Variance and Skewness of Current Fluctuations Experimentally Evidenced in Single-Photon Avalanche Diodes

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***Abstract*—Consistently with recently published theoretical work, we experimentally demonstrate that, in electronic devices exhibiting strongly nonlinear current-voltage characteristics, the white noise cannot be purely Gaussian, as predicted by the stochastic thermodynamic relations. Micro-fabricated silicon single-photon avalanche diodes exemplify this result. Relying on high-accuracy time-domain noise measurements, we go beyond the conventional characterization of fluctuations in terms of mean and variance only by also assessing the third moment (skewness).**

I. Introduction

Noise in electronic devices and circuits is most often modelled and simulated as *Gaussian* [1], [2]. Therefore, characterization of MOS transistor current *fluctuations* usually focuses on *mean* (first moment) and *variance* (second central moment), as well as on the related *power spectral density* (PSD), for noise analysis and compact modelling [3], [4].

This assumption is however challenged from the point of view of thermodynamic consistency by a recently presented contribution [5]. From fundamental *stochastic thermodynamic* relations [6], previous work [5] deduces that, in an arbitrary *nonlinear* device, the variance of *white* noise exceeds the prediction of the extended Johnson-Nyquist's formula [5]. As a consequence of this result, the *skewness* (here defined as the third standardized moment) of the fluctuations cannot be zero, i.e. the noise is *not* rigorously Gaussian. In [5], the variance and skewness of the shot noise in diodes (PN junctions) and of thermal noise in MOS transistors are discussed based on simplified theoretical models only. Skewness of current fluctuations was experimentally reported in a tunnel junction [7], [8], off-the-shelf avalanche diodes [9], and quantum devices [10].

In the present work, we have performed high-accuracy noise measurements of micro-fabricated *single-photon avalanche diodes* (SPAD) [11] in the time domain (Section III). When biased in or near the avalanche regime, the SPAD exhibits a strongly nonlinear current-voltage characteristic and a dominant white noise behaviour that we aim to study. Preliminarily, the stochastic thermodynamic background necessary to rigorously introduce and discuss the theoretical result of [5] is presented in Section II, as well as related experimental work. In Section IV, the non-zero skewness of current fluctuations in SPAD and hence their non-Gaussianity is highlighted. Section V draws conclusions of this paper.

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II. Previous Work

A. Stochastic Thermodynamic Considerations

A SPAD is a dipole that can generally be modelled as a nonlinear dissipative device, here subjected to a constant voltage difference V applied externally. The resulting current $i(t)$ is random since the device is intrinsically noisy at temperature T, and assumed stationary at constant V. The characteristics of the fluctuations may depend on V, in general [5]. We denote by $\Delta q = \int_{t_0}^{t_0+\Delta t} i(t)\,\mathrm{d}t$ the random charge increment flowing through the SPAD over an arbitrary time interval Δt. Laws of stochastic thermodynamics impose constraints on the characteristics (such as statistical moments) of Δq. The *white* noise process was extensively covered in [5]. We summarize the main ingredients here below.

For a linear resistor, mean and variance of Δq are related by the fluctuation-dissipation theorem [12] known in the electronics literature as Johnson-Nyquist formula [13], [14]. Arbitrary nonlinear devices like the SPAD follow a more general relationship linking all moments, the *fluctuation relation* [15]. The fluctuation relation can be recovered from the *local detailed balance relation* [16], [17], itself consequent to the first principle of microscopic reversibility. A notable consequence of the fluctuation relation is the *thermodynamic uncertainty relation* [6].

A few elementary mathematical manipulations (detailed in [5]) lead to the following inequalities involving the first three

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central moments of Δq:

$$2kT\,\Delta t\,\frac{I(V)}{V} \le \mathrm{Var}\{\Delta q\} \le 2kT\,\Delta t\,\frac{I(V)}{V} + \mathbb{E}\left\{\left(\Delta q - I(V)\Delta t\right)^3\right\}\frac{V}{3kT} \quad (1)$$

where k is the Boltzmann constant while $\mathbb{E}$ and Var are the expectation (mean) and variance operators, respectively. We note that the lower bound in (1) is nothing but the extended Johnson-Nyquist formula for a nonlinear device operating out of equilibrium, $I(V)\,\Delta t$ being the mean of Δq. Equation (1) teaches us that any deviation from the Johnson-Nyquist prediction necessarily translates into a non-zero skewness, i.e. implies that the white noise process is non Gaussian.

B. Related Experimental Work

Characterization of higher-order moments like the skewness reveals the existence of non-Gaussian current fluctuations [18], as experimentally reported in a tunnel junction [7], [8] or in metallic wire at cryogenic temperature [18]. In the referenced works, special attention was paid to understanding and modelling the effect of the measurement circuit and of the environment noise, in order to give credence to the intrinsic nature of numerical values reported for variance and skewness.

Higher-order moments also provide quantitative information about the underlying noise mechanism, e.g. the avalanche phenomenon in quantum devices [10] or off-the-shelf avalanche diodes [9]. The super-Poissonian behaviour (Poisson distribution with an effective charge Mq_e, with q_e the elementary charge and M some large number) of the avalanche noise was previously established by evaluating the full counting statistics [9], [10], which quantify the probability of observing a certain number of charge transfer events during a given time interval.

III. Experimental Details

Multiple silicon SPAD samples (schematically shown in Figure 1(a)) were manufactured in UCLouvain Winfab cleanroom facilities. The micro-fabrication process on industrial silicon-on-insulator (SOI) wafers as well as characterizations in the dark and under illumination are exhaustively described in [11]. The width of the active region is 26 µm (Figure 1) or 16 µm, for the geometries considered in the present work. The SPAD pixel embeds a series polysilicon resistor, whose value ranges from 300 to 700 kΩ depending on the sample, passively quenching the avalanche current.

The SPAD current fluctuations in the time domain were measured near and in the avalanche region, at room temperature and in the dark, with a Keysight ALFNA noise analyzer as previously used for MOS transistors [3], [4]. Typical results are shown in Figure 1.

Any noise measurement is necessarily preceded by a DC measurement to retrieve the $I(V)$ characteristics of the device (Figure 1(b)). For a given DC bias point (fixed V), a current noise trace $i(t)$ is recorded with a specified time step Δt (Figure 1(c)). The DC current matches the mean of the random $i(t)$: $I = \mathbb{E}\{i(t)\}$. The empirical distribution can be

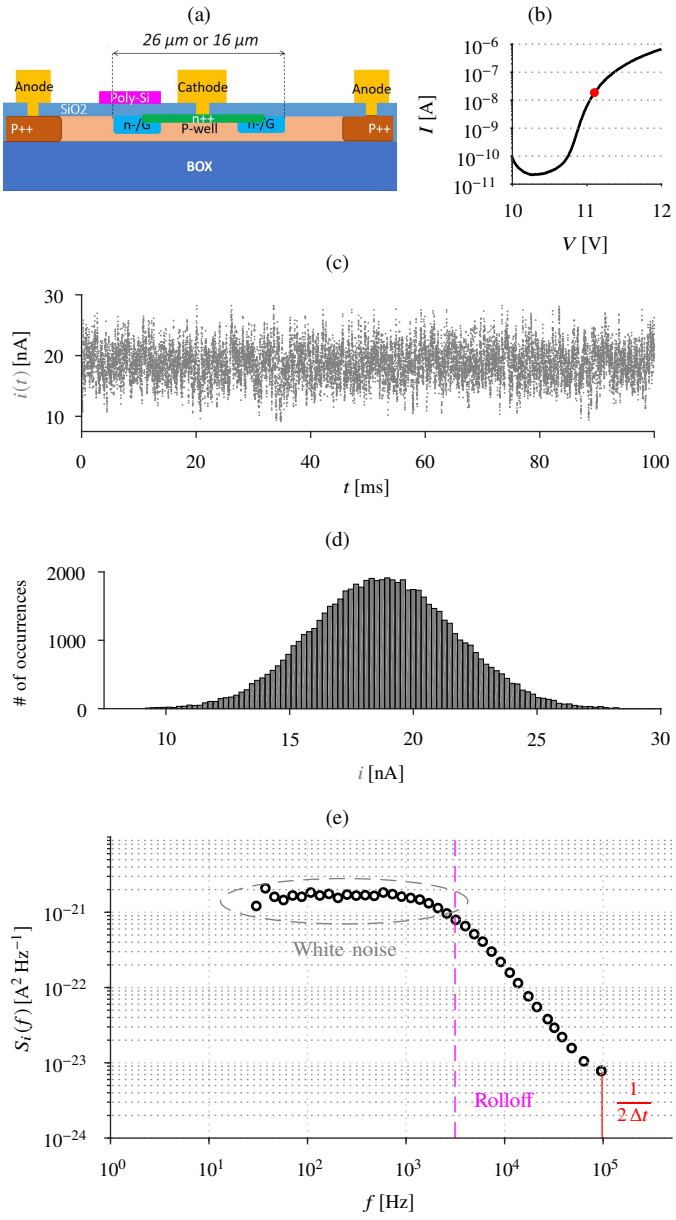

Figure 1. (a) Vertical cross section of the SPAD (from [11]).
(b) Measured $I(V)$ characteristics of a 26 µm-wide resistor-quenched SPAD. Red dot: DC bias point of the noise measurement, in avalanche regime, V = 11.1 V (mean current $I(V) \approx 20$ nA).
(c) Measured current fluctuations $i(t)$ (time step $\Delta t = 5$ µs).
(d) Histogram of i, obtained from $i(t)$ measurement (c).
(e) Estimated current noise power spectral density.

constructed as an histogram (Figure 1(d)). Although its shape seems qualitatively Gaussian, this statement will be revisited quantitatively in Section IV.

A. Noise Power Spectral Density

The nature of the noise can be assessed through spectral estimation [3]. Since the noise is inevitably low-pass filtered by the measurement chain of the equipment, one should observe that the obtained PSD $S_i(f)$ is white up to the rolloff frequency (Figure 1(e)). A dominant white noise behaviour

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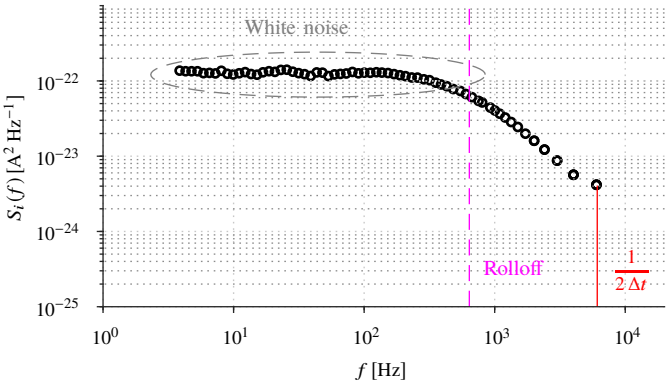

Figure 2. Estimated current noise power spectral density for a resistor-quenched 16 µm-wide SPAD (different sample from Figure 1) biased near avalanche $V = 9.9$ V (mean current $I(V) \approx 5$ nA).
Measurement time step $\Delta t = 80$ µs.

was consistently reported earlier in an avalanche diode biased far from equilibrium [9]. For the avalanche case illustrated in Figure 1(e), the value of the white noise plateau exceeds $2q_eI \approx 6 \times 10^{-27}$ A^2 Hz^{-1} by a factor $\approx 3 \times 10^5$, hence the fluctuations are certainly not attributed to shot noise. This suggests an *avalanche (multiplication) noise* $M\,2q_eI$ (M being the gain or multiplication factor of the SPAD) [19, eq. (8)], consistently with the bias point of the SPAD. Further characterisation and modelling of this noise will be part of future work to support and confirm this preliminary observation. Beyond variance (related to the PSD) and skewness (also addressed in Section IV), the need for a detailed theory capable of explaining the experimentally reported full counting statistics of the avalanche process was motivated for instance by [9].

B. Time Step and (Non)-Whiteness of the Noise

Formally, the theoretical result written as (1) is only valid for white noise. As described above, the intrinsic properties of the noise such as its distribution and its color (whiteness or not) are to some extent altered by the measurement chain, i.e. nonlinearities and low-pass filtering (rolloff in Figure 1(e)). This justifies the former efforts in analytically modelling and correcting these effects [7], [8]. In the present work, particular care was taken in the choice of the measurement time step Δt so as to minimize the colorization of the avalanche white noise by set-up artefacts within the measurement bandwidth.

Another PSD estimated from time-domain $i(t)$ measurement, on a different sample and this time near (slightly before) avalanche, is shown in Figure 2. The use of a larger time step, compared to the case of Figure 1, reduces the measurement bandwidth and thereby limits the undesirable low-pass filtering effect. If the number of recorded time samples remains constant, a larger time step gives a longer duration and hence lower-frequency spectral components can be assessed, as seen in Figure 2. Although the SPAD sample studied in Figure 2 is different from Figure 1, we may still notice that the white noise plateau is lower, by one order of magnitude. The current noise reduction, in terms of variance or PSD, when moving from the avalanche to the near-avalanche region is intuitively expected. The mean current $I(V)$ drops by a factor $\approx 20\,\text{nA}/5\,\text{nA} = 4$. However, the multiplication factor, sometimes referred to as Fano factor [9], $M = S_i/(2q_eI)$ is found about 75 000 near avalanche, compared to $\approx 300\,000$ in avalanche.

Table I
Variance (and lower and upper bounds) of the measured SPAD charge increment fluctuations.

Measurement	$2kT\Delta t\,\frac{I(V)}{V}$	$\mathrm{Var}\{\Delta q\}$	$2kT\Delta t\,\frac{I(V)}{V} + \mathbb{E}\{(\Delta q - I(V)\Delta t)^3\}\,\frac{V}{3kT}$	
In avalanche $\Delta t = 5$ µs (Figure 1)	7×10^{-35}	2×10^{-28}	6×10^{-23}	C^2
Near avalanche $\Delta t = 82$ µs (Figure 2)	4×10^{-34}	8×10^{-28}	9×10^{-22}	C^2

IV. Results and Discussion

A. Variance and its Lower and Upper Bounds

We now evaluate the different quantities of (1) for the performed measurements illustrated in Figures 1 and 2. The lower bound for the variance of the charge increment is the extended Johnson-Nyquist formula, tight for a linear resistor [5]. The upper bound involves the third central moment. Given a measurement of current noise trace (Figure 1(c)) with a time step Δt, we approximate $\Delta q \approx i(t)\,\Delta t$ and, by stationarity and ergodicity, we replace all the statistical averages ($\mathbb{E}$ operator) by time averages to empirically estimate the moments in (1). The actual noise variance and the lower and upper bounds are evaluated in Table I for the two measurements chosen for illustration. The theoretical inequalities of (1) are thus verified experimentally. Several orders of magnitude separate the actual variance from its lower and upper bounds, which was expected for a device as strongly nonlinear as the SPAD biased far from equilibrium [5]. Yet, interestingly, the actual noise variance lies roughly halfway between the bounds on a logarithmic scale for both cases.

Effect of the Time Step: The fact that $\mathrm{Var}\{\Delta q\}$ is reported larger near avalanche that in avalanche in Table I can be disturbing at first glance. We therefore emphasize that, whereas the PSD S_i and the variance $\mathrm{Var}\{i\}$ of the *current* effectively scale with the mean current $I(V)$, the variance $\mathrm{Var}\{\Delta q\}$ of the *charge* increment also increases linearly with Δt. The following relationship holds, assuming (band-limited) white noise:

$$\mathrm{Var}\{\Delta q\} = \mathrm{Var}\{i\}\,\Delta t^2 = S_i\,\Delta t/2. \qquad (2)$$

Thus, the larger time step of the near-avalanche measurement contributes to the variance of the charge increment (2) and more than compensates the reduction of the current PSD that we discussed in Section III.

It can be shown mathematically that the first three central moments of Δq, defined as the integral of some white noise process $i(t)$, are proportional to Δt [5]. Consequently, the

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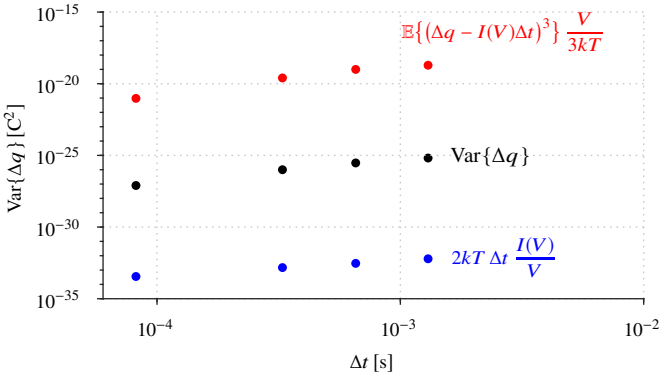

Figure 3. Influence of Δt on the empirical variance of Δq and on the tightness of the lower and upper bounds.
Illustrated case: resistor-quenched SPAD near avalanche of Figure 2.

tightness (or looseness) of the bounds (1) is *independent* of Δt, which arbitrarily but similarly determines the magnitude of the moments. The theoretical argument is supported by the experimental plot of Figure 3 where all the quantities present in (1) vary with Δt. For all the cases, the actual noise variance remains halfway between the lower and upper bounds on a logarithmic scale, i.e. the ratios are well unaffected by the Δt.

B. Skewness

The (Fisher-Pearson's coefficient of) skewness is computed as $\mathbb{E}\{(\Delta q - I(V)\Delta t)^3\}/\mathrm{Var}\{\Delta q\}^{3/2}$, i.e. the third central moment normalized by the cube of the standard deviation, and is often regarded as measuring the *asymmetry* of the fluctuations around the mean. For the noise measured in and near avalanche (Table I), we find 0.02 and 0.05, respectively. These small but non-negligible (as experimentally proven) values highlight that the current fluctuations in the SPAD deviate from a pure Gaussian noise behaviour.

V. Conclusions

Whereas the knowledge of mean and variance of the fluctuations is usually sufficient from a compact modelling and circuit design perspective, the Gaussian noise model is thermodynamically inconsistent for nonlinear devices. Provided with theoretical results derived in [5], we have experimentally evidenced the non-zero skewness of the current fluctuations in µm-scale SPAD [11] biased in or near the avalanche regime. White noise has been measured in the time domain, and the extracted statistical moments verify the predictions. The nature of this non-Gaussian noise, likely a multiplication noise [9], [19], remains to be investigated and modelled as required for future photonic applications.

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