Propagation of Electromagnetic Waves along Cylindrical Metasurface Structure

Davorin Mikulić*, Marko Bosiljevac*, Christophe Craeye†, Zvonimir Šipuš*
* University of Zagreb Faculty of Electrical Engineering and Computing, Zagreb, Croatia,
davorin.mikulic@fer.hr, marko.bosiljevac@fer.hr, zvonimir.sipus@fer.hr
† UCLouvain, Louvain-la-Neuve, Belgium, *christophe.craeye@uclouvain.be*

***Abstract*—This paper explores the design of cylindrical leaky-wave metasurface antennas, addressing the need for antennas that conform to curved cylindrical surfaces. We introduce analytical tools for efficiently extracting the properties of electromagnetic waves propagating along cylindrical metasurfaces, enabling the suppression of curvature effects in the antenna design. A comprehensive design procedure is presented, along with a detailed design example and its corresponding simulation results. Our approach provides a fast and efficient method for designing cylindrical leaky-wave metasurface antennas, enabling the ability to radiate in the desired direction.**

***Index Terms*—cylindrical metasurface, leaky-wave antenna, creeping waves, sheet impedance**

I. Introduction

Planar leaky-wave metasurface antennas have been thoroughly investigated in the past 15 years, and the methods for their analysis and synthesis are well known to the electromagnetic community. Apart from designing leaky wave antennas, planar metasurfaces allow the design of other electromagnetic structures like microwave lenses, polarization converters, beam splitters, and highly directive reflective arrays. Common planar leaky-wave metasurface antenna synthesis, where the objective of the design is to arbitrarily shape the radiation pattern, includes the holographic approach, an approach based on the analysis of sinusoidally modulated reactance surfaces, and the numerical inverse MoM (Method of Moments) approach [1]. With the emergence of 5G radio networks and the advent of 6G, machine-to-machine and human-to-machine communications are gaining more interest. In order to support such communication, planar antennas should be able to curve into a cylindrical shape to attach to a robot arm or human body. Unfortunately, curved metasurfaces have been investigated to a very limited extent, and design procedures for curved leaky-wave metasurface antennas are still in the pioneering stage (one of the attempts is presented in [2]). The design of curved leaky-wave metasurfaces also introduces other challenges not present in the planar case, mainly connected to the curvature of the structure itself, which influences the radiation pattern of such an antenna [2], [3], [4]. In this paper, to successfully design a curved leaky-wave metasurface antenna, we discuss the properties of electromagnetic waves propagating along an arbitrary cylindrical metasurface structure and show how the properties of the wave can be modified to suppress the curvature effects, allowing for the design of an antenna in a similar manner as one would do in the planar case.

The paper is organized as follows: Section II presents the theory and analytical tools required to extract the properties of electromagnetic waves propagating along cylindrical metasurface structures, while Section III provides a detailed description of how to utilize these analytical tools to design a cylindrical leaky-wave metasurface antenna with a beam pointing in an arbitrary direction, along with a full design example and its corresponding simulation results.

II. Creeping wave representation of waves propagating along a cylindrical structure

In this paper we will consider waves propagating in one direction along grounded planar and cylindrical dielectric structures, i.e. we will consider the 2D case in which the excitation is a line current source. Normally polarized surface waves, launched to propagate along a planar grounded dielectric slab, only experience losses introduced by the dielectric layer itself and, therefore, propagate along the surface with very low attenuation. Planar metasurface (MTS) leaky-wave antennas utilize the interaction between the launched, nearly lossless wave propagating along the surface and the quasi-periodic metasurface layer. In this interaction, the slow, non-radiating surface wave excites the radiating $n = -1$ Floquet harmonic (see e.g. [5], [6] and [7]). By bending the infinite planar grounded dielectric slab coated with the MTS structure, a more complex geometry is obtained: a dielectric coated, grounded PEC cylinder, as illustrated in Fig. 1. Since the obtained cylinder structure is infinite in the $\hat{z}$ direction and is excited by the radially oriented currents it is sufficient to analyze propagation in the $\hat{\phi}$-direction, as suggested in Fig. 2. Fig. 2.a shows that the launched non-radiating wave propagating along the surface becomes a radiating creeping wave, with very poor control over the direction of the resulting wave propagation. This introduces additional losses due to the curvature of the structure and the thickness of the dielectric layer itself. To focus the electromagnetic waves in a desired manner, as suggested in Fig. 2.b, one should determine the proper MTS surface impedance profile, which minimizes the creeping wave losses introduced by the curvature and also induces the correct phase profile of the electromagnetic wave to achieve the desired radiation pattern of the cylindrical MTS antenna.

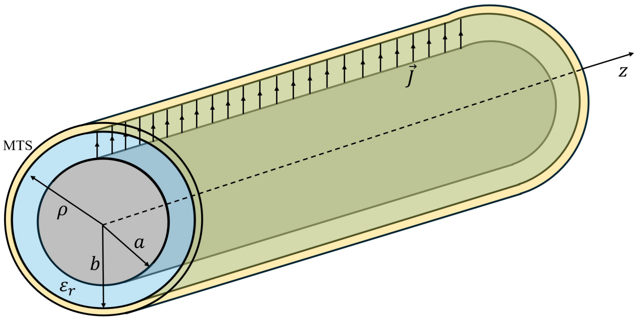


Fig. 1: Sketch of the cylindrical metasurface structure.

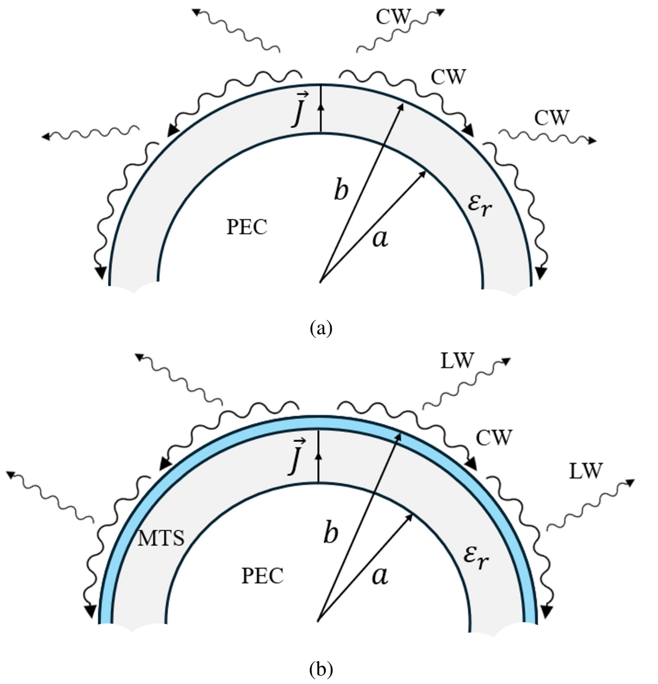


Fig. 2: Propagation of electromagnetic waves along a dielectric coated grounded PEC cylinder. (a) Without the MTS coating; (b) With the MTS coating.

To assess the propagation of creeping waves along the dielectric coated grounded cylinder with the radius of the PEC cylinder (oriented in the z direction) a and the outer radius of the dielectric coat b, and relative permittivity ε_r, covered with an arbitrary MTS of normalized surface impedance C^{sheet}, a dispersion relation must be solved. The dispersion relation can be obtained using the transverse resonance method for the considered 2D case:

$$\frac{1}{Z_{up}} + \frac{1}{Z_{down}} = 0. \tag{1}$$

Z_{up} and Z_{down} are calculated as follows [8]:

$$\frac{Z_{up}}{\eta_0} = j\frac{H_\nu^{'(2)}(k_0 b)}{H_\nu^{(2)}(k_0 b)}, \tag{2}$$

$$\frac{Z_{down}}{\eta_0} = C^h \parallel C^{sheet}, \tag{3}$$

where k_0 is the free space wavenumber, η_0 is the free space impedance, $H_\nu^{(2)}$ and $H_\nu^{'(2)}$ are the Hankel function of the second kind of complex order ν and its derivative [8]. C^h represents the surface impedance of the dielectric-coated, grounded PEC cylinder for TE_z polarization of the launched wave, normalized to η_0, and is equal to [9][10]:

$$C^h = - j\frac{1}{\sqrt{\varepsilon_r}}\frac{H_\nu^{'(2)}(k_1 b)H_\nu^{'(1)}(k_1 a) - H_\nu^{'(2)}(k_1 a)H_\nu^{'(1)}(k_1 b)}{H_\nu^{(2)}(k_1 b)H_\nu^{'(1)}(k_1 a) - H_\nu^{'(2)}(k_1 a)H_\nu^{(1)}(k_1 b)}, \tag{4}$$

where $k_1 = k_0\sqrt{\varepsilon_r}$, $H_\nu^{(1)}$ and $H_\nu^{'(1)}$ are the Hankel function of the first kind of complex order ν and its derivative [8]. Eq. 1 then yields the dispersion relation:

$$H_\nu^{'(2)}(k_0 b) - j\left(C^h \parallel C^{sheet}\right) H_\nu^{(2)}(k_0 b) = 0. \tag{5}$$

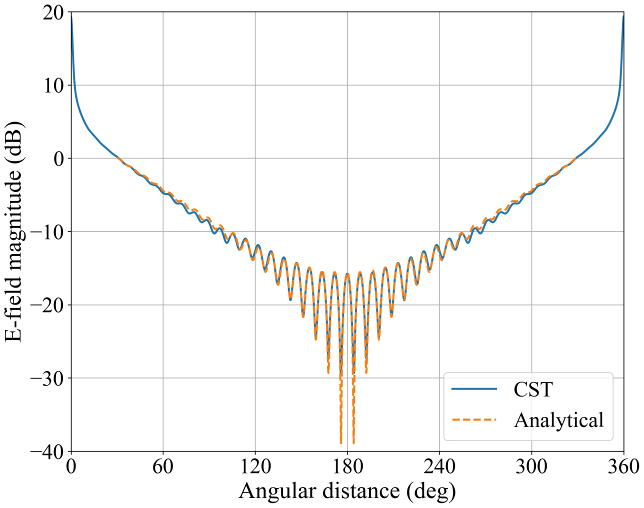
Solving the dispersion relation for the first zero yields complex order ν_1, which describes the propagation constant k_s of the dominant creeping wave mode (the dominant pole of the Green function) as follows:

$$\nu_1 = \nu_{1,r} + j\nu_{1,i} = k_s b. \tag{6}$$

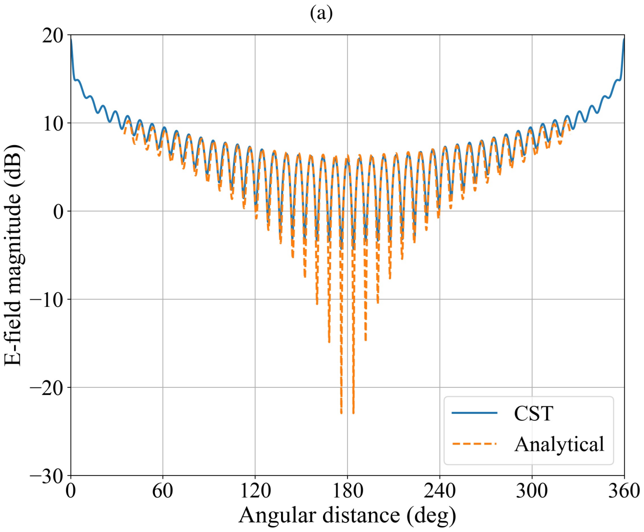
Creeping waves ($E-$ field Green's function due to an radially oriented electric line source, see Fig. 1), traveling in the positive and negative $\hat{\phi}$ directions, are then proportional to [11]:

$$E_\rho(\rho, \phi) \sim \left(e^{-j\nu_1\phi} + e^{-j\nu_1(2\pi-\phi)}\right). \tag{7}$$

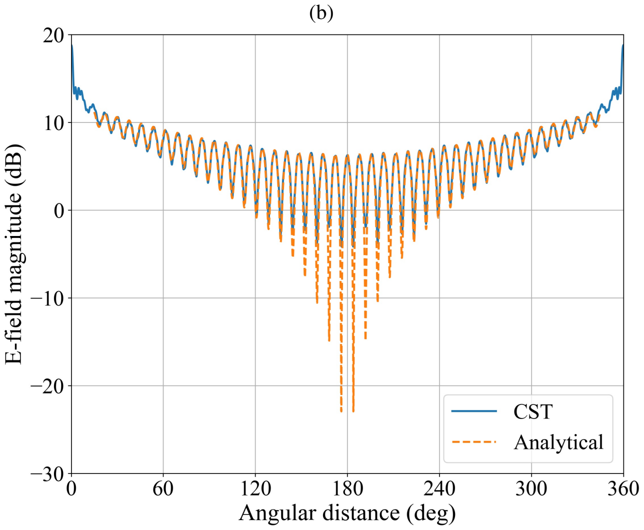
It is then clear that $\nu_{1,r}$ represents the phase dependency of the creeping wave, while $\nu_{1,i}$ represents losses due to the curvature of the structure. By modifying $Z_{sheet} = C^{sheet}\eta_0$, it is possible to control the complex propagation constant of the creeping waves. To verify the validity of the creeping waves representation, the described structure is modeled using the CST Studio Suite. For parameters $k_0 b = 20$, $\varepsilon_r = 4$, $Z_{sheet} = -j200\ \Omega$, and dielectric thickness $h = 0.05\lambda_0$, where λ_0 represents the free space wavelength, the distribution of the $E_\rho-$ field component in the vicinity of the outer surface of the metasurface structure is shown in Fig. 3. Furthermore, the metasurface is modeled using the metasurface sheet impedance approach, as illustrated in Figures 3.b and 3.c. Fig. 3.b presents results where the metasurface is modeled as an ideal sheet impedance layer in CST, while Fig. 3.c shows simulation results where the desired sheet impedance is realized through a realistic unit cell consisting of two metallic strips with a sufficient gap between them. Both results demonstrate very similar values, indicating consistency between the ideal model and the realistic unit cell implementation.



(a)



(b)



(c)

Fig. 3: Comparison of the distribution of the $E_\rho-$ field component at a distance $0.05\lambda_0$ from the outer surface of the MTS structure. (a) Dielectric-coated grounded PEC cylinder only; (b) Structure with a MTS where the MTS is modeled as a sheet surface impedance; (c) Structure with a MTS realized as an array of axially oriented metallic strips.

The structure is modeled using CST as a very thin (in z-direction) dielectric-coated disk with an radially oriented dipole placed in the dielectric layer to excite the surface waves, as suggested in Fig. 2. To ensure that a cylinder of infinite length in the $\hat{z}$ direction is properly modeled, a PMC boundary condition is enforced at the structure's maximum and minimum $\hat{z}$ dimensions. A radiation boundary condition is enforced on the other four planes. As can be seen in Fig. 3, the analytically obtained $E_\rho-$ component field distribution matches extraordinarily well with the values obtained from the CST simulation, hence validating the analytically obtained values of the creeping wave propagation constants. Fig. 3 also reveals that the losses caused by leaky wave radiation can be significantly reduced by covering the dielectric with an MTS of appropriate impedance, which effectively slows down the creeping wave and binds it more closely to the surface of the dielectric.

III. Design of the Cylindrical Metasurface Antenna

To design a cylindrical metasurface antenna, with a narrow lattice in the $E-$plane, a holographic approach is utilized to obtain the total surface reactance distribution $Z_S = jX_S$ along the cylinder [12]:

$$X_S(\phi) = X\left[1 + M\Re(\psi_{rad}\psi^*_{surf})\right], \tag{8}$$

where ψ_{surf} is the creeping wave propagating along the MTS structure, ψ_{rad} is the desired leaky wave, X is the average surface reactance, and M is the modulation factor. The creeping wave propagating along the cylinder surface in the $+\hat{\phi}$ direction is given with:

$$\psi_{surf} = e^{-jk_s b\phi}. \tag{9}$$

The leaky wave radiating in the ϕ_R direction is written as reads:

$$\psi_{rad} = e^{jk_0 b[1-\cos(\phi-\phi_R)]}. \tag{10}$$

The total desired surface reactance of the MTS structure, which ensures phase matching of the plane wave radiating in the ϕ_R- direction, is then:

$$X_S(\phi) = X\left[1 + M\cos\left(k_s b\phi + k_0 b\Delta\right)\right], \quad \Delta = 1 - \cos(\phi - \phi_R). \tag{11}$$

The required impedance Z_{sheet} is calculated from the eq. 3 as follows:

$$\frac{1}{Z_{sheet}} = \frac{1}{jX_S} - \frac{1}{C^h\eta_0}. \tag{12}$$

After describing the required theory, we present the antenna design procedure as follows:

(i) Select the antenna operating frequency f, MTS structure radius b, dielectric thickness h, desired radiation angle ϕ_R, modulation factor M, and length of the antenna.
(ii) Using the dispersion relation given in eq. 5, calculate the creeping wave propagation constants for different Z_{sheet}.
(iii) Calculate the losses due to the imaginary part of the propagation constant and choose the average surface

reactance X for which the losses are sufficiently small for the corresponding length of the antenna. The chosen average surface reactance X will determine the propagation constant k_s.

(iv) Retrieve the surface reactance distribution $X_S(\phi)$ using eq. 11.

(v) Choose a sufficient unit cell size l to properly sample the reactance distribution. Calculate the required MTS sheet impedances using eq. 12.

Antenna parameters are chosen as follows: operating frequency of $f = 24$ GHz, MTS structure radius $b = 101.5$ mm, dielectric thickness $h = 1.5$ mm, and permittivity $\varepsilon = 2.25$, desired radiation angle $\phi_R = 30°$, and modulation factor $M = 0.2$. A sketch of the proposed cylindrical MTS antenna is shown in Fig. 4. The proposed antenna consists of three parts:

(i) modulated MTS radiating part: $\phi \in [-60°, 60°]$,

(ii) uniform MTS guiding part: $\phi \in [180°, 300°]$,

(iii) absorber part: $\phi \in [60°, 180°]$.

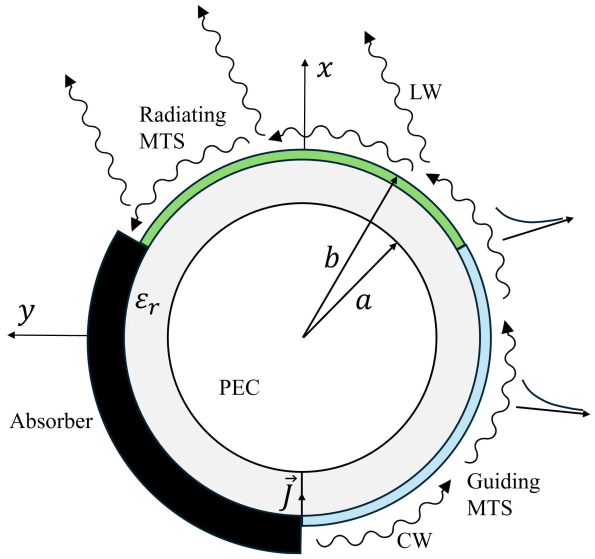


Fig. 4: Sketch of the proposed cylindrical MTS antenna.

The antenna is modeled using CST in the same manner as presented in the section II. The absorber part of the structure is introduced to attenuate the creeping wave launched in the clockwise direction. The absorber is modeled as a lossy dielectric covered with a uniform MTS sheet with an arc length of $2/3\pi b$. Fig. 5 shows the dependence of the loss of the creeping wave propagating along the homogeneous metasurface structure with an arc length of $4/3\pi b$, while Fig. 6 shows the dependence of the creeping wave's phase constant on the selected surface reactance for the proposed MTS antenna. An average surface reactance of 259.9 Ω yields losses of 0.01 dB, which can be considered a lossless case; hence, the imaginary part of k_s can be neglected. For the selected average surface reactance, k_s yields $k_s = 1.229k_0$. The sheet reactance distribution and its sampled values for a unit cell length of $l = 2$ mm and a modulation factor $M = 0.2$ are given in Fig. 7.

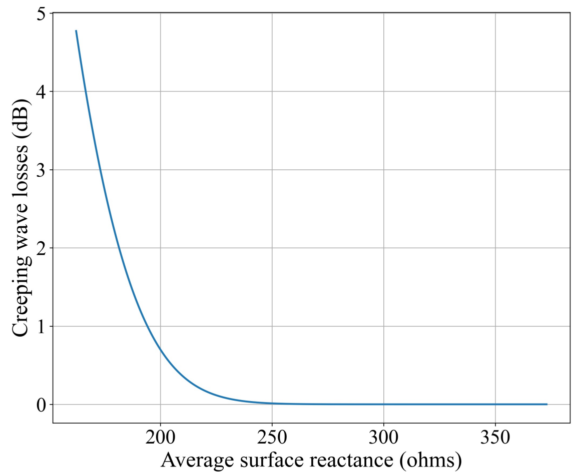


Fig. 5: Dependency of creeping wave losses on the selected average MTS impedance of the proposed antenna.

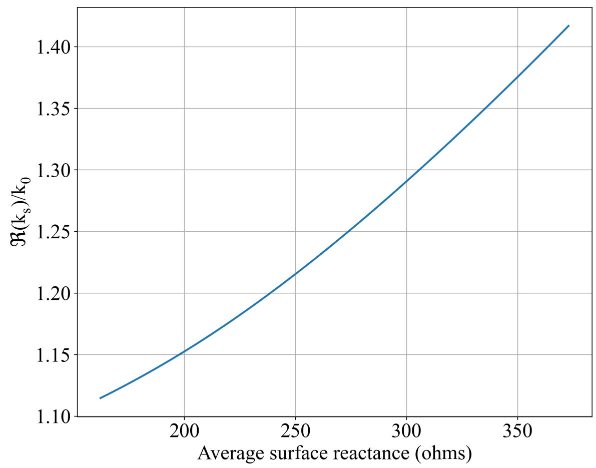


Fig. 6: Phase constant of the creeping waves on the selected average MTS impedance of the proposed antenna.

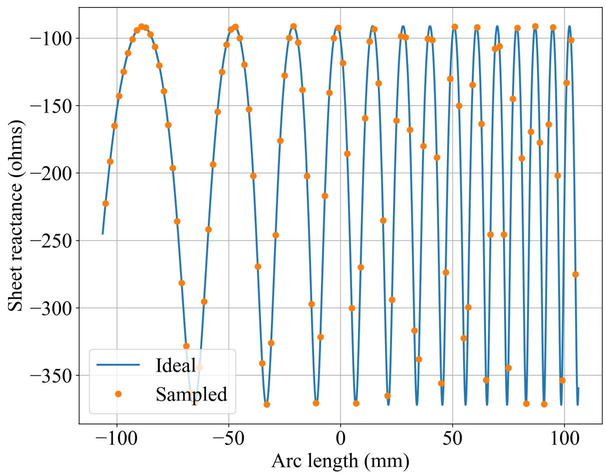


Fig. 7: Capacitive sheet reactance distribution of the proposed MTS antenna.

The described antenna is simulated with CST, and its radiation pattern in polar coordinate systems is shown in Fig. 8. In-time snapshot of the $E-$ field around from the MTS structure is shown in Fig. 9.

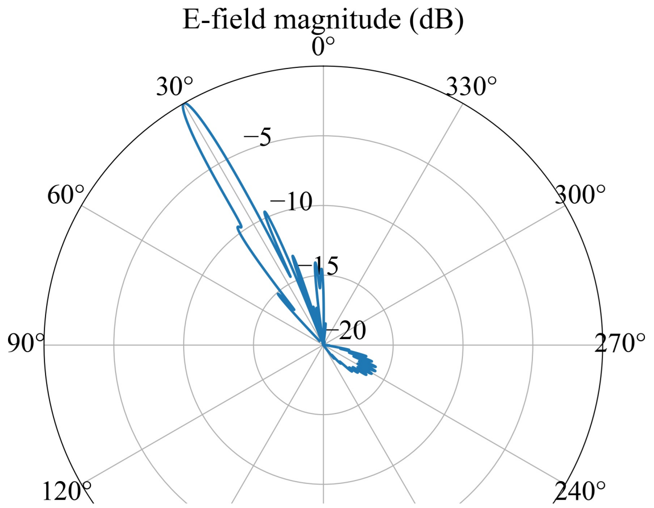


Fig. 8: Polar radiation pattern of the proposed MTS cylindrical antenna.

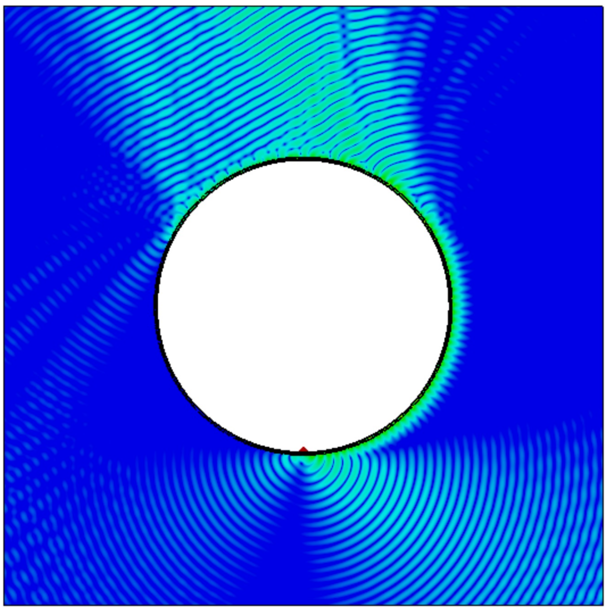


Fig. 9: In-time snapshot of the $E-$ field around the MTS structure.

Fig. 9 shows the desired planar wavefront forming above the radiating part of the structure, while the guiding part remains non-radiating. Strong radiation from the radially oriented dipole is present in the lower part, as the feeder excites more than just surface waves. This is reflected in the dipole-shaped radiation pattern in Fig. 8. The experimental antenna will use a feeder with less unwanted radiation, but for simulation purposes, the current feeder is adequate since the focus is on the radiating region. The radiation pattern results thus confirm the validity of the design procedure for the cylindrical MTS antenna with a beam in an arbitrarily set direction.

IV. Conclusion

We presented the analytical tools to efficiently extract the properties of electromagnetic waves propagating along a cylindrical metasurface structure. These tools are successfully utilized to diminish the curvature effects when designing a cylindrical leaky-wave metasurface antenna. The design procedure for such an antenna is thoroughly presented, and the simulation results confirm its validity. The design procedure presented in the paper does not depend on general computationally costly electromagnetic solvers and is a quick and robust solution for designing cylindrical leaky-wave antennas with a beam in an arbitrarily set direction.

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