AUTOMATIC ADJUSTMENT MECHANISMS IN PUBLIC PENSION SCHEMES TO ADDRESS POPULATION AGEING AND SOCIOECONOMIC DISPARITIES IN LONGEVITY

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Automatic Adjustment Mechanisms in Public Pension Schemes to Address Population Ageing and Socioeconomic Disparities in Longevity

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Abstract

Many pension systems require and continue to need change to maintain long-term financial sustainability. Most developed countries' policy-makers have been pushed to reform their pension systems in order to preserve or re-establish financial sustainability. As longevity heterogeneity is frequently disregarded in the policy chosen, the pursuit of financial sustainability is done at the expense of intra-generational actuarial fairness. We propose a pension system management system based on two adaptation mechanisms:

The first dynamic mechanism is integrated directly into the pension formula and corrects the heterogeneity of longevity when it exists between the agents of the pension scheme.

A steering mechanism for both the contribution rate and the mean benefit ratio respects Musgrave's rule, which makes it possible to distribute the demographic risk between working people and retirees.

In order to capture both effects and incorporate a mortality component into the pension formula, we model longevity heterogeneity and ageing in a pension based on historical data. Keywords: Automatic Adjustment Mechanisms, Longevity heterogeneity, Musgrave, pay-as-you-go, Pension design

1 Introduction

As the population ages and life expectancy continues to increase, countries around the world have been forced to confront the fiscal challenges posed by their pension systems. To ensure their long-term financial sustainability, many countries have implemented systemic and/or gradual parametric reforms to their pension schemes. These reforms have typically aimed to achieve solvency and enhance fiscal sustainability, while also introducing adequacy safeguards through automatic adjustments that keep pace with increasing life expectancy.

Parametric reforms have taken many different forms, including modifying the pension system rules and parameters such as the standard and early retirement rules, qualifying conditions, the contribution rate, the benefit formula, the index for accrued rights and indexation of benefits, and pension decrements and increments. Other reforms have focused on increasing pre-funding (reserve funds), adjusting early-retirement options to enhance work incentives, expanding contribution options, expanding the coverage of private (mandatory or voluntary) pensions, and developing auto-enrolment schemes [OECD (2019)]. Some countries have also reformed first-tier social/guarantee pensions, brought public-sector pension benefits more in line with private-sector benefits, and reformed the taxation of pensions.

While some pension reform approaches have proposed introducing an automatic link between future pensions and developments in life expectancy [Turner(2009)] , it is important to note that this is not the definitive solution. One of the most common responses to population ageing has been to increase standard and early retirement ages in an automatic or scheduled way as life expectancy at the pension age progresses. However, raising the retirement age can have undesirable effects on actuarial fairness. This is because it could lead to lower benefits for those with lower life expectancies and greater benefits for those with higher life expectancies. As a result, policy-makers may need to consider other options, such as increasing contributions or adjusting benefit formulas, to ensure that pension systems remain equitable and sustainable in the face of demographic changes. Countries have pursued various retirement policy strategies, including implementing fixed schedules, targeting a constant expected period in retirement, targeting a constant balance between time spent in work and retirement, targeting a constant ratio of adult life spent in retirement, targeting a stable old-age dependency ratio, setting a target age for retirement, following simple ad-ho rules to share the longevity risk burden between workers and pensioners, and linking the eligibility age for pensions to the eligibility age for other benefits such as public health care. The common thread among these various retirement age policies is their utilization of automatic adjustment mechanisms to effectively address the negative impact of economic and demographic trends on the financial stability of national pension plans. Although these policies are typically intended to align with the ultimate goals of these pension plans, such as ensuring adequacy, long-term sustainability, and intergenerational fairness, this alignment is often insufficient.

Moreover, income and socio-economic status (SE) are important determinants of health and longevity, with income being a commonly used indicator of material resources that is positively associated with longevity [Bosworth (2018)]. While education and social class are also relevant SE indicators, income provides a better long-term measure of SE status due to its wider range of variation. The link between SE status and mortality has implications for social security programs, especially in light of increasing longevity and population ageing. OECD countries have responded to these challenges by increasing the retirement age, but this approach does not necessarily take into account SE differences in mortality. Differences in life expectancy (LE) between high and low SE groups affect the actuarial fairness and progressivity of public pension systems.

The literature on the subject is growing, with researchers increasingly interested in mortality and LE inequalities related to SE status and their impact on social security programs. Evidence suggests that there is a sizeable and possibly growing disparity in late-life longevity in high-income countries. More notably, disparities in old-age mortality measured by SE status have widened in recent years in the Netherlands [Kalwij (2013)] [Wouterse(2021)] ,Germany [Wenau (2019)],the UK [Longevity Panel (2020)], Italy [Belloni et al (2013)] [Lallo (2018)] [Ardito et al (2020)], Sweden [Fors (2021)], Canada [Kleinow (2020)] and the USA [Waldron (2007)] [Orszag (2014)] [Bosley (2018)].

In most European countries, there has been a decline in mortality in lower SE groups, but relative inequalities in mortality have increased due to smaller percentage declines in these groups. These trends have important implications for pension reform and scheme design, as taxes and subsidies may not adequately address the effects of a closer contribution-benefit link, a later formal retirement age, and more individual funding and private annuities. As such, it is important for policy-makers to consider the welfare implications of growing inequality in LE by SE status when designing and implementing social security programs.

The distribution of lifespan among socio-economic classes is not uniform or random [Donkin et al (2002)]. Despite considerable improvements in mortality rates over the years, the health gap between diverse groups has risen, leading to rising inequality. A large body of literature reveals the unequal distribution of health and lifespan among persons of different socio-economic classes, which appears to be rising over time, perpetuating inequality [Marmot(2015)]. This calls the fairness of the social security system into question. Fairness is a complicated notion that incorporates subjective distributive justice values.

The notion of actuarial fairness has recently emerged as a benchmark in the field of pension economics [Borsch-Supan (2006)]. Essentially, it involves ensuring that the internal rate of return for all individuals is equal, regardless of the amount of contributions paid or benefits received. This implies that if two individuals who belong to the same birth cohort paid identical amounts of contributions during their working lives, they should receive equal pension wealth over their lifespan. Similarly, if two individuals paid different contribution amounts, the internal rate of return on their contributions should be equivalent. However, actuarial fairness alone may not suffice in addressing distributive justice, since it does not account for factors beyond an individual's control, such as their gender, family background, or health status. Thus, it may be necessary to introduce a progressive redistribution scheme that allocates more resources to individuals who made lower contributions during their working lives, compensating them for their disparate outcomes and opportunities.

In this paper, we propose a pension system based on two automatic adaptation mechanisms (AAM):

- The first dynamic mechanism is integrated directly into the pension formula and corrects the heterogeneity of longevity when it exists between socio-economic classes (intragenerational fairness principle).

- A steering mechanism for both the contribution rate and the replacement rate respects Musgrave's rule, which makes it possible to distribute the demographic risk between working people and retirees (intergenerational fairness principle).

The pension formula operates on a longitudinal axis and integrates longevity heterogeneity correction via the progressivity in the formula, and the financial sustainability is assured on a transverse axis by piloting the contribution rate and the replacement rate using a risk-sharing invariant inspired by the Musgrave rule. It must be said that other risk sharing mechanisms than the Musgrave rule are possible.

Our main contributions are the dynamic longevity heterogeneity correction directly integrated in the progressive pension formula through an intragenerational correction as well as the design of the Progressive Musgrave system, which operates on both intergenerational and intragenerational levels.

The paper is divided into five main sections. The first section describes the progressive longevity heterogeneity correction and its generalization in a dynamic environment. The second section presents the defined Musgrave system as introduced by Devolder and De Valeriola [Devolder (2020)]. The third section combines the two mechanisms into a unique plan and shows the rules for calculating pension benefits as well as the evolution of the plan's parameters. The fourth section introduces a useful tool that should aid in the transition from traditional pension systems to those outlined in the preceding part and the final section presents the results of the analysis, including the computation of the integrated plan based on historical data gathered from the United States. Overall, the paper aims to provide a comprehensive analysis of the integrated pension plan and its potential benefits for retirees based on historical mortality data.

2 Intra-generational Automatic Adjustment Mechanism

Although not in all nations, the differences in mortality and life expectancy have been growing as global wealth inequality has become more lopsided. The use of income or education quantiles, which do not signal distributional widening or narrowing, has made it challenging to connect widening income distributions to mortality disparities. Ex-ante disparities and ex-post disparities are two categories for lifetime differences. While the latter reveals the random element of mortality occurrences, the former displays variations in the chance of death. Ex-ante inequalities in longevity may cast doubt on the fairness of risk-sharing financial instruments like annuities and pensions. Ex-post inequalities in death ages, however, may lead to substantial variations in the compensation received but do not represent the same threat.

When life expectancy inequalities are linked to income distribution, it leads to an unavoidable regressivity in defined benefit(DB) pension systems. This underlying actuarial unfairness is a major concern, particularly if the public pension scheme aims for redistribution. To address this issue, a progressive factor mechanism was proposed in a previous paper [Diakite (2021)], for a specific pension plan in a static environment. In this section, the study extends this progressivity mechanism for a DB final salary plan in a dynamic environment, presenting the main results for pension transformation, progressive factors, and pension benefits.

2.1 Progressive formula for a final salary plan in a Static environment

We consider a defined-benefit pension system in which agents are differentiated in terms of salary and life expectancy. To study the redistributive features of such a pension system, we consider the lifetime benefit ratio. This ratio for an agent is the discounted value of the pension benefit divided by their last salary. We introduce a progressive transformation of the pension formula that takes into account the life expectancy differential in order to satisfy the lifetime replacement rate equality condition for agents of different salary classes. In the previous paper, we were able to explicitly describe a method to determine the progressive factors in a career average defined benefit.

We implemented the progressive mechanism in the pension formula, aiming to correct the underlying unfairness issue. In order to transform a DB system into a progressive one, we chose to transform the salary in the pension formula using salary bandwidths, and based on those thresholds, we applied the progressive coefficients.

We apply this technique here for a final salary plan in a static environment. In this plan, the final salary is noted S^{x_r}.

The following are the main assumptions about the salary class and mortality:

- Salary levels differentiate the agent classes. There are m classes. We take note of S_j, which is the salary for class j. Let us imagine that the salary of an agent depends on his age (x). This gives :

$$S_j^x = S_j^{x_0} \cdot (1 + s_j)^{(x-x_0)}$$

  Where s_j is the salary growth effect for group j.

  We can write the salary transformation for any class j as such(with $S_0^x = 0$):

$$X(S_j^{x_r}) = X_j^{x_r} = \sum_{i=1}^{j} \lambda_i \cdot (S_i^{x_r} - S_{i-1}^{x_r})$$

- The pension at age x_r for class j agents is noted $P_j^{x_r} = \delta \cdot X_j^{x_r}$ instead of $P^{x_r} = \delta \cdot S^{x_r}$ when we do not take longevity heterogeneity into account.

- Present value of benefit are discounted with the factor $\nu = \dfrac{1}{1+r}$

- We compute the total population periodic mortality table and using the survival probabilities ${}_x p_{x_r}$ we obtain the average annuity rate :

$$\ddot{a}_{x_r} = \sum_{x=x_r}^{\omega} {}_{x-x_r}p_{x_r} \cdot (\frac{1}{1+r})^{x-x_r}$$

- Once the classes are formed, we can compute the periodic mortality table for each class $j \in [1 : m]$ and obtain the stratified survival probabilities ${}_{x}p_{x_r}^{(j)}$ used to compute the stratified annuity rates

$$\ddot{a}_{x_r}^{(j)} = \sum_{x=x_r}^{\omega} {}_{x-x_r}p_{x_r}^{(j)} \cdot (\frac{1}{1+r})^{x-x_r}$$

- We define the inter-class fairness condition using lifetime replacement rate equality.
The lifetime replacement rate measures the present value of benefits divided by the last salary. This indicator is equivalent to the lump sum at retirement expressed in the number of final salaries. Since the progressive transformation affects only pension benefits, this indicator compares efforts between retirees of different classes.
Before taking into account longevity heterogeneity, the LR based on the mean longevity rates for the whole population in the regime is written as follows:

$$\begin{aligned} LR^* &= \frac{1}{S^{x_r}} \sum_{x=x_r}^{\omega} P^{x_r} \cdot {}_{x-x_r}p_{x_r} \cdot (\frac{1}{1+r})^{x-x_r} \\ &= \frac{P^{x_r}}{S^{x_r}} \cdot \sum_{x=x_r}^{\omega} {}_{x-x_r}p_{x_r} \cdot (\frac{1}{1+r})^{x-x_r} \\ LR^* &= \delta \cdot \ddot{a}_{x_r} \end{aligned}$$

This value of the lifetime replacement rate is by definition the same for every agent; this is the target LR value when ignoring longevity heterogeneity.

- When taking into account longevity heterogeneity, we can write the lifetime replacement rate for each class as follows:

$$LR_j = \delta \cdot \frac{X_j^{x_r}}{S_j^{x_r}} \cdot \ddot{a}_{x_r}^{(j)} \tag{1}$$

- We obtain the values of progressive coefficients when the LR equality condition is satisfied :

$$LR_1 = ... = LR_j = ... = LR_m = LR^*$$

Solving this system for $\lambda = (\lambda_1; ...; \lambda_m)$ gives the following form for the progressive coefficients :

$$\lambda_1 = \frac{\ddot{a}_{x_r}}{\ddot{a}_{x_r}^{(1)}} \tag{2}$$

$$\lambda_j = \frac{S_j^{x_r} \cdot \frac{\ddot{a}_{x_r}}{\ddot{a}_{x_r}^{(j)}} - S_{j-1}^{x_r} \cdot \frac{\ddot{a}_{x_r}}{\ddot{a}_{x_r}^{(j-1)}}}{S_j^{x_r} - S_{j-1}^{x_r}} \tag{3}$$

The pension for a complete contribution for an agent of class j is given by:

$$P_j^{x_r} = \delta \cdot X_j^{x_r} = \delta \cdot S_j^{x_r} \cdot \frac{\ddot{a}_{x_r}}{\ddot{a}_{x_r}^{(j)}}$$
$$P_j^{x_r} = \delta \cdot S_j^{x_r} \cdot \theta_j$$

where $\theta_j = \dfrac{\ddot{a}_{x_r}}{\ddot{a}_{x_r}^{(j)}}$. The longevity heterogeneity correction for an agent of class j will be denoted as θ_j.

We notice that if all the classes are identical in terms of mortality after retirement, we come back to a classical DB on a final salary. In this system, the pension benefit would be equal to:

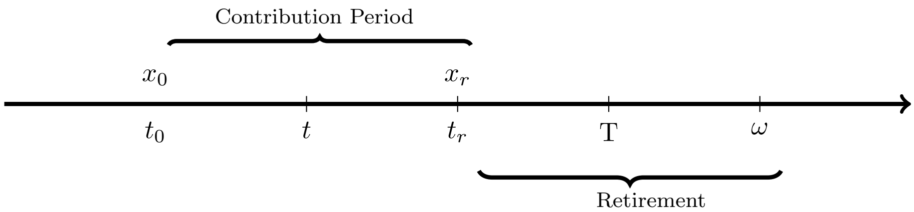
$$P_j^{x_r} = \delta \cdot S_j^{x_r}$$

In this first stationary framework, ,the progressive formula only transforms the salary taken into account in the pension benefit calculation; the replacement rate stays the same. The solution to the lifetime replacement rate fairness condition yields the progressive coefficient value. This condition states that the present value of benefits divided by the last salary of every agent in the regime is the same.

2.2 Progressive formula in a dynamic environment

The estimation of the length of the remaining years of life is evolving and is different between classes; therefore, introducing static mechanisms in the pension formula that incorporate correction based on these estimations is prone to being inaccurate over time and has to be corrected.

We will introduce a dynamic progressive formula that takes into account the evolution of the annuity rate to correct for inaccuracies between the moment an affiliate joins the pension system t_0 and the moment they retire t_r.



- Before retirement we define the salary at age $x < x_r$ for a class i :

$$S_j^{(t;x)} = S_j^{(t_0;x_0)} \cdot (1 + s_j)^{(x-x_0)} \cdot (1 + \gamma)^{(t-t_0)}$$

  Where s_j is the career growth effect for the group j salary and γ the inflation effect depending on time t.

- After retirement, we define $S_j(t;x) = S_j(t;x_r) \cdot (1+\gamma)^{x-x_r}$ for $x > x_r$. Because the concept of salary no longer makes sense after retirement, we consider the retired "notional salary" to be the indexed last salary, with γ representing the indexation rate of pension benefits (wage growth rate).

- The pension benefit at retirement is given by : $P_j^{(t_r;x_r)} = \delta \cdot X_j^{(t_r;x_r)}$
  The pension benefit formula is made up of two parts: the scheme mean benefit rate δ, and a progressive longevity heterogeneity component, which is determined by the salary transformation. $X_i^{(t_r;x_r)}$
- Pension benefit after retirement are indexed with γ as such for $x > x_r$: $P_j^{(T;x)} = P_j^{(t_r;x_r)} \cdot (1+\gamma)^{x-x_r}$
- The progressive factors are solutions to the lifetime replacement rate equality condition. The present value of the pension benefit computed at retirement divided by the last salary should be the same for all agents.
- The progressive transformation at retirement is expressed this way:

$$X_j^{(t_r;x_r)} = \sum_{i=1}^{j} \lambda_i^{t_r} \cdot (S_i^{(t_r;x_r)} - S_{i-1}^{(t_r;x_r)})$$

- Pension formula

The dynamic coefficient formula every period t_j between t_0 and t_r :

$$\lambda_i^{t_j} = \frac{S_i \cdot \dfrac{\ddot{a}_{x_r}(t_j)}{\ddot{a}_{x_r}^{i}(t_j)} - S_{i-1} \cdot \dfrac{\ddot{a}_{x_r}(t_j)}{\ddot{a}_{x_r}^{i-1}(t_j)}}{S_i - S_{i-1}} \tag{4}$$

Where $\ddot{a}_{x_r}(t_j) = \sum_{x=x_r}^{\omega} {}_{x-x_r}p_{x_r}(t_j) \cdot (\dfrac{1+\gamma}{1+r})^{x-x_r}$ represents the life annuity at retirement computed using estimated survival probabilities at time t_j, Every period, the progressive coefficients are communicated to affiliates of the regime. Active workers can evaluate their pension at retirement based on the annual update of the coefficients.

The pension benefit estimated at time t ($t_0 < t < t_r$) is :

$$\bar{\bar{P}}_j^{(t;x)} = \delta \cdot \sum_{i=1}^{j} \lambda_i^{t} \cdot (S_i^{(t;x)} - S_{i-1}^{(t;x)})$$

$$\bar{\bar{P}}_j^{(t;x)} = \delta \cdot S_j^{(t;x)} \cdot \frac{\ddot{a}_{x_r}(t)}{\ddot{a}_{x_r}^{j}(t_0)} = \delta \cdot S_j^{(t;x)} \cdot \theta_j^{t}$$

The pension benefit at retirement is :

$$P_j^{(t_r;x_r)} = \delta \cdot X(\lambda(t_r))$$

$$P_j^{(t_r;x_r)} = \delta \cdot S_j^{(t_r;x_r)} \cdot \frac{\ddot{a}_{x_r}(t_r)}{\ddot{a}_{x_r}^{j}(t_r)} = \delta \cdot S_j \cdot \theta_j^{t_r}$$

The lifetime replacement rate is then given by

$$LR_j(t_r) = \delta \cdot \ddot{a}_{x_r}(t_r)$$

The inter class fairness condition is met : the present value of pension benefits over the last salary (lifetime replacement rate) is the same across all the classes.

- After retirement

In order to avoid too much uncertainty on the amount to pay after retirement, we assume that the progressive factors are fixed at their value at the retirement age for all the agents. The pension benefit at a time $T > t_r$ for an agent of age $x > x_r$ is :

$$P_j^{(T;x)} = \delta \cdot X_j^{(T;x)}$$

, the progressive transformation of the pension benefit is written:

$$\begin{aligned} X_j^{(T;x)} &= \sum_{i=1}^{j} \lambda_i^{t_r} \cdot (S_i^{(T;x)} - S_{i-1}^{(T;x)}) \\ X_j^{(T;x)} &= X_j^{(t_r;x_r)} \cdot (1+\gamma)^{x-x_r} \end{aligned}$$

This means that every new cohort of retirees will have their own pension benefit formula at retirement, but in the year following retirement, the progressive part of the pension benefit is fixed for them.

Remark 1: Fully dynamic system

It is also possible for the progressive system to be fully dynamic even after retirement; such a system does not respect the lifetime replacement rate equality condition and introduces potential uncertainty after retirement. In this scheme, the progressive factors for the entire retiree population evolve dynamically, and even after retirement, the pension level is adjusted with the estimation of the life annuity at retirement.

$$X_j^{(T;x)} = \sum_{i=1}^{j} \lambda_i^{T} \cdot (S_i^{(T;x)} - S_{i-1}^{(T;x)})$$

Remark 2: Communication strategy
Before retirement, the plan doesn't have to be fully dynamic, from an operational standpoint, we present an equivalent system able to reproduce the same pension benefit at retirement with fixed parameters and an ex-post correction at retirement age.

- Pension formula:

The progressive pension benefit formula corrects for longevity heterogeneity ex ante. The progressive factors are computed using mortality data at inception . The estimated pension benefit at time t ($t_0 < t < t_r$) and communicated to the affiliate as the target retirement age is:

$$\begin{aligned} \bar{P}_j^{(t;x_r)} &= \delta \cdot \bar{X}_j^{(t;x_r)} \\ &= \delta \cdot S_j^{(t;x_r)} \cdot \frac{\ddot{a}_{x_r}(t_0)}{\ddot{a}^j_{x_r}(t_0)} \end{aligned}$$

- Balance mechanism:

Once one reaches retirement we apply a ex post correction based on the ratios between the final mortality table and the initial one.

The correction factor is given for an agent of class i by

$$g_j(t_0;t_r) = \frac{\ddot{a}_{x_r}(t_r)}{\ddot{a}^j_{x_r}(t_r)} \cdot \frac{\ddot{a}^j_{x_r}(t_0)}{\ddot{a}_{x_r}(t_0)}$$

The pension benefit is obtained via the correction with:

$$P_j^{(t_r;x_r)} = \bar{P}_j^{(t_r;x_r)} \cdot g_j(t_0;t_r) = \delta \cdot S_j^{(t;x_r)} \cdot \frac{\ddot{a}_{x_r}(t_r)}{\ddot{a}^j_{x_r}(t_r)}$$

The lifetime replacement rate:

$$LR_j(t_r) = \delta \cdot \ddot{a}_{x_r}(t_r)$$

The lifetime replacement rate is the same for every agent, the system respects our fairness criteria.

3 Intergenerational Automatic Adjustment Mechanism

In order to introduce a first form of risk sharing, let us first set up a simple deterministic two-period framework without longevity heterogeneity in which the Musgrave rule [Musgrave (1948)] can be easily defined. The framework is set up as a two-period model with no longevity heterogeneity. In period 0, the system is stable, and each active and retiree individual receives a homogeneous salary S and pension P, respectively. The replacement rate, denoted by δ_0, is the percentage of a retiree's pre-retirement income that is replaced by their pension. The contribution rate, denoted by π_0, is the percentage of an active individual's salary that goes toward the pension system. The dependence ratio, denoted by D_0, is the ratio of retirees to contributors. The pension system is characterized by the budget equation and the pension equation.

The budget equation states that the total amount paid in pensions must equal the total amount contributed to the pension system

$$D_0 \cdot P = \pi_0 \cdot S \tag{5}$$

The pension equation states that the pension received by each retiree (P) is equal to the replacement rate (δ_0) times their pre-retirement income (S).

$$P = \delta_0 \cdot S \tag{6}$$

Leading to the condition

$$\pi_0 = \delta_0 \cdot D_0 \tag{7}$$

Now, suppose a demographic shock occurs in period 1, which changes the dependence ratio from D_0 to D_1 (with $D_1 > D_0$, indicating an ageing population). The impact of this shock on δ_1 and π_1 will depend on the pension architecture.

In a pure defined benefit (DB) scheme, the replacement rate is constant, and the active population bears the whole burden of the demographic shock through an increase in the contribution rate. In this case, δ_1 remains equal to δ_0, while π_1 increases by a factor of (D_1/D_0).

$$\pi_1 = \pi_0 \cdot \frac{D_1}{D_0}$$

On the other hand, in a pure defined contribution (DC) scheme, the contribution rate is constant, and the retiree population bears the whole burden of the demographic shock through a decrease in benefits. In this case, π_1 remains equal to π_0, while δ_1 decreases by a factor of (D_0/D_1).

$$\delta_1 = \delta_0 \cdot \frac{D_0}{D_1}$$

The defined Musgrave architecture aims to provide a form of risk sharing between the active and retired generations by fixing the Musgrave ratio, which represents the ratio between the pension and the salary net of pension contributions. By fixing this ratio, the burden of the demographic shock is shared between the two generations.

To achieve this, the values of δ and π are adjusted in such a way that the Musgrave ratio remains constant at time t=1. The evolution of the replacement and contribution rates is obtained by using the Musgrave ratio equality:

$$M_0 = M_1 = M = \frac{\delta_0}{1-\pi_0} = \frac{\delta_1}{1-\pi_1}$$

Using the budget equation , we can rewrite π_0 in terms of D_0 and P:

$$\pi_0 = \frac{D_0 \cdot P}{S}$$

Substituting this into the expression for M_0, we obtain:

$$M_0 = \frac{\delta_0}{1 - \frac{D_0 \cdot P}{S}}$$

Solving for P and simplifying, we get:

$$P = \frac{S\delta_0}{1 + \pi_0 \cdot \delta_0}$$

Using this expression for P, we can rewrite the Musgrave ratio as:

$$M_0 = \frac{\delta_0}{1-\pi_0} = \frac{\delta_0}{1 - \frac{D_0\delta_0}{1+\pi_0\cdot\delta_0}}$$

Simplifying, we obtain:

$$M_0 = \frac{\delta_0}{1 - \frac{D_0}{1+\pi_0\delta_0}} = \frac{\delta_0(1+\pi_0\cdot\delta_0)}{1+\pi_0\cdot\delta_0 - D_0}$$

Setting this expression equal to M_1 and solving for π_1, we obtain:

$$\pi_1 = \pi_0 \cdot \frac{D_1}{D_0 + \pi_0 \cdot (D_1 - D_0)}$$

Similarly, setting the expression for P at time $t = 1$ equal to the expression for P at time $t = 0$, we obtain:

$$\delta_1 = \frac{\delta_0}{1 + \delta_0 \cdot (D_1 - D_0)}$$

Therefore, the Defined Musgrave architecture provides a way to share the risk of demographic shocks between the active and retired generations by adjusting the replacement and contribution rates in such a way that the Musgrave ratio remains constant over time.

The Musgrave rule is appealing for two reasons. To begin, it means that demographic or economic shocks cause equiproportional changes in pensions and labor wages net of contributions, to the extent that these changes are defined by pension policy. As a result, despite these shocks, inter-generational income inequality will remain unaltered [Schokkaert (2020)]. This could be regarded as good in terms of equity. Second, the Musgrave rule can be viewed as a pragmatic interpretation of an optimal insurance policy in terms of allocating resources to one cohort over its own life cycle [1] . Given reasonable assumptions about individual utility functions, effective inter-generational risk sharing requires that shocks have no effect on the ratio of old to young consumption levels [Mankiw (2007)].

In the event of population ageing, the dependency ratio D rises, therefore the replacement rate lowers, which might make political acceptance harder. This will uniformly cut the level of pensions without taking into account disparities in lifespan, particularly for the lowest pensions, which may lead to non-social acceptability due to injustice and unfairness. In the next section, we propose a mechanism that reconciles risk sharing between generations and social class equity.

[1] For instance, Ball and Mankiw (2007) propose that social security should match this conclusion and derive this result as the anticipated outcome in a hypothetical scenario with fully developed insurance markets.

4 Inter-generational and Intra-generational Automatic Adjustment Mechanisms

This section describes a pay-as-you-go pension system combining a progressive pension benefit formula as well as a demographic risk sharing mechanism piloting the contribution rate and the pension rate.

- Agents only enter the regime at age x_0 .
- Active workers can only exit the regime by dying.
- The effective of workers aged x at period t of class i is given by $Na_i^{(t;x)}$, the number of retirees aged y is $Nr_i^{(t;y)}$
- The effective are given via the relationship $Na_i^{(t;x+1)} = Na_i^{(t;x)} \cdot {}_1p_x^i(t)$

  ${}_1p_x^i(t)$ is the one year survival probability of individuals of age x at time t in class i . We obtain the retiree's number following the same procedure. Since there is no migration, the population function at year t can be obtained from the entry function and a periodic mortality table.
- The total number of active workers at time t is given by Na^t and the total number of retirees Nr^t

4.1 Two period model

This part introduces a mix of intergenerational ageing risk sharing using a Musgrave-like invariant and an intragenerational correction factor for longevity heterogeneity through a progressive pension formula.
In this framework, we are displaying the two mechanisms in action. We imagine a system functioning for a long period of time with the initial parameters for contribution rate replacement and progressive factors, until we add a disturbance in ageing and longevity heterogeneity at time $t = 1$ and observe both of the risk sharing mechanisms in action[2].

- $t = 0$

When we initialize the system, we start computing the progressive transformation for all retirees using the first mortality table. For all ages $x \geq x_r$ the progressive transformation is:

$$X_i^{(0;x)} = \sum_{j=1}^{i} \lambda_j^0 \cdot (S_j^{(0;x)} - S_{j-1}^{(0;x)})$$

The pension benefit for the retirees of class i after retirement is written:

$$P_i^{(0;x)} = \delta_0 \cdot X_i^{(0;x)}$$

[2]This implies that before $t = 0$ we are in a DB system

The budget equation is

$$\pi_0 \cdot \sum_{i=1}^{m} \sum_{x=x_0}^{x_{r-1}} Na_i^{(0;x)} \cdot S_i^{(0;x)} = \delta_0 \sum_{i=1}^{m} \sum_{x=x_r}^{\omega} Nr_i^{(0;x)} \cdot X_i^{(0;x)}$$

We introduce the notation for the average benefit ratio $\bar{\delta}$, which is the ratio between the pension benefits and the retiree's notional salary:

$$\bar{\delta_0} \cdot \sum_{i=1}^{m} \sum_{x=x}^{\omega} Nr_i^{(0;x)} \cdot S_i^{(0,x)} = \delta_0 \sum_{i=1}^{m} \sum_{x=x_r}^{\omega} Nr_i^{(0;x)} \cdot X_i^{(0;x)} \tag{8}$$

The previous expression becomes :

$$\pi_0 = \bar{\delta_0} \cdot \frac{\sum_{i=1}^{m} \sum_{x=x_r}^{\omega} Nr_i^{(0;x)} \cdot S_i^{(0,x)}}{\sum_{i=1}^{m} \sum_{x=x_0}^{x_{r-1}} Na_i^{(0;x)} \cdot S_i^{(0;x)}}$$

We define the economic dependency ratio as :

$$D_0^* = \frac{\sum_{i=1}^{m} \sum_{x=x_r}^{\omega} Nr_i^{(0;x)} \cdot S_i^{(0,x)}}{\sum_{i=1}^{m} \sum_{x=x_0}^{x_{r-1}} Na_i^{(0;x)} \cdot S_i^{(0;x)}}$$

then the budget equation is given by

$$\pi_0 = \bar{\delta_0} \cdot D_0^* \tag{9}$$

Musgrave proposed an invariant that results in risk sharing between active workers and retirees. Let us define the Musgrave invariant (or the invariant risk sharing condition) as the ratio between the average pension and the average salary net of pension contributions:

$$M_0 = \frac{\bar{\delta_0} \cdot \dfrac{\sum_{i=1}^{m} \sum_{x=x_r}^{\omega} Nr_i^{(0;x)} \cdot S_i^{(0,x)}}{Nr^0}}{(1-\pi_0) \cdot \dfrac{\sum_{i=1}^{m} \sum_{x=x_0}^{x_{r-1}} Na_i^{(0;x)} \cdot S_i^{(0;x)}}{Na^0}}$$

We define :

$$\mu_0 = \frac{\dfrac{\sum_{i=1}^{m} \sum_{x=x_r}^{\omega} Nr_i^{(0;x)} \cdot S_i^{(0,x)}}{Nr^0}}{\dfrac{\sum_{i=1}^{m} \sum_{x=x_0}^{x_{r-1}} Na_i^{(0;x)} \cdot S_i^{(0;x)}}{Na^0}} = \frac{D_0^*}{D_0}$$

μ_t is the ratio between the economic dependency ratio and the dependency ratio at time t

The Musgrave ratio becomes :

$$M_0 = \frac{\bar{\delta_0}}{1-\pi_0} \cdot \mu_0 \tag{10}$$

- $t = 1$

The system progresses to the next stage, which is distinguished by another economic dependency ratio, D_1^*.We want to define the relationship between the parameters $\bar{\delta_1}$ and π_1 and $(\lambda_1^1, .., \lambda_m^1)$so that the risk sharing condition stays invariant and the budget equation and the intergenerational fairness condition are respected.

The budget equation becomes:

$$\pi_1 = \bar{\delta_1} \cdot D_1^*$$

and the Musgrave ratio stays invariant, leading to this condition:

$$M_1 = \frac{\bar{\delta_1}}{1 - \pi_1} \cdot \mu_1 = \frac{\bar{\delta_0}}{1 - \pi_0} \cdot \mu_0 = M_0$$

We obtain the values of the contribution rate and average benefit ratio solving the previous system of equations:

$$\bar{\delta_1} = \bar{\delta_0} \cdot \frac{\frac{\mu_0}{\mu_1}}{1 + \bar{\delta_0} \cdot (D_1^* \cdot \frac{\mu_0}{\mu_1} - D_0^*)} \tag{11}$$

$$\pi_1 = \pi_0 \cdot \frac{\frac{\mu_0}{\mu_1} \cdot D_1^*}{D_0^* + \pi_0 \cdot (D_1^* \cdot \frac{\mu_0}{\mu_1} - D_0^*)} \tag{12}$$

4.2 Multi-period model

We can extend those results to any given period of time based on the recurrence property tying the parameters. Here is presented the procedure for the adjustment of the regime parameters at any given time t:

- Intragenerational correction:
  The first step is to adjust the longevity heterogeneity correction . The lifetime replacement rate equality condition allows us to obtain the progressive coefficient. They only depend on the final salary and on the annuity computed with the year's survival probability .

  $$\lambda_i(t) = \frac{S_i^{(t;x_r)} \cdot \frac{\ddot{a}_{x_r}(t)}{\ddot{a}_{x_r}^i(t)} - S_{i-1}^{(t;x_r)} \cdot \frac{\ddot{a}_{x_r}(t)}{\ddot{a}_{x_r}^{i-1}(t)}}{S_i^{(t;x_r)} - S_{i-1}^{(t;x_r)}}$$

  The progressive coefficients allow us to obtain the progressive transformation for all the classes.

- Intergenerational correction:
  The pay-as-you-go equilibrium for year t is obtained through recurrence with the previous year's equilibrium, and the risk sharing is constant. The

relationship between the average benefit ratio and the contribution rate for those years is expressed as such:

$$\pi_t = \pi_{t-1} \cdot \frac{\frac{\mu_{t-1}}{\mu_t} \cdot D_t^*}{D_{t-1}^* + \pi_{t-1} \cdot (D_t^* \cdot \frac{\mu_{t-1}}{\mu_t} - D_{t-1}^*)}$$

$$\bar{\delta}_t = \bar{\delta}_{t-1} \cdot \frac{\frac{\mu_{t-1}}{\mu_t}}{1 + \bar{\delta}_{t-1} \cdot (D_t^* \cdot \frac{\mu_{t-1}}{\mu_t} - D_{t-1}^*)} \tag{13}$$

- Average pension rate:
  When we obtain the average benefit ratio, we can transform it back into a mean pension rate via formula (8) and use it in the pension benefit formula.

$$\delta_t = \bar{\delta}_t \cdot \frac{\sum_{i=1}^{m} \sum_{x=x_r}^{\omega} Nr_i^{(t;x)} \cdot S_i^{(t;x)}}{\sum_{i=1}^{m} \sum_{x=x_r}^{\omega} Nr_i^{(t;x)} \cdot X_i^{(t;x)}} \tag{14}$$

- Pension benefit for new retirees:
  In this scheme , once you reach retirement your progressive transformation is fixed.

$$P_i^{(t_r;x_r)} = \delta_{t_r} \cdot X_i^{(t_r;x_r)} = \delta_{t_r} \cdot \sum_{j=1}^{i} \lambda_j^{t_r} \cdot (S_j^{(t_r;x_r)} - S_{j-1}^{(t_r;x_r)}) \tag{15}$$

- Pension benefit for old retirees:
  The progressive transformation for already retired generations stays the same, for $x > x_r$, only the pension rate and the notional salary change .

$$P_i^{(T;x)} = \delta_T \cdot X_i^{(T;x)} = \delta_T \cdot \sum_{j=1}^{i} \lambda_j^{T-x+x_r} \cdot (S_j^{(T;x)} - S_{j-1}^{(T;x)}) \tag{16}$$

Remark 3: Fully dynamic system

It is also possible to extend those results in the system that does not guarantee the progressive transformation after retirement, we present the results in terms of mean benefit rate in the following.

The progressive transformation after retirement in the fully dynamic progressive factors scheme is:

$$P_i^{(T;x)} = \delta_T \cdot X_i^{(T;x)} = \delta_T \cdot \sum_{j=1}^{i} \lambda_j^{T} \cdot (S_j^{(T;x_r)} - S_{j-1}^{(T;x_r)})$$

The benefit rate becomes :

$$\delta_T = \bar{\delta_T} \cdot \frac{\sum_{i=1}^{m} \sum_{x=x_r}^{\omega} Nr_i^{(T;x)} \cdot S_i^{(T;x)}}{\sum_{i=1}^{m} \sum_{x=x_r}^{\omega} Nr_i^{(T;x)} \cdot X_i^{(T;x)}}$$

This system, although simpler to implement and explain lacks of intergenerational fairness since the lifetime replacement rate equality is never verified when there is longevity heterogeneity.

5 Numerical application

In this section, we aim to project the population by utilizing mortality rates from the United States spanning from 1982 to 2019, segmented by socio-economic quintiles [Barbieri (2020)]. We will further analyze the evolution of pension system parameters, accounting for potential heterogeneity in longevity and demographic risk sharing. Initially, the population was distributed using the mortality rates of 1982. However, we will subsequently introduce demographic modifications to incorporate differences in survival probabilities and mortality rates for active workers and retirees in each salary class, which will be informed by mortality rate estimates segmented by socio-economic quintiles for the United States from 1982 through 2019.

The mortality data highlight two important effects on the population: first, the increase in longevity across all classes; and, most importantly, the fact that the gain in longevity is not equally distributed between poor and rich agents in the regime. These two effects motivate the introduction of an intergenerational risk-sharing mechanism to spread the longevity risk between active workers and retirees and an intragenerational correction mechanism for longevity heterogeneity.

We will start with a defined benefit system offering a mean benefit rate $\delta^{DB} = 60\%$ of the last working salary and observe its reaction to the two previous effects. The objective is to compare this system to one that integrates longevity heterogeneity correction and automatic adaptation mechanisms via the progressivity of the pension formula and a risk sharing rule that steers the contribution rate and the mean benefit rate.

We divided the population in three salary categories, and we suppose that the population is entirely characterized by these three classes.

- Low income class agents represent the first quintile Q_1 of the population.
- Average class composed of the next three quintiles(Q_2, Q_3, Q_4) .
- High income class agent represent the 5th quintile Q_5 of income distribution.

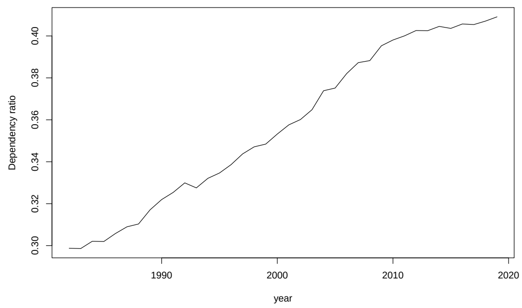
As starting salary for each class we used income quintiles from the United States [US Census Bureau (2023)]

$$S_1^{(1982;x_0)} = 4790$$
$$S_2^{(1982;x_0)} = 20675$$
$$S_3^{(1982;x_0)} = 54720$$

The methodology for projecting the salaries, the population effective and computing the life annuities is explained in Annex A.

Figure 1 presents the evolution of the Dependency ratio (ratio between the number of retirees and the number of active workers)

Figure 1: Evolution of the dependency ratio. *Source*:Author's Calculations



The starting and last values are summarized in *Table 1*

Table 1: Dependency ratio values

	D
1982	0.2987
2019	0.4091

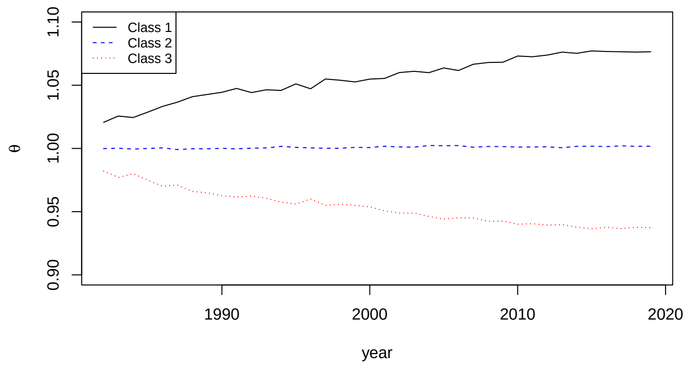
We also compute the annuity rates by class, the life annuity for the entire population and their evolution. The values for the discount factor and indexation rate are specified in the Appendix 1.

$$\ddot{a}^i_{x_r}(t) = \sum_{x=x_r}^{\omega} {}_{x-x_r}p^i_{x_r}(t) \cdot (\frac{1+\gamma}{1+r})^{x-x_r}$$

We present in *Figure 2* the evolution of the longevity heterogeneity correction, defined as the ratio between the entire population life annuity and the life annuity of each class every year.

$$\theta^t_i = \frac{\ddot{a}_{x_r}(t)}{\ddot{a}^i_{x_r}(t)}$$

Figure 2: Evolution of the longevity heterogeneity correction by class. *Source*:Author's Calculations



This parameter describes how the gain in mortality is distributed every year among our three classes.

Table 2: Longevity gap evolution between a salary class and the general population

	1982	2019
Class 1	1.0206	1.0765
Class 2	0.9998	1.0018
Class 3	0.9821	0.9373

Table 2 shows that the longevity heterogeneity gap increases between the low income class and the high income class every year. Each year , θ_i^t is also the intensity of the correction on the pension benefit for class i.

We observe that in both years, Class 1 has the highest value for the correction, followed by Class 2, and then Class 3. This means that gains in mortality are distributed more heavily towards the higher income class (Class 3) compared to the lower income class (Class 1).

However, we can also see that the values for this parameter have changed over time, indicating that the gap in longevity heterogeneity between different income classes has increased. For example, in 1982, the value of the correction for class 1 was 1.0206, while in 2019, it had increased to 1.0765. In contrast, the value for class 3 decreased from 0.9821 in 1982 to 0.9373 in 2019.

We also describe the in *Table 3* evolution of the progressive factors at different periods. They follow the relationship:

$$\lambda_i(t) = \frac{S_i^{(t;x_r)} \cdot \dfrac{\ddot{a}_{x_r}(t)}{\ddot{a}^i_{x_r}(t)} - S_{i-1}^{(t;x_r)} \cdot \dfrac{\ddot{a}_{x_r}(t)}{\ddot{a}^{i-1}_{x_r}(t)}}{S_i^{(t;x_r)} - S_{i-1}^{(t;x_r)}}$$

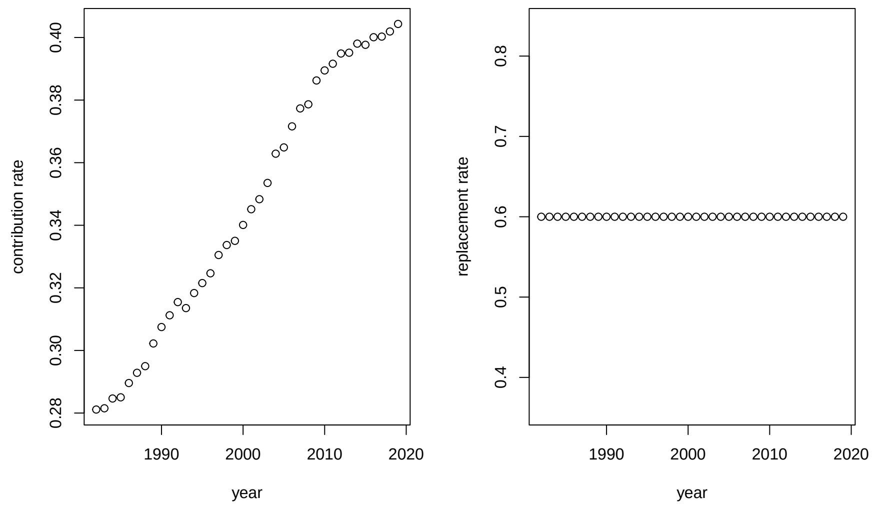
Table 3: Progressive factors evolution

	λ_1	λ_2	λ_3
1982	1.0206	0.9966	0.9740
1992	1.0442	0.9934	0.9452
2002	1.0600	0.9920	0.925
2012	1.0738	0.9899	0.9112
2019	1.0765	0.9901	0.9080

5.1 Pure Defined Benefits system (DB)

In our initial DB system , the replacement rate is fixed at $\delta = 60\%$. There is no longevity heterogeneity correction strategy nor risk sharing mechanism. The replacement rate is constant and the demographic risk is supported by the active workers, we present in *Figure 3* the evolution of the contribution rate in this system ;

Figure 3: Contribution rate and replacement rate in a pure DB system. *Source*:Author's Calculations



The starting and final values of the contribution rate and replacement rate are displayed in *Table 4*:

Table 4: Contribution and replacement rate in the pure DB system

	π
1982	0.2811
2019	0.4043

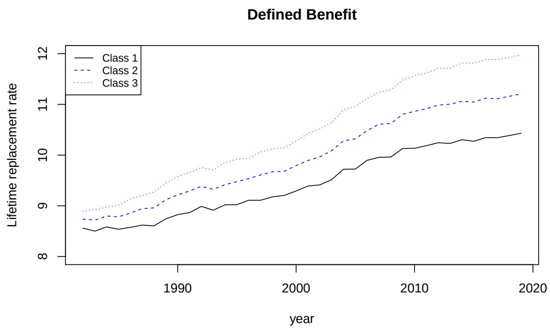
(a) Contribution rate

	δ
1982	0.6
2019	0.6

(b) Replacement rate

We compute in *Figure 4* the lifetime replacement rate at retirement for the three classes, to observe unfairness after retirement between the three classes :

Figure 4: Lifetime replacement rate by salary class in a DB system.*Source*:Author's Calculations



We can compare the relative lifetime replacement rate compared to class 2, $(\dfrac{LR_i}{LR_2})$:

Table 5: Relative lifetime replacement rate in a DB system in 1982 and 2019

	1982	2019
Class 1	97.96%	93.06%
Class 3	101.81%	106.88%

Table 5 suggests that the lifetime pension benefits for Class 1 decreased more compared to Class 2, while the lifetime pension benefits for Class 3 increased more compared to Class 2, indicating a greater increase in lifetime replacement rate among high-income workers. In a Defined benefit system the demographic risk is solely supported by active workers. Since longevity gain are not uniformly distributed among all retirees classes we observe a greater increase in lifetime replacement rate among high income workers.

5.2 Defined Musgrave system

In order to share the demographic risk between retirees and active workers, we introduced the Musgrave rule, as defined in the intergenerational automatic adjustment mechanism section. This rule allows an intergenerational risk sharing mechanism between workers and retirees. *Figure 5* describes the evolution of the contribution rate and replacement rate in this system:

Figure 5: Contribution rate and replacement rate in a DM system. *Source*:Author's Calculations

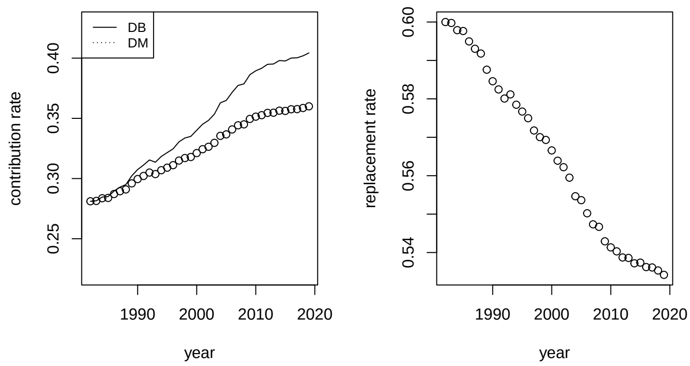


Table 6 shows that the replacement rate for retires decreases as well as the contribution rate increases such that the Musgrave ratio is constant the whole time.

Table 6: Contribution and replacement rate in the DM system

	π
1982	0.2811
2019	0.3599

(a) Contribution rate

	δ
1982	0.6
2019	0.5364

(b) Replacement rate

When taking into account longevity when computing the lifetime replacement rate we observe that the system is not fair. We are going to compare in *Figure 6* the lifetime replacement rate between the three classes in a DM system vs a DB system to assess intra-generational fairness between the classes.

Figure 6: Comparison of lifetime replacement rate in a DM system (left) vs DB system (right). *Source*:Author's Calculations

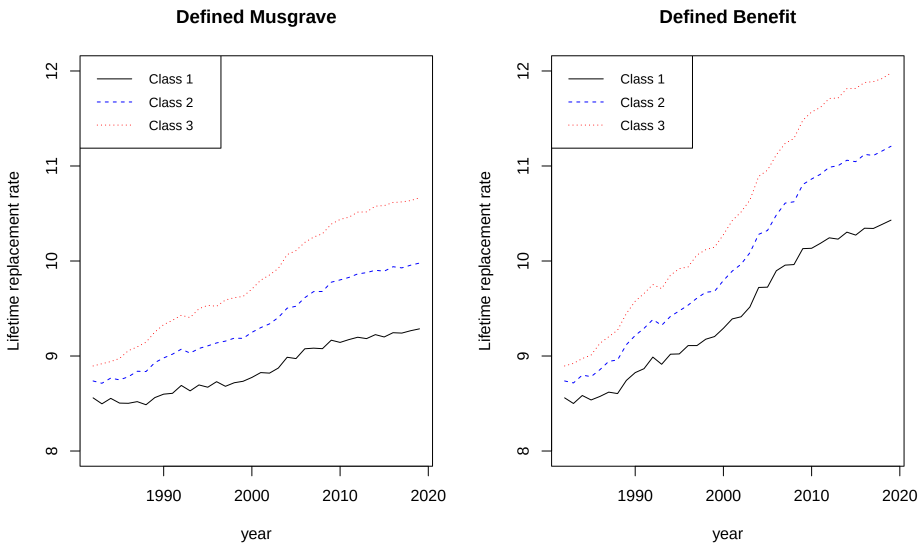


Table 7: Relative Lifetime replacement rate in a DM system

	1982	2019
Class 1	97.96%	93.06%
Class 3	101.81%	106.88%

Table 8: Lifetime replacement rate by class in a DM system

	1982	2019
Class1	8.5595	9.2859
Class2	8.7374	9.9782
Class3	8.8952	10.6650

From *Table 8*, we can see that the lifetime replacement rate in a DM system has increased for all three classes between 1982 and 2019. Additionally, in *Table 7* the relative lifetime replacement rate compared to class 2 is highest for class 3 in both years. This suggests that high-income workers benefit more from a DM system than low-income workers, as they receive a higher lifetime replacement rate and also experience a smaller decrease due to longevity heterogeneity. Thus we need a longevity heterogeneity correction mechanism , as it could help to reduce the penalty for low-income workers.

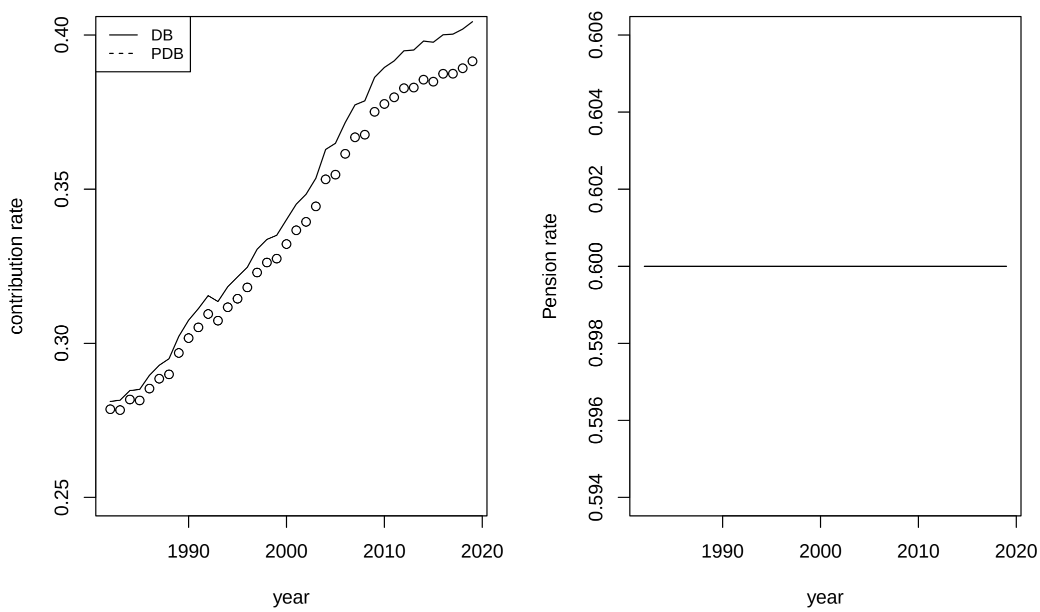
5.3 Progressive DB system (PDB)

We introduce a progressive system without a demographic risk sharing mechanism to analyse the first effect on the pension benefit and the lifetime replacement rate. We define the PDB pension benefit formula as such:

$$P_i^{(t;x_r)} = \delta \cdot X_i^{(t;x_r)}$$

In this system only the progressive part of the pension formula is updated for every new generation that retires. The pension rate is constant, the demographic risk is transferred to the active workers hence the "defined benefit" nomenclature.

Figure 7: Evolution of the contribution rate and mean benefit ratio in the progressive DB system. *Source*:Author's Calculations



The starting and final values of the contribution rate and replacement rate are :

Table 9: Contribution and replacement rate in the progressive DB system

	π
1982	0.2786
2019	0.3914

(a) Contribution rate

	δ
1982	0.6
2019	0.6

(b) Replacement rate

We observe in *Table 9* that the contribution rate in the DB system in 2019 stands at 0.4043, while that of the PDB system is at 0.3914 . The disparity in contribution rates is attributable to differences in redistribution mechanisms employed in the two systems.

The lifetime replacement rate is the same in every class due to the progressive transformation applied in the pension formula. We present the evolution of the lifetime replacement rate in *Table 10*:

Table 10: Lifetime replacement rate in a Progressive DB system

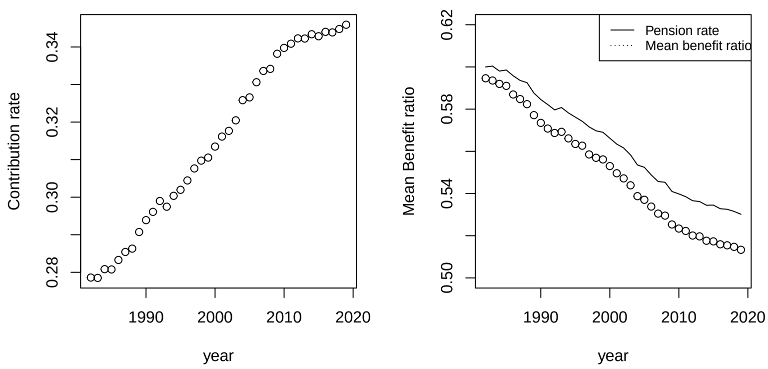
	1982	2019
LR_i	8.73644	11.22801

The intragenerational fairness condition is met although the demographic risk is only supported by active workers.

5.4 Progressive Defined Musgrave system (PDM)

We propose a system that combines the implementation of longevity heterogeneity correction techniques with intergenerational risk sharing. The risk-sharing parameters are in *Figure 8* as follows.

Figure 8: Contribution rate and mean benefit ratio in a PDM system. *Source*:Author's Calculations



There is a steering mechanism between the contribution rate, the mean benefit ratio and the risk sharing invariant.

Table 11: Contribution rate and pension rate in the PDM system

	π
1982	0.2786
2019	0.3459

(a) Contribution rate

	δ
1982	0.6
2019	0.5301

(b) Replacement rate

This system also integrates intragenerational fairness, the lifetime replacement rate at retirement should be the same for every agents in the regime. we present in *Table 12* the initial and final values for the lifetime replacement rate in this system.

Table 12: Lifetime replacement rate values in the PDM system

	1982	2019
Class i	8.73644	9.921065

We define the class replacement rate at retirement as the ratio between the pension benefit over the last working salary:

$$Tr_i^{(t;x_r)} = \frac{P_i^{(t,x_r)}}{S_i^{(t,x_r)}} \tag{17}$$

We present across the projection some values for the risk sharing mean benefit ratio $\bar{\delta}$, the pension formula rate δ and the replacement rate at retirement for each class Tr_1, Tr_2, Tr_3.

Table 13: Comparative table of mean benefit ratio, benefit rate, and replacement rate by class

	$\bar{\delta}$	δ	Tr_1	Tr_2	Tr_3
1982	0.5946298	0.6	0.6123982	0.5999337	0.5892852
1992	0.5686889	0.5796425	0.605263	0.5797883	0.5578465
2002	0.5471703	0.5616006	0.595314	0.5622799	0.5329082
2012	0.5200873	0.5365872	0.5762197	0.5372748	0.5040369
2019	0.5133252	0.5301598	0.5707188	0.5311234	0.4969196

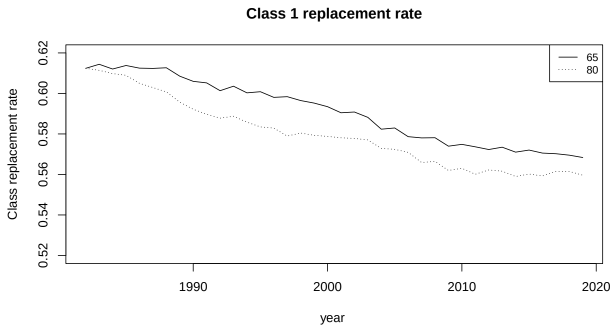
- After retirement

After retirement , the pension benefit evolution is only driven by the evolution of the system mean benefit rate δ_t. The replacement rate for every new generation retiring is:

$$Tr_i^{(T;x_r+(T-t_r))} = \delta_T \cdot \theta_i^{t_r}$$

We will compare in *Figure 9* the evolution of the replacement rate at 65 and 80 between the first generation to retire in 1982 and the one who retires in 2019 for low incomes and high incomes.

Figure 9: Evolution of the replacement rate at 65 and 80 for low incomes(top) and high income (bottom) in the PDM system. *Source*:Author's calculations



The replacement rate in the high income class is higher for older retirees because the newer generation longevity correction increases.

Table 14: Benefit ratio at 65 and 80 for high incomes in the double AAM system

$\delta_T \cdot \theta_i^{t_r}$	65	85
1982	0.5892852	0.5892852
2019	0.4948666	0.499557

The opposite effect appears in the low income class, older generation have a lower benefit ratio because the longevity correction favours more the new retirees.

Table 15: Benefit ratio at 65 and 80 for low incomes in the double AAM system

$\delta_T \cdot \theta_i^{t_r}$	65	85
1982	0.6123982	0.6123982
2019	0.5683609	0.5596287

The proposed formula would represent a significant departure from traditional pension systems, which typically use fixed benefit calculations. Our approach aims to introduce a more flexible system that can adapt to changing demographic trends and ensure the long-term sustainability of pension programs.

5.5 Transition to progressive systems

Starting from the ground up, such a system would be socially difficult to implement; however, a gradual transition from existing systems could be achieved through the use of a convex transformation of mortality intensity. To accomplish this, we propose in the following section a gradual transition mechanism based on a convex transformation of mortality intensity. This mechanism ensures a smooth and controlled transition from the current system to the proposed progressive formula, minimizing any social and economic disruptions. Using this mechanism, the proposed system can be implemented in a fair and transparent manner, with the potential to significantly improve the financial sustainability of pension programs.

The previous numerical application was done using the mortality intensities $\mu^i_{x,t}$ extracted from the tables by socio-economic groups. For the entire population, we used $\mu_{x,t}$, the mortality intensity for all the groups. We are going to test the results under different scenarios for longevity heterogeneity using a transformation inspired by credibility theory . The idea is to use a convex transformation of the mortality intensity specific to a socio-economic class and the mortality intensity of the entire population.

Let us consider the following expression for the estimate of the mortality intensity in each class i,

$$\tilde{\mu}^i_{x,t} = (1-\alpha)\cdot\mu^i_{x,t} + \alpha\cdot\mu_{x,t} \tag{18}$$

α is defined as the progressivity indicator of the transition. Each value of $\alpha \in [0;1]$ defines a new mortality intensity . Consequently each value of α defines a new progressive system given that the pension formula uses a previously defined progressive transformation. The application of the credibility transformation in the longevity adaptation and demographic risk sharing framework also allows for a parallel with the systems previously defined.

In a system without demographic risk sharing :

- $\alpha = 1$ is equivalent to a pure DB system
- $\alpha = 0$ is equivalent to the Progressive DB system.

In a system with demographic risk sharing :

- $\alpha = 1$ is equivalent to a Defined Musgrave system
- $\alpha = 0$ is equivalent a PDM system .

This means that we can allow for the parametrization of a large number of systems depending on the importance we give to mortality intensity relative to the socio-economic classes. The progressivity indicator, α, plays a crucial role in defining the degree of progressivity in the system. The previous numerical application used a fixed value of α, but it is important to note that the progressivity indicator can also evolve over time. In fact, a system described with a decreasing $\alpha(t)$ will become completely progressive when the progressivity indicator reaches its minimal value. Therefore, the proposed pension system can be customized and adapted to changing circumstances by adjusting the value of α.

We are going to present the evolution of the longevity correction, the mean benefit ratio,the replacement rates at retirement in each class and the contribution rates for each system for select values of α that we refer to as the progressivity indicator.

- Longevity heterogeneity correction

The previous application in systems that allowed longevity heterogeneity correction showed that most of the differences existed between the first and last socio-economic classes. We are displaying the evolution of the longevity heterogeneity correction , the average benefit ratio, and the contribution rate for different values of α.

Figure 10: Longevity heterogeneity correction in the low income class (left) and high income class (right)

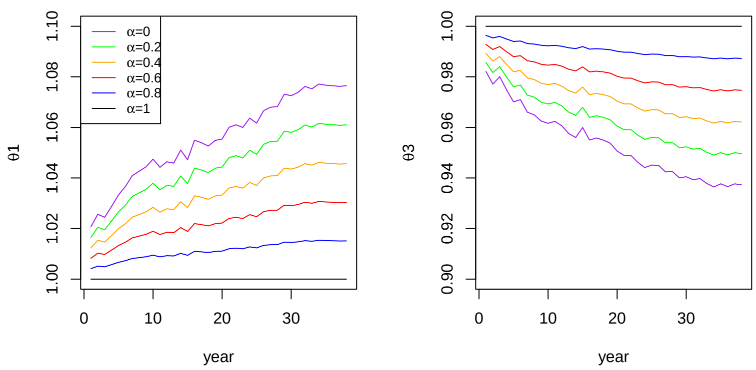


Figure 10 displays the evolution of the longevity heterogeneity correction for both low and high socio-economic classes. When α is close to 0 the correction is more important in both classes.

- Mean benefit ratio and contribution rate

Figure 11: Mean benefit ratio and contribution rate for different values of α . *Source*:Author's calculations

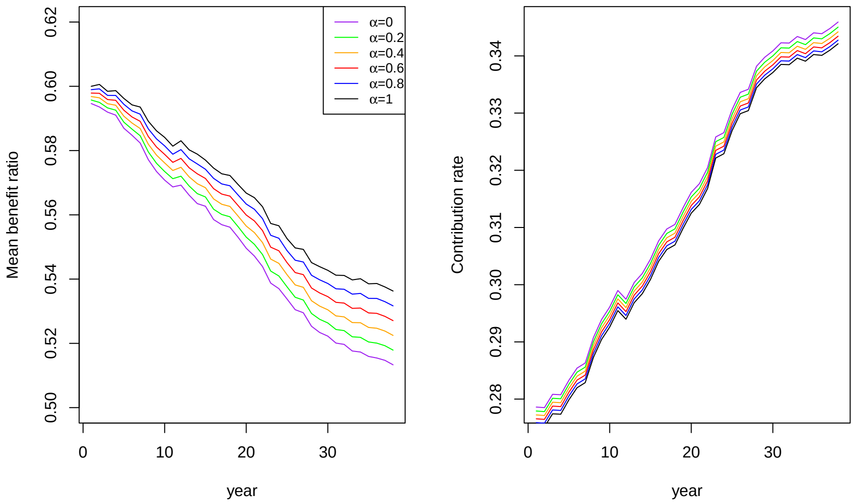


Figure 11 displays the evolution of the average benefit ratio for different values of α. As expected, when $\alpha = 1$ the replacement rate is the highest for all socio-economic classes, which corresponds to a DM system. On the other hand, when $\alpha = 0$, the replacement rate is the lowest, which corresponds to the Progressive DM. The intermediate values of α allow for a parametrization of the system between these two extremes.

We present the evolution of the replacement rate for each class using a sequence for the value of the progressivity indicator

Table 16: Evolution of the class replacement rate for different values of the progressivity indicator

t	α	Tr_1	Tr_2	Tr_3
1982	0.8	0.6025	0.6000	0.5978
1992	0.6	0.5908	0.5807	0.5718
2002	0.4	0.5832	0.5633	0.5456
2012	0.2	0.5691	0.5379	0.5112
2019	0	0.5707	0.5311	0.4969

α	Tr_1	Tr_2	Tr_3
0	0.6124	0.5999	0.5893
0	0.6053	0.5798	0.5578
0	0.5953	0.5623	0.5329
0	0.5762	0.5373	0.5040
0	0.5707	0.5311	0.4969

From the previous tables, we can see the impact of longevity correction and progressivity on the replacement rates for each class. As the progressivity indicator increases, the replacement rates decrease for each class, indicating that a more progressive system leads to a lower replacement rate for the high earners. This is because a more progressive system places a larger burden on high earners to support low earners, which reduces the replacement rate for high earners. We can also see that the impact of progressivity on the replacement rate depends on the level of progressivity indicator and the longevity correction.

For example, when the progressivity indicator decreases from one to zero , a less abrupt transition from a DM system to a fully progressive DM system is possible for different values of α. This suggests that a gradual transition to a more progressive system may be beneficial for minimizing the impact on replacement rates.

It is also possible not to completely correct longevity heterogeneity and only partially apply progressivity in the system. We showcase the example with $\alpha = 0.4$ throughout the evolution of the system.

Table 17: Evolution of the class replacement rate for $\alpha = 0.4$

t	α	Tr_1	Tr_2	Tr_3
1982	0.4	0.6074	0.5999	0.5935
1992	0.4	0.5956	0.5803	0.5671
2002	0.4	0.5832	0.5633	0.5456
2012	0.4	0.5620	0.5386	0.5185
2019	0.4	0.5566	0.5329	0.5120

We see that when only partial progressivity is applied (e.g., $\alpha = 0.4$), the replacement rates still decrease over time, but the rate of decrease is not as steep as in a fully progressive system. This indicates that a partial application of progressivity can still lead to a more adequate system while minimizing the impact on replacement rates.

Overall, the class replacement rates provide important information on the impact of progressivity and longevity correction on the replacement rates for each class, which is crucial for designing a sustainable and equitable pension system.

5.6 Discussion

We are going to compare, through the projection period (1982–2019), the evolution of the lifetime replacement rate and the pension benefit for the three classes between the previous system presented and also the contribution effort of active workers within those systems

- Defined Benefit system -(without AAM)

The base plan is a DB system on a final salary, providing a replacement rate of $\delta = 60\%$ and the dependency ratio in 1982 is $D = 0.27$, thus the equilibrium contribution rate is $\pi = 0.28$.

The dependency ratio increases every year, the pension contribution equilibrium is affected, and by design the replacement rate stays constant, therefore the contribution of active workers increases. In 2019, its final value was $\pi = 0.4$. The replacement rate of every retiree is the same, and the lifetime replacement rate for each class depends on the remaining life expectancy at retirement. Initially, since there is little longevity heterogeneity, we can observe from Table 5 that high incomes have a higher lifetime replacement rate than low incomes, but this difference in last salary received after retirement is less than a year. At the end of the projection , the overall longevity increased, we deduced this observation from the evolution of the lifetime replacement rate in Figure 4, since all the curves are increasing and the replacement rate is constant and longevity is increasing. We also observe that the longevity gain is not evenly distributed among our salary class, meaning that the increase among high income agents is greater than that among low-income agents. This translates into more than a 1-year gap in the lifetime replacement rate differences between those two classes.

- Defined Musgrave system -(inter-generational AAM)

The second system we looked at was the Defined Musgrave system (DM), which includes a risk-sharing parameter M that allows for the evolution of both the replacement rate and the contribution rate in order to share the demographic risk between active workers and retirees. (Table 6) presents the initial and final values of those two parameters, and we observe that the increase in contribution rate is less intense for active workers, the final value of $\pi = 0.36$ is inferior to the DB system, but the replacement rate in the counterpart also decreases, from 60% to 53%. The lifetime replacement are also affected by the piloting of the replacement rate, reducing the lifetime replacement rate in each class. This indicates that the inter-generational risk sharing piloting of the replacement rate penalizes the low-income workers twice, as they not only support the decrease in the replacement rate, but also the longevity heterogeneity.

- Progressive Defined Benefit system - (Intragenerational AAM)

To focus solely on the intra-generational risk-sharing mechanism, we examined the progressive defined benefit (PDB) system, which directly incorporates longevity heterogeneity correction in the pension formula. Like the defined benefit (DB) system, the PDB system lacks a demographic risk-sharing component, placing the demographic risk burden on active workers. We observed that the contribution rate in the PDB system increases from 25% to 36% while the pension rate remains constant. Figure 7 depicts the similarity in the evolution of the contribution rate between the PDB and DB systems. However, the contribution rate in the PDB system is lower than that of the DB system due to the distinct redistribution approaches utilized by both systems. Table 9 shows that the lifetime replacement rate is equal in all three classes, with pension benefits adjusted for longevity heterogeneity correction factors. Although the PDB system would be a favourable alternative, the absence of a demographic risk-sharing component renders it unsuitable for consideration.

- Progressive Defined Musgrave system -(Intergenerational plus Intragenerational AAM)

We combine the two mechanisms into one system, the double AAM system, in order to correct for longevity heterogeneity and demographic risk sharing. In comparison with the previous intra-generational fair systems, the lifetime replacement rate is the lowest, and indeed the pension rateδ is decreasing, driving the LR for all classes down (Table 10). The pension rate communicated to the affiliates is above the mean benefit ratio, $\overline{\delta}$ making it easier to understand since it applies directly to the transformed salary. Two effects influence the replacement rate, for low income agents, the decline in the pension rate is mitigated by an increase in the longevity heterogeneity correction, thereby reducing the loss in replacement rate. However, for high income class agents, both effects are negative, resulting in a lower replacement rate at the end of the projection.

We summarize in terms of class replacement rate for each group the effect of each system.

Table 18: Replacement rate by class comparison

	DB		DM		PDB		PDM	
	1982	2019	1982	2019	1982	2019	1982	2019
Tr_1	0.6	0.6	0.6	0.5364	0.6125	0.6452	0.6125	0.5707
Tr_2	0.6	0.6	0.6	0.5364	0.5997	0.6010	0.5997	0.5311
Tr_3	0.6	0.6	0.6	0.5364	0.5897	0.5630	0.5897	0.4969

Each system that follows the defined benefit framework includes a mechanism for sharing longevity heterogeneity and ageing risks. The defined Musgrave system distributes demographic risk between active workers and retirees by controlling the contribution and replacement rates. As the population ages, the replacement rate declines, leading to a higher share of benefits for the high-income class. On the other hand, the progressive defined benefit system addresses longevity heterogeneity by sharing risks between retirees of different socio-economic classes, with lifetime replacement rates being equalized across retirees. However, this system may result in a loss for active workers. The double Automatic adjustment mechanism system combines both approaches, facilitating wealth redistribution from high to low socio-economic classes while ensuring financial stability amidst population ageing.

6 Conclusion

The aim of this study is to address the challenges faced by national public pension systems in the context of demographic ageing and significant longevity inequalities between socio-economic classes. The paper proposes a pension system that seeks to achieve intergenerational and intragenerational equity, while also ensuring financial sustainability and social adequacy. The first goal of the proposed system is to design a fair system that takes into account the differences in longevity among retirees. This will help to ensure that the system is fair to all categories of retirees. In addition, the system incorporates an intergenerational risk sharing mechanism that will distribute demographic risk more fairly among active workers and retirees. This will help to ensure that the system is financially sustainable over the long-term. Finally, the proposed system aims to be socially acceptable through transparent communication and easy to implement, mainly due to the fact that it is based on salaries. This paper emphasizes the importance of making changes to national public pension systems in order to ensure long-term sustainability in the face of demographic and economic events. To address financial imbalances, most pension schemes have implemented automatic adjustment or stabilization mechanisms, but a difficult trade-off must be made to balance intergenerational costs with intragenerational actuarial fairness and social adequacy. Moreover, we suggest a pension system based on two automatic adaptation mechanisms that ensure intergenerational and intragenerational equity. The first mechanism accounts for longevity heterogeneity among pension scheme agents, while the second distributes demographic risk among working people and retirees. The pension formula incorporates longevity heterogeneity correction and progressivity, while financial sustainability is ensured by the Musgrave rule-inspired risk-sharing invariant. Finally, we argue that constructing pension schemes entirely on actuarial principles is challenging and that a balance must be achieved between intergenerational costs, intragenerational actuarial fairness, and social adequacy. The suggested pension system may provide a solution to these issues, but further research is needed to determine its viability and usefulness in various circumstances. The study underlines the significance of tackling the issues faced by the ageing population as well as preserving the long-term viability and fairness of public pension systems. Overall, this study highlights the importance of preserving the long-term viability and fairness of public pension systems, and suggests a promising path forward to achieve these goals. In conclusion, this paper has proposed a pension system that seeks to achieve intergenerational and intragenerational equity, financial sustainability, and social adequacy in the face of demographic ageing and socio-economic disparities in longevity. While the proposed system is based on a fixed retirement age and final salaries as a basis for benefits, further research could explore alternative rules of indexation and the potential benefits of incorporating progressivity into the system. For instance, alternative rules of indexation could include the use of average salaries or a different measure of inflation to adjust benefits, which could potentially impact the financial sustainability and social adequacy of the system. Furthermore, incorporating progressivity into the system could provide more flexibility in achieving greater equity by adjusting retirement age instead of pension benefits to equalize the lifetime replacement rate across different demographic groups. By continuing to refine and improve the proposed pension system, we can help ensure the long-term viability and fairness of public pension systems in the years to come.

A Methodology for projection

We extracted the mortality rates by quintile of the population from the Mortality by Socio-economic Category in the United States study. The research stratifies the US population into socio-economic groups of similar population size using eleven county-wide factors on education, occupation, employment, income. The study produced a set of comprehensive life tables broken down by socio-economic group and year that may be utilized in mortality models. Details on the data and methodologies used in the development of the life tables are summarized in the research report. [Barbieri (2020)]

- The mortality data range is from 1982 to 2019. From those periodic mortality tables we are able to extract each period the 1 year survival probabilities by socio-economic group j, is ${}_1p_x^j(t)$.
- Agents only enter the regime at age $x_0 = 25$. Retirement age is $x_r = 65$
- Active workers can only exit the regime by dying.
- The effective of workers aged x at period t is given by $Na_i^{(t;x)}$, the number of retirees aged y is $Nr_i^{(t;y)}$
- The effective are given via the relationship $Na_i^{(t;x+1)} = Na_i^{(t;x)} \cdot {}_1p_x^i(t)$

  ${}_1p_x^i(t)$ is the one year survival probability . We obtain the retirees effective following the same procedure. Since there is no migration the population function at year t can be obtained recursively from the entry function and the periodic mortality table.
- The total number of active workers at time t is given by Na^t and the total number of retirees Nr^t.
- The initial number of active workers corresponds to the entry function at age x_0 is $E = 100000$
- The distribution of workers within the salary classes stays the same throughout the whole projection :
  Quintile 1 (Q_1) of salary distribution represents the low salary workers.
  Quintiles 2,3,4 (Q_2, Q_3, Q_4) represent the average salary class.
  Quintile 5 (Q_5) of salary distribution represents the high salary class.
- Low income class agents represent 20% of the population , average class represent 60% ,and high income retiree represent 20% of the population.
- The dependency ratio measures the number of retirees over the number of active workers
- The initial dependency ratio depicts the demography of a system using the mortality table of 1982. Every following year the number of workers and retirees are obtained via the corresponding mortality table this year.

- Using the same data we compute each period the life annuity at retirement by socio-economic quintile using the following formula

$$\ddot{a}^i_{x_r} = \sum_{x=x_r}^{\omega} {}_{x-x_r}p^i_{x_r} \cdot (\frac{1+\gamma}{1+r})^{x-x_r}$$

- The life annuity indexation rate is the same as the pension indexation rate, we used the average rate of salary evolution.

- We defined salary in each class with

$$S_i^{(t;x)} = S_j^{(t_0;x_0)} \cdot (1+s_j)^{(x-x_0)} \cdot (1+h)^{(t-t_0)}$$

  As starting salary for each class we used data on household income quintiles [US Census Bureau (2023)] , in 1982

$$S_1^{(1982;x_0)} = 4790$$
$$S_2^{(1982;x_0)} = 20675$$
$$S_3^{(1982;x_0)} = 54720$$

- The career growth effect in each group is given by

$$s_1 = 0.1\% \ ; \ s_2 = 0.15\% \ ; \ s_3 = 0.2\%$$

- The time effect is obtained by averaging the yearly evolution of salary in the data through our on our projection horizon and we obtained $h = 2.5\%$

- Pension benefits are indexed with the yearly average salary growth rate $\gamma = 2.5\%$.

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References

[1] [Alho (2013)] J Alho, J M Bravo, E Palmer Non financial defined contribution Pension Schemes in a Changing Pension World, volume 2, p. 395 - 436

[Ardito et al (2020)] Ardito C, D'Errico A, Leombruni R, Ricceri F, Costa G. Life expectancy inequalities and their evolution in Italy. How these impact on the equity of the pension system? *Eur J Pub Health.* 2020;30(5):ckaa165.764. https://doi.org/10.1093/eurpub/ckaa165.764.

[Ayuso et al (2017)] Ayuso M, Bravo JM, Holzmann R. On the heterogeneity in longevity among socioeconomic groups: scope, trends, and implications for earnings-related pension schemes. *Glob J Hum Soc SciEcon.* 2016;17(1):33–58.

[Ayuso et al (2021)] Ayuso, Mercedes, Jorge M. Bravo, and Robert Holzmann. 2021. Getting Life Expectancy Estimates Right for Pension Policy: Period versus Cohort Approach. *Journal of Pension Economics and Finance* 20: 212–31.

[Barbieri (2020)] Barbieri Magali. Mortality by Socioeconomic Category in the United States. Society of Actuaries, November 2020.

[Belloni et al (2013)] Belloni M, Alessie R, Kalwij ET, et al. Lifetime income and old age mortality risk in Italy over two decades. Demog Res. 2013;29:1261–98.

[Bosworth (2018)] Bosworth B. Increasing disparities in mortality by socioeconomic status. *Annu Rev Public Health.* 2018;39:237–51.

[Bovenberg et al (2015)] L Bovenberg, S Bilsen, R Mehlkopf Personal pensions with risk sharing affordable, adequate and stable private pensions in Europe Netspar Discussion Paper Series, p. 3 - 2015

[Borsch-Supan (2006)] Borsch-Supan A. What are NDC systems? What do they bring to reform strategies? In: Holzmann R, Palmer E, editors. Pension reform: issues and prospects for ndc schemes. Washington: World Bank Publishing; 2006.

[Bosley (2018)] Bosley T, Morris M, Glenn K. Mortality by Career-Average Earnings Level. In: Actuarial study, 124, Social Security Administration, Office of the Chief Actuary; 2018. $https://www.ssa.gov/OACT/NOTES/pdf_studies/study124.pdf$

[US Census Bureau (2023)] US Census Bureau Historical Income tables. March 2023 https://www.taxpolicycenter.org/statistics/household-income-quintiles

[Donkin et al (2002)] Donkin A, Goldblatt P, Lynch K. Inequalities in life expectancy by social class 1972–1999. *London: Health Statistics Quarterly* no. 15; 2002.

[Devolder (2015)] Devolder, Pierre ; Melis, Roberta. Optimal mix between pay as you go and funding for pension liabilities in a stochastic framework. In: *Astin Bulletin : the journal of the International Actuarial Association*, Vol. 45, no.3, p. 551-575 (2015)

[Devolder (2020)] Devolder, Pierre ; De Valeriola, Sébastien. Pension Design and Risk Sharing: Mixed Solutions Between Defined Benefit and Defined Contribution for Public Pension Schemes. In: Marta Peris-Ortiz, José Álvarez-García, Inmaculada Domínguez-Fabián, Pierre Devolder (eds.), *Economic Challenges of Pension Systems*, Springer International Publishing : Cham 2020, p. 311-339

[Diakite (2021)] Diakite, K.; Devolder, P. Progressive Pension Formula and Life Expectancy Heterogeneity. Risks 2021, 9, 127. https://doi.org/10.3390/risks9070127

[Fors (2021)] Fors S, Wastesson JW, Morin L. Growing income-based inequalities in old-age life expectancy in Sweden, 2006-2015. Demography. 2021;58(6):2117–38.

[Holtzmann (2019)] Ayuso M, Bravo JM, Holzmann R. On the heterogeneity in longevity among socioeconomic groups: scope, trends, and implications for earnings-related pension schemes. *Glob J Hum Soc SciEcon.* 2016;17(1):33–58.

[Jijiie (2021)] Jijiie A, Alonso-García J, Arnold S. Mortality by socio-economic class and its impact on the retirement schemes: how to render the systems fairer? *Eur Actuar J.* 2021.

[Kalwij (2013)] Kalwij A, Alessie R, Knoef M. The association between individual income and remaining life expectancy at the age of 65 in the Netherlands. Demography. 2013;50(1):181–206.

[Kleinow (2020)] Wen J, Kleinow T, Cairns AJG. Trends in Canadian mortality by pension level: evidence from the CPP and QPP. *N Am Act J.* 2020;24(4):533–61.

[Kibele (2013)] Kibele EUB, Jasilionis D, Shkolnikov VM. Widening socioeconomic differences in mortality among men aged 65 years and older in Germany. *J Epidemiol Community Health.* 2013;67:453–7.

[Longevity Panel (2018)] Longevity Science Panel. Life expectancy: Is the socio-economic gap narrowing? 2018. http://www.longevitypanel.co.uk/viewpoint/life-expectancy-is-the-socio-economic-gap-narrowing/

[Longevity Panel (2020)] Longevity Science Panel. An analysis of mortality trends in developed countries, focusing on the recent slowdown in mortality improvements. 2020. $https://www.longevitypanel.co.uk/_files/LSP2020_Report.pdf$

[Lallo (2018)] Lallo C, Raitano M. Life expectancy inequalities in the elderly by socioeconomic status: evidence from Italy. *Popul Health Metrics.* 2018;16(1):7.

[Mackenbach (2016)] Mackenbach JP, Kulhánová I, Artnik B, et al. Changes in mortality inequalities over two decades: register based study of European countries. *Brit Med J.* 2016;352(5):1732.

[Marmot(2015)] Marmot M. The health gap. The challenge of an unequal world. London: Bloomsbury Publishing; 2015.

[Musgrave (1948)] Musgrave, Richard, and Tun Thin. 1948. Income Tax Progression, 1929–1948. *Journal of Political Economy* 56: 498.

[Mankiw (2007)] Mankiw NG, Ball L. Intergenerational Risk Sharing in the Spirit of Arrow, Debreu, and Rawls, with Applications to Social Security Design. Journal of Political Economy. 2007;115 (4) :523-547.

[Orszag (2014)] Goldman DP, Orszag PR. The growing gap in life expectancy: using the future elderly model to estimate implications for social security and medicare. Am Econ Rev. 2014;104(5):230–3.

[OECD (2019)] Pensions at a Glance 2019: OECD and G20 Indicators. Paris: OECD Publishing.

[Schokkaert (2020)] Schokkaert Erik, Devolder Pierre, Hindriks Jean, Vandenbroucke Frank, 2020. "Towards an equitable and sustainable points system. A proposal for pension reform in Belgium," LIDAM Reprints CORE 3121, Université catholique de Louvain, Center for Operations Research and Econometrics (CORE).

[Turner(2009)] Turner, J. Social Security Financing: Automatic Adjustments to Restore Solvency. AARP Research Report 2009-01

[Waldron (2007)] Waldron H. Trends in mortality differentials and life expectancy for male social security-covered workers, by socioeconomic status. Soc Secur Bull. 2007;67(3):1–28.

[Wenau (2019)] Wenau G, Grigoriev P, Shkolnikov V. Socioeconomic disparities in life expectancy gains among retired German men, 1997-2016. *J Epidemiol Community Health.* 2019;73:605–11.

[Wouterse(2021)] Bär M, Wouterse B, Riumallo Herl C, Van Ourti T, Van Doorslaer E. Diverging mortality inequality trends among young and old in the Netherlands. *Fisc Stud.* 2021;42:79–101.