On A Three Way Equivalence

Leonidas C. Koutsougeras*

School of Economic Studies, University of Manchester

First draft: November 17, 1996.
This draft: January 18, 1998

Abstract

In view of the well known core equivalence results in atomless economies, coincidence of market game equilibrium allocations with competitive allocations is tantamount to a three way equivalence between market game mechanisms, competitive equilibria and the core. Based on this idea I propose an equilibrium refinement of market games which allows me to use the core equivalence machinery to provide an exact market game characterization of competitive equilibria.

*Correspondence: School of Economic Studies, University of Manchester, Rm N.5.8 Dover Street Bldg, Manchester M13 9PL, United Kingdom. tel: 44-161-275-4863, email: leon.koutsougeras@man.ac.uk

1 Introduction

In view of the fact that competitive equilibria can be approximated by equilibria of the market game mechanism, it seems that these games can provide the description of price formation that the Arrow-Debreu model lacks. This would be the case if one could establish equality between the sets of competitive (Walrasian) equilibrium allocations and equilibrium allocations resulting from market game mechanisms in ideal models of competition. Attempts in this direction have been moderately successful. In atomless economies competitive equilibria can be obtained as outcomes of a market game, but at the same time market game mechanisms support outcomes dramatically different from the competitive ones. In particular, the no trade allocation is always a market game equilibrium but it is not a competitive allocation unless the distribution of endowments is Pareto optimal.

It has been conjectured[1] that an equilibrium refinement might eliminate those unpleasant equilibria. Elsewhere it has been conjectured that some minimal amount of cooperation is needed in order to get trade started in a market game (Mas Collel [8]). In this note I attempt to combine the above conjectures with the following observations: The construction of market games is based on the concept of trading posts where individuals place their orders. However, there is no reason why we should be confined to a model where each commodity is traded in *exactly* one post. Consequently, in this note I will allow any group of agents to form a trading post and involve in trades independently from others. This idea draws inspiration from [7] where it is pointed out that the number of trading posts postulated at the outset of a market game is crucial. It was suggested there that one way out of this was to leave the number of trading posts unspecified and modify the equilibrium concept accordingly. Another observation that should guide the analysis is that the well known core equivalence results in atomless economies[2], make the equivalence of market game mechanisms and the core *necessary* in order to obtain exact equivalence with the competitive mechanism. In other words, equilibrium allocations of the market game mechanism that remain bounded away from the core cannot converge to competitive equilibria. This observation is also evident in [9], where the author discusses the limit price mechanism. Much of the purpose of this note is to stress those two messages.

Intuitively, I would like to think of agents as being free to aggregate their bids and offers in groups, independently of others. It should not be surprising that with such a formulation of trade, trivial equilibria are automatically eliminated once endowments are not Pareto optimal[3]. I think of agents as setting up posts and propose bids and offers within groups, where they can exercise some market power. This is what gets trading posts going. An equilibrium of such a process of setting up trading posts has been reached when the setup of another post has no potential benefit. Alternatively, one may think of a sustainable market game outcome as one where, disintegrating the set of agents into smaller groups and matching bids and offers within groups gives no potential advantage to some group. In other words there is no 'better' way to match

[1] Personally, I have heard this conjecture from J-F. Mertens.

[2] See [1] for a comprehensive review of core equivalence results.

[3] This is true even in economies with finite numbers of agents.

agents in the game. It turns out that, in an atomless economy, the set of undominated market game allocations coincides with the set of competitive allocations. This is the main result in this note.

Market games in atomless economies have been studied in [4] and in [3]. It is evident in those papers that even in atomless economies the equivalence is problematic. Another paper that is related to the approach taken here is [2] where the authors study exchange within groups of agents via market game mechanisms. In that paper the authors use a similar equilibrium refinement, as I do here, with the difference that they postulate that groups of agents must trade in the same trading post for each commodity. By contrast, in what follows I leave the number of trading posts within any group unspecified. In this way groups of agents are allowed to trade in subgroups, if necessary, in order to exhaust the benefits of using the market game mechanism. Furthermore, the object of study there is the structure of trading groups of agents, which is a meaningful problem in economies with a finite number of agents. In what follows it will become clear that in atomless economies the structure of trading groups is irrelevant in equilibrium, because the trading ratios within any group, as well as across trading posts for the same commodity, are equal. This is a consequence of the equivalence result that we provide here: undominated commodity allocations and the corresponding trading ratios within any group are in fact competitive.

In the section that follows I develop the model in which my equilibrium concept is embedded. Next I proceed to prove an exact equivalence result. An interpretation of the model and some discussion as well as possible extensions are discussed in section four.

2 Model

Let $(A, \mathcal{A}, \mu)$ be a measure space of agents. There are L commodity types in the economy and the consumption set of each agent is identified with $\Re^L_+$. Each individual is characterized by a preference relation, which is representable by a utility function $u_a : \Re^L_+ \to \Re$, and an initial endowment $e(a) \in \Re^L_+$. In order to be able to use some standard results we impose the following assumptions:

Assumption 2.1 *$e(a) >> 0$ ae.*

Assumption 2.2 *Preferences are continuous, strictly monotonic and indifference surfaces through the endowment do not intersect the axis.*

Let $\mathcal{P}$ denote the set of utility functions satisfying the above assumption endowed with the appropriate topology (see [11]). An economy is defined as a measurable mapping $\mathcal{E} : A \to \mathcal{P} \times \Re^L_+$. A standard definition of a competitive equilibrium for such an economy is as follows:

Definition 2.1 *A competitive equilibrium is a price system $p \in \Re^L_+$ and a measurable assignment $x : A \to \Re^L_+$ such that:*

(*i*) $\int_A x(a) \leq \int_A e(a)$

(*ii*) $x(a) \in$ *argmax* $\{u_a((x(a)) : p \cdot x(a) \leq p \cdot e(a)\}$ *ae*

Let $\mathcal{CE}(A)$ denote the set of competitive equilibria for this economy.

Remark 2.1 In what follows it will be important to recall that, due to the monotonicity of preferences, if $(p,x)\in\mathcal{CE}(A)$ then $p\in\Re^L_{++}$.

We now develop another equilibrium concept which will be based on market games. For each $T\in\mathcal{A}$ consider the measure space $(T,\mathcal{T},\nu)$ where $\mathcal{T}=\{T\cap C : C\in\mathcal{A}\}$ and $\nu(D)=\mu(D\cap T)$ for each $D\in\mathcal{A}$. Trade is organized via systems of trading posts where individuals offer commodities for sale and place bids for purchases of commodities. A possible scenario for the rules of exchange which is attributed to [12] is presented below. The interested reader should consult the original source or [13] for a detailed account of this formulation.

In this setup bids are placed in terms of a unit of account. The strategy sets of agents are described by a measurable correspondence $S: T\to 2^{\Re^L_+\times\Re^L_+}$ such that $S(a)=\left\{(b,q)\in\Re^L_+\times\Re^L_+ : q^i_T\le e^i_a,\ i=1,2,\ldots,L\right\}$. A strategy profile is a pair of measurable mappings $b:T\to\Re^L_+$ and $q:T\to\Re^L_+$ such that $(b(a),q(a))\in S(a)$ *ae*. Such mappings exist by Aumann's measurable selection theorem. Given a strategy profile let $B^i_T=\int_{a\in T}b^i(a)$ and $Q^i_T=\int_{a\in T}q^i(a)$.

Allocation assignments are determined as follows:

$$x^i_a(b^i(a),q^i(a),B^i_T,Q^i_T)=\begin{cases} e^i(a)-q^i(a)+b^i(a)\frac{Q^i_T}{B^i_T} & \text{if } B^i_T\neq 0\\ e^i(a)-q^i(a) & \text{if } B^i_T=0\\ 0 & \text{if } \sum_{i=1}^L q^i(a)\frac{B^i_T}{Q^i_T}<\sum_{i=1}^L b^i(a)\end{cases}$$

for $i=1,2,\ldots,L$. Moreover, it is postulated that if $Q^i_T=0$ then $B^i_T/Q^i_T=0$ and all bids are lost.

Consumers are viewed as solving the following problem:

$$\max\ \left\{u_a((x^i_a(b^i(a),q^i(a),B^i_T,Q^i_T))^L_{i=1})\ :\ (b(a),q(a))\in S(a)\right\} \tag{1}$$

An equilibrium for the market game is defined as a strategy profile (b,q) that forms a Nash equilibrium. For each $T\in\mathcal{A}$ the set of Nash equilibria is denoted by $\mathcal{NE}(T)$.

Notice that, due to the bankruptcy rule above, in an equilibrium with positive bids and offers individuals can be viewed as solving the following problem:

$$\max\ \left\{u_a((x^i_a(b^i(a),q^i(a),B^i_T,Q^i_T))^L_{i=1})\ :\ \sum_{i=1}^L(B^i_Tq^i(a)/Q^i_T)\ge\sum_{i=1}^L b^i(a)\right\} \tag{2}$$

Notice that I have used a distribution mechanism that, for any group of agents, it aggregates the orders (bids and offers) in *one* spot for each commodity. This seems to contradict my initial statement that I leave the number of trading posts within any group unspecified. However, this is done without loss of generality as the following fact shows:

Fact 2.1 *Given an atomless measure space $(T,\mathcal{T},\nu)$, the equilibrium allocations which are attainable through any (finite) number of trading posts are also attainable through one trading post for each commodity.*

Proof:

Let K_i be an integer denoting the number of trading posts for each commodity $i = 1, 2, \ldots, L$. The allocation accruing to each agent who places simultaneous orders in each post is given by:

$$x^i(a) = e^i(a) - \sum_{k=1}^{K_i} q_k^i(a) + \sum_{k=1}^{K_i} b_k^i(a) \frac{Q_T^{i,k}}{B_T^{i,k}}$$

From the first order conditions of (2), using the modified allocation rule, one obtains that in equilibrium the following condition should be satisfied for each agent $a \in T$ and each pair of trading posts k_1, k_2 for a commodity:

$$\left(\frac{B_T^{i,k_1}}{Q_T^{i,k_1}}\right)^2 = \frac{B_{-a}^{i,k_1}}{Q_{-a}^{i,k_1}} \cdot \frac{Q_{-a}^{i,k_2}}{B_{-a}^{i,k_2}} \cdot \left(\frac{B_T^{i,k_2}}{Q_T^{i,k_2}}\right)^2$$

It follows from the non-atomicity of $(T, \mathcal{T}, \nu)$ that:

$$\frac{B_T^{i,k_1}}{Q_T^{i,k_1}} = \frac{B_T^{i,k_2}}{Q_T^{i,k_2}}, \ \forall\, k_1, k_2 \tag{3}$$

Define $b^i(a) = \sum_{k=1}^{K_i} b_k^i(a)$ and $q^i(a) = \sum_{k=1}^{K_i} q_k^i(a)$. Since each term in the summation is measurable we have that b^i and q^i are measurable mappings on T so we can define:

$$B_T^i = \int_T b^i(a) = \int_T \sum_{k=1}^{K_i} b_k^i(a), \ \ Q_T^i = \int_T q^i(a) = \int_T \sum_{k=1}^{K_i} q_k^i(a)$$

From (3) above we can conclude that:

$$\frac{\sum_{k=1}^{K_i} B_T^{i,k}}{\sum_{k=1}^{K_i} Q_T^{i,k}} = \frac{B_T^i}{Q_T^i}$$

Hence, with the strategy profile $\left(b^i(a), q^i(a)\right)_{a\in T}$ each agent receives the same allocation $x^i(a)$ as before. We leave it to the reader to verify that this profile of strategies forms a Nash equilibrium with one trading post for each commodity $\square$

Thus, augmenting the number of trading posts within any group makes no difference. It remains to ensure that the group which trades in each trading post is irrelevant. This is the idea behind the following definition, which states that a given 'market structure' is in equilibrium if there is no group of agents can derive a potential benefit from a trading post.

Definition 2.2 *A trading posts equilibrium is a profile of strategies (b, q) such that:*

(*i*) $(b, q) \in \mathcal{NE}\left(\bar{A}\right)$ *for some* $\bar{A} \subset A$ *of full measure*

(*ii*) $\nexists\, T \in \mathcal{A}$ *where* $\mu(T) > 0$ *and* $\left(\hat{b}, \hat{q}\right) \in \mathcal{NE}(T)$ *such that*

$$u_a((y_a^i(\hat{b}^i(a), \hat{q}^i(a), \hat{B}_T^i, \hat{Q}_T^i))_{i=1}^L) > u_a((x_a^i(b^i(a), q^i(a), B_A^i, Q_A^i))_{i=1}^L) \ \ ae.$$

Let $\mathcal{TPE}(A)$ denote the set of trading posts equilibria. With this setup I am now ready to proceed to the main argument of this paper.

3 Results

I begin with a result parallel to the ones obtained in [3] and in [4].

Proposition 3.1 *Let $x : A \to \Re^L_+$ be a competitive allocation, i.e., $(p, x) \in \mathcal{CE}(A)$ for some $p \in \Re^L_{++}$. There exists a profile of strategies $(b, q) \in \mathcal{TPE}$ such that $\left(x^i_a(b^i(a), q^i(a), B^i_A, Q^i_A)\right)_{i=1}^L = x(a)$, ae.*

Proof: I start by reporting that, by a standard result, since x is a competitive assignment it belongs to the core.

Let $L_1 = \{i : x^i(a) \neq e^i(a)\}$ and $L_2 = \{i : x^i(a) = e^i(a)\ ae\}$. For each $i \in L_1$ define $q^i(a) = -\left(x^i(a) - e^i(a)\right)^-$ and $b^i(a) = p^i\left(x^i(a) - e^i(a)\right)^+$. For each $i \in L_2$ extract from S a measurable selection $q^i : A \to \Re^L_+$ so that $e^i(a) \geq q^i(a) > 0$ on a set of positive measure (for example $q^i(a) = e^i(a)\ ae$) and define $b^i(a) = p^i q^i(a)$. Clearly, b and q are measurable and by construction $q^i(a) \leq e^i(a)$ for $i = 1, 2, \ldots, L$ so $\left(b^i(a), q^i(a)\right) \in S(a)$, ae. Let $B^i_A = \int_A b^i(a)$ and $Q^i_A = \int_A q^i(a)$. Notice that $B^i_A/Q^i_A = p^i > 0$. For this profile of strategies we have:

For $i \in L_1$:

$$\begin{aligned} x^i_a(b^i(a), q^i(a), B^i_A, Q^i_A) &= e^i(a) - q^i(a) + b^i(a) \cdot \frac{Q^i_A}{B^i_A} \\ &= e^i(a) + \Big(x^i(a) - e^i(a)\Big)^- + \Big(x^i(a) - e^i(a)\Big)^+ \\ &= x^i(a) \end{aligned}$$

For $i \in L_2$:

$$x^i_a(b^i(a), q^i(a), B^i_A, Q^i_A) = e^i(a) - q^i(a) + b^i(a) \cdot \frac{Q^i_A}{B^i_A} = e^i(a)\ ae$$

Thus, we have that $\left(x^i_a(b^i(a), q^i(a), B^i_A, Q^i_A)\right)_{i=1}^L = x(a)$.

I claim that $(b, q) \in \mathcal{TPE}$. To this end we have:

$$\begin{aligned} \sum_{i=1}^L \frac{B^i_A}{Q^i_A} \cdot q^i(a) &= \sum_{i \in L_1} \frac{B^i_A}{Q^i_A} \cdot q^i(a) + \sum_{i \in L_2} \frac{B^i_A}{Q^i_A} \cdot q^i(a) \\ &= \sum_{i \in L_1} -p^i \Big(x^i(a) - e^i(a)\Big)^- + \sum_{i \in L_2} p^i q^i(a) \\ &= \sum_{i \in L_1} p^i \Big(x^i(a) - e^i(a)\Big)^+ + \sum_{i \in L_2} b^i(a) \\ &= \sum_{i \in L_1} b^i(a) + \sum_{i \in L_2} b^i(a) = \sum_{i=1}^L b^i(a) \end{aligned}$$

Let $\bar{A} \subset A$ be the set of agents for which x is a competitive allocation. Consider any $a \in \bar{A}$. Suppose that for some $\left(\hat{b}^i, \hat{q}^i\right) \in S(a)$, the corresponding allocation, given by $\hat{x}^i = e^i(a) - \hat{q}^i + \hat{b}^i Q^i_A / B^i_A$, is such that $u_a(\hat{x}) \geq u_a(x(a))$. By definition of a competitive

equilibrium it follows that $p{\cdot}\hat{x} \geq p{\cdot}e(a)$ which implies that $\sum_{i=1}^{L} \hat{b}^i(a) \geq \sum_{i=1}^{L} \frac{B_A^i}{Q_A^i}{\cdot}\hat{q}^i(a)$.
Same reasoning establishes that $u_a(\hat{x}) > u_a(x(a))$ implies $\sum_{i=1}^{L} \hat{b}^i(a) > \sum_{i=1}^{L} \frac{B_A^i}{Q_A^i}{\cdot}\hat{q}^i(a)$.
Thus, we conclude that $\left(b^i(a), q^i(a)\right)$ solves (2) for all $a \in \bar{A}$, i.e., $(b,q) \in \mathcal{NE}\left(\bar{A}\right)$.
To complete the proof suppose that for some $T \subset A$ where $\mu(T) > 0$ there exists $\left(\hat{b}, \hat{q}\right) \in \mathcal{NE}(T)$ such that the corresponding assignment $y : T \to \Re_+^L$ is improving for T, i.e., $u_a((y_a^i(\hat{b}^i(a), \hat{q}^i(a), \hat{B}_T^i, \hat{Q}_T^i))_{i=1}^L) > u_a((x_a^i(b^i(a), q^i(a), B_A^i, Q_A^i))_{i=1}^L)$, *ae* in T.
Since $\left(\hat{b}, \hat{q}\right) \in \mathcal{NE}(T)$ we have $\int_T y(a) \leq \int_T e(a)$. But then x cannot be in the core, which contradicts our initial statement.
Thus, we conclude that $(b,q) \in \mathcal{TPE}(A)$ □

The result that follows establishes that the converse is also true. It is a consequence of the equivalence of competitive equilibria and the core.

Proposition 3.2 *Let* $(b,q) \in \mathcal{TPE}$ *and* $x : A \to \Re_+^L$ *be the corresponding commodity assignment* $x(a) = \left(x_a^i(b^i(a), q^i(a), B_A^i, Q_A^i)\right)_{i=1}^L$, *ae. Then* x *is a competitive allocation, i.e., there exists* $p \in \Re_+^L$ *such that* $(p,x) \in \mathcal{CE}(A)$.
Proof: Let $(b,q) \in \mathcal{TPE}(A)$ and denote by x the corresponding commodity assignment. Suppose that x is not a competitive allocation, i.e., there is no $p \in \Re_+^L$ such that $(p,x) \in \mathcal{CE}(A)$. Then x is not in the core of the economy so there is $T \in \mathcal{A}$, where $\mu(T) > 0$, and a measurable assignment $\hat{x} : T \to \Re_+^L$ such that $u_a(\hat{x}(a)) > u_a(x(a))$, *ae* in T.
By a result of Mas Collel [11] [proposition 7.3.2, p.269] it can be assumed that $\hat{x}$ is a competitive allocation for the coalition T, i.e., there is a $\hat{p} \in \Re_+^L$ so that $(\hat{p}, \hat{x}) \in \mathcal{CE}(T)$.
Clearly, by a construction similar to the one in the previous proposition we can find $\left(\hat{b}, \hat{q}\right) \in \mathcal{NE}(T)$ such that $\left(x_a^i(b^i(a), q^i(a), B_A^i, Q_A^i)\right)_{i=1}^L = \hat{x}(a)$, *ae*.
Thus, we have found $T \in \mathcal{A}$ and $\left(\hat{b}, \hat{q}\right) \in \mathcal{NE}(T)$ so that:

$$u_a((x_a^i(\hat{b}^i(a), \hat{q}^i(a), B_T^i, Q_T^i))_{i=1}^L) = u_a(\hat{x}(a)) > u_a(x(a)), \ ae \text{ in } T.$$

which contradicts the fact that $(b,q) \in \mathcal{TPE}(A)$ □

Putting together the conclusions of the last two propositions we can state the following result.

Theorem 3.1 *The set of allocations which are trading posts equilibria equals the set of competitive allocations.*

In this sense the trading posts mechanism developed here is equivalent to the competitive mechanism. In order to highlight the fact that the equilibrium refinement defined herein avoids 'trivial' equilibria of the market game mechanism I report the following result:

Theorem 3.2 $(0,0) \in \mathcal{TPE}(A)$ *if and only if the endowment assignment is Pareto optimal.*
Proof: - $(0,0) \in \mathcal{TPE}(A) \Rightarrow$ Endowments are Pareto optimal.
Notice that the allocation resulting from $(0,0)$ is precisely the endowment allocation, which by the previous theorem is a competitive allocation so by the first welfare theorem it is Pareto optimal.

- Endowments are Pareto optimal $\Rightarrow (0,0) \in \mathcal{TPE}(A)$.

Since endowments are Pareto optimal, by the second welfare theorem they can be supported as a competitive assignment for some prices and hence the endowment assignment is in the core. Suppose now that $(0,0) \notin \mathcal{TPE}(A)$. Clearly, $(0,0) \in \mathcal{NE}(A)$. Then there must be some $T \in \mathcal{A}$ and $\left(\hat{b}^i, \hat{q}^i\right) \in \mathcal{NE}(T)$, so that the corresponding assignment, say $\hat{x}^i$, is preferable: $u_a(\hat{x}(a)) > u_a(e^i(a))$, ae in T. Since $\int_T \hat{x}(a) \leq \int_T e(a)$ we conclude that the endowment assignment is not in the core, a contradiction to the initial statement. $\square$

4 Concluding remarks

In this note I have established the equivalence of the competitive mechanism with non cooperative trade within groups of agents. This equivalence has been obtained via equality of the equilibrium allocations corresponding to the two mechanisms. I believe that there are two points that validate my approach: First, that one may replace the *hypothesis* of a single trading post where all bids and offers are placed, with the hypothesis that any group of traders can replicate the mechanism by which prices and allocations are calculated. In that case the definition of equilibrium must require that the structure of trading posts be in equilibrium. Although the structure of trading posts is not explicitly described, an equilibrium structure of trading posts is implicit in my definition of a trading posts equilibrium. Second that, in view of the well known core equivalence results, the quest for exact equivalence between market game mechanisms and the competitive mechanism is, in fact, a pursuit of a three way equivalence between market game mechanisms, the core and the competitive mechanism. This fact has motivated the refinement of market game equilibria chosen and, of course, drives the result from a technical point of view.

I think of agents coming together in groups, proposing bids and offers and calculating the corresponding allocations accruing to them. This may be interpreted as agents agreeing to meet at spot and trade or as groups of agents meeting randomly and propose bids and offers. This is the meaning of exclusion of some agents from economic activity in a trading post. An equilibrium in all this process occurs when no group can use the trading mechanism to achieve improving allocations. Notice that the structure of equilibrium trades is abstract: Agents could end up either trading all together in the same trading post or in groups trading in different posts or trading simultaneously in many posts for each commodity. In other words the number of trading posts where commodities are exchanged is indeterminate in equilibrium. This is natural in view of the equivalence result that I showed: One trading post is as good as a hundred, since the bid-offer ratios are the same in all of them. Equilibrium in the structure of trading posts has the meaning that no group of traders should be interested in establishing an *extra* trading post and involve in trades among themselves.

Finally, it should be noted that in this paper, I have used a rather crude model to drive the points made in the introduction. I believe that the modeling of my ideas can be improved in two ways: One might try to exploit the equivalence between competitive equilibria and the f-core (see [10]) in economies with anonymous externalities. The market game structure involved in our definition makes the f-core an appealing concept. Alternatively one might think of a 'trading post' as a bilateral trade and develop an elaborate model of exchange as in Gale [5]. That model seems even more interesting in that it provides a matching process by which trading posts (understood as bilateral trades) are set up.

References

[1] Anderson, R. (1992) The Core in Perfectly Competitive Economies, in Handbook of Game Theory, Vol. I, ed. R.J. Aumann & S. Hart, Elsevier Science Publishers B.V 1992.

[2] Block F. & S. Goshal (1994) Stable Trading Structures in Bilateral Oligopolies, C.O.R.E. discussion paper.

[3] Codognato G. & S. Goshal (1994) Cournot Equilibria in Markets with a Continuum of Traders, C.O.R.E. discussion paper 9459.

[4] Dubey, P. & L. Shapley (1994) Non-cooperative General Exchange with a Continuum of Traders: Two Models, *Journal of Mathematical Economics*, **23**, 253-293.

[5] Gale D. (1986) Bargaining and Competition Part I: Characterization, *Econometrica*, Vol.**54**, No 4, 807-818.

[6] Koutsougeras, L.C (1995) Cooperative Cournot Equilibria, (mimeo)

[7] Koutsougeras, L. (1996) Market Games With Multiple Trading Posts I: some remarks, mimeo, University of Manchester.

[8] Mas-Collel, A. (1982) The Cournotian Foundations of Walrasian Equilibrium Theory: An Exposition of Recent Theory, in Advances in Economic Theory, ed. W. Hildenbrand, Cambridge University Press, 183-224.

[9] Mertens, J.F (1996) The Limit Price Mechanism, C.O.R.E discussion paper 9650.

[10] Hammond P., M.Kaneko & M.H. Wooders (1989) Continuum Economies with Finite Coalitions: Core, Equilibria and Widespread Externalities, *Journal of Economic Theory*, **44**, 113-134.

[11] Mas-Collel A. (1985) The Theory of General Economic Equilibrium: A Differentiable Approach, ed. F. Hahn, Cambridge University Press.

[12] Pazner E. & Schmeidler D. (1976) Non-Walrasian Equilibria and Arrow-Debreu Economies, mimeo University of Illinois at Urbana-Champaign.

[13] Peck, J., K. Shell & S. Spear (1992) The Market Game: Existence and Structure of Equilibrium, *Journal of Mathematical Economics*, **21**, 271-299.