Parameter Estimation of Hyperbolic Model of Counterflow Heat Exchanger

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Abstract: The aim of this paper is to estimate the parameters of a counterflow heat exchanger in the case where the dynamics are described by a system of two hyperbolic partial differential equations (PDEs). This study is performed using data collected on a laboratory test bench. First we study the practical identifiability of the system, the objective of which is to verify the possibility of determining the parameters of the system in a unique way from the available data. Next the parameter estimation is performed by a nonlinear optimization method, typically by the Levenberg-Marquardt algorithm on a set of ordinary differential equations obtained from the reduction of the original PDEs by the method of lines. Since the Levenberg-Marquardt algorithm may only converge with a reasonable estimate of the parameters, we first consider a least squares parameter estimation approach based on the global difference model. Finally, the results of this identification are validated by using laboratory data, and by verifying some statistical properties.

Keywords: Identifiability, system identification, parameter estimation, heat exchangers, distributed parameter systems.

1. INTRODUCTION

The derivation of of a mathematical model consists in representing a system by a mathematical model that best describes its dynamical behavior. The development of such a model can be carried out either by considering exclusively physical principles governing the behavior of the process (white-box model), by considering approximations on the time response to a test signal (black-box model), or by combining the two approaches (grey-box model).

In this work we are interested in the identification of a counterflow heat exchanger model. Heat exchangers have a fundamental role in most industrial processes (chemical, biochemical, food processing industry, etc.) since heat is essential there. In industry they are generally used in a counterflow configuration, because it offers the possibility of having a high efficiency, unlike the parallel-flow configuration. The dynamics of such type of systems is largely due to transport phenomena (convection and possibly diffusion), leading to the derivation of hyperbolic or parabolic partial differential equations from the internal energy balance of the system (Maidi et al. (2009), Burns and Cliff (2014a), Kazaku et al. (2020, 2022a)). The parameters of these models are generally poorly known. Indeed, an important property of heat exchangers is that the heat transfer coefficients of the fluids depend on the operating points of the mass flows and the inlet temperatures likely to vary over time (Jonsson et al. (1992), Jonsson and Palsson (1994)).

In the literature, heat exchangers have already been the subject of identification studies. In Jacobsen and Skogestad (1994), Kazaku et al. (2018) and Pfortner et al. (2022), the authors propose an approach which consists in estimating the model from the inputs-outputs data. The resulting models, known as the "black-box model", are easy to handle in the context of control design and applications (Babuška (1998)). However, the estimated parameters may not have any physical interpretation. In Jonsson et al. (1992), Jonsson and Palsson (1994), Mutambara and Al-Haik (1999) and Jonsson et al. (2007), the heat exchanger parameters are estimated recursively (by Kalman filtering and recursive least squares) from a lumped parameter model, obtained by splitting the heat exchanger into supposedly homogeneous volumes. The algorithms used are interesting because they allow the operating point variations to be monitored over time. And moreover, the (on-line) estimated parameters have a physical meaning. However, the model that serves as a calibration does not take into account the distributed nature of heat exchangers. Farah et al. (2016) proposed an approach for estimating the parameters of a hyperbolic model of a co-current heat exchanger, that consists in reducing the PDEs of the model using reinitialized partial moments. This approach works well, yet it requires a significant number of thermocouples to be placed along the heat exchanger, so as to characterize temperature gradients. In the same PDEs context, Burns et al. (2018) proposes an approach that consists in transforming the parabolic PDEs of a counterflow heat exchanger into a set of ordinary differential equations (ODEs) obtained by finite element, on which the Levenberg-Marquart algorithm (see Marquardt (1963)) is used to estimate the parameters. The Levenberg-Marquardt algorithm has good convergence properties (espe-

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cially for ill-conditioned problems). Given that it is a local algorithm, it may converge only if we have a reasonable first estimate. So the search for this first estimate is a big challenge for its use. However, the context in which it is used by Burns et al. (2018) does not pose any initialization problem because the algorithm is applied to a simulator whose true values of the parameters are known a priori. This does not correspond to the real situation for which, in general, there is no a priori knowledge of the order of magnitude of the parameters.
In this work the Levenberg-Marquardt algorithm is used to estimate the parameters of the hyperbolic model of a counterflow heat exchanger, from laboratory data. In our approach, the first estimate is obtained by replacing the spatial derivative operators by global differences, which leads to a system of two sets of linear ODEs with respect to the parameters on which the least squares method is used. This study will be preceded by that of practical identifiability. The practical identifiability study is an important step in modelling because it allows to know if the model parameters can be uniquely determined from the available data (Walter and Pronzato (1997), Vanrolleghem et al. (1995)).
The paper is organized as follows. In Section 2, we introduce the PDEs of the hyperbolic model of counterflow heat exchangers, that we transform in Section 3 by method of lines into a set of ODEs. In Section 4 we study the practical identifiability of the system by considering the steady-state model. Section 5 deals with the estimation and the validation of the heat exchanger parameters.

2. SYTEM MODEL

We are interested in a tubular counterflow heat exchanger as depicted in the Figure 1 involving a heat transfer phenomenon between two fluids. The system model is established using the first law of thermodynamics, considering constant pressure (Kazaku et al. (2022b), Maidi et al. (2009)):

$$\frac{\partial x}{\partial t} = \underbrace{\begin{pmatrix} -v_1 & 0 \\ 0 & v_2 \end{pmatrix}}_{\Lambda} \frac{\partial x}{\partial z} + \underbrace{\begin{pmatrix} -\alpha_1 & \alpha_1 \\ \alpha_2 & -\alpha_2 \end{pmatrix}}_{M} x(z,t), \tag{1}$$

with $x(z,t) = (x_1(z,t)\ x_2(z,t))^\top$ represent the distributed state vector that represent the temperatures of each fluid, v_j and $\alpha_j = \frac{A_s U_g}{\rho_j S_j c_{p_j}}$ are the velocity and heat transfer coefficient of component j, respectively. $A_s, U_g, \rho_j, c_{p_j}$ and S_j represent the heat transfer area, the overall transfer coefficient, density, heat capacity and section of the tube of fluid j, respectively.
The system (1) is completed by the following boundary conditions:

$$x_1(0,t) = x_{1,in}(t),\ x_2(1,t) = x_{2,in}(t), \tag{2}$$

and the vector of the measured outputs:

$$y_m(t) = \begin{pmatrix} x_1(1,t) = x_{1,out}(t) = y_m^1(t) \\ x_2(0,t) = x_{2,out}(t) = y_m^2(t) \end{pmatrix}. \tag{3}$$

3. MODEL APPROXIMATION

In this part, we shall reduce the equations (1)-(2) of the heat exchanger to a system of ordinary differential equations. There are several possibilities for this. In our case we opt for the solution by the method of lines (MOL) (Wouwer et al. (2001), Maidi et al. (2008)). This method is simple to implement and avoids the complexity of the computations encountered by adopting other approaches such as weighted residual methods and variational methods (Burns and Cliff (2014b), Burns and Cliff (2014a), Burns et al. (2013)). The principle of the method consists in discretizing the spatial differential operator by using a numerical scheme of finite differences. Consider a forward and backward discretization scheme for the first order derivatives with respect to z by taking n finite elements (with $n = \frac{L=1}{\Delta z}$, and Δz the discretization step), (1)-(2) becomes:

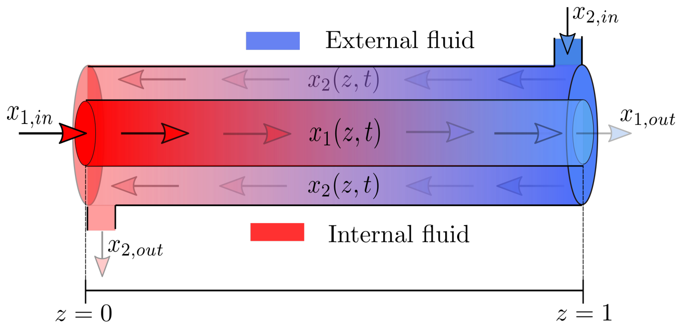


Fig. 1. A counter-current heat exchanger

$$\begin{cases} \dfrac{dx_{1,j}}{dt} &= -v_1 \dfrac{x_{1,j} - x_{1,j-1}}{\Delta z} + \alpha_1(x_{2,j} - x_{1,j}) \\ & \text{with } j = 1,\cdots,n, \\ \dfrac{dx_{2,j}}{dt} &= v_2 \dfrac{x_{2,j+1} - x_{2,j}}{\Delta z} + \alpha_2(x_{1,j} - x_{2,j}) \\ & \text{with } j = 0,\cdots,n-1. \end{cases} \tag{4}$$

Considering $x(t) = [x_{1,1}(t),\cdots,x_{1,n}(t),x_{2,1}(t),\cdots,x_{2,n}(t)]^\top$ we can rewrite (4) in the matrix form:

$$\frac{dx(t)}{dt} = A_n x(t) + B_1 x_{1,in}(t) + B_2 x_{2,in}(t). \tag{5}$$

The matrices A_n, B_1 and B_2 are expressed as follows:

$$A_n = \begin{pmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{pmatrix},$$

where $K_{11} = K_{11}^1 + K_{11}^2$, with $K_{11}^1 = -\left(\frac{v_1}{\Delta z} + \alpha_1\right) \cdot I$, and $K_{11}^2 = \frac{v_1}{\Delta z}\mathscr{O}_1$, $K_{12} = \alpha_1 \mathscr{O}_2$, $K_{21} = \alpha_2 \mathscr{O}_1$, $K_{22} = K_{22}^1 + K_{22}^2$, with $K_{22}^1 = -\left(\frac{v_2}{\Delta z} + \alpha_2\right) \cdot I$ and $K_{22}^2 = \frac{v_2}{\Delta z}\mathscr{O}_2$. The matrices $\mathscr{O}_1$ and $\mathscr{O}_2$ are given by:

$$\mathscr{O}_1 = \begin{pmatrix} 0 & 0 & 0 & \cdots & 0 \\ 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 & 0 \end{pmatrix},$$

$$\mathscr{O}_2 = \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{pmatrix}$$

and $B_1 = \begin{pmatrix} \frac{v_1}{\Delta z} & \cdots & \alpha_2 & \cdots & 0 \end{pmatrix}^\top$, $B_2 = \begin{pmatrix} 0 & \cdots & \alpha_1 & \cdots & \frac{v_2}{\Delta z} \end{pmatrix}^\top$.
Thus, we obtain a system that can be solved easily by using for example the Matlab subroutine *ode*15s.

4. IDENTIFIABILITY

We study in this section the practical identifiability of the heat exchanger model (1)-(2), i.e. if the model parameters can in fact be recovered uniquely from the given data. The approach that will be used is based on the sensitivity functions of the outputs of the heat exchanger. The analysis will be carried out by considering the stationary model corresponding to (1)-(2). The analytical approach will be compared to the numerical approach derived from the approximate model (5). Indeed, with regard to hyperbolic systems, it happens that the analytical results do not corroborate the numerical results (Trefethen and Embree (2005)). In this case, the interest of this comparison is to ensure that the identifiability of the system is not lost by switching to the discrete model.
The stationary problem of (1)-(2) is defined by

$$\begin{cases} \Lambda \dfrac{dx}{dz} + Mx(z) = 0 \\ x_1(0) = x_{1,in},\ x_2(1) = x_{2,in} \end{cases} \tag{6}$$

whose solutions $\bar{x}(z)$ represent the profile of temperatures at equilibrium which are given for all $x(z)$ and $z \in [0,1]$ by $\bar{x}(z) = \Phi(z)x(0)$, with $\Phi(z) = \exp\left(-\Lambda^{-1}Mz\right)$ given by:

$$\Phi(z) = \frac{1}{\beta_2 - \beta_1} \begin{bmatrix} \beta_2 - \beta_1 e^{(\beta_2-\beta_1)z} & \beta_1 \left(e^{(\beta_2-\beta_1)z} - 1\right) \\ \beta_2 \left(1 - e^{(\beta_2-\beta_1)z}\right) & \beta_2 e^{(\beta_2-\beta_1)z} - \beta_1 \end{bmatrix}, \tag{7}$$

where $\beta_1 = \frac{\alpha_1}{v_1}$ and $\beta_2 = \frac{\alpha_2}{v_2}$. Taking into account the configuration of the system, the boundary condition is expressed by

$$\begin{pmatrix} x_1(0) \\ x_2(0) \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ (0\ \ 1)\cdot\exp(-\Lambda^{-1}M) \end{pmatrix}^{-1} \begin{pmatrix} x_{1,in} \\ x_{2,in} \end{pmatrix}. \tag{8}$$

Thus the stationary solution of (6) is:

$$\begin{aligned} \bar{x}_1(z) &= \frac{1}{\beta_1 - \beta_2 e^{(\beta_2-\beta_1)}} \left(\beta_1 e^{(\beta_2-\beta_1)z} - \beta_2 e^{(\beta_2-\beta_1)}\right) x_{1,in} \\ &\quad + \frac{\beta_1}{\beta_1 - \beta_2 e^{(\beta_2-\beta_1)}} \left(1 - e^{(\beta_2-\beta_1)z}\right) x_{2,in}, \\ \bar{x}_2(z) &= \frac{\beta_2}{\beta_1 - \beta_2 e^{(\beta_2-\beta_1)}} \left(e^{(\beta_2-\beta_1)z} - e^{(\beta_2-\beta_1)}\right) x_{1,in} \\ &\quad + \frac{1}{\beta_1 - \beta_2 e^{(\beta_2-\beta_1)}} \left(\beta_1 - \beta_2 e^{(\beta_2-\beta_1)z}\right) x_{2,in}. \end{aligned} \tag{9}$$

Evaluating these expressions at the locations $z = 1$ and $z = 0$ for $x_1(z)$ and $x_2(z)$, respectively, yields the input-output relationship:

$$\begin{cases} x_{1,out} = \dfrac{(\beta_1 - \beta_2)\,\mathrm{e}^{(\beta_2-\beta_1)}}{\beta_1 - \beta_2\,\mathrm{e}^{(\beta_2-\beta_1)}} x_{1,in} + \dfrac{\beta_1\left(1 - \mathrm{e}^{(\beta_2-\beta_1)}\right)}{\beta_1 - \beta_2\,\mathrm{e}^{(\beta_2-\beta_1)}} x_{2,in}, \\ x_{2,out} = \dfrac{\beta_2\left(1 - \mathrm{e}^{(\beta_2-\beta_1)}\right)}{\beta_1 - \beta_2\,\mathrm{e}^{(\beta_2-\beta_1)}} x_{1,in} + \dfrac{\beta_1 - \beta_2}{\beta_1 - \beta_2\,\mathrm{e}^{(\beta_2-\beta_1)}} x_{2,in}. \end{cases} \tag{10}$$

Likewise, the output of the approximated model (5) is given by:

$$y = -CA_n^{-1}\left(B_1 x_{1,in} + B_2 x_{2,in}\right), \tag{11}$$

with $C = \begin{pmatrix} 0 & \cdots & 1 & 0 & \cdots & 0 \\ 0 & \cdots & 0 & 1 & \cdots & 0 \end{pmatrix} \in \mathbb{R}^{2\times n}$ the output matrix.
The output sensitivity functions can be obtained using the Symbolic Math Toolbox of Matlab© dedicated to symbolic computation. We can observe by analyzing the above relationships that the sensitivity functions depend on the value of the model parameters.

Figures 2a to 2h illustrate the output sensitivity functions obtained by the two approaches. One can observe that these functions differ from each other and are independent. This indicates that the parameters of the heat exchanger model are not correlated, and that they can be identified in practice (a unique estimate would be possible). As mentioned in Vanrolleghem et al. (1995) and (Petersen, Britta, 2000, Chapter 4), the sensitivity functions emphasize the experimental conditions under which the dependency of the outputs on the parameters is the largest, and thereby under which conditions most information can be obtained on the parameters. For example, Figures 2a, 2d, 2f and 2g show peaks indicating that at some point the sensitivity with respect to the parameters is important. Thus a possible approach for improving the information contained in the data is to choose the sampling instants corresponding to the zones where the parameters have a prominent influence, i.e. when the sensitivity is high. This could be very helpful to avoid large estimation errors. On the other hand, the Figures 2b, 2c, 2e and 2h do not exhibit any peak. In this case the information contained in the data is distributed homogeneously; and so the information provided by the data set is better exploited.
Furthermore note the existence of significant differences between the numerical approach and the analytical approach. The main differences relate to the sensitivities with respect α_1 and α_2. That is mainly due to the low number of discretization points considered for the simulation.

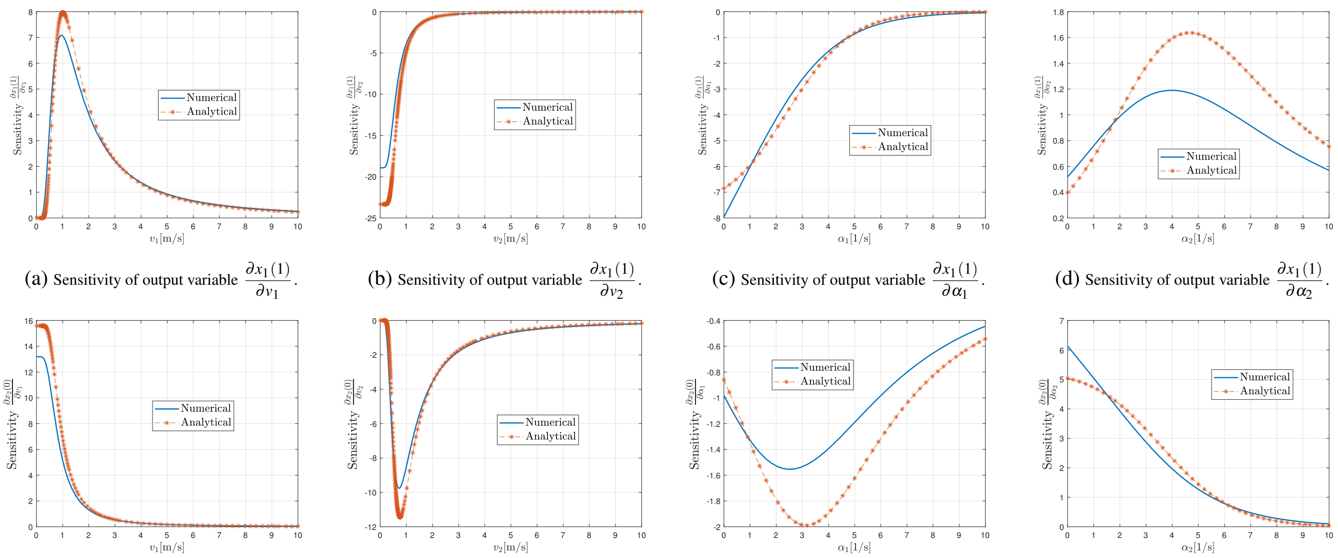
5. MODEL CALIBRATION AND VALIDATION

5.1 Parameter identification

The model (1)-(2) presented above involves unknown parameters $\theta = [v_1, v_2, \alpha_1, \alpha_2]^\top$. These must be determined so that the model best represents the real system. This results in expressing that the difference between the simulated outputs and the measured outputs, which are by definition the only accessible part of the system, must be minimal, and this for any input sequence. The parameter vector θ is determined by minimizing the following criterion:

$$J(\theta) = \sum_{i=1}^{N} \left(y_m(t_i) - y(t_i,\theta)\right)^\top \left(y_m(t_i) - y(t_i,\theta)\right), \tag{12}$$

where N is the number of measurements.
In this work, the minimization procedure will be considered by using the Levenberg-Marquardt algorithm (Marquardt (1963)). This method makes it possible to combine the convergence properties guaranteed by the gradient descent and Gauss-Newton's algorithm. Basically the gradient method estimates the parameters θ by a sequence of iterations: $\theta(i+1) = \theta(i) - \eta D$, where η is the relaxation step and D is the search direction and step. Levenberg-Marquardt's algorithm proposes the following formulation: $D = (H + \gamma I)^{-1} G$ with $(H + \gamma I)^{-1}$ non-negative defined, where $H = \frac{\partial^2 J}{\partial\theta\partial\theta^\top}$ is the Hessian of $J(\theta)$, $G = \frac{\partial J}{\partial\theta}$ is the gradient, and γ is the damping parameter adjusted at each new iteration.
The heat exchanger used in this case study is a small water to water tubular heat exchanger from Armfield, type HT31X, with dimensions $160 \times 630 \times 393$ mm, and consists of two concentric (coaxial) tubes carrying the hot and cold fluids. It is a part of a heat exchanger setup located at the Department of Electromechanics, Polytechnics faculty, University of Lubumbashi in DR Congo. A partial view of the system is given in Kazaku et al. (2018). To identify the parameters of the system, we launched



(a) Sensitivity of output variable $\frac{\partial x_1(1)}{\partial v_1}$. (b) Sensitivity of output variable $\frac{\partial x_1(1)}{\partial v_2}$. (c) Sensitivity of output variable $\frac{\partial x_1(1)}{\partial \alpha_1}$. (d) Sensitivity of output variable $\frac{\partial x_1(1)}{\partial \alpha_2}$.

(e) Sensitivity of output variable $\frac{\partial x_2(0)}{\partial v_1}$. (f) Sensitivity of output variable $\frac{\partial x_2(0)}{\partial v_2}$. (g) Sensitivity of output variable $\frac{\partial x_2(0)}{\partial \alpha_1}$. (h) Sensitivity of output variable $\frac{\partial x_2(0)}{\partial \alpha_2}$.

Fig. 2. Sensitivity functions of output variables with respect to parameter $\theta = [v_1, v_2, \alpha_1, \alpha_2]^\top$ obtained by an analytical approach and by numerical calculation ($n = 15$), for numerical values: $v_1 = 1m/s, v_2 = 1.1m/s, \alpha_1 = 3.5s^{-1}$ and $\alpha_2 = 3s^{-1}$.

an acquisition campaign of 3400 samples of the variables of the heat exchanger. During this test, the heat exchanger was excited over a certain operating range by modifying the compression ratio of the cold water circuit and the heating power of the hot water circuit, so as to vary the inlet temperatures. The temperature measurements are in degrees Celsius and the accuracy is in the second decimal. The sampling interval was one second and constant throughout the experiment. On these data, two thirds will be used for the calibration stage and simple-validation, and one thirds for the (cross)-validation of the model.
The Levenberg-Marquardt algorithm being local, it may only converge if we have at the start a first reasonable estimate of the parameters. To obtain a first approximation of the parameters to be identified, first we approximate the model (1)-(2) by global differences (see Dochain (1994), (Renou, 2000, Chapter 3)). This model reduction technique consists in expressing the energy balances of the PDEs model at the outlet of the heat exchanger. This leads to the following equations:

$$\begin{cases} \left.\dfrac{\partial x_1}{\partial t}\right|_{z=1} = -v_1 \left.\dfrac{\partial x_1}{\partial z}\right|_{z=1} + \alpha_1 \left(x_2(z,t) - x_1(z,t)\right)|_{z=1} \\ \left.\dfrac{\partial x_2}{\partial t}\right|_{z=0} = v_2 \left.\dfrac{\partial x_2}{\partial z}\right|_{z=0} + \alpha_2 \left(x_1(z,t) - x_2(z,t)\right)|_{z=0}. \end{cases} \tag{13}$$

At the heat exchanger outlet, the partial spatial derivatives can be approximated by the following overall differences:

$$\begin{cases} \left.\dfrac{\partial x_1}{\partial z}\right|_{z=1} \approx x_1(1,t) - x_1(0,t) = y_1(t) - x_{1,in}(t) \\ \left.\dfrac{\partial x_2}{\partial z}\right|_{z=0} \approx x_2(1,t) - x_2(0,t) = x_{2,in}(t) - y_2(t). \end{cases} \tag{14}$$

These global differences are then introduced into the energy balances. This allows to obtain the following linear reduced model:

$$\begin{cases} \dfrac{dy_1(t)}{dt} = -v_1 \left(y_1(t) - x_{1,in}(t)\right) + \alpha_1 \left(x_{2,in}(t) - y_1(t)\right) \\ \dfrac{dy_2(t)}{dt} = v_2 \left(x_{2,in}(t) - y_2(t)\right) + \alpha_2 \left(x_{1,in}(t) - y_2(t)\right). \end{cases} \tag{15}$$

Since the obtained model is linear in parameters θ, these can be estimated by least squares as follows:

$$\begin{cases} \begin{pmatrix} \hat{v}_1 \\ \hat{\alpha}_1 \end{pmatrix} = \left[\sum_{i=1}^{N} T \phi_i \phi_i^\top\right]^{-1} \sum_{i=1}^{N} T \phi_i y_m^1(i+1) \\ \begin{pmatrix} \hat{v}_2 \\ \hat{\alpha}_2 \end{pmatrix} = \left[\sum_{i=1}^{N} T \varphi_i \varphi_i^\top\right]^{-1} \sum_{i=1}^{N} T \varphi_i y_m^2(i+1), \end{cases} \tag{16}$$

with T the sampling period, and

$$\phi_i = \begin{pmatrix} y_m^1(i) - y_m^1(i-1) - x_{1,in}(i) \\ x_{2,in}(i) - y_m^1(i) + y_m^1(i-1) \end{pmatrix}$$
$$\varphi_i = \begin{pmatrix} x_{2,in}(i) - y_m^2(i) + y_m^2(i-1) \\ x_{1,in}(i) - y_m^2(i) - y_m^2(i-1) \end{pmatrix}.$$

So the first estimate of the parameters is equal to $\theta_0 = [0.0914\ 0.0949\ 0.9029\ 0.9103]^\top$.
System (5) was solved using the following initial condition Besson et al. (2006):

$$\begin{cases} x_1(z) = x_{1,in}\left[\left(\mathrm{e}^1 \dfrac{x_{2,in}}{x_{1,in}} - 1\right) z^2 + 1\right] \mathrm{e}^{-z^2}, \\ x_2(z) = x_{2,in} \exp\left[(1-z)^2 \ln\left(\dfrac{x_{1,in}}{x_{2,in}}\right)\right]. \end{cases} \tag{17}$$

The results of the simulation with the estimated parameters by Levenberg-Marquardt'algorithm are illustrated in Figure 3, on the data set used for calibration. It can be observed that the dynamics of the outputs of the heat exchanger are well reproduced. The distributed temperatures corresponding to this experience are represented in Figure 4 and 5.

5.2 Model validation

The last step in the identification process is cross validation, whose the aim is to demonstrating the predictive capability of the estimated model on the (one-third) dataset not having been used for the estimation of the parameters. This can be seen in Figure 6. We have still to assess the confidence in

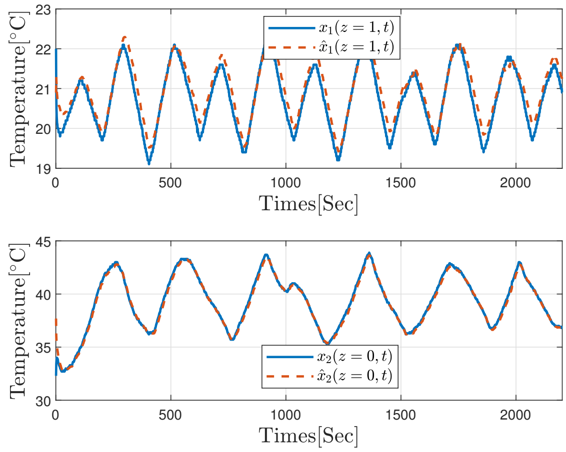


Fig. 3. Fitting simple-validation.

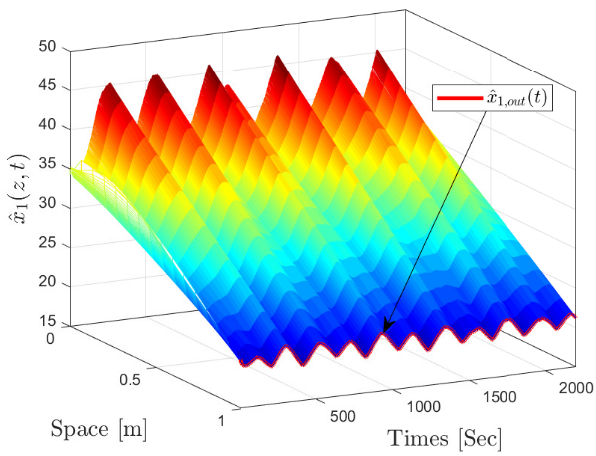


Fig. 4. Distributed predicted hot fluid temperature.

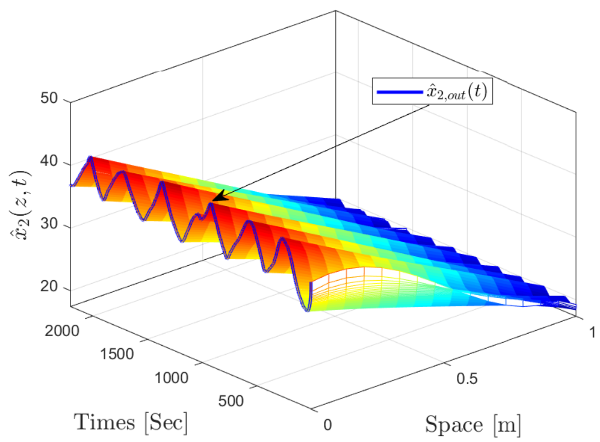


Fig. 5. Distributed predicted cold fluid temperature.

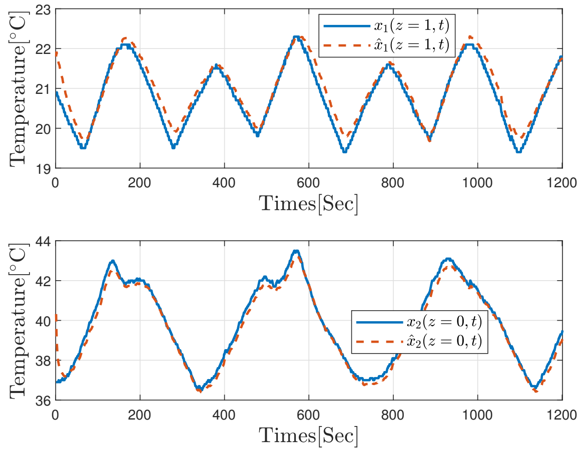


Fig. 6. Fitting cross-validation.

the numerical values of the estimated parameters. This can be obtained by using the Fisher information matrix (Vanrolleghem et al. (1995)). The confidence intervals are obtained by using the following expression:

$$\hat{\theta} \pm t_{\alpha;N-p}\mathrm{COV}(\hat{\theta})^{1/2}, \tag{18}$$

with $\mathrm{COV}(\hat{\theta}) = F^{-1}$ the covariance matrix of the estimate $\hat{\theta}$, p the number of parameter, α is a parameter (usually taken equal to 95%) that expresses the confidence level, F is the Fisher information matrix and t is obtained from Student's distributions (Dochain, 2008, Chapter 3).
The Fisher information matrix corresponding with this experiment is given by:

$$F = \begin{pmatrix} 4,0604\cdot 10^7 & 1,8244\cdot 10^7 & 1,8145\cdot 10^7 & 1,9548\cdot 10^7 \\ * & 3,4635\cdot 10^7 & 1,7566\cdot 10^7 & 1,8610\cdot 10^7 \\ * & * & 3,4635\cdot 10^7 & 1,7538\cdot 10^7 \\ * & * & * & 3,5538\cdot 10^7 \end{pmatrix}. \tag{19}$$

Note that the coefficients of this matrix are very large. By the fact that the inverse of this matrix corresponds to the covariance matrix of $\hat{\theta}$, we can therefore expect more confidence in the estimated values of the parameters. This is confirmed by Table 1 where we note very small confidence intervals, which reflect a good confidence of the estimated parameters.

6. CONCLUSION

The aim of this article was to estimate the parameters of a counterflow heat exchanger, in the case where the dynamics is described by a system of two hyperbolic partial differential equations. This study was carried out using data collected on a laboratory test bench. Before estimating the parameters involved in the heat exchanger model, a study of the uniqueness of the parameters (or practical identifiability) with respect to

Table 1. Estimated values and performance coefficients.

Parameters	Unity	Parameter value $\pm$ confidence intervals
v_1	m/s	$0.0938 \pm 4\cdot 10^{-4}$
v_1	m/s	$0.0946 \pm 5\cdot 10^{-4}$
α_1	s^{-1}	$0.9011 \pm 3\cdot 10^{-4}$
α_2	s^{-1}	$0.9091 \pm 3\cdot 10^{-4}$

the experimental data was carried out by evaluating the outputs sensitivity functions. Next one of the nonlinear optimization methods, precisely that of Levenberg-Marquardt, was used on a set of ordinary differential equations obtained by the method of lines to estimate the parameters of the model. To circumvent the initialization problem of the latter, the least squares method was used on the global difference model. Finally the results of the proposed identification approach were validated using laboratory data and checking some statistical properties.

REFERENCES

Babuška, R. (1998). *Fuzzy Modeling for Control*. Kluwer Academic Publishers, Boston, USA.

Besson, T., Tchousso, A., and Xu, C.Z. (2006). Exponential stability of a class of hyperbolic pde models from chemical engineering. In *Proceedings of the 45th IEEE Conference on Decision and Control*, 3974–3978. doi: 10.1109/CDC.2006.377214.

Burns, J.A., Herdman, T.L., and Zietsman, L. (2013). Approximating parabolic boundary control problems with delayed actuator dynamics. In *2013 American Control Conference*, 2080–2085. doi:10.1109/ACC.2013.6580142.

Burns, J. and Cliff, E. (2014a). Control of hyperbolic pde systems with actuator dynamics. In *53th IEEE conference on decision ans control*, 2864 – 2869. California, USA.

Burns, J. and Cliff, E. (2014b). Numerical methods for optimal control of heat exchangers. In *American control conference*, 14467821. Portland, USA.

Burns, J.A., Cliff, E.M., and Herdman, T.L. (2018). Parameter estimation of a thermal fluid system. In *2018 Annual American Control Conference (ACC)*, 122–127. doi: 10.23919/ACC.2018.8431435.

Dochain, D. (1994). *Contribution to the analysis and control of distributed parameter systems with application to (bio)chemical processes and robotics*. Ph.D. thesis, Thèse d'agrégation. Université Catholique de Louvain.

Dochain, D. (2008). *Bioprocess control*. ISTE, London.

Farah, M., Mercère, G., Ouvrard, R., and Poinot, T. (2016). Combining least-squares and gradient-based algorithms for the identification of a co-current flow heat exchanger. *International Journal of Control*, 92, 1–19. doi: 10.1080/00207179.2016.1234717.

Jacobsen, E.W. and Skogestad, S. (1994). Identification of ill-conditioned plants — a benchmark problem. In R.S. Smith and M. Dahleh (eds.), *The Modeling of Uncertainty in Control Systems*, 367–376. Springer Berlin Heidelberg, Berlin, Heidelberg.

Jonsson, G. and Palsson, O.P. (1994). An Application of Extended Kalman Filtering to Heat Exchanger Models. *Journal of Dynamic Systems, Measurement, and Control*, 116(2), 257–264. doi:10.1115/1.2899218. URL `https://doi.org/10.1115/1.2899218`.

Jonsson, G., Palsson, O.P., and Sejling, K. (1992). Modeling and Parameter Estimation of Heat Exchangers—A Statistical Approach. *Journal of Dynamic Systems, Measurement, and Control*, 114(4), 673–679. doi:10.1115/1.2897740. URL `https://doi.org/10.1115/1.2897740`.

Jonsson, G.R., Lalot, S., Palsson, O.P., and Desmet, B. (2007). Use of extended kalman filtering in detecting fouling in heat exchangers. *International Journal of Heat and Mass Transfer*, 50(13), 2643–2655. doi: https://doi.org/10.1016/j.ijheatmasstransfer.2006.11.025.

Kazaku, J.K., Dochain, D., Kahilu, M.M., and Kasongo, J.K.K. (2022a). P-i boundary control of countercurrent heat exchanger. *IFAC-PapersOnLine*, 55(40), 49–54. doi:https://doi.org/10.1016/j.ifacol.2023.01.047. 1st IFAC Workshop on Control of Complex Systems COSY 2022.

Kazaku, J.K., Dochain, D., Kahilu, M.M., and Kaunde Kasongo, J.K. (2022b). Thermodynamics analysis of the distributed parameter model of counterflow heat exchanger. In *To appear*.

Kazaku, J.K., Dochain, D., Winkin, J., Kahilu, M.M., and Kaunde Kasongo, J.K. (2020). Port-hamiltonian sliding mode observer design for a counter-current heat exchanger. *IFAC-PapersOnLine*, 53(2), 4910–4915. doi: https://doi.org/10.1016/j.ifacol.2020.12.1066. 21th IFAC World Congress.

Kazaku, J., Kasongo, J., Banka, R., and Kadinda, F. (2018). Contribution à l'identification par logique floue d'un échangeur de chaleur à tube concentrique. In *4ème conférence sur les applications pratiques de l'intelligence artificielle*, pp:33 – 38. Nancy, France.

Maidi, A., Diaf, M., and Corriou, J. (2009). Boundary geometric control of a counter-current heat exchanger. *Journal of process control*, 19, 297 – 313.

Maidi, A., Diaf, M., and Corriou, J.P. (2008). Optimal linear pi fuzzy controller design of a heat exchanger. *Chemical Engineering and Processing: Process Intensification*, 47(5), 938–945. doi:https://doi.org/10.1016/j.cep.2007.03.008.

Marquardt, D.W. (1963). An algorithm for least-squares estimation of nonlinear parameters. *SIAM Journal on Applied Mathematics*, 11(2), 431–441. doi:10.1137/0111030. URL `http://dx.doi.org/10.1137/0111030`.

Mutambara, A. and Al-Haik, M. (1999). Ekf based parameter estimation for a heat exchanger. In *Proceedings of the 1999 American Control Conference (Cat. No. 99CH36251)*, volume 6, 3918–3922 vol.6. doi:10.1109/ACC.1999.786251.

Petersen, Britta (2000). *Calibration, identifiability and optimal experimental design of activated sludge models*. Ph.D. thesis, Ghent University. URL `http://dx.doi.org/1854/397`.

Pfortner, B., Hadad, W.A., Schick, V., Maillet, D., Zacharie, C., and Rémy, B. (2022). Transient detection of either maldistribution or flowrate change in a counter current plate-fin heat exchanger using an arx model. *International Journal of Heat and Mass Transfer*, 182, 121987. doi: https://doi.org/10.1016/j.ijheatmasstransfer.2021.121987.

Renou, S. (2000). *Commande non-linéaire d'un système décrit par des équations paraboliques: Application au procédé de blanchiment*. Ph.D. thesis, Université de Montréal.

Trefethen, L.N. and Embree, M. (2005). Spectra and pseudospectra : the behavior of nonnormal matrices and operators.

Vanrolleghem, P.A., Daele, M.V., and Dochain, D. (1995). Practical identifiability of a biokinetic model of activated sludge respiration. *Water Research*, 29(11), 2561–2570. doi: https://doi.org/10.1016/0043-1354(95)00105-T.

Walter, E. and Pronzato, L. (1997). *Identification of Parametric Models from Experimental Data*. Springer-Verlag, Heidelberg.

Wouwer, A.V., Saucez, P., and Schiesser, W.E. (2001). *Adaptive Method of Lines*. Chapman & Hall/CRC Press. doi: https://doi.org/10.1201/9781420035612.