

Development of time-efficient optimal design approaches based on sensitivity analyses — Application to synchronous reluctance motors

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Abstract

Synchronous reluctance motors are a type of electric motor that can achieve higher efficiencies than induction motors without using permanent magnets. The torque in these machines is generated by the magnetic anisotropy between their direct and quadrature axes. To increase the saliency ratio of these machines, a multilayered structure is used, which interleaves flux guides and flux barriers. However, this topology choice results in a more complex rotor design. To maintain conformity with industry standards, where identical laminations are punched and axially stacked, the flux guides are held together by small ribs using the same lamination sheet. The ribs in the rotor design must be wide enough to sustain the mechanical loading of the rotor, but narrow enough to limit their negative impact on the magnetic performance.

Optimizing these machines typically involves a high number of parameters and the use of finite element models to accurately describe and account for the complex rotor geometry. Simultaneously optimizing all parameters can result in computing times of several months due to long evaluation times and the curse of dimensionality. To achieve feasible optimization times, a common approach is to decompose the complex optimization problem into smaller, more manageable subproblems. The decomposition is usually carried out at the objectives level, optimizing the flux barriers and flux guides for magnetic performance, and then adding ribs to ensure structural integrity. While this design strategy may effectively allow for reasonable optimization time, it neglects potential parameter interactions. Therefore, optimizing all parameters together may still yield substantial performance improvements.

This work proposes developing time-efficient optimal design approaches that account for parameter interactions using the outputs of an extensive variance-based sensitivity analysis. The design approaches are benchmarked against those inspired by the literature. The application framework focuses on evaluating the feasibility of replacing the rotor of a commercially available induction motor to convert it into a synchronous reluctance machine. The optimization objectives are to maximize the mean torque, limit the relative torque ripple to below 10%, and ensure the structural integrity of the rotor. Therefore, the design process involves developing numerical models to evaluate the quantities of interest, discussing the possibilities

of parametrization, conducting sensitivity analyses, and developing time-efficient design approaches.

To accommodate the intricate rotor geometry and nonlinearity of ferromagnetic materials, the mean torque and torque ripple are evaluated by solving the magneto-static equations in 2D using finite element method, with the mesh generator Gmsh and the solver GetDP. The evaluation time is reduced by employing the moving band method and limiting resolution domain to a pole. The accuracy of this model is verified by experimental data collected from a full-scale 300kW SynRM that shares the same stator as the one being modeled. Iron losses, calculated using Bertotti's decomposition in post-processing, are also validated experimentally, albeit using the induction version of the motor. Similarly, structural integrity is assessed by solving equations of solid mechanics and applying the von Mises yield criterion to detect mechanical failure. The interplay between the magnetic and mechanical models, represented by magnetic forces, is evaluated and addressed through a conservative safety factor on the maximum allowable stress. Overall, it takes approximately 11 minutes to evaluate mean torque and torque ripple at 150 positions, while the mechanical model requires only 45 seconds.

After describing the numerical models, the possibilities of parametrization are discussed. In a preliminary study, rectangular, circular, hyperbolic, and Joukowski flux barriers are compared on mean torque and torque ripple. While circular flux barriers achieve slightly better magnetic performance, Joukowski flux barriers are ultimately selected due to their balance between parameter count and magnetic performance. Furthermore, the study reveals that faster optimization convergence could be achieved by employing a rectangular design space, preventing overlap between flux barriers. This parametrization involves rescaling the individual thickness of flux barriers and flux guides to fit the rotor yoke, eliminating explicit constraints between parameters. A second parametrization is proposed, emphasizing the magnetic insulation of the rotor as an explicit parameter in the design space. This re-parametrization is expected to favor optimization convergence, as this parameter has been linked to the torque capability of SynRM in previous studies. Lastly, considerations for structural integrity prompts the addition of radial and tangential ribs, characterized by their thicknesses and fillet radii.

A comprehensive sensitivity analysis is then conducted on both parametrizations to elucidate the effects of parameters on the outputs of interest. Sobol indices are chosen as sensitivity indicators, as they decompose the variance of the output into contributions stemming from variations of individual parameters alone (first-order effects), variations of two parameters simultaneously (second-order effects), and so on. To facilitate interpretation, these indices are depicted using nested pie charts and chord diagrams. Accurately estimating these indices for rotors with four flux barriers necessitated approximately seventy thousand samples of the search space. The evaluation of these quasi-Monte Carlo samples is fully parallelizable, benefiting from recent advancements in computing power and facilitated through cluster computing. Results indicate that the second parametrization proves highly effective for mean torque and iron losses, as parameter interactions are minimized. In particular, over 95% of the variance in mean torque can be attributed to the magnetic

insulation ratio alone. However, torque ripple and equivalent stress remain notably influenced by tangential ribs and distribution of air and iron within the flux barriers and flux guides, with the magnetic insulation ratio only playing a marginal role.

The first time-efficient design approaches explored in this study draw inspiration from existing literature and involve optimizing the ribs separately from the flux guides and flux barriers. Combinations of two formulations for magnetic optimization and three formulations for mechanical optimization are benchmarked against a reference solution obtained by optimizing all parameters simultaneously. This reference solution could be achieved in 2 weeks through efficient parallelization of the workload on a dedicated workstation. To address the stochastic nature of the optimization algorithm, three independent runs of each combination are conducted. Results indicate that total optimization time can be effectively reduced by over half with negligible impact on mean torque when the optimization problem is separated. The most effective approach involves pre-dimensioning the radial ribs within the magnetic optimization and minimizing the total surface area of the ribs within the subsequent mechanical optimization, followed by one iteration of the process. Despite these efforts, significant variations in torque ripple are observed across the independent runs, which results from unaccounted parameter interactions.

Utilizing the results of sensitivity analysis, parameters can be categorized into different sets based on their impact across various outputs of interest. By prioritizing parameters with negligible impacts on other outputs, optimization constraints can be sequentially satisfied by optimizing subsets of parameters. If the search space is insufficient to meet the constraint, additional parameters are included to expand it, albeit with penalties applied to changes in these parameters. Using this methodology, mean torque is first maximized using the entire search space. Subsequently, structural integrity is ensured by minimizing changes to tangential ribs. Torque ripple is then minimized by redistributing air and iron between flux barriers and flux guides. Finally, a reassessment of structural integrity is performed to accommodate these changes. This approach proves highly effective, reducing total optimization time to two-fifths of the reference optimization time, while achieving a torque ripple below 5%. Consequently, the objective of developing a time-efficient optimal design approach is successfully accomplished.

Finally, the database gathered to perform sensitivity analysis can be reused to compare different topologies across various objectives without requiring resource-intensive optimizations. Indeed, the numerous samples across the search space can provide some statistical information on what topology is more likely to outperform others, even if the optimum was unlikely to be captured in the database. This is illustrated by comparing histograms of topologies featuring one or four internal flux barriers, with or without a cut-off barrier, in terms of iron losses, mean torque, torque ripple, power factor, structural integrity, and suitability for sensorless control. For competing topologies across two objectives, the comparison methodology can be extended using density plots. Moreover, it is shown that the identification of the switching point, denoting when one topology surpasses another, can be approximated more precisely through cost-effective metamodel-based optimizations. For rotors featuring a single inner flux barrier, the comparison methodology is validated

using Pareto fronts obtained with finite element models. The analysis uncovers that rotor configurations with a cut-off barrier generally exhibit higher mean torque and lower iron losses, whereas those without tend to have slightly lower equivalent stresses, reduced torque ripple, and enhanced sensorless capability.

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Nomenclature

List of symbols

f	Fillet radius ($\in \mathbf{x}_{\text{mech}}$)
g_m	Machine gain for sensorless control
i	Stator current
j	Stator current density
K	Virtual spring stiffness ($\in \mathbf{x}_{\text{mag}}$)
L_d, L_q	d - and q -axis inductances
$L_{dd}, L_{qq}, L_{dq}, L_{qd}$	Incremental d - and q -axis inductances
L_{Σ}, L_{Δ}	Average and difference of L_d and L_q
$L_{\Delta\Delta}$	Difference of L_{dd} and L_{qq}
n	Number of flux barriers
p	Number of pole pairs
$\bar{p}$	Time derivative operator d/dt
q	Number of flux guides
R_{in}, R_{out}	Inner and outer radii of rotor yoke
R_s	Stator resistance
S_i, S_{ij}, S_{T_i}	First, second and total Sobol indices
t^R, t^T	Radial and tangential ribs thicknesses ($\in \mathbf{x}_{\text{mech}}$)
T_{em}	Electromagnetic torque
$\bar{T}, \tilde{T}$	Mean torque and torque ripple
$\tilde{T}_{\text{max}}$	Target maximum torque ripple
v	Stator voltage

$\mathbf{x}_{\text{mag}}, \mathbf{x}_{\text{mech}}$	Magnetic and mechanical parameters
δ	Load angle
ζ	Saliency ratio
θ_e	Electrical angle of the stator currents ($\in \mathbf{x}_{\text{mag}}$)
$\theta_{\text{em}}, \theta_{\text{m}}$	Rotor electrical and mechanical positions
$\tilde{\theta}, \tilde{\theta}^*$	Instantaneous and final estimation error of the (dq) frame
λ	Magnetic flux
μ	Magnetic permeability
ξ	Air-to-rotor ratio ($\in \mathbf{x}_{\text{mag}}$)
σ_y	Yield stress of the material
σ_{VM}	Equivalent von Mises stress
σ_{max}	Target maximum von Mises stress
ϕ_i	Internal power angle
ψ_f	Permanent magnet flux linkage
ω_e, ω_m	Rotor electrical and mechanical speeds

Notations

Unless specified otherwise, the following notations are used:

$\mathbf{x}$	Vector
$[\mathbf{x}]$	Matrix
x_α, x_β	Components of $\mathbf{x}$ in the $(\alpha\beta)$ frame
x_d, x_q	Components of $\mathbf{x}$ in the (dq) frame
$\hat{x}$	Estimator
x^0	Initial value of parameter x
x^*	Parameter x has been subjected to an optimization
x_i or x_j	Subset of all parameters of type x . E.g., $t_i^T = \{t_1^T, t_2^T, \dots\}$.

List of optimization problems

Following table provides an overview of the optimization problems solved in this research.

Name	Objective	Constraints	Search space (1 st param.)	Search space (2 nd param.)	Comment
F	Mean torque	Torque ripple Equivalent stress	$K_j, \theta_e, t_i^T, t_i^R, f$ 34 param.	$\xi, K_i^{fb}, K_j^{fg}, \theta_e, t_i^T, t_i^R$ 17 param.	Reference optimization
A	Mean torque	Torque ripple	K_j, θ_e 10 param.	$\xi, K_i^{fb}, K_j^{fg}, \theta_e$ 9 param.	
B	Mean torque	Torque ripple	K_j, θ_e 10 param.	$\xi, K_i^{fb}, K_j^{fg}, \theta_e$ 9 param.	Pre-dim. of radial ribs
I	Mean torque	Torque ripple Equivalent stress	t_i^T, t_i^R, f 24 param.	/	Requires mag. model
II	Rib surface	Equivalent stress	t_i^T, t_i^R, f 24 param.	t_i^T, t_i^R 8 param.	
III	L_q	Equivalent stress	t_i^T, t_i^R 24 param.	/	L_q estimated analytically
V	Change of tan. ribs	Equivalent stress	t_i^T, t_i^R 8 param.	/	
$\mathcal{P}_1$	Mean torque	/	/	ξ, θ_e 2 param.	
$\mathcal{P}_2$	Change of tan. ribs	Equivalent stress	/	t_i^T, t_i^R 8 param.	
$\mathcal{P}_3$	Torque ripple	/	/	K_i^{fb}, K_j^{fg} 7 param.	Alternative termination condition: $\tilde{T} < T_{\max} \rightarrow \mathcal{P}_3^*$

Introduction

1

Electricity stands as one of the most vital form of energy in the contemporary world, finding versatile applications such as lighting, heating, and actuation systems. Notably, more than half of the global electricity consumption is attributed to motor-driven systems, which is twice as much as lighting [1, 2]. Given their role in converting the majority of electrical energy, the efficiency of electric machines is a frequent focus of regulatory measures. In Europe, the latest regulation EU 2019/1781, already in effect since July 2023, mandates that electric machines meet efficiency requirements falling within the IE4 class. This regulation supersedes the previous EC 640/2009 regulation, which stipulated the IE3 efficiency level.

While induction motors currently dominate the industry due to their relative affordability, reliability, and line-starting capabilities, their mechanism of torque production inevitably involves losses at the rotor, thereby limiting their efficiency. Although careful design considerations can enable them to meet the IE4 efficiency requirements, they may struggle to comply with future norms.

Conversely, synchronous motors, either with a wound rotor or permanent magnets, stand out for their unparalleled torque density, efficiency, and power factor. Despite these remarkable attributes, the manufacturing cost of these machines is notably higher. The inclusion of slip rings, essential for supplying power to the field winding, incurs substantial maintenance expenses. Additionally, the use of permanent magnets, while advantageous in terms of performance, comes with a considerable environmental footprint and susceptibility to geopolitical pressures.

Synchronous reluctance motors strategically position themselves between the efficiency and cost spectrum, bridging the gap between induction motors on one end and conventional synchronous motors on the other. Synchronous reluctance motors leverage the magnetic anisotropy of the rotor to produce reluctant torque, as depicted in Fig. 1.1. The intrinsic efficiency of synchronous reluctance motors surpasses that of induction motors, a result of not relying on induced current for torque generation. Furthermore, the absence of permanent magnets in synchronous reluctance motors eliminates the risk of demagnetization, facilitating operation in higher-temperature environments and concurrently reducing manufacturing costs.

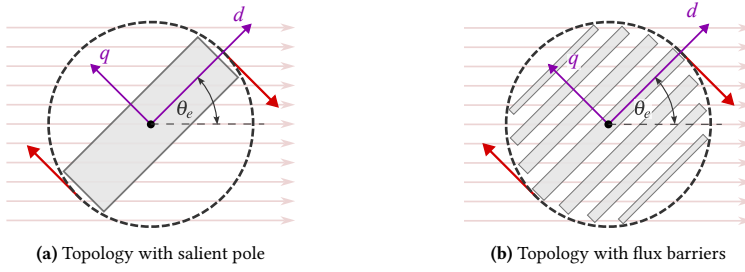


Figure 1.1: Working principle of a synchronous reluctance motor.

This unique combination of enhanced efficiency, reduced costs, and increased reliability positions synchronous reluctance motors as a promising alternative to other types of electrical machines. The working principle of SynRM and key parameters of interest are elucidated in the subsequent section 1.1, along with an exploration of the physical limitations inherent to this machine. Within the same section, crucial geometrical features of the rotors are also presented.

Section 1.2 delves into a comprehensive review of the methods employed in the optimal design of these machines, accompanied by an examination of the challenges faced. Subsequently, sections 1.5 and 1.6 detail the objectives and organizational structure of this research, providing a roadmap for the subsequent chapters.

1.1 Theory of synchronous reluctance motors

1.1.1 Equivalent circuit model

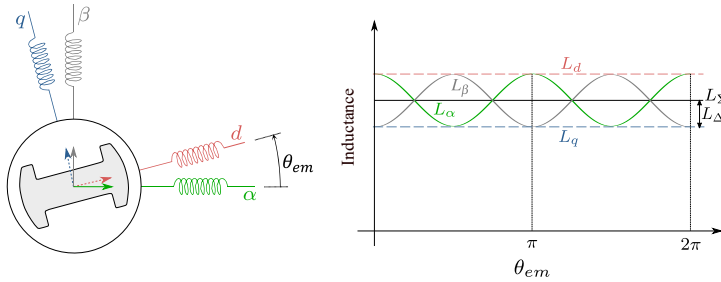


Figure 1.2: Evolution of the inductances in $(\alpha\beta)$ and (dq) coordinate systems.

The equivalent circuit model of the motor establishes the relationship between the voltages $\mathbf{v}$ and the currents $\mathbf{i}$, phase stator resistance R_s and magnetic fluxes $\boldsymbol{\lambda}$:

$$\mathbf{v} = R_s \mathbf{i} + \frac{d\boldsymbol{\lambda}}{dt}. \quad (1.1)$$

The electrical equations of SynRM are commonly established in the stator reference frame $(\alpha\beta)$, or in the rotor reference frame (dq) , as illustrated in Fig. 1.2. These transformations are performed using the space vectors transformations detailed in appendix A.

In the stator reference frame ($\alpha\beta$)

In this frame, the α axis is aligned with the magnetic axis of phase a . The equations are:

$$v_\alpha = R_s i_\alpha + \frac{d\lambda_\alpha}{dt} \quad (1.2)$$

$$v_\beta = R_s i_\beta + \frac{d\lambda_\beta}{dt} \quad (1.3)$$

$$\begin{bmatrix} \lambda_\alpha \\ \lambda_\beta \end{bmatrix} = \begin{bmatrix} L_\Sigma + L_\Delta \cos(2\theta_{\text{em}}) & L_\Delta \sin(2\theta_{\text{em}}) \\ L_\Delta \sin(2\theta_{\text{em}}) & L_\Sigma - L_\Delta \cos(2\theta_{\text{em}}) \end{bmatrix} \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix}, \quad (1.4)$$

with θ_{em} being the electromechanical position of the rotor. The terms L_Σ and L_Δ represent the averaged inductance and variation of inductance, and can be expressed using the maximum and minimum values of the inductances, L_d and L_q :

$$L_\Sigma \triangleq (L_d + L_q)/2 \quad L_\Delta \triangleq (L_d - L_q)/2. \quad (1.5)$$

In the rotor reference frame (dq)

Assuming isotropic and homogeneous properties of the ferromagnetic materials, the d axis and q axis of minimum and maximum reluctance paths are purely geometrical and define the (dq) coordinate system, attached to the rotor. In this reference frame, the electric equations become:

$$v_d = R_s i_d + \frac{d\lambda_d}{dt} - \omega_e \lambda_q \quad (1.6)$$

$$v_q = R_s i_q + \frac{d\lambda_q}{dt} + \omega_e \lambda_d \quad (1.7)$$

$$\begin{bmatrix} \lambda_d \\ \lambda_q \end{bmatrix} = \begin{bmatrix} L_d & 0 \\ 0 & L_q \end{bmatrix} \begin{bmatrix} i_d \\ i_q \end{bmatrix}, \quad (1.8)$$

with ω_e being the electromechanical speed of the rotor, equal to the number of pole pairs p times the mechanical speed of the rotor ω_m . Expression (1.8) shows that, in the absence of magnetic saturation, the components of the fluxes are decoupled from each other when expressed in this frame of reference. The electromagnetic torque is given by:

$$T_{\text{em}} = \frac{3}{2}p |\lambda_{dq} \times \mathbf{i}_{dq}| = \frac{3}{2}p (\lambda_d i_q - \lambda_q i_d) \quad (1.9)$$

$$= \frac{3}{2}p (L_d - L_q) i_d i_q. \quad (1.10)$$

Equations (1.6) to (1.9) can be used to establish the operation diagram of the motor under current and voltage limitations. Such analyses typically involve saliency ratio ζ , current angle θ_e , and load angle δ , which are all dimensionless parameters:

$$\zeta \triangleq \frac{L_d}{L_q} \quad \tan(\theta_e) \triangleq \frac{i_q}{i_d} \quad \tan(\delta) \triangleq \frac{\lambda_q}{\lambda_d}. \quad (1.11)$$

The maximum torque per ampere (MTPA) is reached for $i_d = i_q$, which implies $\tan(\theta_e) = 1$, i.e., $\theta_e = 45^\circ$. This is a major difference from a surface mounted permanent magnet synchronous motor, which uses $i_d = 0$. The maximum torque per volt (MTPV) is reached for $\lambda_d = \lambda_q$, which implies $\tan(\delta) = 1$. The angles δ and θ_e are interconnected together through the saliency ratio ζ :

$$\tan(\delta) = \frac{1}{\zeta} \cdot \tan(\theta_e). \quad (1.12)$$

We also derive the internal power angle ϕ_i , being the angle between the back-EMF and the current vectors. The cosine of this angle corresponds to the internal power factor (IPF):

$$\phi_i = \frac{\pi}{2} + \delta - \theta_e \quad \text{IPF} \triangleq \cos \phi_i. \quad (1.13)$$

Higher IPF is desirable as it lowers the reactive power, leading to a reduction of the rating of the inverter. This form of control is called maximum torque per volt-ampere (MTPVA). As with MTPA and MTPV, the IPF is limited by the reachable saliency ratio ζ which entirely depends on the design of the motor. Using (1.12) in (1.13), the maximum reachable IPF is:

$$\max \text{IPF} = \frac{\zeta - 1}{\zeta + 1} \quad \text{at } \tan \theta_e = \sqrt{\zeta}. \quad (1.14)$$

Therefore, maximizing the saliency ratio allows maximizing the internal power factor.

1.1.2 Effects of ferromagnetic saturation

The saturation of ferromagnetic materials results in non-linear relationships between fluxes and currents. In general, this can be expressed as:

$$\lambda_d = \lambda_d(i_d, i_q) \quad \lambda_q = \lambda_q(i_d, i_q) \quad (1.15)$$

$$\Rightarrow L_d = L_d(i_d, i_q) \quad \Rightarrow L_q = L_q(i_d, i_q). \quad (1.16)$$

The effects of saturation in the q axis are usually small, as this axis is essentially constituted of air and, therefore, already saturated. Conversely, the d axis is extremely sensitive to saturation because this axis is mainly constituted of ferromagnetic materials. For MTPA control, the result is a shift in the optimal angle θ_e towards 90° , as a marginal current increase is more valuable in the q axis than in the d axis.

In addition, the interdependence of d axis flux with respect to the q axis current and vice-versa is called cross-saturation. Although these axes are orthogonal, this effect exists because these axes share the same iron elements. This cross-coupling is known to have significant effects in sensorless control [3, 4].

1.1.3 Theoretical limits to the saliency ratio

The maximum theoretical value of the saliency ratio is constrained by the presence of the airgap in the d axis and the finite, non-zero permeability of air in the q axis.

Assuming an ideal rotor, infinitely permeable along the flux lines and completely impermeable across them, and an ideal stator, equivalent to a current sheet, a theoretical upper limit of $\zeta = 60$ can be established [5]. However, when considering additional limiting factors, such as slotting effects, the practical saliency ratio hardly exceeds 10. Consequently, this limitation results in an internal power factor (IPF) lower than 0.82, which is inferior to the typical values encountered in induction motors at full load. Despite the increased efficiency and advantageous life cycle cost, the power factor remains a significant drawback of SynRMs.

1.1.4 Key geometrical features

The key geometrical features of the rotor of a SynRM have been refined over the last century to address the challenge of poor power factor in these machines. In 1923, Kotsko highlighted the limitations of salient pole topologies, Fig. 1.1 (a). He emphasized that any decrease in the q axis inductance L_q by widening the cut-off implies a reduction of the pole arc, leading to an unwanted decrease in the d axis inductance L_d [6]. This effect is mitigated in a multi-layered structure, where one or more flux barriers for the q axis flux are interleaved with flux guides for the d axis flux, as depicted in Fig. 1.1 (b), and in Fig. 1.3 for a 4-pole rotor.

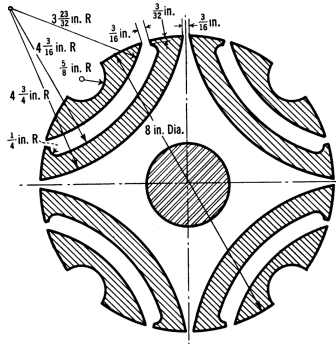


Figure 1.3: Rotor of a SynRM as described by Kotsko [6].

While the multi-layered rotor structure offers significant magnetic advantages, the manufacturing and assembly of segments with different shapes, as illustrated in Fig. 1.4 (a), makes its construction considerably more challenging. To capitalize on existing tooling and maintain a low manufacturing cost, the rotor geometry is adapted by connecting the segments together as shown in Fig. 1.4 (b).

By employing these connecting bridges, referred to as ribs in this text, the construction aligns with industry standards, involving the straightforward process of stacking identical laminations axially after cutting them into the appropriate shape.

The significantly improved manufacturability of TLA justifies the widespread use of ribs in SynRMs. Nevertheless, these ribs also create a short-circuit for a

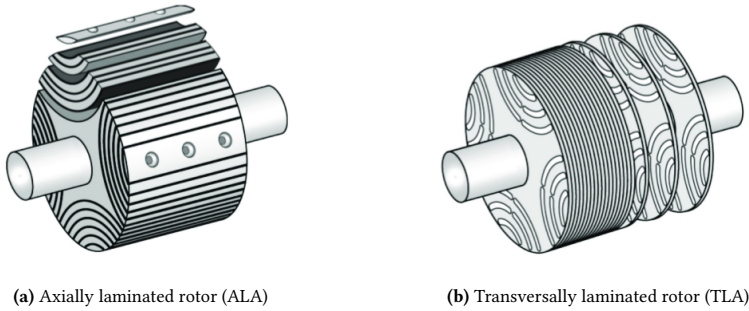


Figure 1.4: Construction of the rotor of a SynRM [7]

portion of the flux in the q axis, as they are constructed from the same ferromagnetic material as the flux guides. This results in a reduction of the magnetic anisotropy of the rotor. Therefore, precise dimensioning of the ribs is essential to withstand the mechanical load to which the rotor is subjected while minimizing their negative impact on magnetic performance.

1.2 Optimal design and challenges

The specifications for designing a SynRM can be as diverse as maximizing mean torque, power density, or efficiency, or minimizing torque ripple, acoustic noise, or ensuring structural integrity. The optimal design is typically obtained by performing a black-box optimization, as schematized in Fig. 1.5. In this scheme, an optimization algorithm explores the search space iteratively, provided the information gradually obtained.

Black-box optimizations have the advantage of being flexible in application, but implies a high number of iterations. The issue of high computational time stands out as a major challenge in the optimal design of SynRM, where the complex rotor geometry demands a considerable number of parameters, and addressing the torque ripple requires evaluating a non-linear magnetic model at different rotor positions. Due to the extended evaluation time required by numerical models, it is not uncommon to encounter optimization times spanning several months of CPU time, even for the simplest problems of optimizing the rotor on a single operating point [9, 10].

This unreasonable computational time has been identified as one of the limiting factor in addressing larger-scale problems, such as stator optimization, consideration of additional objectives, or optimization on several operating points [11–15]. To address this challenge, motor designers typically intervene at the model level to minimize evaluation time. Alternatively, they break down the optimization problem into smaller, more manageable problems, employing what are commonly referred to as time-efficient (or computationally-efficient) design approaches [16–18]. The

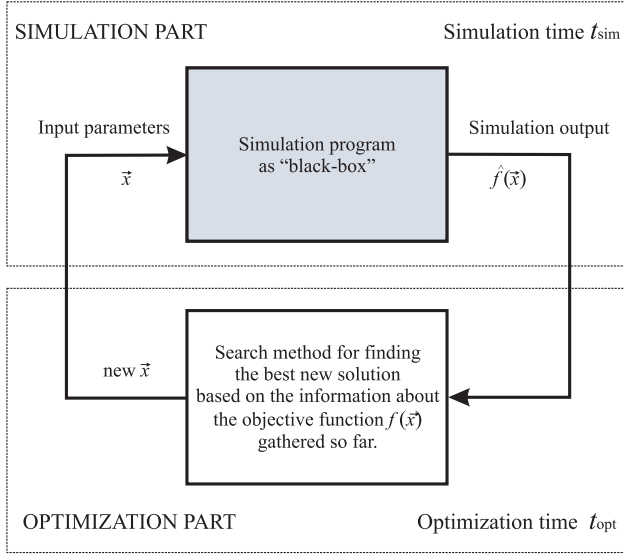


Figure 1.5: Black-box optimization [8].

subsequent subsection provides an in-depth review of these strategies.

1.2.1 Lowering the evaluation time

With the goal of reducing the evaluation time, a semi-analytical approach using Fourier-based modeling was proposed in [19], with strict assumptions on the shape of flux barriers, the linearity of ferromagnetic materials, and the absence of ribs. The reported evaluation time was similar to the one of a finite element (FE) model, partly attributed to the lack of code optimization in the implementation but also to the high number of regions. As a viable alternative, Magnetic Equivalent Circuits (MEC) present themselves as promising candidates for addressing electro-magnetic evaluations. Nevertheless, the accurate modeling of reluctance motors using MEC poses a challenge due to the pronounced effects of highly localized saturation and flux fringing [20, 21]. Using finite element methods (FEM), Pellegrino et al. proposed an evaluation method for the fundamental and third harmonic components of the torque ripple. Their approach involves only 6 distinct rotor positions to capture these harmonics, achieved by introducing a random initial position offset during the optimization process [22, 23].

Adhering to FEM, the evaluation time can be significantly reduced to a negligible amount by incorporating a metamodel. However, achieving an accurate metamodel across the entire search space remains challenging, especially near an optimum that was not especially captured by the design of experiments (DoE). Consequently, FE calls are often performed within the optimization routine to fit and update a metamodel around the region of interest. In return, the metamodel can provide the direction of the steepest descent to the optimization algorithm. Among the

various local metamodeling techniques, the Response Surface Method (RSM) with a second order polynomial stands out as a popular choice in the literature. In [24], this approach has been applied for the 3-objectives problem of maximizing the torque, torque density and efficiency of a SynRM. In [25], the weighted objectives were the efficiency, power factor, and torque ripple. Similarly, torque density and torque ripple have been optimized using RSM in [26]. If researches accounting for the structural integrity within the optimization process are less common, there are still examples assessing for its feasibility using RSM [27].

1.2.2 Lowering the complexity of the optimization

Another approach to lower the optimization time consists in simplifying the optimization problem itself, one possibility being to decompose the optimization problem into smaller-sized optimization problems. This approach is entirely compatible with the methods that aim at reducing the evaluation time, so that the two strategies can be adopted simultaneously [10].

In the realm of optimization, these approaches operate by relaxing what is commonly referred as the curse of dimensionality, which posits that the search space of large-scale problems grows exponentially with an increase of dimension. A prominent paradigm inspired by this divide-and-conquer principle is the Cooperative Coevolution (CC) [28], which interfaces well with popular population-based derivative-free search algorithms. These frameworks are typically employed to tackle optimization problems having very large dimensionality. Notably, it has been demonstrated that a perfect variable grouping strategy can exponentially reduce the search space without losing optimization accuracy [29]. This approach is, however, sensible to the decomposition strategy, and a poor grouping will reduce the optimization accuracy [30, 31].

While the dimensionality of the optimization problem of a SynRM remains low despite its complexity, a comparable divide-and-conquer strategy can be implemented without relying on specialized frameworks like Cooperative Coevolution. These dividing strategies typically leverage a priori knowledge of the physical interactions between parameters and objectives. The decomposition of the problem is usually performed at the objectives level, given that the contributing factors are easily identified. An additional advantage is to reduce the number of models involved in evaluations as long as they are not coupled, effectively reducing the evaluation time. This reduction is particularly notable when the structural integrity is part of a magnetic optimization problem, as the evaluation time is highly asymmetrical between these objectives. Indeed, the structural integrity is inherently a linear problem, while the evaluation of torque ripple or mean torque involves several calls of a non-linear magnetic model.

Relying on these principles, R. R. Moghaddam addressed in [32] the challenge of maximizing mean torque while minimizing torque ripple. In his approach, the mean torque is first maximized using the magnetic insulation ratio of the rotor ξ , corresponding to the thickness of air in the q axis divided by the total thickness

of the rotor lamination. Subsequently, torque ripple is minimized by finding the optimal rotor slot pitch in the d axis, keeping ξ constant. This study, considered a pioneering work in the field of SynRM, successfully addressed the initial problem through a simple and rapid design procedure.

Following this work, the consideration of mechanical integrity is usually performed in the final step. In [33], a multi-physics optimization of a SynRM across several operating points was achieved, with the mechanical analysis conducted in the final step of the design approach. Similarly, RSM was employed in [34] to minimize the torque ripple of a PMA-SynRM, with the structural analysis performed post-optimization. Additionally, in [35], the mechanical design of the magnetically-optimized rotor was ensured using topology optimization, resulting in a geometry featuring radial and tangential ribs.

While the reviewed approaches effectively address the initial problem through a rapid and straightforward design procedure, they often fall short in providing sufficient guarantees regarding the quality of the solution it generates. Beyond the macroscopic physical reasoning concerning parameter interactions on the target objectives, there is a need for more robust assurances as optimizing all parameters collectively may still yield a substantial performance improvement.

1.3 Positioning of the research

Looking at the problem of optimizing a SynRM from a different perspective, it is evident that the operating principles of this machine intricately tie to the rotor geometry. Consequently, any alterations made to the geometry will inevitably influence the objectives to some extent. The success of the aforementioned approaches that separate the optimization problem to reduce the overall optimization time hinges on two critical conditions:

1. Proper identification of the primary influencing factors for each objective.
2. Negligible impact of parameters optimized for other objectives.

In the domain of electric motors, these assumptions are commonly validated through parametric analyses, also referred to as One-At-a-Time (OAT). This method involves moving each parameter between its minimum and maximum values while observing the effects on the outputs, keeping the other parameters at their nominal values. However, this conventional approach is widely recognized within the field of sensitivity analyses as prone to significant errors [36]. This is primarily due to its underlying assumption of linear dependence between objectives and input parameters, an assumption that does not hold true for SynRM [32, 37, 38].

Although alternative, more comprehensive methods for sensitivity analyses exist [39], they remain significantly underutilized in the domain of electric motors. This underutilization can be attributed to the substantial computational cost associated with such analyses, often requiring a number of model evaluations comparable to those needed for optimizing the entire search space [40, 41]. As a result, current

research efforts aimed at developing time-efficient design approaches seem to be trapped in a deadlock. Indeed, the resulting designs from these approaches cannot be effectively benchmarked, as the design obtained from optimizing the entire search space remains unknown, and the validity of the assumptions underpinning these approaches cannot be adequately demonstrated.

When considering the trends in computational power over the last 50 years, as illustrated in Fig. 1.6, there is little hope in the foreseeable future of attaining sufficient computing power to solve these complex optimization problems in reasonable times. Solving an optimization inherently presents a sequential challenge, where gains from an increasing number of logical cores are marginal compared to those from clock frequency or single-thread performance. However, the evolution of clock frequency has remained stagnant over the last two decades, and a comparable saturation in single-thread performance is becoming evident.

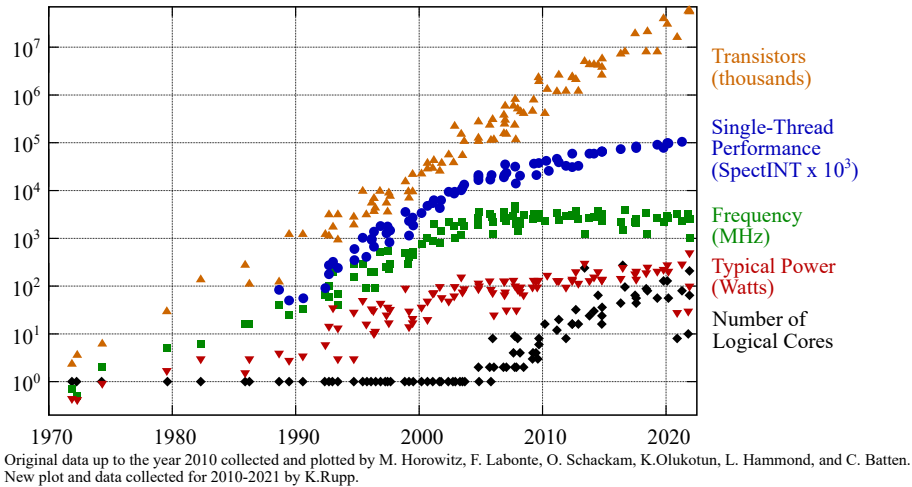


Figure 1.6: 50-year trends in microprocessors [42].

Conversely, the number of logical cores has experienced a remarkable increase since 2010, favoring tasks that are fully parallelizable. The evaluation of quasi-Monte-Carlo samples used in sensitivity analyses falls within the class of embarrassingly parallel problems, extremely appropriate for parallel computing, with the added advantage of neither requiring the implementation of message-passing interfaces (MPI) nor to manage shared resources (OpenMP). If the number of logical cores remains insufficient for conducting sensitivity analyses on a dedicated workstation, this capability can already be unlocked by utilizing cloud-computing services that have been made available since 2010. These services provide virtual access to an incredibly high number of CPU cores at a reasonable cost, which is expected to continue decreasing over time.

As a consequence, we anticipate that this shift in the nature of computing power will fundamentally influence the approach to designing electric machines. The current research aims at showcasing the potential of integrating sensitivity analyses

at the core of electrical machine design, with the primary objective of developing time-efficient design approaches.

Additionally, we envisage the possibility to develop time-efficient design approaches in the presence of non-negligible parameters across several objectives through a proper penalization or iteration procedure. This reliance hinges on a single critical condition:

«Precise quantification of the primary impacting factors, and their interactions, for each objective. »

This condition is naturally satisfied by performing the sensitivity analysis. Consequently, the proposed methodological framework is expected to address the major limitations of existing time-efficient design approaches, which include their lack of assurance they provide in the quality of the output design, and their difficulty in extending to additional objectives.

1.4 Case study

Our application framework centers on evaluating the feasibility of replacing the rotor of a commercially available induction motor used in railway applications to convert it into a synchronous reluctance machine. This transition, as highlighted in the introductory section, holds the promise of enhancing motor efficiency, reliability, and overall life-cycle cost reduction.

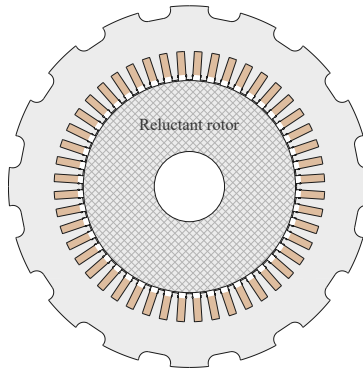


Figure 1.7: Active part of the investigated motor.

Our specific objective involves designing a reluctance rotor for use with a conventional 48 slots 3-phase stator with short-pitched windings, as illustrated in Fig. 1.7. The primary characteristics of the variable frequency inverter-fed induction motor are detailed in Table 1.1, with M400-50A ferromagnetic materials employed for both the stator and the rotor.

Consequently, we aim at identifying the rotor design that maximizes the mean torque while adhering to torque ripple and structural integrity constraints. Iron

losses, power factor, and sensorless capability are additional quantities of interest, but that will not be optimized.

Table 1.1: Main parameters of the motor

Param.	Description	Value	Unit
p	Number of pole pairs	2	—
P_n	Rated power	300	kW
j_n	Rated current density	8.5	A/mm ²
N_n	Rated speed	1800	rpm
$N_{\max}$	Overspeed limit	4800	rpm

The objective of mean torque maximization, alongside torque ripple and structural integrity constraints, mirrors typical objectives encountered in synchronous reluctance motor designs. Following the pioneering work of R. R. Moghaddam in developing time-efficient design approaches, this case study represents a modest extension by incorporating a structural integrity constraint. However, the simultaneous consideration of torque ripple and structural integrity presents considerable challenges in developing time-efficient design approaches, as both constraints are highly sensitive to parameter variations. Consequently, the optimization problem under examination provides an excellent test case to showcase the potential benefits of integrating sensitivity analyses into the design process of synchronous reluctance motors.

1.5 Objectives of this research

Building upon existing works, our primary motivation is to introduce a comprehensive methodology for the optimal design of the rotor of a synchronous reluctance machine, specifically addressing the challenges of torque ripple, mean torque, and structural integrity. At the core of this study is an extensive sensitivity analysis aimed at quantifying the impact of parameters and their interactions on crucial outputs of interest. Within this framework, our objectives are two-fold:

1. To address goals specific to the design of synchronous reluctance motors, focusing on the unique aspects of this machine.
2. To identify and address more methodological objectives that go beyond the specificities of synchronous reluctance machines, providing insights applicable to a broader range of electrical machines.

These objectives are detailed in the next two subsections.

1.5.1 Analysis of synchronous reluctance motors

The primary goal of developing time-efficient approaches for the optimal design of SynRM encompasses several key sub-objectives:

- Propose a comprehensive parametrization of the rotor's geometry. This involves a discussion on the shape and number of flux barriers, the management of non-overlapping constraints between these barriers, and the exploration of a parametrization incorporating the magnetic insulation ratio ξ as an explicit design parameter.
- Quantify the effects of the parameters on critical performance metrics such as mean torque, torque ripple, and structural integrity.
- Discuss and benchmark different implementations of time-efficient design approaches, assessing both the quality of solutions generated and the total optimization time.
- Propose and benchmark improved implementations of time-efficient design approaches, based on sensitivity analyses.

In addition to these, the pursuit of finding the optimal design of SynRM introduces two supplementary goals:

- Derive a design metric associated to the motor's suitability for sensorless control, offering advantages such as cost reduction, increased reliability, easier integration, higher control bandwidth, and enhanced compatibility with harsh environments.
- Explore the benefits of incorporating a cut-off barrier at the periphery of the rotor, as illustrated in Fig. 1.3.

1.5.2 Development of methodological frameworks

Following the goals set for synchronous reluctance motors, our investigation extends to diverse methodologies with broader scopes:

- Providing a comprehensive review of methods involved in sensitivity analysis, and detailing the interpretation of key metrics derived from such analyses.
- Utilizing the results of sensitivity analyses to propose time-efficient design approaches.
- Comparing motor topologies across several outputs of interest, while avoiding resource-intensive optimizations.

1.6 Organization of the thesis

To achieve the outlined objectives, this work is structured as follows:

- **Chapter 2** describes the numerical models used to evaluate the quantities of interest, such as the mean torque, the iron losses, or the torque ripple. The effects of the magnetic forces on the mechanical stresses are discussed. Prioritizing the accurate modelling of the motor, the evaluation of magnetic quantities of interest is validated experimentally.

- **Chapter 3** reviews the shapes of flux barriers and possibilities in parametrization to prevent them from overlapping during the automated exploration of the search space. Two parametrizations are proposed, the first involves a direct representation of the geometry while solving non-overlapping constraints. The second parametrization proposes a more complex construction of the geometry, so that the magnetic insulation ratio ξ can be an explicit parameter of the search space, following the works of R. R. Moghaddam.
- **Chapter 4** presents the results of the sensitivity analyses of the two parametrizations. Central to this work, the focus is put in selecting adequate metrics and presenting the results in a comprehensive manner. The results permit to gauge the success of the parametrizations, and are used to discuss and propose design approaches in the subsequent chapters.
- **Chapter 5** delves into the possibilities and challenges associated with developing time-efficient approaches for the optimal design of SynRM. Consistent with the literature, these approaches ground their foundations using physical reasoning. These approaches are benchmarked by comparing the quality of solutions and total optimization times.
- **Chapter 6** introduces a methodology for deriving design strategies directly from the results of the sensitivity analysis. The aim is to establish a more systematic approach for developing a time-efficient design strategy, that leverages the advantages of the parametrization. This methodology is illustrated for both parametrizations.
- **Chapter 7** investigates the benefits of integrating a cut-off barrier to the rotor geometry, recognizing its potential for improved power factor and suitability for sensorless control. Beyond the comparison of different rotor configurations, this chapter tackles the objectives of formulating a design metric for sensorless suitability and devising a methodology to compare topologies across diverse objectives without relying on resource-intensive optimizations. The principles of the methodology are demonstrated by incorporating rotors featuring a single inner-flux barrier into the comparative analysis.
- **Chapter 8** summarizes findings, providing final remarks and perspectives for future work.

Numerical models

2

Numerical models are used to predict the performance of a given motor. In this research, they are employed in optimization loops to find the configuration that maximizes the performance, or in sensitivity analyses to gather information on the design space. For models, there are no distinctions between objectives or constraints of an optimization, referring to them as outputs of interest. This chapter aims to provide a detailed account of the magnetic model, solving Maxwell's equations to calculate torque, magnetic flux, and iron losses. Additionally, the mechanical model, employing the equations of solid mechanics to assess the structural integrity of the rotor, is outlined.

A common challenge in modeling is finding the right balance between model accuracy and evaluation time. In this research, the focus is on reducing optimization time by lowering the complexity of the optimization rather than modifying the models, although the two approaches are compatible. Therefore, the emphasis is on avoiding modeling inaccuracies to ensure the validity of the results. Consequently, the inherent non-linearity of ferromagnetic materials and the complexities of rotor geometry are addressed by utilizing finite elements.

A second challenge specific to SynRM revolves around incorporating considerations of structural integrity during the design of the active part of the rotor. The cross-coupling between the magnetic and mechanical models, manifested as magnetic forces, complicates the derivation of time-efficient design approaches. This complexity arises from the need for systematic evaluation of magnetic forces when assessing structural integrity. Consequently, this chapter explores possibilities for accounting for these forces without directly evaluating them with the magnetic model.

To achieve these goals, this chapter is organized as follows. First, the magnetic model is detailed in section 2.1, and validated experimentally. The extraction of magnetic forces, used to assess the cross-coupling, is derived in subsection 2.1.5. Subsequently, the mechanical model is detailed in section 2.2, along with the structural integrity criterion. From this model, the effects of magnetic forces are assessed and discussed in subsection 2.2.3.

2.1 Magnetic Model

2.1.1 Fundamental equations

The electromagnetic field, from which the electromagnetic forces can be derived, is governed by the Maxwell's equations:

$$\nabla \cdot \mathbf{D} = q \quad (2.1)$$

$$\nabla \cdot \mathbf{B} = 0 \quad (2.2)$$

$$\nabla \times \mathbf{E} = -\partial_t \mathbf{B} \quad (2.3)$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \partial_t \mathbf{D}, \quad (2.4)$$

with $\mathbf{D}$ the electric flux density, q the electric charge, $\mathbf{E}$ the electric field, $\mathbf{H}$ the magnetic field, and $\mathbf{B}$ the magnetic flux density. The relations between the different physical quantities are described using constitutive laws. These relations are specific to a medium and can be anisotropic, nonlinear, dependent on other physical quantities, or exhibit some memory effect leading to hysteresis. In the simple case of isotropic linear materials, we have:

$$\mathbf{D} = \varepsilon \mathbf{E} \quad \mathbf{B} = \mu \mathbf{H} \quad \mathbf{E} = \rho \mathbf{J}, \quad (2.5)$$

where ε , μ , and ρ are the permittivity, permeability, and resistivity of the material, respectively.

In electromagnetism, the Maxwell's equations are usually approximated by performing a dimensional analysis on the characteristic lengths and times involved in the problem. For electric motors, the dimensional analysis shows that the magneto-quasi-static approximation of the Maxwell equations applies [43]. From this approximation, we also neglect the contribution of eddy currents on the $\mathbf{H}$ field, so that the field entirely results from the prescribed current density $\mathbf{j}_s$ in the windings. Under these conditions, the magnetic field can be described using the magnetostatic equations only:

$$\nabla \cdot \mathbf{B} = 0 \quad (2.6)$$

$$\nabla \times \mathbf{H} = \mathbf{j}_s. \quad (2.7)$$

The windings, air, and rotor shaft are constituted of non-magnetic materials. They obey the constitutive law (2.5) using the constant permeability of free space¹. Nevertheless, this law is not sufficient to describe accurately the behavior of ferromagnetic materials under the magnetic fields encountered in electric motors. While isotropy can be assumed for non-grain oriented electrical steel, the constitutive equation follows a non-linear law:

$$\mathbf{B} = \mathcal{B}(\mathbf{H}). \quad (2.8)$$

Assuming it exists, we write $\mathcal{H}$ the inverse mapping: $\mathcal{H} := \mathcal{B}^{-1} : \mathbf{B} \rightarrow \mathbf{H}$:

$$\mathbf{H} = \mathcal{H}(\mathbf{B}). \quad (2.9)$$

¹ $\mu_0 = 4\pi 10^{-7}$ (H/m)

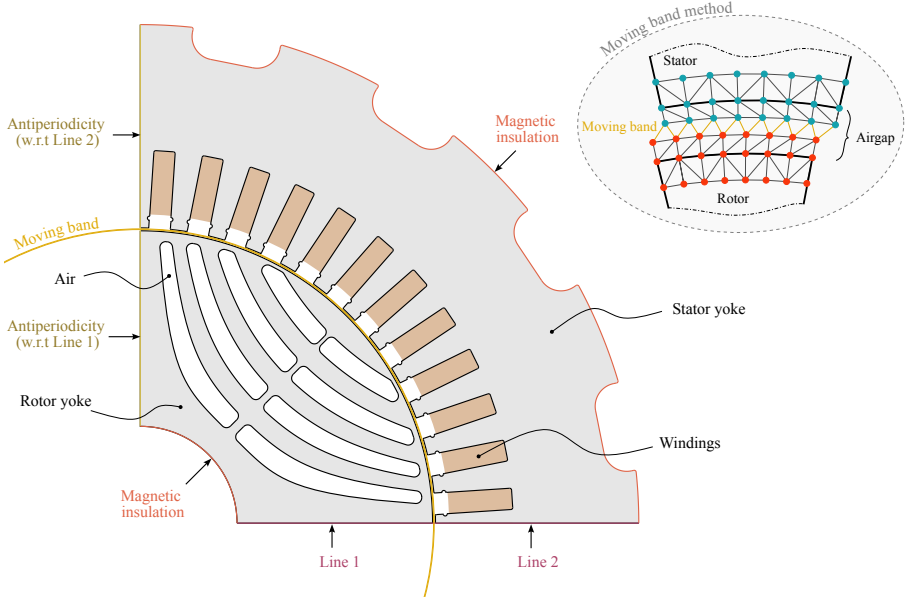


Figure 2.1: Boundary conditions of the magnetic model.

Since $\mathbf{B}$ is divergence-free (2.6), it derives by definition from a vector potential $\mathbf{A}$, such that $\mathbf{B} = \nabla \times \mathbf{A}$. Using the constitutive equation (2.9) in (2.7), we have:

$$\nabla \times (\mathcal{H}(\nabla \times \mathbf{A})) = \mathbf{j}_s. \quad (2.10)$$

The partial differential equation (2.10) is solved using finite elements to accommodate the complex geometry and nonlinearity of ferromagnetic materials.

2.1.2 Resolution

Given the small airgap length and high permeability of the ferromagnetic materials, we neglect the end-effects and reduce the domain of resolution to the 2D plane of the motor. The domain is further reduced to a single pole by employing appropriate anti-periodicity conditions, as illustrated in Fig. 2.1. We assume that the magnetic field remains confined within the rotor and stator yokes, whose inner and outer dimensions will not be altered in this study. This eliminates the need for meshing additional air around the geometry.

Assuming perfect sinusoidal current control, the z-axis current densities $\mathbf{j}_s = j_s^i \hat{\mathbf{e}}_z$ of the three-phase windings $i = \{a, b, c\}$ are imposed as:

$$\begin{pmatrix} j_s^a \\ j_s^b \\ j_s^c \end{pmatrix} = j_s \begin{pmatrix} \sin(\theta_e + p\theta_m) \\ \sin(\theta_e + p\theta_m - 2\pi/3) \\ \sin(\theta_e + p\theta_m + 2\pi/3) \end{pmatrix}, \quad (2.11)$$

with j_s being the peak current density, θ_e the current angle defined in (1.11), and θ_m the mechanical angle.

The relative position of the rotor with respect to the stator is handled using the moving band method. This method dissociates the rotor mesh from the stator mesh, removing the need to remesh the full geometry each time the position of the rotor is altered. Ergo, a position increment involves rotating the rotor mesh and then stitch it back with the stator mesh by inserting triangular mesh elements in the middle of the airgap. For this method to be compatible with magnetic resolution over a pole, the moving band must be complete. This entails that the moving band arcs, one for the stator and one for the rotor, cover the full revolution. Anti-periodicity boundary conditions are applied on the successive pole segments of the arcs.

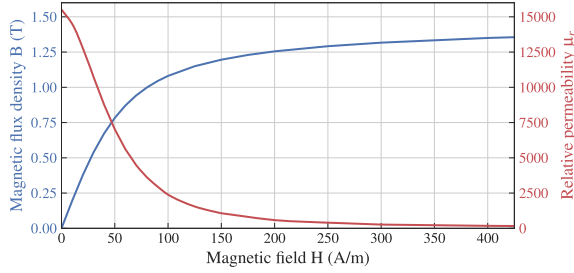


Figure 2.2: Magnetic properties of the silicon steel M400-50A.

The non-linearity of the ferromagnetic materials is handled by an iterative Newton-Raphson resolution scheme on a piecewise linear interpolation of the an-hysteretic $B - H$ curve. Associated with the moving-band method, the solution of the previous position is used as starting point for the nonlinear resolution. That lowers the number of iterations needed in the Newton-Raphson (NR) resolution scheme, which benefits the global resolution time.

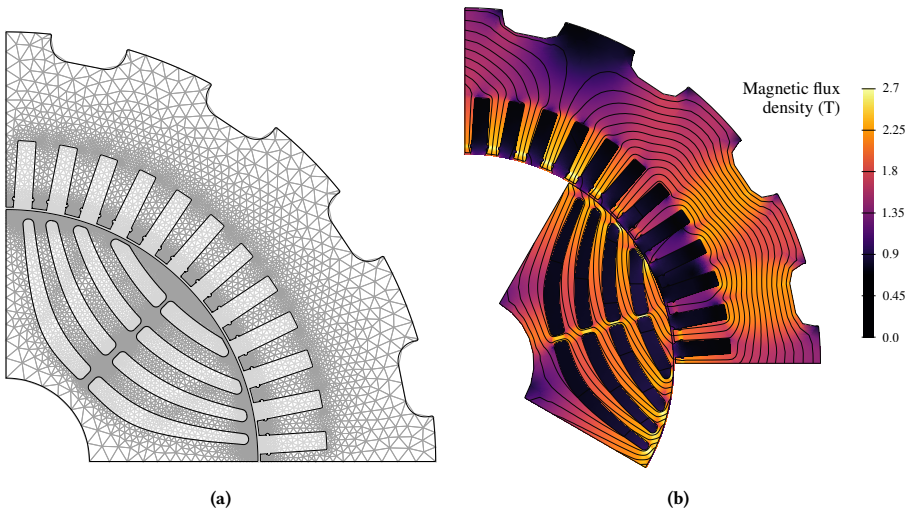


Figure 2.3: Resolution of (2.10) at nominal current, with $\theta_e=45^\circ$ and $\theta_m=-30^\circ$. (a) Mesh, with ~ 28000 elements (b) Flux density and flux lines.

An example of mesh and resulting flux density is provided in Fig. 2.3, utilizing the properties of the M400-50A electrical steel as indicated in Fig. 2.2. This model utilizes the mesh generator Gmsh [44] and the solver GetDP [45]. The mesh is refined in the ribs, in the airgap, and at the bottom of the stator slots to accurately capture the effects of magnetic saturation and changes in magnetic field. It takes around 10 seconds (CPU time) with 16 NR iterations to perform an evaluation that has not been initialized. Evaluating the magnetic field at a small position increment of one percent of the pole span takes around 3 seconds with 5 NR iterations. This highlights the benefits of employing the moving band method, allowing to reuse the previous solution as a starting point for the NR iterations.

2.1.3 Magnetic flux

The magnetic flux λ going through a surface S can be calculated using:

$$\lambda \triangleq \iint_S \mathbf{B} \cdot d\mathbf{S} = \iint_S (\nabla \times \mathbf{A}) \cdot d\mathbf{S} = \oint_{\partial S} \mathbf{A} \cdot d\mathbf{l}. \quad (2.12)$$

For an infinitesimal wire in our 2D model, this equation is reduced to:

$$\lambda^{wire} = \oint_{\partial S} \mathbf{A} \cdot d\mathbf{l} = L_a (A_z^+ - A_z^-), \quad (2.13)$$

with L_a being the axial length of the motor, A_z the z-component of the vector potential, and the superscript $+$ and $-$ designating the regions of go and back conductors. For a winding constituted of N_t turns, expression (2.13) is averaged over the surface of the slots S_L to get the averaged magnetic field of a turn, and then multiplied by N_t to get the flux λ^i of phase $i = \{a, b, c\}$:

$$\lambda^i = N_t \cdot L_a \cdot \left(\frac{1}{S_L^{i+}} \int_{S_L^{i+}} A_z^{i+} ds - \frac{1}{S_L^{i-}} \int_{S_L^{i-}} A_z^{i-} ds \right). \quad (2.14)$$

These fluxes are used in the electric equations of the motor, and required to calculate inductances.

2.1.4 Global magnetic forces

In electrical machines, the common approach to extract the magnetic forces acting on a body consists in integrating the Maxwell stress tensor T_M along a surface Γ placed in the air and enclosing this part [46]. Neglecting the contribution of the electric field, this tensor is reduced to:

$$T_M = \frac{1}{\mu_0} B_i B_j - \frac{1}{2\mu_0} B^2 \delta_{ij}, \quad (2.15)$$

allowing the calculation of the torque vector $\boldsymbol{\tau}$ as follows:

$$\boldsymbol{\tau} = \int_{\Gamma} \mathbf{r} \times (T_M \cdot \mathbf{n}) d\Gamma. \quad (2.16)$$

In this expression, $\mathbf{n}$ is the normal component of the surface of integration and $\mathbf{r}$ the lever arm, i.e., the vector that connects the origin to the segment $d\Gamma$. The moving-band arcs on the rotor-side are chosen as the surface of integration². In this case, the torque T is simply the z-component of $\boldsymbol{\tau}$, multiplied by L_a to account for the 3D geometry.

The mean torque $\bar{T}$ and torque ripple $\tilde{T}$ require the evaluation of the torque by sweeping N rotor positions $\theta_m = \theta_i$ over a pole span:

$$\bar{T} = \frac{1}{N} \sum_{i=1}^N T(\theta_i) \quad (2.17)$$

$$\tilde{T} = \frac{\max(T(\theta_i)) - \min(T(\theta_i))}{\bar{T}}. \quad (2.18)$$

In the following, we set $N = 150$ to properly capture the torque ripple.

2.1.5 Local magnetic forces

The local magnetic forces acting on the rotor are extracted using the discrete formulation of virtual work principles [47–49]. The increased computational cost associated to this method is not seen as a drawback, as the local magnetic forces are not evaluated during optimizations. Instead, their effects are incorporated into the structural integrity criterion for the mechanical design, as it will be discussed in subsection 2.2.3.

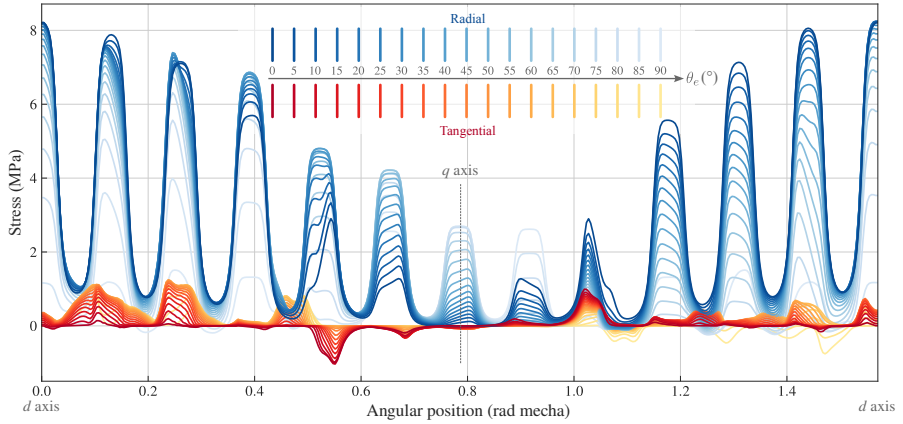


Figure 2.4: Radial and tangential magnetic stress exerted on the periphery of the rotor, at the nominal current and $\theta_m = 0^\circ$, as a function of the observation point and for different electric angles θ_e .

The radial and tangential components of the magnetic stress at the iron-air interface, considering the geometry in Fig. 2.3 (a), are provided in Fig. 2.4. The tangential stress, responsible for generating torque when $\theta_e \neq n\pi/2$ ($n \in \mathbb{Z}$), constitutes only

²A line, in 2D.

a minor portion of the radial stress. This indicates that the stator predominantly exerts a pulling force on the rotor, and only a slight imbalance contributes to the torque.

In accordance with the principle of action-reaction, these pulling forces also induce time-varying forces on the material points situated in the stator teeth when the rotor is rotating. By exciting the natural frequencies of the stator, these forces are one of the reasons behind the acoustic noise in electric machines [50, 51]. However, this aspect has been placed in the perspective of future works.

The worst-case scenario for the structural integrity of the rotor is identified at $\theta_e=0^\circ$, meaning that all the current is injected into the d axis. This is logical as, by definition, the d axis is the one that maximizes the generation of the flux and, consequently, radial forces. Conversely, the best-case scenario is encountered at $\theta_e=90^\circ$, meaning that all the current is injected into the q axis. In addition to having overall reduced radial forces, this configuration aligns these forces with the radial ribs, making it favorable for structural integrity. Therefore, we adopt the conservative configuration $\theta_e=0^\circ$ in section 2.2.3 to assess the mechanical stresses resulting from magnetic forces.

2.1.6 Iron losses

Iron losses frequently stand as the second major source of power losses in SynRM [52], behind the Joule losses in the stator windings [53]. An overview of the physical mechanisms behind iron losses and their modeling is provided in appendix B. In this work, we estimate the iron losses p_{Fe} through a posteriori models, following the iron loss decomposition of Bertotti:

$$p_{Fe} = p_h + p_{cl} + p_{exc}, \quad (2.19)$$

with p_h the static hysteresis losses, p_{cl} the classical eddy current losses, and p_{exc} the excess losses. The magnetic field is extracted on the elements of the position-stepping model developed in section (2.1), so that the total iron losses are calculated by summing up the contribution of each element. Using FEM, the formulae that integrate the instantaneous losses are favored due to the unavailability of direct harmonics information from the field. However, the implementation complexity of the iron loss models based on a frequency decomposition remains low, offering the added advantage of better accounting for the rotational nature of the field. In the end, we implement both possibilities to enable the cross-verification of the iron loss estimations through the two methods.

Using the instantaneous losses

The iron losses components are computed using the model detailed in [54]:

$$p_h = \frac{1}{T} \int_0^T \left| H_{irr} \frac{dB}{dt} \right| dt \quad (2.20)$$

$$H_{irr}(B, \hat{B}) = \frac{k_h \cdot \hat{B}}{\pi} \cdot \cos \left(\arcsin \left(\frac{B}{\hat{B}} \right) \right) \quad (2.21)$$

$$p_{cl} = \frac{k_{cl}}{2\pi^2 T} \int_0^T \left(\left| \frac{dB_x}{dt} \right|^2 + \left| \frac{dB_y}{dt} \right|^2 \right) dt \quad (2.22)$$

$$p_{exc} = \frac{k_{exc}}{8.76 T} \int_0^T \left(\left| \frac{dB_x}{dt} \right|^2 + \left| \frac{dB_y}{dt} \right|^2 \right)^{0.75} dt. \quad (2.23)$$

The source [54] also provides the coefficients k_h, k_{cl}, k_{exc} for the M400-50A:

$$k_h = 25.9 \cdot 10^{-3} \quad [\text{W kg}^{-1} \text{ s T}^{-2}] \quad (2.24)$$

$$k_{cl} = 125.7 \cdot 10^{-6} \quad [\text{W kg}^{-1} \text{ s}^2 \text{ T}^{-2}] \quad (2.25)$$

$$k_{exc} = 779 \cdot 10^{-6} \quad [\text{W kg}^{-1} \text{ s}^{1.5} \text{ T}^{-1.5}] \quad (2.26)$$

The derivatives and integrations are evaluated numerically.

Using the frequency decomposition of the field

For this model, a Fourier transform is applied on the individual components of the 2D magnetic field to extract both the amplitude and the phase shift of the harmonics. The representation of each harmonic takes the form of a rotated ellipse, with the major and minor axes' lengths recoverable from the amplitude and phase shift components of the harmonic. Using this representation, the iron losses induced by the rotating field are calculated as follows [55–57]:

$$p_{Fe} = p_h(B_{\parallel}) \cdot \gamma_1(B_{\perp}/B_{\parallel}) + p_{cl}(B_{\parallel}) + p_{cl}(B_{\perp}) + p_{exc}(B_{\parallel}) \cdot \gamma_2(B_{\perp}/B_{\parallel}). \quad (2.27)$$

For each frequency component f_i , the following formulae are utilized for the separation of the losses:

$$p_{h,i} = k_h \cdot f_i \cdot B^2 \quad (2.28)$$

$$p_{cl,i} = k_{cl} \cdot (f_i \cdot B)^2 \cdot (1 + a_3 B^{a_4}) \quad (2.29)$$

$$p_{exc,i} = k_{exc} \cdot (f_i \cdot B)^{1.5}. \quad (2.30)$$

The additional term in $a_3 B^{a_4}$ is a minor correction suggested in [58] to better account for the classical eddy current losses within a saturated ferromagnetic material. This reference provides the values $a_3 = 0.307$ and $a_4 = 2.975$ for the material M800-50A, which has the same lamination thickness as the M400-50A.

We employ the two functions $\gamma_1(B_{\parallel}/B_{\perp})$ and $\gamma_2(B_{\parallel}/B_{\perp})$ proposed in [59] for a silicon steel with a similar composition (3.2% Si). These functions are illustrated in Fig. 2.5 for a pure rotational field. Their values are reduced to zero for higher magnetic fluxes, resulting in the absence of hysteresis and excess losses when the field is strong enough. Counterintuitively, this phenomenon arises from the fact that, under saturation, all the magnetic moments of the atoms constituting the

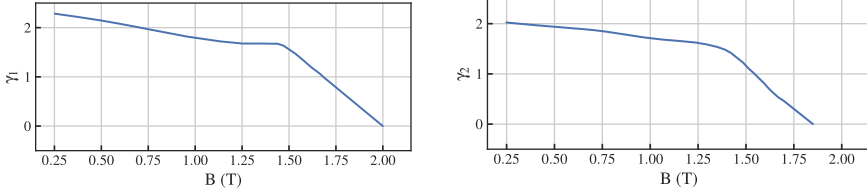


Figure 2.5: Functions γ_1 and γ_2 for a pure rotating field ($B_{\perp}/B_{\parallel} = 1$), in a FeSi electrical steel [59].

sample align in the same direction, effectively forming a single domain. With the absence of domain walls, changing the orientation of the excitation field without affecting its strength does not induce other losses than the classical eddy-current losses.

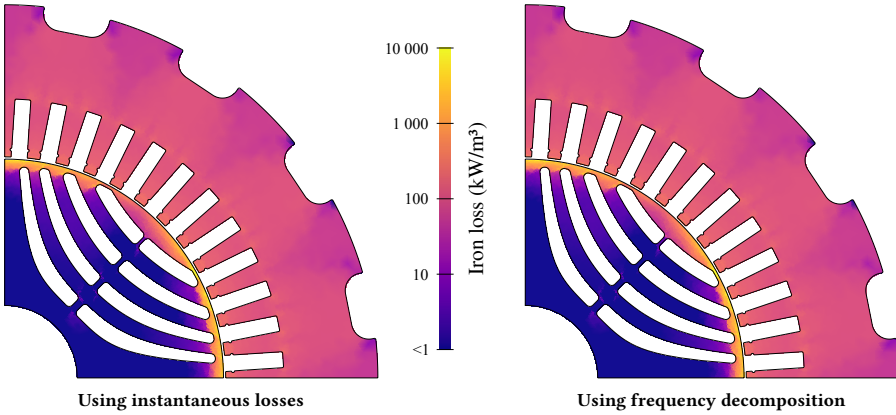


Figure 2.6: Distribution of the iron losses inside the motor, with a supply frequency of 70 Hz and at nominal current, using the two models.

The distribution of the iron losses in the motor is provided in Fig. 2.6. Visually, the two models exhibit similar behavior, though a more detailed comparison is conducted in the subsequent experimental validation. Despite the highest density of iron losses localized in the rotor's periphery, attributed to slot harmonics of the field, the stator still accounts for two-thirds of the total losses because of the rotating field.

2.1.7 Experimental validation

The rotor geometry of the experimental motor has been optimized to answer specifications of high torque and low torque ripple, using the models of this chapter. The geometry and associated search space correspond to the ones of the first parametrization that is described in the following chapter 3, with one minor geometrical adaptation to better distribute the stresses due to tight fitting the magnetic sheets on the shaft. Specifically, the innermost edge of the first flux barrier is given a curvature that follows more closely the one of the shaft by

substituting $R'_{in} = 0.9 R_{in}$ in (3.3). The resulting rotor geometry is visible in Fig. 2.7.

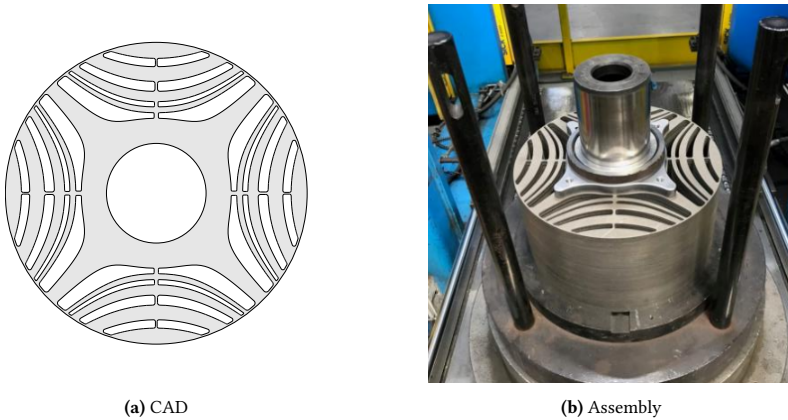


Figure 2.7: Rotor of the prototype used for experimental validation.

In the assembly process, the rotor laminations are cut, stacked, annealed, and tight-fitted on the shaft. The rotor is balanced under the G level 2.5 of grade ISO 2194-11, up to the designed overspeed limit.

The predicted characteristics of the motor are given on Fig. 2.8. The saliency ratio, Fig. 2.8 (a), increases sharply with i_q as a result of the saturation of the ribs, and decreases with i_d as a consequence of the saturation of the flux guides. Under MTPA operation, its nominal values range from 5 to 6, leading to an internal power factor of 0.7. Interestingly, the relative torque ripple, Fig. 2.8 (b), has minima that coincides with the MTPA line even though the enforcement was explicitly applied only for the nominal current. This is a nice feature that provides a smooth torque over a wide range of operating points. The constant torque hyperbolae and constant flux ellipses are given in Fig. 2.8 (c) and (d). Neglecting the resistive voltage drop, the intersection of the two graphs would give the MTPV trajectory. The efficiency map, Fig. 2.8 (e), is derived considering the DC supply voltage limitation. It accounts for the winding Joule losses at 100°C , computed using the measured stator resistance, and the iron losses, estimated using the instantaneous losses model developed in section 2.1.6.

In these graphs, the white dots correspond to operating points that have been experimentally characterized. To enhance identification, the operating points that have been established under the same experimental conditions, such as MTPA or constant flux, are connected. The experimental setup uses a torque-controlled load, and an extra torque-meter is added between the motor and the load for cross-validation. The flux is deduced from the applied voltages and resulting currents at the terminal level using (1.1). Consequently, the reported measured values are corrected to account for the experimentally characterized end-winding inductances.

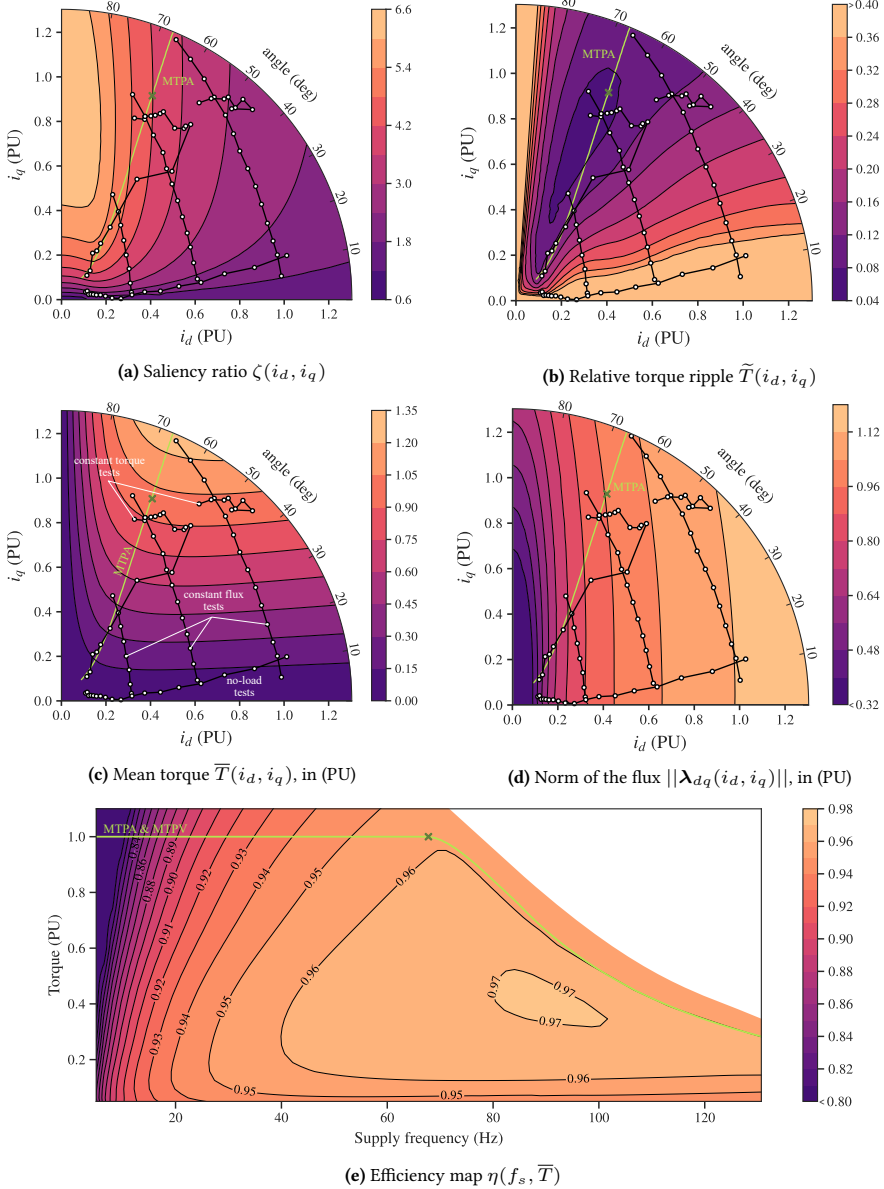


Figure 2.8: Predicted characteristics of the motor. The white dots represent experimentally characterized operating points. The cross denotes the operating point used for the design of the motor. The currents, mean torque and fluxes are normalized based on the characteristics of this reference point.

No-load tests

The no-load measurements are performed by controlling the flux $||\lambda_{dq}(i_d, i_q)||$ at a supply frequency of 32 Hz. The resulting current and voltages are measured and reported in Fig. 2.9 as a function of the controlled flux, with excellent agreement with the predictions. The effects of the magnetic saturation is visible in the flux-current characteristic, whereas the constant speed operation results in a linear voltage-flux characteristic.

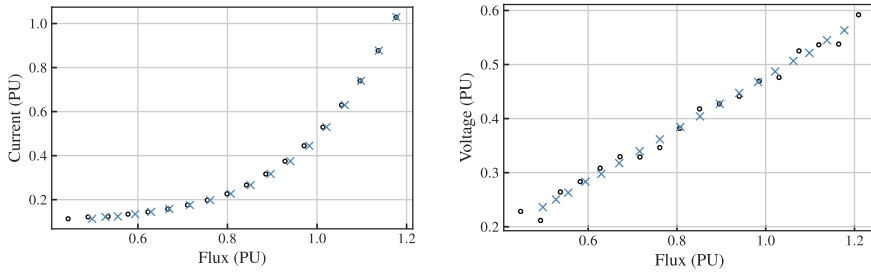


Figure 2.9: Comparison between the measured (black) and simulated (blue) characteristics of the experimental points, for the no-load tests. The voltages are expressed in PU of the maximum AC output voltage of the inverter.

Constant flux tests

As for the no-load test, the flux $||\lambda_{dq}(i_d, i_q)||$ is controlled at a supply frequency of 32 Hz. The load torque is then progressively increased. This control aims at keeping the input voltage constant regardless of the level of the load. The measurements, reported in Fig. 2.10, are performed for 3 levels of reference flux. The resulting torque is accurately predicted by the models.

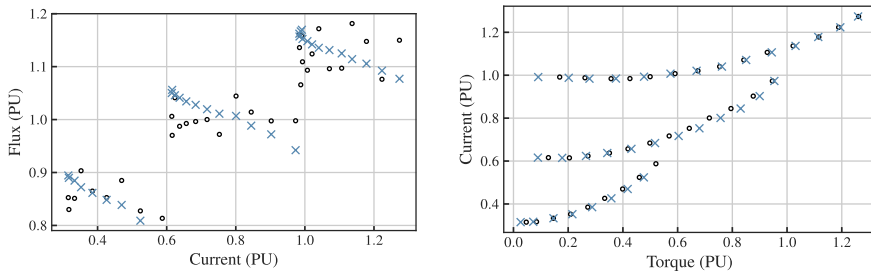


Figure 2.10: Comparison between the measured (black) and simulated (blue) characteristics of the experimental points, for the constant flux tests.

Constant torque tests

In these tests, the controller aims at maintaining a constant torque for increasing rotor speeds. The measurements are performed for 90% and 100% of the nominal

torque. The designed controller does not follow the MTPA or MTPV trajectories, which can be seen in Fig. 2.8. Nonetheless, the measured operating points can still be simulated and compared with the acquisitions.

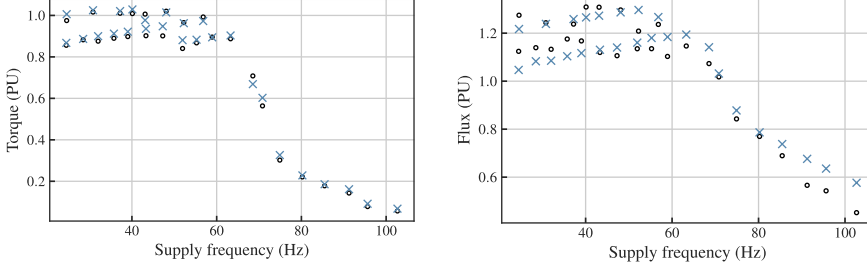


Figure 2.11: Comparison between the measured (black) and simulated (blue) characteristics of the experimental points, for the constant torque tests.

The results are reported in Fig. 2.11, with excellent agreement with the predictions on all the speed range. The small discrepancies on the flux are attributed to the end-winding effects and parasitic inductances on the experimental setup.

Iron losses

Accurate measurements of the iron losses were not feasible with our experimental setup. Fortunately, we have precise experimental data from an induction motor with a stator and airgap length identicals to our prototype. This induction motor was meticulously modeled using the same finite-element framework employed for the SynRM, allowing us to extract the magnetic field used for computing the iron losses.

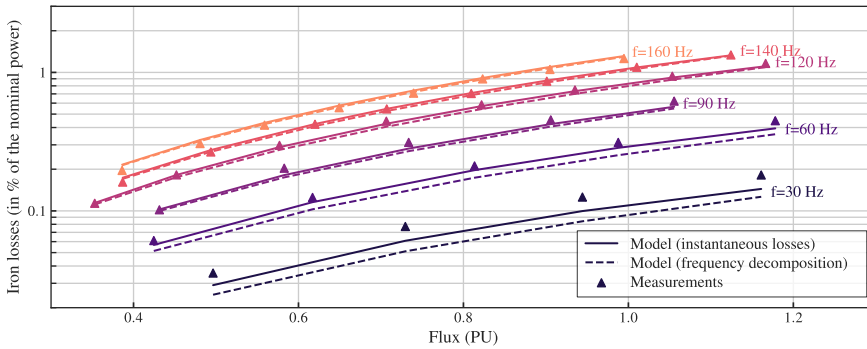


Figure 2.12: Validation of the iron loss models using experimental data gathered from an induction motor, that reuses the same stator as the prototype of the SynRM. The flux is normalized by the nominal characteristics of the SynRM prototype, and the iron losses are normalized by the nominal output power of the SynRM prototype.

The measurements are performed under no-load conditions for frequencies ranging from 30 Hz to 160 Hz. To isolate the iron losses, we subtracted mechanical losses from the overall losses, while neglecting rotor Joule losses. The resulting iron losses are reported in Fig. 2.12.

Both models give good predictions of the iron losses, especially at higher speeds when these losses are significant. The model that is based on the frequency decomposition of the field tends to underestimate the iron losses and requires higher post-processing on the magnetic field. For these reasons, in the following, the iron losses will be estimated exclusively by the model based on the instantaneous losses.

2.2 Mechanical Model

2.2.1 Fundamental equations

The mechanical stresses are physical quantities that are described by the theory of solid continuum mechanics. The effects of the rotation of the rotor can be modeled using static equations by working in a co-rotating frame of reference and imposing an equivalent centrifugal force density $\mathbf{f}$, function of the position vector $\mathbf{r}$:

$$\mathbf{f} = \bar{\rho} \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}), \quad (2.31)$$

with $\bar{\rho}$ being the mass density and $\boldsymbol{\omega}$ the rotational speed. Equation (2.31) assumes a small displacement vector $\mathbf{u}$, so that the body forces in the deformed configuration are the same as those in the initial configurations:

$$\mathbf{r} + \mathbf{u} \approx \mathbf{r}. \quad (2.32)$$

In those conditions, the static equilibrium and the infinitesimal strain theory give the following equations for the stress and strain tensors $\boldsymbol{\sigma}$ and $\boldsymbol{\epsilon}$:

$$\nabla \cdot \boldsymbol{\sigma} = -\mathbf{f} \quad (2.33)$$

$$\boldsymbol{\epsilon} = \frac{1}{2} (\nabla \mathbf{u} + \nabla \mathbf{u}^T). \quad (2.34)$$

The linear constitutive law that relates the stresses to the strains is known as Hooke's law and involves a fourth-order tensor. For materials that are both isotropic and orthotropic, it reduces to:

$$\boldsymbol{\sigma} = \bar{\lambda} \text{tr}(\boldsymbol{\epsilon}) \mathbf{I} + 2\bar{\mu} \boldsymbol{\epsilon}, \quad (2.35)$$

with $\bar{\lambda}$ and $\bar{\mu}$ being the Lamé's first and second parameters. In engineering, the stress-strain relationship is characterized by the Young's modulus E and Poisson's ratio ν , obtained under an uniaxial loading by measuring the resulting axial and transverse deformations. They are related to the Lamé's parameters through:

$$\bar{\lambda} = \frac{E \nu}{(1 + \nu)(1 - 2\nu)} \quad \bar{\mu} = \frac{E}{2(1 + \nu)}. \quad (2.36)$$

The Navier-Lamé equation is obtained by substituting the stresses for the displacements in the equilibrium equation (2.33), with the constitutive law (2.35) and the displacement-strain relationship (2.34):

$$\bar{\mu}\Delta\mathbf{u} + (\bar{\lambda} + \bar{\mu})\nabla(\nabla \cdot \mathbf{u}) = -\mathbf{f}. \quad (2.37)$$

As for the magnetic model, we use FEM to solve this equation to account for the complex geometry of the rotor.

2.2.2 Resolution and post-processing

We consider the plane-strain assumption due to the use of clamping plates that prevent the axial deformations of the lamination stack, so that the domain of resolution can be reduced to 2D. This domain can further be reduced to one pole by implementing periodic boundary conditions. In the specific case of our two pole pairs rotor, this condition finds a straightforward implementation by aligning geometry to the Cartesian axes in which the unknowns $\mathbf{u} = (u_x, u_y)$ are expressed, and enforcing the normal component to equal zero. This is illustrated in Fig. 2.13 (a), and valid as long as the applied magnetic force density does not generate a torque.

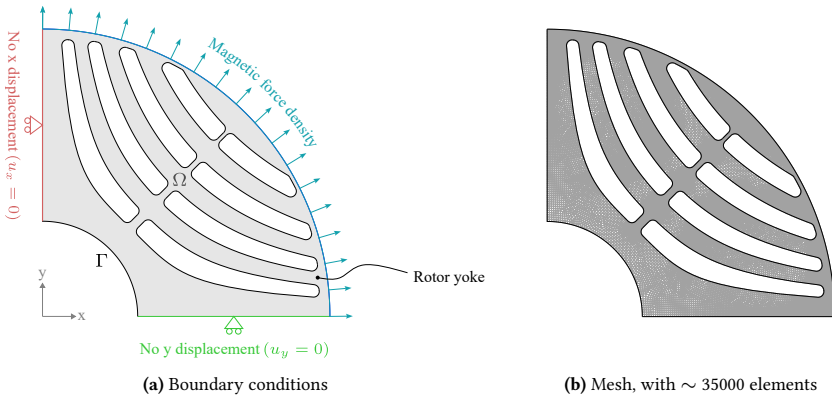


Figure 2.13: Domain of resolution and mesh used to solve (2.37).

The associated mesh is provided in Fig. 2.13 (b). To generate the hybrid quad/triangle mesh, we employ the recombination algorithm [60] integrated into Gmsh. Particularly, the mesh is refined around the fillets to accurately capture the effects of stress concentration.

We employ the distortion energy theory to evaluate structural integrity. This theory posits that failure occurs due to the distortion of a part, and not due to volumetric changes in the part. In particular, the von Mises yield criterion predicts that yielding begins when the distortion energy under a multiaxial stress equals the distortion energy at yield in a uniaxial tension specimen [61]. In terms of applied stresses, this criterion is:

$$\text{Failure if } \sigma_{\text{VM}} \geq \sigma_y, \quad (2.38)$$

where σ_y is the yield stress of the material under uniaxial loading, and σ_{VM} the equivalent von Mises stress, also called equivalent stress:

$$\sigma_{VM} = \sqrt{\frac{(\sigma_{11} - \sigma_{22})^2 + (\sigma_{22} - \sigma_{33})^2 + (\sigma_{33} - \sigma_{11})^2}{2} + 3(\sigma_{12}^2 + \sigma_{23}^2 + \sigma_{31}^2)}, \quad (2.39)$$

where σ_{ij} represents the components of the tensor $\boldsymbol{\sigma}$. In practical applications, a safety factor $S_F > 1$ is typically applied to accommodate unexpected loads, modifying (2.39) to:

$$\text{Failure if } \sigma_{VM} \geq \sigma_y / S_F. \quad (2.40)$$

The equivalent stress for the geometry shown in Fig. 2.13 is represented in Fig. 2.14 (a). We confirm that the displacement field, Fig. 2.14 (b), is significantly smaller than the airgap length. This model, using this mesh, requires approximately 40 seconds (CPU time) to run.

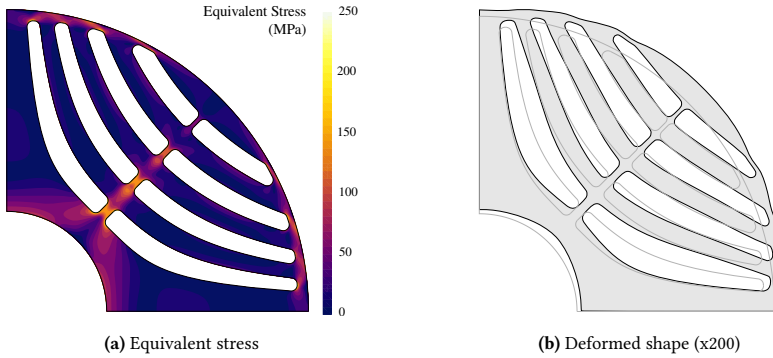


Figure 2.14: Evaluation of the equivalent stress and displacement field at the target overspeed limit, without accounting for the magnetic forces.

2.2.3 Effects of the magnetic forces

We are now assessing the impact of the magnetic forces on equivalent stress. The magnetic force density obtained in section 2.1.5, at nominal and using $\theta_e = 0^\circ$, is imposed as boundary condition as depicted in Fig. 2.13 (a). The stresses resulting from rotation alone, magnetic forces alone, and the combination of both are presented in Fig. 2.15. The maximum stresses induced by the magnetic forces alone are of a similar order of magnitude as those caused by the rotation of the rotor. When considering both sources, the maximum stress increases by a notable 20% compared to the stresses resulting from rotation alone. A similar order of magnitude was reported in a prior study [62], where the author suggested correcting the safety factor to account for the magnetic forces. Consequently, in (2.40), we decided to use a safety factor of 1.25 for the design of the rotor. While this approach may be considered rudimentary, it proves to be a prudent step in addressing the influence of magnetic forces, offering an improvement over neglecting them entirely [63–67].

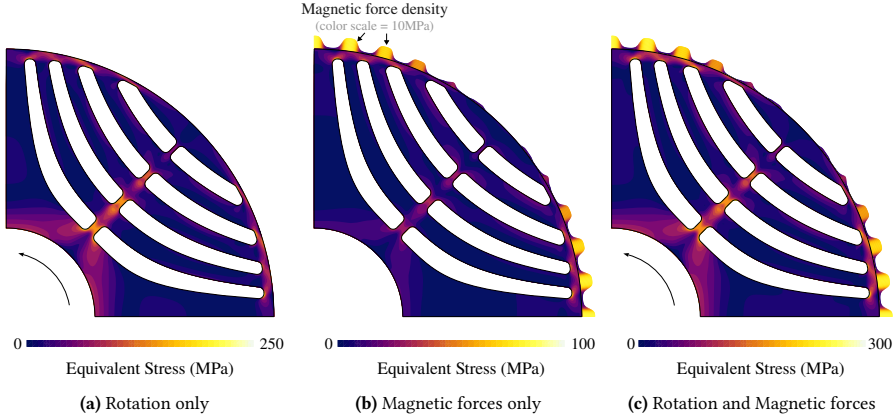


Figure 2.15: Evaluation of the combined actions of the rotation and magnetic forces on the equivalent stress.

If we have demonstrated that the stresses resulting from the magnetic force can be in the same order of magnitude as the stresses induced by rotation, it is essential to emphasize that, under typical operating conditions, the motor is not intended to be controlled with full current at zero current angle, i.e., $\theta_e = 0^\circ$. The MTPA, MTPV, or MTPVA control strategies all involve $\theta_e \geq 45^\circ$, approaching 90° with increasing currents, see Fig. 2.8 and reference [68]. In these scenarios, there is an overall reduction in magnetic forces as they align with the q-axis, proving mechanically advantageous due to the presence of radial ribs.

Under normal circumstances, the only plausible scenario in which the full current might be applied in the d-axis is during an open-loop start-up at no-load, such as the V/f [69] or I-f [70] strategies. During start-up, rotational speed is low, resulting in lower stresses due to rotation. In other words, the worst-case scenario for magnetic force density and the worst-case scenario for rotational forces do not coincide, making our safety factor of 1.25 conservative in these aspects.

2.3 Summary

In this chapter, we have presented the numerical models employed to derive key parameters of interest, encompassing the mean torque, torque ripple, von Mises stresses, and iron losses. The fundamental equations of the magnetic and the mechanical models are solved using finite elements in 2D to ensure accurate representation of the complex rotor geometry. Furthermore, we properly accounted for the non-linear properties of the M400-50A ferromagnetic material, as validated by experimental verification.

Additionally, we investigated the cross-coupling between the magnetic and mechanical models, specifically through magnetic forces. By incorporating a conservative safety factor in the motor design, the structural integrity can be evaluated independently of the magnetic model outputs, effectively decoupling the

two models. This decoupling will prove helpful when separating the optimization problems at the objective level.

To improve computational efficiency without sacrificing accuracy, we implemented boundary conditions to reduce the resolution domain to a pole, and utilized the moving band technique in the magnetic model to account for movement. This technique allows for the solution of the previous position to serve as starting point for the next nonlinear resolution, further accelerating the resolution. Despite these considerations, the evaluation time remains significantly asymmetrical between the magnetic and the mechanical models. While assessing the magnetic model takes approximately 11 minutes (CPU time) to cover 150 positions for torque ripple evaluation, the mechanical model requires only 45 seconds. This results from the inherent nonlinearity of the magnetic model, and the necessity to sweep rotor positions to account for torque ripple.

In the subsequent chapters, mesh characteristics will remain unchanged to preserve model accuracy.

Geometry and Parametrization

3

Deriving a search space for a parametric optimization involves selecting a geometry, defining design parameters, and specifying variation boundaries. This process reduces the infinite-dimensional search space to a manageable dimensionality, thereby allowing its exploration for finding an optimal solution within a finite amount of time.

This reduction of dimensionality is essentially a postulate where we hypothesize the location of the optimum among all possible geometrical configurations. This postulate draws from the experience of previous studies and intuition regarding the interactions between geometry and objectives. While the pursuit of a search space that perfectly encapsulates the true global optimum may be deemed utopian, we aim to find a search space whose optimum closely approximates the performance of the true solution. The goal is to ensure that, in practical implementation, no perceivable difference exists between the two solutions.

Furthermore, exploring the search-space for an optimum is a time-consuming process, typically addressed by automated optimization algorithms through iterative procedures. Depending on the parametrization, the algorithm may encounter challenges such as falling into local minima or inefficiently exploring the search space, affecting the optimization convergence rate. Therefore, another challenge for the engineer is to provide a parametrization suitable for optimization.

To address these considerations, we begin by discussing, in section 3.1, the number and shape of flux barriers likely to meet our expectations in terms of electromagnetic performance. In section 3.2, we explore possibilities for preventing overlapping between flux barriers through parametrization, rather than incorporating these constraints at the optimizer level during search space exploration. Shapes and parametrizations are compared in the bi-objective problem of mean torque maximization and torque ripple minimization.

From the selected geometry and parametrization, two search spaces are derived and summarized in section 3.4. The first adopts a frontal approach to define the geometry, extending the one applied in the preliminary study in section 3.3 to account

for the ribs. The second proposes a reparametrization with the goal of emphasizing the main impacting factors on mean torque. Through this reparametrization, an improvement in the convergence rate during optimizations is expected, as discussed in subsequent chapters by comparing optimizations on both search spaces.

3.1 Geometries

3.1.1 Number of flux barriers

Increasing the number of flux barriers has been deemed beneficial for the mean torque in a number of studies, through the reduction of the circulating component of the q axis flux [5, 68]. This component represents the portion of the flux that circulates across the segment ends through the stator, as illustrated in Fig. 3.1, and is totally analogous to the zigzag inductance of induction motors.

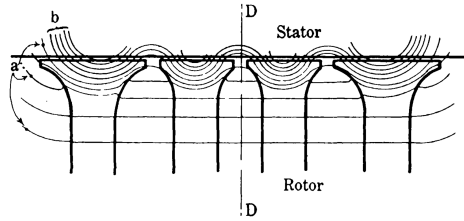


Figure 3.1: Circulating component of the flux [6].

Using the analogy of induction motors, the ratio between the circulating inductance L_{cq} and L_d can be approximated by [71, 72]:

$$\frac{L_{cq}}{L_d} \approx \frac{\pi^2}{12n_r^2}. \quad (3.1)$$

This ratio quickly decreases with the number of rotor "slots" per pole n_r , which is twice the number of flux barriers. For $n_r = 9$, L_{cq} is reduced to 1% of L_d , indicating that there are no significant gains on the saliency ratio ζ when the number of flux barriers exceeds 4. This observation has been confirmed in numerous studies [5, 68, 73] using finite element analysis.

3.1.2 Shape of flux barriers

When considering TLA SynRM, it has been established that a topology with a few flux barriers, along with radial and tangential ribs, meets the requirements for mean torque, torque ripple, power factor and structural integrity. To maximize the saliency ratio, the flux barriers must obstruct a maximum of the q axis flux while minimally impacting the permeance path of the d axis. Achieving this objective involves addressing the shape and insulation of each flux barrier, and it can be attained by employing numerous thin layers to carve the flux barriers alongside the d axis flux lines of a solid rotor. This approach was suggested in [68], where it was observed that the q axis flux lines are orthogonal to the d axis flux lines, especially when the shaft

is made from a magnetic material, as shown in Fig. 3.2 (a). Even when the shaft is non-magnetic, a certain degree of orthogonality is maintained for flux lines far from the surface of the shaft, as depicted in Fig. 3.2 (b). Consequently, carving the flux barriers alongside the d axis flux lines remains an effective solution for maximizing the saliency ratio.

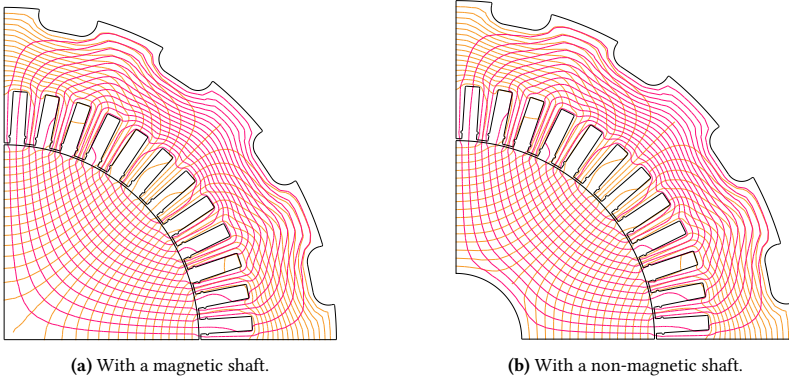


Figure 3.2: Flux lines in a solid rotor, when supplying currents in the d ($\theta_e = 0^\circ$) and q axes ($\theta_e = 90^\circ$).

The shape that conforms to the flux lines in a solid rotor has gained increasing popularity over the last decade, thanks to the works of R. R. Moghaddam in the field [68]. By combining conformal mapping concepts with the Joukowski airfoil potential function, he was able to provide a simple analytical formulation to describe the flux lines in a solid rotor with a non-magnetic shaft:

$$f(C, \theta) = R_{in} \cdot \sqrt[p]{\frac{C + \sqrt{C^2 + 4 \sin^2(p\theta)}}{2 \sin(p\theta)}}, \quad (3.2)$$

with C being a constant, function of the point coordinates (r_i, θ_i) the flux line is passing through:

$$C(r_i, \theta_i) = \sin(p\theta_i) \frac{\left(\frac{r_i}{R_{in}}\right)^{2p} - 1}{\left(\frac{r_i}{R_{in}}\right)^p}. \quad (3.3)$$

This shape is called after the conformal map it employs, i.e., Joukowski¹, or fluid shape, as it draws inspiration from concepts applied in aerodynamics.

Other frequently investigated shapes for a four poles rotor are depicted in Fig. 3.3. The rectangular shape [74] (a) holds the advantage of having a straightforward geometrical representation. Additionally, its flat surfaces make it suitable for permanent-magnets assisted synchronous reluctance motors (PMaSynRM) due to geometric constraints on the shape of the magnets. However, in the absence of PM, the presence of corner points makes it suboptimal for guiding flux in the d axis, unlike conic flux barriers, such as the circular [75] (b) or hyperbolic [76] (c) flux barriers.

¹Other transliterations being Joukowski, Joukovsky or Zhukovsky.

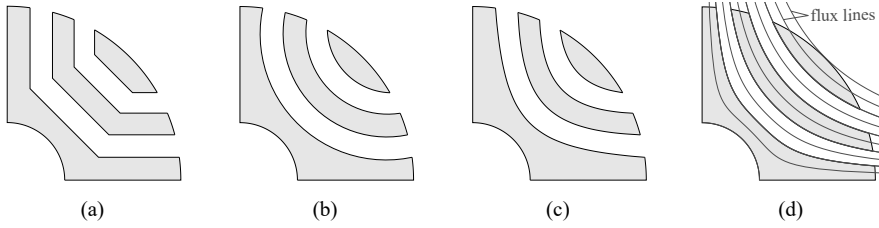


Figure 3.3: Typical shape of flux barriers for SynRMs. (a) Rectangular. (b) Circular. (c) Hyperbolic. (d) Following the flux lines in a solid rotor (Joukowski).

Two pole pairs rotor with circular flux barriers, also known as round flux barriers or C-shape flux barriers, are well-represented in the literature. A smaller number of publications cover hyperbolic flux barriers, whereas examples of elliptical or parabolic shapes are almost non-existent.

The circular and hyperbolic shapes have an advantage in the number of possible geometrical configurations over the Joukowski shape. A Joukowski line is uniquely defined by a single point with coordinates (r_i, θ_i) , as seen through the constant C (3.3). In contrast, a conical shape requires two points for complete constraint, accounting for the symmetry in the rotor. While this larger search space statistically has a higher likelihood of containing a better optimum, its exploration might be more challenging. We evaluate this trade-off in section 3.3.

3.2 Parametrizations

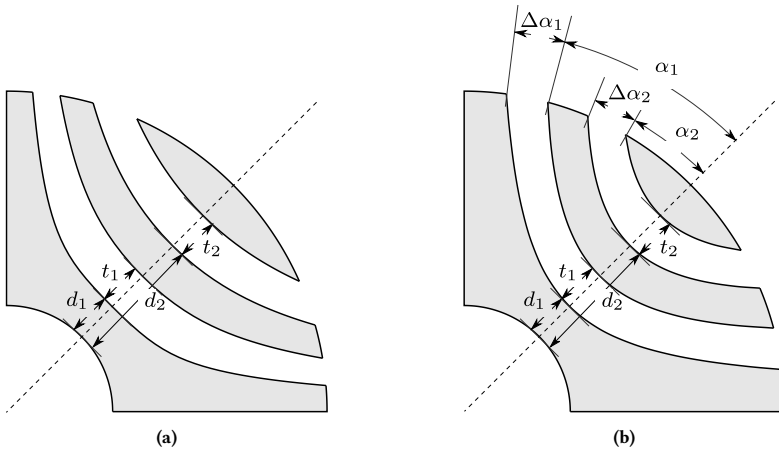


Figure 3.4: Direct parametrization of (a) Joukowski flux barriers. (b) Hyperbolic flux barriers (also applicable to circular and rectangular flux barriers).

A direct and simple parametrization of a Joukowski flux barrier involves assigning one parameter d_i for its position within the rotor yoke and another for its

thickness t_i , as depicted in Fig. 3.4 (a). For round, hyperbolic, or rectangular shapes, the parametrization can be extended by controlling the opening angles in the airgap through the parameters α_i and $\Delta\alpha_i$, as illustrated in Fig. 3.4 (b).

We aim to maintain a large variation range for these parameters to explore all potential solutions. However, using this straightforward parametrization, there is a risk for the flux barriers to overlap each other within the optimization loop. Without changing the parametrization, several adaptations can be applied:

- The flux barriers can be merged when overlapping occurs. This leads to areas of the search space where a small variation of the parameters does not change the geometry and, as a consequence, does not change the evaluated objectives.
- The search space can be restricted to areas where there is no overlapping. In the parametrization of Fig. 3.4, this implies to add the following constraints to the optimization problem:

$$\begin{cases} d_i + t_i < d_{i+1} & \text{if } i \neq n \\ d_i + t_i < R_{out} & \text{otherwise} \end{cases}, \quad \begin{cases} \alpha_i + \Delta\alpha_i < \pi/(2p) & \text{if } i = 1 \\ \alpha_i + \Delta\alpha_i < \alpha_{i-1} & \text{otherwise} \end{cases} \quad (3.4)$$

- The search space can be reduced so that (3.4) is always satisfied within the variation range of the parameters. As a drawback, this can potentially exclude the global optimum.
- A system to detect the overlapping and change the construction of the geometry can be implemented. For instance, the positions and thicknesses could be saturated to prevent the overlap when it is detected.

The lack of sensitivity, holes in the search space, or additional constraints are considered detrimental for the optimization convergence. Another solution consists in adopting a different parametrization that makes the overlapping not possible, as documented in [77]. The rotor is seen as a succession of flux guides and flux barriers of different thicknesses, such that, stacked together, the specified outer radius is met. To satisfy this constraint, it is possible to rescale the individual thicknesses of these elements to exactly fit the rotor yoke:

$$t'_i = t_i \cdot \frac{R_{out} - R_{in}}{\sum t_k} \quad (3.5)$$

As a consequence of the rescaling operation, the thicknesses t_i involved in the right-hand side of (3.5) can be multiplied by any factor κ without affecting the resulting thickness t'_i :

$$t'_i = \kappa t_i \cdot \frac{R_{out} - R_{in}}{\sum \kappa t_k} \quad (3.6)$$

$$= K_i \cdot \frac{R_{out} - R_{in}}{\sum K_k} \quad (3.7)$$

We arbitrarily select a factor κ with the units of reciprocal length, so that $\kappa t_i = K_i$ becomes dimensionless. Introducing the parameters K_i has the advantage to prevent any ambiguity on the nature of the design space when dealing with the rescaling operation. This approach is analogous to finding the individual length of a network of springs, as illustrated in Fig. 3.5.

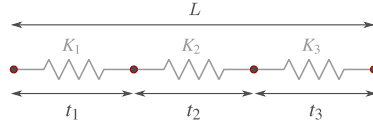


Figure 3.5: Analogy of the rescaling operation with a network of springs, connected in series.

Another implication of (3.7) and (3.6) is an over-parametrization of the geometry, i.e., there are infinite combinations of K_i that lead to the same geometries. This remains mostly true for a constrained search space with finite upper bounds². Assigning an arbitrary value of one of the stiffnesses will fix this form of redundancy in the parametrization, with the cost of a minor restriction of the search space. Nonetheless, from our experience, the extra parameter did seem to favor the optimization convergence for larger networks of virtual springs.

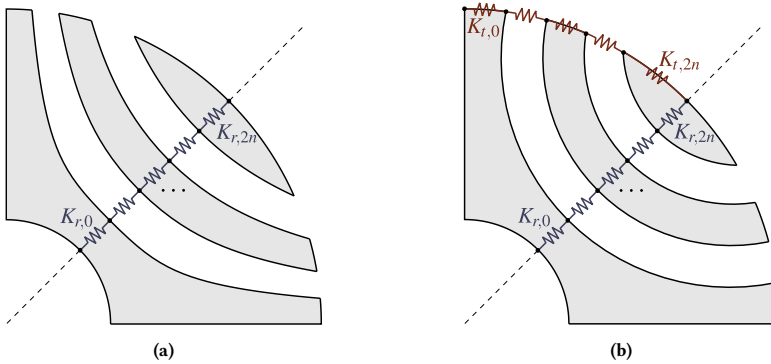


Figure 3.6: Indirect parametrization of (a) Joukowski flux barriers. (b) Circular flux barriers (also applicable to hyperbolic and rectangular flux barriers).

The substitution of the direct parametrization to the parametrization that solves the non-overlapping constraints using networks of virtual springs is illustrated in Fig. 3.6. This parametrization is referred as indirect parametrization in the following.

3.3 Selection of the geometry and parametrization

In a preliminary comparative study, we optimized rotors featuring rectangular, circular, hyperbolic, or Joukowski flux barriers, to address the bi-objective

²The only exceptions, uniquely defined, are located at the edges of the search space. They correspond to the geometrical configurations that are described by at least one stiffness taking its lower bound value and another one taking its upper bound value.

problem of maximizing the mean torque while minimizing the torque ripple. The optimizations were conducted using both the direct parametrization, incorporating the constraints (3.4), and the indirect parametrization shown in Fig. 3.6. This study is thoroughly discussed in [77], and the key findings are summarized here.

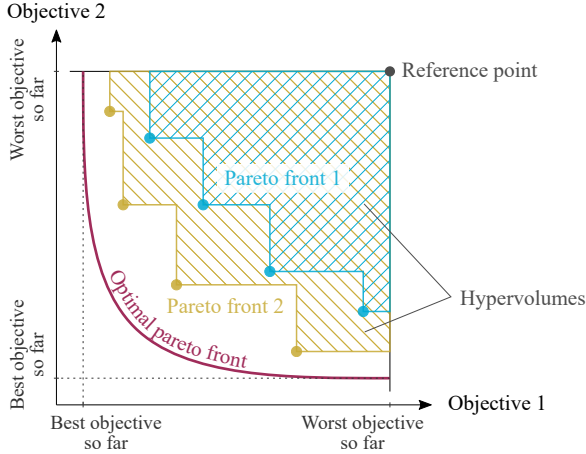


Figure 3.7: Hypervolume indicator as a performance metric for indicating the quality of a Pareto front.

The comparison of the obtained Pareto fronts is performed using the hypervolume indicator [78], a scalar metric depicted in Fig. 3.7. This metric gauges both the distribution of solutions across the objective space and the proximity of the current Pareto front to the optimal Pareto front, and is maximized for the optimal Pareto front. This indicator is computed on successive Pareto fronts obtained at each step of the optimization algorithm to assess optimization convergence, with the results reported in Fig. 3.8.

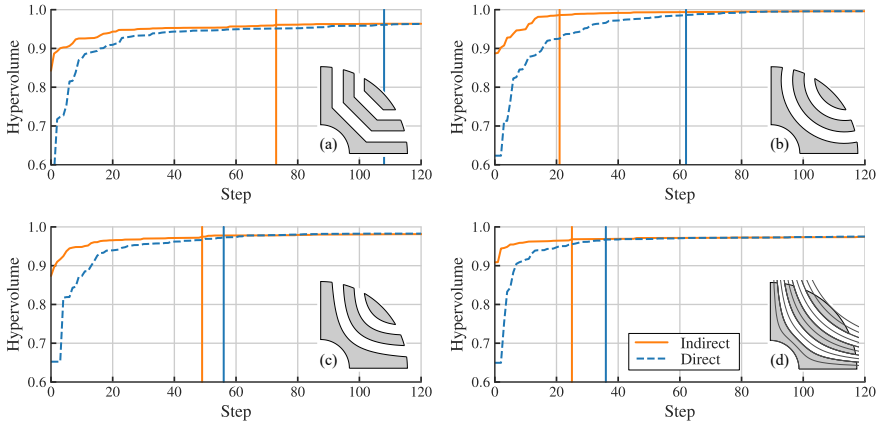


Figure 3.8: Evolution of the hypervolume during the optimization, for the two parametrizations and the four shapes of flux barriers, Fig. 3.3. The vertical lines denote the optimization steps where 99.9% of the hypervolume at the end of optimization is achieved, serving as an indicator of convergence.

From this figure, it is evident that the indirect parametrization using networks of virtual springs consistently reaches convergence faster than the direct parametrization that incorporates the constraints (3.4). Notably, both two parametrizations achieve the same hypervolume at the end of the optimization, indicating that the obtained Pareto fronts have equivalent quality regardless of the chosen parametrization.

Addressing the question of the optimal shape, round flux barriers elevate the hypervolume higher than all the other shapes. However, the disparity between hypervolumes is marginal, at less than 5% when compared to rectangular flux barriers, being the worst-performing shape. This aligns with the findings of G. Pellegrino et al. in [14], where no significant difference was observed between circular and rectangular flux barriers on a similar problem.

Regarding joukowski flux barriers, they exhibit overall superior convergence properties while achieving more than 98% of the hypervolume of circular flux barriers. In a subsequent study, this gap was found to shrink to negligible amounts when ribs are added to the geometry, accounting for the structural integrity [79]. Consequently, given their smaller design space and better convergence rate, Joukowski flux barriers are selected in the continuation of this study. Another advantage of this shape is its adaptability to the curvature of the shaft and applicability to any number of pole pairs.

3.4 Design spaces

Following the concept of virtual springs to address the non-overlapping constraints between flux barriers, we develop two parametrizations to characterize a rotor with n Joukowski flux barriers, incorporating radial and tangential ribs. The first parametrization simply extends the indirect parametrization, as presented in Fig. 3.6, to accommodate the ribs. In the second parametrization, the air-to-rotor ratio ξ , also known as insulation ratio, becomes an explicit parameter in the design space. It is defined as:

$$\xi = \frac{\text{Thickness of air in the } d \text{ axis}}{\text{Thickness of the rotor yoke}}, \quad (3.8)$$

and has been closely related to the torque capabilities of the machine [5, 68]. Hence, utilizing it as an explicit design parameter is expected to facilitate deriving time-efficient design approaches [32] and improve optimization convergence.

3.4.1 First parametrization

The first parametrization, illustrated in Fig. 3.9, uses a single network of virtual springs to determine the individual thicknesses of flux guides and flux barriers alongside the q axis. The Joukowski lines (3.2) are constructed from the radial positions of these points. The radial and tangential ribs are then added to the geometry, with their thicknesses used as design parameters. Circular fillets are used to prevent stress concentration at the connecting points with the flux guides.

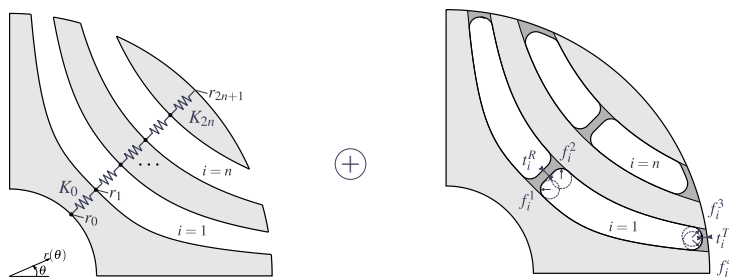


Figure 3.9: First parametrization, with the flux barriers parametrized separately from the ribs. The non-overlapping constraint is solved using the concept of virtual springs. Radial and tangential ribs are added in the flux barriers to ensure the structural integrity, and the corners are rounded with circular fillets. This parametrization can be applied to generate any desired number n of flux barriers, as illustrated for $n = 2$.

These fillets are expected to be detrimental for the mean torque due to the increased amount of ferromagnetic material put in the flux barriers. However, they might act like pole shoes and alleviate the flux concentration on the corners of the flux guides so that less torque ripples are generated [80, 81].

With this parametrization, there remains a risk for overlap between fillets situated on both the inner and outer sides of the flux barrier, especially if the barrier is thin. Unlike the approach taken with the flux barriers, we address this scenario by detecting and saturating the radius of the fillets when such overlap occurs. This reduced sensitivity is considered less critical than that of the flux barriers. Nonetheless, we acknowledge that implementing this mechanism ultimately proved more complex than simply preventing overlaps by reutilizing the concept of virtual springs.

Table 3.1: Design space of the first parametrization
($i = 1, \dots, n$), ($j = 0, \dots, 2n$)

Param.	Description
θ_e	Current angle
K_j	Stiffness of the virtual spring
t_i^R	Thickness of the radial rib
t_i^T	Thickness of the tangential ribs
$f_i^{1 \rightarrow 4}$	Radius of the fillet (x4)

The design space of this first parametrization is provided in Table 3.1. In total, accounting for the ribs adds 6 parameters per flux barrier to the design space, two for the thicknesses and four for the fillets. In the case of a geometry with 4 flux barriers, the overall number of parameters is 34.

3.4.2 Second parametrization

Using the air-to-rotor ratio ξ , the distribution of iron and air within the flux guides and flux barriers is accomplished through two separate sub-networks of virtual

springs, as depicted in Fig. 3.10. Following interleaving, the construction of the geometry remains identical to that of the first parametrization.

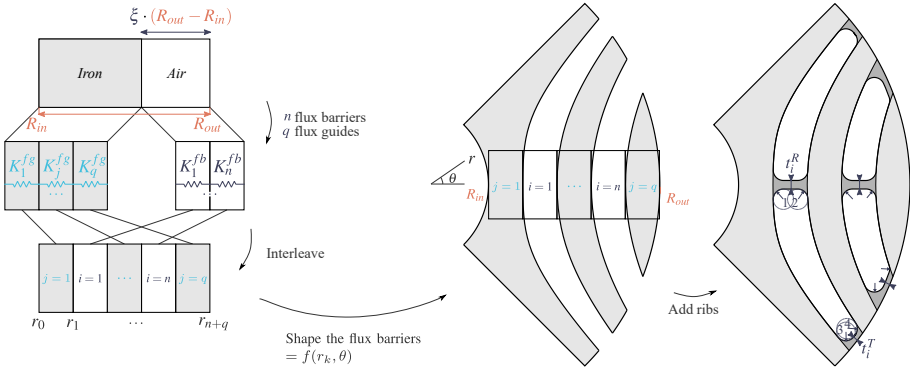


Figure 3.10: Second parametrization. The amount of magnetic insulation, i.e., the air-to-rotor ratio ξ , is an explicit parameter. The iron and air are then subdivided into the flux guides and the flux barriers, before shaping the flux barriers and adding the ribs.

The last layer of the rotor will be a flux guide if the number of flux barriers is one less than the number of flux guides, i.e., $n = q - 1$. Conversely, the last layer of the rotor becomes a flux barrier if the number of flux barriers equals the number of flux guides, $n = q$. Hence, this parametrization naturally accommodates the presence of a cut-off barrier, as illustrated in Fig. 3.11.

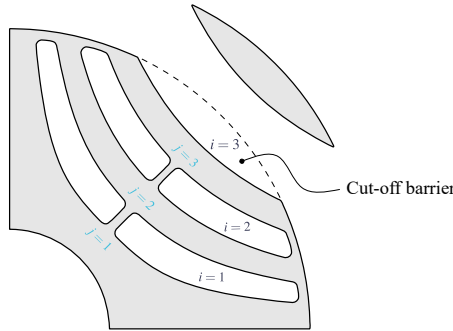


Figure 3.11: Geometry with two inner Joukowski flux barriers and one cut-off barrier.

A cut-off barrier was already proposed by Kotsko, as illustrated in Fig. 1.3, and it has been subject of discussion in various studies [82–84]. The advantages and disadvantages of this topology are thoroughly examined in chapter 7 using a specific comparison methodology.

The design space of this second parametrization is provided in Table 3.2, which results in 35 parameters in the case of a geometry with 4 flux barriers. The increased number of parameters compared to the first parametrization results from the redundancy in each sub-network of virtual springs. For this parametrization, we address

Table 3.2: Design space of the second parametrization
 $(i = 1, 2, \dots, n), (j = 1, 2, \dots, q)$

Param.	Description
θ_e	Current angle
ξ	Air-to-rotor ratio
K_j^{fg}	Stiffness of the virtual spring, for a flux guide
K_i^{fb}	Stiffness of the virtual spring, for a flux barrier
t_i^R	Thickness of the radial rib
t_i^T	Thickness of the tangential ribs
$f_i^{1 \rightarrow 4}$	Radius of the fillet (x4)

it by setting $K_q^{fg} = K_n^{fb} = 1$. In the case of a rotor with a cut-off barrier, we assign $K_q^{fg} = K_{n-1}^{fb} = 1$ instead, ensuring that the depth of the cut-off barrier remains a degree of freedom.

3.5 Summary

In this chapter, we made key decisions regarding the shape of flux barriers and the parameterization to characterize the geometry. Initially, we compared rectangular, circular, hyperbolic, and Joukowski flux barriers in a preliminary study, focusing on mean torque and torque ripple. While circular flux barriers achieved slightly better magnetic performance, Joukowski flux barriers were ultimately selected due to their excellent balance between parameter count and magnetic performance.

Additionally, we evaluated the optimization convergence of two strategies of parametrization. Results indicated that the indirect parametrization, utilizing networks of virtual springs to prevent overlap between flux barriers, achieved faster convergence. This parametrization offers the advantage of a rectangular search space, essential for many sensitivity analysis methods.

Following the concept of virtual springs, we proposed two parametrizations. The second parametrization introduced the air-to-rotor ratio, ξ , as an explicit parameter of the design space. This latter parametrization is expected to favor the optimization convergence, as ξ has been linked to the torque capability of SynRM in prior studies.

Lastly, considerations for structural integrity prompted the addition of radial and tangential ribs, characterized by their thicknesses and fillet radii.

Sensitivity Analysis

4

Sensitivity analysis serves as a crucial tool for evaluating the impact of parameter variations on a designated output. Integral to our research, advanced sensitivity analysis methods offer a quantitative understanding of parameter interactions. In the context of this research, our interest in sensitivity analyses stems from several pivotal reasons.

Firstly, the second parametrization introduced in chapter 3 shapes the input design space differently from the first, specifically emphasizing the air-to-rotor ratio ξ as a primary contributor to mean torque. By subjecting mean torque to sensitivity analysis, we not only validate but also quantify the efficacy of this second parametrization.

Secondly, the time-efficient design approaches for SynRM separate the complex optimization problem into smaller, more manageable problems through physical reasoning on parameter interactions. The validity of these assumptions can be evaluated using sensitivity analysis.

Finally, by providing a deep understanding between parameter-objective interactions, sensitivity analyses pave the way for seamlessly deriving alternative design approaches.

Consequently, driven by the need for precise quantification of output sensitivity to parameters, we begin this chapter by presenting a comprehensive overview of sensitivity analysis methods in section 4.1. The Ishigami function [85], a recurrent test case for sensitivity analyses, serves as a practical illustration for interpreting key metrics. Notably, within the same section, we address the challenge of summarizing these metrics on a single graph.

Following this groundwork, we proceed to conduct sensitivity analyses for the first and second parametrizations in section 4.2 and 4.3, respectively. The outputs of interest under analysis encompass mean torque, torque ripple, iron losses, and the maximum equivalent stress.

4.1 Theoretical background

4.1.1 Introduction

Sensitivity analyses (SA) aim at determining how the outputs of a mathematical or numerical model are related to its inputs. They share strong similarities with uncertainty analyses (UA) in their execution, involving the sampling of the input space, model evaluations, and the application of statistical metrics to the outputs. However, uncertainty analyses distinguish themselves by associating a probability function with the input variables, the goal being to describe the set of possible outcomes and their respective probabilities. Sensitivity analyses, conducted as a preliminary step before uncertainty analyses, play a crucial role in identifying the primary factors influencing the outputs of the model.

The evaluation of models as black-box entities makes SA transversal to many engineering disciplines. Historically, one of its primary application domains was the modeling of complex environmental systems, such as the Soil & Water Assessment Tool (SWAT) model [86]. This model was developed to predict the impact of land management on water, sediment, and agricultural chemicals in watersheds. Environmental systems of this nature often involve numerous parameters challenging to measure accurately. Therefore, sensitivity analysis is a valuable tool to select the parameters to include in a calibration procedure [87]. Drawing a parallel, sensitivity analyses find utility in the realm of electric motors to help identifying areas for improvement in manufacturing processes. This is particularly pertinent when faced with limitations in accuracy stemming from manufacturing tolerances or alterations in material properties [88].

However, it is noteworthy that in most applications involving standard manufacturing techniques, the uncertainties remain sufficiently small to be disregarded. The magnetic and mechanical properties of commercial ferromagnetic laminations are well-established, and the geometric tolerances achieved through sheet cutting are typically orders of magnitude smaller than the main dimensions of the elements being cut¹. The information gathered by conducting an extensive sensitivity analysis hardly seem to overweight the computational cost associated to it. This observation possibly contributes to the limited application of sensitivity analyses in the field of electric motors, in contrast to its prevalence in other engineering domains.

As a result, when conducted, most studies in electric motors involve observing the effects on model outputs by varying one parameter at a time around an arbitrary design point. This approach leans more towards a parametric analysis than a sensitivity analysis. Such analyses are relatively straightforward to execute, with results often presented with as many graphs as there are parameters in the design space. While interpretation is direct, these analyses are qualitative and primarily serve to reflect an *a priori* broad understanding of the motor's physical behavior.

¹Exceptions exist, such as in the radial and tangential ribs where tolerances can approach a fraction of their width.

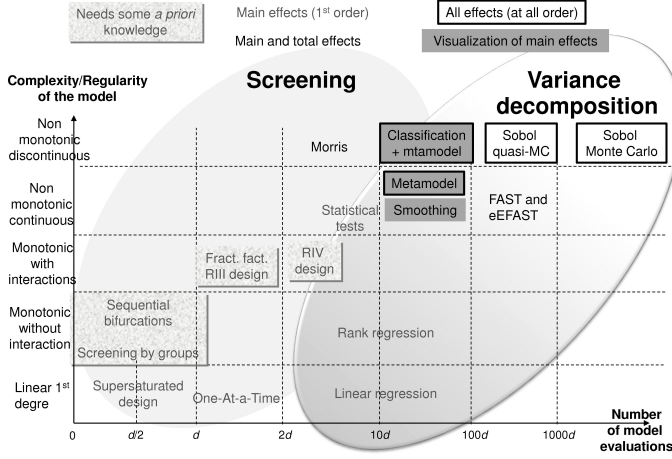


Figure 4.1: Popular methods used in sensitivity analyses [39].

One evident limitation of this method, known as one-at-a-time (OAT), is its inability to account for interactions between parameters. In other words, it falls short when simultaneous changes in two or more parameters result in a non-additive impact on the output. More problematically, the exploration of the design space by an hypercross is extremely inefficient. For instance, over 99.75% of the search space remains totally unexplored when no more than 10 parameters are involved [36].

Fortunately, there are alternative methods that offer more robust insights than OAT, albeit at a cost of a higher number of model evaluations. The most popular methods are reported in Fig. 4.1. The choice of a sensitivity analysis method depends on the extractable metrics and the complexity of the model. In our context, our focus is on quantifying the effects of a parameter alone as well as its interactions. Sobol indices, a metric derived by decomposing the variance of the output of interest, facilitate this analysis, as elaborated in the following section.

4.1.2 Variance-based methods

In variance-based methods, the output Y , function of d variable model inputs $\mathbf{X} = \{X_1, X_2, \dots, X_d\}$, is decomposed into a series of orthogonal functions [89]:

$$Y = f(\mathbf{X}) \quad (4.1)$$

$$= f_0 + \sum_i^d f_i(X_i) + \sum_i^d \sum_{j>i}^d f_{ij}(X_i, X_j) + \dots + f_{12\dots d}(X_1, X_2, \dots, X_d), \quad (4.2)$$

$$\text{with:} \quad f_0 = \mathbb{E}(Y), \quad (4.3)$$

$$f_i(X_i) = \mathbb{E}(Y|X_i) - f_0, \quad (4.4)$$

$$f_{ij}(X_i, X_j) = \mathbb{E}(Y|X_i, X_j) - f_i(X_i) - f_j(X_j) - f_0, \quad (4.5)$$

$$\dots \quad (4.6)$$

Using this decomposition, the variance follows:

$$\text{Var}[Y] = \sum_i^d V_i(Y, X_i) + \sum_i^d \sum_{j>i}^d V_{ij}(Y, X_i, X_j) + \cdots + V_{12\dots d}, \quad (4.7)$$

$$V_i = \text{Var}[f_i(X_i)] = \text{Var}[\mathbb{E}(Y|X_i)], \quad (4.8)$$

$$V_{ij} = \text{Var}[f_{ij}(X_i, X_j)] = \text{Var}[\mathbb{E}(Y|X_i, X_j)] - V_i - V_j, \quad (4.9)$$

$$\dots \quad (4.10)$$

This expression decomposes the variance of the output as the contributions due to the variations of the parameters alone, the 1st order effects, majored by the contributions due to the variations of two parameters simultaneously, the 2nd order effects, up to the contributions due to the variations of all the parameters simultaneously, the d^{th} order effects. By normalizing the left-hand and right-hand sides of the equations by $\text{Var}[Y]$, we obtain the so-called adimensional Sobol indices:

$$\underbrace{\sum_i^d S_i}_{1^{\text{st}} \text{ order effects}} + \underbrace{\sum_i^d \sum_{j>i}^d S_{ij}}_{2^{\text{nd}} \text{ order effects}} + \cdots + \underbrace{S_{12\dots d}}_{d^{\text{th}} \text{ order effects}} = 1. \quad (4.11)$$

We also define the total order index, S_{T_i} , which represents the effect of a parameter accounting for all its interactions on the variance of the output:

$$S_{T_i} = 1 - \frac{\text{Var}[\mathbb{E}(Y|X_{\sim i})]}{\text{Var}[Y]}, \quad (4.12)$$

where the notations $X_{\sim i}$ designs all the other variables except X_i . These indices have the following properties:

- S_{T_i} is always greater or equal to the effects of parameter i at all orders, and to the sum of the individual contributions S_i .
- The model is said to be additive in the absence of interactions between parameters. In this case, we have:

$$\sum_i^d S_{T_i} = \sum_i^d S_i = 1. \quad (4.13)$$

The additive property is interesting in a context of optimization, as it indicates the inputs can be optimized separately.

- All the indices are positive, and symmetrical (e.g. $S_{ij} = S_{ji}$). It follows that the higher order interactions of parameter i are accounted $n!$ times in the total index:

$$S_{T_i} = 1 \cdot S_i + 2! \cdot \sum_{j \neq i}^d S_{ij} + \cdots + d! \cdot \sum S_{12\dots d}. \quad (4.14)$$

Most models follow the sparsity-of-effects principle, also called hierarchical ordering principle. This principle states that the main effects tend to be larger

than two-parameter interactions, being themselves larger than three-parameter interactions, and so on. The representation of the first and second order indices are often sufficient to analyze the model, with the total indices giving a good indication of the higher-order interactions not accounted for.

As suggested by (4.8) and (4.9), these indices can either be extracted from the functional decomposition of Y , or approached using the mathematical expectation of a sufficiently large number of samples. The first method requires to fit a known function to the observations Y , which is called metamodeling. Famous metamodels include Kriging and Polynomial Chaos Expansions (PCE), the later one being written:

$$Y = \sum_{i \in \mathbb{N}} c_i \Psi_i(\mathbf{X}), \quad (4.15)$$

where c_i are coefficients usually determined using least-square regression, and Ψ_i the polynomial basis functions. Using metamodels, the extraction of Sobol indices is straightforward and often analytical, so that they do not suffer from convergence issues. The fit itself, however, can be cumbersome especially when a large number of parameters is involved. For a PCE, the number of coefficients D grows exponentially with the number of parameters d and the degree of the polynomial P [90]:

$$D = \frac{(d + P)!}{d!P!}. \quad (4.16)$$

The coefficients are, however, generally sparse. One common strategy is to extract the Sobol indices from the raw samples, i.e., using the mathematical expectation, to broadly determine the main impacting parameters, so that the number of parameters represented by the PCE can be lowered. From that perspective, the two methods are extremely similar as they both aim at extracting the same metrics by relying on the same samples.

The approximation of the expectations using Monte Carlo method, i.e., with purely randomly generated samples, shows a convergence rate in $O(1/\sqrt{N_k})$, N_k being the number of samples. A convergence rate in $O(1/N_k)$ can be obtained by better spreading the samples through the use of quasi-random sequences. These sequences introduce a statistical bias in the estimation of the expectation, that is bounded by a value called discrepancy. Halton sequence, Saltelli's extension of the Sobol Sequence [91], and Latin Hypercube Sampling (LHS), are common examples of low-discrepancy methods suitable for sensitivity analysis. Conversely, grid sampling is a high discrepancy method. These methods are represented visually in Fig 4.2.

Most mathematical methods and available tools to sample and extract the Sobol indices require the input space to be rectangular, meaning that no constraints should exist between the parameters. The two parametrizations proposed in chapter 3 are rectangular, thanks to the use of virtual-spring stiffnesses to solve the non-overlapping constraints.

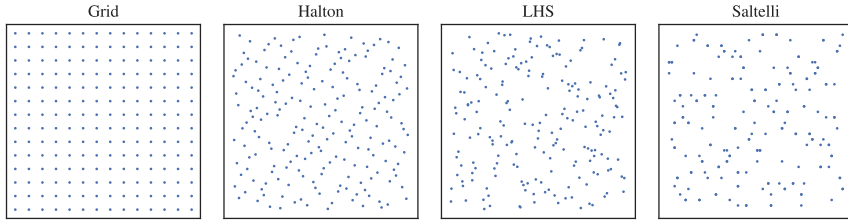


Figure 4.2: Sampling of a 2-D dimensional space using different methods. Each dot represents a sample, with 200 samples by input space.

4.1.3 Interpretation of Sobol' indices

In this section, we provide an example to interpret the Sobol indices using the Ishigami function [85, 92]:

$$g(a, b, c) = \sin(a) + 7 \sin(b)^2 + 0.1c^4 \sin(a), \quad (4.17)$$

where a, b, c are three parameters $\in [-\pi, \pi]$. This function is non-linear and non-monotonic. For this test function, the Sobol' decomposition variances are analytical. The first order indices are:

$$\mathbf{S}_1 = \begin{bmatrix} S_a & S_b & S_c \\ 0.313, & 0.442, & 0.0 \end{bmatrix} \quad (4.18)$$

$$\sum S_i = 0.756. \quad (4.19)$$

From these indices, we observe that the parameter c alone has no influence on the output g . The reason is that it is multiplied by $\sin(a)$ in (4.17). Because the sum of the elements S_i is less than one, we can conclude for parameter interactions. The total order indices are:

$$\mathbf{S}_T = \begin{bmatrix} S_{T_a} & S_{T_b} & S_{T_c} \\ 0.557, & 0.442, & 0.243 \end{bmatrix} \quad (4.20)$$

$$\sum S_{T_i} = 1.243. \quad (4.21)$$

Since $S_{T_b} = S_b$, we conclude the parameter b does not interact with any other parameter, in contrast to a and c . The parameter a contributes the most to the variance of g , followed by parameters b and, finally, c .

We can display the interactions between two parameters, i.e., the second order indices $\mathbf{S}_2$, in the form of a symmetric matrix, analogous to a correlation matrix:

$$\mathbf{S}_2 = \begin{pmatrix} a & b & c \\ S_a & 0.0 & 0.243 \\ 0.0 & S_b & 0.0 \\ 0.243 & 0.0 & S_c \end{pmatrix} \begin{array}{l} a \rightarrow \sum_j S_{aj} = S_a + 0.243 \\ b \rightarrow \sum_j S_{bj} = S_b + 0.0 \\ c \rightarrow \sum_j S_{cj} = S_c + 0.243. \end{array} \quad (4.22)$$

From this matrix, we observe that the only two-parameter interactions are between a and c . Furthermore, by observing that the sums of rows in (4.22) correspond to the total order indices (4.20), we can also conclude there are no interactions between 3 parameters.

While the Sobol indices are easy to interpret, their raw representation in (4.18), (4.20), and (4.22) makes their analysis impractical. This becomes even more pronounced when a large number of parameters are involved, as their contributions to the total index quickly gets diluted in the matrices. In an effort to provide a graphical tool to ease their interpretation, and taking advantage of the properties of the Sobol indices, we propose the representation depicted in Fig. 4.3, which is a mix between an incomplete pie chart for the first order indices $\mathbf{S}_1$, and a chord diagram for the second and total order indices, $\mathbf{S}_2$ and $\mathbf{S}_T$.

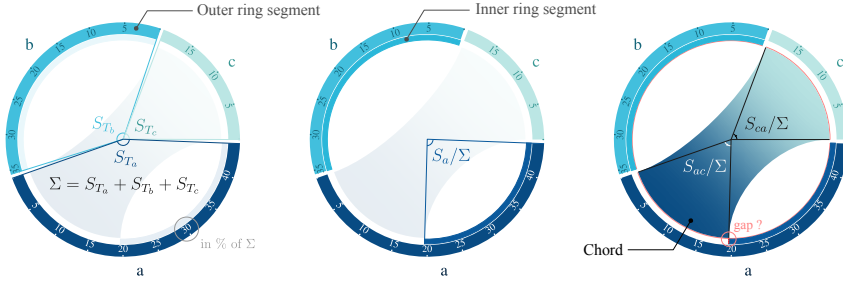


Figure 4.3: Representation of the first, second, and total order Sobol indices of the Ishigami function.

In these graphs, each outer ring segment represents the relative contribution of the parameter i to the cumulative total order index, i.e., $S_{T_i} / \sum S_{T_i}$. An inner ring segment represents the relative impact of the parameter alone, i.e., $S_i / \sum S_{T_i}$. An arc between two parameters i and j represents the second order interactions relative to the cumulative total order index, i.e., $S_{ij} / \sum S_{T_i}$. It follows that the higher order interactions for the parameter i correspond to the gap left between the inner ring, $S_i / \sum S_{T_i}$, and the arcs, $\sum_j S_{ij} / \sum S_{T_i}$.

Using this graph, the previous observations are easily reaffirmed: the parameter b does not interact with other parameters, as indicated by the outer ring having the same angular span as the inner ring segment. Conversely, parameter c has no effect at first order, indicated by the absence of inner ring segment, but exhibits a substantial effect at the second order with a . Finally, the absence of a gap between the first and second-order indices signifies the absence of interactions between 3 parameters.

While the idea to use such chord diagrams to represent the Sobol indices is not new, their construction typically relies on $\mathbf{S}_2$ only, thereby neglecting the higher order-interactions by assuming $S_{T_i} \approx S_i + \sum_j S_{ij}$. Consequently, the arc span intended to represent the total index, S_{T_i} , is derived from the values of the first and second indices instead of its definition (4.12). Although higher-order

effects are often negligible, making such a representation mathematically valid, the convergence of estimators for total order indices is usually better than that of the first order indices, which, in turn, is better than that of second-order indices, and so forth. The partial convergence of these indices can have a non-negligible impact when added together.

The construction of the diagrams presented in Fig. 4.3 starts by the definition of S_{T_i} , what lowers the chances of being misled by partially converged second order indices. The partial convergence of the indices can still clash with the construction of the graph when $\sum_i \sum_j S_{ij} > \sum S_{T_i}$, in which case the chords do not fit the area allocated by the 360° of the total order indices. This case is managed by downscaling the full matrix S_{ij} . Similarly, the condition $\sum_j S_{ij} > S_{T_i}$ might not be respected, meaning the chords of parameter i do not fit the area allocated by the total order index of this parameter. This case is managed by downscaling the i^{th} row and column of S_{ij} .

4.2 Application to the first parametrization

Conversely to the analysis performed for the Ishigami function, the outputs of interest are not analytical, meaning the Sobol indices cannot be derived analytically. Instead, we have to go through the process of sampling the design space and estimating the Sobol indices, either from the raw samples, or from a metamodel trained on these samples.

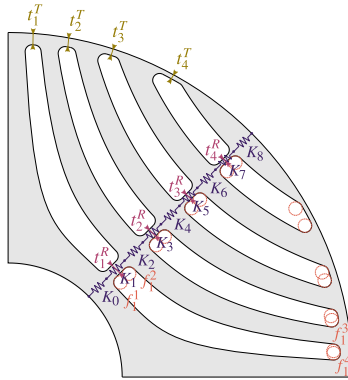


Figure 4.4: Summary of the geometric parameters for a rotor having 4 flux barriers, using the first parametrization

Consequently, for the following analysis, we use Saltelli's method to sample the search space. It has the advantage to be sequential, so that the list of samples can be extended without affecting the existing evaluated samples. This is helpful to enrich the analysis with new samples in case of partial convergence of Sobol indices. This method outputs $N(2d + 2)$ samples, with N being the size of the samples that we set at 1024.

While the parametrizations detailed in chapter 3 are applicable to rotors with any number n of flux barriers, we will essentially consider the case of a rotor with 4 flux

barriers. This choice is a decent trade-off between parameter count and magnetic performance. Hence, for the first parametrization having $d=34$, 71680 samples are generated within the boundaries of the search space provided in Table 4.1, and with the design space represented in Fig. 4.4. We authorize the removal of radial ribs by including zero thicknesses in the search space. The potential discontinuities it introduces is not considered a problem for the sensitivity analysis, as Sobol indices are indicated for discontinuous models, Fig. 4.1.

Table 4.1: Boundaries of the design space for the first parametrization ($i = 1, \dots, n$), ($j = 0, \dots, 2n$)

Param.	Description	Lower bound	Upper bound	Unit
θ_e	Current angle	45	75	$^\circ$
K_j	Stiffness of the virtual spring	1	10	—
t_i^R	Thickness of the radial rib	0	5	mm
t_i^T	Thickness of the tangential ribs	0.3	8	mm
$f_i^{1 \rightarrow 4}$	Radius of the fillet (x4)	1	10	mm

With a broad estimation of 15 minutes evaluation time per sample using the accurate FE models presented in chapter 2, the full analysis exceeds 4 years CPU time. Fortunately, as discussed in the introductory chapter, the evaluation of quasi-Monte-Carlo samples falls in the class of embarrassingly parallel problems. The results are made available within a few days thanks to the supercomputing capabilities of the *Consortium des Équipements de Calcul Intensif* (CÉCI), consortium of which UCLouvain is a member. We opted for local cluster computing over cloud computing services as they are provided free of charge for our university. However, we estimated the cost of conducting the same study on third-party cloud services to be a few hundred euros.

4.2.1 Mean torque

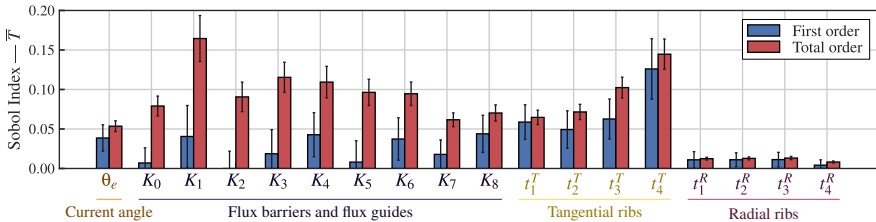


Figure 4.5: Summary of the Sobol indices obtained by performing the sensitivity analysis on the mean torque, using the first parametrization. The 95% confidence interval is shown. The parameters associated to the fillets are not displayed because of their negligible impact.

The first and total Sobol indices for the mean torque are depicted in Fig. 4.5. The 95% confidence interval has been determined through bootstrapping method, wherein the indices are recomputed using subsets of the complete dataset. The

narrow span of the confidence interval attests the robust convergence of these metrics. This conclusion is reinforced by plotting the evolution of these indices with the sample size, as performed in Fig. 4.6.

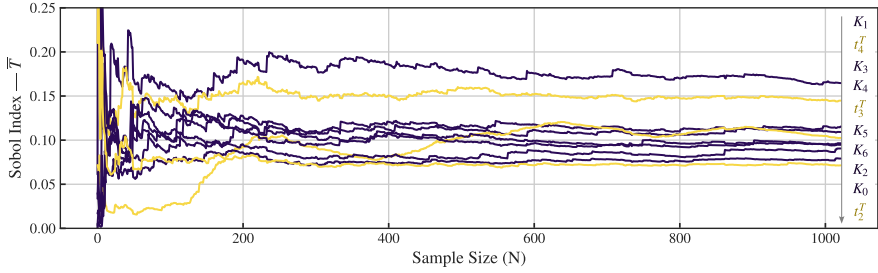


Figure 4.6: Evolution of the total order index S_T with the sample size N , for the 10 most influential parameters.

Parameters sharing the same physical meaning are grouped by color, in these figures and subsequent ones. The identified groups include the flux barriers and flux guides (without differentiation), tangential ribs, radial ribs, fillets, and the current angle.

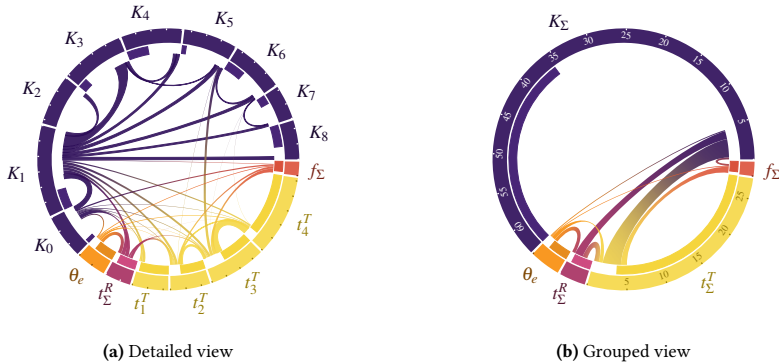


Figure 4.7: Representation of Sobol indices for the mean torque, using the first parametrization. The subscript Σ indicates that the parameters having the same physical meaning have been grouped together in the visualization. The ticks in the detailed view (a) mark a 2% interval.

Individually, none of these parameters significantly influences the mean torque production of the motor. This is evidenced by the outer ring segments of Fig. 4.7, which represents the effects of a parameter accounting for all its interactions. Indeed, the most influential parameter, K_1 , only covers around 12% of the total outer ring. Notably, the tangential ribs exhibit a more pronounced effect on torque compared to the radial ribs. This observation can be partly attributed to a more significant impact on the circulating component of the q axis inductance, L_{cq} . Additionally, it may result from a larger variation range for the tangential ribs, as indicated in Table 4.1, coupled with the fact that there are twice as many tangential ribs as there are radial ribs.

Moreover, the representation of the Sobol indices in Fig. 4.7 indicates substantial interactions, primarily among parameters characterizing the flux barriers and flux guides. This is evident through the presence of chords within the group K_i and, indirectly, by the gaps between the first indices (inner ring segments) and the second order indices (chords). Indeed, the second order indices represent the effects of interactions between two parameters while the first order indices represent the effects of a parameter alone. Consequently, each gap signifies unaccounted higher-order interactions and is expected to manifest symmetrically for all parameters involved. Given that these gaps predominantly appear for parameters K_i it is reasonable to conclude that higher-order interactions are confined within this group K_Σ , as highlighted in the group view (b).

Furthermore, the absence of significant interactions between parameters belonging to different groups suggests that the design space can be partitioned for optimizations on the mean torque. Specifically, the ribs can be optimized independently of the flux barriers based on this observation. Finally, the combined influence of the 16 parameters related to the fillets amounts to less than 2% of the cumulative total order index. This suggests these parameters do not need optimization.

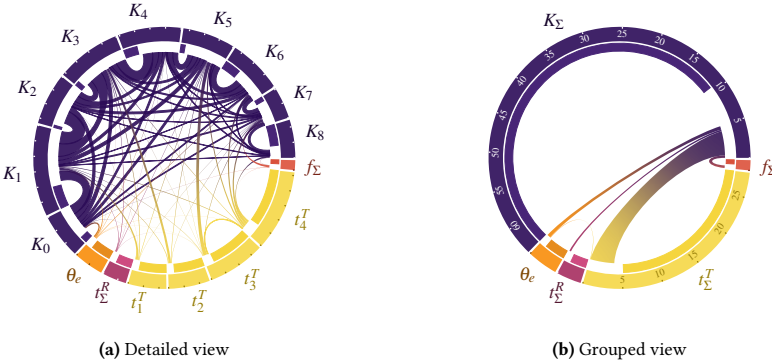


Figure 4.8: Same as Fig. 4.7, but using a 4th degree PCE to extract the Sobol indices analytically.

By employing metamodels, the Sobol indices can be evaluated analytically, eliminating the need for estimators. We succeeded to fit a 4th degree PCE on the mean torque, after reducing its basis by using the 16 most influential parameters. The first 70% samples has been used as training dataset, and the remaining 30% as validation dataset. The predictivity factor, Q^2 , is calculated on this last dataset as follows:

$$Q^2 = 1 - \frac{\sum_k^{N_k} (Y_k - \text{PCE}(\mathbf{X}_k))^2}{N_k \cdot \text{Var}[Y]}. \quad (4.23)$$

We obtain $Q^2 = 96\%$, what indicates this metamodel is a good substitute to the model. The Sobol indices are extracted analytically from this PCE, and presented in Fig. 4.8. This figure indicates that what was initially identified as higher-order interactions might actually be second-order interactions, as evidenced by the chords. Interestingly, we observe some small interactions between the tangential ribs and the

corresponding flux barrier, i.e., between t_i^T and K_{2i-1} . Overall, these results align entirely with the previous analysis of Fig. 4.7.

4.2.2 Relative torque ripple

A similar analysis can be conducted on the torque ripple, with the main results summarized in Fig. 4.9. Higher-order interactions involving three parameters or more contribute significantly to the total order indices, as indicated by the substantial gaps between the inner ring segments and the chords. This indicates complex interactions between parameters, rendering optimization of this objective challenging.

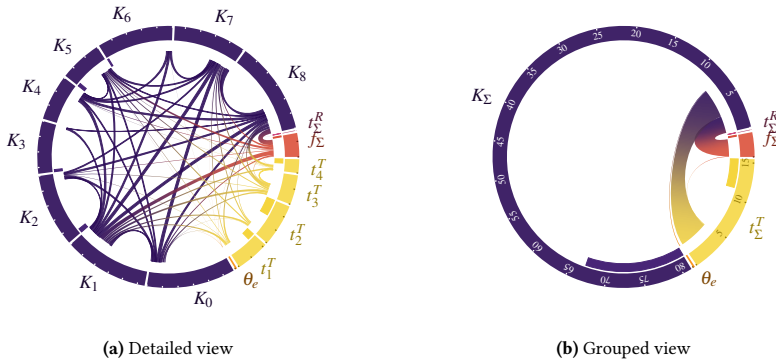


Figure 4.9: Representation of Sobol indices for the relative torque ripple, using the first parametrization.

The primary factors driving variance are the position and thicknesses of the flux barriers, as evidenced by the outer ring segment of the group K_Σ collectively accounting for over 80% of the total variance. The second main contributing factors are the thicknesses of the tangential ribs. In contrast, the influence of the fillets, radial ribs, and electric angle is found to be nearly negligible.

4.2.3 Iron losses

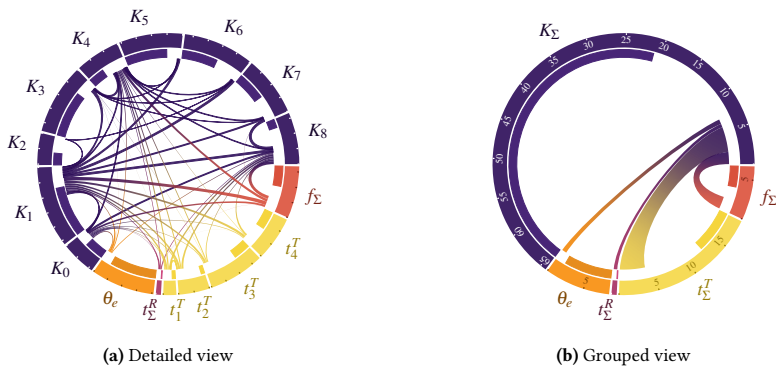


Figure 4.10: Representation of Sobol indices for the iron losses, using the first parametrization.

The primary parameters influencing the total iron losses, as illustrated in Fig. 4.10, include the positions and thicknesses of the flux barriers, the thicknesses of the tangential ribs, and the electric angle θ_e . Due to the essentially constant magnetic field within the rotor, iron losses are predominantly localized in its periphery, attributed to slot harmonics, thereby rendering the role of radial ribs insignificant.

Notably, across all simulated samples, the stator contributes to approximately two-thirds of the total iron losses. The majority of these losses are associated with the average magnetic flux within the stator. Similar to mean torque, the averaged magnetic flux is highly contingent on the level of magnetic insulation in the rotor, parameterized through the variables K_i .

4.2.4 Equivalent stress

The wide range of geometrical configurations sometimes led to mesh-induced stress concentration, bringing outliers in the evaluations. As a consequence, the 95% confidence interval of the Sobol indices was too wide for these indices to be meaningful, due to the vast contribution of these outliers to the variance of the maximum equivalent stress. Saturating these outputs at an arbitrary level, such as the ultimate tensile stress of the material, did fix the problem. Nonetheless, we preferred to re-evaluate all the samples by, this time, averaging the local equivalent stresses over their mesh elements, before taking the maximum. This smoothed out the outliers in a less brutal way than applying arbitrary saturations to the evaluated samples, and brought back the 95% confidence interval to reasonable levels.

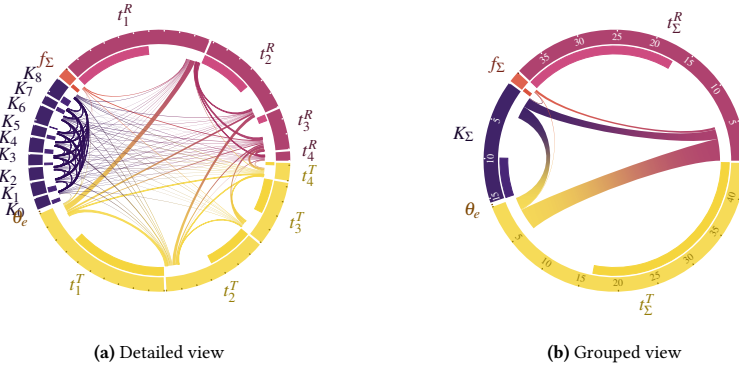


Figure 4.11: Representation of Sobol indices for the maximum local equivalent stress averaged over an element, using the first parametrization.

The Sobol indices of the maximum local equivalent stresses, averaged over an element, are summarized in Fig. 4.11. Consistent with expectations, the predominant influencers are identified as the tangential ribs and the radial ribs. The position and thicknesses of the flux barriers and flux guides also play a significant role by altering the mass distribution in the rotor. While the fillets appear to have a negligible impact on this specific objective, it is essential to consider the possibility that their influence may be obscured by the averaging process applied to stresses across the element, potentially overlooking stress concentration effects.

4.3 Application to the second parametrization

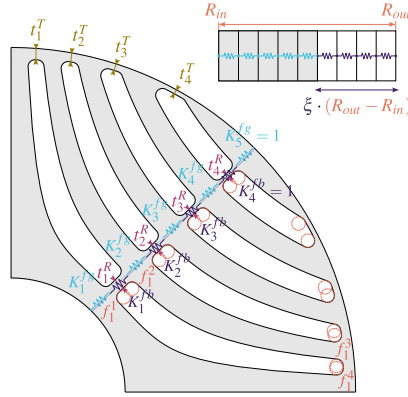


Figure 4.12: Summary of the geometric parameters for a rotor having 4 flux barriers, using the second parametrization

As for the first parametrization, we use Saltelli's method to sample the search space of the second parametrization, setting $N=1024$. This results in 69632 samples for the $d=33$ parameters, within the boundaries of the search space provided in Table 4.2, and with the design space represented in Fig. 4.12. The search space for parameters associated to the ribs, lower part of the table, is the same as for the first parametrization. The search space for the magnetic parameters, however, has been narrowed.

Table 4.2: Boundaries of the design space, for the second parametrization ($i = 1, 2, \dots, n$), ($j = 1, 2, \dots, q$)

Param.	Description	Lower bound	Upper bound	Unit
θ_e	Current angle	55	70	$^\circ$
ξ	Air-to-rotor ratio	0.2	0.8	—
K_j^{fg}	Stiffness of the virtual spring, for a flux guide	0.5	2	—
K_i^{fb}	Stiffness of the virtual spring, for a flux barrier	0.5	2	—
t_i^R	Thickness of the radial rib	0	5	mm
t_i^T	Thickness of the tangential ribs	0.3	8	mm
$f_i^{1 \rightarrow 4}$	Radius of the fillet (x4)	1	10	mm

4.3.1 Mean torque

This second parametrization aims at constructing the geometry around the main contributing factor of the mean torque, which is expected to be the air-to-rotor ratio ξ . As for the first parametrization, we perform a metamodeling fit using a 4th degree PCE on a reduced basis, which results on a 99% predictivity factor Q^2 .

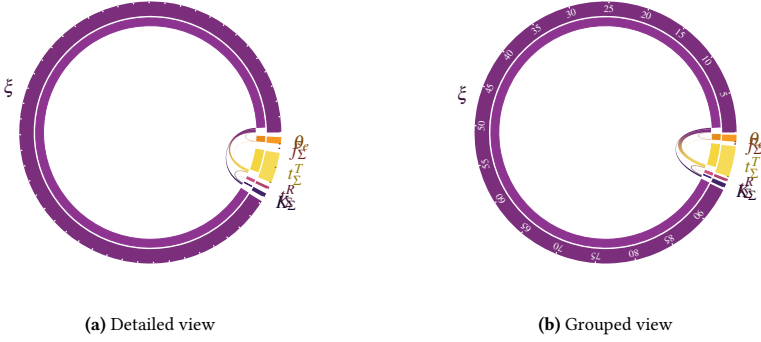


Figure 4.13: Representation of Sobol indices for the mean torque, using the second parametrization.

The Sobol indices extracted analytically from this PCE are summarized in Fig. 4.13, and shows the quality of this parametrization. Indeed, interactions between two or more parameters are minimized, with the inner ring segments nearly equalizing the outer ring segments. This indicates that the output variance is primarily attributable to the effects of varying each parameter alone.

Remarkably, nearly 95% of the variance in mean torque can be attributed to variations in a single parameter, with the remaining 33 parameters collectively contributing only 5%. The tangential ribs emerge as the second most influential factors affecting mean torque, while the individual distributions of air and iron between flux barriers and flux guides exhibit marginal importance. Additionally, the limited interactions between parameters suggest an additive model, implying that parameters can be optimized independently when mean torque is the sole objective.

4.3.2 Relative torque ripple

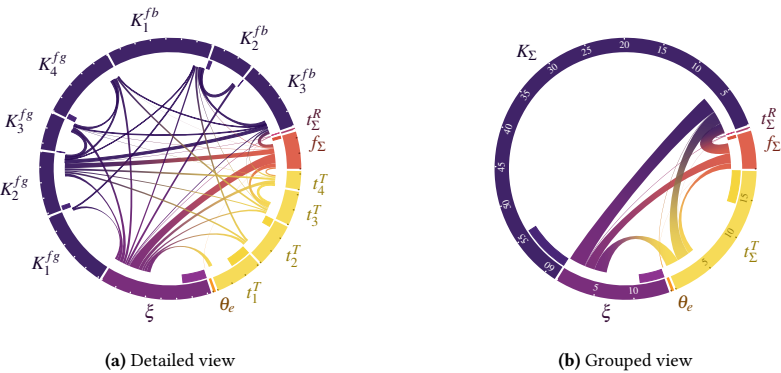


Figure 4.14: Representation of Sobol indices for the torque ripple, using the second parametrization.

The Sobol indices for relative torque ripple, as depicted in Fig. 4.14, exhibit strong similarities with those from the first parametrization. Surprisingly, the air-to-rotor

ratio ξ , expected to assume greater significance due to the normalization of torque ripple by mean torque, does not play a more prominent role. This underscores the presence of strong interactions, particularly between the individual distribution of air and iron among the flux barriers, flux guides, and the tangential ribs.

4.3.3 Iron losses

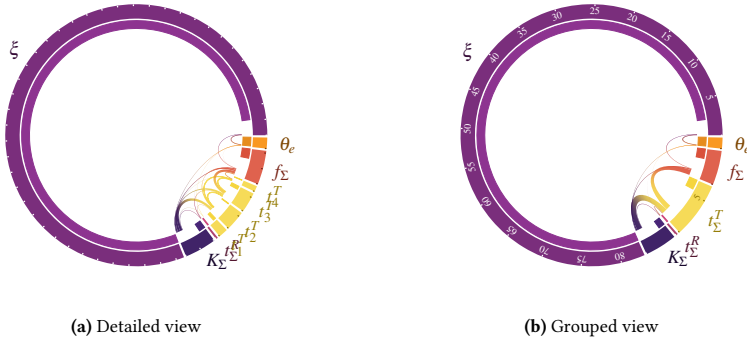


Figure 4.15: Representation of Sobol indices for the iron losses, using the second parametrization.

With this parametrization, over 80% of the variations in iron losses, as illustrated in Fig. 4.15, can be attributed to changes in the insulation ratio of the rotor alone. This finding aligns with observations from the first parametrization, with the iron losses essentially localized in the stator. The thicknesses of the tangential ribs and the individual distribution of air and iron between the flux guides and flux barriers emerge as the second most influential factors affecting iron losses.

4.3.4 Equivalent stress

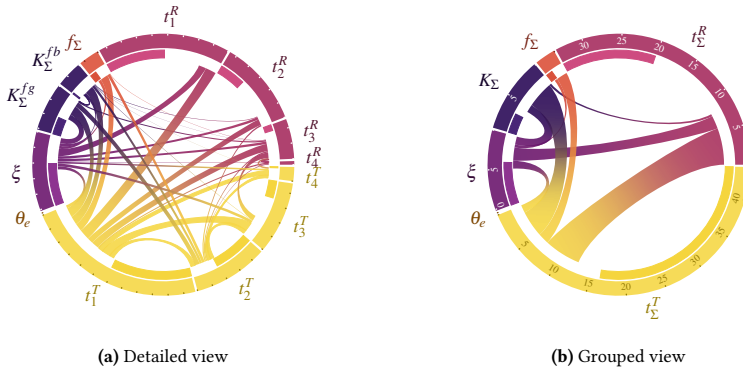


Figure 4.16: Representation of Sobol indices for the maximum local equivalent stress averaged over an element, using the second parametrization.

The Sobol indices for the maximum local equivalent stress, averaged over an element, are depicted in Fig. 4.16. The figure highlights that the quantity and

distribution of iron in the rotor contribute equally to the observed variations. Similar to the torque ripple, the Sobol indices derived from the second parametrization closely resemble those obtained from the first.

4.4 Summary

In this chapter, we provided an overview on the extraction and interpretation of the Sobol indices. To facilitate their interpretation, we represented total and first-order indices using nested pie charts, and depicted interactions between two parameters using a chord diagram.

In continuation, we conducted an extensive sensitivity analysis for both parametrizations on the key outputs of interest, including mean torque, torque ripple, iron loss, and equivalent stress. Quasi-random samples, obtained through Sobol Sequence, were evaluated on clusters using the models of chapter 2.

The Sobol indices for mean torque and iron losses exhibit similarities across the two parametrizations. The second parametrization, utilizing the air-to-rotor ratio ξ as an explicit parameter, successfully disentangles complex parameter interactions observed in the first parametrization. Consequently, the second parametrization proves highly effective for mean torque, indicating an additive model where input parameters can be independently optimized. In particular, over 95% of the variance in mean torque can be attributed to the air-to-rotor ratio ξ .

However, this re-parametrization is not as effective for torque ripple and equivalent stress. These outputs are notably influenced by tangential ribs and distribution of air and iron within the flux barriers and flux guides, with ξ playing a marginal role. This will complicate efforts to partition the optimization problem into smaller, more manageable sub-problems, as there exists a significant cross-coupling between these two outputs.

Intuition-based design approaches

5

In our work, the high computational cost associated to maximizing the mean torque under a torque ripple constraint and a structural integrity constraint could be managed through proper parallelization of the workload on a dedicated workstation. Given the iterative nature of solving optimization problems, workstations are more suitable than clusters thanks to their higher clock speed. Despite these considerations, we still observe wall-clock optimization times of several weeks.

This observation further underscores the need for time-efficient approaches, as they are indispensable for addressing more complex problems like stator optimization or optimization across several operating points. While time-efficient design approaches are widely used with many different implementations [10, 32–35], they often lack benchmarks on the computational advantages they bring. Moreover, by intervening at the optimization level, these approaches are expected to yield a degraded solution. Although this degradation is commonly accepted to be negligible, this aspect has not been comprehensively assessed in existing research. Consequently, the goal of this chapter is to address these topics.

This chapter is organized as follows. First, in section 5.1, we propose several approaches in line with the literature to tackle the optimal design of a SynRM that maximizes the mean torque under both a torque ripple and a structural integrity constraints. These approaches are formulated as a sequence of smaller optimization problems to solve. Subsequently, a detailed description of our optimization algorithm is provided in section 5.2. Then, the effectiveness of these approaches is evaluated on both parametrizations, with the results provided in sections 5.3 and 5.4. Finally, we summarize and discuss the findings in the concluding section 5.5.

5.1 Proposed design approaches

As mentioned in the introductory chapter, we deal with the problem of finding the rotor design that maximizes the mean torque under both a torque ripple and a structural integrity constraints. Mathematically, the problem is written:

$$\begin{aligned}
& \max_{\mathbf{x}} \quad \bar{T}(\mathbf{x}) \\
& \text{s.t.} \quad \max(\sigma_{\text{VM}}(\mathbf{x})) < \sigma_{\text{max}} \\
& \quad \quad \tilde{T}(\mathbf{x}) < \tilde{T}_{\text{max}},
\end{aligned} \tag{5.1}$$

where $\tilde{T}_{\text{max}}$ and σ_{max} are the maximum admissible values of the torque ripple and equivalent von Mises stress. The target maximum von Mises stresses and torque ripple are:

$$\tilde{T}_{\text{max}} = 10 \% \tag{5.2}$$

$$\sigma_{\text{max}} = 0.8\sigma_y = 270 \text{ MPa}, \tag{5.3}$$

with σ_y being the yield strength of the material.

Dividing this magneto-mechanical optimization problem into two parts, initially optimizing the flux barriers to achieve the desired magnetic performance, and subsequently optimizing the ribs to ensure the structural integrity, appears to be a natural approach for breaking down the optimization problem. Since the ribs are primarily introduced to withstand the mechanical load and may conflict with the magnetic performance, it seems logical to exclude these parameters from the magnetic optimization and optimize them specifically to support the mechanical load.

In the studies reviewed in the introductory chapter, the adjustment of the ribs to satisfy the structural integrity constraint is always performed in the last step of the optimization process. Frequently labeled as 'fine-tuning' in these studies, this term suggests a manual, designer-driven adjustment rather than being explicitly formulated as an optimization problem. That poses an additional challenge for the automated design of SynRM. Consequently, maintaining the principle of separating the optimization of the flux barriers from the optimization of the ribs, we also discuss various formulations for sizing the ribs as an optimization problem.

The workflow of the approaches is summarized in Fig. 5.1, where the zero superscript indicates the initial guess and the star superscript indicates the parameters that

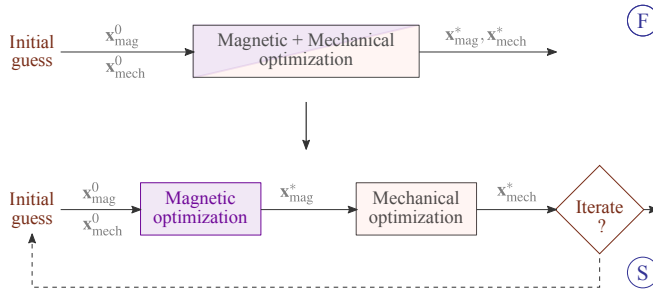


Figure 5.1: Principle scheme of the two different design approaches. A single full optimization can be carried out (F), or it can be split in two successive optimizations (S).

have been optimized. The first approach (F) consists of carrying a single optimization on the full search space, addressing the optimization problem (5.1). This approach serves as a benchmark for both the total optimization time and quality of the solution. The second approach (S) involves two successive optimizations that split the initial search space $\mathbf{x}$ into parameters associated with the flux barriers $\mathbf{x}_{\text{mag}}$, and the parameters associated with the ribs $\mathbf{x}_{\text{mech}}$. These subsets correspond to the upper and lower half of the tables presented in Tables 3.1 and 3.2. Although not related to the flux barriers, the current angle θ_e is also a parameter to be optimized for the mean torque and torque ripple, so that it belongs in $\mathbf{x}_{\text{mag}}$. The magnetic optimization uses the subset $\mathbf{x}_{\text{mag}}$ to maximize the mean torque under the torque ripple constraint, while the mechanical optimization uses $\mathbf{x}_{\text{mech}}$ to ensure the structural integrity of the rotor:

$$\text{Magnetic optimization} = \begin{cases} \max_{\mathbf{x}_{\text{mag}}} & \bar{T}(\mathbf{x}_{\text{mag}}, \mathbf{x}_{\text{mech}}^0) \\ \text{s.t.} & \tilde{T}(\mathbf{x}_{\text{mag}}, \mathbf{x}_{\text{mech}}^0) < \tilde{T}_{\text{max}} \end{cases} \quad (5.4)$$

$$\text{Mechanical optimization} = \begin{cases} \min_{\mathbf{x}_{\text{mech}}} & g(\mathbf{x}_{\text{mech}}, \mathbf{x}_{\text{mag}}^*) \\ \text{s.t.} & \max(\sigma_{\text{VM}}(\mathbf{x}_{\text{mech}}, \mathbf{x}_{\text{mag}}^*)) < \sigma_{\text{max}}. \end{cases} \quad (5.5)$$

The separation of the mono-objective multi-constraints optimization problem (5.1) into two parts results in an arbitrary choice for the function $g(\mathbf{x}_{\text{mech}}, \mathbf{x}_{\text{mag}}^*)$. This function, designed to uphold the magnetic performance of the previously optimized motor, is subject to various formulations explored and discussed in subsection 5.1.2.

Provided an ideal global search algorithm, the choice of the initial guess in the full optimization (F) should not affect the outcome of the optimization, i.e., the optimized parameters $\mathbf{x}_{\text{mag}}^*$ and $\mathbf{x}_{\text{mech}}^*$. Contrarily, the choice of the initial guess is expected to impact the outcome of the second approach (S) due to second and higher order interactions that exist between the parameters that are not simultaneously optimized.

Ideally, the initial guess $\mathbf{x}_{\text{mech}}^0$ should come as close as possible to the optimal value of the mechanical parameters $\mathbf{x}_{\text{mech}}^*$ obtained from the mechanical optimization. This would ensure that the magnetic optimization finds the optimal values of $\mathbf{x}_{\text{mag}}$ by accounting for the effects of the ribs. Since the optimal values of $\mathbf{x}_{\text{mag}}^*$ and $\mathbf{x}_{\text{mech}}^*$ are not known a priori, iterative can be employed. When iterating, the values of $\mathbf{x}_{\text{mech}}^*[i-1]$ obtained at step $i-1$ are used within the initial population of the magnetic optimization, and the solution $\mathbf{x}_{\text{mag}}^*[i-1]$ is injected as one individual of the initial population. Another possibility consists in predicting the values $\hat{\mathbf{x}}_{\text{mech}}^*(\mathbf{x}_{\text{mag}})$ within the magnetic optimization, in an attempt to compensate the cross-coupling that exists between the magnetic and the mechanical optimization. On these bases, we also derive two different formulations for the magnetic optimization.

5.1.1 Magnetic optimization

The two approaches investigated to perform the magnetic optimization are schematized in Fig. 5.2. The first formulation is the direct implementation of (5.4) without

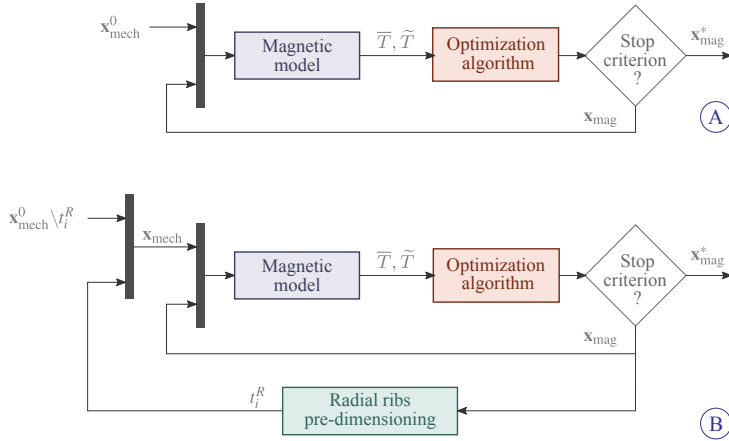


Figure 5.2: Approaches investigated to perform the magnetic optimization.

modification:

$$\text{Magnetic optimization } \underline{A} = \begin{cases} \max_{\mathbf{x}_{\text{mag}}} & \bar{T}(\mathbf{x}_{\text{mag}}, \mathbf{x}_{\text{mech}}^0) \\ \text{s.t.} & \tilde{T}(\mathbf{x}_{\text{mag}}, \mathbf{x}_{\text{mech}}^0) < \tilde{T}_{\text{max}} \end{cases} \quad (5.6)$$

This implementation is straightforward, but it ignores the effects of the mechanical optimization on the ribs, and so on the magnetic performance of the motor. In the second implementation, the radial ribs are pre-dimensioned analytically within the magnetic optimization. This approach has been suggested in [93] to tackle the design of a high-speed SynRM with a similar rotor structure, illustrated in Fig. 5.3.

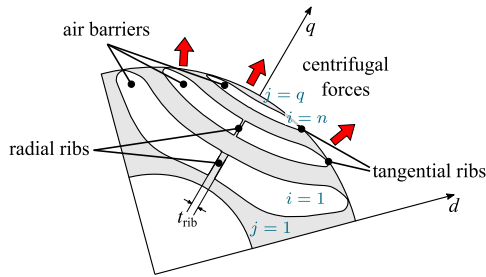


Figure 5.3: Sketch of the SynRM investigated in [93].

Using this approach, the magnetic optimization problem becomes:

$$\text{Magnetic optimization } \underline{B} = \begin{cases} \max_{\mathbf{x}_{\text{mag}}} & \bar{T}(\mathbf{x}_{\text{mag}}, t_i^R(\mathbf{x}_{\text{mag}}), \mathbf{x}_{\text{mech}}^0 \setminus t_i^R) \\ \text{s.t.} & \tilde{T}(\mathbf{x}_{\text{mag}}, t_i^R(\mathbf{x}_{\text{mag}}), \mathbf{x}_{\text{mech}}^0 \setminus t_i^R) < \tilde{T}_{\text{max}} \end{cases}, \quad (5.7)$$

with $i = \{1, \dots, n\}$. By neglecting the contribution of the tangential ribs, the radial rib i sustain the centrifugal load of all the flux guides located after the rib, i.e., for $j > i$. The equivalent centrifugal force F_i is:

$$F_i(\mathbf{x}_{\text{mag}}) = M_i(\mathbf{x}_{\text{mag}}) \omega_m^2 R_i(\mathbf{x}_{\text{mag}}), \quad (5.8)$$

with ω_m the rotational speed of the rotor, M_i the mass the i^{th} rib has to withstand that are the masses of the flux guides located after the rib, and R_i the radius of its center of gravity. They are related to the position of the center of gravity r_j and mass m_j of the flux guide j through:

$$M_i = \sum_{j=i+1}^q m_j \quad R_i = \frac{\sum_{j=i+1}^q m_j r_j}{M_i}, \quad (5.9)$$

so that the radial rib i can be dimensioned using the maximal admissible stress:

$$t_i^R(\mathbf{x}_{\text{mag}}, \sigma_{\text{max}}) = \frac{F_i(\mathbf{x}_{\text{mag}})}{\sigma_{\text{max}} L} \cdot \text{SF}, \quad (5.10)$$

with SF a safety factor, arbitrarily set to 1.5, and L the axial length of the lamination stack.

By pre-dimensioning the ribs within the magnetic optimization, the configurations with smaller flux guides and closer to the shaft are favored. Indeed, these configurations decrease the thicknesses of the radial ribs and therefore the negative impact they have on the mean torque. Essentially, pre-dimensioning the ribs within the magnetic optimization allows us to consider the effect of the rotor mass distribution on the von Mises stresses, which contributes to approximately 15 % of the variance in the maximum, averaged, von Mises stresses (Fig. 4.11 and Fig. 4.16). Consequently, the subsequent mechanical optimization is expected to result in smaller ribs, thereby minimizing the negative impact of the mechanical optimization on the magnetic performance.

5.1.2 Mechanical optimization

The objective function $g(\mathbf{x}_{\text{mech}}, \mathbf{x}_{\text{mag}}^*)$ in the optimization problem (5.5) is arbitrarily defined to maintain the magnetic performance achieved in the motor optimization preceding this stage. We present three distinct implementations, illustrated in Fig. 5.4.

The first implementation involves reusing the identical objective and constraints utilized in the magnetic optimization:

$$\text{Mechanical optimization I} = \begin{cases} \max_{\mathbf{x}_{\text{mech}}} & \bar{T}(\mathbf{x}_{\text{mech}}, \mathbf{x}_{\text{mag}}^*) \\ \text{s.t.} & \max(\sigma_{\text{VM}}(\mathbf{x}_{\text{mech}}, \mathbf{x}_{\text{mag}}^*)) < \sigma_{\text{max}} \\ & \tilde{T}(\mathbf{x}_{\text{mech}}, \mathbf{x}_{\text{mag}}^*) < \tilde{T}_{\text{max}} \end{cases} \quad (5.11)$$

This optimization problem resemble as closely as possible to the initial optimization problem (5.1). However, it requires magnetic model evaluations within the mechanical optimization, which is suboptimal due to the significant asymmetry in evaluation times between the magnetic and mechanical models.

Alternatively, a more intuitive approach involves minimizing the total volume of the ribs, recognizing their detrimental impact on mean torque. This objective closely

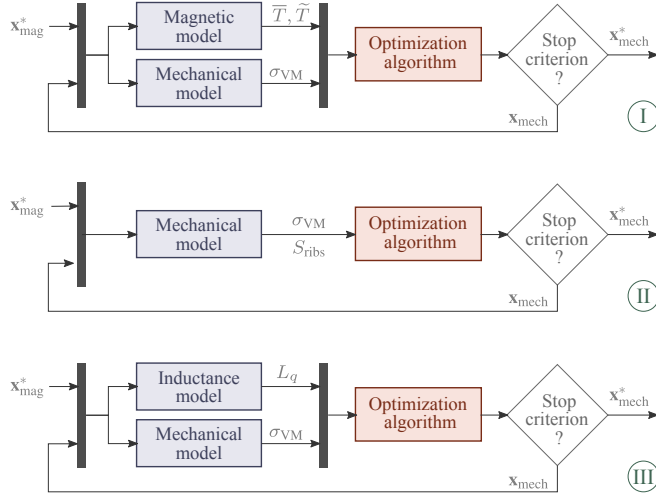


Figure 5.4: Approaches investigated to perform the mechanical optimization.

aligns with the goal of the topology optimization performed for the ribs in [35]. In 2D, this problem involves minimizing the total surface of the ribs S_{ribs} :

$$\text{Mechanical optimization II} = \begin{cases} \min_{\mathbf{x}_{\text{mech}}} & S_{\text{ribs}}(\mathbf{x}_{\text{mech}}, \mathbf{x}_{\text{mag}}^*) \\ \text{s.t.} & \max(\sigma_{\text{VM}}(\mathbf{x}_{\text{mech}}, \mathbf{x}_{\text{mag}}^*)) < \sigma_{\text{max}} \end{cases} \quad (5.12)$$

The total surface area of the ribs is computed as the difference between the total surface of the electrical steel with and without the ribs:

$$S_{\text{ribs}}(\mathbf{x}_{\text{mech}}, \mathbf{x}_{\text{mag}}^*) = S_{\text{iron}}(\mathbf{x}_{\text{mech}}, \mathbf{x}_{\text{mag}}^*) - S_{\text{iron}}(\mathbf{x}_{\text{mech}} = \mathbf{0}, \mathbf{x}_{\text{mag}}^*) \quad (5.13)$$

The last term being a constant, the objective $S_{\text{ribs}}(\mathbf{x}_{\text{mech}}, \mathbf{x}_{\text{mag}}^*)$ can be replaced by $S_{\text{iron}}(\mathbf{x}_{\text{mech}}, \mathbf{x}_{\text{mag}}^*)$ in (5.12) with no consequence for the optimization. Computing the surface S_{iron} is trivial using FE by integrating 1 over all the iron mesh elements.

A refined version of the optimization (5.12) can be derived using the torque equation (1.10):

$$\begin{aligned} \max_{\mathbf{x}_{\text{mech}}} & \quad \frac{3}{2} p (L_d(\mathbf{x}_{\text{mech}}, \mathbf{x}_{\text{mag}}^*) - L_q(\mathbf{x}_{\text{mech}}, \mathbf{x}_{\text{mag}}^*)) i_d i_q \\ \text{s.t.} & \quad \max(\sigma_{\text{VM}}(\mathbf{x}_{\text{mech}}, \mathbf{x}_{\text{mag}}^*)) < \sigma_{\text{max}} \end{aligned} \quad (5.14)$$

While the ribs have a negligible effect on the d axis inductance L_d , their impact on the q axis inductance L_q is substantial. Consequently, minimizing the q axis inductance within the mechanical optimization is posited to minimize the influence of the ribs on mean torque. Using this hypothesis, the optimization problem (5.14) is equivalent to:

$$\text{Mechanical optimization III} = \begin{cases} \min_{\mathbf{x}_{\text{mech}}} & L_q(\mathbf{x}_{\text{mech}}, \mathbf{x}_{\text{mag}}^*) \\ \text{s.t.} & \max(\sigma_{\text{VM}}(\mathbf{x}_{\text{mech}}, \mathbf{x}_{\text{mag}}^*)) < \sigma_{\text{max}} \end{cases} \quad (5.15)$$

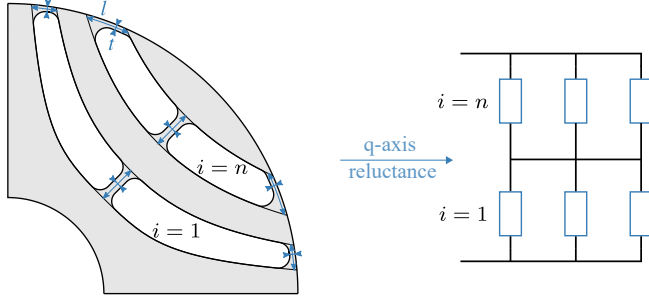


Figure 5.5: Equivalent q-axis reluctance. The flux guides are considered infinitely permeable, and the ribs are approximated by rectangular elements.

Despite its connection to magnetism, the q-axis inductance L_q can be quickly estimated through a simple model with negligible computational cost. Assuming flux guides as perfect magnetic conductors and flux barriers as perfect magnetic insulators, the q-axis inductance is determined by solving a magnetic circuit formed by the ribs. Radial and tangential ribs are parallel reluctance elements within the same flux barrier, arranged in series with the elements of the subsequent flux barrier, as illustrated in Fig. 5.5. Each rib is approximated by a rectangular element of length l and cross-section $S = L \cdot t$, having a magnetic permeability μ . The reluctance of a rib is given by:

$$\mathcal{R} = \frac{l}{\mu \cdot S}. \quad (5.16)$$

Assuming uniform permeability across all elements, the solution to the optimization problem (5.15) remains independent of the magnetic permeability μ .

5.1.3 Summary of the approaches

We derived 2 implementations for the magnetic optimizations and 3 for the mechanical optimizations, leading to 6 combinations for the split design approach (S). To differentiate them, we use the letter of the magnetic implementation (A or B) followed by the roman number of the mechanical implementation (I, II, or III), which can be repeated in case of iterations. For instance, the approach AIII-AIII consists in iterating once on the scheme where the magnetic optimization A is paired with the mechanical optimization III. The number of iterations is limited to one as it drastically increases the total optimization time. The initial guess is chosen as a rotor having flux barriers the same thicknesses as the flux guides without any ribs, i.e., $\mathbf{x}_{\text{mech}}^0 = \mathbf{0}$.

The second parametrization is anticipated to result in a reduced optimization time compared to the first parametrization when optimizing the mean torque, i.e., in optimizations A, B, and (F). This advantage stems from its additive property, highlighted in section 4.3.1 through the sensitivity analysis. Despite the expected efficiency of one parametrization over the other, the observations for each approach are projected to be analogous between the two parametrizations. Indeed, both parametrization separate the ribs from the flux barriers in a consistent manner, and

the level of interdependence among the parameters for the torque ripple and von Mises stresses remain high in both cases. Consequently, separating the magnetic optimization from the mechanical optimization is likely to show the same effects on both parametrizations. Hence, to mitigate the computational load associated with the duplication of the results, the proposed approaches are benchmarked using the first parametrization, which aligns more closely with parametrizations found in the literature. In a subsequent phase, the best-performing approach will be applied using the second parametrization to verify the extent of the results. The full optimization (F) is solved for both parametrizations to allow the comparison.

5.2 Optimizations

In the case of SynRM and other types of electric machines, the exploration of the search space often encounters numerous local minima on the typical magnetic objectives such as mean torque or torque ripple [94, 95], a consequence from the complex interactions among the numerous parameters describing the geometry. Additionally, accounting for structural integrity introduces discontinuities in constraints as the geometrical location where maximum equivalent stress occurs may jump during the optimization. Consequently, the exploration of the search space is typically performed by stochastic global search algorithms [96]. Such algorithms include the well-known Non-Dominated Sorting Genetic Algorithm-II (NSGA-II) [97], the Particle Swarm Optimization (PSO) [98], or the Differential Optimization (DE) [99].

The choice of the best algorithm remains a subject of lively debate within the scientific community [100–102]. However, the consensus tends to align with the no free lunch theorem, which posits that no algorithm is best across all scenarios [103]. Consequently, determining the most suitable optimization algorithm, along with its configuration settings, becomes wholly contingent on the optimization problem. This necessitates testing different algorithms on the same problem, a process that is time-consuming and impractical.

Nonetheless, all these algorithms serve the same fundamental purpose, which is to find an optimum within a finite amount of time. In this regard, all optimizations in this study are conducted using the generic NSGA-II algorithm, chosen among others, and configured with the settings reported in Table 5.1. These settings correspond to the default values commonly encountered in other optimization frameworks, such as jMetal [104] or pymoo [105]. While there may exist other settings or optimization algorithms that could potentially achieve convergence faster, these considerations are reserved for future perspectives of this work.

A termination criterion on the objective is applied when the relative improvement over the last 30 generations does not exceed 0.1%:

$$\text{terminate if } \text{abs} \left(\frac{\text{objective}[k] - \text{objective}[k-30]}{\text{objective}[k]} \right) < \frac{0.1}{100}. \quad (5.17)$$

Table 5.1: Configuration settings of the NSGAI.

Parameter name	Value
Population size	100
Population generator	Random, except for 1 initial guess
Selection operator	Tournament (with size 2)
Crossover operator	Simulated Binary Crossover (SBX)
Crossover probability	1
Crossover distribution index	15.0
Mutation operator	Polynomial Mutation
Mutation probability	$1/d$ (d = number of variables)
Mutation distribution index	20.0

The relevance of this termination criterion is illustrated in Fig. 5.6 for the reference optimization problem F (5.1), where this criterion was not applied. The objective evolves only slightly after the theoretical application of the termination criterion (5.17), suggesting that continuing the optimization is not worth the extra optimization time. The relative improvement on the objective diminishes steadily, indicating that the window frame of 30 generations is sufficiently long to prevent the termination criterion to trigger prematurely.

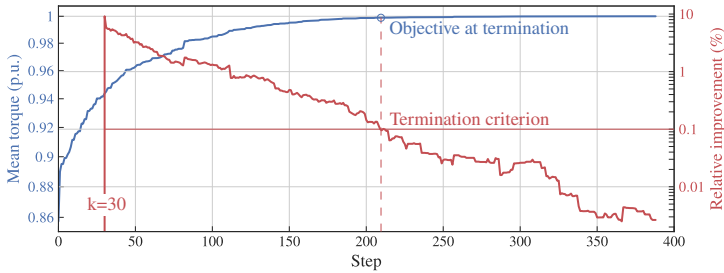


Figure 5.6: Evolution of the objective over the optimization, for the reference optimization F (5.1).

To account for the cross-coupling between $\mathbf{x}_{\text{mag}}$ and $\mathbf{x}_{\text{mech}}$ alongside the stochastic nature of the optimization algorithm, the approaches that consider successive optimizations, i.e., approaches AI, AII, AIII, BI, BII, BIII, and the additional iteration, are applied in 3 independent runs using the same initial conditions. The average values of the quantities of interest are used for comparison purpose, while the dispersion gives information on how repeatable the design approach is.

The calculations are carried out using the Intel i9-7940X CPU (14 cores, 4.3 GHz) with 64 GB of RAM. Parallel evaluations of individuals belonging to the same generation are performed to reach a near 100% CPU load. On average, it takes 11 minutes CPU time for the magnetic model to evaluate the torque characteristics of the machine, and 45 seconds CPU time for the mechanical model to evaluate the local von Mises stresses.

5.3 Results using the first parametrization

The reference result of carrying the optimization (5.1), within the search space presented in Table 4.1, is reported in Table 5.2. It takes over 12 days in elapsed real time for this optimization to converge, with the convergence graph provided in Fig. 5.6. While dropping the torque ripple constraint reduces the optimization time by half, the marginal improvement in mean torque is deemed insufficient to justify the additional torque ripple.

Table 5.2: Performance metrics and total optimization time obtained by carrying the optimization (5.1). In the second row, the same optimization has been carried out with the torque ripple constraint dropped, aiming to assess the convergence of the initial optimization.

Design approach	Mean torque	Torque ripple	Total opti. time	
	p.u.	%	Hours	p.u.
Ⓕ	1	10.00	302.4	1
Ⓕ' (w/o $\tilde{T}$)	1.01	36.60	151.0	0.5

This design obtained by approach Ⓕ outputs around 3% higher torque than the prototype used for the experimental validation presented in chapter 2. Its output torque characteristic, geometry and von Mises stresses are displayed in Fig. 5.7. The torque ripple and maximum von Mises stress perfectly meet the constraints (5.3) and (5.2).

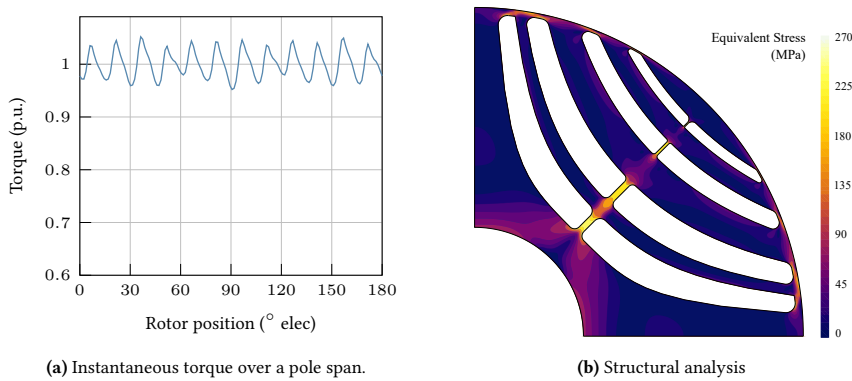


Figure 5.7: Characteristics of the reference solution.

The mean torque, torque ripple, and total optimization time of the investigated approaches are presented in Table 5.3, and the underlying parameters are summarized in appendix C. All approaches conclude with an optimization step addressing structural integrity of the rotor to ensure adherence to the constraint. Notably, approaches B consistently result in lower total optimization times compared to approaches A. The optimization I explicitly accounts for the torque ripple during the mechanical optimization, ensuring perfect adherence to the constraint for approaches AI and BI, along with their iteration. However, the mean torque in these

Table 5.3: Quantities of interest for the proposed design approaches, averaged over three independent runs. The upper and lower deviations are indicated. The mean torque and total optimization time are expressed in p.u. of the reference solution (Table 5.2).

Design approach	Mean torque (p.u.)	Torque ripple (%)	Total opti. time (p.u.)
AI	0.91 $\begin{smallmatrix} +0.05 \\ -0.05 \end{smallmatrix}$	10.0 $\begin{smallmatrix} +0.0 \\ -0.0 \end{smallmatrix}$	0.97 $\begin{smallmatrix} +0.35 \\ -0.24 \end{smallmatrix}$
AI-AI	0.93 $\begin{smallmatrix} +0.03 \\ -0.02 \end{smallmatrix}$	9.9 $\begin{smallmatrix} +0.1 \\ -0.1 \end{smallmatrix}$	1.50 $\begin{smallmatrix} +0.45 \\ -0.32 \end{smallmatrix}$
AII	0.96 $\begin{smallmatrix} +0.02 \\ -0.04 \end{smallmatrix}$	17.2 $\begin{smallmatrix} +4.5 \\ -2.7 \end{smallmatrix}$	0.48 $\begin{smallmatrix} +0.23 \\ -0.25 \end{smallmatrix}$
AII-AII	0.90 $\begin{smallmatrix} +0.01 \\ -0.03 \end{smallmatrix}$	14.6 $\begin{smallmatrix} +10.0 \\ -11.1 \end{smallmatrix}$	0.77 $\begin{smallmatrix} +0.28 \\ -0.23 \end{smallmatrix}$
AIII	0.95 $\begin{smallmatrix} +0.03 \\ -0.03 \end{smallmatrix}$	23.0 $\begin{smallmatrix} +3.1 \\ -1.6 \end{smallmatrix}$	0.52 $\begin{smallmatrix} +0.20 \\ -0.25 \end{smallmatrix}$
AIII-AIII	0.91 $\begin{smallmatrix} +0.01 \\ -0.01 \end{smallmatrix}$	11.9 $\begin{smallmatrix} +2.9 \\ -3.3 \end{smallmatrix}$	0.74 $\begin{smallmatrix} +0.18 \\ -0.28 \end{smallmatrix}$
BI	0.92 $\begin{smallmatrix} +0.03 \\ -0.04 \end{smallmatrix}$	10.0 $\begin{smallmatrix} +0.0 \\ -0.0 \end{smallmatrix}$	0.41 $\begin{smallmatrix} +0.01 \\ -0.01 \end{smallmatrix}$
BI-BI	0.94 $\begin{smallmatrix} +0.03 \\ -0.04 \end{smallmatrix}$	9.9 $\begin{smallmatrix} +0.1 \\ -0.1 \end{smallmatrix}$	1.07 $\begin{smallmatrix} +0.07 \\ -0.13 \end{smallmatrix}$
BII	0.97 $\begin{smallmatrix} +0.02 \\ -0.03 \end{smallmatrix}$	17.5 $\begin{smallmatrix} +1.3 \\ -1.5 \end{smallmatrix}$	0.17 $\begin{smallmatrix} +0.01 \\ -0.01 \end{smallmatrix}$
BII-BII	1.00 $\begin{smallmatrix} +0.01 \\ -0.01 \end{smallmatrix}$	13.9 $\begin{smallmatrix} +3.9 \\ -7.3 \end{smallmatrix}$	0.55 $\begin{smallmatrix} +0.21 \\ -0.12 \end{smallmatrix}$
BIII	0.97 $\begin{smallmatrix} +0.01 \\ -0.02 \end{smallmatrix}$	20.8 $\begin{smallmatrix} +3.2 \\ -4.7 \end{smallmatrix}$	0.18 $\begin{smallmatrix} +0.01 \\ -0.02 \end{smallmatrix}$
BIII-BIII	0.97 $\begin{smallmatrix} +0.02 \\ -0.01 \end{smallmatrix}$	13.3 $\begin{smallmatrix} +3.9 \\ -4.4 \end{smallmatrix}$	0.43 $\begin{smallmatrix} +0.01 \\ -0.02 \end{smallmatrix}$

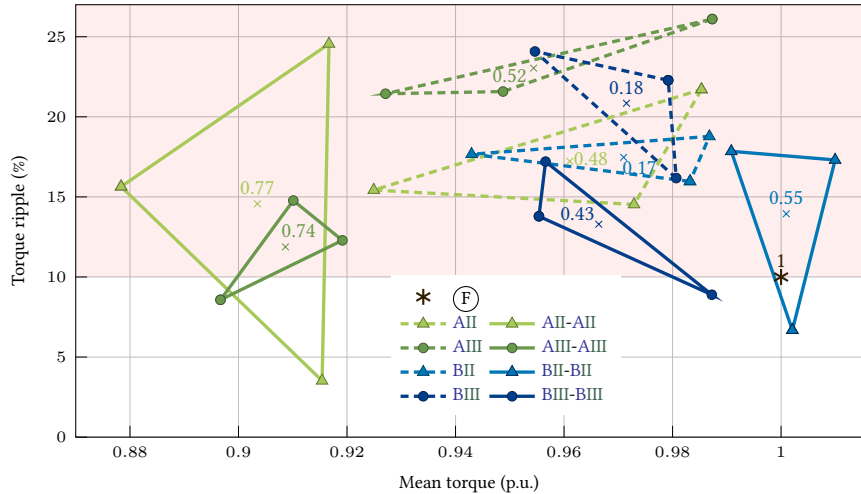


Figure 5.8: Performance of the obtained solutions. The area of the triangles formed by three independent runs of the same approach can be related to the reproducibility of the results, smaller being better. The center of gravity, marked with an $\times$, corresponds to the averaged mean torque and torque ripple presented in Table 5.3. The numbers correspond to the total optimization time averaged over the three independent runs and relative to the reference optimization F. For the sake of clarity, the combinations with the optimization I are omitted from this graph.

cases is not as high as that of the other approaches, and the total optimization time compares to the one of the reference optimization (F). Consequently, we exclude the approach I from the next analyses.

To facilitate the comparison between the solutions, a visualization of the electromagnetic performance of the solutions is provided in Fig. 5.8. The $x - y$ coordinates of the center of gravity of the triangle represent the averaged mean torque and torque ripple over the independent runs, and the inverse size of the triangle is indicative of the repeatability of the approach. The dashed triangles correspond to the results of AII, BII, AIII, and BIII, revealing their overall similar performance. Iterating on the scheme enhances the torque ripple for all the approaches, as shown by the solid triangles. By iterating, the mean torque deteriorates for the approaches A and improves for the approaches B, which pre-dimension the radial ribs within the magnetic optimization.

The reasons of this deterioration can be elucidated by examining the rotor geometries depicted in Fig. 5.9. The iterations on approaches A tend to bring the first flux barrier closer to the shaft, leading to the formation of larger ribs in the subsequent optimization II or III, so that the mean torque is decreased. The shift in the position of the first flux barrier is the consequence of the cross-coupling between the tangential ribs t_i^T and the position and thicknesses of the flux barriers K_j . Indeed, as quantified in Fig. 4.8, these interactions account for 5% of the variance in the mean torque, so that the optimal values K_j^* change when the initial configuration $t_i^{T,0}$ is altered during the iterations. In optimizations B, the configurations with the flux barriers closer to the shaft are penalized by larger radial ribs. This penalization is likely to have a greater impact on the mean torque than the interactions between K_j and $t_i^{T,0}$, lowering the sensitivity of the solutions K_j^* with respect to the initial guess. This could explain the better performances of approaches B over A upon iterating. On a side note, the last radial rib is entirely removed in some of the optimized geometries such as in AII #3. This is attributed to the lack of sensitivity of the objectives and constraints with respect to the parameter t_4^R .

Minimizing the total surface of the ribs in optimization II or minimizing the estimated q axis inductance in optimization III lead to similar solutions and total optimization time. The optimization II, however, keeps an advantage in its ease of implementation. With an optimization time that is halved and a similar mean torque, the approach BII-BII is effectively time-efficient for tackling the design of the rotor of a SynRM when compared to the reference optimization (F). Moreover, in Fig. 5.8, one solution of this approach slightly dominates the reference solution, what indicates an imperfect convergence of optimization (F) despite its high optimization time.

An evident limitation of the investigated approaches is their inability to consistently limit the torque ripple while preserving a low optimization time. The adaptations made in optimizations II and III to preserve the mean torque are not sufficient to always guarantee a low ripple. By interchanging the mechanical optimization with the magnetic optimization in the optimization scheme, the targets in the magnetic

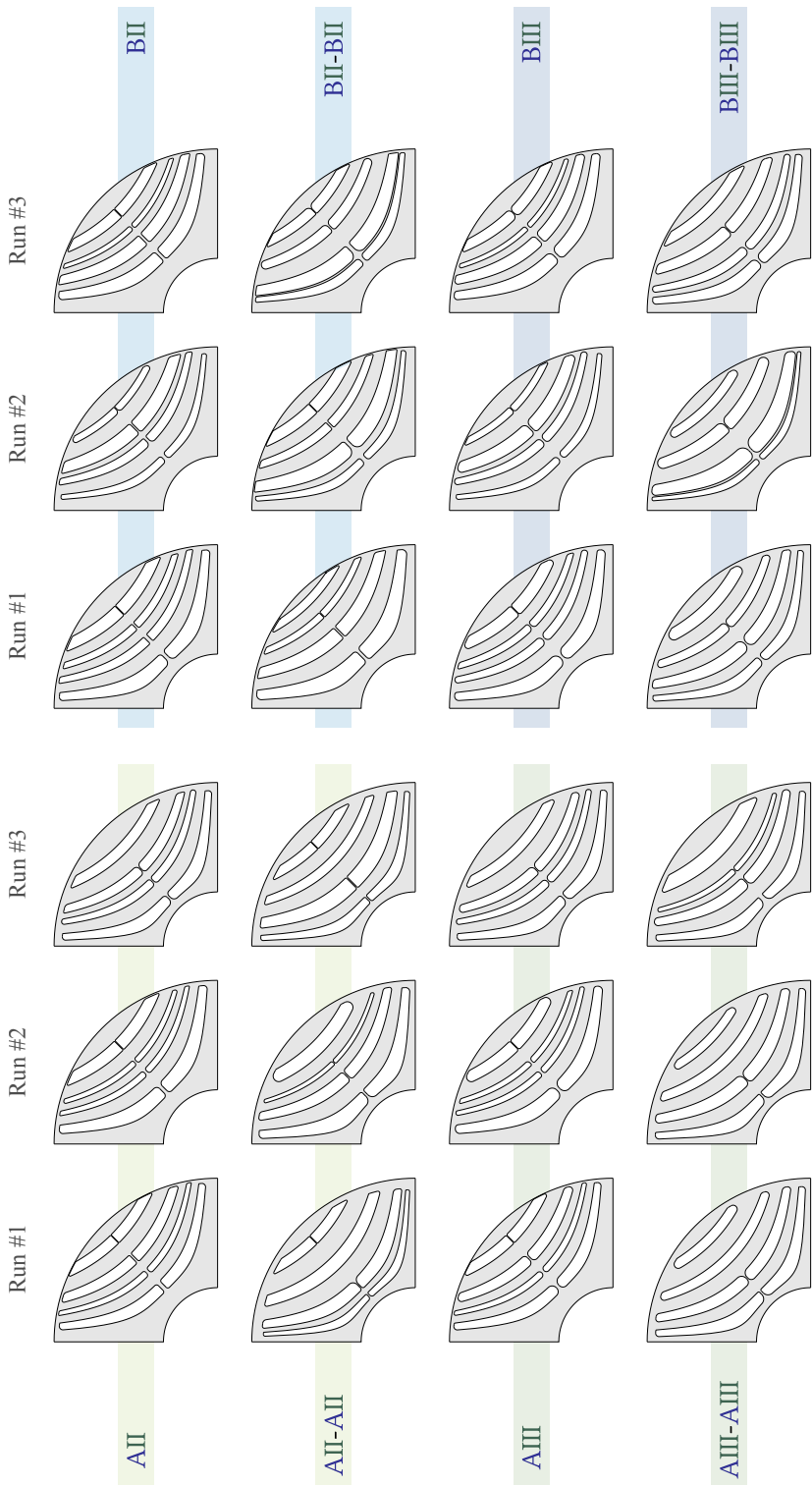


Figure 5.9: Optimized geometries using approaches AII, AIII, BII, BIII, and an additional iteration.

objectives can be met. Nonetheless, this change is also likely to lead to solutions that break the structural integrity constraint, which is considered a harder constraint than the torque ripple. Hence, altering the order of the optimizations is not an option.

5.4 Results using the second parametrization

In this section, the approach [BII-BII](#) is applied using the second parametrization and benchmarked against the full optimization [\(F\)](#), both using the search space presented in Table 4.2. This parametrization highlights the main impacting factors on the mean torque and has a smaller search space, so that the optimizations are expected to converge faster. With this idea of smaller search space, the chapter 4 has shown the radii of the fillets $f_i^{1 \rightarrow 4}$ to be negligible. Hence, for this parametrization, the fillets are excluded from the optimizations and are assigned a constant value of 3 mm. Doing so, the number of mechanical parameters $\mathbf{x}_{\text{mech}}$ reduces from 24 to 8, and the number of magnetic parameters $\mathbf{x}_{\text{mag}}$ is reduced from 10 to 9¹. The settings of the optimization algorithm are kept identical, except for the mutation probability that is dynamically related to the number of variables.

The reference solution is presented in Table. 5.4. The conversion factors from the results of section 5.3 and these results are:

$$\frac{\text{Mean torque (p.u., first parametrization)}}{\text{Mean torque (p.u., second parametrization)}} = 1.001, \quad (5.18)$$

$$\frac{\text{Total optimization time (p.u., first parametrization)}}{\text{Total optimization time (p.u., second parametrization)}} = 1.520. \quad (5.19)$$

This means the optimization algorithm was able to find an equivalent solution in the second search space within 2/3 of the time required to find the solution in the first search space. From this result alone, it is not possible to dissociate the time gains from the smaller search space, the re-parametrization, or the exclusion of $f_i^{1 \rightarrow 4}$ in the optimization.

The results obtained by separating the optimization problem using approach [BII](#) are presented in Table 5.4 and Fig. 5.10. The gains on the total optimization time are in line with the results of the first parametrization, with a fifth of the reference optimization time without iterating, and a half after iterating. The iteration keeps being beneficial for the mean torque, which is brought close to the reference. As for the first parametrization, this method remains unreliable to lower the torque ripple below the limit $\hat{T}_{\text{max}}$.

The number of iteration steps for each optimization to reach 99.9% convergence are given in Table 5.6, for the two parametrizations. Using this table, it becomes possible to dissociate the effects of the smaller search space and lower number of parameters on the optimizations.

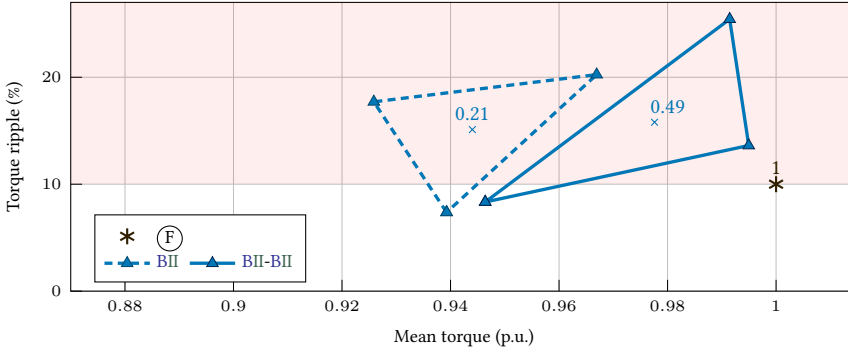
¹This is a consequence of fixing the redundancy of the parametrization by assigning $K_q^{fg} = K_n^{fb} = 1$

Table 5.4: Performance metrics and total optimization time obtained by carrying the optimization (5.1), using the second parametrization.

Design approach	Mean torque	Torque ripple	Total opti. time	
	p.u.	%	Hours	p.u.
(F)	1	10.00	199.0	1

Table 5.5: Quantities of interest averaged over three independent runs. The upper and lower deviations are indicated. The mean torque and total optimization time are expressed in p.u. of the reference solution (Table 5.4).

Design approach	Mean torque (p.u.)	Torque ripple (%)	Total opti. time (p.u.)
BII	0.94 $\begin{smallmatrix} +0.02 \\ -0.02 \end{smallmatrix}$	15.1 $\begin{smallmatrix} +5.1 \\ -7.7 \end{smallmatrix}$	0.21 $\begin{smallmatrix} +0.02 \\ -0.01 \end{smallmatrix}$
BII-BII	0.98 $\begin{smallmatrix} +0.02 \\ -0.03 \end{smallmatrix}$	15.8 $\begin{smallmatrix} +9.6 \\ -7.5 \end{smallmatrix}$	0.49 $\begin{smallmatrix} +0.15 \\ -0.17 \end{smallmatrix}$

**Figure 5.10:** Performance of the solutions obtained in Table 5.5.**Table 5.6:** Breakdown of the approach BII-BII, for the two parametrizations. The values are the number of iteration steps until convergence, for each optimization.

Design approach	Run #1	Run #2	Run #3	Mean	(hours)
<u>B</u> (1 st param.)	34	31	36	34	(44)
<u>B</u> (2 nd param.)	28	33	28	30	(39)
<u>BII</u> (1 st param.)	63	78	66	69	(6)
<u>BII</u> (2 nd param.)	40	33	33	35	(3)
<u>BII-B</u> (1 st param.)	63	132	52	82	(108)
<u>BII-B</u> (2 nd param.)	44	60	18	41	(54)
<u>BII-BII</u> (1 st param.)	68	75	96	80	(7)
<u>BII-BII</u> (2 nd param.)	43	37	20	33	(3)

When optimizing the ribs in [BII](#) or in [BII-BII](#), the smaller number of parameters in the 2^{nd} parametrization greatly benefits the convergence rate. This cannot be attributed to the reparametrization, as the parameters $\mathbf{x}_{mag}$ are not optimized in the mechanical optimization. However, the mechanical optimization only represents a fraction of the total optimization time due to its significant smaller evaluation time.

The two optimizations are comparable when optimizing the mean torque and torque ripple in the first optimization [B](#) with the initial guess $\mathbf{x}_{mech}^0 = \mathbf{0}$. In this scenario, the second parametrization does not exhibit clear advantages over the first, despite the smaller search space and better parametrization for the mean torque. One possible explanation is that meeting the torque ripple constraint for a rotor without tangential ribs is challenging, causing the convergence rate to be primarily dictated by the torque ripple. Both parametrizations have a similar level of entanglement for this output, resulting in a comparable optimization time. Conversely, after the iteration, the second parametrization consistently exhibits a significantly faster convergence rate.

5.5 Summary and discussion

Several approaches to tackle the design of the rotor of a SynRM have been implemented and benchmarked. These approaches dissociate the optimization of the flux barriers from the optimization of the ribs, to first optimize the magnetic performance and then ensure the structural integrity. Overall, the optimization time can be reduced to a fraction of the optimization time required to optimize all the parameters together, approach [\(F\)](#), at the cost of a slightly lower mean torque.

In particular, the approach [AII](#) consists in minimizing the total surface area of the ribs after the frontal optimization of the mean torque. This approach is representative to the majority of the approaches involved in the design of a SynRM [[33–35, 106–108](#)], including those that involve a fine-tuning of the ribs as a final step of the optimization process. The optimization time is halved, and the mean torque of the solutions remain acceptable.

By pre-dimensioning the radial ribs within the magnetic optimization, as suggested in [[93](#)], the approach [BII](#) further diminishes the optimization time to a fifth of the reference optimization time [\(F\)](#). The magnetic torque can be brought in line with the reference design by iterating on the scheme. By accounting for the extra computational cost of the iteration, the total optimization time is still half of the reference optimization time. Similar observations are performed after the reparametrization of the geometry with a smaller search space, that also excludes the radii of the fillets from the optimizations. With this second search space, the total optimization time of approaches [BII](#), [BII-BII](#), and [\(F\)](#) were overall reduced by a factor 1.5. Altogether, by reparametrizing and separating the optimization problem, the total optimization time reported in Table 5.2 has been divided by a factor ranging from 8 to 3, depending on the design approach.

While these approaches are time-efficient to obtain a high mean torque and to ensure the structural integrity, the inconsistent torque ripple remains a limitation.

The adaptations in optimizations I, II, III are not sufficient to preserve a low relative torque ripple, despite preserving the mean torque. Such variability in this output is explained by the absence of penalization mechanism on the tangential ribs. Indeed, the sensitivity analysis performed in chapter 4 informs us a quite significant role of the tangential ribs on all the objectives, including on the von Mises stresses, the iron losses, and the torque ripple. To prevent the negative impacts of the mechanical optimization from the magnetic objectives, it should be guided towards solutions that favor the radial ribs more than the tangential ribs. The radial ribs are also very potent in reducing the von Mises stresses and, conversely from the tangential ribs, are negligible contributors to the other outputs. Following these guidelines, we could propose the following optimization:

$$\text{Mechanical optimization IV} = \begin{cases} \min_{\mathbf{x}_{\text{mech}}} & \sum_{i=1}^n (a_i t_i^T + b_i t_i^R) \\ \text{s.t.} & \max(\sigma_{\text{VM}}(\mathbf{x}_{\text{mech}}, \mathbf{x}_{\text{mag}}^*)) < \sigma_{\text{max}} \end{cases}, \quad (5.20)$$

with the weighting coefficients a_i and b_i , such that $a_i \gg b_i$. In a refined version, the changes from an initial configuration 0 are penalized instead of the values themselves, because we want to minimize the effects of the mechanical optimization on the magnetic objectives:

$$\text{Mechanical optimization V} = \begin{cases} \min_{\mathbf{x}_{\text{mech}}} & \sum_{i=1}^n (a_i |t_i^T - t_i^{T,0}| + b_i |t_i^R - t_i^{R,0}|) \\ \text{s.t.} & \max(\sigma_{\text{VM}}(\mathbf{x}_{\text{mech}}, \mathbf{x}_{\text{mag}}^*)) < \sigma_{\text{max}} \end{cases}, \quad (5.21)$$

By replacing the optimization II by the optimization V in the scheme BII-BII, i.e., BV-BV, we expect to observe less variations on the magnetic objectives, while keeping the advantage in total optimization time of this approach.

In hindsight, this new scheme arises to correct the results from commonly applied design approaches that, intuitively, dissociate the ribs from the magnetic performance. The results gathered in this chapter demonstrated that such hypothesis is not always valid, especially when considering the torque ripple as one of the outputs to be optimized. The corrections proposed in (5.20) and (5.21) are based on mathematical evidences brought by the sensitivity analysis. From a methodological point of view, the same sensitivity analysis can also be used to derive a design approach that dissociates the optimizations at the level of the outputs, which is the focus of the next chapter.

Sensitivity-based design approaches

6

The design approaches proposed in chapter 5 involve successive optimizations on two distinct partitions of the initial design space: the parameters associated to the magnetic performance, $\mathbf{x}_{\text{mag}}$, and those associated to the structural integrity, $\mathbf{x}_{\text{mech}}$. These approaches have proven effective in reducing the total optimization time when the mean torque is the primary objective, as they can identify designs that output at least 95% of the mean torque of the best-known design. The performance gap can be further reduced to negligible levels by pre-dimensioning the radial ribs within the magnetic optimization, and performing a single iteration of the approach. However, these approaches exhibit two major drawbacks:

- Challenges persist in consistently limiting the torque ripple during the mechanical optimization focused on the ribs, despite explicit constraints imposed in preceding magnetic optimizations. This highlights the intricate cross-interactions among parameters influencing multiple objectives, and reveals a clear limitation of partitioning the optimization problem based solely on physical intuition.
- While the second parametrization effectively isolated the primary factors influencing mean torque, the efforts in reparametrization did not result in significant reductions in the total optimization time.

Motivated by the need to better account for parameter interactions and to make more effective use of the parametrization, this chapter proposes to rely exclusively on the results of the sensitivity analysis to dissect the optimization problem. The methodology is detailed in section 6.1 and is applied to address the same optimization problem introduced in the previous chapter, specifically, mean torque maximization under both a torque ripple and a structural integrity constraints.

The application of this methodology using the search space of the second parametrization is detailed in section 6.2. This parametrization is considered more potent in benefiting from this method than the first. Nonetheless, our curiosity extends to understanding the design approach that can be derived using this methodology on the first parametrization. Therefore, in section 6.3, a design approach is similarly derived for the first parametrization, emphasizing the strong parallel of

this approach to those of the previous chapter. Finally, a summary and discussion of the results are provided in the concluding section 6.4.

6.1 Methodology

To help escape the local minima and reduce the total optimization time, our goal is to identify a sequence of smaller optimization problems $\mathcal{P}_i$ designed to minimize¹ an objective $f(\mathbf{x})$ under one or more inequality constraints of the form $g(\mathbf{x}) \leq g_{\max}$.

The analysis starts by making no distinction between objectives and constraints, collectively referring to them as outputs. Through the application of sensitivity analysis, the parameters of the optimization problem can be categorized into three distinct sets:

- $\mathcal{S}_1$: Parameters that exclusively impact a single output, but not the others,
- $\mathcal{S}_2$: Parameters that concurrently impact multiple outputs,
- $\mathcal{S}_3$: Parameters with negligible impact on all the outputs.

The set $\mathcal{S}_3$ can be excluded from the optimization problem as parameters within this set do not impact the outputs. The presence of the set $\mathcal{S}_1$ indicates that the sequence of optimization problems $\mathcal{P}_i$ can be as straightforward as optimizing the outputs separately. However, the existence of the set $\mathcal{S}_2$ mitigates this possibility due to cross-coupling between multiple outputs. Following this categorization, the methodology for deriving smaller optimization problems is quite straightforward:

1. First, optimize the main objective $f(\mathbf{x})$ using the search space $\mathcal{S}_1^f \cup \mathcal{S}_2^f$:

$$\min_{\mathbf{x} \in \mathcal{S}_1^f \cup \mathcal{S}_2^f} f(\mathbf{x}). \quad (6.1)$$

The superscript f designates the subset of parameters in $\mathcal{S}$ that are relevant to the output f .

2. Then, for each inequality constraint g_i , minimize the output using the relevant search space $\mathcal{S}_1^{g_i}$:

$$\min_{\mathbf{x} \in \mathcal{S}_1^{g_i}} g_i(\mathbf{x}). \quad (6.2)$$

The partial convergence $g_i(\mathbf{x}) \leq g_{i,\max}$ can be used as a stopping criterion for that optimization, as it already provides a valid solution to the initial problem. Depending on the requirements, the extra optimization time required for complete convergence can be worth the performance gain on the solution.

By following this approach, the main objective benefits from the entire relevant search space, and the remaining orthogonal parameters are successively adjusted to satisfy the constraints. These adjustments will not impact the other outputs, by definition of the set $\mathcal{S}_1$. The underlying, strong, hypothesis is the following: the

¹Or, similarly, maximize.

constraint $g_i(\mathbf{x}) \leq g_{i,\max}$ can be satisfied using the subset $\mathcal{S}_1$ alone.

The design freedoms offered by the subset $\mathcal{S}_1$ might not be sufficient to fulfill some constraints g_i , thereby violating the previous hypothesis. In such instances, some parameters from $\mathcal{S}_2$ are introduced to the search space, and the optimization formulation of the constraint g_i is changed to:

$$\begin{cases} \min_{\mathbf{x} \in \mathcal{S}_1^{g_i} \cup \mathcal{S}_2^{g_i}} & \sum_{\forall \mathbf{x}_i \in \mathcal{S}_2^{g_i}} a_i |\mathbf{x}_i - \mathbf{x}_i^0| \\ \text{s.t.} & g_i(\mathbf{x}) \leq g_{i,\max} \end{cases} \quad (6.3)$$

This strategy is similar to the one discussed for mechanical optimization V (5.21), involving weighting coefficients a_i . This formulation penalizes parameters belonging to $\mathcal{S}_2$ to mitigate their impact on the outputs optimized prior the current optimization. Achieving full convergence in this optimization is necessary to minimize changes to the other outputs. Moreover, if only one constraint requires the adaptation (6.3), the optimization associated with that constraint should be prioritized over the others to accommodate the new values of parameters $\mathbf{x} \in \mathcal{S}_2^{g_i}$ in subsequent optimizations.

6.2 Application to the second parametrization

We now apply this methodology to tackle the optimization problem (5.1) using the second parametrization. As previously mentioned, this parametrization serves as an excellent case to illustrate, step by step, the decomposition of the optimization problem, with the results provided in the following subsections.

6.2.1 Decomposition of the optimization

The first and total order Sobol indices are reported in Fig. 6.1. This visualization facilitates the categorization of parameters into the three sets $\mathcal{S}_1$, $\mathcal{S}_2$, and $\mathcal{S}_3$. The radii of the fillets, $f_i^{1 \rightarrow 4}$, have a negligible impact on all the outputs and do not require optimization. They are assigned the arbitrary value of 3 mm in the subsequent optimizations.

Following the methodology, the initial optimization aims to maximize the mean torque using the design space $\mathcal{S}_1^{\bar{T}} \cup \mathcal{S}_2^{\bar{T}} = \{\theta_e, \xi, t_i^T\}$. However, the tangential ribs are known to be detrimental for the mean torque. The optimization for these parameters can be expedited by assigning their lower bound values, $t_i^{T,*} = 0.3$ mm. Therefore, the optimization simplifies to:

$$\mathcal{P}_1 = \max_{\mathbf{x}_1 = \{\theta_e, \xi\}} \bar{T}(\mathbf{x}_1), \quad (6.4)$$

The flux barriers and flux guides are equally distributed by assigning $K_i^{fb} = 1$ and $K_j^{fg} = 1$. The thicknesses of the radial ribs t_i^R are set to their lower bound value of 0 mm, indicating the absence of radial ribs in this initial optimization.

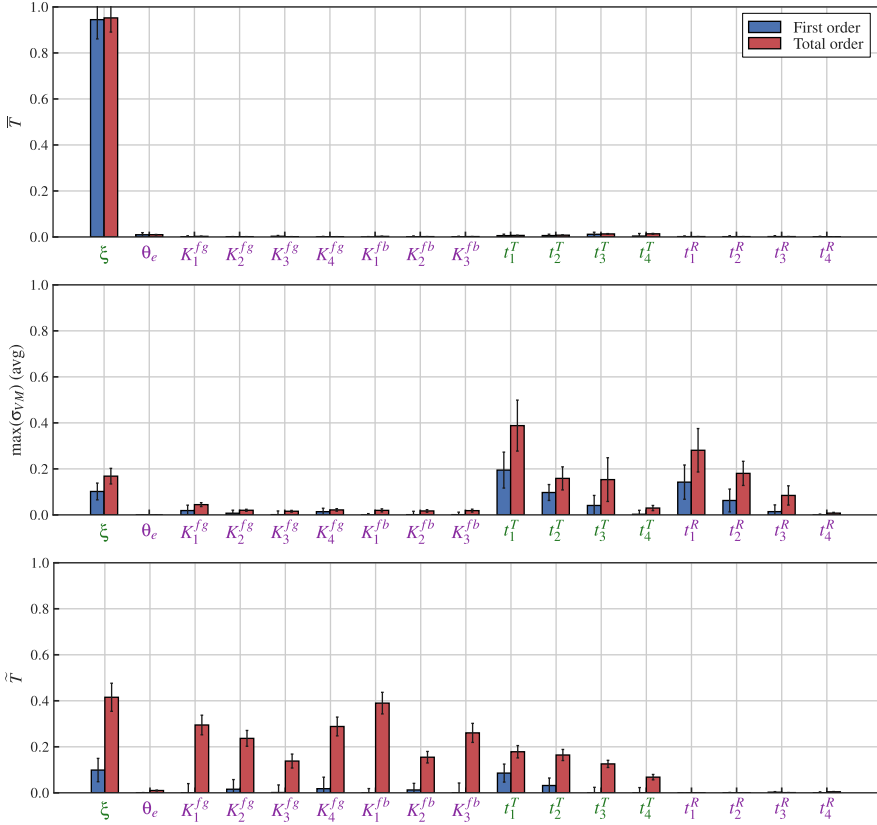


Figure 6.1: Sobol indices for the mean torque $\bar{T}$, the maximum von Mises stress σ_{VM} , and the torque ripple $\tilde{T}$. The parameters highlighted in purple belong to $\mathcal{S}_1$, and those highlighted in green belong to $\mathcal{S}_2$. The parameters belonging to $\mathcal{S}_3$, negligible, are not represented.

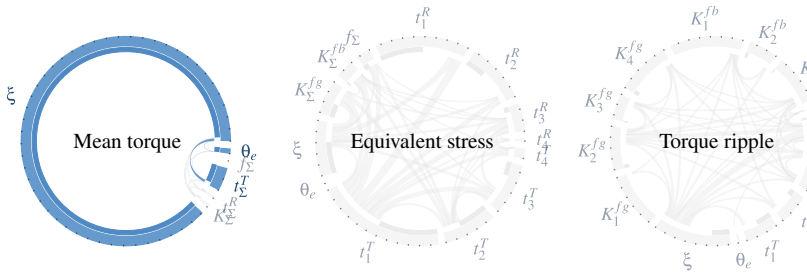


Figure 6.2: Effects of the optimization $\mathcal{P}_1$ on the outputs with, in blue, the parameters that are optimized for the output and, in grey, the parameters that have not been optimized for the output. The ticks mark a 2% interval.

Using the chord diagrams introduced in chapter 4, it is possible to keep track of the effects of successive optimizations. In Fig. 6.2, the mean torque is optimized using its entire relevant design space highlighted in blue, with the tangential

ribs assigned their lower bound values. The other objectives have not undergone optimization at this point, and as such, they are grayed out.

Considering the maximum equivalent von Mises stresses, it is improbable to bring it below the elasticity limit with parameters belonging in $\mathcal{S}_1^{\sigma_{VM}}$ alone, i.e., with the thicknesses of the radial ribs t_i^R . The thicknesses of the tangential ribs are introduced to the search space, and are penalized accordingly as per (6.3):

$$\mathcal{P}_2 = \begin{cases} \min_{\mathbf{x}_2 = \{t_i^R, t_i^T\}} & \sum_{i=1}^n \text{abs}(t_i^T - t_i^{T,0}) \\ \text{s.t.} & \max(\sigma_{VM}(\mathbf{x}_2)) < \sigma_{\max} \end{cases} \quad (6.5)$$

We employ $a_i = 1$, although assigning decreasing weights for the tangential ribs farther from the shaft is also a valid strategy.

As depicted in Fig. 6.3, this optimization benefits from most of the design space, making it likely to satisfy the constraint. There are no penalizations on the radial ribs, so the previously optimized mean torque will be impacted within the variation range attributed to these parameters. The variation range, highlighted in red, is negligible, as expected by sorting the parameters t_i^R into the set $\mathcal{S}_1$. Conversely, the tangential ribs account for 4 % of the variance observed in the mean torque, but the changes made to these parameters are kept minimal using (6.5).

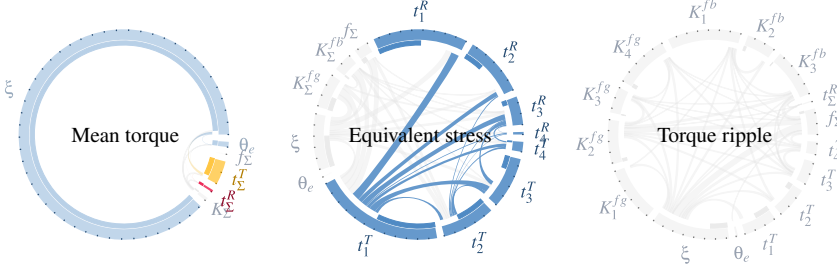


Figure 6.3: Effects of the optimization $\mathcal{P}_2$ on the outputs. In yellow, the parameters that are changed to optimize an other output, with penalization. In red, the parameters that are changed to optimize an other output, without penalization. In lighter colors, the memory of the previous optimizations.

The torque ripple constraint is likely to be satisfied with parameters belonging to set $\tilde{\mathcal{S}}_1^{\tau}$ only, specifically the parameters K_i^{fb} and K_j^{fg} . Indeed, these parameters account for over 60% of the variance observed in the torque ripple. Therefore, similar to the mean torque, we proceed to its minimization using that subset of parameters:

$$\mathcal{P}_3 = \min_{\mathbf{x}_3 = \{K_i^{fb}, K_j^{fg}\}} \tilde{T}(\mathbf{x}_3). \quad (6.6)$$

As discussed previously, the partial convergence $\tilde{T}(\mathbf{x}_3) \leq \tilde{T}_{\max}$ can serve as a stopping criterion for the optimization algorithm. Both possibilities will be implemented and subsequently compared.

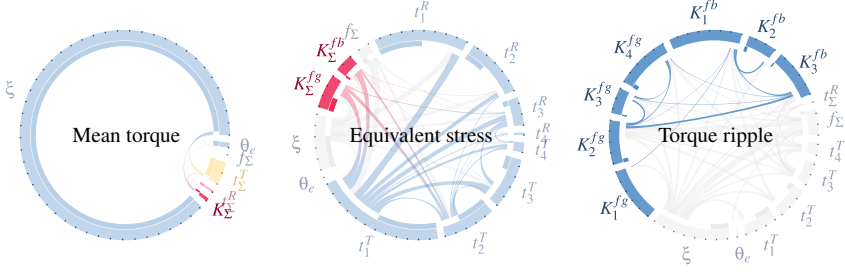


Figure 6.4: Effects of the optimization $\mathcal{P}_3$ on the outputs.

The effects of this optimization on all the outputs are visible on Fig. 6.4. The sorting of the parameters K_i^{fb} and K_j^{fg} into $\mathcal{S}_1$ is not perfect, with these parameters accounting for a bit more than 8% of the total order index $\sum S_{T_i}$ in the mechanical stresses. In practice, the variations are expected to fall in the 4% range rather than 8%, as most of the total order index comes from the second-order interactions between K^{fb} and K^{fg} and the parameters t_i^T and t_i^R . These cross-interactions between the flux barriers/guides and the ribs cannot manifest because the ribs are not changed within $\mathcal{P}_3$.

Nonetheless, the structural integrity remains a hard constraint. To enforce it, the optimization $\mathcal{P}_2$ can simply be repeated after performing $\mathcal{P}_3$. The effects of this optimization on the other outputs are represented in Fig. 6.5. For this last optimization, the initial guess on the tangential ribs $t_i^{T,0}$ is anticipated to be very close to the optimal values $t_i^{T,*}$ since they were optimized in the previous step. Hence, the impact on the other outputs is expected to be minimal.

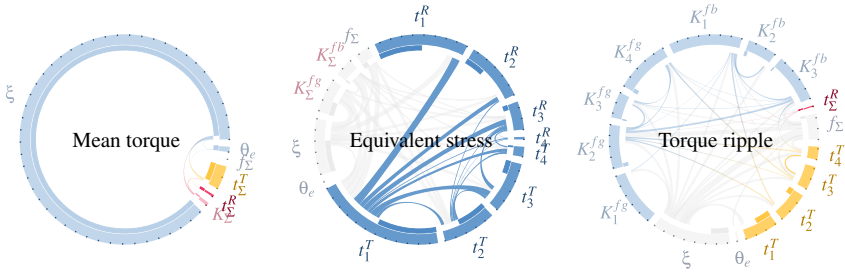


Figure 6.5: Effects of the (second) optimization $\mathcal{P}_2$ on the outputs.

To summarize, the proposed design approach consists in applying the sequence of optimizations $\mathcal{P} = \mathcal{P}_1 \rightarrow \mathcal{P}_2 \rightarrow \mathcal{P}_3$, with a potential re-optimization $\mathcal{P}_2$ if the structural integrity constraint is no longer satisfied. To discuss the interest of aborting the optimization $\mathcal{P}_3$ upon reaching partial convergence, the approach is forked to $\mathcal{P}^*$ when the criterion $\tilde{T}(\mathbf{x}_3) \leq \tilde{T}_{\max}$ is met. Therefore, the two approaches

reference mean torque while reaching a lower torque ripple.

While providing superior solutions than approach BII-BII, the approach $\mathcal{P}$ also reduces the total optimization time to two-fifths of the reference optimization time. The implementation of the partial convergence termination condition in $\mathcal{P}_3^*$ further decreases the optimization time, at the cost of a higher torque ripple. These design approaches exhibit good repeatability, with very little dispersion observed on the mean torque and torque ripple. A similar observation is made upon examining the optimized rotor geometries, as shown in Fig 6.7. These geometries share the same air-to-rotor ratio ξ and current angle θ_e , taking the values of 0.38 and 65.5° , respectively. The individual distribution of air and iron between flux barriers and flux guides through the optimization $\mathcal{P}_3$ demonstrates far better consistency than what was obtained using the design approaches investigated in chapter 5.

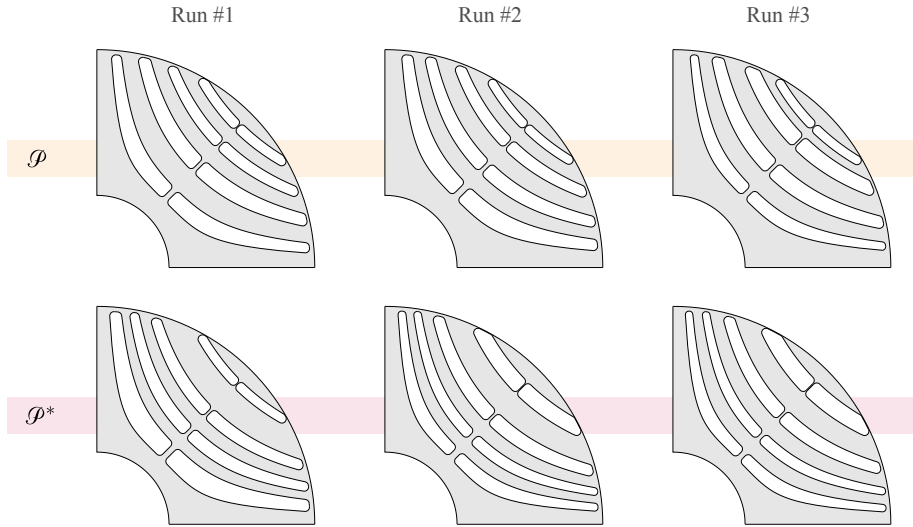


Figure 6.7: Optimized geometries by approaches $\mathcal{P}$ and $\mathcal{P}^*$, for three independent runs.

The number of iteration steps before convergence of the optimizations involved in $\mathcal{P}$ and $\mathcal{P}^*$ are given in Table 6.2 and Table 6.3, respectively. The first two lines are identical, which is the consequence of forking the approach $\mathcal{P}$ to $\mathcal{P}^*$ from optimization $\mathcal{P}_3$. Notably, the termination condition (5.17) was systematically satisfied by the initial population of $\mathcal{P}_1$, i.e., within a population of 99 randomly generated sets $\mathbf{x}_1 = \{\theta_e, \xi\}$. We confirm that this solution differs from the initial guess injected in the initial population, ruling out its influence on the repeatability observed across different runs. This suggests that the mean torque optimization $\mathcal{P}_1$ is relatively straightforward to solve, and that the optimization algorithm could have been configured with a smaller population size. Such an adjustment would further benefit the proposed approaches, considering the time-penalty associated with the evaluation of the entire generation.

Similarly, we perform the same observations when lowering the torque ripple

Table 6.2: Breakdown of the approach $\mathcal{P}$. The values are the number of iteration steps until convergence, for each optimization. The structural integrity constraint was already satisfied in runs #1 and #2 after the optimization $\mathcal{P}_3$.

Step	Run #1	Run #2	Run #3	Mean	(hours)
$\mathcal{P}_1$	0	0	0	0	(1)
$\mathcal{P}_2$	138	127	146	137	(12)
$\mathcal{P}_3$	60	34	52	49	(64)
$(\mathcal{P}_2)$	—	—	81	27	(2)

below its constraint in optimization $\mathcal{P}_3^*$. With this parametrization, it implies that the suggested decomposition of the magnetic optimization is effective, consistently finding designs nearly as good as the best-known design with only a few randomly generated samples, i.e., without suffering from poor convergence rate.

Table 6.3: Breakdown of the approach $\mathcal{P}^*$. The values are the number of iteration steps until convergence, for each optimization.

Step	Run #1	Run #2	Run #3	Mean	(hours)
$\mathcal{P}_1$	0	0	0	0	(1)
$\mathcal{P}_2$	138	127	146	137	(12)
$\mathcal{P}_3^*$	0	0	0	0	(1)
$\mathcal{P}_2$	122	138	214	158	(14)

Using these results, the total optimization time of the approach is no longer attributed to the high evaluation time of the magnetic model or the magnetic optimization. Instead, it is now attributed to the mechanical optimization. This is noteworthy, as it opens up possibilities for reducing this optimization time. One straightforward approach is to decrease the evaluation time of the mechanical model by implementing a boundary condition of symmetry, effectively halving the domain of resolution. Another solution could involve the application of a gradient-based optimization algorithm, starting from the configuration of the initial guess.

6.3 Application to the first parametrization

Similar to the approach taken in the previous section, we now address the problem of decomposing the optimization using the first parametrization. This section is motivated by the curiosity of comparing the design approach derived for the first parametrization with that of the second parametrization. This comparative analysis will provide insights into the importance of parametrization in the success of the proposed methodology.

6.3.1 Decomposition of the optimization

The first and total order Sobol indices are presented in Fig. 6.8. Similar to the second parametrization, the radii of the fillets, $f_i^{1 \rightarrow 4}$, have negligible impact on all the

outputs. Consequently, there is no need for optimization, and these parameters are assigned the arbitrary value of 3 mm in the subsequent analyses.

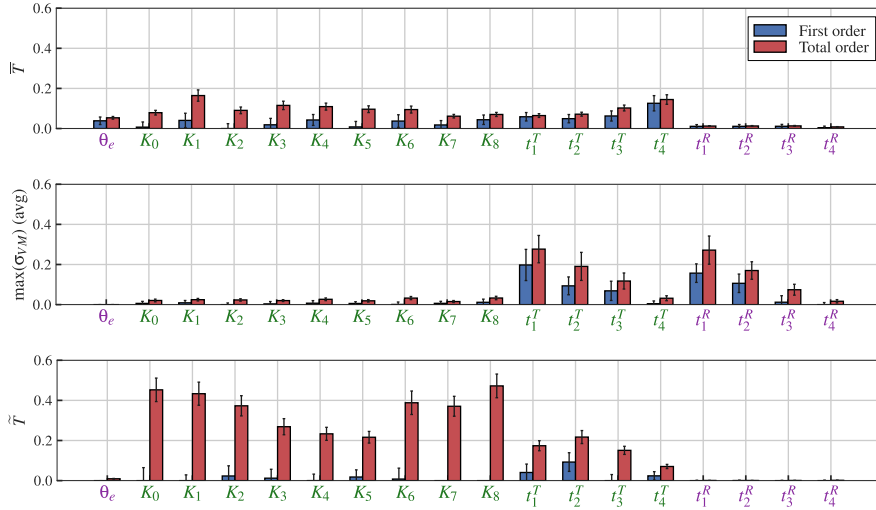


Figure 6.8: Sobol indices for the mean torque $\bar{T}$, the maximum von Mises stress σ_{VM} , and the torque ripple $\hat{T}$. The parameters highlighted in purple belong to $\mathcal{S}_1$, and those highlighted in green belong to $\mathcal{S}_2$. The parameters belonging to $\mathcal{S}_3$, negligible, are not represented.

In contrast to the second parametrization, the primary influencing factors on the mean torque are not distinctly identified. All the impactful factors belong to the set $\mathcal{S}_2$ except for the current angle θ_e . Both the torque ripple and the mean torque require optimization considering the stiffnesses K_j . Therefore, these two outputs need to be optimized concurrently using the design space relevant to the mean torque $\mathcal{S}_1^{\bar{T}} \cup \mathcal{S}_2^{\bar{T}} = \{\theta_e, K_j, t_i^T\}$. Since the tangential ribs are known to be detrimental to the mean torque, we exclude these parameters from the optimization and assign them their lower bound values. Hence, the optimization problem becomes:

$$\mathcal{O}_1 = \begin{cases} \max_{\mathbf{x}_1 = \{\theta_e, K_j\}} & \bar{T}(\mathbf{x}_1) \\ \text{s.t.} & \tilde{T}(\mathbf{x}_1) < \tilde{T}_{\max} \end{cases}. \quad (6.8)$$

Building on the earlier analysis, optimizing both the radial and tangential ribs is necessary to satisfy the structural integrity constraint. Following the methodology, a penalization mechanism must be implemented since the tangential ribs belong to the set $\mathcal{S}_2$:

$$\mathcal{O}_2 = \begin{cases} \min_{\mathbf{x}_2 = \{t_i^R, t_i^T\}} & \sum_{i=1}^n \left(a_i |t_i^T - t_i^{T,0}| + b_i |t_i^R - t_i^{R,0}| \right) \\ \text{s.t.} & \max(\sigma_{VM}(\mathbf{x}_2)) < \sigma_{\max} \end{cases}. \quad (6.9)$$

Using $a_i = 100$ and $b_i = 1$, a small penalization is imposed on the radial ribs since their impact on the mean torque is not totally negligible, as visible in Fig. 6.8. Similarly to

the approach proposed for the second parametrization, we suggest performing this optimization twice - before and after addressing the torque ripple constraint. This approach ensures that the initial guesses on the tangential ribs $t_i^{T,0}$ used in $\mathcal{O}_1$ come closer to their optimal values. The sequence of optimization problems becomes:

$$\mathcal{O} = \mathcal{O}_2 \rightarrow \mathcal{O}_1 \rightarrow (\mathcal{O}_2). \quad (6.10)$$

With the parentheses indicating the last optimization might be avoided if the structural integrity constraint is still satisfied after $\mathcal{O}_1$.

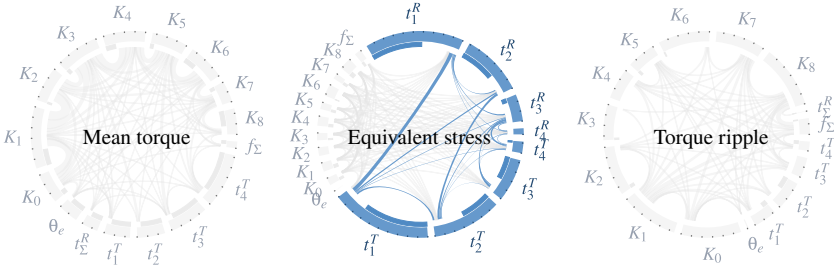


Figure 6.9: Effects of the optimization $\mathcal{O}_2$ on the outputs with, in blue, the parameters that are optimized for the output and, in grey, the parameters that have not been optimized for the output. The ticks mark a 2% interval.

For the first optimization, the initial guess is chosen using the lower bound values of radial and tangential ribs with equally distributed flux barriers and flux guides, i.e., $t_i^{T,*} = 0.3 \text{ mm}$, $t_i^R = 0 \text{ mm}$, $K_j = 1$. The effects of this optimization $\mathcal{O}_2$ on the outputs is visible in Fig. 6.9.

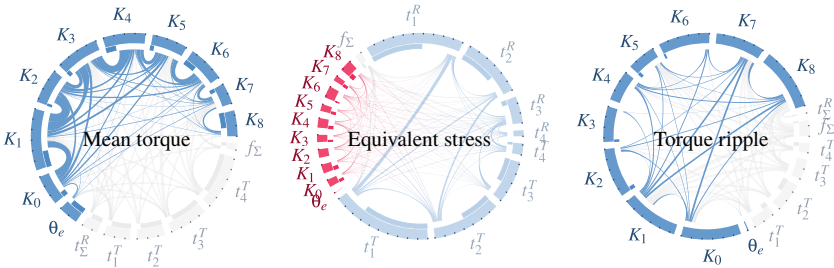


Figure 6.10: Effects of the optimization $\mathcal{O}_1$ on the outputs. In red, the parameters that are changed to optimize an other output, without penalization. In lighter colors, the memory of the previous optimizations.

The second optimization $\mathcal{O}_1$ benefits both the torque ripple and the mean torque, as visible in Fig 6.10. Considering that the previous optimization aims at minimizing the thicknesses of the ribs, we exploit the entire relevant design space to maximize the mean torque, and a substantial portion to reduce the torque ripple below the

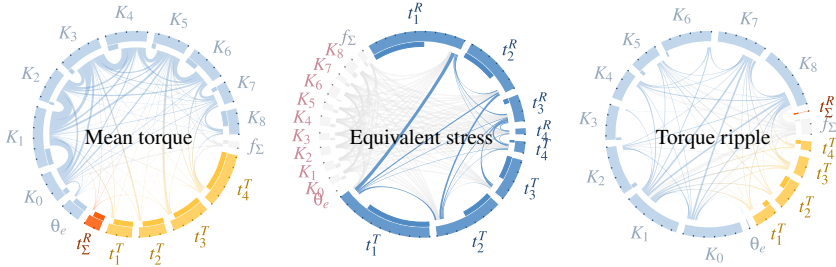


Figure 6.11: Effects of the (second) optimization $\mathcal{O}_2$ on the outputs. In yellow, the parameters that are changed to optimize an other output, with strong penalization. In orange, the parameters that are changed to optimize an other output, with small penalization.

constraint.

The effects of these optimizations on the equivalent stress is not negligible, and the reason behind performing the subsequent optimization $\mathcal{O}_2$. The effects of this last optimization are illustrated in Fig. 6.11.

The proposed scheme $\mathcal{O}$ has strong similarities with the schemes investigated in chapter 5. Indeed, the optimization is separated into a magnetic optimization and a mechanical optimization, the first acting on $\mathbf{x}_1 = \mathbf{x}_{\text{mag}}$ to maximize the mean torque under a torque ripple constraint, and the second on $\mathbf{x}_2 = \mathbf{x}'_{\text{mech}}$ (without the fillets) to ensure the structural integrity. The mechanical optimization $\mathcal{O}_2$ corresponds to the optimization $\underline{\mathbf{V}}$ described in chapter 5, whereas the magnetic optimization $\mathcal{O}_1$ is identical to the optimization $\underline{\mathbf{A}}$. Continuing with the similarities, there is a form of iteration in performing the optimization $\mathcal{O}_2$ before and after the magnetic optimization. The only difference in the scheme $\mathcal{O}$ over the design approaches investigated in chapter 5 is the economy of the first magnetic optimization.

Following the nomenclature of chapter 5, we will refer to this approach as $\mathbf{VAV}$. Additionally, we will explore the approach $\mathbf{VBV}$ as it does not require any extra implementation effort. However, it's worth noting that pre-dimensioning the tangential ribs within the magnetic optimization, instead of the radial ribs, would be more appropriate based on the results presented in Fig. 6.11.

6.3.2 Results of the optimizations

The search space of the optimizations $\mathbf{VAV}$ and $\mathbf{VBV}$ excludes the radii of the fillets, in contrast to the optimizations reported in section 5.3. Consequently, the optimization time for the mechanical optimization $\mathbf{V}$ is expected to be generally lower than for previous optimizations $\mathbf{I}$, $\mathbf{II}$ and $\mathbf{III}$. This expectation also holds for the reference optimization $\mathbf{F}$, with an optimization time anticipated to be lower in this smaller search space. While this might appear to create an unfair comparison, it can be argued that identifying and excluding the least influential parameters is part of the methodology, made possible by conducting the sensitivity analysis. Additionally, by combining the results of Table 5.6 with the reference solutions of

each parametrization in Table 5.2 and Table 5.4, it is safe to conclude that the radii of the fillets are not the primary reasons for the high optimization time encountered in the reference optimization (F).

Table 6.4: Quantities of interest averaged over three independent runs. The upper and lower deviations are indicated. The mean torque and total optimization time are expressed in p.u. of the reference solution obtained in section 5.3 (Table 5.2). For readability, the main results of section 5.3 are reminded.

Design approach	Mean torque (p.u.)	Torque ripple (%)	Total opti. time (p.u.)
(F)	1	10	1 (302.4 h)
AII-AII	0.90 ^{+0.01} _{-0.03}	14.6 ^{+10.0} _{-11.1}	0.77 ^{+0.28} _{-0.23}
VAV	0.91 ^{+0.01} _{-0.01}	13.6 ^{+3.6} _{-6.5}	0.25 ^{+0.04} _{-0.05}
BII-BII	1.00 ^{+0.01} _{-0.01}	13.9 ^{+3.9} _{-7.3}	0.55 ^{+0.21} _{-0.12}
VBV	0.99 ^{+0.01} _{-0.01}	8.91 ^{+1.0} _{-0.8}	0.28 ^{+0.06} _{-0.06}

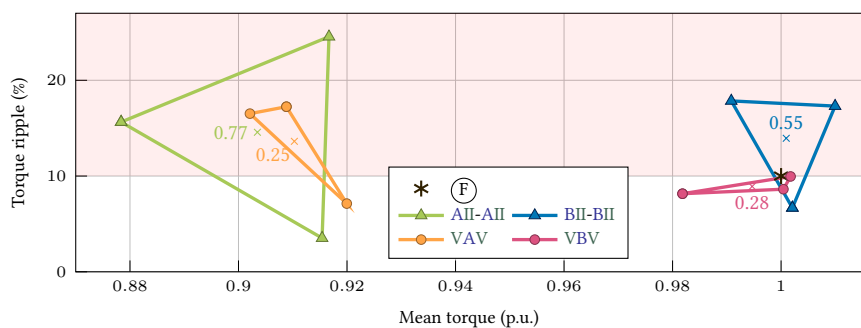


Figure 6.12: Performance of the solutions obtained in Table 6.4.

The results of optimizations VAV and VBV are reported in Table 6.4 and visually represented in Fig. 6.12.

Approach VAV shares similar challenges with other approaches involving optimization A, exhibiting a significant decrease in mean torque and a torque ripple that violates the constraint. However, the total optimization time is significantly reduced, and this approach proves to be more consistent in the solutions it produces compared to previously explored approaches for this parametrization.

Approach VBV retains the advantages of significantly reduced total optimization time and consistency in the solutions. Moreover, it provides excellent solution quality, consistently meeting the torque ripple constraint. While it compares favorably in terms of optimization time with approach P, it achieves a slightly higher torque ripple.

Once again, the decline in mean torque observed in approaches **A** can be elucidated by examining the rotor geometries resulting from the different optimizations, as depicted in Fig. 6.13. This figure displays the mechanical stresses within the rotor after the final mechanical optimizations for schemes **VAV** and **VBV**. The magnetic optimizations redistribute the flux barriers and the flux guides very differently from the initial configuration, which employed even thicknesses for flux barriers and flux guides. Notably, optimizations **A** bring the first flux barrier very close to the shaft, resulting in high stress concentration in tangential ribs. This stress concentration can only be mitigated by oversizing these ribs, regardless of the penalization mechanism introduced in $\mathcal{O}_2$ (6.9), leading to an overall deterioration of the mean torque. In contrast, approaches **B** are less susceptible to this effect, as they penalize flux barriers closer to the shaft by increasing the thicknesses of the radial ribs within the magnetic optimization.

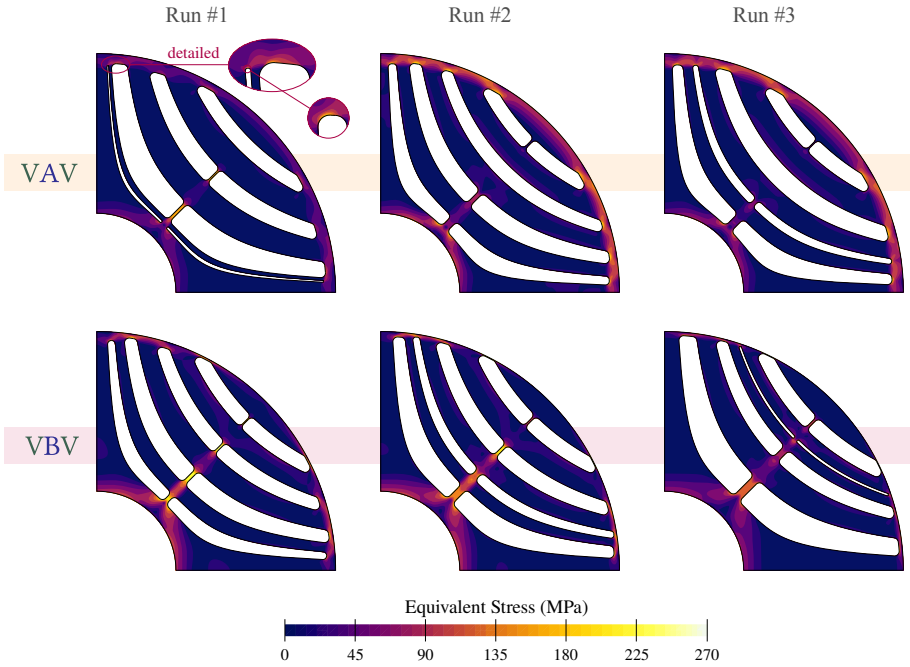


Figure 6.13: Structural analysis of the optimized geometries by approaches **VAV** and **VBV**, for three independent runs.

While proving to be an effective solution in finding configurations of flux barriers preventing stress concentration in tangential ribs, the optimization **B** addresses the problem indirectly by pre-dimensioning the radial ribs so that flux barriers are pushed further from the shaft. Instead, referring to Fig. 6.10, one could consider penalizing the changes in parameters K_j within $\mathcal{O}_1$ (6.8) to limit the chances of introducing additional stress concentration. However, this approach would also constraint the optimization algorithm to find solutions locally, which is not recommended if the goal is to leverage the full search space to maximize the mean torque.

Alternatively, as previously suggested, pre-dimensioning the tangential ribs instead of the radials ribs in the optimization (6.8) is likely to yield satisfying results. Another possibility is to softly incorporate the structural integrity within the objective of the magnetic optimization. This integration could be achieved, for instance, using:

$$\mathcal{O}'_1 = \begin{cases} \max_{\mathbf{x}_1 = \{\theta_e, K_j\}} & a \bar{T}(\mathbf{x}_1) + b \max(\sigma_{VM}(\mathbf{x}_1)) \\ \text{s.t.} & \tilde{T}(\mathbf{x}_1) < \tilde{T}_{\max} \end{cases}. \quad (6.11)$$

This would favor configurations of the flux barriers and flux guides better suited for the subsequent mechanical optimization. Naturally, this approach slightly increases the evaluation time due to the mechanical model. The overall convergence rate of this optimization is expected to be the same as that of optimization $\mathcal{O}_1$, as the search space remains identical and the constraint is only embedded softly.

6.4 Summary and conclusion

In this chapter, we introduced a method to decompose an optimization problem involving a single objective and multiple inequality constraints into a series of smaller optimization problems that are easier to solve. In contrast to the approaches employed in the preceding chapter, this method depends on the results of the sensitivity analysis to define the objectives, constraints, and search space for the successive optimizations. We applied this methodology to break down the problem of mean torque maximization under both a torque ripple constraint and a structural integrity constraint.

The similarities in the optimization scheme derived using this methodology and the ones applied in chapter 5 have been highlighted when using the first parametrization. Specifically, the optimization scheme **VAV** divides the design space into $\mathbf{x}_{\text{mech}}$ and $\mathbf{x}_{\text{mag}}$, where mean torque and torque ripple are optimized together using $\mathbf{x}_{\text{mag}}$, and the structural integrity constraint is addressed through parameters in $\mathbf{x}_{\text{mech}}$. While the results align with previous approaches, featuring a torque ripple constraint that is not consistently met and a mean torque lower than the reference solution, this method demonstrates improved repeatability and a lower overall total optimization time.

The key advantage of this methodology lies in its ability to formulate an optimization strategy using tangible metrics rather than intuitive reasoning. Furthermore, post-analysis of the results enables straightforward identification of potential enhancements to mitigate drawbacks. For instance, we suggest implementing a mechanism within the magnetic optimization to prevent stress concentration in the tangential ribs, such as pre-dimensioning the tangential ribs or softly incorporating the maximum stress constraint into this optimization. Notably, although indirect, pre-dimensioning the radial ribs within the magnetic optimization proves effective, as evidenced by the results of the design approach **VBV**. This variant significantly reduces the total optimization time while maintaining mean

torque and torque ripple levels comparable to the reference solution.

The efforts in reparametrization using the second parametrization are well rewarded by this methodology. By isolating the main contributing factors to the mean torque, the single-objective and two-constraint optimization problem could be decomposed into three different optimization problems. Consequently, the mean torque is maximized using the air-to-rotor ratio and the electric angle ($\mathcal{P}_1$), and the torque ripple is minimized by adjusting the individual distribution of iron and air between flux guides and flux barriers ($\mathcal{P}_3$). As for the previous mechanical optimizations, the ribs are adapted to comply with the structural integrity constraint, while penalizing changes in the thicknesses of the tangential ribs ($\mathcal{P}_2$).

Using the sequence $\mathcal{P} = \mathcal{P}_1 \rightarrow \mathcal{P}_2 \rightarrow \mathcal{P}_3 \rightarrow \mathcal{P}_2$, we demonstrate that the torque ripple can be reduced to as low as 4%, which is significantly below the initial requirement of 10%. This achievement comes with an averaged total optimization time of 80 hours, a substantial improvement from the 200 hours required to optimize the initial problem.

By implementing a stopping criterion on optimization $\mathcal{P}_3$ once the torque ripple reaches the target of $T_{\max}$, we have shown that the two optimizations on the magnetic objectives $\mathcal{P}_1$ and $\mathcal{P}_3^*$ were almost trivial to solve. Indeed, over 99.9% convergence was reached by the randomly generated initial population. With this approach, the total optimization time is almost entirely attributed to the mechanical optimization. Hence, there are promising prospects to further reduce the remaining optimization time, averaging 28 hours, by symmetrizing the mechanical model and/or using specialized algorithms for structural optimization.

Hence, we have demonstrated that, through careful parametrization and leveraging the results of sensitivity analysis, we can derive a time efficient optimization approach that produces a solution comparable to the best-known design. Despite the computational cost associated with sensitivity analysis, it differs in nature from the optimization problem itself. The evaluation of quasi-random samples is fully parallelizable, whereas optimization is inherently sequential. Moreover, the sensitivity analysis can only be refined by increasing the number of the samples, while an optimization algorithm can be trapped in local minima and, therefore, may fail to extract useful information from additional evaluations. In the following chapter, we will further leverage the database gathered for sensitivity analysis to compare different topologies across various objectives.

Benefits of the cut-off barrier

7

The main goal of this chapter consists in exploring the benefits of integrating a cut-off barrier to the rotor geometry. From a mechanical standpoint, the use of a cut-off barrier offers advantages by reducing the peripheral mass of the rotor. This reduction holds the potential to decrease the size of the ribs, thereby leading to an increase in the mean torque. Furthermore, marking the geometry with a salient pole is expected to improve the internal power factor through a possible increase of the saliency ratio ζ [109, 110]. Additionally, this rotor topology presents opportunities for sensorless¹ control using saliency-tracking-based methods [111–113]. These methods leverage the magnetic saliency of the rotor to estimate its position, with the anticipation that augmenting the saliency ratio will also enhance the motor's suitability for this type of control.

Hence, a second objective of this chapter is to establish a design metric for evaluating the motor's suitability for sensorless control, extending the comparisons of rotors with and without cut-off barrier to this criterion. To achieve this objective, we offer an overview of the working principles of sensorless control methods to elucidate the associated challenges and their connection to motor design, particularly in the simplified scenario of linear ferromagnetic materials. Subsequently, we undertake a mathematical derivation of a design metric that accommodates saturation and cross-saturation effects inherent in SynRM, as introduced in section 1.1.2. Notably, this development explores the potential of leveraging saturation effects to enhance the sensorless control scheme, departing from the conventional approach that typically treats saturation as a perturbation [111, 114, 115]. The derived design metric takes into account these modifications in the sensorless scheme.

The final challenge addressed in this chapter directly lies in the methodology employed for comparing different topologies across multiple objectives. Achieving a fair comparison necessitates optimization, yet optimizing topologies for all objectives proves impractical. Introducing dimensionality to the optimization problem, for instance, by comparing topologies based on trade-offs between two objectives as seen in Pareto fronts, further complicates optimization and renders it unrealistic.

¹More appropriately referred to as "self-sensing".

Therefore, the third objective of this chapter is to develop a methodology suitable for comparing topologies across multiple objectives without relying on resource-intensive optimizations. The effectiveness of this methodology is demonstrated by incorporating rotors with a single inner flux barrier, with and without a cut-off barrier, into the comparison. This move serves two purposes:

1. The outcomes of the methodology can be benchmarked against a bi-objective problem, using Pareto fronts obtained through resource-intensive optimizations as reference results. The reduced design space of rotors with a single inner flux barrier allows for optimization convergence within a few days, a significant advantage compared to the anticipated longer convergence time for rotors with four flux barriers.
2. While it is established that a higher number of flux barriers enhances torque capabilities by reducing circulating inductance, refer to chapter 3, the benefits of an increased number of flux barriers on other objectives, such as iron losses, internal power factor, or sensorless capability, remain to be assessed.

To address these three objectives, the structure of this chapter is outlined as follows. First, section 7.1 provides a comprehensive review of sensorless control methods, with the aim of deriving a design metric for this specific purpose. Subsequently, section 7.2 introduces and discusses the proposed comparison methodology. The outcomes of comparing rotors, featuring either one or four inner flux barriers, with or without cut-off barriers, are detailed in section 7.3. Finally, the key findings are succinctly summarized in the concluding section 7.4.

7.1 Sensorless control

Field oriented control (FOC) is a widely employed control technique for synchronous motors that need to operate smoothly across their full speed range. This control method, depicted in Fig. 7.1, relies on the knowledge of the rotor mechanical speed ω_m for control and rotor position θ_m for executing the $(\alpha\beta)$ to (dq) transforms, as detailed in appendix A.

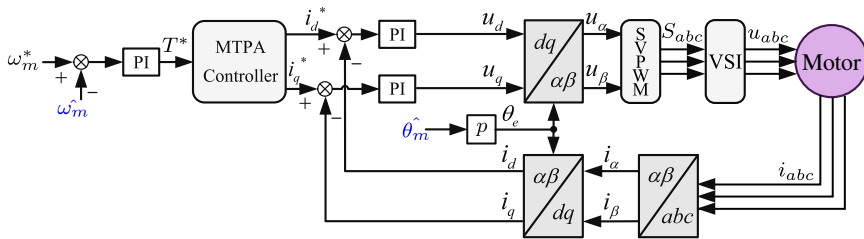


Figure 7.1: Field oriented control of a 3-phase synchronous motor.

The position and speed information is typically acquired by equipping the motor with a resolver, a sensor comprising a stator and a rotor that are magnetically coupled. As illustrated in Fig. 7.2, the rotor is excited by a high-frequency sinusoidal signal, inducing a voltage on two secondary windings in the stator. This voltage is amplitude-modulated by the position of the rotor and in quadrature for the two

windings. The envelope of each signal is recovered through a synchronized sample and hold circuit. Subsequently, software demodulation is employed to extract both the position and the speed.

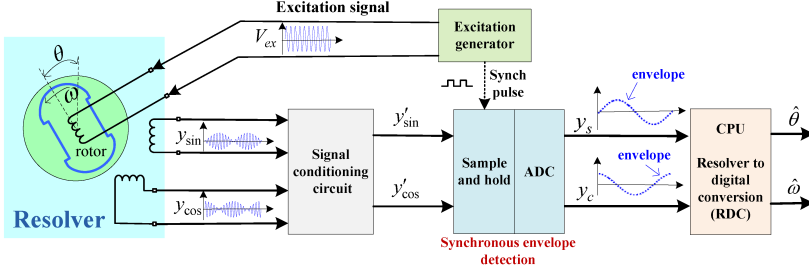


Figure 7.2: Operating principle of a resolver [116].

The hardware components of a resolver closely resemble those needed for motor control, such as current sensing, analogous to voltage sensing employed in resolver, a voltage source inverter (VSI), and the motor itself. Leveraging the electrical motor equations, the estimation the speed and position of the reluctance rotor can be integrated within the control algorithm, eliminating the need for an additional sensor. This integration, often termed "sensorless control", might be more accurately described as "self-sensing". The direct advantages of forgoing the resolver include cost reduction, increased reliability, easier integration, higher control bandwidth, and enhanced compatibility with harsh environments.

7.1.1 Overview of the sensorless methods

An overview of the methods employed in sensorless control is presented in Fig. 7.3. To grasp the principles of these methods, it is advantageous to begin with the $(\alpha\beta)$ equations of an interior permanent magnet synchronous motor (IPMSM) which is general to both a SynRM and a surface-mounted permanent magnet synchronous motor (SPMSM):

$$\begin{bmatrix} v_\alpha \\ v_\beta \end{bmatrix} = R_s \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} + \frac{d}{dt} \left([\mathbf{L}_{\alpha\beta}] \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} \right) + \underbrace{\omega_e \psi_f \begin{bmatrix} -\sin \theta_{em} \\ \cos \theta_{em} \end{bmatrix}}_{\mathbf{e}_{\alpha\beta}} \quad (7.1)$$

$$[\mathbf{L}_{\alpha\beta}] = \begin{bmatrix} L_\alpha & M_{\alpha\beta} \\ M_{\beta\alpha} & L_\beta \end{bmatrix} = \begin{bmatrix} L_\Sigma + L_\Delta \cos(2\theta_{em}) & L_\Delta \sin(2\theta_{em}) \\ L_\Delta \sin(2\theta_{em}) & L_\Sigma - L_\Delta \cos(2\theta_{em}) \end{bmatrix}. \quad (7.2)$$

The equations of the SynRM are derived by setting the permanent magnet flux linkage ψ_f to zero in (7.1). Similarly, the equations of the SPMSM are obtained by setting $L_\Delta = 0$. Utilizing the back-EMF term $\mathbf{e}_{\alpha\beta}$ of a SPMSM, the rotor position can be determined by considering the applied voltages $\mathbf{v}_{\alpha\beta}$, measured currents $\mathbf{i}_{\alpha\beta}$, and motor characteristics. In the case of SynRM or IPMSM with strong saliency, the contribution of inductance terms to the back-EMF cannot be neglected. Introducing the

concept of extended back-EMF (EEMF), (7.1) is rewritten [117]:

$$\begin{aligned} \begin{bmatrix} v_\alpha \\ v_\beta \end{bmatrix} &= R_s \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} + \begin{bmatrix} \bar{p}L_d & \omega_e(L_d - L_q) \\ -\omega_e(L_d - L_q) & \bar{p}L_d \end{bmatrix} \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} \\ &+ \underbrace{((L_d - L_q)(\omega_e i_d - \dot{i}_q) + \omega_e \psi_f)}_{\mathbf{ee}_{\alpha\beta}} \begin{bmatrix} -\sin \theta_{em} \\ \cos \theta_{em} \end{bmatrix}, \end{aligned} \quad (7.3)$$

where $\bar{p}$ is the time derivative operator d/dt . The time derivative operator $\dot{\cdot}$ applied on i_q means that it is only effective for i_q and not on $\sin \theta_{em}$ and $\cos \theta_{em}$.

Back-EMF-based sensorless methods are widely applicable, as all electric motors exhibit a back-EMF when operating at non-zero speeds. In practice, reversing equations (7.1) or (7.3) is rarely performed due to unmodeled dynamics, inverter non-linearity affecting the applied voltages $\mathbf{u}$, sensor noise or calibration offset affecting the measured currents $\mathbf{i}$, or uncertainty in estimating machine parameters R_s , L_d , L_q , and ψ_f . Instead, the system is typically represented with a state-space formulation, such as:

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{Ax} + \mathbf{Bu} & \dot{\mathbf{x}} &= \mathbf{f}(\mathbf{x}, \mathbf{u}) \\ \mathbf{y} &= \mathbf{Cx} + \mathbf{Du} & \mathbf{y} &= \mathbf{h}(\mathbf{x}) \end{aligned} \quad \begin{array}{l} \text{(linear)} \\ \text{(nonlinear)} \end{array} \quad (7.4)$$

with the inputs $\mathbf{u}$, states $\mathbf{x}$, and measurable outputs $\mathbf{y}$ being:

$$\mathbf{u} = \begin{bmatrix} v_\alpha \\ v_\beta \end{bmatrix} \quad \mathbf{x} = \begin{bmatrix} i_\alpha \\ i_\beta \\ ee_\alpha \\ ee_\beta \end{bmatrix} \quad \mathbf{y} = \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix}. \quad (7.5)$$

In this context, the challenge of estimating the back-EMF is tackled through well-established state observer theories. These methods encompass the Model Reference Adaptive Method (MRAS) [118], the Sliding Mode Observer (SMO) [113], or the Extended Kalmann Filter (EKF) [119].

The influence of the back-EMF on the applied voltage is inherently significant at higher speeds. Consequently, there are no additional specific metrics to consider when designing a motor for sensorless control in the upper-speed range. Conversely, the insignificance of the back-EMF at lower speeds and its complete absence at standstill renders back-EMF based methods unsuitable for the low-speed region.

The sole viable approach to maintain control in the lower speed range is to adopt a method that retrieves position information through the inductance matrix (7.2). This necessitates having $L_\Delta \neq 0$, implying the presence of magnetic saliency. By applying a known alternative voltage on $\mathbf{v}_{\alpha\beta}$ and measuring the current response $\mathbf{i}_{\alpha\beta}$, the position information can be extracted. For instance, the signal can take the form of a rotating sinusoidal voltage:

$$\mathbf{v}_{inj} = \begin{bmatrix} v_{\alpha_h} \\ v_{\beta_h} \end{bmatrix} = V_{inj} \begin{bmatrix} -\sin(\omega_h t) \\ \cos(\omega_h t) \end{bmatrix}. \quad (7.6)$$

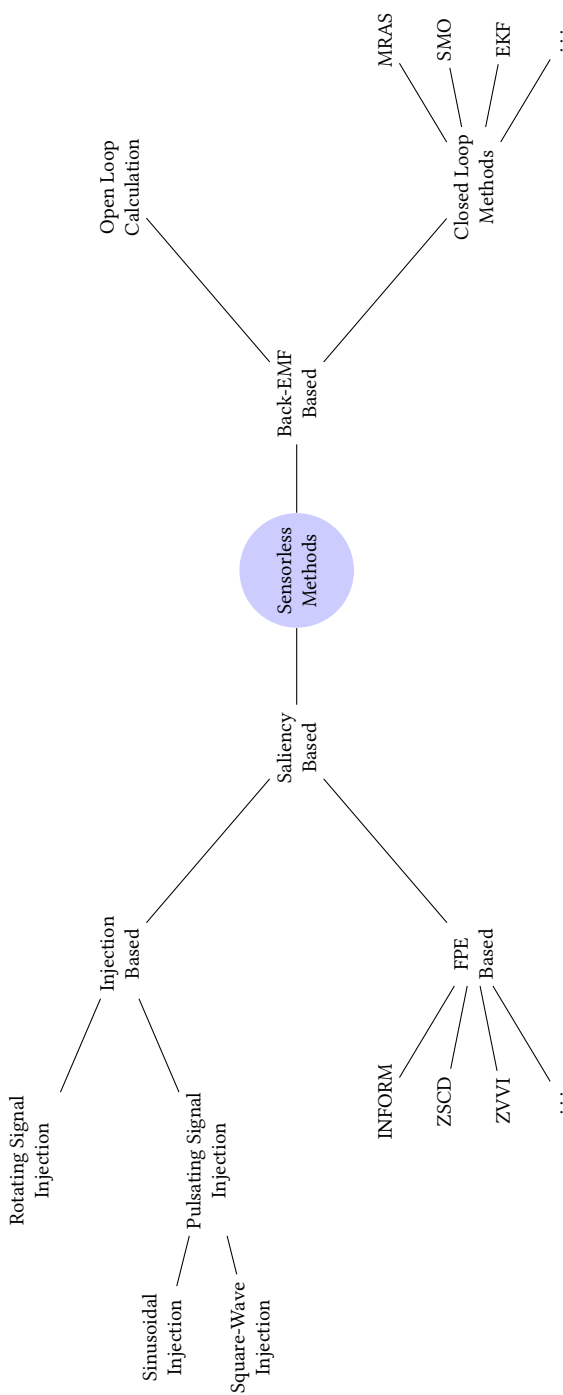


Figure 7.3: Overview of the methods used in sensorless control.

Injecting (7.6) in (7.1), the response currents are [120]:

$$\mathbf{i}_{\text{inj}} = \begin{bmatrix} i_{\alpha_h} \\ i_{\beta_h} \end{bmatrix} = \begin{bmatrix} I_0 \cos(\omega_h t) + I_1 \cos(2\theta_{\text{em}} - \omega_h t) \\ I_0 \sin(\omega_h t) + I_1 \sin(2\theta_{\text{em}} - \omega_h t) \end{bmatrix}, \quad (7.7)$$

with the gains I_0 and I_1 given by:

$$I_0 = \frac{L_\Sigma}{L_d L_q} \cdot \frac{V_{\text{inj}}}{\omega_h} \quad I_1 = \frac{L_\Delta}{L_d L_q} \cdot \frac{V_{\text{inj}}}{\omega_h}. \quad (7.8)$$

This results from neglecting the resistance drop, assuming that the injection frequency ω_h is sufficiently high. Through rotating signal injection, the position information is encoded in the phase of the response signal (7.7), which can be demodulated after applying a synchronous reference frame filter. In execution, this scheme closely resembles that of a resolver.

A significant departure from the resolver is that the current sensors employed to measure the response currents are the same as those used in the motor control. These currents must be of sufficient magnitude to be accurately measured, introducing additional Joule losses and torque ripple. To mitigate the ripple generated by injecting a rotating field, a solution involves injecting the signal within the estimated synchronous frame of the rotor. This strategy is commonly referred to as pulsating signal injection.

An other consequence of sharing the same hardware for control and injection introduces the need to separate these signals at the software level to prevent interference. The frequency of the injected signal must be set outside the bandwidth of the control loop, but the PWM frequency imposes an upper limit on the achievable injection frequency. By employing a square signal instead of a sinusoid, this frequency can be pushed to half of the PWM frequency, as illustrated in Fig. 7.4. The use of square-wave injection (SWI) allows for the separation of control and injection signals without requiring additional filters [121].

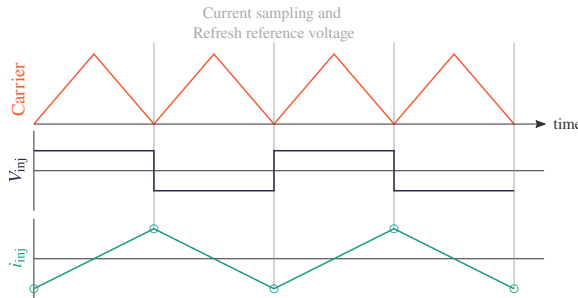


Figure 7.4: Synchronization between PWM carrier, injection voltage and response current measurement.

Alternatively, certain methods operate at the PWM level to alternate between injection signals and control signals, simplifying their separation within the control loop. These methods fall under the category of Fundamental PWM Excitation-Based

methods (FPE). However, they come with the drawback of requiring extra additional current-derivative sensors. Notable implementations include the Indirect Flux Detection by On-Line Reactance Measurement (INFORM) [122], the Zero Sequence Current Derivatives measurements method (ZSCD) [123], or the Zero Voltage Vector Injection (ZVVI) [124]. Historically, these methods were prevalent when the low-computing power of microprocessors posed a constraint, benefitting from the absence of software filters. In contemporary applications, they are being replaced with injection-based method that are less sensitive to the quality of measurements.

In all these methods, the primary challenge is to recover a meaningful current response given an injection voltage. Using (7.8), we can easily readily derive a metric for designing a machine suitable for rotational signal injection:

$$\max_{\mathbf{x}} \left| \frac{L_{\Delta}(\mathbf{x})}{L_d(\mathbf{x})L_q(\mathbf{x})} \right|. \quad (7.9)$$

The absolute value accounts for reverse saliency that can occur in IPMSM. By using the saliency ratio ζ , (7.9) is reformulated as:

$$\max_{\mathbf{x}} \left| \frac{1}{L_q(\mathbf{x})} \left(\frac{\zeta(\mathbf{x}) - 1}{\zeta(\mathbf{x})} \right) \right|. \quad (7.10)$$

This equation suggests that the saliency ratio ζ needs to be maximized, which is a natural target when designing a SynRM. However, the above analysis is limited by the saturation effects inherent in SynRM. The injection signal, by its nature, has a small amplitude in contrast to the control signal. The saturation level of the motor is primarily determined by the operating point, impacting the apparent inductances tested by signal injection methods. The goal of the next subsection is to derive a design metric for SynRM that accounts for magnetic saturation and is well-suited for Square-Wave Injection.

7.1.2 Design metric for saturated Square-Wave Injection

Unlike the rotating signal injection method described in (7.6), square-wave injection involves injecting a square-wave signal along the d axis of the estimated ($\widehat{dq}$) frame:

$$\mathbf{v}_{\text{inj}} = \begin{bmatrix} v_{\widehat{d}_{-h}} \\ v_{\widehat{q}_{-h}} \end{bmatrix} = \begin{bmatrix} \pm V_{\text{inj}} \\ 0 \end{bmatrix}, \quad (7.11)$$

with the plus or minus sign indicating the sign flips at each PWM period, as illustrated in Fig. 7.4. The original scheme, depicted in Fig. 7.5, was initially developed for IPMSM, with the intention of limiting torque ripple by injecting in the d axis.

In SynRM, torque is generated by both the d and q components of the currents so that this advantage is lost. Therefore, after deriving the equations for the square-wave injection while accounting for the saturation, we will also discuss the decision to align the co-rotating frame of the injection with the reference (dq) frame used for control.

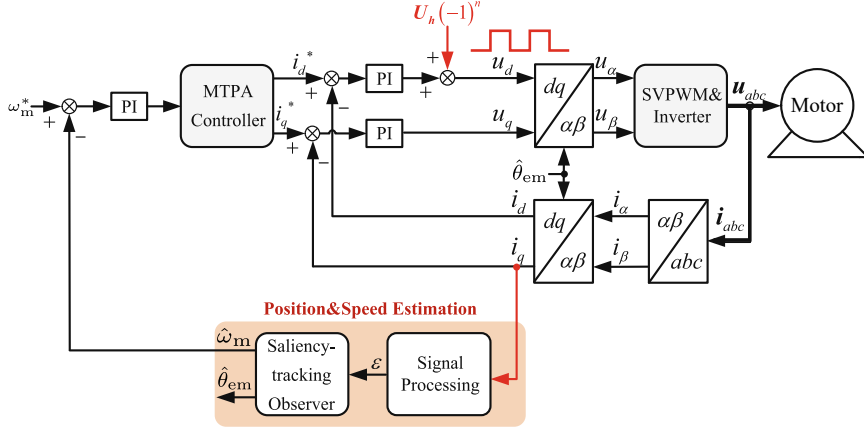


Figure 7.5: Square-wave injection inside the sensorless FOC loop [113].

The motor equations in the (dq) frame are given by (1.6) to (1.8), restated here to emphasize the effects of saturation:

$$\begin{aligned} v_d &= R_s i_d + \frac{d\lambda_d(i_d, i_q)}{dt} - \omega_e \lambda_q(i_d, i_q) \\ v_q &= R_s i_q + \frac{d\lambda_q(i_d, i_q)}{dt} + \omega_e \lambda_d(i_d, i_q). \end{aligned} \quad (7.12)$$

The fluxes λ_{dq} can be linearized around the operating point (i_d^*, i_q^*) using Taylor polynomials. This linearization aids in constructing a small-signal model that accurately represents the behavior of the non-linear system when a small signal is injected near an operating point:

$$\begin{aligned} \lambda_d(i_d, i_q) &\approx \lambda_d(i_d^*, i_q^*) + (i_d - i_d^*) \underbrace{\left. \frac{\partial \lambda_d}{\partial i_d} \right|_{(i_d^*, i_q^*)}}_{L_{dd}} + (i_q - i_q^*) \underbrace{\left. \frac{\partial \lambda_d}{\partial i_q} \right|_{(i_d^*, i_q^*)}}_{L_{dq}} \\ \lambda_q(i_d, i_q) &\approx \lambda_q(i_d^*, i_q^*) + (i_d - i_d^*) \underbrace{\left. \frac{\partial \lambda_q}{\partial i_d} \right|_{(i_d^*, i_q^*)}}_{L_{qd}} + (i_q - i_q^*) \underbrace{\left. \frac{\partial \lambda_q}{\partial i_q} \right|_{(i_d^*, i_q^*)}}_{L_{qq}} \end{aligned} \quad (7.13)$$

The expressions in (7.13) use the concepts of incremental inductances, defined as follows:

$$\begin{aligned} L_{dd} &\triangleq \left. \frac{\partial \lambda_d}{\partial i_d} \right|_{(i_d^*, i_q^*)} & L_{dq} &\triangleq \left. \frac{\partial \lambda_d}{\partial i_q} \right|_{(i_d^*, i_q^*)} \\ L_{qq} &\triangleq \left. \frac{\partial \lambda_q}{\partial i_q} \right|_{(i_d^*, i_q^*)} & L_{qd} &\triangleq \left. \frac{\partial \lambda_q}{\partial i_d} \right|_{(i_d^*, i_q^*)}. \end{aligned} \quad (7.14)$$

Injecting (7.14) and (7.13) in (7.12), we obtain:

$$\begin{aligned} v_d &= R_s i_d + L_{dd} \frac{di_d}{dt} + L_{dq} \frac{di_q}{dt} - \omega_e \lambda_q(i_d, i_q) \\ v_q &= R_s i_d + L_{qq} \frac{di_q}{dt} + L_{qd} \frac{di_d}{dt} + \omega_e \lambda_d(i_d, i_q). \end{aligned} \quad (7.15)$$

These equations are separated into a low-frequency part for the currents and voltages associated with control, and a high-frequency part for those associated with signal injection. In the high-frequency regime, inductive effects dominate over resistive effects. Additionally, we disregard the contribution of the back-EMF to the applied voltage as this scheme is utilized at low-speeds. Consequently, the high-frequency components, denoted by the suffixes h , are:

$$\begin{bmatrix} v_{d_h} \\ v_{q_h} \end{bmatrix} = \underbrace{\begin{bmatrix} L_{dd} & L_{dq} \\ L_{qd} & L_{qq} \end{bmatrix}}_{[\mathbf{Z}]} \bar{p} \begin{bmatrix} i_{d_h} \\ i_{q_h} \end{bmatrix}. \quad (7.16)$$

In square-wave injection, the signals are measured and injected in a co-rotating estimated frame ($\widehat{dq}$) at angle $\hat{\theta}_{\text{SWI}}$. The error angle $\tilde{\theta}$ between the (dq) frame and injection frame ($\widehat{dq}$) is:

$$\tilde{\theta} = \theta_{\text{em}} - \hat{\theta}_{\text{SWI}}. \quad (7.17)$$

Using this notation, we implicitly distinguish the frame used for signal injection, at angle $\hat{\theta}_{\text{SWI}}$, from the frame used for control, at angle $\hat{\theta}_{\text{em}}$. The resulting scheme is illustrated in Fig. 7.6. In the initial scheme in Fig. 7.5, the $(\alpha\beta)$ to (dq) transforms are shared between the control loop and the injection loop, enforcing the condition $\hat{\theta}_{\text{em}} = \hat{\theta}_{\text{SWI}}$.

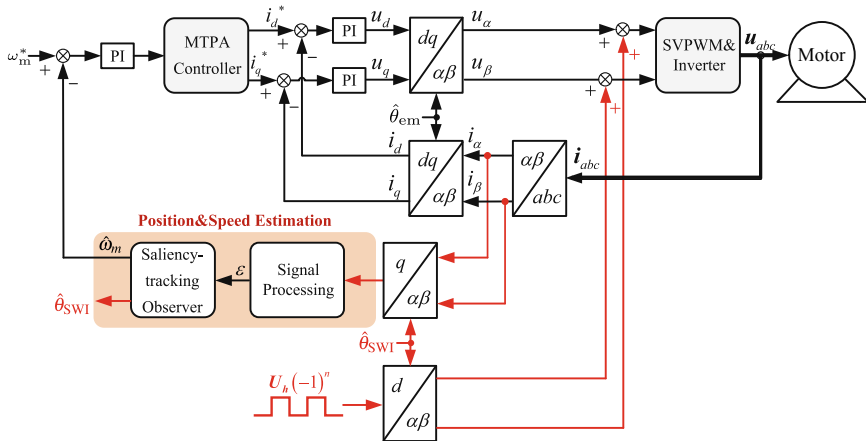


Figure 7.6: Square-wave injection using $\hat{\theta}_{\text{SWI}} \neq \hat{\theta}_{\text{em}}$.

Similarly to the $(\alpha\beta)$ to (dq) transform (A.3), the $(\widehat{dq})$ to (dq) transform is:

$$\mathbf{x}_{dq} = [\mathbf{R}(\tilde{\theta})] \mathbf{x}_{\widehat{dq}} \quad (7.18)$$

$$\begin{bmatrix} x_d \\ x_q \end{bmatrix} = \begin{bmatrix} \cos \tilde{\theta} & \sin \tilde{\theta} \\ -\sin \tilde{\theta} & \cos \tilde{\theta} \end{bmatrix} \begin{bmatrix} x_{\hat{d}} \\ x_{\hat{q}} \end{bmatrix}, \quad (7.19)$$

so that the signals injected and measured in the estimated frame are, in the reference (dq) frame:

$$\begin{aligned} [\mathbf{R}(\tilde{\theta})] \begin{bmatrix} v_{\hat{d}_h} \\ v_{\hat{q}_h} \end{bmatrix} &= [\mathbf{Z}] \bar{p} [\mathbf{R}(\tilde{\theta})] \begin{bmatrix} i_{\hat{d}_h} \\ i_{\hat{q}_h} \end{bmatrix} \\ \Rightarrow \bar{p} \begin{bmatrix} i_{\hat{d}_h} \\ i_{\hat{q}_h} \end{bmatrix} &= [\mathbf{R}^{-1}(\tilde{\theta})] [\mathbf{Z}^{-1}] [\mathbf{R}(\tilde{\theta})] \begin{bmatrix} v_{\hat{d}_h} \\ v_{\hat{q}_h} \end{bmatrix}. \end{aligned} \quad (7.20)$$

This development overlooks the time derivative of the rotation matrix, assuming that the position of the rotor remains constant throughout the PWM period. This assumption holds true as SWI is utilized at low-speeds.

As illustrated in Fig. 7.4, the time derivative di/dt corresponds to the slope $\Delta i/\Delta T$ for the short SWI period of length ΔT . Considering the injected signal (7.11), (7.20) successively becomes:

$$\begin{bmatrix} \Delta i_{\hat{d}_h} \\ \Delta i_{\hat{q}_h} \end{bmatrix} = \frac{\Delta T}{\det(\mathbf{Z})} \begin{bmatrix} c & -s \\ s & c \end{bmatrix} \begin{bmatrix} L_{qq} & -L_{dq} \\ -L_{qd} & L_{dd} \end{bmatrix} \begin{bmatrix} c & s \\ -s & c \end{bmatrix} \begin{bmatrix} \pm V_{inj} \\ 0 \end{bmatrix} \quad (7.21)$$

$$= \frac{\Delta T}{\det(\mathbf{Z})} \begin{bmatrix} cL_{qq} + sL_{qd} & -(cL_{dq} + sL_{dd}) \\ sL_{qq} - cL_{qd} & -sL_{dq} + cL_{dd} \end{bmatrix} \begin{bmatrix} \pm cV_{inj} \\ \mp sV_{inj} \end{bmatrix} \quad (7.22)$$

$$= \frac{\pm V_{inj} \Delta T}{\det(\mathbf{Z})} \begin{bmatrix} c^2 L_{qq} + scL_{qd} + scL_{dq} + s^2 L_{dd} \\ scL_{qq} - c^2 L_{qd} + s^2 L_{dq} - scL_{dd} \end{bmatrix} \quad (7.23)$$

$$= \frac{\pm V_{inj} \Delta T}{L_{dd}L_{qq} - L_{dq}L_{qd}} \begin{bmatrix} c^2 L_{qq} + scL_{qd} + scL_{dq} + s^2 L_{dd} \\ scL_{qq} - c^2 L_{qd} + s^2 L_{dq} - scL_{dd} \end{bmatrix}, \quad (7.24)$$

with the notations s and c to represent $\sin \tilde{\theta}$ and $\cos \tilde{\theta}$, respectively. By considering a lossless magnetic circuit, the conservation of energy leads to the following reciprocity condition [125, 126]:

$$L_{dq} = L_{qd}, \quad (7.25)$$

so that (7.24) finally simplifies to:

$$\begin{bmatrix} \Delta i_{\hat{d}_h} \\ \Delta i_{\hat{q}_h} \end{bmatrix} = \frac{\pm V_{inj} \Delta T}{L_{dd}L_{qq} - L_{dq}^2} \begin{bmatrix} s^2 L_{dd} + c^2 L_{qq} + 2scL_{dq} \\ -scL_{\Delta\Delta} + L_{dq}(s^2 - c^2) \end{bmatrix}, \quad (7.26)$$

with the difference of d and q incremental inductances defined as:

$$L_{\Delta\Delta} \triangleq L_{dd} - L_{qq}. \quad (7.27)$$

This expression is analyzed without and with the presence of cross-saturation, i.e., $L_{dq} = L_{qd} = 0$ and $L_{dq} = L_{qd} \neq 0$.

Without cross-saturation

In this case, equation (7.26) becomes:

$$\begin{bmatrix} \Delta \dot{i}_{\hat{d}_{-h}} \\ \Delta \dot{i}_{\hat{q}_{-h}} \end{bmatrix} = \frac{\pm V_{\text{inj}} \Delta T}{L_{dd} L_{qq}} \begin{bmatrix} \textcolor{red}{s}^2 L_{dd} + \textcolor{red}{c}^2 L_{qq} \\ -\textcolor{red}{s} \textcolor{red}{c} L_{\Delta\Delta} \end{bmatrix}. \quad (7.28)$$

Starting from an initial position from which the position error $\delta\tilde{\theta}$ is small, the sine and cosine in (7.28) can be linearized using $\sin \delta\tilde{\theta} \approx \delta\tilde{\theta}$ and $\cos \delta\tilde{\theta} \approx 1$:

$$\Rightarrow \Delta \dot{i}_{\hat{q}_{-h}} = \mp V_{\text{inj}} \Delta T \underbrace{\frac{L_{\Delta\Delta}}{L_{dd} L_{qq}}}_{g_m} \delta\tilde{\theta}. \quad (7.29)$$

In the conventional SWI, the saliency-tracking observer enforces $\Delta \dot{i}_{\hat{q}_{-h}} \rightarrow 0$ by acting on $\hat{\theta}_{\text{SWI}}$ using a PI controller [121, 127]. In this case, the final estimation error $\tilde{\theta}^*$ at $\Delta \dot{i}_{\hat{q}_{-h}} = 0$ becomes 0. From (7.17), that means the estimated ($\widehat{dq}$) frame is aligned with the reference (dq) frame, making it entirely reasonable to use the frame ($\widehat{dq}$) for current control:

$$\tilde{\theta}^* = 0 \Rightarrow \hat{\theta}_{\text{SWI}} = \theta_{\text{em}} = \hat{\theta}_{\text{em}}. \quad (7.30)$$

Moreover, using incremental inductances, the motor gain g_m has an expression similar to (7.8). This expression holds true for small to moderate levels of magnetic saturation.

With cross-saturation

The original square-wave injection scheme does not account for cross-saturation, so that the saliency-tracking observer still enforce $\Delta \dot{i}_{\hat{q}_{-h}} \rightarrow 0$. In this case, (7.26) becomes:

$$\Delta \dot{i}_{\hat{q}_{-h}} = 0 \Leftrightarrow -\textcolor{red}{s} \textcolor{red}{c} L_{\Delta\Delta} + L_{dq} (\textcolor{red}{s}^2 - \textcolor{red}{c}^2) = 0 \quad (7.31)$$

$$\Rightarrow L_{\Delta\Delta} - L_{dq} \left(\frac{\textcolor{red}{s}}{\textcolor{red}{c}} - \frac{\textcolor{red}{c}}{\textcolor{red}{s}} \right) = 0. \quad (7.32)$$

The condition $\textcolor{red}{s} \textcolor{red}{c} \neq 0$ is satisfied if $\tilde{\theta} \neq n\pi/2$ ($n \in \mathbb{Z}$), i.e., if we commit a small error in the estimation. This error can be calculated by solving (7.32):

$$\Rightarrow L_{\Delta\Delta} \tan^2 \tilde{\theta}^* - L_{dq} \tan^2 \tilde{\theta}^* + L_{dq} = 0 \quad (7.33)$$

$$\Rightarrow \tan^2 \tilde{\theta}^* - \frac{L_{\Delta\Delta}}{L_{dq}} \tan^2 \tilde{\theta}^* - 1 = 0, \quad (7.34)$$

which has minimal solution $|\tilde{\theta}^*|$ at:

$$\tilde{\theta}^* = \begin{cases} \arctan \left(\frac{1}{2L_{dq}} \left(L_{\Delta\Delta} - \sqrt{L_{\Delta\Delta}^2 + 4L_{dq}^2} \right) \right) & \text{if } L_{\Delta\Delta} > 0, \\ \arctan \left(\frac{1}{2L_{dq}} \left(L_{\Delta\Delta} + \sqrt{L_{\Delta\Delta}^2 + 4L_{dq}^2} \right) \right) & \text{otherwise.} \end{cases} \quad (7.35)$$

This last equation shows that an error in the estimation of the angle is introduced in presence of cross-saturation. This error can be compensated in software if the machine parameters are known, so that there are no particular reasons to design the machine to minimize this error². Such compensation mechanism can be implemented with the scheme in Fig. 7.6 by enforcing:

$$\hat{\theta}_{\text{em}} = \hat{\theta}_{\text{SWI}} + \tilde{\theta}^*, \quad (7.36)$$

so that, using (7.17), we recover the reference frame (dq):

$$\hat{\theta}_{\text{em}} = \theta_{\text{em}}. \quad (7.37)$$

Despite the apparent complexity of Fig. 7.6 over Fig. 7.5, this scheme does not create additional delay for the control loop. The only drawback is a slight increase in FPGA resource utilization due to the additional $(\alpha\beta)$ to $(\hat{q})$ and $(\hat{d})$ to $(\alpha\beta)$ transforms.

The injection gain g in presence of cross-saturation can be retrieved after linearizing $\Delta i_{\hat{q}_h}$ around the equilibrium position error $\tilde{\theta}^*$, for small additional position error $\delta\tilde{\theta}$:

$$\Delta i_{\hat{q}_h}(\tilde{\theta}^* + \delta\tilde{\theta}) = \underbrace{\Delta i_{\hat{q}_h}(\tilde{\theta}^*)}_{\triangleq 0} + \underbrace{\left. \frac{d\Delta i_{\hat{q}_h}}{d\tilde{\theta}} \right|_{\tilde{\theta}^*}}_g \delta\tilde{\theta}. \quad (7.38)$$

Hence, the gain g is given by:

$$g = \left. \frac{d\Delta i_{\hat{q}_h}}{d\tilde{\theta}} \right|_{\tilde{\theta}^*} = \frac{\pm V_{\text{inj}} \Delta T}{L_{dd} L_{qq} - L_{dq}^2} \left(L_{\Delta\Delta} (\sin^2 \tilde{\theta}^* - \cos^2 \tilde{\theta}^*) + 4 L_{dq} \sin \tilde{\theta}^* \cos \tilde{\theta}^* \right). \quad (7.39)$$

Injecting the expression of $\tilde{\theta}^*$ (7.35) in (7.39) and simplifying it using:

$$\sin(\arctan x) = \frac{x}{\sqrt{1+x^2}} \quad \cos(\arctan x) = \frac{1}{\sqrt{1+x^2}}, \quad (7.40)$$

we finally obtain:

$$g = \left. \frac{d\Delta i_{\hat{q}_h}}{d\tilde{\theta}} \right|_{\tilde{\theta}^*} = \mp V_{\text{inj}} \Delta T \underbrace{\frac{\sqrt{L_{\Delta\Delta}^2 + 4L_{dq}^2}}{L_{dd} L_{qq} - L_{dq}^2}}_{g_m}. \quad (7.41)$$

The expression of the machine gain g_m is more general than (7.29), which can be retrieved by setting $L_{dq} = 0$ in (7.41). We now have a metric associated to the sensorless controllability of the motor. A higher gain g_m is desirable as it results in an increased response current for a given injection signal and position error $\delta\tilde{\theta}$. Expressing it differently, a higher gain g_m reduces the smallest detectable position error.

²Because incremental inductances depend on the operating points, such implementation would necessitate a look-up table. In this discussion, we assume the control algorithm possesses precise knowledge of machine parameters at all times.

Continuing the analysis of SWI, we are also interested in determining the value of the final position error $\tilde{\theta}'$ that maximizes the gain g . By separating the injection angle from the control angle (Fig. 7.6), we can intentionally introduce a position error to maximize the gain. Restarting from (7.39), we have:

$$\max_{\tilde{\theta}'} \left(g = \frac{d\Delta i_{\hat{q}_h}}{d\tilde{\theta}} \Big|_{\tilde{\theta}'} \right) \quad (7.42)$$

$$\Rightarrow 0 = \frac{d}{d\tilde{\theta}'} \left(g = \frac{d\Delta i_{\hat{q}_h}}{d\tilde{\theta}} \Big|_{\tilde{\theta}'} \right) \quad (7.43)$$

$$\Rightarrow 0 = L_{\Delta\Delta}(\sin \tilde{\theta}' \cos \tilde{\theta}') + L_{dq}(\cos^2 \tilde{\theta}' - \sin^2 \tilde{\theta}') \quad (7.44)$$

$$\Rightarrow 0 = -\sin \tilde{\theta}' \cos \tilde{\theta}' L_{\Delta\Delta} + L_{dq}(\sin^2 \tilde{\theta}' - \cos^2 \tilde{\theta}'). \quad (7.45)$$

Interestingly, this last expression is identical to (7.32), which was used to derive $\tilde{\theta}^*$. Therefore:

$$\tilde{\theta}' = \tilde{\theta}^*. \quad (7.46)$$

Hence, we demonstrated that the best use of the injection signal occurs when injecting at an angle $\hat{\theta}_{SWI}$ such that the response signal $\Delta i_{\hat{q}_h}$ is zero, as proposed in the original SWI scheme. This remains true even in the presence of saturation and cross-saturation in SynRM. The response currents are proportional to the metric g_m and the estimated position error $\delta\tilde{\theta}$ (around $\tilde{\theta}^*$):

$$\Delta i_{\hat{q}_h}(\tilde{\theta}^* + \delta\tilde{\theta}) = \mp V_{inj} \Delta T \cdot g_m \cdot \delta\tilde{\theta} \quad (7.47)$$

$$\text{with } g_m = \frac{\sqrt{L_{\Delta\Delta}^2 + 4L_{dq}^2}}{L_{dd}L_{qq} - L_{dq}^2}. \quad (7.48)$$

The evaluation of the incremental inductances involves partial derivatives, which are approached numerically:

$$\begin{aligned} L_{dd} &\approx \frac{\lambda_d|_{(i_d^* + \Delta i_d, i_q^*)} - \lambda_d^*}{\Delta i_d} & L_{dq} &\approx \frac{\lambda_d|_{(i_d^*, i_q^* + \Delta i_q)} - \lambda_d^*}{\Delta i_q} \\ L_{qq} &\approx \frac{\lambda_q|_{(i_d^*, i_q^* + \Delta i_q)} - \lambda_q^*}{\Delta i_q} & L_{qd} &\approx \frac{\lambda_q|_{(i_d^* + \Delta i_d, i_q^*)} - \lambda_q^*}{\Delta i_d}. \end{aligned} \quad (7.49)$$

The incremental inductances are determined using the magnetic model developed in chapter 2, at the cost of two additional simulations with perturbed currents $(i_d^* + \Delta i_d, i_q^*)$ and $(i_d^*, i_q^* + \Delta i_q)$. These perturbations are set at 1% of the nominal current.

7.2 Comparison methodology

Through the comparison of many objectives, one topology is anticipated to outperform the other in certain aspects while underperforming for others. This inherent trade-off manifests as an N-dimensional Pareto front, whose unique solution results on assigned weights to the considered objectives. Although an exhaustive

exploration of all possible weight combinations is impractical, we can establish a comparison methodology with the primary goal of recommending the most suitable topology for a given application, rather than precisely quantifying the performance differentials associated with designing one topology over another.

Recognizing this, we propose a comparison methodology that does not rely directly on optimizations involving accurate yet time-consuming models. Instead, the suggested approach leverages a second valorization of the samples gathered during sensitivity analyses. The broad identification of the optimal topology is achieved by comparing histograms of the output of interest. While the shape of the histogram is intrinsically tied to the search space, comparing histograms provides valuable insights into which topology is likely to outperform another within its search space. Importantly, it is a readily available source of information, incurring no extra computational cost after the completion of the sensitivity analysis. For multi-output problems, this methodology can be extended through density plots.

To enhance confidence in the selection of the topology, pre-optimization on each topology using metamodels can still be performed. The goal of these pre-optimizations is to compare the optimized objectives and select the topology that requires optimization using accurate models. The binary nature of the selection makes it robust to evaluation errors, mitigating the criticality of metamodel accuracy compared to conventional optimization.

While inherently leveraging a computational advantage, this comparison methodology ensures good reusability of results for other applications, as the search space is thoroughly explored without bias from specific outputs. Furthermore, as additional outputs of interest are evaluated, they can be systematically stored and repurposed in subsequent studies. In contrast, optimization algorithms ideally seek solutions with minimal exploration of the search space, limiting their ability to provide meaningful information on other quantities of interest unless explicitly defined as objectives or constraints in the optimization problem.

Construction of the databases

As with the sensitivity analyses performed in chapter 4, the databases for the comparison are constructed by sampling the search space using Saltelli's method. In comparison to previous analyses, we decided to extend the boundaries of the search space, and remove the radius of the fillets from the design space, assigning them the value of 3 mm. The updated boundaries are provided in Table 7.1

With a sample size N of 1024, that results in 14336 and 12688 samples of the search space for one inner flux barrier, with and without-cutoff, respectively. For geometries having 4 inner flux barriers, i.e., $q = 5$, that leads to 38912 and 36864 for geometries with and without cut-off, respectively. The evaluation of these samples is dispatched using the clusters of CÉCI. For reference, the results of the sensitivity analysis of a single inner flux barrier rotor with and without cut-off barrier are reported in appendix D.

Table 7.1: Updated boundaries of the search space ($i = 1, 2, \dots, n$), ($j = 1, 2, \dots, q$).

Param.	Description	Lower bound	Upper bound	Unit
θ_e	Current angle	55 45	70 75	°
ξ	Air-to-rotor ratio	0.2	0.8	—
K_j^{fg}	Stiffness of the virtual spring (flux guide)	0.5	2	—
K_i^{fb}	Stiffness of the virtual spring (flux barrier)	0.5	2	—
t_i^R	Thickness of the radial ribs	0.0 1	5 10	mm
t_i^T	Thickness of the tangential ribs	0.3 1	8 10	mm
$f_i^{1 \rightarrow 4}$	Radius of the fillet (x4)	1	10	mm

7.3 Results

7.3.1 Using one inner flux barrier

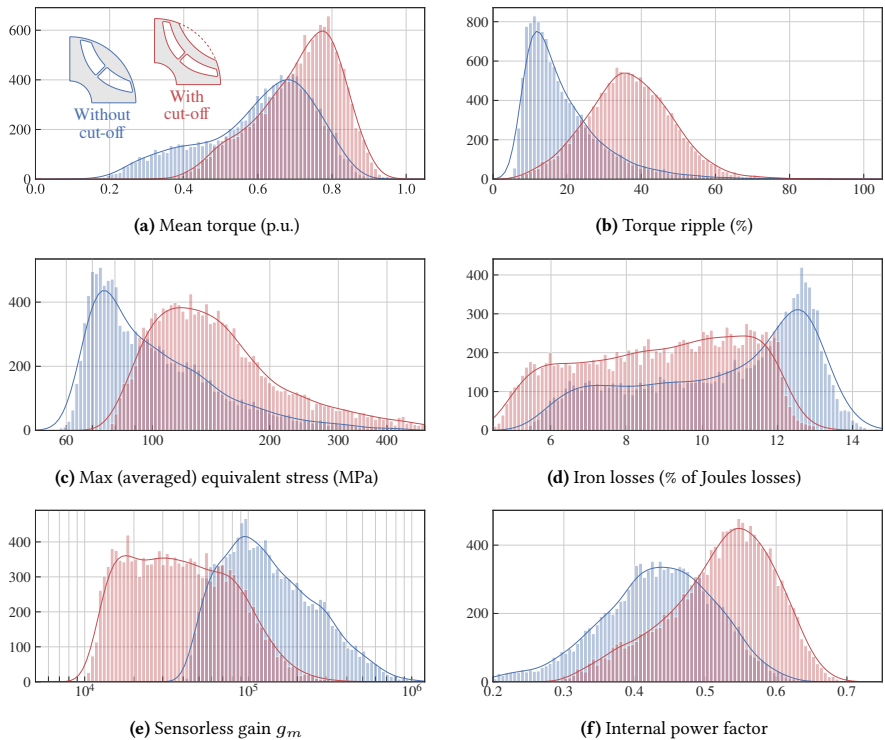


Figure 7.7: Number of occurrences of each objective, for rotors with and without cut-off barrier.

Histograms of several objectives of interest are depicted in Fig. 7.7. These histograms are not normalized, so there is a slight unbalance in the total count between the two geometries. Overall, rotors with a cut-off barrier tend to have

higher mean torque, higher internal power factor, and lower iron losses. Conversely, smooth rotors tend to exhibit lower torque ripple, lower maximum (averaged) equivalent stress, and higher sensorless gain g_m . These results are counterintuitive, as the cut-off barrier was expected to bring an advantage in both the local stress by removing some of the peripheral mass and improving the sensorless gain by marking the apparent saliency. This highlights the complex parameter interactions that make some objectives more difficult to predict than expected, and underscores the need for metrics to better select an appropriate topology.

For conflicting outputs where the choice of the topology is not clear, such as considering simultaneously the mean torque and the torque ripple, the adequate topology will depend on the selected trade-off between the outputs. The histograms can be extended using density plots, such as those presented in Fig. 7.8 for the mean torque and sensorless gain, and the mean torque and the torque ripple. These graphs are obtained using a Gaussian kernel density estimator (KDE) of the samples that smoothen out the data. It is the same method that was applied in Fig. 7.7 to obtain the smooth lines. Similar to the histograms, these graphs give us an information of what should be the adequate topology given a trade-off between two outputs. On these figures, we outlined in black a simili-Pareto front, with the symbol $\times$ marking the estimated location where one topology becomes better than the other.

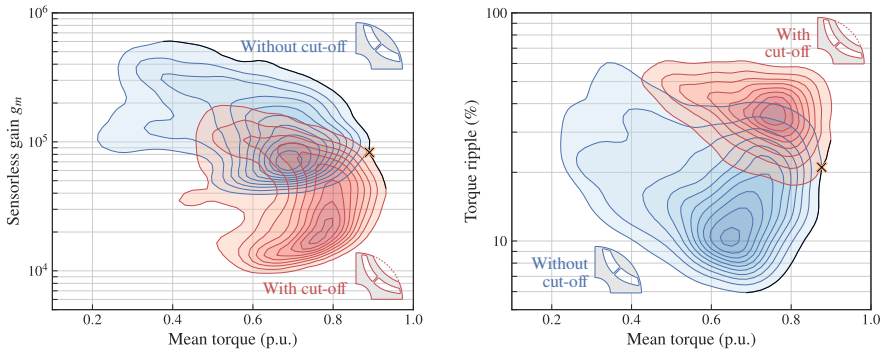


Figure 7.8: Density plots of (left) the mean torque and the sensorless gain, (right) the mean torque and the torque ripple. The simili-Pareto front is outlined.

To refine the estimation of the point $\times$, we propose to perform a metamodel-based optimization to maximize the mean torque and sensorless gain. The metamodels use Polynomial Chaos Expansions (4.15) trained on 70% of the available samples, and validated using the predictivity factor Q^2 on the remaining 30% of the samples. Conversely to the coefficient of determination R^2 , the predictivity factor Q^2 is robust to over-fitting thanks to the use of a validation set.

The resulting fits are provided in Fig. 7.9. For these fits, the degree of the polynomial is gradually increased until Q^2 reaches a maximum. Due to the large range of variation of the gain g_m , the fit has been performed by taking the logarithm instead.

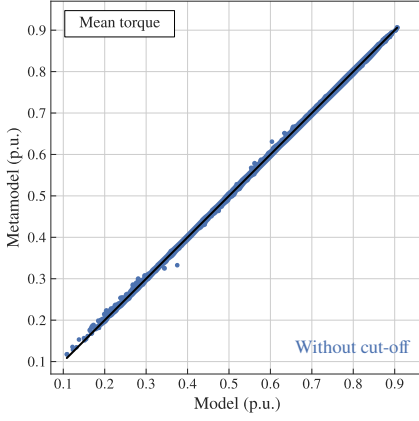
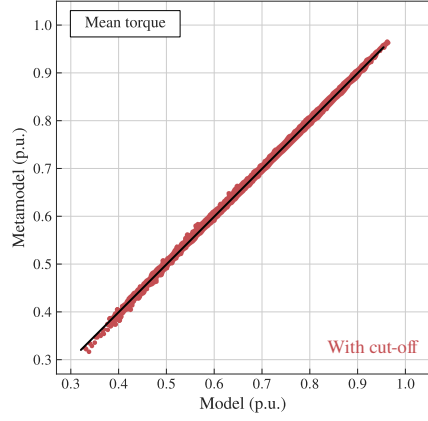
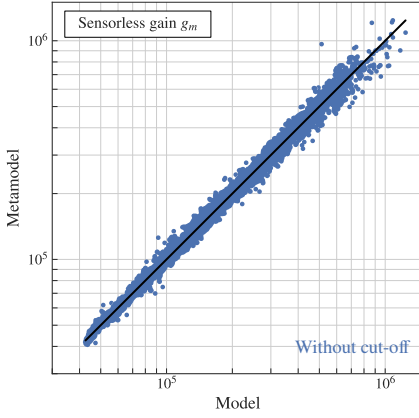
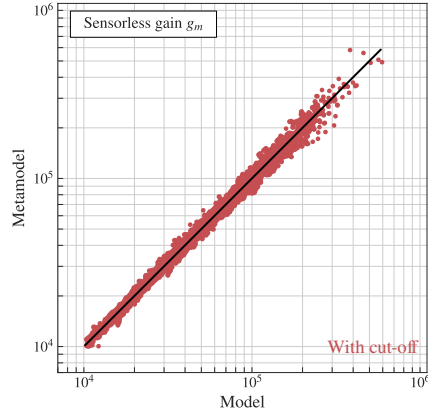
(a) Mean torque without cut-off ($P=5$, $Q^2=99.978\%$)(b) Mean torque with cut-off ($P=4$, $Q^2=99.917\%$)(c) Sensorless gain without cut-off ($P=7$, $Q^2=99.444\%$)(d) Sensorless gain with cut-off ($P=6$, $Q^2=99.596\%$)

Figure 7.9: Goodness of prediction for the fitted PCEs. The ideal case is the one-to-one correspondence outlined in black.

Using these metamodels, we generate the Pareto fronts reported in continuous lines in Fig. 7.10. The newly found switching point $\times'$ at which one topology becomes better than the other falls close to the estimate using density plots.

For the sake of validating the methodology only, we perform the same optimizations using the real models. These reference Pareto fronts are reported in the same figure and validate the good location of the point $\times'$. After reevaluating the metamodel Pareto fronts using the real models, we can also highlight the ability of the metamodels to find the optimal solutions at higher mean torques. Nonetheless, we also observe notable differences at higher gain g_m on the solutions found by the metamodels and the models. Despite the favorable conditions of having a low number of variables and an overall good quality of the fits, these results show the metamodel is not always a good substitute for finding the optimal design of the motor.

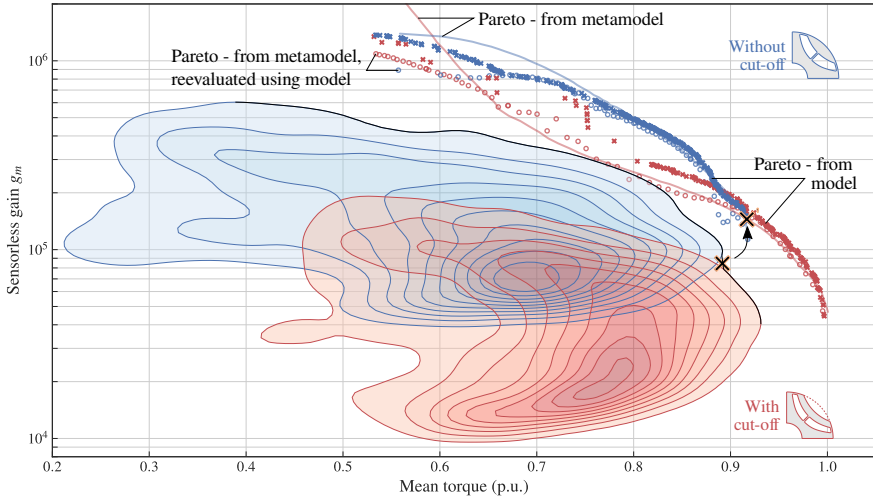


Figure 7.10: Density plots of the mean torque and the sensorless gain, with the Pareto fronts obtained by performing optimizations.

To conclude this section, we have been able to compare rotors having a single inner flux barrier with and without a cut-off barrier. We showed that the selection of the optimal topology depends on a trade-off between considered objectives and we cannot conclude that one topology outperforms the other overall. The selection of the adequate topology for a given problem can be made by comparing density plots, reusing the same dataset that is used to perform a sensitivity analysis. A better refinement in the selection of the appropriate topology can be obtained by performing inexpensive metamodel-based optimizations, with the metamodel trained on the same dataset. This comparison methodology allows to determine which topology should be optimized without requiring costly model-based optimization for each topology. Moreover, the unbiased exploration of the search space provides a good reusability of the results for other application case.

In particular, it was found that, for rotors having a single inner flux barrier, the cut-off barrier tends to increase the mean torque, the power factor, and to lower iron losses. Conversely rotors without cut-off barrier tends to have lower von Mises stresses, lower torque ripple, and higher sensorless capability.

7.3.2 Using four inner flux barriers

The 1D histograms of the objectives of interest are reported in Fig. 7.11. We also include the previous results for a single inner flux barrier to enable the comparison between four and one inner flux barriers. Because of the high difference in sample count between the two scenarios, these histograms are normalized so that their integral is one.

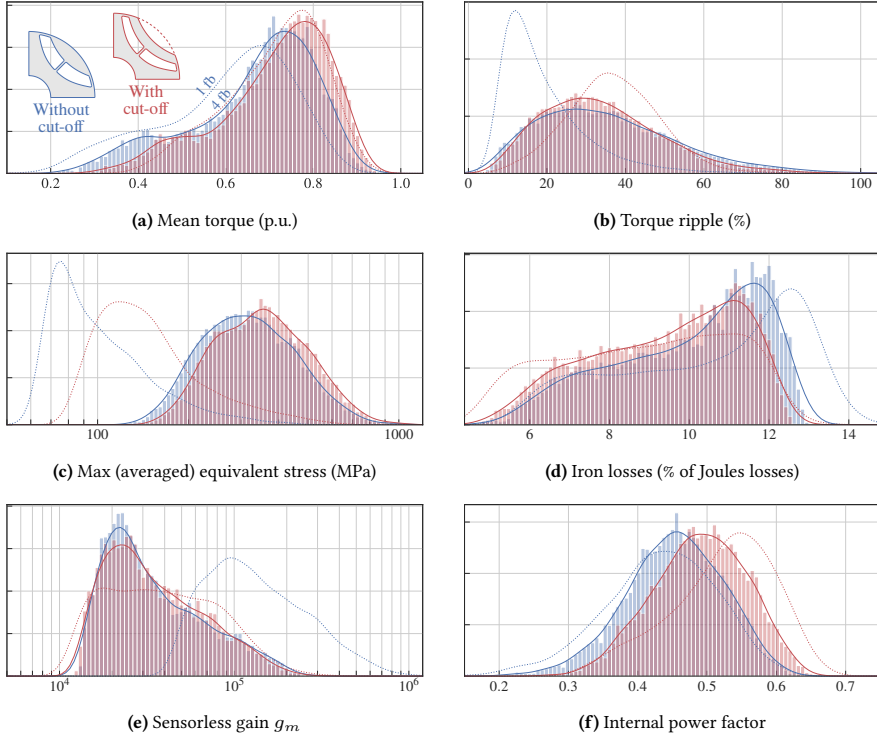


Figure 7.11: Normalized number of occurrences of each objective, for rotors with and without cut-off barrier. The dashed lines correspond to the results for rotors having a single inner flux barrier, and the continuous lines to rotors having four inner flux barriers.

For topologies with four inner flux barriers, the presence of a cut-off barrier tends to slightly increase the internal power factor and the mean torque, despite the air-to-rotor ratio ξ having the same variation range for both topologies. This implies that redistributing the flux barriers with one at the periphery of the rotor can be beneficial, although the gain is more pronounced for the internal power factor than for the mean torque. Rotors without a cut-off barrier have a slight tendency to have higher iron losses, which can be attributed to more peripheral material in which slot harmonics induce eddy currents, as seen in Fig. 2.6. By carving a cut-off barrier, we might have amputated a portion of the iron losses. Besides, there are no significant differences in the other objectives.

When comparing one and four inner flux barriers, we arrive at the typical conclusions that a single flux barrier results in slightly lower mean torque than four, attributed to circulating inductances discussed in chapter 3. However, topologies with a single inner flux barriers exhibit significantly reduced equivalent stress despite rotating at the same speed. Lastly, we observe that the topology with a single inner flux barrier and without cut-off is significantly more suitable for sensorless control at our operating point. In applications where the signal-to-noise ratio of the current sensors is low, such topology is better indicated.

7.4 Summary

This chapter systematically addresses three core objectives: the formulation of a metric for the motor's suitability for sensorless control, establishing a methodology free from extensive optimizations, and conducting a comprehensive comparison between rotors with and without a cut-off barrier, featuring either one or four inner flux barriers.

Following the presentation of methods pertinent to the sensorless control of SynRM, we questioned the choice of aligning the injection frame with the control frame (dq) during square-wave injection. We demonstrated that, in the presence of cross-saturation, co-aligning the injection frame with the control frame induces an error $\tilde{\theta}^*$ on the position used for the control. This introduced error angle can be effectively compensated in software by decoupling the injection frame from the control frame. Consequently, we proceeded to derive the mathematical expression for the injection angle that maximizes the current response, crucial for detecting small errors in the estimated position. Notably, we discovered that this angle coincides with $\tilde{\theta}^*$, at which the current response is proportional to a gain g_m , calculated using the incremental inductances of the motor.

The proposed comparison methodology involves a comprehensive exploration of the design space and extraction of valuable information from evaluations. Leveraging histograms or density plots, this methodology proves effective in quickly comparing rotor topologies for various objectives. The identification of the switching point, denoting when one topology surpasses another, can be approximated more precisely through cost-effective metamodel-based optimizations.

Significantly, the analysis uncovers that rotor configurations with a cut-off barrier generally exhibit higher mean torque and lower iron losses, whereas those without tend to have slightly lower equivalent stresses, reduced torque ripple, and enhanced sensorless capability. The recommendation of opting for a lower number of flux barriers is particularly pertinent in applications where structural integrity is a concern or in sensorless control scenarios with a low signal-to-noise ratio in current sensors.

We also caution of a potential dependency of the comparison outcome on the operating point. For instance, it is conceivable that, at zero currents, a topology featuring a cut-off barrier may be more suitable for sensorless control than topologies without, owing to the absence of saturation in tangential ribs. To address this, the same methodology could be applied after extending the databases to new operating points, providing a more nuanced understanding of the comparative performance across varying conditions.

Conclusion

8

8.1 Content overview

In this study, we addressed the intricate process of designing the rotor for a synchronous reluctance motor. To overcome the challenge of high optimization times, we devised an approach grounded in the outcomes of a sensitivity analysis. After describing the numerical models to evaluate the quantities of interest in chapter 2, the pivotal insights of each chapter are summarized below:

- **Chapter 3** explored various flux barrier shapes and parametrizations, introducing a strategy to solve the non-overlapping constraints between flux barriers. This approach involves parametrizing individually the thickness of each flux guide and flux barrier and performing a simple rescaling operation to fit the rotor yoke. This parametrization is analogous to using virtual springs between points that cannot meet, and proved highly effective in optimization convergence. By resolving the non-overlapping constraints, the design space was rendered rectangular which was absolutely crucial to perform the subsequent sensitivity analysis. Building on the concept of virtual springs, a second parametrization was developed, explicitly incorporating the magnetic insulation ratio ξ as a design parameter. This alternative parametrization aligned more closely with the works of R. R. Moghaddam.
- **Chapter 4** introduced the fundamental metrics employed in sensitivity analysis. It featured an illustrative interpretation of Sobol' indices using the Ishigami test function. Acknowledging the challenges in their interpretation, the chapter proposed a visual representation, combining a pie chart for first-order indices and a chord diagram for the rest. The presented results encompassed mean torque, torque ripple, iron losses, and equivalent stress for the two parametrizations. Throughout sample evaluations, numerous other relevant quantities were extracted and stored for potential use in future studies. We highlighted the non-negligible impact of the tangential ribs on all the considered objectives, with radial ribs significantly influencing equivalent stress. Also, we emphasized the importance of the magnetic insulation ratio ξ on both the mean torque and the iron losses. Relative torque ripple and maximum equivalent stresses, however, exhibited sensitivity to the remaining parameters.

- **Chapter 5** delved into the challenges of deriving time-efficient design approaches for synchronous reluctance motors, addressing the intricacies of balancing magnetic optimization with structural integrity considerations. The considered approaches optimize flux barriers for mean torque and torque ripple, and ribs for structural integrity. Proposing two formulations for the magnetic optimization problem and three for the mechanical optimization problem, the chapter benchmarked all possible combinations in terms of solution quality and total optimization time using the first parametrization. While these approaches effectively reduced total optimization time and achieved high mean torque, they faced challenges in consistently limiting torque ripple across independent runs of the same approach. The best-performing approach was further tested using the second parametrization, yielding similar observations. The inconsistent torque ripple was attributed to a lack of mechanisms within the mechanical optimization to control changes made to the tangential ribs, as indicated by the results of the sensitivity analysis.
- **Chapter 6** introduced a methodology to derive time-efficient approaches for the design of synchronous reluctance motors, exclusively leveraging the results of the sensitivity analysis. This methodology offers improved consideration of parameter interactions and makes more effective use of parametrization. When applied to the second parametrization, the approach demonstrated exceptional effectiveness, providing high-quality solutions, significantly reducing total optimization time, and ensuring excellent reproducibility of results. On the other hand, when applied to the first parametrization, the methodology revealed a scheme similar to that explored in chapter 5. The key advantage of this methodology lies in its ability to derive approaches systematically, outperforming all investigated methods in chapter 5 for this particular parametrization.
- **Chapter 7** introduced a methodology for comparing different rotor topologies, utilizing the same database as the one employed for sensitivity analysis. This chapter provided an alternative application of the sensitivity analysis database, adding further justification for conducting such analyses. The comparison involved rotors with and without cut-off barriers, featuring one and four inner flux barriers. Multiple objectives were considered, including mean torque, torque ripple, equivalent stress, iron losses, internal power factor, and sensorless capability of the motor. The methods related to sensorless control were reviewed to derive a metric associated to sensorless capability. Through this methodology, the comparison was executed swiftly, avoiding resource-intensive evaluations. The findings indicated that, for rotors with four flux barriers, the inclusion of a cut-off barrier holds limited advantages, except for enhancing the internal power factor. A lower number of flux barriers was deemed beneficial for applications prioritizing structural integrity or sensorless capability.

8.2 Original contributions

The contributions stemming from this thesis span both the domain of synchronous reluctance motors and a methodological perspective. These contributions will be

systematically explored in the subsequent subsections 8.2.1 and 8.2.2.

In addition to these scientific contributions, we introduce the Python framework developed to interface the tools utilized in this thesis, which includes sensitivity analysis, optimization algorithms, and data visualization. This framework, named *optimeed* [128], is an open-source module designed as an API. Its applicability extends beyond the realm of electric motors, making it suitable for a variety of problems. The development of *optimeed* prioritizes efficient utilization of available CPU cores, minimal RAM and disk usage, and real-time monitoring of ongoing computations. Interactive graphs and automatic archiving of results in human-readable format are other key features, facilitating the reusability and interpretation of the results. The maturity and stability of *optimeed* are demonstrated through results presented in this thesis, obtained from extensive optimizations spanning weeks and large-scale sensitivity analyses deployed on clusters.

8.2.1 Analysis of synchronous reluctance motors

Parametrization and time-efficient optimization approaches

Through the application of the virtual-spring technique, we have introduced a parametrization for the SynRM rotor that effectively addresses non-overlapping constraints among flux barriers while maintaining a rectangular design space. This parametrization inherently accommodates the inclusion of additional flux barriers or a cut-off barrier, exhibits an improved optimization convergence rate, and aligns seamlessly with common sensitivity analysis methods. In chapter 3, we presented a second parametrization focusing on the air-to-rotor ratio, drawing inspiration from foundational works spanning decades while retaining the benefits associated with a rectangular design space. Leveraging sensitivity analysis, we identified this parameter as a key contributor not only to mean torque but also to iron losses.

We discussed the time-efficient optimization approaches that separate the magnetic optimization problem from the mechanical one. By appropriately parallelizing the optimization algorithm, we conducted a comprehensive benchmark of various implementations against the reference solution obtained through a singular optimization across the entire search space. Although we achieved a significant reduction in total optimization time, our analysis underscored the challenges in consistently maintaining a low torque ripple following mechanical optimization. Sensitivity analysis revealed that this issue could be attributed to changes in the sizing of tangential ribs resulting from the mechanical optimization. While pre-dimensioning radial ribs within the magnetic optimization offered some improvement, as suggested in the literature, our recommendation is to consider pre-dimensioning the tangential ribs instead.

Through a comprehensive analysis of sensitivity results, the sequential approach developed in chapter 6 demonstrated exceptional time efficiency in finding solutions that exhibit both high mean torque and low torque ripple while meeting structural integrity constraints. Specifically, approach $\mathcal{P}$ involves a stepwise process. First, $\mathcal{P}_1$ focuses on maximizing mean torque by adjusting the air-to-rotor ratio (ξ) and

current angle (θ_e) at minimal rib thickness. Subsequently, $\mathcal{P}_2$ ensures structural integrity by minimizing changes to tangential ribs. Torque ripple is then minimized in $\mathcal{P}_3$ through redistribution of air and iron between flux barriers and flux guides. Finally, a re-evaluation of structural integrity is performed in $\mathcal{P}_2$ to accommodate these changes. This approach significantly reduces total optimization time to two-fifths of the reference optimization time, concurrently achieving a more than 50% reduction in torque ripple from the reference solution. Partial convergence of $\mathcal{P}_3$ can be employed as a stopping criterion to further reduce optimization time, albeit with a less impressive decrease in torque ripple. It is noteworthy that, in these approaches, the optimization time is now primarily attributed to mechanical optimization rather than magnetic optimization.

Moreover, we can draw a parallel between approach $\mathcal{P}$ and $\mathcal{P}^*$ and the approach proposed by R. R. Moghaddam in his pioneering work [32]. In his approach, the initial step involves maximizing mean torque by adjusting the air-to-rotor ratio ξ . Subsequently, torque ripple is minimized by determining the optimal rotor slot pitch in the d axis, while maintaining ξ constant. This closely aligns with design approach $\mathcal{P}$ when structural integrity considerations are excluded, resulting to the sequence $\mathcal{P}_1 \rightarrow \mathcal{P}_3$. While the methodology to derive design approach $\mathcal{P}$ differs from that of R. R. Moghaddam, employing sensitivity analysis instead of physical reasoning on the machine, it mirrors R. R. Moghaddam's approach for the magnetic objectives.

A significant departure from previous studies lies in our consideration of both torque ripple and structural integrity as target constraints in the design approach. Unlike in prior studies, these two outputs exhibit substantial cross-coupling due to alterations in rotor geometry. Consequently, developing satisfactory time-efficient design approaches by independently addressing mechanical objectives, through the ribs, and magnetic objectives, through the flux barriers, proves challenging. By placing sensitivity analysis at the core of the study, we managed to identify suitable optimization sequences and penalization procedures to mitigate the effects of this cross-coupling. Hence, sensitivity analyses have been instrumental in developing time-efficient design approaches that effectively balance both mechanical and magnetic objectives.

Sensorless capabilities and design metric

After presenting an overview of methods related to the sensorless control of SynRM, we questioned the choice of aligning the injection frame with the control frame ($\widehat{dq}$) during square-wave injection. While this alignment is logical for IPMSM, where injecting in the d axis mitigates torque ripple generation, the same does not apply to SynRM.

We demonstrated mathematically that, in the presence of cross-saturation, co-aligning the injection frame with the control frame induces an error $\tilde{\theta}^*$ on the position used for the control. This error depends solely on machine parameters. By dissociating the control frame from the injection frame, this error can be compensated in software for the control loop, given that the machine parameters are known.

Building on this foundation, we derived mathematically the injection angle that would maximize the current response for detecting a small error in the estimated position. Interestingly, we discovered that this angle coincides with $\tilde{\theta}^*$. At this angle, the current response is proportional to the error angle, characteristics of the injection signal, and a gain g_m associated with the design of the motor. This gain g_m , calculated using incremental inductance, depends on the operating point of the motor.

Comparative study between topologies

Our final contribution involved an extensive comparison of rotors with and without cut-off barrier, with a single inner flux barrier or four. These four distinct geometries underwent a comprehensive comparison across multiple objectives, including torque ripple, mean torque, internal power factor, structural integrity, iron losses, and suitability for sensorless control.

Based on our selected operating point, the results revealed that the topology without a cut-off barrier and with a single flux barrier demonstrates advantages in both sensorless control and structural integrity. In the case of a topology featuring a single inner flux barrier and a cut-off barrier, the internal power factor surpassed that of configurations with four flux barriers. However, the increased number of flux barriers remains advantageous to output higher mean torque.

Furthermore, our study demonstrated that the optimal topology can depend on the trade-off chosen between objectives. This was illustrated in the case of a single inner flux barrier, particularly in the bi-objective problem where the aim was to maximize both the motor's suitability for sensorless control and the mean torque.

8.2.2 Development of methodological frameworks

By showcasing the versatility and reusability of the database generated for sensitivity analysis, whether for the sensitivity analysis itself, metamodel training, topology comparisons, or finding time-efficient optimization approaches, we aim to encourage engineers to embrace this valuable tool for addressing the design challenges of electrical machines. Indeed, as discussed in the introductory chapter, constructing this database differs in nature from optimization in terms of computing power, and we anticipate that design approaches relying on parallelization will become increasingly favored with advancements in computational science. That large array of applications, as summarized in Fig. 8.1, has been demonstrated throughout this work and will be briefly reviewed in this section.

Assessing the efficacy of a parametrization

Sensitivity analyses serve as an essential tool for quantifying the impact of parameter changes on a target output. Emphasizing the identification of primary influencing factors, we have demonstrated the interpretation of Sobol indices as a means to assess the effectiveness of a parametrization, offering both qualitative understanding

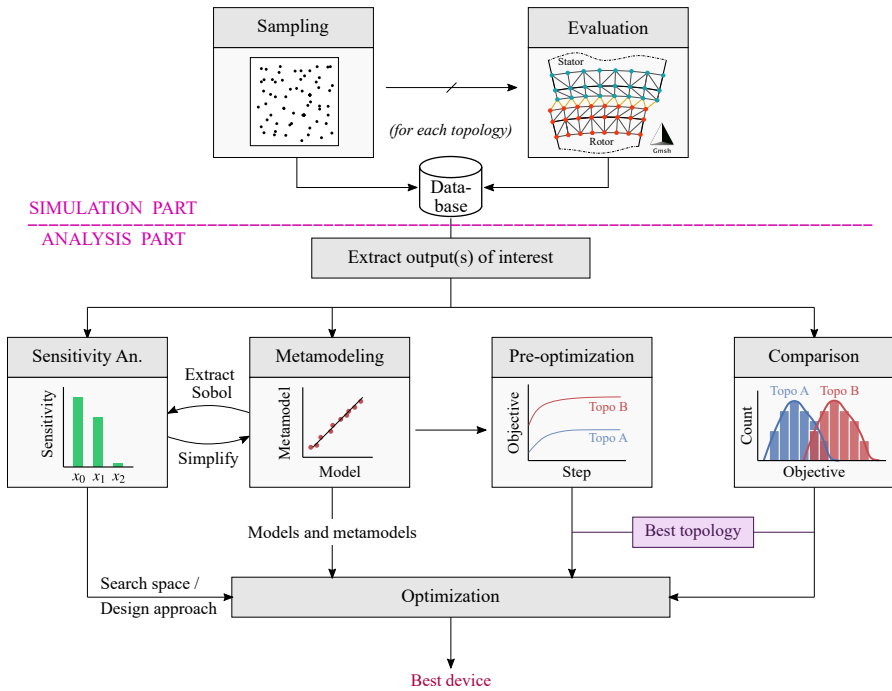


Figure 8.1: Leverage a computationally intensive database collected by quasi-random sampling of the search space.

and quantitative measures. The graphical representation of Sobol indices simplifies their interpretation, providing profound insights into the physical behavior of the machine.

Developing time-efficient design approaches

Drawing on Sobol indices, we have illustrated the seamless interpretation of these results to derive time-efficient design approaches. This methodology, characterized by the exact quantification of parameter interactions on specified objectives, surpasses intuition-based design approaches. By relying on tangible metrics, the success and quality of solutions derived through these design approaches are no longer based on postulations about parameter interactions.

Metamodel training for model substitution

While not fully explored in this work, we demonstrated the potential of using the database samples as a dataset for training a metamodel, eventually simplified by sensitivity analysis. We showcased how this metamodel can be utilized to analytically derive Sobol indices, providing a departure from the convergence limitations of estimators. Additionally, it can serve as a surrogate model within the optimization loop, effectively reducing optimization time. This practice aligns with prevalent approaches in electric motor design [129–131].

Comparing topologies without resource-intensive optimizations

We have demonstrated that the identification of optimal topologies can be swiftly executed without resorting to resource-intensive optimizations. By harnessing the database compiled for each device topology, we can compare them across multiple objectives and trade-offs without relying on computationally expensive optimization processes. This approach holds greater generality compared to optimization, which often focuses on specific applications and trade-offs. For enhanced confidence in topology selection, cost-effective metamodel optimizations remain a viable option. The binary nature of the selection means that precise metamodeling is not as crucial as it would be for optimization tasks aimed at finding the single best solution.

8.3 Discussion on the construction of the database

While we have showcased the extensive reusability of the database, a cornerstone of our work, the question arises about what precisely is stored within it. Logically, one would consider storing data that are time-consuming to evaluate, such as the solutions of finite-elements in our case, encompassing both magnetic and mechanical aspects. Storing these solutions ensures the complete reusability of results with only minor post-processing. However, this approach involves handling a colossal amount of data that may surpass the limits of typical hard disk storage capacities. In response to this challenge, we opted to store post-processing quantities, a decision not devoid of complexity. Defining relevant quantities for post-processing is non-trivial, given the inherently open-ended nature of potential future applications.

During the construction of the database, we anticipated the possible utility of iron losses at different supply frequencies. Consequently, we performed the post-processing of the magnetic model to extract iron losses over a wide frequency range, storing this information in the database. The computational cost of this post-processing step is negligible compared to the magnetic model evaluation. Extending the database to include different supply current amplitudes, the computation of these iron losses might prove valuable to draw efficiency maps.

However, there are aspects where our foresight fell short. Notably, we missed the opportunity to extract and store the magnetic force density at each resolution of the magnetic model, as we did not foresee the potential interest in future analyses that would include acoustic assessments. Evaluating the magnetic forces is crucial for assessing stator vibrations and, consequently, the noise of the motor.

The situation is less critical for the mechanical model, given its lower evaluation time. Reevaluating the entire database to include an additional output of interest, as we did during sensitivity analyses to evaluate the maximum equivalent stress averaged over an element, is a more manageable task in this context.

8.4 Perspectives

8.4.1 Analysis of synchronous reluctance motors

A logical next step involves the exploration of other operating points and expanding the search space to include the stator. This expansion would require a similar approach to parametrize the entire motor, incorporating factors like the ratio between the outer radius of the rotor and the outer diameter of the stator. Other objectives could cautiously be considered with the inclusion of the degrees of freedom of the stator, such as thermal aspects and acoustic noise.

Furthermore, we have demonstrated that mechanical optimization of the rotor is systematically dissociated from magnetic optimization in time-efficient design approaches. In the realm of these optimizations, there exists an opportunity to explore more specialized algorithms, such as topology optimization. This approach has the potential not only to significantly reduce mechanical optimization time but also to provide avenues for better stress distribution in the rotor. Leveraging topology optimization, it may be possible to minimize stress concentrations at the tangential ribs without resizing them. By excluding the domain of the tangential ribs from this optimization, we anticipate preserving the previously optimized magnetic performance.

Finally, there is potential to manage the cross-coupling between the magnetic and mechanical models more elegantly than the constant safety factor employed in this thesis. For instance, leveraging the database, the increase in local stresses due to magnetic forces could have been estimated using a metamodel and dynamically accounted for in the safety factor. By tightening the safety factor, we would logically observe performance gain on the magnetic objectives.

8.4.2 Extending the usage of the database

In the pursuit of developing time-efficient design approaches, optimizing the settings of the optimizer can bring a better trade-off between convergence rate and solution quality. Traditionally, this involves fine-tuning the optimizer settings through multiple iterations of the optimization problem, which can be impractical with high evaluation times. However, employing metamodels trained on the database offers an alternative for tuning the optimizer, even if it is not deemed accurate enough to directly solve the optimization problem.

Other potential usage of the database involves finding the main contributing factors of a target objective through a combination of design parameters. For instance, using the database of the first parametrization, we can recover the magnetic insulation ratio by simply combining the parameters:

$$\xi(K_j) = \frac{\sum_{j \text{ odd}} K_j}{\sum K_j}. \quad (8.1)$$

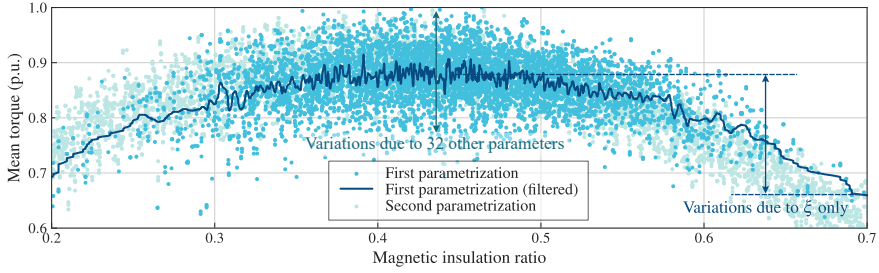


Figure 8.2: Evolution of the mean torque with respect to the magnetic insulation ratio ξ , using the databases collected for sensitivity analyses.

To establish a correlation between $\xi(K_j)$ and the mean torque $\bar{T}$, one possibility consists in representing these functions in a XY chart, as performed in Fig. 8.2. After ordering the points of the first parametrization, a Gaussian filter is applied to enable a visual interpretation of the curve. Based on this result, we would infer a strong (non-linear) correlation between the mean torque and the magnetic insulation ratio. Subsequently, we might adjust the parametrization to align with the second parametrization, also presented on the same graph.

Finally, our analyses has primarily utilized straightforward visual tools, like histograms and density plots. The examination of such extensive databases falls within the realm of big data analytics. Consequently, more advanced statistical and deep learning tools could be applied to leverage the existing data more effectively.

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Appendix

Space vector transformations

A

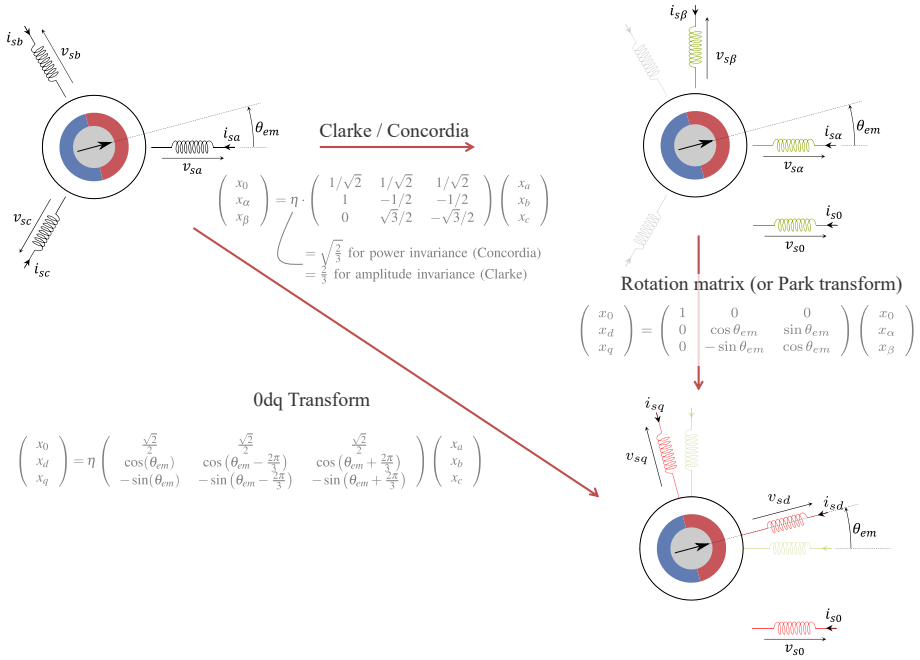


Figure A.1: Space vector transformations.

The summary of space vector transformations employed in this thesis is provided in Fig. A.1, and described in the next sections.

A.1 (abc) to $(\alpha\beta)$ transform

Assuming a balanced system, the space vector $\mathbf{x}_{abc} = (x_a, x_b, x_c)$ can be expressed in a 2D coordinate system using space vector transformations. This facilitates the interpretation and representation of the variables. The stationary reference frame $(\alpha\beta)$ is obtained using the Clarke transform, which is amplitude invariant. We have:

$$\mathbf{x}_{\alpha\beta} = [\mathbf{T}_{\alpha\beta}] \mathbf{x}_{abc} \quad (\text{A.1})$$

$$\begin{bmatrix} x_\alpha \\ x_\beta \end{bmatrix} = \frac{2}{3} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} x_a \\ x_b \\ x_c \end{bmatrix}. \quad (\text{A.2})$$

A.2 $(\alpha\beta)$ to (dq) transform

Starting from the $(\alpha\beta)$ coordinate system, the components of the vectors are expressed in the (dq) coordinate system by performing a passive (alias) coordinate system rotation. For a counter-clockwise rotation of an angle θ_{em} , the vector components in the new coordinate system are:

$$\mathbf{x}_{dq} = [\mathbf{R}(\theta_{\text{em}})] \mathbf{x}_{\alpha\beta} \quad (\text{A.3})$$

$$\begin{bmatrix} x_d \\ x_q \end{bmatrix} = \begin{bmatrix} \cos \theta_{\text{em}} & \sin \theta_{\text{em}} \\ -\sin \theta_{\text{em}} & \cos \theta_{\text{em}} \end{bmatrix} \begin{bmatrix} x_\alpha \\ x_\beta \end{bmatrix}. \quad (\text{A.4})$$

Iron losses

B

The multitude of iron loss models outlined in the literature can pose a challenge in selecting the most appropriate one without a profound understanding of the physical mechanisms behind these losses and the distinctions between the models. The next sections are dedicated to addressing these aspects.

B.1 Physical origin

The physical mechanisms governing iron losses are attributed to eddy currents occurring at various scales. On the macroscopic scale, a varying magnetic field induces eddy currents driven by the conductivity of the material. This component, known as classical eddy current losses, prevails at higher frequencies. Conversely, the hysteresis and excess losses occur at the microscopic scale and are tied to the magnetization process of ferromagnetic materials.

These materials can be subdivided in uniformly magnetized regions called magnetic domains. The boundaries between two adjacent domains are marked by a tiny region, the domain wall, where the magnetic moment of the atoms are gradually reoriented. The presence of these regions is a consequence of a delicate balance between various types of energies, all contributing to the internal energy W_{in} :

$$W_{in} = W_{ms} + W_{an} + W_{ex} + W_{me}. \quad (\text{B.1})$$

- The magnetostatic energy, W_{ms} , is the potential energy of the external magnetic field generated by the material. It encourages configurations that result in no net magnetic moment, with magnetic domains aligned parallel to the boundaries of the sample.
- The magnetocrystalline anisotropy energy, W_{an} , is due to the existence of preferred magnetization directions inside the crystal lattice. In the absence of external magnetic field, the magnetic moments align themselves along the easy axes of magnetization.
- The exchange energy, W_{ex} , is a consequence of the magnetic coupling between adjacent atoms. This energy, with the magnetocrystalline energy, are responsible of the thickness of the walls.

- The magnetoelastic energy, W_{me} , accounts for the elastic strain induced by magnetostriction. It promotes the formation of smaller magnetic domains.

When a ferromagnetic material is placed in an external magnetic field, the domains closely aligned with the external field expand in size at the expense of the other domains. Within a perfect crystal, the displacement of the domain walls is smooth and reversible, with the domains reverting to their initial configuration once the excitation field is removed. The large-scale, continuous movement of the domain walls is dampened by eddy currents. The losses associated to these currents, called excess losses, can be analytically computed using Maxwell's equations. This involves simplifying assumptions regarding the placement and movement of the domain walls [132].

The domain walls cannot move as freely if the crystal contains impurities such as cavities or elements of a different type. These impurities introduce a magnetostatic energy that contributes to the internal energy of the crystal. The energy minimization results in the formation of closure domains that cling to the imperfection, representing a local energy minimum. These closure domains abruptly snap off when the applied field gets strong enough, leading to a sudden, discontinuous change in the magnetic flux at the microscopic level. This phenomenon, known as the pinning effect, explains the Barkhausen jumps observed in the $B - H$ curve of the material. The sudden change of the magnetic flux induces tiny eddy currents, giving rise to the hysteresis losses. These losses are independent to the global magnetization rate and show the irreversibility of the magnetization of the material. The large number of impurities and the dynamic behaviour inherently make these losses challenging to model.

B.2 Literature overview

The conventional setup to measure the magnetic properties of the ferromagnetic materials consists in applying an external H field through a coil and measuring the resulting B field through an other coil. Manufacturers provide the iron losses resulting from unidirectional H field that varies sinusoidally over time, for different amplitudes and frequencies. The availability of these data drives the historical development of the models used to predict the iron losses. An overview of the models that will be discussed in the next paragraphs is provided in Fig. B.1.

The Steinmetz and the loss separation models belong to the class of post-processing models. They neglect the impact iron losses have on the settling point of the $B - H$ curve, so that the losses can be computed based on the knowledge of the B field. In this case, the magnetic field is obtained by solving Maxwell's equations using the anhysteretic $B - H$ magnetization curve. This hypothesis is extensively adopted in the literature due to its capacity to provide a reasonably accurate approximation of the iron losses through a simple and lightweight model. For more accurate predictions, a representation of the hysteretic $B - H$ curve provided by the hysteresis models can be used as constitutive law in the Maxwell's equations in (2.9).

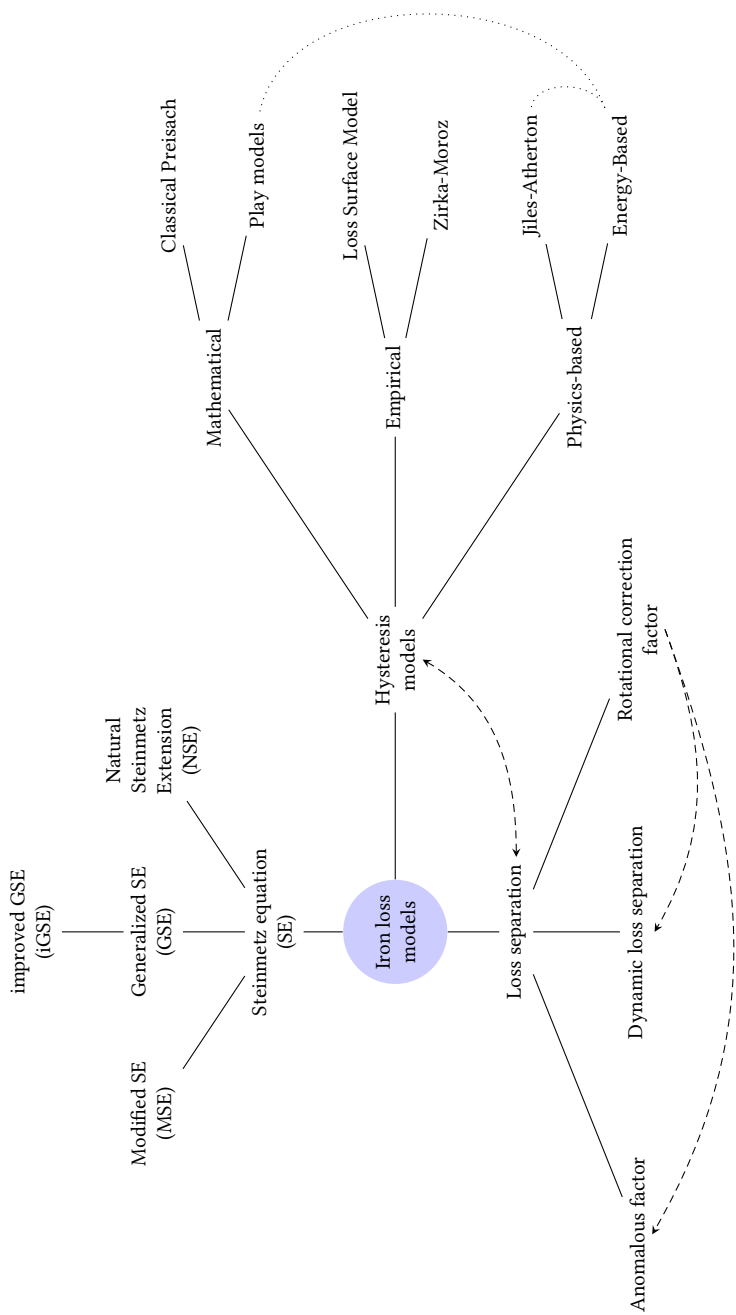


Figure B.1: Overview of the existing models used to predict the iron losses.

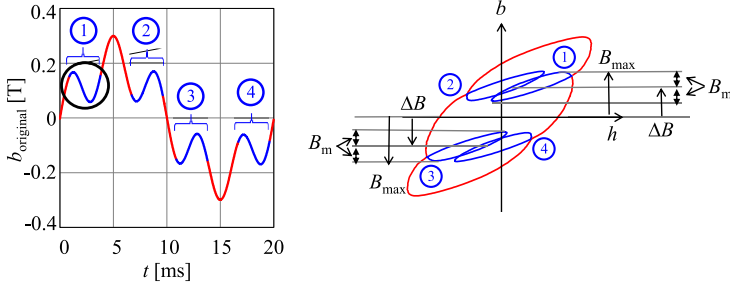


Figure B.2: Minor loops due to flux reversal [137].

Steinmetz models

One of the earliest model to evaluate these losses was proposed by Steinmetz in 1892, and consists in fitting an analytical function on the measurements:

$$p_{Fe} = C_{SE} f^\alpha \hat{B}^\beta. \quad (\text{B.2})$$

In this equation, p_{Fe} is the iron losses per unit volume, $\hat{B}$ the peak amplitude of the sinusoidal magnetic field with f its frequency, and C_{SE} , α , β the fitted Steinmetz coefficients. This equation has later been extended to better predict the iron losses resulting from non-sinusoidal magnetic fields. The Modified Steinmetz Equation (MSE) [133] uses the averaged magnetization rate dM/dt as the frequency term in (B.2). In the Generalized Steinmetz Equation (GSE) [134], the iron losses are recovered by integrating the instantaneous losses expressed as a time function of the field and its rate dB/dt :

$$p_{Fe} = \frac{1}{T} \int_0^T C_{SE} \left| \frac{dB}{dt} \right|^\alpha |B(t)|^{\beta-\alpha} dt. \quad (\text{B.3})$$

The Natural Steinmetz Extension (NSE) [135] complies with (B.3) and is especially suited for non-sinusoidal waveforms with fast edges, when there is no reversal of the flux. An improved version of the GSE model (iGSE) is proposed in [136] to better handle the minor loops due to the flux reversal, as illustrated in Fig. B.2.

These modifications of the original Steinmetz Equation provide better accuracy, but they all lack the physical basis to perform accurate prediction of the iron losses outside the identification range.

Loss separation models

A second group of models decomposes the iron losses from their physical origins. First proposed by Jordan in 1924 [138], the iron losses were decomposed into two components, the static hysteresis losses, p_h , and the classical eddy current losses, p_{cl} . This model was refined by Pry and Bean to account for the excess losses through the multiplication of p_{cl} by a constant factor η_a [132]:

$$p_{Fe} = p_{hyst} + \eta_a \cdot p_{cl} \quad (\text{B.4})$$

$$= k_h f \hat{B}^2 + \eta_a \cdot k_{cl} (f \hat{B})^2 \quad \text{for a sinusoidal field} \quad (\text{B.5})$$

With k_h and k_{cl} , two material-dependent coefficients. This model fails to predict the non-linear increasing of the losses per cycle with respect to the frequency, that were observed at intermediate frequencies in FeSi materials. From his statistical interpretation of the iron losses, Bertotti could identify the excess losses per cycle to be proportional to $\sqrt{f}$ instead of f [139]. This led to the modern and widely popular decomposition of the iron losses:

$$p_{Fe} = p_h + p_{cl} + p_{exc} \quad (\text{B.6})$$

$$= k_h f \hat{B}^2 + k_{cl} (f \hat{B})^2 + k_{exc} (f \hat{B})^{1.5} \quad \text{for a sinusoidal field.} \quad (\text{B.7})$$

The weaker effect of the excess losses, p_{exc} , is due to irregularities in the wall motion such as domain nucleation or wall bowing [140]. The coefficient k_{exc} can be derived analytically using the micro-structural properties of the material, and the coefficient k_{cl} from the conductivity and thickness of the laminations [139]. An other possibility consists in fitting the three coefficients k_h , k_{cl} and k_{exc} on the available measurements.

For non-sinusoidal fields, the dynamic loss are recovered either by summing up the contribution of each frequency component or by integrating the instantaneous losses over a time period. The terms p_{cl} and p_{exc} in (B.6) become:

$$p_{cl} = k_{cl} \sum (f_i \hat{B}_i)^2 = \frac{k_{cl}}{2\pi^2 T} \int_0^T \left| \frac{dB}{dt} \right|^2 dt \quad (\text{B.8})$$

$$p_{exc} = k_{exc} \sum (f_i \hat{B}_i)^{1.5} = \frac{k_{exc}}{8.76 T} \int_0^T \left| \frac{dB}{dt} \right|^{1.5} dt. \quad (\text{B.9})$$

The harmonic content of the flux only increases the hysteresis losses if they lead to multiple peaks in the flux density waveform, so that less hysteresis losses would be generated with a square waveform than with its fundamental component only. As a result, summing up the contribution of each frequency component by disregarding their phase would lead to an overestimation of the hysteresis losses. Using the $B-H$ characteristic of the material, the extra losses due to the flux reversals correspond to the total area enclosed by the minor loops. These can be estimated using hysteresis models, or with simpler analytical formulae. In [141, 142], the following formula was applied:

$$p_h = k_h f \hat{B}^2 \left(1 + \frac{0.65}{\hat{B}} \sum_{i=1}^n \Delta B_i \right), \quad (\text{B.10})$$

with ΔB_i being the amplitudes of the flux reversals. In [54, 143, 144], the magnetic field H was decomposed in a reversible and irreversible part, H_{rev} and H_{irr} , the later one being approximated by an ellipse:

$$p_h = \frac{1}{T} \int_0^T \left| (H_{rev} + H_{irr}) \frac{dB}{dt} \right| dt = \frac{1}{T} \int_0^T \left| H_{irr} \frac{dB}{dt} \right| dt \quad (\text{B.11})$$

$$H_{irr}(B, \hat{B}) = \frac{k_h \cdot \hat{B}}{\pi} \cdot \cos \left(\arcsin \left(\frac{B}{\hat{B}} \right) \right).$$

Adaptations to the above formulae can be made to account for 2D fields, also known as rotating fields, which denote magnetic fields evolving in two spatial directions. These fields result from the superposition of two orthogonal alternative fields. In the context of purely sinusoidal alternative fields, the locus of the 2D field is an ellipse characterized by major and minor axes denoted as $B_{\parallel}$ and $B_{\perp}$. The assessment of iron losses can be calculated based on this elliptical representation [55–57]:

$$p_{Fe}^{tot} = p_h(B_{\parallel}) \cdot \gamma_1(B_{\perp}/B_{\parallel}) + p_{cl}(B_{\parallel}) + p_{cl}(B_{\perp}) + p_{exc}(B_{\parallel}) \cdot \gamma_2(B_{\perp}/B_{\parallel}), \quad (B.12)$$

where γ_1 and γ_2 are two functions to account for the physics of iron losses in rotating fields [59, 145]. For non-sinusoidal 2D fields, the contribution of each frequency component is estimated using the above formula. When this decomposition is not directly available, the 2D norm of the rate of change in flux density is injected in equations (B.8) and (B.9):

$$p_{cl} = \frac{k_{cl}}{2\pi^2 T} \int_0^T \left(\left| \frac{dB_x}{dt} \right|^2 + \left| \frac{dB_y}{dt} \right|^2 \right) dt \quad (B.13)$$

$$p_{exc} = \frac{k_{exc}}{8.76 T} \int_0^T \left(\left| \frac{dB_x}{dt} \right|^2 + \left| \frac{dB_y}{dt} \right|^2 \right)^{0.75} dt. \quad (B.14)$$

For the hysteresis losses, the formula (B.11) can be applied for each direction, using the peak amplitude of the 2D norm of the flux.

Conversely to (B.12), these adaptations do not account for the magnetization process in rotating fields. Nonetheless, they are frequently applied to estimate the losses in electrical machines.

Hysteresis models

The hysteresis models aim at providing the constitutive law applicable in (2.8) or (2.9). These models can be categorized in three different groups based on their approach to provide this representation:

- **Mathematical models:** These models describe the hysteresis through system identification, which is a general approach applicable to any physical system exhibiting hysteresis. Renowned examples include classical Preisach [146] and Play models [147], which decompose the overall hysteresis curve into smaller elementary entities called hysterons. The accuracy of these models can be adjusted by manipulating the number of hysterons.
- **Empirical models:** These models directly leverage experimental data with minimal processing. Notable examples include the Loss Surface [148, 149] and the Zirka-Moroz [150–152] models. The Loss Surface model utilizes measurements from triangular induction-controlled signals as a lookup-table to predict $H(B, dB/dt)$. Zirka and Moroz reconstruct the minor loops by congruency from available measurements on the static major loop and first-order reversal curves. Drawbacks of these models are the loss of accuracy when extrapolating data outside of the measurement range, and the increased storage requirements.

- **Physics-based models:** These models describe the magnetization process from an understanding of the underlying phenomena. The widely used Jiles-Atherton model [153] depicts the pinning effect using a dry friction force and decomposes the total magnetization into a reversible and an irreversible components. Energy-based models [154–156] build on the simplicity of Jiles-Atherton model but consider the magnetization process as the contribution of a large number of pseudo-particles. The state of these pseudo-particles is determined by locally minimizing the Landau free energy. Similar to Preisach models, the precision of energy-based models can be adjusted by the number of pseudo-particles. This class of models is expected to provide better predictions of the iron losses for magnetic excitations that differ from the ones used to identify the parameters of the model.

Hysteresis models, in general, offer superior accuracy in estimating the losses compared to loss-separation or Steinmetz models. Nevertheless, their implementation complexity, the need for extensive characterization of the material, and the associated numerical evaluation time remain their primary drawbacks.

Results of the optimizations

C

The tables below, Table C.1 and Table C.2, compile the values of the parameters at the conclusion of each optimization outlined in this study.

Table C.1: Values of the parameters at the end of the optimization, for the first parametrization

Opti.	θ_e °	K_0 /	K_1 /	K_2 /	K_3 /	K_4 /	K_5 /	K_6 /	K_7 /	K_8 /	t_1^T mm	t_2^T mm	t_3^T mm	t_4^T mm	t_1^R mm	t_2^R mm	t_3^R mm	t_4^R mm
Ⓔ	64.91	5.79	6.85	3.45	7.25	8.14	4.77	4.77	2.62	4.05	2.92	2.04	1.46	0.62	3.04	2.96	1.23	0.50
AI #1	64.64	8.18	7.21	7.31	4.17	6.41	7.15	9.99	7.16	6.49	4.23	2.95	2.68	1.70	3.37	2.79	1.93	0.50
AI #2	65.04	5.45	7.35	7.32	3.42	4.45	3.12	7.27	6.61	8.15	4.75	6.67	4.40	3.49	3.10	2.77	2.15	0.53
AI #3	65.24	1.14	3.78	5.05	2.92	2.77	3.74	6.01	3.88	6.92	6.85	5.45	6.35	6.65	4.16	2.50	1.61	4.88
AI #1	64.64	8.18	7.21	7.31	4.17	6.41	7.15	9.99	7.16	6.49	4.08	1.96	2.30	0.49	4.02	3.08	2.16	0.50
AI #2	65.04	5.45	7.35	7.32	3.42	4.45	3.12	7.27	6.61	8.15	4.25	2.70	3.54	1.54	3.43	2.83	2.16	0.67
AI #3	65.24	1.14	3.78	5.05	2.92	2.77	3.74	6.01	3.88	6.92	7.35	5.15	5.29	4.87	2.05	1.51	1.10	0.00
AI #1	64.64	8.18	7.21	7.31	4.17	6.41	7.15	9.99	7.16	6.49	3.36	1.99	2.18	0.44	3.35	2.94	1.84	0.50
AI #2	65.04	5.45	7.35	7.32	3.42	4.45	3.12	7.27	6.61	8.15	4.74	3.21	4.17	1.89	3.05	2.76	2.49	0.60
AI #3	65.24	1.14	3.78	5.05	2.92	2.77	3.74	6.01	3.88	6.92	6.89	4.65	5.08	4.61	2.64	1.51	1.21	0.00
AI-AI #1	65.90	1.72	8.10	7.02	5.26	9.03	7.72	9.37	5.73	8.34	5.65	4.03	3.71	3.21	3.34	2.18	1.24	0.00
AI-AI #2	65.81	1.47	6.42	3.91	1.78	6.48	6.23	6.50	5.05	8.49	6.80	7.44	5.32	4.89	2.02	2.55	0.98	0.00
AI-AI #3	66.60	1.00	3.72	4.15	4.78	5.23	3.82	4.45	3.10	7.03	6.24	4.74	5.11	5.12	4.04	2.16	0.97	0.00
AI-AI #1	65.18	1.00	2.37	2.36	4.81	6.96	5.18	7.50	4.12	4.50	10.58	9.07	7.73	5.41	1.62	1.23	0.00	0.57
AI-AI #2	65.10	1.19	6.13	7.34	5.68	4.86	2.16	6.10	6.61	7.88	6.42	4.50	5.91	5.01	2.30	1.42	1.01	0.00
AI-AI #3	65.39	1.00	3.15	4.30	5.94	5.98	3.84	5.38	3.97	5.13	8.11	6.65	6.29	5.71	1.62	1.12	0.00	0.50
AI-AI #1	65.69	1.00	5.10	5.81	5.82	7.32	5.07	7.87	4.89	5.65	7.60	6.36	6.25	5.51	2.03	1.31	0.00	0.00
AI-AI #2	66.12	1.00	6.33	8.33	7.90	9.03	6.62	9.16	5.44	6.77	7.32	5.83	5.98	5.37	2.07	1.01	0.00	0.00
AI-AI #3	65.00	1.00	5.03	7.34	6.14	3.80	3.00	8.38	8.31	9.74	7.92	4.91	6.00	4.43	4.64	1.39	1.20	0.00
BI #1	65.32	2.95	6.30	6.78	4.40	4.64	3.73	5.59	6.16	7.76	9.00	3.67	3.80	7.27	4.76	4.39	1.93	1.75
BI #2	65.19	6.59	4.38	8.74	5.30	3.57	8.05	8.74	4.23	8.52	6.49	3.43	3.01	4.20	2.67	1.76	1.29	0.00
BI #3	65.14	9.03	8.48	6.26	7.89	4.87	3.33	7.29	6.97	9.30	3.62	5.98	2.86	1.59	3.34	2.93	2.63	0.66
BI #1	65.32	2.95	6.30	6.78	4.40	4.64	3.73	5.59	6.16	7.76	4.53	2.49	3.49	1.41	3.51	3.65	2.14	0.67
BI #2	65.19	6.59	4.38	8.74	5.30	3.57	8.05	8.74	4.23	8.52	5.89	2.58	2.66	2.08	3.73	2.76	1.89	0.78
BI #3	65.14	9.03	8.48	6.26	7.89	4.87	3.33	7.29	6.97	9.30	3.43	1.86	2.59	1.31	3.16	2.73	2.01	0.76
BI #1	65.32	2.95	6.30	6.78	4.40	4.64	3.73	5.59	6.16	7.76	4.35	2.70	2.88	1.94	2.93	2.24	1.70	0.59
BI #2	65.19	6.59	4.38	8.74	5.30	3.57	8.05	8.74	4.23	8.52	5.63	2.28	2.39	1.17	3.80	2.32	1.97	0.66
BI #3	65.14	9.03	8.48	6.26	7.89	4.87	3.33	7.29	6.97	9.30	3.56	2.06	2.84	0.62	3.03	2.77	2.88	0.74
BI-BI #1	63.96	1.64	1.00	4.75	9.94	8.13	5.36	1.00	1.85	7.65	14.64	10.06	8.33	14.96	13.51	1.18	0.00	11.37
BI-BI #2	65.31	3.07	4.54	5.96	8.74	8.75	5.66	6.57	6.32	6.82	6.17	3.88	3.22	2.45	2.46	1.86	0.92	0.00
BI-BI #3	65.41	4.74	9.46	3.88	1.00	6.54	7.73	8.65	6.57	8.73	5.67	5.23	4.49	3.57	2.47	4.31	1.15	0.00
BI-BI #1	65.05	3.64	7.83	8.46	7.08	6.78	3.50	5.68	4.25	2.71	4.03	2.59	2.39	1.10	3.09	2.03	0.81	0.00
BI-BI #2	64.51	5.60	4.83	4.39	10.0	5.77	8.00	7.52	6.50	3.08	1.26	1.49	0.48	4.38	3.45	2.08	0.52	
BI-BI #3	64.37	5.53	4.51	1.06	7.93	9.97	5.53	6.85	5.53	5.65	3.07	2.53	2.00	0.76	3.71	4.11	1.75	0.51
BI-BI #1	65.36	3.51	6.19	7.48	8.73	9.36	6.50	9.47	8.58	7.92	5.04	3.29	3.88	2.94	2.76	1.98	1.08	0.00
BI-BI #2	64.92	5.07	3.18	1.23	8.72	9.93	7.02	7.61	5.13	5.55	4.15	3.24	2.86	2.87	3.34	2.61	1.18	0.00
BI-BI #3	64.99	4.30	6.74	4.51	5.70	9.03	7.41	9.59	6.67	6.30	4.62	3.34	2.95	1.96	2.50	2.30	1.14	0.00
VAV #1	65.17	2.54	1.01	1.45	9.10	9.97	6.98	9.38	5.62	6.56	7.45	5.89	7.14	4.37	4.80	1.87	0.96	0.00
VAV #2	66.26	1.09	5.38	7.41	7.41	8.56	5.90	8.37	5.86	6.98	6.27	4.75	4.76	5.16	4.74	2.79	0.00	0.99
VAV #3	65.70	1.06	5.92	5.01	3.04	7.34	7.05	8.94	5.44	6.45	6.95	6.05	5.27	5.63	4.97	2.62	0.00	0.72
VBV #1	65.77	2.54	3.87	4.54	6.13	7.12	5.13	6.42	5.01	5.30	5.38	2.65	4.00	1.09	2.77	2.38	1.38	0.58
VBV #2	65.61	3.67	7.08	4.09	3.83	6.78	6.24	8.34	5.63	5.76	3.05	1.82	2.01	0.37	4.64	3.99	1.92	1.13
VBV #3	65.27	4.93	9.99	8.97	5.69	3.83	1.08	4.35	5.79	6.60	2.12	1.46	1.49	0.30	5.00	3.95	2.70	1.34

Table C.2: Values of the parameters at the end of the optimization, for the second parametrization

Opti.	ξ %	θ_e mm	K_1^{fg} /	K_2^{fg} /	K_3^{fg} /	K_4^{fg} /	K_1^{fb} /	K_2^{fb} /	K_3^{fb} /	t_1^T mm	t_2^T mm	t_3^T mm	t_4^T mm	t_1^R mm	t_2^R mm	t_3^R mm	t_4^R mm
Ⓔ	42.65	65.58	1.02	1.38	1.08	0.73	1.84	1.53	0.51	4.30	2.71	2.99	1.87	2.30	1.38	0.57	0.17
BI #1	39.83	66.04	0.65	1.05	1.08	1.13	0.87	1.17	0.98	6.56	4.80	4.70	5.04	1.44	0.97	0.03	0.41
BI #2	40.65	65.89	0.66	0.57	1.30	1.30	1.05	0.51	1.15	6.78	5.92	5.18	5.36	1.35	1.17	0.02	0.03
BI #3	40.70	66.44	0.92	1.23	1.35	1.56	1.42	1.08	1.46	5.48	3.56	4.59	1.97	2.23	1.75	0.90	0.01
BI-BI #1	42.52	65.26	0.50	0.88	1.05	0.80	0.65	1.18	0.91	6.20	4.21	4.03	5.18	1.52	1.14	0.04	0.01
BI-BI #2	42.67	65.51	0.65	0.69	0.90	0.81	0.71	0.87	0.80	4.55	2.87	3.05	2.05	2.06	1.71	0.82	0.01
BI-BI #3	42.54	64.97	0.78	0.80	0.88	1.13	0.85	0.71	0.87	4.34	2.72	3.06	1.79	2.14	1.76	1.02	0.00
Ⓕ #1	38.77	65.60	1.72	1.81	1.22	0.98	1.52	1.58	1.13	3.04	1.85	1.78	0.34	4.03	2.76	1.68	0.71
Ⓕ #2	38.84	65.57	1.95	1.25	1.38	0.99	1.90	1.60	1.37	2.85	1.82	1.75	0.33	4.77	2.93	1.72	0.56
Ⓕ #3	38.84	65.57	1.94	1.38	1.45	0.59	1.40	1.80	1.71	2.91	1.88	1.87	0.43	3.85	2.67	1.83	0.73
Ⓕ* #1	38.77	65.60	1.36	0.84	0.94	1.99	1.94	1.39	1.35	3.17	1.96	2.27	0.40	4.19	2.89	1.91	0.71
Ⓕ* #2	38.84	65.57	1.26	0.74	0.89	1.59	0.77	0.70	0.90	2.86	1.82	1.98	0.33	4.77	3.98	1.96	0.56
Ⓕ* #3	38.84	65.57	1.28	0.99	1.12	1.52	0.67	0.64	0.82	2.91	1.88	1.88	0.37	3.85	2.81	1.83	0.72

Opti.	f_1^1 mm	f_2^1 mm	f_3^1 mm	f_4^1 mm	f_1^2 mm	f_2^2 mm	f_3^2 mm	f_4^2 mm	f_1^3 mm	f_2^3 mm	f_3^3 mm	f_4^3 mm	f_1^4 mm	f_2^4 mm	f_3^4 mm	f_4^4 mm	$\bar{T}$ p.u.	$\hat{T}$ %	
Ⓐ	3.03	2.78	1.38	1.00	3.64	2.27	1.07	9.22	8.49	4.15	3.61	1.26	1.00	5.25	7.33	7.11	1.00	10.00	
AI #1	2.55	4.59	1.84	1.53	2.87	4.99	1.46	1.52	4.87	1.05	3.78	5.41	1.80	2.68	4.89	5.16	0.96	10.00	
AI #2	4.83	3.85	1.44	8.62	8.78	3.12	1.29	2.35	5.24	6.88	1.12	4.73	1.00	4.43	3.41	1.22	0.91	9.99	
AI #3	8.01	8.72	5.75	8.21	1.28	2.56	7.71	5.29	6.94	8.18	8.58	8.65	1.00	1.13	6.31	1.19	0.86	9.99	
AI# #1	2.20	7.68	1.86	1.00	2.39	5.00	1.47	1.00	6.20	1.35	2.16	1.05	1.00	7.64	9.55	1.14	0.99	21.71	
AI# #2	3.37	2.43	9.15	1.05	3.92	2.14	4.56	1.00	3.34	1.48	5.72	1.00	1.01	3.95	2.51	8.26	0.97	14.53	
AI# #3	9.07	9.65	8.03	1.62	6.34	5.71	9.53	7.14	4.82	4.84	1.01	1.00	1.00	7.10	7.30	6.67	0.92	14.43	
AI#-AI# #1	4.78	2.98	3.96	1.35	5.33	2.90	3.71	1.66	3.19	2.84	1.00	1.00	1.00	5.56	5.62	6.02	0.99	26.10	
AI# #2	9.03	3.35	2.69	2.03	8.95	3.82	6.23	1.87	6.92	8.40	1.00	5.52	4.49	1.80	2.38	9.20	0.95	21.58	
AI# #3	6.57	5.51	4.98	5.53	2.96	5.24	8.53	7.94	8.23	5.25	9.50	1.90	2.05	4.22	8.10	4.74	0.93	21.44	
AI-AI #1	9.25	6.42	1.00	1.76	1.86	4.61	1.00	3.72	6.60	6.51	1.03	1.01	1.01	2.83	4.97	4.24	0.96	7.70	
AI-AI #2	8.67	7.59	4.42	6.59	5.61	2.71	8.57	2.78	5.10	2.89	4.99	7.93	1.31	4.12	7.74	6.18	0.91	9.77	
AI-AI #3	7.16	9.11	4.36	6.78	1.00	2.45	6.17	3.98	8.06	4.10	4.75	6.04	1.03	5.99	5.27	6.35	0.92	10.00	
AI-AI# #1	4.26	3.38	2.78	1.00	1.83	2.86	1.06	1.05	2.66	2.91	1.80	1.00	1.00	5.95	1.00	6.45	0.88	15.64	
AI-AI# #2	7.34	2.99	3.25	2.38	2.52	8.92	5.75	1.37	3.27	3.26	7.86	6.74	1.04	5.18	6.03	3.41	0.92	24.55	
AI-AI# #3	2.08	1.51	1.07	1.00	1.45	1.32	1.18	1.00	3.50	1.87	1.34	1.00	1.01	1.03	1.08	4.02	0.92	3.52	
AI-AI#-AI# #1	7.66	6.70	3.41	8.34	1.79	9.02	1.16	8.19	8.71	4.38	4.09	7.80	1.08	7.87	5.35	5.76	0.90	8.58	
AI#-AI# #2	9.42	5.13	5.32	1.70	1.81	6.57	4.41	2.85	3.52	5.29	3.04	4.11	1.00	4.76	9.08	7.74	0.91	14.77	
AI#-AI# #3	7.45	7.04	7.43	3.55	1.16	4.68	8.64	5.44	7.22	6.49	7.54	1.89	2.06	6.15	6.37	4.74	0.92	12.29	
BI #1	9.37	5.96	2.60	6.29	3.24	5.00	6.35	9.84	6.19	6.70	7.88	7.29	8.23	2.11	7.88	8.66	0.84	14.07	
BI #2	4.18	1.77	2.72	4.58	5.67	1.43	1.81	7.48	7.70	1.38	1.00	2.71	1.07	1.92	6.16	7.65	0.93	9.98	
BI #3	2.72	3.25	8.97	5.40	2.27	2.11	3.29	8.38	7.48	6.64	1.00	9.93	1.01	3.54	5.81	4.52	0.94	9.99	
BI# #1	5.06	1.26	1.58	1.26	3.05	1.18	1.25	1.00	3.28	1.00	1.00	1.00	1.00	2.34	1.93	2.70	0.98	15.97	
BI# #2	3.62	7.48	2.51	7.17	2.86	2.71	2.06	1.1	6.88	2.00	1.04	2.59	2.27	4.94	4.41	1.81	0.94	17.66	
BI# #3	2.40	2.16	6.23	1.00	2.74	1.77	7.96	1.00	3.58	1.00	1.06	1.00	1.00	3.81	5.90	4.71	0.99	18.88	
BI#I #1	5.82	5.28	8.12	2.48	6.03	5.03	4.18	2.14	6.40	8.21	1.05	3.22	3.34	4.61	3.77	9.09	0.98	22.28	
BI#I #2	6.71	9.13	3.11	2.54	6.35	8.94	4.78	1.79	2.69	9.60	4.55	1.01	1.05	7.60	6.56	3.60	0.95	24.08	
BI#I #3	4.37	2.85	1.24	4.43	5.01	3.62	1.99	3.97	9.29	2.94	2.89	1.44	5.16	2.22	2.14	5.16	0.98	16.17	
BI-BI #1	3.75	1.12	6.97	8.14	3.12	1.02	3.75	1.68	6.51	9.56	2.90	1.65	5.66	1.39	8.90	6.31	0.78	6.49	
BI-BI #2	3.86	7.34	1.00	3.93	3.54	6.20	1.00	6.08	3.46	1.43	1.08	1.95	5.57	6.88	4.25	6.78	0.97	9.82	
BI-BI #3	3.06	5.27	2.41	2.98	1.83	4.31	1.85	1.00	4.88	6.53	1.00	2.19	1.10	4.33	2.81	8.27	0.95	9.80	
BI#-BI# #1	3.18	1.85	1.00	1.43	3.12	1.60	1.00	1.01	5.52	1.00	1.05	1.00	1.00	3.90	4.35	1.00	1.00	6.68	
BI#-BI# #2	4.89	8.38	1.43	1.26	3.19	3.46	1.22	1.07	6.97	1.00	1.00	1.00	1.01	1.09	7.29	1.14	1.01	17.31	
BI#-BI# #3	3.80	8.17	3.90	1.61	3.13	2.67	6.50	5.94	2.77	1.00	7.50	1.00	6.28	1.22	6.98	2.21	0.99	17.85	
BI#I-BI#I #1	4.55	7.07	5.69	1.27	2.58	7.13	5.63	1.02	5.10	2.86	7.46	6.50	1.00	4.25	4.83	3.01	0.96	17.79	
BI#I-BI#I #2	5.48	6.70	3.47	2.16	4.44	9.64	6.86	3.99	9.05	5.60	7.43	8.35	5.18	7.43	7.65	3.80	0.96	13.20	
BI#I-BI#I #3	8.10	6.58	3.10	5.29	9.19	5.69	3.09	1.45	5.89	7.42	2.15	1.13	1.41	9.47	5.83	3.31	0.99	8.89	
VAV #1	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	0.91	17.24
VAV #2	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	0.92	7.12
VAV #3	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	0.90	16.52
VBV #1	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	0.98	8.15
VBV #2	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	1.00	9.96
VBV #3	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	1.00	8.63

[illegible]

Sensitivity analyses for a single inner flux barrier

D

The sensitivity analyses for a single inner flux barriers, without and with a cut-off barriers, are provided in Fig. D.1 and Fig. D.2. The search space is defined in Table 7.1. The following observations can be made:

- **Mean torque (a), internal power factor (c), and iron losses (d):** These objectives are primarily influenced by the air-to-rotor ratio ξ . For these outputs, the thickness of the tangential ribs becomes the second most influential factor. The presence of the cut-off barrier, represented by the parameter K_2^{fb} , does not significantly impact these objectives. There are minimal parameter interactions.
- **Relative torque ripple (b) without cut-off barrier:** The torque ripple is significantly affected by the tangential ribs, with the air-to-rotor ratio being the second contributing factor. The analysis suggests three-order interactions, possibly involving the air-to-rotor ratio ξ , the tangential ribs t_1^T , and the repartition of iron K_1^{fg} . Despite these interactions, their impact remains small.
- **Relative torque ripple (b) with cut-off barrier:** The torque ripple is mainly influenced by interactions between the air-to-rotor ratio ξ , repartition of iron K_1^{fg} , and repartition of air K_1^{fb} . Three-order interactions are evident, as seen by the gaps between inner ring segments and chords in the analysis.
- **Motor's suitability for sensorless control (e):** This objective is primarily impacted by the air-to-rotor ratio ξ , current angle θ_e , and thickness of the tangential ribs t_1^T . There are significant two-order interactions, particularly between ξ and θ_e , and ξ and t_1^T . The repartition of air or iron has negligible impact, regardless of the presence of the cut-off barrier. Second-order interactions suggest that parameters ξ , θ_e , and t_1^T should be optimized simultaneously for this objective.
- **Maximum equivalent stress (f):** The stress in the rotor, averaged over a mesh element, is primarily influenced by the thickness of the radial rib t_1^R and the air-to-rotor ratio ξ . Combined actions of these factors play a significant role. The repartition of iron, represented by the parameter K_1^{fg} , impacts stress, whereas the presence of the cut-off barrier has a negligible effect, as reflected by the parameter K_2^{fb} .

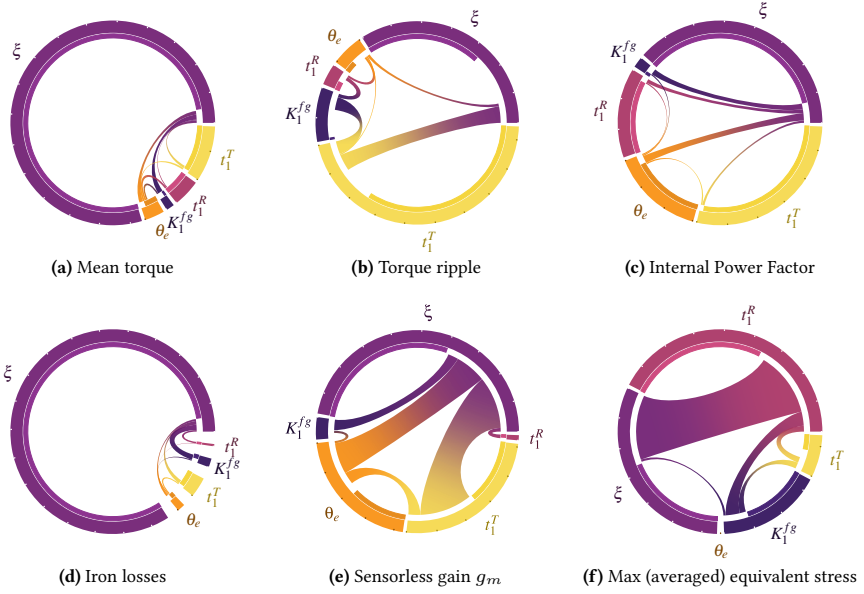


Figure D.1: Results of the sensitivity analysis, without cut-off barrier.

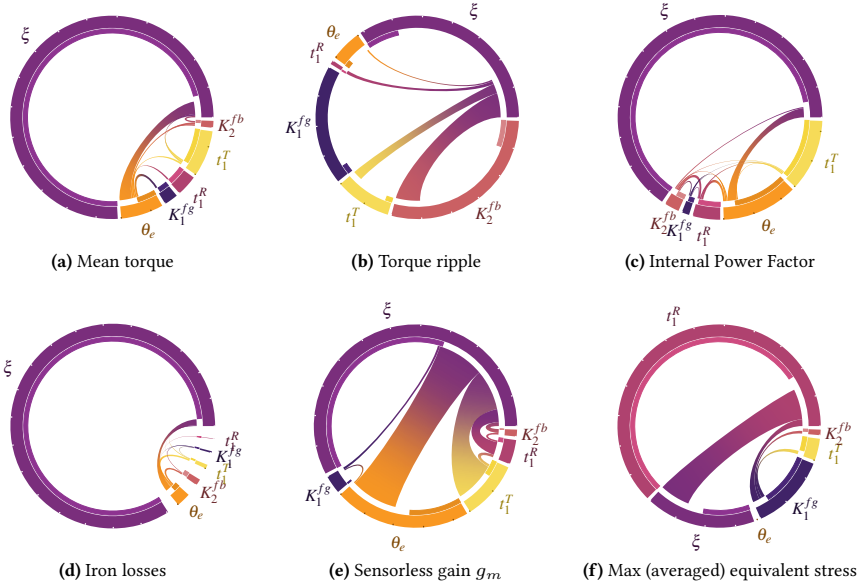


Figure D.2: Results of the sensitivity analysis, with cut-off barrier.