Chapter 12

Conclusions and Perspectives

Now this is not the end. It is not even the beginning of the end.
But it is, perhaps, the end of the beginning.
- Winston Churchill

This final chapter presents the main conclusions of the study as well as a number of recommendations for possible future work. The principal features of the extension of spectral element method are first summarized along with the main results from numerical investigation performed on benchmark problems. Directions of research for improving and extending the method are then suggested.

12.1 Summary and Conclusions

In this dissertation, we treated numerical methods for wave propagation in exterior domains using *Spectral Element Method*. The thesis is divided in two main parts: *Acoustics* (chapters 3–7) and *Elastodynamics* (chapters 8–11).

The analysis of wave propagation problems in unbounded domains requires the handling of appropriate radiation conditions (Sommerfeld), which impose that only an outward flow of energy is allowed at infinity and ensure the uniqueness of the solution of the given problem.

We have presented three numerical strategies, able to simulate the Sommerfeld radiation condition underlying the dynamics of unbounded domains: *DtN*, *PML* and *IEM*. These methods were developed for the Helmholtz and the Navier wave propagation coupled to Legendre and Chebyshev spectral element method. For the computational work, their implementation in MATLAB® was one of the main areas of the research. We have shown how to construct spectral element implementation for all schemes and classes of problems.

The accuracy of the developed formulations is assessed through the study of numerous benchmarks. Different boundary conditions and a large range of frequency have been analyzed. The computed results are shown to be in good agreement with the analytic solutions and exponential convergence with the interpolation order is improved. Computational cost, total error, speed of convergence and conditioning are compared in detail for the different methods.

During the computational experiments we observed that globally the DtN and the IEM are more accurate than the PML method. This is, in part, due to the choice of the PML parameters that considerably affect the minimization of the error. An optimal choice of these parameters still appear to be open question.

A list of pertinent literature was furnished for each addressed methodology.

It is believed that the methods presented in this dissertation constitute a spectrum of computational tools that can be used to solve a variety of wave propagation problems in a very efficient and accurate manner.

12.2 Scope of Further Work

There is still much scope for future work in this project.

One aspect which is of great interest to industry is the extension of the techniques to three dimensional problems. This would require an extension of the current set of two dimensional basis functions and with larger problem sizes envisaged in three dimensions. Iterative methods can be used to solve the large resulting linear system. However, such a solving is only efficient when the linear system is well-conditioned. In this optic, the studies on conditioning of previous schemes can be helpful tools.

Another recommended future research would be the extension to transient waves [68].

A direction of research could be the use of mortar element for its numerical efficiency.

The IEM coupled to the SEM is an attractive development in elastodynamics. The method can be based on the conjugated infinite element of Pinto [75] or alternative techniques to take into account the stress boundary conditions.

Modern high-order local non-reflecting boundary conditions are based on the introduction of auxiliary variables on the artificial boundary [101, 216]. The use of auxiliary variables eliminate all the high-order derivatives that occur and can be incorporated in a spectral element scheme.

The quality of the PML scheme depends on a number of parameters and their optimal choice is necessary. A determinist method able to select efficient parameters can be based on the study of numerical dispersion of plane waves in the layer [49].