

The identification of preferences from equilibrium prices under uncertainty ¹²

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Abstract

The competitive equilibrium correspondence, which associates equilibrium prices of commodities and assets with allocations of endowments, identifies the preferences and beliefs of individuals under uncertainty; this is the case even if the asset market is incomplete.

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1 Introduction

Explanation and prediction require the behavior of individuals, which is observable, to identify, possibly within a class, their characteristics, which are not.

In a market, it is competitive equilibria that are observable. The theory fails to specify out of equilibrium behavior and, as a consequence, demand at arbitrary prices and incomes is not observable. Experimental observations may be less restrictive.

The preferences of individuals are unobservable; beliefs are unobservable as well, even though they may not be exogenous and might vary with equilibrium prices or aggregate behavior.

Observations may involve different degrees of aggregation. At the most disaggregated level, one can observe the demand of individuals as prices and the allocation of endowments or revenue varies. More appropriately, with less disaggregation, observations are restricted to equilibrium prices as the allocation of endowments varies. The endowments of individuals may be in part unobservable, though redistributions of revenue are mostly observable; production possibilities may be observable or not.

In Chiappori et al (1999), under certainty, the competitive equilibrium correspondence, which associates equilibrium prices of commodities and assets with allocations of endowments, identifies the preferences and beliefs of individuals.

Identification obtains in economies with uncertainty, even if the asset market is incomplete.

Under uncertainty, with an incomplete asset market, the identification of preferences from observed behavior has strong positive as well as normative implications. A competitive equilibrium allocation is not optimal; more pertinently, according to Geanakoplos and Polemarchakis (1986), redistributions of portfolios of assets can result in a Pareto improvement; alternatively, following Herings and Polemarchakis (1998), the regulation of prices and the imposition of rationing in order to attain market clearing can be Pareto improving. The question of the informational requirements for the determination of improving redistributions of portfolios of assets or regulation of prices immediately arises.

The identification of preferences from the equilibrium correspondence allows possibly counterintuitive distributional effects of financial innovation as in Hart (1975) to be predicted. Similarly it allows for the determination of the investment decisions of firms as in Drèze (1974) without recourse to unobservable characteristics.

With an incomplete asset market, the identification of preferences even from individual demand behavior, though possible, is not evident: the first order conditions for individual optimization do not determine, at least immediately, the marginal rate of substitution between consumption at different states of the world. With one commodity, restrictions on the structure of payoffs of assets, as in Dybvig and Polemarchakis (1981) and in Polemarchakis and Rose (1984), or on the utility function that represents the preferences, as in Green, Lau and

Polemarchakis (1979) permit identification. However, without such assumption local identification is impossible when only prices and first- period incomes vary. In contrast to this, with multiple commodities, the variation of relative price of commodities at each state of the world permits identification when preferences are state - separable, as in Geanakoplos and Polemarchakis (1990).

The argument of Geanakoplos and Polemarchakis extends to the more restrictive settings where only aggregate demand as a function of individual first period incomes and all prices is observable. This implies directly that the equilibrium correspondence identifies preferences when they are state - separable and when there are several goods traded at each state.

In the limiting case of one commodity identification from aggregate asset demand is in general not possible. Nevertheless, since along the equilibrium correspondence consumption is not restricted to lie in a subspace of lower dimension than the commodity space, identification from the equilibrium correspondence is possible. An example shows that the assumption of separable utility is needed identify preferences when there is only one good.

The argument for recoverability is local. Given any profile of endowments with equilibrium prices, one identifies the associated consumption allocation as well as preferences over consumption in a neighborhood of this allocation. This argument extends immediately to preferences over the whole consumption set if additional assumptions on preferences assure that the associated allocations are attained at some equilibrium.

The identification of unobservable characteristics here assumes that the behavior of individuals is derived from the maximization of utility subject to budget constraints; which distinguishes it from the issue of the integrability of demand functions

As in Chiappori et al (1999), identification implies that there are local restrictions on the equilibrium correspondence since the argument for identification does not use all restrictions utility maximization imposes on individual demand.

Concerning further research, in a more applicable argument, one needs to take into account that for time series data, prices and incomes might be part of one, intertemporal equilibrium, and not points on an equilibrium correspondence. According to K'ubler (1999), the assumption of time separable expected utility restores global restrictions in an intertemporal model. For identification, the assumption of separable utility is not enough, since sufficiently complete asset markets allow individuals to smooth their expenditure across dates and states of the world. Also, optimizing individuals take all prices and dividends into account when choosing their portfolio and consumption plans. However, observations can only consist of one sample path, and it seems unlikely that identification of preferences is possible without any knowledge of equilibrium prices at nodes that do not lie on the sample path.

2 The economy

Individuals are $i \in \mathcal{I} = \{1, \dots, I\}$, a finite, non - empty set.

States of the world, exhaustive and exclusive descriptions of the environment, are $s \in \mathcal{S} = \{1, \dots, S\}$, a finite, non - empty set.

Commodities are $l \in \mathcal{L} = \{1, \dots, L\}$, a finite, non - empty set.

At the state of the world s , commodities are $(l, s) \in \mathcal{L} \times \{s\}$, and a bundle of commodities is $x_s = (\dots, x_{l,s}, \dots)'$; across states of the world, commodities are $(l, s) \in \mathcal{L} \times \mathcal{S}$, and a bundle of commodities is $x = (\dots, x'_s, \dots)' = (\dots, x_{l,s}, \dots)'$.

Assets for the transfer of revenue across states of the world are $a \in \mathcal{A} = \{1, \dots, A\}$, a finite set, and a portfolio of assets is $y = (\dots, y_a, \dots)'$. The payoff of asset a at the state of the world s is $r_{a,s}$; across states of the world, the payoffs of an asset are $r_a = (\dots, r_{a,s}, \dots)'$. The payoffs of assets at the state of the world s are $R_s = (\dots, r_{a,s}, \dots)$; across states of the world, the matrix of payoffs of assets is

$$R = (\dots, r_a, \dots) = (\dots, R_s, \dots)'.$$

The payoff of a portfolio of assets, y , at state of the world s is $r_s y$; across states of the world, the payoffs of a portfolio of assets are

$$Ry = (\dots, r_s y, \dots).$$

The column span of the matrix of payoffs of assets, $[R]$, is the subspace of attainable reallocations of revenue across states of the world.

Assumption 1 *The asset structure is such that*

1. *there are at least two assets, $A \geq 2$.*
2. *the matrix of payoffs of assets has full column rank;*
3. *the payoff of asset $a = 1$ is positive: $r_1 > 0$.*
4. *at every state of the world, the payoffs of assets do not vanish: $R_s \neq 0$.*

Redundant assets, whose payoffs are linear combination of the payoffs of other assets can be priced by arbitrage.

The asset market is either complete: $A = S$, or incomplete: $A < S$.

A portfolio of assets with positive payoffs serves to eliminate satiation over portfolios; that this portfolio consist of only one asset, $a = 1$, simplifies the exposition.

The preferences of an individual are described by the utility function, w^i , with domain the consumption set.

The preferences of the individual admit a representation that is additively separable across states of the world, $\{u_s^i : s \in \mathcal{S}\}$: the consumption set has a product structure; the cardinal utility function, at a state of the world, is u_s^i , and

the utility function is $w^i = \sum_{s \in \mathcal{S}} u_s^i$. The preferences may, but need not admit a von Neumann - Morgenstern representation, (u^i, μ^i) : the consumption sets at different states of the world and the cardinal utility functions, u^i , coincide; $\mu^i = (\dots, \mu_s^i, \dots)$, is a subjective probability measure on the set of states of the world, and the utility function is ¹ $w^i = E_\mu u^i$.

Utility functions, w_1^i and w_2^i , are cardinally equivalent if w_2^i is a monotonically increasing, affine transformation of w_1^i .

The endowment of an individual in commodities is e^i , a bundle of commodities across states of the world; his endowment of assets is f^i , a portfolio of assets.

The effective endowment of the individual in commodities is ² $\tilde{e}^i = (\dots, \tilde{e}_s^i, \dots)'$, where $\tilde{e}_s^i = e_s^i + \mathbf{1}_1^L R_s f^i$ is the effective endowment in commodities at a state of the world.

Assumption 2 *For every individual and for every state of the world,*

1. *the consumption set is the set of non - negative bundles of commodities;*
2. *the cardinal utility function, u_s^i , is continuous and concave; in the interior of the consumption set, the utility function is differentiable strictly monotonically increasing: $Dv_s^i(x) \gg 0$, and strictly concave: $y \neq 0 \Rightarrow y' D^2 u_s^i(x) y < 0$; for a sequence of strictly positive consumption bundles, $(x_{s,n} \gg 0 : n = 1, \dots)$, and for $\bar{x}_s$, a consumption bundle on the boundary of the consumption set, $(\lim_{n \rightarrow \infty} x_{s,n} = \bar{x}_s) \Rightarrow (\lim_{n \rightarrow \infty} (\|Du^i(x_{s,n})\|)^{-1} Du^i(x_{s,n}) \bar{x}_{s,n}) = 0)$, while $\lim_{n \rightarrow \infty} \|Du^i(x_{s,n})\| = \infty$;*
3. *$\tilde{e}^i \gg 0$: the effective endowment in commodities is a consumption bundle in the interior of the consumption set.*

The distinction between endowments in commodities and endowments in assets simplifies the exposition, but is not essential.

The profile of utilities functions is

$$u^{\mathcal{I}} = (\dots, \{u_s^i : s \in \mathcal{S}\}, \dots),$$

and the allocation of endowments is

$$(e^{\mathcal{I}}, f^{\mathcal{I}}) = (\dots, (e^i, f^i), \dots).$$

Profiles of utility functions, $u_1^{\mathcal{I}}$ and $u_2^{\mathcal{I}}$, are cardinally equivalent if, for every individual, the utility functions w_1^i and w_2^i are cardinally equivalent.

The profile of utility functions, $u^{\mathcal{I}}$, and the matrix of payoffs of assets, R , are fixed, while the allocation of endowments varies.

¹ " E_μ " denotes the expectation with respect to the probability measure μ .

² " $\mathbf{1}_k^K$ " denotes the k - th unit vector of dimension K .

The aggregate endowment in commodities is $e^a = \sum_{i \in \mathcal{I}} e^i$, and the aggregate endowment in commodities is $f^a = \sum_{i \in \mathcal{I}} f^i$; the aggregate endowment is (e^a, f^a) .

At a state of the world, prices of commodities are $p_s = (\dots, p_{l,s}, \dots) \gg 0$; Across states of the world, prices of commodities are $p = (\dots, p_s, \dots)$.

Across states of the world, the expenditures associated with bundle of commodities, x , at prices of commodities p are

$$p \otimes x = (\dots, p_s x_s, \dots).$$

Prices of assets are $q = (\dots, q_a, \dots)$.

Prices of assets do not allow for arbitrage if $Ry > 0 \Rightarrow qy > 0$; this is the case if and only if $q = \pi R$, for some $\pi = (\dots, \pi_s, \dots) \gg 0$.

Prices are a pair, (p, q) , of prices of commodities and of prices of assets.

The optimization problem of an individual is

$$\begin{aligned} \max \quad & w^i(x), \\ \text{s.t} \quad & p \otimes x \leq p \otimes e^i + Ry, \\ & qy \leq qf^i. \end{aligned}$$

The solution of the individual optimization problem, $(x^i, y^i)(p, q, e^i)$, exists and is unique, and the consumption plan, $x^i(p, q, e^i)$, lies in the interior of the consumption set; it defines (x^i, y^i) , the demand function of the individual for consumption plans and portfolios of assets.

The demand function is continuously differentiable; price effects are

$$\begin{aligned} D_{p_t} x_s^i &= (\dots, \frac{\partial x_{l,s}^i}{\partial p_{k,t}}, \dots), D_q x_s^i = (\dots, \frac{\partial x_{l,s}^i}{\partial q_a}, \dots), \\ D_{p_{l,s}} y^i &= (\dots, \frac{\partial y_a^i}{\partial p_{l,s}}, \dots), D_q y^i = (\dots, \frac{\partial y_a^i}{\partial q_b}, \dots); \end{aligned}$$

income effects are

$$\begin{aligned} D_{e_{1,t}^i} x_s^i &= (\dots, \frac{\partial x_{l,s}^i}{\partial e_{1,t}^i}, \dots), D_{f_1^i} x_s^i = (\dots, \frac{\partial x_{l,s}^i}{\partial f_1^i}, \dots), D_{e_{1,s}^i} y^i = (\dots, \frac{\partial y_a^i}{\partial e_{1,s}^i}, \dots), \\ D_{f_1^i} y^i &= (\dots, \frac{\partial y_a^i}{\partial f_1^i}, \dots). \end{aligned}$$

For cardinally equivalent utility functions, the demand functions coincide.

Associated with the individual optimization problem, at each state of the world, there is a conditional optimization problem

$$\begin{aligned} \max \quad & u_s^i(z_s), \\ \text{s.t} \quad & p_s z_s \leq p_s e_s^i + R_s y^i, \end{aligned}$$

where $y^i > 0$ is a fixed portfolio of assets, such that $p_s e_s^i + R_s y^i > 0$.

The solution of the auxiliary optimization problem, $z_s^i(p_s, e_s^i, y^i)$, exists, is unique, and lies in the interior of the consumption set; it defines z_s^i , the conditional demand function of the individual.

The conditional demand functions are continuously differentiable; price effects are

$$D_{p_s} z_s^i = (\dots, \frac{\partial z_{l,s}^i}{\partial p_{k,s}}, \dots);$$

income effects are

$$D_{e_{1,s}^i} z_s^i = (\dots, \frac{\partial z_{l,s}^i}{\partial e_{1,s}^i}, \dots),$$

and

$$D_{y^i} z_s^i = D_{e_{1,s}^i} z_s^i R_s.$$

Assumption 3 *For every individual and for every state of the world,*

1. *the vectors $z_s^i = (\dots, z_{l,s}^i, \dots)$ and $D_{e_{1,s}^i} z_s^i = (\dots, \partial z_{l,s}^i / \partial e_{1,s}^i, \dots)$ are linearly independent, and*
2. *$(D_q y^i + D_{f_1^i} y^i (y^i - f^i)') R_s \neq 0$,*

In the conditional demand function, the vector of income effects and the vector of demands are not co-linear. This excludes homothetic utility functions. For this case identification is still possible but the argument has to be modified - the remark at the end of this section clarifies this point.

In the asset demand function, the sum of the matrix of price effects and the matrix formed by the product of the column vector of income effects and the transpose of the column vector which is the portfolio of assets does not vanish; this sum is the matrix of substitution effects in the demand for assets.

To simplify notation it is assumed throughout that all derivatives are evaluated at a price system which satisfies $p_{1,s} = 1$ for all $s \in \mathcal{S}$.

Lemma 1 (Geanakoplos and Polemarchakis (1990)) *The demand function for consumption plans and portfolios of assets identifies the utility function of the individual up to cardinal equivalence.*

Proof The argument is developed in two steps.

Step 1 The demand function for consumption plans and portfolios of assets, (x^i, y^i) , determines the conditional demand function, z_s^i , at every state of the world. By the supporting hyperplane theorem, given prices of commodities and a portfolio of assets revenue, (p_s, y^i) , there exist commodity prices, p_t , for $t \in \mathcal{S} \setminus \{s\}$, states of the world other than s , and prices of assets, q , such that $y^i(p, q) = y^i$. It follows that $x_s^i(p, q) = z_s(p_s, e_s^i, y^i)$.

The necessary and sufficient conditions for a solution of the conditional individual optimization problem at a state of the world are

$$Du_s^i - \lambda_s^i p_s = 0,$$

$$p_s z_s^i - p_s e_s^i - R_s y^i = 0.$$

Differentiating the first order conditions and setting

$$\begin{pmatrix} K_s^i & -v_s^i \\ -v_s^{i'} & b_s^i \end{pmatrix} = \begin{pmatrix} D^2 u_s^i & -p_s' \\ -p_s & 0 \end{pmatrix}^{-1},$$

and

$$S_s^i = \lambda_s^i K_s^i,$$

yields, by the implicit function theorem, that

$$D_{p_s} z_s^i = S_s^i - v_s^i (z_s^i - e_s^i)',$$

$$D_{e_{1,s}^i} z_s^i = v_s^i,$$

$$D_{y^i} z_s^i = v_s^i R_s,$$

$$D_{p_s} \lambda_s^i = -\lambda_s^i v_s^{i'} + b_s^i (z_s^i - e_s^i)',$$

$$D_{y^i} \lambda_s^i = -b_s^i R_s,$$

and, as a consequence

$$dz_s^i = (S_s^i - v_s^i (z_s^i - e_s^i)') dp + v_s^i (p_s de_s^i + R_s dy^i),$$

$$d\lambda_s^i = (-\lambda_s^i v_s^{i'} + b_s^i (z_s^i - e_s^i)') dp_s - b_s^i v_s^i (p_s de_s^i + R_s dy^i).$$

The matrix, S_s^i , of substitution effects, $s_{l,k,s}^i = (\partial x_{l,s}^i / \partial p_{k,s})_{\bar{v}_s^i}$, is symmetric and negative semi-definite, it has rank $(L-1)$, and satisfies $p_s S_s^i = 0$, and the vector, v_s^i , of income effects, $v_{l,s}^i = \partial x_{l,s}^i / \partial t_s^i$, satisfies $p_s v_s^i = 1$. The marginal utility of revenue, $\lambda_s^i = \partial v_s^i / \partial t_s^i$, decreases with revenue: $\partial \lambda_s^i / \partial t_s^i = -b_s^i < 0$.

The necessary and sufficient first order conditions for a solution to the individual portfolio choice problem are

$$\lambda^i R = \mu^i q,$$

$$qy - qf^i = 0,$$

where, across states of the world, $\lambda^i = (\dots, \lambda_s^i, \dots)$ are the marginal utilities of revenue obtained from the conditional optimization with $y^i = y$.

Differentiating the first order conditions and setting

$$\begin{pmatrix} K^i & -v^i \\ -v^{i'} & b^i \end{pmatrix} = \begin{pmatrix} -\sum_{s \in \mathcal{S}} b_s^i R_s R_s' & -q \\ -q' & 0 \end{pmatrix}^{-1},$$

and

$$S^i = \lambda^i K^i,$$

yields by the implicit function theorem that

$$D_{p_s} y^i = S^i R_s (\lambda_s^i v_s^{i'} - b_s^i z_s^{i'}),$$

$$D_q y^i = S^i - v^i (y^i - f^i)'$$

$$D_{f_1^i} y^i = v^i,$$

where, for each state of the world, v_s^i and b_s^i are the income effects and the derivative of the marginal utility of revenue, respectively, obtained from the conditional optimization with $y^i = y$.

Step 2 The demand function for assets, y^i , and its derivatives with respect to revenue, $D_{f_1^i} y^i$, the prices of assets, $D_q y^i$, determine the vector of income effects v^i , and the S^i , matrix of substitution effects.

The conditional demand function for commodities at a state of the world, z^i , and its derivative with respect to revenue, $D_{e_{1,s}^i} z_s^i$, or, alternatively, $D_{y^i} z_s^i$, determine the vector of income effects, v_s^i — in the latter case, since $R_s \neq 0$.

The derivatives with respect to the prices of commodities of the demand for assets, $D_{p_s} y^i$, determines the marginal utility of revenue, λ_s^i and its first derivative b_s^i ; this is the case, since, by assumption, the vectors $D_{e_{1,s}^i} z_s^i = v_s^i$ and z_s^i and linearly, while $S^i R_s = (D_q y^i + D_{f_1^i} y^i) R_s \neq 0$.

By the separating hyperplane theorem, the demand function for commodities is surjective. Since, at a solution of the individual optimization problem, the gradient of the utility function is co-linear with the vector $\lambda^i \otimes p$, the demand function for consumption plans and portfolios of assets identifies the utility function up to a monotonically increasing transformation.

Since the utility function is additively separable across states of the world, of which there are, at least, two, the demand function for consumption plans and portfolios of assets identifies the utility function up to a monotonically increasing, affine transformation. $\square$

Remark The argument for the identification of the utility function does not require variations in the endowments of the individual in commodities at each state of the world; variations in the endowment of assets suffice.

With a single commodity endowments of individuals at each states have to vary to allow for identification.

Across individuals,

$$(x^a, y^a)(p, q, e^{\mathcal{I}}) = \sum_{i \in \mathcal{I}} (x^i, y^i)(p, q, e^i),$$

which defines (x^a, y^a) , the aggregate demand function for consumption plans and portfolios of assets.

For cardinally equivalent profiles of utility functions, the aggregate demand functions coincide.

At each state of the world, for $y^{\mathcal{I}} = (\dots, y^i, \dots)$, a fixed allocation of portfolios of assets, such that $p_s e_s^i + R_s y^i > 0$, for every individual,

$$z_s^a(p_s, e_s^{\mathcal{I}}, y^{\mathcal{I}}) = \sum_{i \in \mathcal{I}} z_s^i(p_s, e_s^i, y^i),$$

which defines z_s^a , the aggregate, conditional demand function.

Assumption 4 *For every individual,*

1. *the income effect for every asset, $\partial y_a^i / \partial f_1^i$, is a twice differentiable function of revenue, f_1^i ; Furthermore*

$$\frac{\partial^2 y_a^i}{\partial (f_1^i)^2} \neq 0,$$

2. *there exist assets, d and e , other than the numeraire, such that*

$$\frac{\partial}{\partial f_1^i} \left(\ln \frac{\partial^2 y_d^i}{(\partial f_1^i)^2} \right) \neq \frac{\partial}{\partial f_1^i} \left(\ln \frac{\partial^2 y_e^i}{(\partial f_1^i)^2} \right);$$

for every state of the world,

3. *the income effect in the conditional demand for every commodity, $\partial z_{l,s}^i / \partial e_{1,s}^i$, is a twice differentiable function of revenue, $e_{1,s}^i$; and*

$$\frac{\partial^2 z_{m,s}^i}{\partial (e_{1,s}^i)^2} \neq 0,$$

4. *there exist commodities, m and n , other than the numeraire, such that*

$$\frac{\partial}{\partial e_{1,s}^i} \left(\ln \frac{\partial^2 z_{m,s}^i}{(\partial e_{1,s}^i)^2} \right) \neq \frac{\partial}{\partial e_{1,s}^i} \left(\ln \frac{\partial^2 z_{n,s}^i}{(\partial e_{1,s}^i)^2} \right).$$

This is the analogue of the condition of non - vanishing income effects that was employed in the argument under certainty in Chiappori et al (1999). However, while this assumption is naturally fulfilled as long as preferences are not homothetic, it is not straightforward to translate the assumption of non-vanishing income effects in the demand for securities to an assumption on utility functions.

Lemma 2 *The aggregate demand function for consumption plans and portfolios of assets identifies the profile of utilities up to cardinal equivalence.*

Proof It suffices that the aggregate demand function identify, for every individual, the demand functions for portfolios of assets, y^i , and z_s^i , the conditional demand function for commodities, at every state of the world.

The argument is developed in two steps.

Step 1 The aggregate demand function for consumption plans and portfolios of assets, (x^a, y^a) , determines the aggregate, conditional demand function, z_s^a , at every state of the world.

For the aggregate, conditional demand function,

$$D_{p_s} z_s^a = \sum_{s \in \mathcal{S}} (S_s^i - v_s^i (z_s^i - e_s^i)'),$$

$$D_{e_{1,s}^i} z_s^a = v_s^i,$$

$$D_{y^i} z_s^a = v_s^i R_s,$$

and, as a consequence

$$dz_s^a = \sum_{s \in \mathcal{S}} (S_s^i - v_s^i (z_s^i - e_s^i)') dp + \sum_{s \in \mathcal{S}} v_s^i (p_s de_s^i + R_s dy^i).$$

Since $D_{e_{1,s}^i} z_s^a = v_s^i$, or, alternatively, $D_{y^i} z_s^a = v_s^i R_s$, the aggregate, conditional demand function identifies, v_s^i , the income effects of every individual — in the latter case, since $R_s \neq 0$.

The functions

$$f_{j,k,s} = \frac{\partial z_{j,s}^a}{\partial p_{k,s}} - \frac{\partial z_{k,s}^a}{\partial p_{j,s}} - \sum_{i \in \mathcal{I}} (v_{j,s}^i e_{k,s}^i - v_{k,s}^i e_{j,s}^i), \quad j, k \in \mathcal{L} \setminus \{1\}, j \neq k,$$

for pairs of distinct commodities other than the numeraire, are identified by the aggregate demand function.

By direct substitution and the symmetry of the matrices of substitution effects,

$$f_{j,k,s} = \sum_{i \in \mathcal{I}} (v_{k,s}^i e_{j,s}^i - v_{j,s}^i e_{k,s}^i).$$

As in the proof of lemma 2 in Chiappori et al (1999), the first and second derivatives of the functions $f_{j,k,s}$ with respect to revenue, $e_{1,s}^i$, or, alternatively, y_i , identify z_s^i , the conditional demand function of the individual.

Step 2 For the aggregate demand function for portfolios of assets,

$$D_q y^a = \sum_{i \in \mathcal{I}} (S^i - v^i (y^i - f^i)'),$$

$$D_{f_1^i} y^a = v^i,$$

and, as a consequence

$$dy^a = \sum_{i \in \mathcal{I}} (S^i - v^i(y^i - f^i))' dp + \sum_{i \in \mathcal{I}} v^i df_1^i.$$

Since $D_{f_1^i} y^a = v^i$, the aggregate demand function for portfolios of assets identifies v^i , the income effects of every individual.

The functions

$$f_{b,c} = \frac{\partial y_b^a}{\partial q_c} - \frac{\partial y_c^a}{\partial q_b} - \sum_{i \in \mathcal{I}} (v_b^i f_c^i - v_c^i f_b^i), \quad b, c \in \mathcal{A} \setminus \{1\}, b \neq c,$$

for pairs of distinct assets other than the numeraire, is identified by the aggregate demand function.

By direct substitution and the symmetry of the matrices of substitution effects,

$$f_{b,c} = \sum_{i \in \mathcal{I}} (v_c^i f_b^i - v_b^i f_c^i).$$

As in the proof of lemma 2, the first and second derivatives of the functions $f_{b,c}$ with respect to revenue, f_1^i , identify y^i , the demand function of the individual for portfolios of assets. $\square$

Competitive equilibrium prices are such that

$$(x^a(p, e^{\mathcal{I}}), y^a(p, e^{\mathcal{I}})) - (e^a, f^a) = 0.$$

The competitive equilibrium correspondence associates competitive equilibrium prices of commodities and assets to profiles of endowments,

$$\omega(e^{\mathcal{I}}, f^{\mathcal{I}}) =$$

$$\{(p, q) : (x^a(p, e^{\mathcal{I}}, y^a(p, e^{\mathcal{I}})) - (e^a, f^a) = 0, \text{ and } p_{1,s} = 1, s \in \mathcal{S}, q_1 = 1\}.$$

For cardinally equivalent profiles of utility functions, the competitive equilibrium correspondences coincide.

Proposition 1 *The competitive equilibrium correspondence on an open set of endowments identifies the associated subset of the consumption sets of individual and the profile of utility functions, up to ordinal equivalence, on this set.*

Proof It suffices that the competitive equilibrium correspondence identify the profile of demand functions.

The argument is developed in two steps.

Step 1 For an allocation of portfolios of assets, $y^{\mathcal{I}} = (\dots, y^i, \dots)$, the aggregate portfolio of assets is $y^a = \sum_{i \in \mathcal{I}} y^i$.

The graph of the competitive equilibrium correspondence determines the graph of the conditional competitive equilibrium correspondence at every state of the world, which assigns competitive equilibrium prices of commodities to allocations of endowments and portfolios of assets,

$$\omega_s(e_s^{\mathcal{I}}, y^i) = \{p_s : z_s^a(p_s, e_s^{\mathcal{I}}, y^{\mathcal{I}}) - e_s^a - \mathbf{1}_1^L R_s y^a = 0, \text{ and } p_{1,s} = 1\}.$$

The graph of the conditional competitive equilibrium correspondence has the structure of a continuously differentiable manifold.

The tangent space to the conditional competitive equilibrium manifold is defined by

$$dz_s^a = de_s^a + \mathbf{1}_1^L R_s dy^a,$$

and, as a consequence, by

$$\sum_{i \in \mathcal{I}} (S_s^i - v_s^i (z_s^i - e_s^i)') dp_s + \sum_{i \in \mathcal{I}} (v_s^i p - I) de_s^i + \sum_{i \in \mathcal{I}} (v_s^i - \mathbf{1}_1^L) R_s dy^i = 0.$$

The conditional competitive equilibrium correspondence determines the competitive equilibrium manifold and, consequently, everywhere, its tangent space.

As in the proof of proposition 1, at every state of the world, the graph of the conditional competitive equilibrium correspondence identifies the conditional demand function of every individual.

Step 2 By substitution, the tangent space to the competitive equilibrium manifold satisfies

$$\sum_{s \in \mathcal{S}} \left(\sum_{i \in \mathcal{I}} S_s^i R_s (\lambda_s^i v_s^i - b_s^i z^{i'}) \right) dp_s + \sum_{i \in \mathcal{I}} (S^i - v^i (y^i - f^i)') dq + \sum_{i \in \mathcal{I}} (v^i q - I) df^i = 0$$

As in the proof of proposition 1, the graph of the competitive equilibrium correspondence identifies the demand function for assets of every individual and therefore, with step 1, the entire demand function. $\square$

Remark If for every individual, for every commodity and for every state of the world, for any sequence $((p_{s,n}, e_{s,n}^i, y_s^i) : n = 1, \dots)$, of prices of commodities, endowments of commodities and portfolios of assets,

$$\lim_{n \rightarrow \infty} e_{1,s,n}^i + R_s y_s^i = \infty \quad \Rightarrow \quad \lim_{n \rightarrow \infty} z_{l,s,n}^i(p_{s,n}, e_{s,n}^i, y_s^i) = \infty, \quad l \in \mathcal{L},$$

i.e. for every individual, every commodity is normal in a strong sense then the competitive equilibrium correspondence identifies the utility functions of individuals on their entire domain of definition.

Remark It is an open question whether identification under uncertainty and an incomplete asset market extends to non-separable preferences when there are several commodities. It is shown below that this is impossible if there is only one commodity.

2.1 The special case of a finance economy

The identification result requires that there are at least 3 commodities at each state of the world.

As is the case under certainty the case of 2 commodities remains an open question.

The case of 1 commodity, vacuous under certainty, is indeed of interest under uncertainty.

Though recoverability from individual demand requires additional assumptions as pointed out in the earlier literature, recoverability from the competitive equilibrium correspondence is not problematic.

This is due to the freedom afforded by the equilibrium correspondence, namely the variation in the endowments of individuals across states of the world.

Step 2 of the proof of proposition 2 implies that — as long as parts (1) and (2) of assumption 6 are satisfied — individual asset demand can be identified from the equilibrium manifold even if $L = 1$.

In this framework individual asset demand as a function of individual endowments at all states and of prices does identify the utility:

If $L = 1$, the individual asset demand function is a solution to

$$\begin{aligned} \max \quad & w^i(Ry), \\ \text{s.t.} \quad & qy \leq qf^i. \end{aligned}$$

Differentiating the first order conditions and setting

$$\begin{pmatrix} K^i & -v^i \\ -v^{i'} & b^i \end{pmatrix} = \begin{pmatrix} -R'_s D^2 w^i R_s & -q \\ -q' & 0 \end{pmatrix}^{-1},$$

one obtains that $\partial^2 u_s^i$ can be recovered by variation of e_i^s after the identification of K^i and v^i from variation in prices and endowments in portfolios, since $D^2 w^i$ is a diagonal matrix.

When preferences are not separable identification is no longer possible. The following example clarifies this.

Example There is a single individual. Observing the equilibrium correspondence is equivalent of observing the supporting asset prices at all possible consumption vectors. The matrices H_1 and H_2 are distinct, negative definite symmetric $S \times S$ matrices, with

$$R'H_1 = R'H_2;$$

it is clear that for A much smaller than S these matrices exist.

If $h = (\dots, h_s, \dots)' \geq 0$ is large enough, the utility functions

$$w^1(x) = h'x - x'H_1x \quad \text{and} \quad w^2(x) = h'x - x'H_2x$$

generate identical asset prices for all possible allocations; the equilibrium correspondence under w^1 is indistinguishable from the equilibrium correspondence under w^2 even though these utility functions represent different preferences.

Remark The example shows that it is possible that even though equilibrium allocations are inefficient a planner who can observe the equilibrium correspondence but not individual preferences might not be able to introduce Pareto-improving assets - a firm might not be able to choose a constrained efficient production plan.

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