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Inter and Intra-generational fairness for public pension systems in multi-population mortality models

Keivan Diakite [a], Pierre Devolder[a] and Massimilianno Menzietti [b]

[a] Institut de Statistique, Biostatistique et Sciences Actuarielles, UCLouvain; [b]Department of Economics and Statistics, University of Salerno

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ABSTRACT

As populations age, ensuring the sustainability and fairness of pension systems is increasingly crucial. This paper introduces in a pay-as-you-go (PAYG) the Progressive Defined Musgrave (PDM) system, which combines intergenerational and intragenerational adjustment mechanisms. The system aims at long-term financial stability while promoting fairness by addressing disparities in both longevity and income. Intergenerational fairness is maintained through the Musgrave mechanism, which balances economic risks between workers and retirees. Meanwhile, intragenerational fairness is achieved through a progressive pension benefit formula, offering higher replacement rates for lower-income individuals and lower rates for higher-income individuals. The system adapts to changing demographic and economic conditions by adjusting contribution rates and pension benefits annually. The paper explores the mathematical models underlying this design, including multi-population mortality models that capture longevity inequality. Finally, risk analysis using Value-at-Risk demonstrates the adaptability of the PDM system in ensuring financial sustainability and fairness in the face of evolving demographic dynamics.

KEYWORDS

Longevity Heterogeneity, Intergenerational and Intragenerational Fairness, Progressive Defined Musgrave System

CONTACT K. Diakite. Email: keivan.diakite@gmail.com

1. Introduction

Longevity inequalities significantly impact retirement policies, as individuals reach statutory retirement age with varying work abilities and life expectancies [Strozza & al, 2022]. These disparities stem from unequal exposure to occupational risks, living conditions, and chronic distress [Forster & al, 2018], leading to greater economic disadvantages [Ardito & Costa, 2022] and complicating pension systems by redistributing resources from the less well-off to the better-off [Holzmann et al., 2020].

The link between life expectancy and pension policies has gained attention recently [Arnold & Jijie, 2020]. Life expectancy determines the duration of pension receipt and affects eligibility and benefits across countries. Many OECD pension systems incorporate automatic adjustment mechanisms, such as linking benefits to life expectancy or adjusting the statutory retirement age. However, increasing the age of the state pension overlooks factors such as declining employment rates among older women and regional inequalities in disability-free life expectancy [TUC report, 2013]; [Sundberg & al, 2023]; [Wilmoth & al, 2023], challenging the sustainability of extended working lives.

These mechanisms impact pension wealth distribution, as individuals with lower-than-average life expectancy receive benefits for a shorter duration, resulting in a relative loss of pension wealth. Occupational inequality in mortality can offset income redistribution in pension systems, with significant impacts observed in the French and German PAYG systems. OECD reports [David, 2017] highlight that the gap in remaining life expectancy reduces total pensions received by low earners by 13% compared to high earners.

Given the within-generation inequalities and socioeconomic disparities in life expectancy [Shi & Kolk, 2022], it is crucial to explore multi-population mortality modeling in pension systems. While existing literature extensively covers single-population mortality modeling, it often neglects the inherent interconnections between different populations and the resulting unfairness in Defined Benefits (DB) systems. Incorporating multipopulation models can reveal variations in mortality patterns across socioeconomic classes [Antonio & al, 2015]; [Jevtić & al, 2023], highlighting the correlation between longevity and social advantage [Majer & al, 2011]; [Dudel & Schmied, 2019]. Higher Socio-Economic Class (SEC) individuals benefit more from pension systems, potentially perpetuating unfairness [Queisser & Whitehouse, 2006].

Several studies have examined the implications of longevity heterogeneity on pension systems. [Holzmann et al., 2020] discuss how it challenges the contribution-benefit link in NDC systems, proposing alternative adjustments. [Arnold & Jijie, 2020] determine the optimal retirement age in PAYG and NDC schemes, advocating for different retirement ages based on socioeconomic class. [Jijiie & al., 2022] assess fairness in DB and DC schemes, suggesting adjustments to improve equity. [Boado-Penas et al., 2022] analyze NDC systems using stratified mortality projections, showing the impact of demographic and economic factors on pension benefits.

The sustainability and fairness of pension systems are critical concerns for an aging population. To address these challenges, we propose a pension system design that incorporates both inter- and intragenerational adjustment mechanisms, aiming for long-term stability, equitable distribution, and adequate retirement income. Intergenerational fairness is based on the well-known Musgrave mechanism [Musgrave & Thin, 1948], while the intragenerational dimension is taken into account through a progressive pension benefit formula [Devolder & Diakite , 2021].

Operating on a pay-as-you-go (PAYG) basis, the system relies on current workers' contributions to finance retiree pensions. The Musgrave invariant, reflecting the ratio between average pension and average salary net of contributions, distributes economic risks between workers and retirees. Annual adjustments to system parameters maintain intergenerational risk sharing and financial equilibrium, adapting to changing demographic and economic conditions.

We consider a DB pension system where benefits are calculated using progressive coefficients that distribute benefits across income levels, providing higher replacement rates for lower-income individuals and lower rates for higher-income individuals to mitigate income inequality. An intragenerational correction factor addresses longevity heterogeneity among retirees, ensuring pension benefits reflect life expectancy differences. The adoption of progressive coefficients and intragenerational correction factors ensures pension actuarial fairness while improving the adequacy of benefits for lower socioeconomic classes.

This proposed system, termed the Progressive Defined Musgrave (PDM), aims to ensure simultaneously fairness and financial sustainability by adjusting contribution rates and pension benefits based on demographic factors and income differentials.

In the numerical application, we perform a comprehensive analysis of the PDM pension system defined previously, focusing on its mean evolution and risk characteristics. We examine the contribution rate, the pension rate, and the class replacement rate to gain insight into the system's dynamics. Furthermore, we assess the variability introduced by the longevity heterogeneity correction using multi-population models for socioeconomic group mortality. Finally, by comparing results under two alternative mortality projection models, we illustrate the system's resilience and adaptability, independent of the specific mortality model used. In general, our analysis aims to highlight the importance of considering risk measurement and variability in pension systems alongside mean evolution. By examining various risk indicators and projection scenarios, we contribute to a deeper understanding of the PDM system's implications for different stakeholders.

The main contributions of the paper with respect to the literature can be summarized in three points. First, we extend in a dynamic framework the progressive systems as introduced in a static environment by([Devolder & Diakite , 2021]) , using stochastic models of mortality. This generalization enables us to study the effects of heterogeneity not only in terms of level but also in terms of trends. On the other hand, we generalize in this stochastic framework the Musgrave rule. Finally, we mix the two mechanisms to create a new system combining intra and intergenerational risk sharing through the application of the Musgrave rule in a progressive multi class pension system. This combination of two adaptation mechanisms generates a pension system financially sustainable in the long run, ensuring a fair distribution of the efforts between the classes and improving the adequacy of benefits for lower socio-economic classes compared to standard uniform adaptation. Financial sustainability, social adequacy and fairness are joined and not opposed.

The remainder of the paper is organized as follows. Section 2 explores the proposed pension system that incorporates progressivity in the pension formula and intergenerational risk sharing based on demographic factors. It introduces key conceptual innovations, notably a multiclass Musgrave-like inter-generational risk-sharing mechanism and a progressive pension benefit formula, both adapted to a dynamic environment. Section 3 describes the multi-population mortality models and approaches used to model longevity inequality. Provides a comprehensive definition of the models and data sources used to compare and validate these models [Antonio & al, 2015]; [Jevtić & al, 2023]. By incorporating multi-population models, we capture cross-population dependence and observe variations in mortality patterns among different socioeconomic classes. In section 4, we perform a risk analysis of our proposed Progressive Defined Musgrave system. As a risk indicator, we use a value-at-risk analysis to assess potential risks and uncertainties associated with these mechanisms. The numerical application demonstrates how the proposed system can adapt to changing demographic and economic conditions, ensuring fairness and financial sustainability. Section 5 summarises the main findings and policy implications.

2. Progressive defined Musgrave system

2.1. Assumptions and mechanisms

We adopt the following demographic and economic assumptions, commonly used in the literature for PAYG systems [Settergren & Mikula, 2005], [Ventura-Marco & Vidal-Meliá ,2014], [Alonso-Garcia & Al, 2018].

- *Entry and retirement ages:* Agents only enter the regime at age x_0, the retirement age is x_r. Entry and retirement ages are the same for all classes.
- *Population differentiation:* Salary levels differentiate agent classes. There are m classes. We take note of $S_i^{(x,t)}$, which is the salary for workers aged x (with $x_0 \leq x \leq x_r$) at period t in class i such that

$$S_1^{(x,t)} < ... < S_i^{(x,t)} < ... < S_m^{(x,t)}.$$

- *Workers' salary evolution:* Before retirement, the salary in any class at a given age x is obtained as:

$$S_i^{(x,t)} = S_i^{(x_0,t_0)} \cdot (1+s_i)^{(x-x_0)} \cdot (1+\gamma)^{t-t_0},$$

with γ representing the annual revaluation of the salary and s_i is the career salary growth rate for class i such that

$$s_1 < ... < s_i < ... < s_m.$$

There is no transition between the classes during the career.

- *Notional salary:* After retirement, we define the notional salary for $x > x_r$

$$S_i^{(x,t)} = S_i^{(x_r-1,t+x_r-1-x)} \cdot (1+\gamma)^{(x-x_r+1)} \tag{1}$$

We consider the retired notional salary to be the indexed last pre-retirement salary.

- *Population Dynamics:* The effective number of workers aged x at period t in class i is denoted by $Na_i^{(x,t)}$, and the number of retirees aged y by $Nr_i^{(y,t)}$. The total number of active workers at time t is denoted as Na^t, and the total number of retirees as Nr^t. The number of workers is obtained by the population equation:

$$Na_i^{(x+1,t+1)} = Na_i^{(x,t)} \cdot p_x^i(t) \tag{2}$$

where $p_x^i(t)$ is the one-year survival probability of individuals of age x at time t in class i. The number of retirees is obtained using a similar procedure. Formula(2) shows that the population at time t is determined by the survival of individuals, implying an absence of migration[1].

When not taking into account the socioeconomic class longevity heterogeneity, we can compute the total population mortality table where $p_x(t)$ is the one-year survival probability of all individuals of age x at time t.

[1]Building on [Settergren & Mikula, 2005], we exclude migration from our analysis. However, in reality, migration plays a crucial role in shaping population dynamics across most European countries ([Eurostat, 2011]).

- *Entry function:* The entry function at any time t giving the number of workers entering the regime is written as $L(x_0;t) = E(t)$. At the inception of the system, the population is already composed of active workers and retirees. The numbers of individuals at any age in each class is determined solely by the entry function and survival probabilities.
- The dependency ratio each period is the ratio between the number of retirees over the number of active workers :

$$D_t = \frac{Nr^t}{Na^t} \tag{3}$$

- We define as weighted dependency ratio, the ratio between the weighted notional salaries of retirees and the weighted salaries of active workers.

$$D_t^* = \frac{\sum_{i=1}^{m} \sum_{x=x_r}^{\omega} Nr_i^{(x,t)} \cdot S_i^{(x,t)}}{\sum_{i=1}^{m} \sum_{x=x_0}^{x_{r-1}} Na_i^{(x,t)} \cdot S_i^{(x,t)}} \tag{4}$$

2.2. Progressivity in the pension formula

In a traditional DB pension system, benefits are often directly linked to the individual's earnings, which can disproportionately favor higher earners due to longer life expectancy and accumulated contributions. To address this issue, we extend the progressive salary transformation proposed by [Devolder & Diakite , 2021] in a time-dynamic framework. This mechanism modifies the income base used in the pension benefit calculation, through a transformation of salaries, where different progressive coefficients are applied to different income brackets, to achieve a more equitable distribution of pension benefits across socioeconomic classes. Progressivity in this context ensures that lower-income individuals receive relatively higher pension benefits compared to their earnings, while higher-income individuals contribute proportionally more but receive a relatively lower replacement rate.

The pension benefit for the class i in the year t at age x_r is expressed via:

$$P_i^{(x_r,t)} = \delta_t \cdot X_i^{(x_r,t)} \tag{5}$$

The pension benefit formula is made up of two parts: the pension rate δ_t, and the progressive component, which is determined by the salary transformation $X_i^{(x,t)}$ for class i at time t and age x_r, defined as follows:

$$X_i^{(x_r,t)} = \sum_{j=1}^{i} \lambda_j^t \cdot (S_j^{(x_r,t)} - S_{j-1}^{(x_r,t)}), \tag{6}$$

where λ_i^t are the progressive coefficients. We first define in this section this progressive component (intragenerational aspect). The adaptation of the pension rate will be detailed in section 2.3 (intergenerational aspect). The progressive coefficients account for longevity heterogeneity and are obtained by imposing an equality constraint of lifetime replacement (LR) rates between individuals belonging to different socioeconomic classes. The LR rate is a key indicator used to assess the adequacy of pension benefits, measuring the present value of benefits relative to the last salary received. It provides a means to compare the benefits received by retirees from different socioeconomic classes, as it focuses solely on pension benefits. The condition imposed on LR rates aims at preventing inequalities caused by different life expectancies.

The LR rate is computed assuming a constant pension rate after retirement. Before accounting for longevity heterogeneity, the LR rate for the specific retirement age x_r and period t, based on identical survival probabilities for all classes (denoted by ${}_{x-x_r}p_{x_r}(t)$) can be expressed as:

$$\begin{aligned}
LR^{(x_r,t)} &= \frac{1}{S^{(x_r-1,t)}} \sum_{x=x_r}^{\omega} P^{(x_r,t)} \cdot {}_{x-x_r}p_{x_r}(t) \cdot \left(\frac{1+\gamma}{1+r}\right)^{x-x_r} \\
&= \frac{P^{(x_r,t)}}{S^{(x_r,t)}} \cdot \sum_{x=x_r}^{\omega} {}_{x-x_r}p_{x_r}(t) \cdot \left(\frac{1+\gamma}{1+r}\right)^{x-x_r} \\
&= \delta_t \cdot \ddot{a}_{x_r}(t)
\end{aligned}$$

where γ is the annual revaluation rate of pension, assumed to be equal to the revaluation of salary, r is the discount rate and

$$\ddot{a}_{x_r}(t) = \sum_{x=x_r}^{\omega} {}_{x-x_r}p_{x_r}(t) \cdot (\frac{1+\gamma}{1+r})^{x-x_r}$$

is the average annuity rate at retirement for the total population.

When considering longevity heterogeneity, the LR rate for socioeconomic class i can be expressed as:

$$LR_i^{(x_r,t)} = \delta_t \cdot \frac{X_i^{(x_r,t)}}{S_i^{(x_r,t)}} \cdot \ddot{a}^i_{x_r}(t) \tag{7}$$

In this equation, the socioeconomic groups annuity rates are given by:

$$\ddot{a}^i_{x_r}(t) = \sum_{x=x_r}^{\omega} {}_{x-x_r}p^i_{x_r}(t) \cdot (\frac{1+\gamma}{1+r})^{x-x_r}$$

By satisfying the condition of equal lifetime replacement rates:

$$LR_1^{(x_r,t)} = \ldots = LR_i^{(x_r,t)} = \ldots = LR_m^{(x_r,t)} = LR^{(x_r,t)}$$

we can determine the values of the progressive coefficients. We obtain:

$$\lambda_i^t = \frac{S_i^{(x_r,t)} \cdot \dfrac{\ddot{a}_{x_r}(t)}{\ddot{a}^i_{x_r}(t)} - S_{i-1}^{(x_r,t)} \cdot \dfrac{\ddot{a}_{x_r}(t)}{\ddot{a}^{i-1}_{x_r}(t)}}{S_i^{(x_r,t)} - S_{i-1}^{(x_r,t)}} \tag{8}$$

The pension benefit at retirement for an agent of class i at time t is given by:

$$P_i^{(x_r,t)} = \delta_t \cdot X_i^{(x_r,t)} = \delta_t \cdot S_i^{(x_r,t)} \cdot \frac{\ddot{a}_{x_r}(t)}{\ddot{a}^i_{x_r}(t)} = \delta_t \cdot S_i^{(x_r,t)} \cdot \theta_i^t \tag{9}$$

Here, θ_i^t represents the longevity heterogeneity correction factor for class j. It quantifies the longevity gain in each class relative to the total population.

The class replacement rate[2] at retirement for an agent of class i is defined as the ratio between the pension benefit and the last working salary:

$$Tr_i^{(x_r,t)} = \frac{P_i^{(x_r,t)}}{S_i^{(x_r,t)}} = \delta_t \cdot \theta_i^t \tag{10}$$

For retirees of age $x > x_r$, the progressive coefficient remains the same. The progressive salary transformation becomes:

$$X_i^{(x,T)} = \sum_{j=1}^{i} \lambda_j^{T-x+x_r} \cdot (S_j^{(x,T)} - S_{j-1}^{(x,T)}) \tag{11}$$

In this equation, $X_i^{(x,T)}$ represents the pension benefit for old retirees in class i at time T and age x.The pension benefit is given by:

$$P_i^{(x,T)} = \delta_T \cdot X_i^{(x,T)} \tag{12}$$

2.3. Inter-generational risk sharing:

We now turn to the dynamics of the first component of the pension benefit formula: the pension rate. Our system introduces an intergenerational risk-sharing mechanism based on the Musgrave invariant, ensuring a fair allocation of economic risks between active workers and retirees. Unlike traditional approaches, our framework extends the Musgrave principle to a multiclass setting, allowing for differentiated treatment across socioeconomic groups while maintaining systemic balance.

The formulation we present is novel, as it generalizes the Musgrave invariant to a progressive, multiclass pension system, ensuring both equity and sustainability. This approach not only preserves intergenerational fairness but also accounts for heterogeneity in longevity and income levels, making it a more robust adaptation of risk-sharing principles in pension design.

[2]The replacement rate is the ratio of a pensioner's retirement income to their preretirement earnings. In a DB system, the replacement rate is typically a percentage of final average earnings. In a progressive system, the pension rate is a proportion of the progressive transformation of the salary, the replacement rate differs for each class in such system

Let us highlight the link between the average benefit ratio $\bar{\delta}_t$ [3], and the pension rate δ_t

$$\bar{\delta}_t \cdot \sum_{i=1}^{m} \sum_{x=x_r}^{\omega} Nr_i^{(x,t)} \cdot S_i^{(x,t)} = \delta_t \sum_{i=1}^{m} \sum_{x=x_r}^{\omega} Nr_i^{(x,t)} \cdot X_i^{(x,t)} \tag{13}$$

The contribution rate π_t is then given by the budget equation:

$$\pi_t = \bar{\delta}_t \cdot D_t^* \tag{14}$$

The Musgrave invariant is the ratio between the average pension and the average salary net of pension contributions :

$$M_t = \frac{\bar{\delta}_t \cdot \frac{\sum_{i=1}^{m} \sum_{x=x_r}^{\omega} Nr_i^{(x,t)} \cdot S_i^{(x,t)}}{Nr^t}}{(1-\pi_t) \cdot \frac{\sum_{i=1}^{m} \sum_{x=x_0}^{x_r} Na_i^{(x,t)} \cdot S_i^{(x,t)}}{Na^t}} \tag{15}$$

$$M_t = \frac{\bar{\delta}_t}{1-\pi_t} \cdot \mu_t \tag{16}$$

We define :

$$\mu_t = \frac{\frac{\sum_{i=1}^{m} \sum_{x=x_r}^{\omega} Nr_i^{(x,t)} \cdot S_i^{(x,t)}}{Nr^t}}{\frac{\sum_{i=1}^{m} \sum_{x=x_0}^{x_r} Na_i^{(x,t)} \cdot S_i^{(x,t)}}{Na^t}} = \frac{D_t^*}{D_t} \tag{17}$$

μ_t is the ratio between the weighted dependency ratio and the dependency ratio at time t.

The pay-as-you-go equilibrium for year t is established iteratively by leveraging a recurrence relationship with the preceding year's equilibrium, thereby intertwining the average benefit ratio, pension rate, and demographic factors. Additionally, the principle of constant risk sharing is upheld throughout this process. We obtain then easily:

[3]The average benefit ratio is the ratio between the average pension for retirees and the average salary of contributors. The average benefit ratio is proportional to the notional salary of retirees while the pension rate relies on the progressive transformation

$$\pi_t = \pi_{t-1} \cdot \frac{\dfrac{\mu_{t-1}}{\mu_t} \cdot D_t^*}{D_{t-1}^* + \pi_{t-1} \cdot (D_t^* \cdot \dfrac{\mu_{t-1}}{\mu_t} - D_{t-1}^*)} \tag{18}$$

$$\bar{\delta}_t = \bar{\delta}_{t-1} \cdot \frac{\dfrac{\mu_{t-1}}{\mu_t}}{1 + \bar{\delta}_{t-1} \cdot (D_t^* \cdot \dfrac{\mu_{t-1}}{\mu_t} - D_{t-1}^*)} \tag{19}$$

The pension formula at time t uses the pension rate δ_t. When we obtain the average benefit ratio from the recurrence (19), we can transform it back into a pension rate via relation (13).

3. Multi-population mortality modelling

The existence of differences in life expectancy between different socio-economic classes of a population has long been documented (see e.g. [Mackenbach & al, 1997] and [Balia & Jones, 2008]), but only recently the availability of time series of data on mortality by class has allowed to develop stochastic models of mortality by sub-populations that allowed to estimate the trend of the phenomenon and to project it. In [Villegas & al., 2005], the authors propose several multi-population extensions of the Lee-Carter model to fit and predict the socio-economic mortality differentials in England. In [Cairns & al., 2019], socio-economic differences in Danish mortality are studied using a affluence index and adopting a multi-population gravity model. In [Wen & al., 2020] 11 multi-population mortality models are fitted to the mortality of Canadian pensioners subdivided by pension level, while in [Wen & al., 2021] a similar set (12) of multi-population mortality models are fitted to 10 distinct socio-economic groups in England.

In order to represent the mortality evolution of pension system members, taking into account the heterogeneity between different economic classes, a multi-population mortality model should be selected. We consider six well-known mortality models that can represent different types of heterogeneity in mortality between the reference sub-populations: absence of heterogeneity, level difference, trend difference. The models considered are reported in Table 1, where $m^i_{x,t}$ is the central mortality rate for a person aged x, in the year t, in the class i.

Model (Acronym)	**Specification**	**Reference**
Common Lee–Carter (CLC)	$\log m^i_{x,t} = \alpha^P_x + \beta^P_x \kappa^P_t$	[Lee & Carter, 1992]
Common Factor (CF)	$\log m^i_{x,t} = \alpha^i_x + \beta^P_x \kappa^P_t$	[Li & Lee, 2005]
Joint-κ (JK)	$\log m^i_{x,t} = \alpha^i_x + \beta^i_x \kappa^P_t$	[Lee & Carter, 1992]
Common Age Effect (CAE)	$\log m^i_{x,t} = \alpha^i_x + \beta^P_x \kappa^i_t$	[Kleinow, 2015]
Augmented Common Factor (ACF)	$\log m^i_{x,t} = \alpha^i_x + \beta^P_x \kappa^P_t + \beta^i_x \kappa^i_t$	[Li & Lee, 2005]
Independent Lee–Carter (ILC)	$\log m^i_{x,t} = \alpha^i_x + \beta^i_x \kappa^i_t$	[Lee & Carter, 1992]

Table 1.: Summary of mortality models for multi-population analysis

The first model (CLC) assumes that the same Lee-Carter model is applied to all the m classes, therefore any form of heterogeneity in mortality between classes is neglected. The common factor model (CF) implies different mortality levels but the same mortality improvements for all sub-population at all times. In order to relax this assumption we consider the joint-κ model (JK), where the time index driving the mortality change is the same for all the sub-populations while the age-specific mortality pattern and the age-specific responses to changes in the level of mortality are population specific. The fourth model considered is a simplified version of the Common-Age-Effect model proposed by ([Kleinow, 2015]). In the CAE model the age-specific responses to changes in the level of mortality is the same for all the sub-populations while the age-specific mortality pattern and the time index driving the mortality change are population specific. In the augmented common factor model (ACF) the term $\beta^i_x \kappa^i_t$ represents deviations in mortality evolution of the sub-population i from the general common trend represented by $\beta^P_x \kappa^P_t$. Finally, we consider to use independent Lee–Carter models for each sub-population (ILC). Note that under this approach the dependence between sub-populations can be represent by appropriately modelling the processes followed by the time indices κ^i_t.

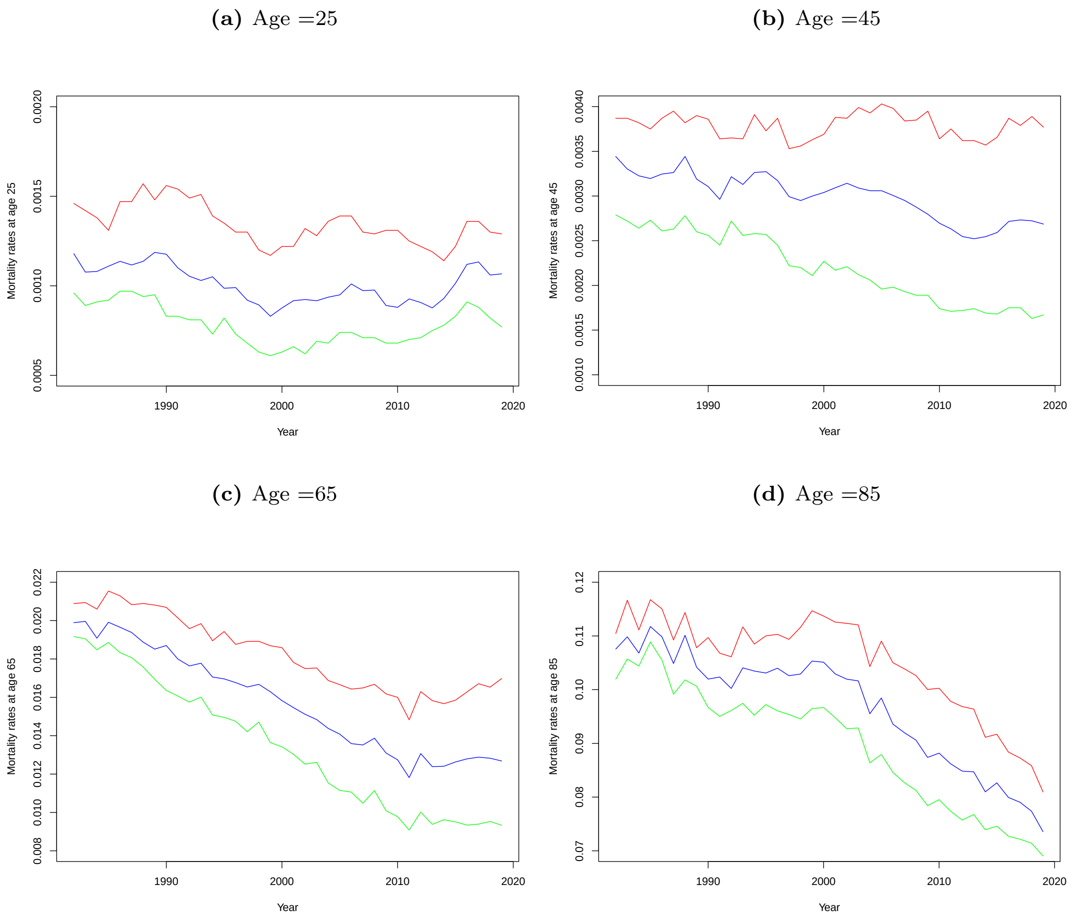
3.1. Mortality data and model comparison

The six mortality models have been compared on mortality data by socioeconomic category in the United States. Data are taken from a study realized by Barbieri and published by the Society of Actuaries (SOA) in 2021 (see [Barbieri, 2021]) that presents mortality rate estimates for the United States by year (from 1982 to 2019) separately by socioeconomic quintile and decile. The mortality indicators are obtained "for groupings of U.S. counties based on their socioeconomic characteristics as measured by county-wide variables on education, occupation, employment, income and housing price and quality."([Barbieri, 2021])

In our application, we take the data by quantile and group the quantiles from 2 to 4 in a single class so that 3 socioeconomic classes are considered[4]: the first class coincides with the first socioeconomic quantile, in the second class there are the three socioeconomic quintiles from the second to the fourth, and the third class coincides with the fifth socioeconomic quantile. Since we are interested in the mortality of people enrolled in a pension system, we do not consider the mortality at young ages, and we restrict our analysis to the age range 25-100. Figure 1 shows the evolution over time of the observed mortality rates for the three classes considered at four ages.

[4]When estimating mortality intensities across five socioeconomic classes, notable disparities predominantly surfaced between the first and last quintiles. Conversely, the three middle quintiles exhibited closely aligned estimated values, prompting our decision to aggregate them.

Figure 1.: Mortality rates by socioeconomic classes (in red the first class, in blue the second, in green the third) at different ages.



The six models considered have been fitted on the mortality data previously described by maximum likelihood estimation (MLE). In order to compare them, we consider both the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC) The log-likelihood, the AIC and the BIC for each model are reported in the Table 2.

The results show that the CLC model consistently yielded the poorest results, highlighting the existence of distinct mortality classes. Additionally, the bad results of the CF model indicates that the differences between classes evolve over time. The results also show that, even taking into account criteria penalising models with muore parameters, the ACF model is consistently and by far the best model.

Table 2.: Log-likelihood and the BIC for the six mortality models (the ranking of the models is reported in brackets).

Model	CLC	CF	JK	CAE	ACF	ILC
Log-Like	-193,494 (6)	-82,518 (5)	-66,106 (3)	-68,979 (4)	-55,495 (1)	-65,383 (2)
AIC	-193,682 (6)	-82,934 (5)	-66,599 (2)	-69,396 (4)	-56,171 (1)	-67,940 (3)
BIC	-194,346 (6)	-84,404 (5)	-68,341 (3)	-70,869 (4)	-58,559 (1)	-67,940 (2)

We also perform a backtest analysis to compare the models. Specifically, we fit the models of the first 28 years data (from 1982 to 2009), then we forecast the death rates for the following 10 years and compare the forecasted values with the observed ones in the years 2010-2019. Mean error (ME), mean squared error (MSE), mean percentage error (MPE) and mean absolute percentage error (MAPE) are then determined to evaluate the goodness of the forecasts. The backtesting procedure as not applied to the 2 models that perform worse in terms of log-likelihood and BIC: the CLC and the CF models. In order to project the mortality rates, assumptions should be made on the processes followed by the time indexes. in particular:

- κ_t^P is modelled as a random walk with drift (ARIMA(0,1,0)) in all the models;
- κ_t^i is modelled as a random walk with drift (ARIMA(0,1,0)) in the CAE model and in the ILC model;
- in the ACF model we made two different assumptions for κ_t^i: in the first case it is modelled as a first-order autoregressive process (AR(1)), while in the second case it is modelled as an ARIMA(1,1,0) process.

The assumption of a random walk with drift process for κ_t^P in all the models and κ_t^i in the CAE and ILC models, is in line with the prevailing literature (see e.g. [Cairns & al., 2011]). With reference to κ_t^i in the ACF model, Le and Lee ([Li & Lee, 2005]) suggest to assume an AR(1) process in order to avoid possible divergence in the mortality evolution of the sub-populations. While it seems appropriate that a multi-population mortality model verifies this condition of no divergence in the mortality evolution of two distinct sub-groups of the same population, on the other hand increasing gaps in life expectancy between different socio-economic groups are observed in several countries: se e.g. [Mackenbach & al, 2003] for 6 Western European Countries including Finland, Sweden, Norway, Denmark, England/Wales, and Italy; [Cristia, 2009] for U.S.; [Villegas & al., 2005] for England; [Cairns & al., 2019] for Denmark). We therefore consider that limiting the analysis to the case of an AR(1) process alone is not appropriate. Moreover, the Augmented Dickey-Fuller test show that the κ_t^i series in the ACF model is not stationary for the first and the third socio-economic groups and the auto.arima function in R suggests second-order ARIMA models for that series. Otherwise, [Haberman & Renshaw, 2009] suggests avoid the use of second order ARIMA(p,2,q) processes, because they could produce excessively wide prediction intervals. In light of these considerations, we found it useful to consider as an alternative hypothesis to the AR(1) process that of an ARIMA(1,1,0) process.

ME, MSE, MPE and MAPE for the five models tested are reported in Table 3.

Table 3.: ME, MSE, MPE and MAPE in backtesting (the ranking of the models is reported in brackets).

Model	JK	CAE	ACF.0	ACF.1	ILC
ME	-0.0035604 (5)	-0.0031407 (2)	-0.0031476 (3)	-0.0025604 (1)	-0.0035300 (4)
MSE	1.0739e-04 (5)	9.2100e-05 (2)	1.0210e-04 (3)	8.5479e-05 (1)	1.0277e-04 (4)
MPE	0.043568 (3)	0.037282 (1)	0.043048 (2)	0.046467 (5)	0.044170 (4)
MAPE	0.098619 (3)	0.10047 (5)	0.096421 (2)	0.094774 (1)	0.099521 (4)

The ACF.1 with the sub-population-specific time index modeled as a random walk with drift is the best mortality model for the populations considered for ME, MSE, and MAPE measures, while the CAE model is the best for the MPE one. Henceforth, we maintain the ACF.1 model as our reference model in the numerical application, with the CAE model serving as the alternative model for comparison. In Table 4 we show the evolution of life expectancy at age 65 for each socioeconomic class from 1982 to 2019 and its projected value to 2039 (best estimate and quantiles at 10 and 90 percent) with the two selected mortality models. We can see that the observed increase in life expectancy was greater for class 3 than for class 2 and greater for the latter than for class 1. Both adopted mortality models predict a greater difference between class 1 and class 2 by 2039, while the difference between class 2 and class 3 decreases slightly with the ACF.1 model and increases slightly with the CAE model.

Table 4.: Life expectancies at age 65: observed (in 1982 and 2019) and projected (for 2039 with ACF.1 and CAE model). $Q_{\alpha\%} := \alpha\%$ quantile; $BE :=$ Best estimate; $SEC :=$ Socio-Economic Class; $\Delta\%^{***} :=$ Percentage change from the last observed value.

	Observed			Projected					
				2039					
Year (model)	1982	2019		(ACF.1)			(CAE)		
				$Q_{10\%}$	BE	$Q_{90\%}$	$Q_{10\%}$	BE	$Q_{90\%}$
$SEC = 1$	16.43	18.67		19.08	19.60	20.13	19.02	19.56	20.10
			$\Delta\%$	*2.2%*	*5.0%*	*7.8%*	*1.9%*	*4.8%*	*7.7%*
$SEC = 2$	16.74	19.86		21.03	21.52	22.02	20.82	21.29	21.76
			$\Delta\%$	*5.9%*	*8.4%*	*10.9%*	*4.8%*	*7.2%*	*9.6%*
$SEC = 3$	16.95	21.00		22.09	22.52	22.95	22.45	22.87	23.28
			$\Delta\%$	*5.2%*	*7.2%*	*9.3%*	*6.9%*	*8.9%*	*10.9%*

4. Risk analysis of Progressive Defined Musgrave schemes

4.1. Trend analysis

We performed a comprehensive numerical analysis of the Progressive Defined Musgrave pension system defined in Section 2, focusing on its mean evolution and risk characteristics.

We extracted the mortality rates by quintile of the population from the Mortality by Socio-economic Category in the United States study ([Barbieri, 2021]). The study produced a set of comprehensive life tables broken down by socio-economic group and year. Details on the data and methodologies used in the development of life tables are summarized in the research report. We made the following initial assumptions.

- The distribution of workers within the salary classes stays the same throughout the whole projection (there is no transition between classes):

  Quintile 1 (Q_1) of salary distribution represents the low salary class.

  Quintiles 2,3,4 (Q_2, Q_3, Q_4) represent the average salary class.

Quintile 5 (Q_5) of salary distribution represents the high salary class.

- As starting salary for each class we used data on household income quintiles [US Census Bureau (2023)], in 1982

$$S_1^{(1982;x_0)} = 4790$$
$$S_2^{(1982;x_0)} = 20675$$
$$S_3^{(1982;x_0)} = 54720$$

- Census data also categorizes people based on their age bracket. The career growth impact refers to the average increase in household income for all working-age groups in 1982. It is given by

$$s_1 = 1\% \; ; \; s_2 = 1.5\% \; ; \; s_3 = 2\%$$

- The time effect is obtained by averaging the yearly evolution of salary in the data through our on our projection horizon, $h = 2.5\%$
- Pension benefits are indexed with the yearly average salary growth rate assumed to be equal to the time effect: $\gamma = 2.5\%$.[5]
- Agents only enter the regime at age $x_0 = 25$. Retirement age is $x_r = 65$.[6]
- $\omega = 100$ is the highest age to which it is possible to survive
- The entry function is assumed constant with $E = 100000$.
- The effective of workers aged x at period t, $Na_i^{(t;x)}$, and the number of retirees aged y, $Nr_i^{(t;y)}$, are given via the relationship $Na_i^{(t;x+1)} = Na_i^{(t;x)} \cdot p_x^i(t)$. Since there is no migration the population function at year t can be obtained recursively from the entry function and the periodic mortality table.
- The observed mortality data ranges form 1982 to 2019. From these periodic mortality tables we obtain the 1 year survival probabilities for socio-economic group j, $p_x^j(t)$.
- We chose the ACF.1 model as the main model to explain subpopulation mortal-

[5]Indexing pensions to wages, is a standard approach in the analysis of PAYG systems, as it maintains the link between pensions and the general evolution of earnings [Alonso-Garcia & Al, 2018]

[6]The entry age and retirement age are in line with [Boado-Penas et al., 2022]

ity, as it is the best of the six models considered. From this model we forecast the central death rates in each class over a period of 20 years and we construct the sub-population periodic mortality tables from the mortality rates. Starting from 2020, the 1 year survival probabilities used in the population effective function are obtained via: ${}_kp^i_x(t) = \exp\left(-\int_x^{x+k} m^i_{s,t} ds\right)$

Starting from the previous assumptions, we construct the demographic dynamics of the system using an entry function. Specifically, we assume that the initial cohort of follows the 1982 mortality table at every age from the entry age up to the maximum age. From 1983 onwards, survival probabilities are updated each year according to the corresponding annual mortality table, until 2019. Entry occurs only at the entry age, while exits are exclusively due to death. The number of actives and retirees in 2019 is $Na = 3908438$ and $Nr = 1657177$, respectively. Consequently, the initial dependency ratio for 2019 is: $D_{2019} = 0.4240$ [7]. The weighted dependency ratio is: $D^*_{2019} = 0.6505$.

The current parameter values result from assuming an initial value of $\delta_{1982} = 0.6$[8]. To maintain equilibrium, the system required a contribution rate of $\pi_{1982} = 0.2786$ and an average benefit ratio of $\bar{\delta}_{1982} = 0.5946$. Based on these starting values, we simulated the evolution of the parameters up to 2019. In 2019, the contribution rate is $\pi_{2019} = 0.3450$, the pension rate[9] is $\delta_{2019} = 0.5435$ and average benefit ratio is $\bar{\delta}_{2019} = 0.5303$. In the starting year, the longevity heterogeneity correction factor in each class are: $\theta_1^{2019} = 1.0667$, $\theta_2^{2019} = 1.0007$ and $\theta_3^{2019} = 0.9454$.

In the following, we present the results of our simulation on the pension system.

- **Dependency ratio**

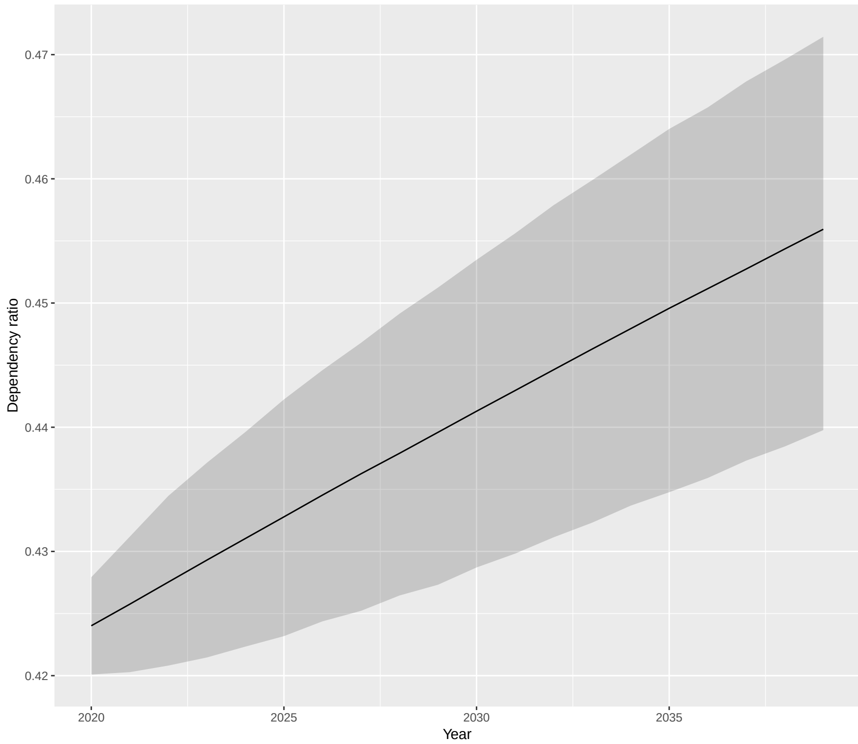
The evolution of the dependency ratio (3) is subject to the projection of mortality intensities forecasted by the ACF.1 model. The forecasted dependency ratio is reported in Figure 2:

[7] The effective of active workers and retirees at any age before the start of the projection, is determined solely by the entry function and 1 year survival probabilities.

[8] The starting value for the replacement rate is fixed like in [Alonso-Garcia & Al, 2018], this is also in line with the Belgian formula

[9] The initial pension rate is a consequence of the PAYG equilibrium condition and is proportional to the choice of the contribution rate

Figure 2.: Forecast of the dependency ratio ACF model

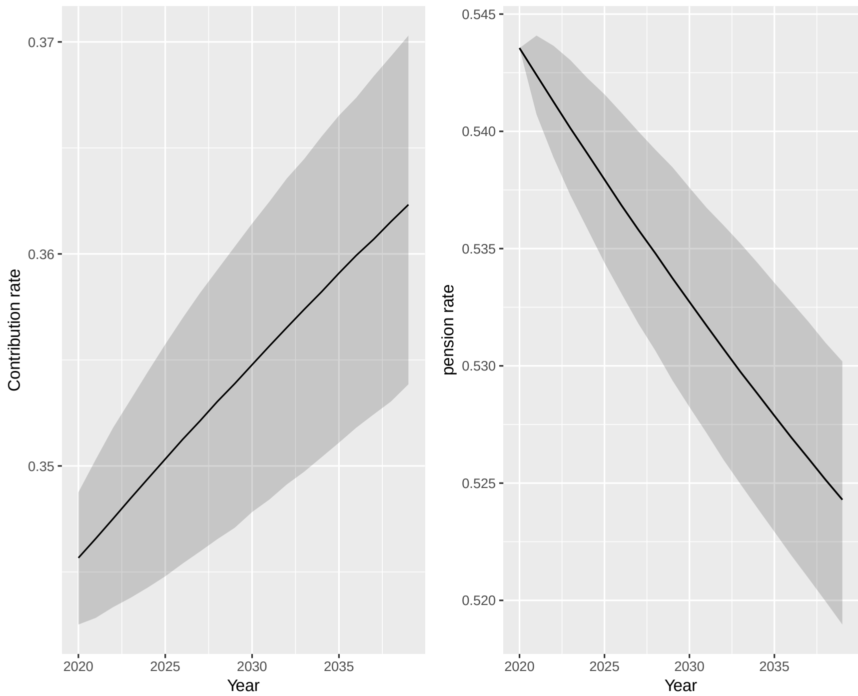


The dependency ratio is expected to increase over the projection horizon. This increase is driven by the mortality evolution as well as the constant entry function hypothesis. The grayed-out area limits represent the 2.5% and 97.5% quantiles of the dependency ratio distribution. Regardless of the evolution of mortality in the sub-populations, the dependency ratio is not very volatile; this is also a consequence of the demographic stability assumption. This has a direct effect on the variations in the inter-generational risk sharing parameters.

- Contribution rate and pension rate

The contribution and pension rates are functions of the dependency ratio via (18), (13) and (19).

Figure 3.: Forecast of the contribution rate and pension rate ACF model



The contribution rate and pension rate reflect both the financial contributions of active workers to the pension system and the standard of living of retirees. They are driven by the evolution of the number of active workers and retirees. In Table 5 is reported their average value through the forecast.

Table 5.: Average contribution rate and pension rate evolution ACF model

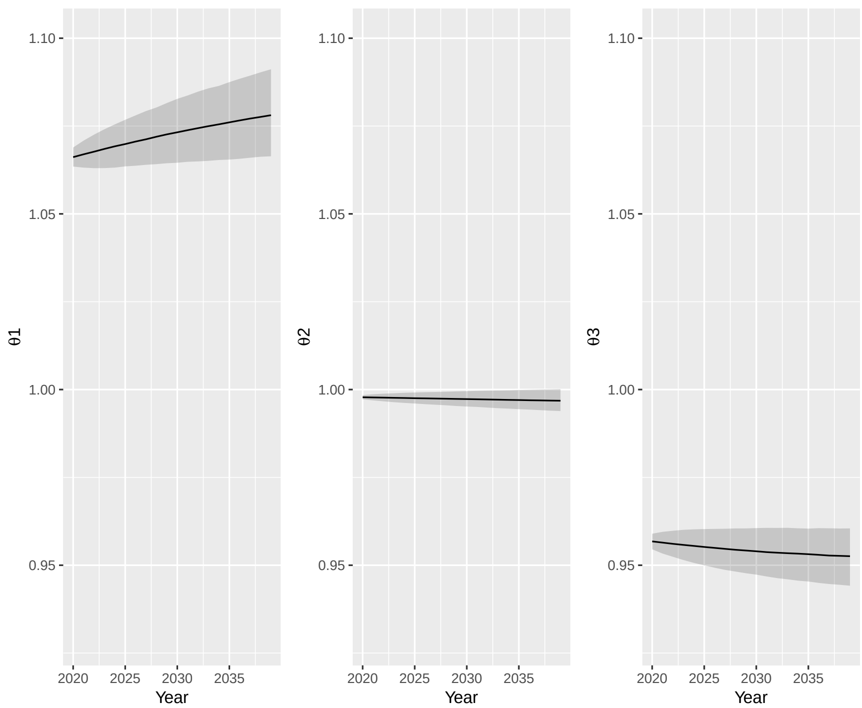
t	π_t	δ_t
2020	0.3456	0.5435
2024	0.3494	0.5390
2039	0.3622	0.5243

The numerical application of the contribution rate and pension rate reveals important insights about the financial dynamics within the pension system. The contribution rate is expected to increase in the ACF average forecast slightly from 34.56% in 2020 to 36.22% by 2039, highlighting the growing financial burden on contributors over time. Similarly, the pension rate is projected to decrease from 54.35% to 52.43% by 2039, reflecting the sensitivity of retirees to changes in the pension rate and its potential impact on post-retirement income. However, it is worth noting that the contribution rate and pension rate are less sensitive to the model induced variations due to the evolution of the dependency ratio and the demographic assumptions.

- longevity heterogeneity correction factor

The forecasted evolution of θ_i^t from eq.(9), is reported in Figure 4

Figure 4.: Forecast of the longevity heterogeneity correction factor ACF model



The grayed-out area limits represent the 2.5% and 97.5% quantiles of the longevity heterogeneity correction factor distribution. Table 6 summarizes the average value for the factor in each class.

Table 6.: Evolution of the average longevity heterogeneity correction factor in each class ACF model

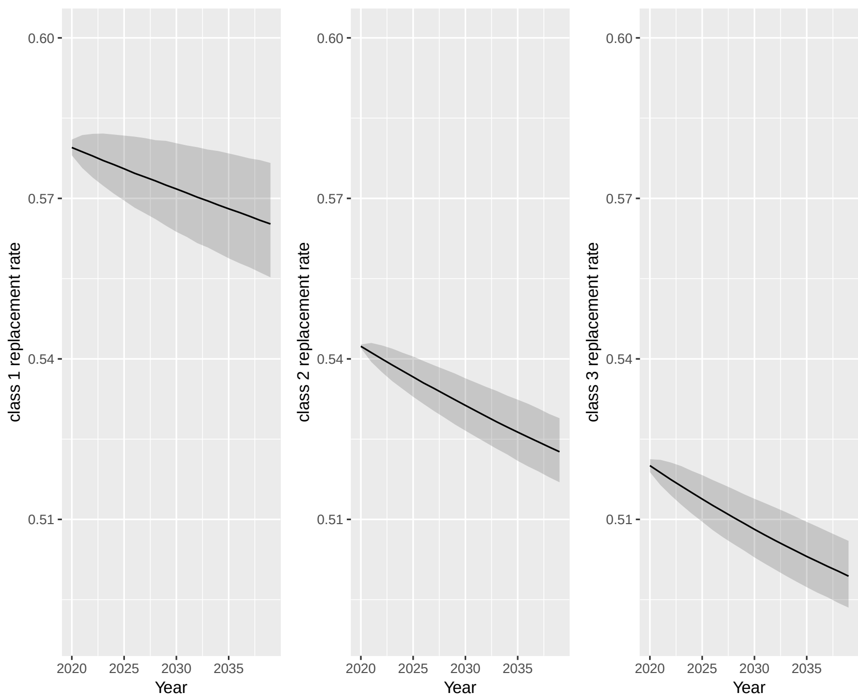
	θ_1	θ_2	θ_3
2020	1.0661	0.9977	0.9567
2024	1.0692	0.9974	0.9553
2039	1.0781	0.9967	0.9424

The mortality intensity model has more influence on the variations of the longevity heterogeneity correction factor than on the dependency ratio.

- Class replacement rate

The class replacement rate at retirement is defined via equation 10. It expresses how the longevity differences between classes affect the replacement rate in every class.

Figure 5.: Forecast of the class replacement rate ACF model



The class replacement rate at retirement of each sub-population combines the two adjusting effects. Its distribution is directly influenced by the mortality model selected for the forecast.

Table 7.: Average class replacement rate at 65

	ACF		
	$Tr_1^{(65,t)}$	$Tr_2^{(65,t)}$	$Tr_3^{(65,t)}$
2020	0.5795	0.5423	0.5200
2024	0.5763	0.5377	0.5150
2039	0.5654	0.5227	0.4994

When comparing the evolution of the class replacement rate over the projection horizon to the initial pension rate $\delta_{2020} = 0.5435$, we note that the positive longevity correction notably elevates the class replacement rate for the low-income group. Despite the reduction in pension rate following the Musgrave rule, their class replacement rate remains higher than the initial level. Conversely, for the high-income group, the class replacement rate decreases twice: first due to the impact of the longevity heterogeneity correction factor, and then due to the pension rate adjustment.

In order to analyse the mortality evolution effect on the class replacement rate we are looking into the class replacement rate differences between two classes i and j presented as such:

$$Tr_i^{(x_r,t)} - Tr_j^{(x_r,t)} = \delta_t \cdot (\theta_i^t - \theta_i^t) = \delta_t \cdot d_{i,j}^t, \tag{20}$$

with $d_{i,j}^t = \theta_i^t - \theta_j^t$ measuring the difference in longevity heterogeneity correction factor between the two classes.

Figure 6.: Forecast of the longevity heterogeneity correction factor distances ACF model

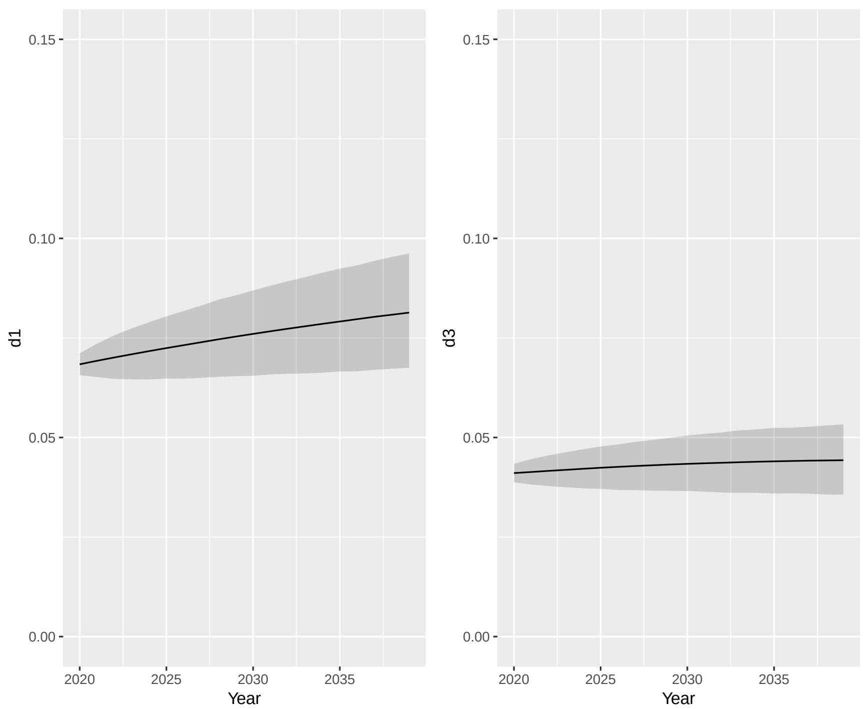


Figure 6 shows the projected difference in longevity heterogeneity correction factor between socioeconomic class 1 and 2 and class 2 and 3 in the ACF model. The trend for the low income class is that the difference is widening over time, meaning that the progressive nature of the pension system is becoming more pronounced. Conversely, the disparities for the high-income group relative to the average group appear to increase at a slower rate over time.

4.2. Quantile analysis

The Progressive Defined Musgrave system is characterized every period t by the triplet $(\pi_t; \delta_t; \theta_t)$, representing respectively the contribution rate, the pension rate, and the longevity heterogeneity correction factor. While Section 4.1 analyzed the mean evolution of these quantities, in a stochastic framework it is important to assess the risk exposure associated with their stochastic dynamics. More specifically, the purpose of this section is to analyze and monitor the level of risk imposed by a progressive pension formula, as well as compare risk exposure for the two stakeholders: contributors are sensitive to an increase in the contribution rate, and retirees are sensitive to a decrease of the pension rate.

- **Contribution rate and pension rate**

We first examine the potential variations in the contribution and pension rates by comparing the Value-at-Risk (VaR) at a specified yearly confidence level of $u = 0.995$. To compute the long-term quantiles, we employ the maturity approach, as presented by [Devolder & Lebègue, 2016], which has also been incorporated within the SII (Solvency 2) legislation through the duration-based equity risk sub-module. This approach is particularly suited for the analysis of assets and liabilities with prolonged time horizons and distinctive characteristics.

In our analysis, parameter quantiles are computed employing a risk measure, while the confidence level employed is regarded as a probability, consistent with the Probabilistic criterion proposed by Hürlimann [Hürlimann, 2010].

The corrected quantiles at maturity represent the probability that the longevity correction does not exceed a particular quantile, adjusted based on a confidence level of 0.995 raised to the power of T. Accordingly, we evaluate the 5-year and the 20-year Value-at-Risk [10] of the contribution rate via

$$VaR_{u^T}(\pi_T) = \inf_{l \in \mathbb{R}} \ \mathbb{P}[\pi_T \leq l] > u^T \tag{21}$$

For the pension rate, we are looking at scenarios corresponding to a loss for the retirees. We evaluate the 5-year and the 20-year Value-at-Risk of the pension rate rate via

$$VaR_{1-u^T}(\delta_T) = \inf_{l \in \mathbb{R}} \ \mathbb{P}[\delta_T \leq l] < (1 - u^T) \tag{22}$$

.

Table 8.: Value-at-risk evolution for the contribution rate and pension rate ACF model

$VaR_{0.975}(\pi_5)$	$VaR_{0.90}(\pi_{20})$	π_{20}	$VaR_{0.025}(\delta_5)$	$VaR_{0.1}(\delta_{20})$	δ_{20}
0.3544	0.3675	0.3622	0.5358	0.5207	0.5243

The Value-at-Risk results reported in Table 8 show that both the contribution rate and the pension rate exhibit very limited tail variability. The difference between the VaR and the expected value is below 1% even at the 20-year horizon. This implies that the automatic Musgrave adjustment stabilizes the PAYG equilibrium, effectively absorbing demographic shocks. From an economic point of view, this low volatility means that contributors face a limited risk of sudden increases in contribution rates, while retirees are protected against large downward corrections in benefits. In probabilistic terms, the system's sustainability risk is small, confirming the capacity of the Progressive Defined Musgrave mechanism to maintain intergenerational balance under uncertainty. Nevertheless, the variability of these rates over time poses financial challenges for both contributors and retirees.

[10]The probability level is adjusted according to the horizon,$1 - 0.995^5 \approx 0.025$ and $1 - 0.995^{20} \approx 0.1$

- **longevity heterogeneity correction factor**

The variations of the longevity heterogeneity correction factor both on the left and right tail of the distribution affect the pension benefit level within all classes. We will focus on the quantiles ($Q_{u^T/2}$ and $Q_{1-u^T/2}$)of their distribution for a probability level u^T given by the VaR adjusted confidence level.

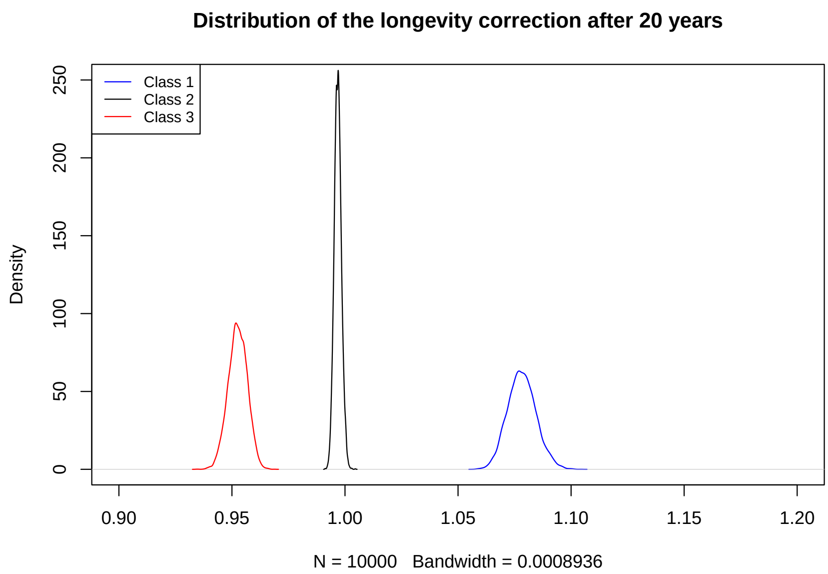
Table 9.: 0.1 and 0.9 Quantiles of the longevity correction ACF model

	$\theta_1(20)$	$\theta_2(20)$	$\theta_3(20)$
$Q_{0.1}$	1.0691	0.9941	0.9474
$Q_{0.9}$	1.0875	0.9996	0.9575

Table 9 shows the 0.1 and 0.9 quantiles of the longevity correction in the ACF model for three socioeconomic classes. The inter-quantile range is wider for the low income group, indicating that the distribution of longevity heterogeneity is more spread out for the low-income class than for the average and high-income classes.

In the PDM system, the variations in the longevity correction are more important for the low-income group, as the longevity gains in this group are less important relative to the total population. This translates into a more advantageous correction for the low-income group. In Figure 7, we present the distribution of correction at the projection horizon ($T = 20$):

Figure 7.: Distribution of the longevity heterogeneity correction factor after 20 years, ACF model

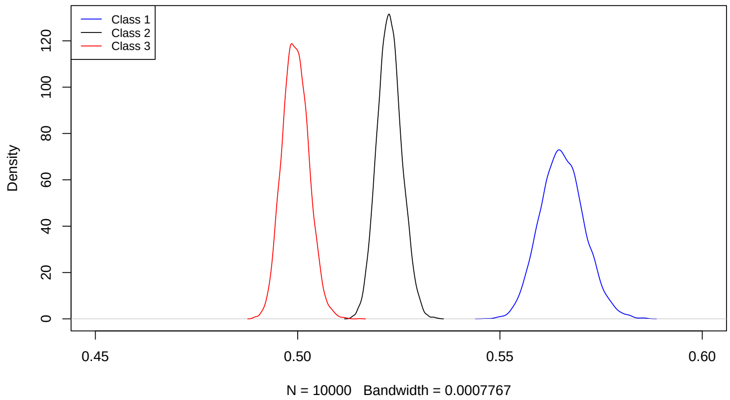


The support of the distribution of the correction factor is clearly disjoint for the three classes at the projection horizon.

- **Class replacement rate**

We next analyze how the two adjustment mechanisms proposed propagate to the replacement rates by class. Analyzing the distribution of class replacement rates at the projection horizon (Figure 8) provides insights into the fairness and variability within the pension system.

Figure 8.: Class replacement rate distribution after 20 years, ACF model



When comparing the Figure 7 and Figure 8, we observe that the shape of each distribution are similar due to the proportionality between the longevity correction factor and the class replacement rate. We also focuses on the distribution of the class replacement rate differences. We presented in figure 6 that over time the evolution differed between low and high income classes. Figure 9 shows the distribution of the differences at the horizon of projection.

Figure 9.: Class replacement rate distance

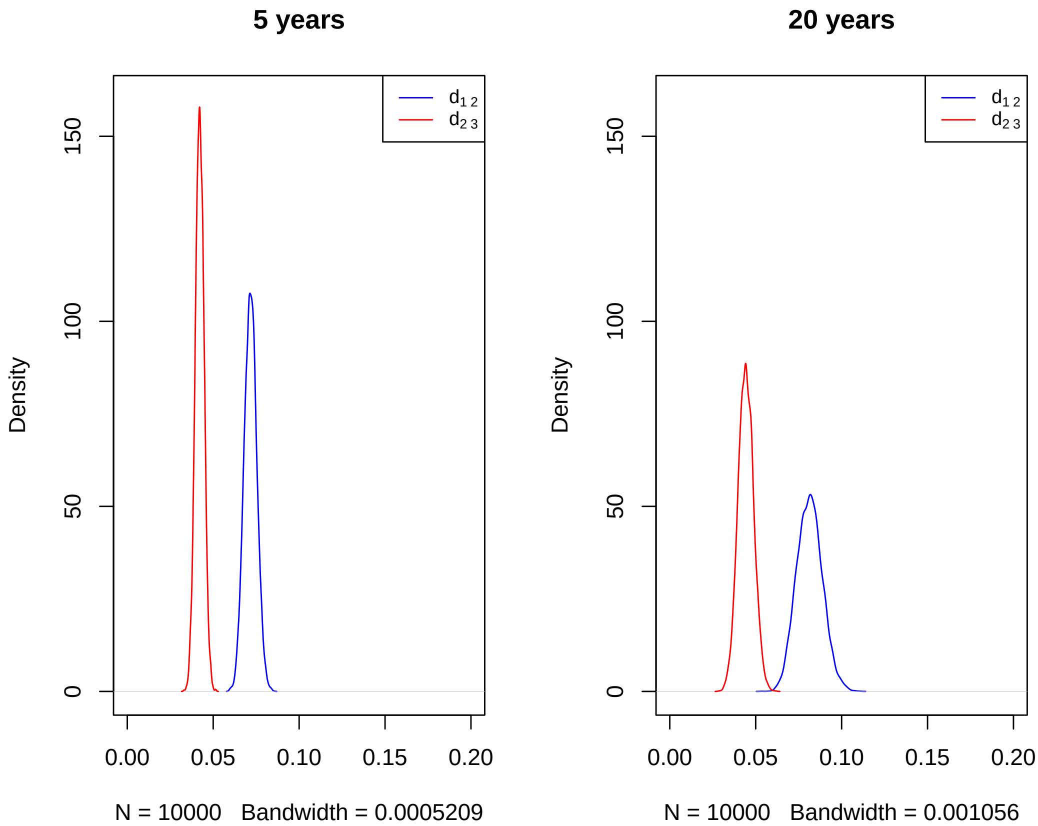


Table 10 reports the interdecile gaps of the class replacement rate distributions at the projection horizon.

Table 10.: interdecile gap in the ACF model at the projection horizon

	$Q_{0.1}$	$Q_{0.9}$	$Q_{0.9} - Q_{0.1}$
$Tr_1^{(65,2039)}$	0.5640	0.5794	1.54%
$Tr_2^{(65,2039)}$	0.5234	0.5409	1.75%
$Tr_3^{(65,2039)}$	0.5000	0.5185	1.85%

It is notable that the interdecile gaps increase with income class, revealing that high-income groups experience greater variability in replacement outcomes. This pattern is economically consistent: higher-income individuals face higher exposure to longevity and return uncertainty, while lower-income individuals are partially protected by the progressive transformation in the benefit formula. Consequently, the quantile analysis confirms that the system not only remains stable overall but also preserves progressivity under uncertainty, ensuring that risk is redistributed in a socially equitable manner.

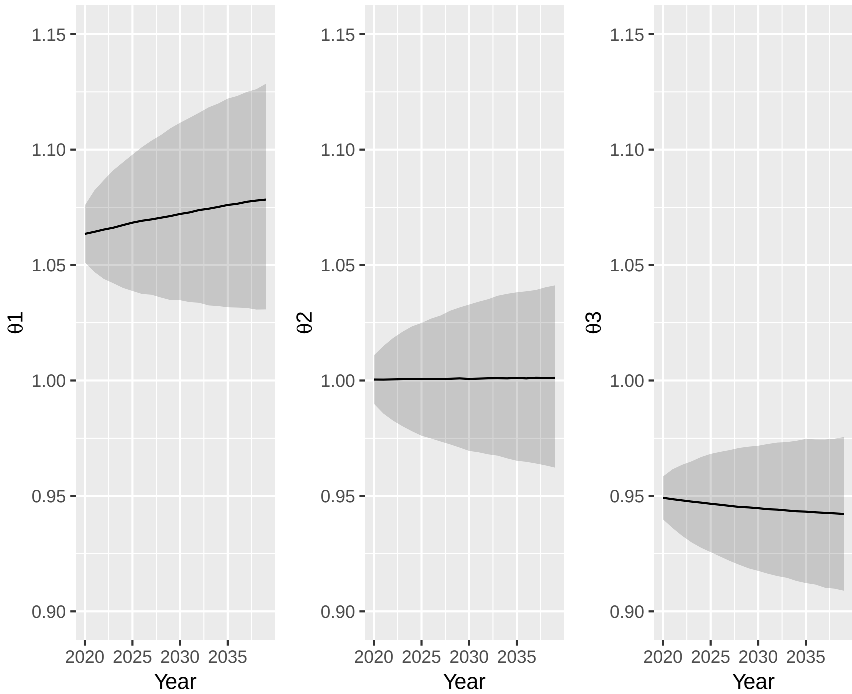
Therefore, the quantile analysis highlights two complementary results. At the macro level (Table 8), the PDM system exhibits strong stability and limited systemic risk. At the micro level (Table 10), progressivity is preserved even under stochastic conditions: risk is redistributed toward higher-income groups, while low-income groups retain higher and more stable replacement rates. Together, these findings confirm that the PDM system achieves its dual objective of financial sustainability and social fairness under demographic uncertainty.

4.3. Risk-model analysis

In order to test the sensitivity of the results to alternative demographic evolution, we present in this section the results of the PDM system using for the mortality pattern the CAE model instead of the ACF.1 one.

- **longevity heterogeneity correction factor**

Figure 10.: Forecast of the longevity heterogeneity correction factor CAE model



The forecast of the longevity heterogeneity correction factor in the CAE model, as shown in Figure 10, reveals a noticeable dispersion compared to the ACF model. Unlike the ACF model, the extremes quantiles of the correction distribution in the CAE model appear to overlap at the projection horizon, indicating potential variations in the correction for each socioeconomic class. To further investigate these variations, we analyze the 20-year quantiles of the longevity correction for the CAE model, as presented in Table 11. The $Q_{0.1}$ and $Q_{0.9}$ quantile values provide insights into the potential range of losses or gains in the correction for each socioeconomic class after 20 years. Contrary to expectations, the quantile analysis reveals consistent correction outcomes across different income classes. Although the quantiles of the distribution in Figure 10 appeared to intersect with this probability level, indicating a possible overlap, our findings affirm the persistent progressive nature of the system across classes at the projection horizon.

- **Class replacement rate**

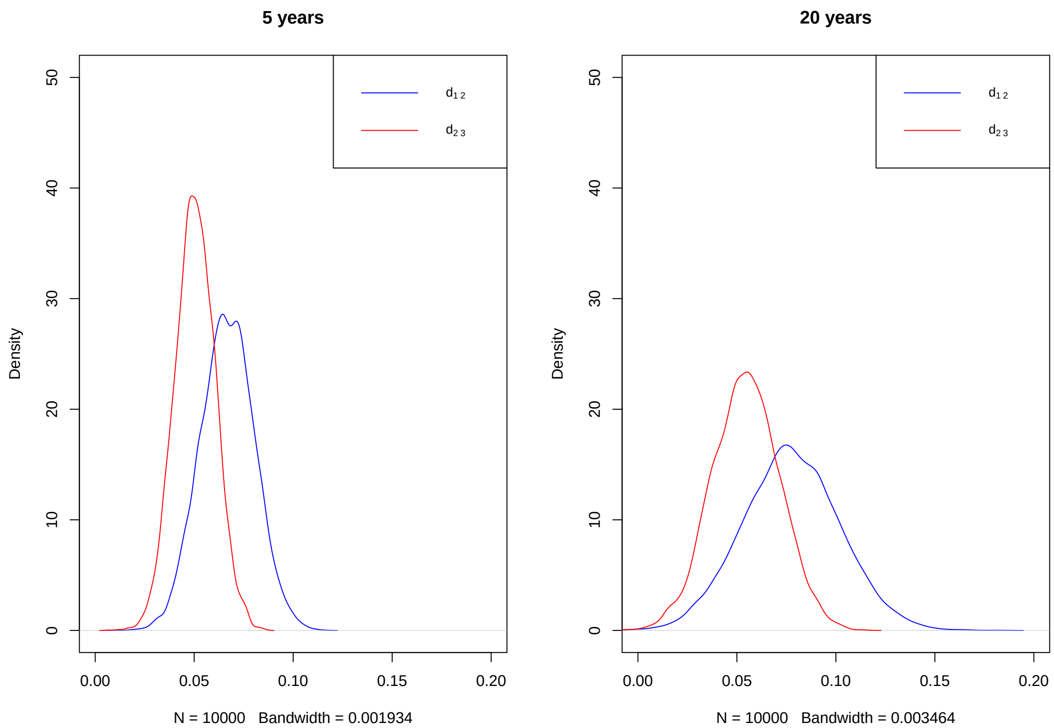
From Table 11 we notice that the CAE model underestimates the mortality for the

Table 11.: 20-year Quantiles of the longevity correction CAE model

	$\theta_1(20)$	$\theta_2(20)$	$\theta_3(20)$
$Q_{0.1}$	1.0470	0.9761	0.9206
$Q_{0.9}$	1.1110	1.0268	0.9636

low-income population relative to the total population and overestimates the mortality for the high-income population. When observing the differences in class replacement rate projected in the CAE model in Figure 11, the distances are not as pronounced as in the ACF model for the low income group compared to the high income group.

Figure 11.: Class replacement rate distance distribution, CAE model



This indicates that, despite increased variability in the CAE model, the system remains progressive, as evidenced by consistently positive distances. Even at the left tail of the distribution of the class replacement rate distance, the PDM system ensures progressivity in the class replacement rate, which is reassuring for the acceptability of such a system.

Table 12.: Interdecile gap in the CAE model at the projection horizon

	$Q_{0.1}$	$Q_{0.9}$	$Q_{0.9} - Q_{0.1}$
$Tr_1^{(65,2039)}$	0.5498	0.5821	2.60%
$Tr_2^{(65,2039)}$	0.5107	0.5400	2.66%
$Tr_3^{(65,2039)}$	0.4904	0.5081	2.76%

The interdecile gap in the CAE model at the projection horizon (Table 12) is greater that in the ACF.1 model. This confirms the more important variability already observed in the longevity heterogeneity correction factor.

We observe that the longevity heterogeneity correction factor is directly affected by the choice of mortality model. The ACF.1 model shows limited overlaps in the distribution of the correction, indicating a progressive aspect of the class replacement rates. On the other hand, when comparing the results with the CAE model, the overlaps in the distribution become more evident. This highlights the risk associated with distribution overlap, as it leads to a loss of the progressive nature of the class replacement rates.

The variations in the longevity heterogeneity correction factor have important implications for the pension system. They can affect the benefit levels for different socioeconomic classes, potentially leading to disparities in pension outcomes.

Those observations underscore the importance of carefully selecting the mortality model and considering the implications of the longevity heterogeneity correction. It highlights the need for equitable pension systems that account for variations in longevity and socioeconomic factors while ensuring fair and progressive class replacement rates for retirees.

5. Conclusion

In many countries, demographic and economic evolutions put an important pressure on the sustainability of PAYG pension schemes. Fairness between successive generations should force the public authorities to find a better equilibrium and risk sharing in terms of contributions and benefits. However, the implementation of pension reforms should also integrate important socio-economics disparities. Figures in many countries show indeed important differences in terms of life expectancy between socio-economic classes. Therefore, financial sustainability, inter-generational risk-sharing and intra-generational fairness should be simultaneously taken into account.

This paper has proposed an answer to this double challenge, by mixing two automatic adjustment mechanisms: the first one based on the Musgrave rule for the intra generational financial equilibrium; the second one based on progressive pension rates for the socio-economic aspect.

The two mechanisms are introduced in a stochastic framework in which, consistent with empirical evidence, the heterogeneity of longevity between different socio-economic classes is represented not only by different levels of initial survival rates but also by different trends.

The projections highlight that applying the Musgrave rule enables a more equitable sharing of the financial impact of demographic ageing between workers and retirees. Over time, contribution rates adjust upward while pension benefits adjust modestly downward, reflecting a gradual and balanced redistribution of costs. This approach supports the long-term sustainability of the pension system without placing undue strain on the active population or undermining the adequacy of retirement income, offering a pragmatic path for policymakers seeking fairness and financial stability.

Moreover, implementing progressive pension rates based on socioeconomic status enables differentiated replacement rates across income groups, enhancing the social acceptability of benefit adjustments. In this framework, lower-income individuals retain relatively higher replacement rates compared to their more affluent counterparts, helping to preserve retirement adequacy for the most vulnerable while aligning the system with principles of equity.

From a policy perspective, several insights emerge from this framework. First, automatic adjustment mechanisms offer a pragmatic alternative to politically sensitive ad hoc reforms, enabling smoother adaptation to demographic shifts. Second, explicitly accounting for longevity disparities is essential to ensure fairness within generations. Third, the persistence of outcome disparities under different mortality assumptions highlights the need for differentiated policy tools rather than uniform solutions. In particular, recent experiences of pension reform in Europe (for instance France or Belgium) illustrate the importance of this feeling of fairness between social classes, in order to get acceptance of a new pension system. Increase of life expectancy is often advocated by regulators to postpone uniformly the retirement age or cut linearly the benefits. Acceptance of these principles can be threatened because of the existence of different evolutions of life expectancy between classes. The double mechanism proposed in this paper tries to answer this dilemma by simultaneously:

- taking into account sustainability constraints by a global adaptation of contributions and benefits;
- avoiding unfair consequences through a segmented socio-economic approach based on observable and available data (salaries).

Finally, transparent communication about these mechanisms is crucial to enhance public understanding and social legitimacy. In summary, sustainable pension reform requires the integration of financial stability, intergenerational solidarity, and intragenerational social justice. These goals can be achieved through dynamic, differentiated, and transparent tools such as those proposed in this study.

List of Figures

List of Tables

References

[Alonso-Garcia & Al, 2018] Alonso-García, Jennifer, María del Carmen Boado-Penas, and Pierre Devolder. "Adequacy, fairness and sustainability of pay-as-you-go-pension-systems: defined benefit versus defined contribution." The European Journal of Finance 24.13 (2018): 1100-1122.

[Antonio & al, 2015] Antonio, K., Bardoutsos, A., Ouburg, W. (2015). Bayesian Poisson log-bilinear models for mortality projections with multiple populations. Eur. Actuar. J. 5(2), 245–281 (2015)

[Ardito & Costa, 2022] Ardito, C., Costa, G. (2022). Could a fairer retirement age mitigate health inequalities? Evidence and decision-making. Frontiers in Public Health, 10, 965140.

[Arnold & Jijie, 2020] Arnold, S., Jijiie, A. (2020). "Retirement Ages by Socio-Economic Class" Risks 8, no. 4: 102. https://doi.org/10.3390/risks8040102

[Balia & Jones, 2008] Balia, S., Jones, A.M. (2008). Mortality, lifestyle and socio-economic status. Journal of Health Economics, 27(1), 1–26.

[Barbieri, 2021] Barbieri, M. (2021). Mortality by Socioeconomic Category in the United States. SOA Research Institute.

[Boado-Penas et al., 2022] Boado-Penas, M., Haberman, S., Naka, P. (2022). Fairness and annuity divisors for notional defined contribution pension schemes. Journal of Pension Economics & Finance, 21(2), 143-167. doi:10.1017/S1474747220000311

[Cairns & al., 2011] Cairns, A.J.G., Blake, D., Dowd, K., Coughlan, G.D., Epstein, D., Khalaf-Allah, M. (2011). Mortality Density Forecasts: An Analysis of Six Stochastic Mortality Models. Insurance: Mathematics and Economics, 48(3), 355–367.

[Cairns & al., 2019] Cairns, A.J.G., Kallestrup-Lamb, M., Rosenskjold, C., Blake, D., Dowd, K. (2019). Modelling socio-economic differences in mortality using a new affluence index. ASTIN Bulletin, 49(3), 555–590.

[Cristia, 2009] Cristia, J.P. (2009). Rising mortality and life expectancy differentials by lifetime earnings in the United States. Journal of Health Economics, 28, 984–995.

[Delbon & al., 2011] Debón, A., Montes, F., Martínez-Ruiz, F. (2011). Statistical methods to compare mortality for a group with non-divergent populations: an application to Spanish regions. European Actuarial Journal 1, 291–308.

[Devolder & Lebègue, 2016] Devolder, P., Lebègue, A. (2016). Iterated VaR or CTE measures: A false good idea?, *Scandinavian Actuarial Journal* pp. 1–32. DOI: 10.1080/03461238. 2015.1126343

[Devolder & Diakite , 2021] Diakite, K.; Devolder, P. Progressive Pension Formula and Life Expectancy Heterogeneity. Risks 2021, 9, 127. https://doi.org/10.3390/risks9070127

[Dudel & Schmied, 2019] Dudel, C., Schmied, J. (2019). Pension adequacy standards: An empirical estimation strategy and results for the United States and Germany (MPIDR Working Paper No. WP 2019-003). Rostock, Germany: Max Planck Institute for Demographic Research.

[Eurostat, 2011] Migrants in Europe A statistical portrait of the first and second generation, Statis- tical Books, Luxembourg: OECD Publishing. ISBN 978-92-79-16231-2 doi:10.2785/5318

[Forster & al, 2018] Forster, T., Kentikelenis, A., Bambra, C. (2018). Health inequalities in Europe: setting the stage for progressive policy action.

[Haberman & Renshaw, 2009] Haberman, S., Renshaw, A. (2009). On Age-Period-Cohort Parametric Mortality Rate Projections. Insurance: Mathematics and Economics 45: 255–270.

[Holzmann et al., 2020] Holzmann, R., Alonso-Garcia, J., Labit-Hardy, H., Villegas, A. (2020). NDC schemes and heterogeneity in longevity: Proposals for redesign in Holzmann, R., Palmer, E., Palacios,R., & Sacchi, S. (Eds.) *Progress and Challenges of Nonfinancial Defined Contribution Pension Schemes, Volume 1:*

Addressing Marginalization, Polarization, and the Labor Market (pp. 307-332). Washington, DC: World Bank. doi:10.1596/978-1-4648-1453-2.

[Hürlimann, 2010] Hürlimann, W. (2010). Solvency risk capital for the short and long term: probabilistic versus stability criterion. *Belg. Actuar. Bull*, 9(1), 8-16.

[Jevtić & al, 2023] Jevtić, P., Qin, C., Zhou, H. (2023). Multi-population mortality modeling with Lévy processes. Decisions in Economics and Finance, 1-27.

[Jijiie & al., 2022] Jijiie, A., Alonso-García, J., Arnold, S. (2002). Mortality by socio-economic class and its impact on the retirement schemes: how to render the systems fairer?. European Actuarial Journal, 12, 701–743.

[Kleinow, 2015] Kleinow, T. (2015). A common age effect model for the mortality of multiple populations. Insurance: Mathematics and Economics, 63(1): 147–152. doi: 10.1016/j.insmatheco.2015.03.023

[Lee & Carter, 1992] Lee, R., Carter, L. (1992). Modeling and forecasting U.S. mortality. Journal of the American Statistical Association, 87: 659–671.

[Li & Lee, 2005] Li, N., Lee, R.D. (2005). Coherent mortality forecasts for a group of populations: An extension of the Lee-Carter method. Demography, 42(3): 575–594.

[Mackenbach & al, 1997] Mackenbach, J.P., Kunst, A.E., Cavelaars, A.E., Groenhof, F., Geurts, J.J. (1997). Socioeconomic inequalities in morbidity and mortality in western Europe. The Lancet, 349(9066), 1655–1659.

[Mackenbach & al, 2003] Mackenbach, J.P., Bos, V., Andersen, O., Cardano, M., Costa, G., Harding, S., Reid, A., Hemström, Ö., Valkonen, T., and Kunst, A.E. (2003). Widening socioeconomic inequalities in mortality in six Western European countries. International Journal of Epidemiology, 32: 830-837.

[Majer & al, 2011] Majer, I.M., Nusselder, W.J., Mackenbach, J.P., Kunst, A. E. (2011). Socioeconomic inequalities in life and health expectancies around official retirement age in 10 Western-European countries. Journal of Epidemiology and Community Health, 65, 972–979.

[Musgrave & Thin, 1948] Musgrave, R.A., Thin, T. (1948). Income tax progression, 1929-48. Journal of political Economy, 56(6), 498-514.

[David, 2017] David, G. R. U. B. B. "Preventing Ageing Unequally-Key findings for

France.” (2017).

[Queisser & Whitehouse, 2006] Queisser, M., Whitehouse, E. (2006). Neutral or fair?: actuarial concepts and pension-system design.

[Shi & Kolk, 2022] Shi, J., Kolk, M. (2022). How does mortality contribute to lifetime pension inequality? Evidence from five decades of Swedish taxation data. Demography, 59(5), 1843-1871.

[Sheshinski & Caliendo, 2021] Sheshinski, E., Caliendo, F. N. (2021). Social security and the increasing longevity gap. Journal of Public Economic Theory, 23(1), 29-52.

[Settergren & Mikula, 2005] Settergren, Ole, and Boguslaw D. Mikula. ”The rate of return of pay-as-you-go pension systems: a more exact consumption-loan model of interest.” Journal of Pension Economics & Finance 4.2 (2005): 115-138.

[Strozza & al, 2022] Strozza, C., Vigezzi, S., Callaway, J., Kashnitsky, I., Aleksandrovs, A., Vaupel, J. W. (2022). Socioeconomic inequalities in survival to retirement age: a register-based analysis

[Sundberg & al, 2023] Sundberg L., Agahi N., Wastesson J.W., Fritzell J., Fors, S. (2023). Increasing inequalities in disability-free life expectancy among older adults in Sweden 2002-2014. Scand J Public Health. 2023 Aug;51(6):835-842. doi: 10.1177/14034948211062309. Epub 2021 Dec 29. PMID: 34965792; PMCID: PMC10350709.

[TUC report, 2013] Trades Union. (2013). Life expectancy inequalities and state pension outcomes. TUC.

[US Census Bureau (2023)] US Census Bureau Historical Income tables. March 2023 `https://www.taxpolicycenter.org/statistics/household-income-quintiles`

[Ventura-Marco & Vidal-Meliá ,2014] Ventura-Marco, Manuel, and Carlos Vidal-Meliá. ”An Integrated Notional Defined Contribution (NDC) Pension Scheme with Retirement and Permanent Disability” `https://www.scor.com/sites/default/files/2014_es_anintegratednotionaldefinedcontributionpensionschemewithretirementandpermanentdisabi venturayvidal.pdf`

[Villegas & al., 2005] Villegas, A., Haberman, S., Kaishev, V., Millossovich, P., (2017). A comparative study of two-population models for the assessment of basis risk in longevity hedges. ASTIN Bulletin: The Journal of the IAA, 47(3): 631-679. doi:10.1017/asb.2017.18

[Wen & al., 2020] Wen, J., Kleinow, T., Cairns, A.J.G. (2020). Trends in Canadian Mortality by Pension Level: Evidence from the CPP and QPP, North American Actuarial Journal. 24:4. 533-561. DOI: 10.1080/10920277.2019.1679190

[Wen & al., 2021] Wen, J., Cairns, A.J.G., Kleinow, T. (2021). Fitting multi-population mortality models to socio-economic groups. Annals of Actuarial Science 15, 144–172. DOI: 10.1017/S1748499520000184

[Wilmoth & al, 2023] Wilmoth, J. R., Bas, D., Mukherjee, S., Hanif, N. (2023). World Social Report 2023: Leaving No One Behind in an Ageing World. UN.