CoLSI:
Continuous Lippmann-Schwinger Intensity diffraction tomography

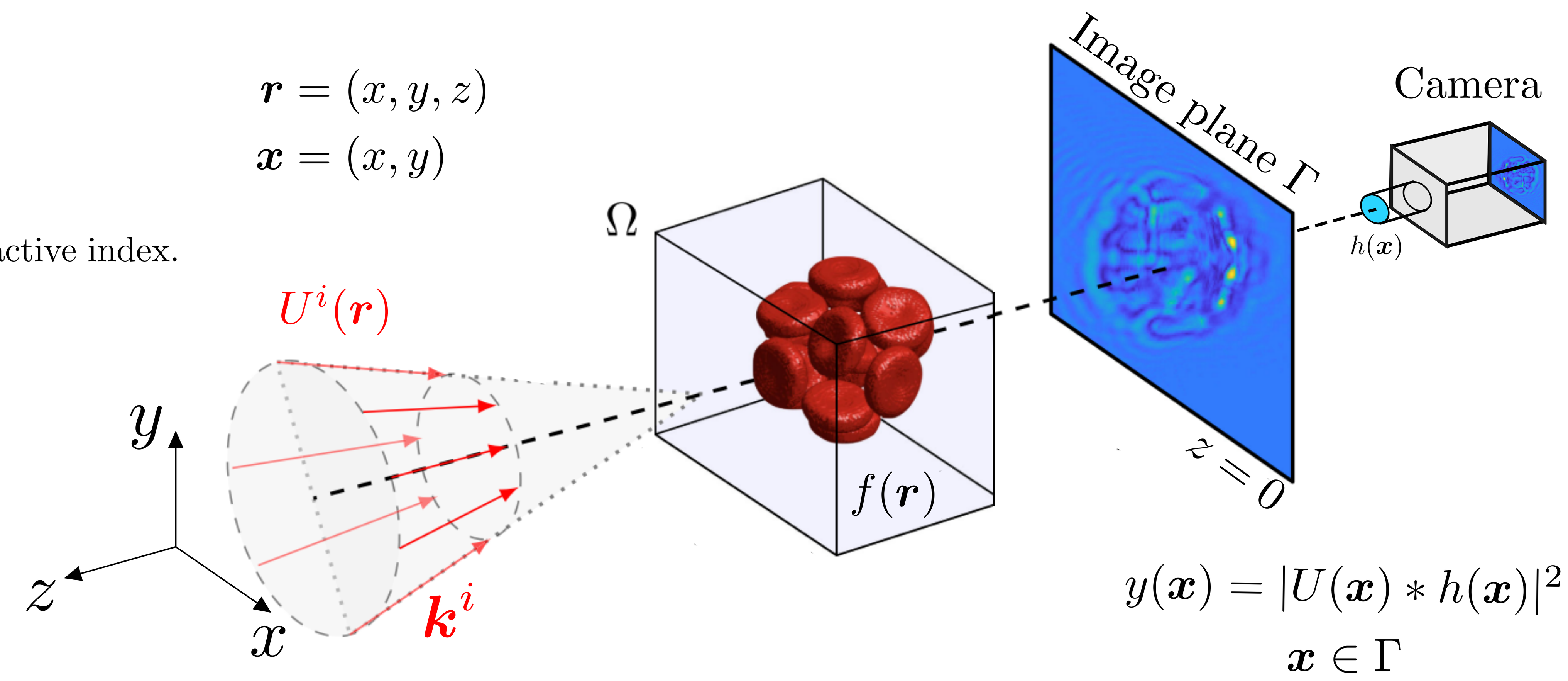
1. Context: 3D Intensity diffraction tomography

▻ **Goal:** given $U^i(\boldsymbol{r})$ and $\boldsymbol{y}$, recover $f(\boldsymbol{r})$, for $\boldsymbol{r} \in \Omega$.

- Incident planewave $U^i(\boldsymbol{r}) = e^{\mathrm{i}\boldsymbol{k}^i \cdot \boldsymbol{r}}$.
- scattering potential $f(\boldsymbol{r}) = k_\mathrm{b}^2 \left[\left(\frac{n(\boldsymbol{r})}{n_\mathrm{b}} \right)^2 - 1 \right]$ with $n(\boldsymbol{r})$ the refractive index.
- pupil of the camera: $h(\boldsymbol{x})$.

▻ Applications: morphogenesis, oncology, biochemistry.

▻ Intensity measurements:
- easy and cheap acquisition --- no interferometry setup needed.
- phase recovery problem.



2. Forward model

a. Lippmann-Schwinger equation

▻ Comes from the wave equation.

$$U(\boldsymbol{r}) = U^i(\boldsymbol{r}) + \int_\Omega U(\boldsymbol{r}')f(\boldsymbol{r}')G(\boldsymbol{r} - \boldsymbol{r}')\mathrm{d}\boldsymbol{r}'$$

▻ Intuitive: $f(\boldsymbol{r}) = 0 \longrightarrow U(\boldsymbol{r}) = U^i(\boldsymbol{r})$

▻ **Discrete form**

$$\boldsymbol{U} = \left(\boldsymbol{I} - \boldsymbol{G}\,\mathrm{diag}(\boldsymbol{f})\right)^{-1}\boldsymbol{U}^i$$

$G(\boldsymbol{r}) := \frac{e^{\mathrm{i}k_\mathrm{b}\|\boldsymbol{r}\|}}{4\pi\|\boldsymbol{r}\|} \longrightarrow$ Singularity, hard numerical computations. solution in [1].

▻ $\boldsymbol{G}$ makes the convolution with $G(\boldsymbol{r})$.

▻ solved with:
- BiConjuguate Gradient Stabilized Method.
- Neumann series $\boldsymbol{U} = \sum_{k=0}^{\infty} (\boldsymbol{G}\,\mathrm{diag}(\boldsymbol{f}))^k \boldsymbol{U}^i$

b. Intensity measurements

▻ $\boldsymbol{R}_\Gamma$ restricts $\boldsymbol{U}$ to camera plane Γ.

$$\boldsymbol{y}' = |\boldsymbol{P}\boldsymbol{R}_\Gamma\boldsymbol{U}|^2 - |\boldsymbol{P}\boldsymbol{R}_\Gamma\boldsymbol{U}^i|^2$$

▻ $\boldsymbol{P}$ applies the convolution with the pupil function $h(\boldsymbol{x})$.

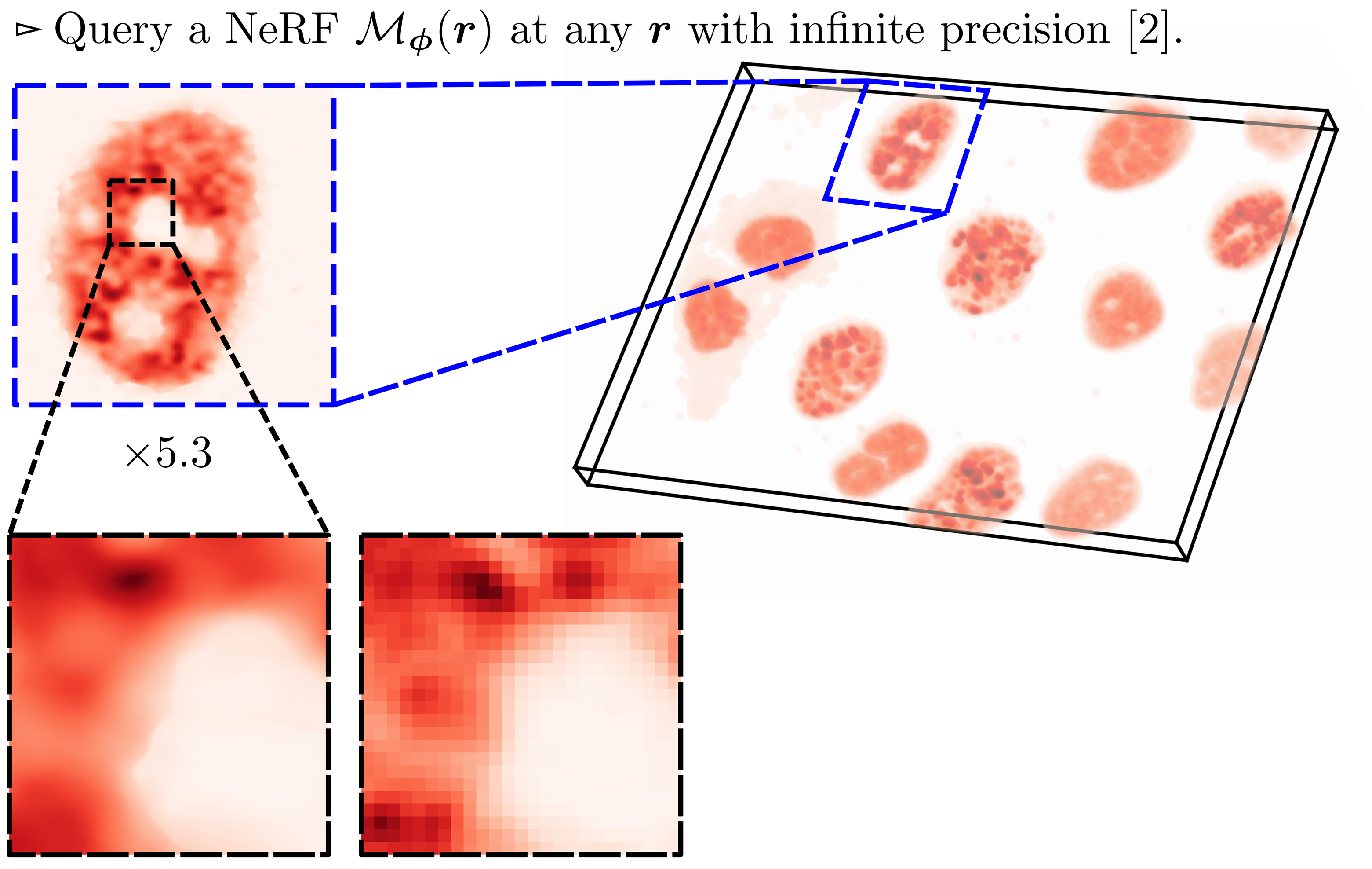
▻ remove the background intensity.

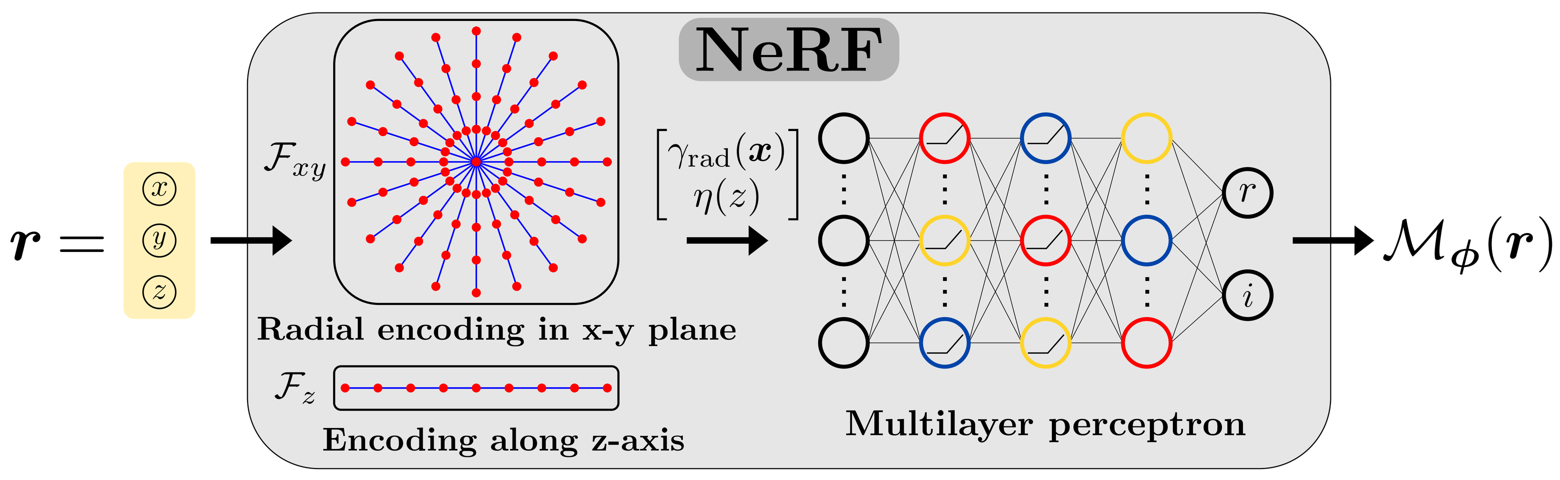
$$\boldsymbol{y}' = \mathcal{A}(\boldsymbol{f})$$

3. Inverse problem

a. Continuous representation

▻ Query a NeRF $\mathcal{M}_\phi(\boldsymbol{r})$ at any $\boldsymbol{r}$ with infinite precision [2].





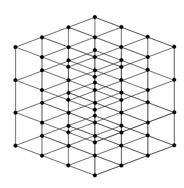
with

- $\gamma_\mathrm{rad}(\boldsymbol{x}) = \begin{pmatrix} \sin(2^0\pi\boldsymbol{R}_{\boldsymbol{\theta}}\boldsymbol{x}), \cos(2^0\pi\boldsymbol{R}_{\boldsymbol{\theta}}\boldsymbol{x}), \\ \vdots \\ \sin(2^{L-1}\pi\boldsymbol{R}_{\boldsymbol{\theta}}\boldsymbol{x}), \cos(2^{L-1}\pi\boldsymbol{R}_{\boldsymbol{\theta}}\boldsymbol{x}) \end{pmatrix}$
- $\eta(z) = \begin{pmatrix} \sin(2^0\pi z), \cos(2^0\pi z), \\ \vdots \\ \sin(2^{L_z-1}\pi z), \cos(2^{L_z-1}\pi z) \end{pmatrix}$

with $\boldsymbol{R}_{\boldsymbol{\theta}}$ a set of K rotation matrices.

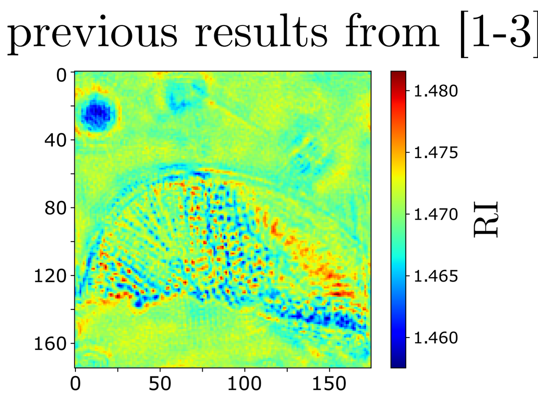
b. NeRF-parameterized inverse problem

$$\hat{\phi} = \arg\min_\phi \underbrace{\tfrac{1}{2}\|\boldsymbol{y} - \mathcal{A}(\boldsymbol{f})\|_2^2}_{\text{Data fidelity}} + \underbrace{\mathcal{R}(\boldsymbol{f})}_{\text{Regularization}} \quad \text{s.t. } \boldsymbol{f} = \{\mathcal{M}_\phi(\boldsymbol{r})\}_{\boldsymbol{r}\in\mathcal{G}}$$

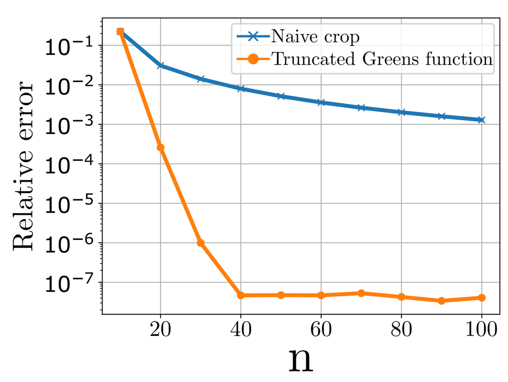
▻ Solved with automatic differentiation. ▻  Regular grid $\mathcal{G}$.

4. Numerical experiments

▻ validated previous results from [1-3].

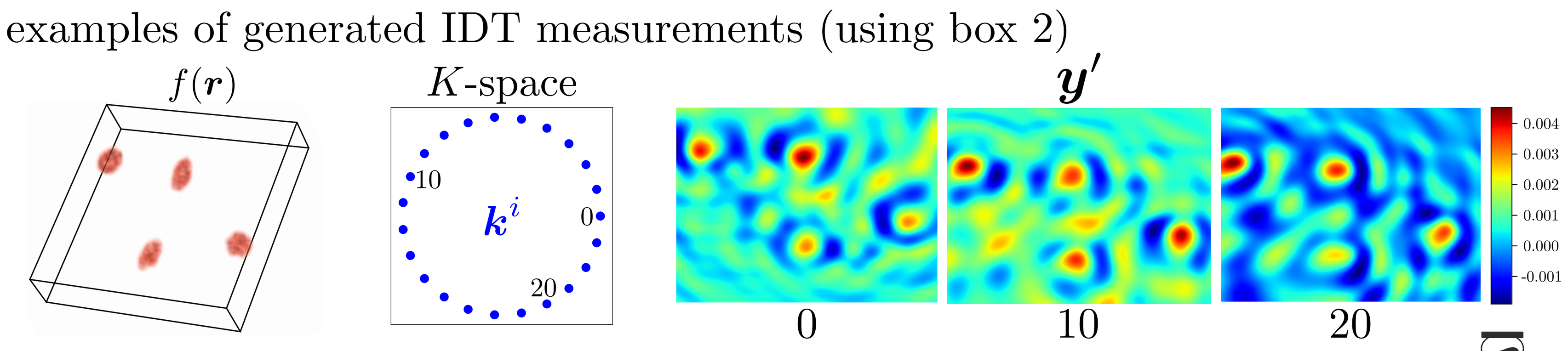


• Reconstruction with FB model and Tikhonov reg.



• Efficient discrete convolution G.

▻ examples of generated IDT measurements (using box 2)



▻ reconstructions and comparisons with First-Born model → ongoing research

References

[1] T. Pham, E. Soubies, A. Ayoub, J. Lim, D. Psaltis, and M. Unser, Three-Dimensional Optical Diffraction Tomography With Lippmann-Schwinger Model, IEEE Trans. Comput. Imaging, 6 (2020), p. 727–738

[2] R. Liu, Y. Sun, J. Zhu, L. Tian, and U. Kamilov, Recovery of Continuous 3D Refractive Index Maps from Discrete Intensity-Only Measurements using Neural Fields, Nat Mach Intell 4, (2022), p. 781–791

[3] J. Li, A. Matlock, Y. Li, Q. Chen, C. Zuo, L. Tian, High-speed in vitro intensity diffraction tomography, Adv. Photon., 1, (2019)

Conclusion

▻ Stabilized discrete physical models.

▻ Efficient continuous representation with NeRF.

▻ Upcoming reconstruction results.

▻ Perspectives
- Combine continuous representation with PINN-like architecture to get direct latent representation to discrete measurements mapping.
- Replace NeRF by COIN++ approach to avoid one NeRF per reconstruction.

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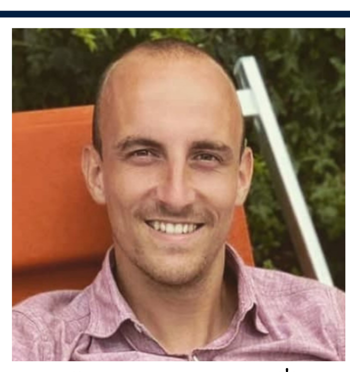

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