

Fitting affine jump-diffusions to market curves using change of time

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Abstract

We address the so-called *calibration problem* which, in finance, consists of fitting in a tractable way a given model x to a specified continuous term structure like, e.g., yield or default probability curve noted G . Homogeneous affine jump-diffusions (HAJDs) $x = (x_t)_{t \geq 0}$ like Vasicek, Cox-Ingersoll-Ross (CIR) or CIR coupled with compounded Poisson jumps (JCIR), are very popular tractable processes. Unfortunately, they have limited flexibility as they only feature time-homogeneous parameters, represented by the vector Ξ (for the JCIR model, Ξ is the vector composed of the speed of mean reversion, long-term mean, instantaneous volatility as well as the jump rate and size parameters). Hence, considering a HAJD process, it is not possible in general to find parameters Ξ such that the term-structure implied by the model, $P^x(t; \Xi)$, would match an actual (exogenously given) market curve $G(t)$.

The deterministic shift extension consists of considering instead a process $x = (x_t)_{t \geq 0}$ that is built by adding a deterministic function φ to a latent HAJD process y with parameter Ξ , i.e. $x_t := y_t + \varphi(t)$. This leads to the well-known Hull-White, CIR++ or JCIR++, depending on whether y is Vasicek, CIR or JCIR. This is a simple but yet efficient solution, that is widely used by both academics and practitioners. Essentially, it is equivalent to allowing some parameters in Ξ to become time-dependent. This method is fully flexible in the sense that for every parameter set Ξ associated to the latent process y , there exists a shift $\varphi(t) = \varphi(t; \Xi, G)$ such that the curve implied by the model agree with the given market curve: $P^x(t; \Xi) = P(t)$. However, relying on the shift approach is not always appropriate. This is particularly true when the process x needs to be positive. This is a common constraint when x aims at describing credit spreads or default intensities. In this case indeed, it is not enough to start from a positive process y (e.g. CIR or JCIR) as there is no guarantee that the shift function $\varphi(\cdot; \Xi, G)$ is non negative; it depends on both the market curve G to be calibrated as well as on the parameter Ξ of the latent process y . Consequently, one either faces the risk to generate negative values for x (which, in many cases, is unacceptable), or needs to put additional constraints on the latent process parameters Ξ so as to make sure that $\varphi(t; \Xi, G) \geq 0$ for all $t \geq 0$. The problem however is that adding a positivity requirement on the shift function leads to parameters Ξ that are very restrictive in terms of the variance of y (hence of x).

In this paper, we tackle this problem by adopting a different approach. Instead of shifting the HAJD process y by a deterministic function φ , we define x using $x_t = \theta(t)y_{\Theta(t)}$ where $\Theta(t)$ is a special time-change function (called a clock) and $\theta = \Theta'$ is the clock rate. On the top of providing an elegant solution to the calibration problem under positivity constraint, our model features additional interesting properties in terms of implied volatilities. It is compared to the shift extension on various credit risk applications such as credit default swap, credit default swaption and credit valuation adjustment under wrong-way risk. The time change approach is able to generate much larger volatility and covariance effects under the positivity constraint. Our model offers an appealing alternative to the shift in such cases. Although our calibration technique is discussed in a finance context, it can be used in many other applications like, e.g. mortality or data-degradation modeling.