are done. The evolution of the relative error for the accumulated plastic strain is presented in figure 8. It can be observed that the relative error decreases when the parameter η decreases to the value of 50 MPa.s and then it increases when η tends towards zero, that is when the effect of viscosity on the plastic domain is neglected.

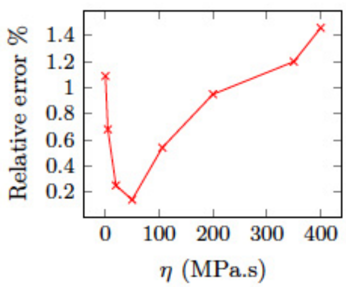


Fig. 8. Evolution of the relative error

5. Conclusion

In this work, an asymptotic temporal homogenization formulation for a coupled VE-VP constitutive model has been presented. Temporal homogenization approach has been tested and implemented and has been found to be in good agreement with the reference solution as long as the scaling parameter T remains small. A significant reduction in the amount of computation time in addition to a small relative error between time homogenized and non homogenized models are obtained. Although the present work does not include fatigue, the theory might enable an approximate prediction of the fracture time t_f of the structure and an evaluation of macro and micro times corresponding to it. For instance, if we consider that fracture occurs when the accumulated plastic strain at a given point exceeds a certain threshold p_{0max} , we can estimate when this threshold is obtained by solving the following equation:

$$p_{0\max} = \int_0^{t_f} \overline{C}\left(\bar{x}, \boldsymbol{\sigma}_0, p_0\right). \qquad (50)$$

Currently, the VE-VP model is being coupled with a fatigue damage model. In our future work the present temporal homogenization scheme will be extended to VE-VP coupled with a fatigue damage, and will then be generalized to fatigue analysis of heterogeneous solids, which are characterized by multiple temporal and spatial scales.

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