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http://ascelibrary.org/doi/abs/10.1061/%28ASCE%29GT.1943-5606.0000983

A modified Case Method for piles with section step changes

Henri de Chaunac[1]; and Alain E. Holeyman[2]

ABSTRACT

Applied during pile driving, the Case Method offers an immediate estimate of the static resistance to driving (SRD) after each hammer blow. It has been used in its original form both in the onshore and offshore piling industry for more than 40 years to provide an indication of the pile static capacity. The Case Method requires measurements of force and velocity near the pile head as the hammer strikes the pile and produces an analytical estimate of the SRD, using a number of assumptions. One of them requires the pile to be of constant impedance (or cross section) along its length. However, for reason of economy, driven piles are often composed of several sections of different cross sections. The Case Method provides in that case an inaccurate estimate of the SRD.

This article presents an improved version of the original Case Method which takes into account possible variations of impedance along the pile. A numerical validation shows that for piles displaying impedance changes, the modified Case Method presented herein provides an estimate closer to the actual SRD than the original Case Method. That conclusion is further validated by applying the modified Method to pile driving records and comparing its results to SRD estimates obtained through more reliable modelling.

Keywords: Case Method; pile dynamic testing; static resistance to driving; SRD; wave equation; dynamic analysis; impedance change; damping; pile driving; pile foundations

INTRODUCTION

During pile driving, the pile head is often instrumented with accelerometers and strain

[1]Ph.D. Student, Dept. of Civil and Environmental Engrg., Institute of Mechanics, Materials, and Civil Engrg., Université catholique de Louvain, Place du Levant 1, 1348 Louvain-la-Neuve, Belgium (corresponding author). E-mail: henri.dechaunac@uclouvain.be.

[2]Professor, Dept. of Civil and Environmental Engrg., Institute of Mechanics, Materials, and Civil Engrg., Université catholique de Louvain, Place du Levant 1, 1348 Louvain-la-Neuve, Belgium.

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transducers. Using the wave equation theory, the force and velocity signals obtained from these sensors provide, a. o., a information about the soil resistance surrounding the pile. In particular, the static resistance to driving (which is an estimation of the pile static capacity made during driving, abbreviated as SRD) can be derived from the signals by two means (Holeyman 1992): direct (rapid but requiring many assumptions, e.g. the Case Method) or inverse modelling (slower and more accurate, e.g. CAPWAP, DLTWAVE or SIMBAT). Herein, the focus is placed on to the Case Method (Rausche et al. 1985), an analytical direct method used to estimate the SRD. One important assumption is a constant pile cross section along its full length. Yet, many piles present section changes: piles with a driving shoe, long offshore pipe piles of different sections welded together, etc. This alters the accuracy of the Case Method estimate of the SRD for pile of varying impedance.

We introduce herein a modified version of the *original* Case Method, which will be henceforth referred to as the *modified* Case Method, which takes into account possible impedance changes along the pile. The modified Case Method essentially adopts all the other assumptions postulated by the original one. However, in order to produce an analytically acceptable formula —like the original Case Method— an assumption is extended, namely that all the pile resistance is lumped at the base instead of part of it for the original Case Method.

The paper is organized as follows. Firstly, a short background section covers the wave equation principles used hereunder. Then, the original Case Method and its assumptions are presented. Afterwards, the modified Case Method for piles of stepped impedance is demonstrated. Following is a comparison of the results provided by the original and the modified Case Methods using a synthetic example of a pile with one section change. Finally, a practical application of the modified Case Method to an offshore pile driven in the Persian Gulf is introduced.

BACKGROUND

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Wave equation

Introduced by de Saint-Venant (1867) for elastic rods, the application of the wave equation theory to pile driving was set forth by Isaacs (1931) and offered a new mind-set: the pile was not considered to act as a rigid unit anymore. Using the wave equation, the pile is considered as a one-dimensional unconstrained elastic rod. The hammer blow creates a downward propagating stress wave while any irregularity (due to soil resistance or impedance changes) creates upward propagating waves. These waves are solutions to the 1D wave equation:

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial z^2} \tag{1}$$

where u = displacement (downward positive), t = time, z = vertical coordinate (downward positive), and c = wave propagation speed. The latter is given by $c = \sqrt{E/\rho}$ where E and ρ are respectively the Young modulus and density of the pile.

The solution of the wave equation (Eq. (1)) is the sum of a downward and an upward propagating displacement wave (d'Alembert 1747) which follow characteristic lines of the wave equation in the z-t plane. These waves induce corresponding velocity (v, downward positive) and internal force (F, positive in compression) waves which can also be split up into their downward (subscript d) and upward (subscript u) travelling components:

$$v = v_d + v_u \tag{2b}$$

$$F = F_d + F_u \tag{2c}$$

Two relationships are useful for the developments presented below. The first one links the downward/upward force and velocity waves:

$$F_d = I v_d \tag{3a}$$

$$F_u = -I v_u \tag{3b}$$

where I = pile impedance ($I = EA/c$ with A = pile cross-sectional area). The second relationship links the downward and upward moving force waves to the total internal force F and velocity v:

$$2F_d = F + Iv \tag{4a}$$

$$2F_u = F - Iv \tag{4b}$$

Consistent with conventions generally used in pile driving, equilibrium forces applied on a free body are positive when being applied downward except for the resistance forces which are positive upward while the velocities are positive downward.

Effects of an impedance change on an incoming wave

Here, the effects of an impedance change are presented and the coefficients of the reflected and transmitted waves relative to the incident wave are formulated.

Let a pile be composed of n impedance changes (thus $n+1$ segments). The k-th impedance change (for $k = 1 \dots n$) is defined by the impedance ratio $\imath_k = I_k/I_{k-1}$ where I_k and I_{k-1} are respectively the impedances of the pile below and above the impedance discontinuity (Fig. 1). The factor $\imath_k$ has the same meaning as the β factor used in low-strain integrity testing (Rausche and Goble 1979).

When a wave encounters an impedance change, part of it is reflected while the other part is transmitted (Fig. 1). Equilibrium of forces and continuity of velocity at the impedance change yield the coefficients of the reflected (superscript r) and transmitted (superscript t) wave relative to the incident wave (superscript i). For a downward force wave (Fig. 1a), the reflected and transmitted internal force waves are:

$$F_u^r = \frac{\imath_k - 1}{1 + \imath_k} F_d^i \qquad F_d^t = \frac{2\imath_k}{1 + \imath_k} F_d^i \tag{5}$$

and for the upward incoming force wave (Fig. 1b), these are:

$$F_d^r = \frac{1-\imath_k}{1+\imath_k} F_u^i \qquad F_u^t = \frac{2}{1+\imath_k} F_d^i \tag{6}$$

THE CASE METHOD

Using the wave equation theory, Rausche et al. (1985) developed the original Case Method: an analytical estimate of the static resistance to driving (SRD). In this section the original Case Method and associated assumptions are discussed. The modified Case Method, that will be introduced in the following section, borrows most of the assumptions of the original one, which are therefore presented first.

We consider a pile of length L. A hammer blow on the pile head induces a downward force wave $\Lambda(t)$, which reaches its peak $\Lambda(t_0)$ at time t_0. Fig. 2 depicts in the z-t plane the characteristic lines corresponding to $\Lambda(t_0)$. The internal force and velocity at coordinate z and time t are respectively $F(z,t)$ and $v(z,t)$. The shaft resistance is discretized into a finite number of upward acting forces (which add up to R_{shaft}) while a single force R_{base} acting upward on the pile base stands for the base resistance. For clarity's sake, the shaft resistance considered on Fig. 2 is lumped into one resistance force R_{shaft} located at coordinate z^*.

The assumptions of the Case Method are the following: (i) The pile is an elastic rod; (ii) The pile impedance is constant along its length; (iii) The hammer blow is axial; (iv) The pile is free except for soil resistance which is represented as a finite number of upward external forces R located along the pile; (v) A resistive force considered at coordinate z^* is activated ("switched on") over a time span extending at least from $t = t_0 + z^*/c$ to $t = t_0 + (2L - z^*)/c$ (i.e. the hammer blow is strong enough to fully mobilise the soil resistance during the time it takes $\Lambda(t_0)$ to travel down the pile and back); (vi) As proposed by Rausche et al. (1985), and inspired by the Smith (1960) model, a resistive force R is split up into a displacement dependent (*static*) component R_s and a velocity dependent (*damping* or dynamic) part R_d, the former *is* the static resistance to driving (SRD) while the latter is assumed to behave linearly viscous:

$$R_d = j_c I v(L, t_0 + L/c) \tag{7}$$

where j_c is the Case damping factor. Eq. (7) implies that all of the damping resistance R_d is lumped at the pile base, where the relevant velocity is evaluated.

From theses assumptions, the original Case Method formula which estimates the SRD is obtained:

$$R_s = (1 + j_c)F_u(0, t_0 + 2L/c) + (1 - j_c)F_d(0, t_0) \tag{8}$$

Although the Case Method is usually expressed in terms of total force F and velocity v, we express it here in terms of downward and upward forces because this leads to a simpler formulation of the modified Case Method. Nevertheless, using Eqs. (4a) and (4b), the two formulations are interchangeable.

It is important to keep in mind that the Case Method (original or modified) requires a set of restricting assumptions to yield its result. The most criticised one relates to the empiric definition of the damping resistance (assumption (vi)) which implies that damping effects depend on the pile impedance and that therefore j_c is not a soil property (Holeyman 1992; Svinkin and Abe 1992; Zhang et al. 2001). Restrictive also is the assumption (v) which implicitly requires (a) a downward movement of the pile during the time required for the wave induced by the hammer blow to travel down the pile and back up again; and (b) that the pile capacity gets fully mobilised by the delivered blow.

CASE METHOD MODIFIED FOR IMPEDANCE CHANGES

This section presents the centrepiece of the article: a modified Case Method which takes into account impedances changes along the pile. For the sake of clarity, the demonstration of the modified Case Method is made for one impedance discontinuity only. After that, the modified Case Method is presented for n impedance changes.

The assumptions of the modified Case Method are the same as the original Case Method ones except for assumption (ii) which is removed. However, in order to produce an acceptable form of the modified Case Method, an additional assumption has to be made. While for the original Case Method, it is enough to suppose that only the damping resistance is lumped at the pile base (Eq. 7), for the modified Case Method, all the resistance (static and damping) has to be lumped at the base, so $R_{base} = R$.

Furthermore, the Case damping factor is still used but redefined as follows, *in lieu of* Eq. (7):

$$R_d = j_c I_n v(L, t_0 + L/c) \tag{9}$$

where I_n is the impedance of the last segment (*i.e.* the pile base impedance).

One impedance change

The demonstration of the modified Case Method for a pile with one impedance change ($n = 1$) at coordinate $z = z_s$ (Fig. 3) is presented here. To keep notations concise, the impedance ratio $\imath_1 = I_1/I_0$ will be denoted as $\imath$ in this subsection.

The demonstration requires two steps (de Chaunac and Holeyman 2012). The first step is used to express the total resistance R while the second one allows us to detail its damping component R_d. The static resistance is then obtained by subtraction: $R_s = R - R_d$.

The demonstration follows the approach taken by Salgado (2008) (in terms of force) rather than the one used by Rausche et al. (1985) (in terms of velocity) because it facilitates its mathematical developments.

The first step is to formulate the upward force wave at the pile head at time $t_0 + 2L/c$:

$$F_u(0, t_0 + 2L/c) = -\frac{4\imath}{(1+\imath)^2}\Lambda(t_0) + \frac{2}{1+\imath}R + \frac{\imath - 1}{1+\imath}\Lambda(t_s) \tag{10}$$

As indicated on Fig. 3, $F_u(0, t_0{+}2L/c)$ is composed of two parts: (a) the reflection of $\Lambda(t_0)$ on the pile base (the plain line); and (b) the reflection of $\Lambda(t_s)$ on the impedance change (the dotted line). The following expression of the total resistance can then be obtained by rearranging terms coming from Eq. (10):

$$R = \frac{1+\imath}{2} F_u(0, t_0 + 2L/c) + \frac{2\imath}{1+\imath} \Lambda(t_0) - \frac{\imath - 1}{2} \Lambda(t_s) \tag{11}$$

where $t_s = t_0 + 2(L - z_s)/c$ is the time at which originates the wave $\Lambda(t_s)$ that meets, at coordinate z_s, the $\Lambda(t_0)$ wave which has been reflected upward (Fig. 3).

In order to attain the second step, the velocity at pile base at time $t_0 + L/c$ is expressed by first using Eqs. (2b) and (3) to formulate the velocity in terms of downward and upward force waves, and then by using the results shown on Fig. 3 to explicit the downward and upward force waves:

$$\begin{aligned} v(L, t_0 + L/c) &= \frac{1}{I_1} \left(F_d(L, t_0 + L/c) - F_u(L, t_0 + L/c) \right) \\ &= \frac{1}{I_1} \left(\frac{4\imath}{1+\imath} \Lambda(t_0) - R \right) \end{aligned} \tag{12}$$

Then, the damping resistance (Eq. (9)) can be formulated as follows:

$$R_d = j_c \left(\frac{4\imath}{1+\imath} \Lambda(t_0) - R \right) \tag{13}$$

Finally, the estimated SRD is obtained by subtracting Eq. (11) from Eq. (13):

$$R_s = (1 + j_c) \frac{1+\imath}{2} F_u(0, t_0 + 2L/c) + (1 - j_c) \frac{2\imath}{1+\imath} F_d(0, t_0) - (1 + j_c) \frac{\imath - 1}{2} F_d(0, t_s) \tag{14}$$

Eq. (14) is the modified Case Method formula for one impedance change. For $\imath = 1$, it collapses into the original formula (Eq. (8)). Next, the result for n changes of impedance is presented.

Multiple impedance changes

Using the same development as presented in the previous subsection, the results for one impedance change can be generalised for n impedance changes (de Chaunac and Domange

2011). The modified Case Method formula for n impedance changes becomes:

$$
\begin{aligned}
R_s = {} & (1+j_c)\frac{1}{2^n}\prod_{k=1}^{n}(1+\imath_k)F_u(0,t_0+2L/c) + (1-j_c)2^n\prod_{k=1}^{n}\frac{\imath_k}{1+\imath_k}F_d(0,t_0) \\
& -(1+j_c)\frac{1}{2^n}\prod_{k=1}^{n}(1+\imath_k)\left\{\frac{\imath_1-1}{1+\imath_1}F_d(0,t_{s,1}) + \underbrace{\sum_{p=2}^{n}\left(\frac{2^{2(p-1)}(\imath_p-1)}{1+\imath_p}\prod_{r=1}^{p-1}\frac{\imath_r}{(1+\imath_r)^2}\right)F_d(0,t_{s,p})}_{\text{if } n>1}\right\}
\end{aligned}
\tag{15}
$$

where the impedance ratio $\imath_k$ and the time $t_{s,k}$ are defined as follows:

$$\imath_k = \frac{I_k}{I_{k-1}} \qquad t_{s,k} = t_0 + \frac{2(L-z_{s,k})}{c} \tag{16}$$

where $z_{s,k}$ is the coordinate where the pile impedance changes from I_{k-1} to I_k.

It can be noted that if we set $\imath_k = 1$ for $k = 1\ldots n$, Eq. (15) collapses into the original Case Method (Eq. (8)).

PARAMETRIC STUDY ON SYNTHETIC EXAMPLE FEATURING ONE IMPEDANCE CHANGE

In the following section, we apply the original and the modified Case Methods on numerical simulations of a dynamic blow on a pile presenting one impedance change at varying depth.

The numerical tool used to model the pile is based on the method of characteristics. It is capable of simulating a downward initial force wave on a pile featuring resistive forces and impedance changes. The resistive forces are modelled as rigid plastic with a damping component set at $j_c = 0.2$ (Eq. (9)). The pile considered has one impedance change characterised by $\imath = 0.5$. Other pile characteristics are summarized in Table 1. To eliminate issues related to partial mobilisation of the resistance, the force wave imposed at head $\Lambda(t)$ is a 10 ms wide rectangular pulse of 5 MN amplitude. An output of force and velocity at pile head is shown on Fig. 4 for a pile with a 140 m deep section change and 2 MN base (Fig. 4a) or 2 MN shaft (Fig. 4b) resistance.

This numerical setup allows for the resistive forces to be mobilized in accordance with the Case Method assumptions. In particular, the resistive forces are activated once the incoming wave $\Lambda(t_0)$ arrives at their level and until the part of $\Lambda(t_0)$ reflected from the pile base leaves that level (assumption (v) in "The Case Method" Section).

For the example depicted on Fig. 4a, the following paragraph explains the calculation of the estimated static resistance R_s^{est} by both the original and modified Case Methods, *per* respectively Eqs. (8) and (15). Since the initial wave is rectangular, time t_0 is chosen as the first time increment: here, 0.07 ms due to the time discretization. From Eq. (16), time $t_{s,1} =$ 23.84 ms. The downward and upward force components of interest can now be assessed from force and velocity at pile head, depicted on Fig. 4a (the transformation between force and velocity to downward and upward force is made using Eqs. (4)): $F_d(0, t_0) = 5{,}000$ kN, $F_u(0, t_0 + 2L/c) = -741$ kN, and $F_d(0, t_s) = 0$. Bearing in mind that $j_c = 0.2$, the static resistances estimated from the original and modified Case Methods can then be determined:

$$\begin{aligned} \text{Original Case: } R_s^{\text{est}} &= 1.2 \cdot (-741) + 0.8 \cdot 5{,}000 = 3{,}111 \text{ kN} \\ \text{Modified Case: } R_s^{\text{est}} &= 1.2 \cdot 3/4 \cdot (-741) + 0.8 \cdot 2/3 \cdot 5{,}000 - 1.2 \cdot 1/4 \cdot 0 \\ &= 2{,}000 \text{ kN} \end{aligned} \tag{17}$$

The relative error is defined as $(R_s^{\text{est}} - R_s^{\text{true}})/R_s^{\text{true}}$ where R_s^{est} and R_s^{true} are respectively the estimated and reference static resistance considered for the pile. In this section, R_s^{true} is set at 2 MN by the numerical program. For the example depicted on Fig. 4a (section change at 140 m coordinate), the relative errors are therefore 56% for the original Case Method and 0% for the modified Case Method.

Running different pile configurations wherein only the coordinate of the section change is varied, within the 0 m to 200 m range, has allowed us to investigate the effect of the impedance change coordinate on the relative error. The results for two resistance distribution are presented: an end bearing pile (Fig. 5a; 2 MN base resistance) and a friction pile (2 MN shaft resistance uniformly distributed; Fig. 5b).

The modified Case Method perfectly predicts the static capacity of the end bearing simulation (Fig. 5a), while the error made by the original Case Method varies from −44% to 56%. The jumps in the error of the original Case Method correspond to harmonic positions of the impedance change, *i.e.* when a reflection of the initial force wave triggered by the impedance change arrives at the pile head at $t_0 + 2L/c$.

For the friction bearing pile (Fig. 5b), the static resistance base lumping assumption of the modified Case Method is violated. The error reaches −13% when the section change approaches the pile base. As was found for the end bearing pile, the error made by the original Case Method is much larger and varies between −62% and 95%. We define such values as *maximum* (negative and positive) errors.

For both considered resistance distributions, the error made by the original Case Method significantly varies with the impedance change coordinate, illustrating its lack of consideration for the reflected waves coming from the section change. As a consequence of that shortcoming, the error made by the original Case Method depends both on the impedance change ratio $\imath$ and on the amplitude of the wave $\Lambda(t_0)$ imposed at the pile head. Fig. 6 illustrates this pattern by showing the maximum errors made by both Case Methods versus a normalized imposed force $\frac{\Lambda(t_0)}{R_s}\frac{2\imath}{1+\imath}$, which quantifies the downward force below the section change relative to the pile resistance. On the other hand, the error made by the modified Case Method shows no dependence on the incoming wave $\Lambda(t_0)$ but slightly varies with the impedance ratio $\imath$ only when the base lumping assumption is not met (Fig. 6b).

A numerical study for a pile featuring two impedance changes is presented in the Appendix to substantiate that the above noted performance of the modified Case Method can be extended to cases involving more than one impedance change.

PRACTICAL APPLICATION

In order to illustrate the practical applicability of the modified Case Method, its application to an actual pile driven in the Persian Gulf and PDA monitored by Fugro (2008; personnal communication) is presented below. The considered pile is a 70 m long offshore open ended steel pipe pile with a 42″ outer diameter (106.68 mm) and made of sections with alternating wall thicknesses (1″ or 1.5″; *i.e.* 25.4 mm or 38.1 mm). The pile is assembled from two pieces; the first one was driven to a penetration of 15 m below sea floor, after which an add-on was welded. Driving resumed after a delay of about 27 hours and proceeded to approximately 30 m of penetration. The pile make-up is schematized on Fig. 7. The PDA monitored pile was driven using a hydraulic hammer operated at a maximum energy of about 300 kJ while the blow count was recorded at 0.25 m intervals.

Soil stratigraphy can be schematized as follows. The upper 8 m consist predominantly of firm to very stiff clay interbedded with medium dense sand and sandstone layers. Very dense silica sand is encountered below 8 m depth, with an interbed of slightly cemented clayey calcareous silica sand between approximately 17 m and 21 m depth. A simplified soil profile is depicted on the right side of Fig. 8.

The left side of Fig. 8 shows three different estimates of the SRD (static resistance to driving). Firstly, CAPWAP inferred SRD, which is derived from force and velocity measurements taking into account pile make-up, is provided for one blow selected towards the end of driving. Secondly, "blow count" inferred SRD is converted from blow count records and PDA monitored Enthru (transferred energy computed from pile head measurements), using energy fitted GRLWEAP analyses. Finally, the SRD estimated by the modified Case Method from PDA acquired force and velocity signals is displayed for 13 blows covering the monitored driving depth range. The modified Case Method is applied following the RMX procedure, *i.e.* searching the t_0^* that produces the largest Case Method result, within $t_0 < t_0^* < 0.9 \cdot 2L/c$ (Likins and Hussein 1988).

The section changes input in the modified Case Method were: for the first pile piece, 4 section changes (Fig. 7a), and for the assembled pile, 5 section changes (Fig. 7b).

It can be concluded from inspecting Fig. 8 that the modified Case Method SRD follows the general trend portrayed by the SRD profile converted by Fugro from blow count records and ENTHRU measurements. This rather satisfactory match is achieved provided the j_c value does not differ significantly from that calibrated by the CAPWAP analysis performed towards the end of driving. The agreement is best within the last 8 m of driving, where the selected j_c value (0.55) can be applied with more confidence, considering the geotechnical stratigraphy. It is worth noting that, in accordance with Eq. (9), the j_c value relates to the impedance of the last modelled pile section (in this case, the 1.5″ thick driving shoe).

CONCLUSION

The Case Method efficiently provides an analytical estimate of the static resistance to driving (SRD) based on measurements made at the pile head during driving. It uses a number of assumptions, including that the pile is of constant impedance. This paper presents a modified Case Method free from this assumption. In order to produce an analytically acceptable form of the modified Case Method, one of the assumptions is extended, namely that all the pile resistance is lumped at the pile base instead of only the damping resistance for the original Case Method.

The comparison between the modified and original Case Methods on numerical simulations of a driven pile with one impedance change evidences advantages of the Modified Case Method: not only does it offer a better estimation of the SRD but it is independent of the impedance change ratio or the relative amplitude of the incident wave (when the base lumping assumption is met). When the base resistance lumping assumption is not fulfilled, numerical comparisons show that the benefit gained by taking into account one impedance change is greater than the penalty brought by the more restrictive resistance distribution assumption. Since that numerical study implements the restricting assumptions associated with the Case Methods, it can only display the theoretical accuracy of the modified Case Method in comparison to the original one with regards to impedance changes.

The practical applicability of the modified Case Method has been illustrated by comparing it to two other SRD estimates derived from a PDA instrumented pile: a SRD profile established from ENTHRU measurements coupled to blow count records and and a CAPWAP computation. Although the agreement is promising, it is advised not to rely solely on the modified Case Method presented herein for a design-orientated estimation of the SRD, but to carry out a thorough signal matching procedure at relevant depths.

Finally, although hampered by stringent assumptions, the modified Case Method offers the key advantage of an immediate and straightforward output in terms of SRD estimate upon each blow as driving proceeds.

ACKNOWLEDGMENTS

The authors express their appreciation to Fugro for providing anonymous PDA records and geotechnical data pertaining to the practical case presented herein.

APPENDIX. TWO IMPEDANCE CHANGES ON SYNTHETIC EXAMPLE

This section presents an evaluation of the performance of both original and modified Case Methods based on force and velocity signals computed by the numerical program presented in the Parametric Study Section of this paper. We consider this time a pile featuring two impedance changes (Fig. 9) and a 5 MN amplitude rectangular force wave lasting for 30 ms. Two resistance distributions have been assumed: an end bearing pile and friction pile, both for which $j_c = 0.2$. The first impedance change is located at $z_{s,1} = 100$ m (with $\imath_1 = I_1/I_0 = 0.5$) and the second one at $z_{s,2} = 180$ m (with $\imath_2 = I_2/I_1 = 2$ and thus $I_2 = I_0$). The modified Case Method (Eq. (15)) for two impedance changes reads:

$$\begin{aligned} R_s = {} & (1+j_c)\frac{1}{4}(1+\imath_1)(1+\imath_2)F_u(0,t_0+2L/c) + (1-j_c)4\frac{\imath_1}{1+\imath_1}\frac{\imath_2}{1+\imath_2}F_d(0,t_0) \\ & -(1+j_c)\frac{1}{4}(1+\imath_1)(1+\imath_2)\left\{\frac{\imath_1-1}{1+\imath_1}F_d(0,t_{s,1}) + 4\frac{\imath_2-1}{1+\imath_2}\frac{\imath_1}{(1+\imath_1)^2}F_d(0,t_{s,2})\right\} \end{aligned} \quad (18)$$

while the original Case Method remains given by Eq. (8).

As explained earlier, $t_0 = 0.07$ ms. The two other times needed for the modified Case Method are given by Eq. (16) and for the example considered are $t_{s,1} = 39.67$ ms and $t_{s,2} =$

7.99 ms. The transformation from force and velocity signals to downward and upward force is made using Eqs. (4).

Fig. 9a is the force and velocity output for the 2 MN end bearing pile, from which the following values can be deduced:

$$\begin{aligned} &F_u(0, t_0 + 2L/c) = -226 \text{ kN} \\ &F_d(0, t_0) = 5{,}000 \text{ kN} \\ &F_d(0, t_{s,1}) = 1{,}667 \text{ kN} \\ &F_d(0, t_{s,2}) = 5{,}000 \text{ kN} \end{aligned} \tag{19}$$

and the estimates and errors made by both Case Methods are:

$$\begin{aligned} \text{Original Case: } R_s &= 3{,}728 \text{ kN} \qquad \text{Relative error: } 87\% \\ \text{Modified Case: } R_s &= 2{,}000 \text{ kN} \qquad \text{Relative error: } 0\% \end{aligned} \tag{20}$$

Fig. 9b is the force and velocity output for the friction pile, for which the 2 MN have been uniformly distributed along the pile length. The following values can be inferred from Fig. 9b:

$$\begin{aligned} &F_u(0, t_0 + 2L/c) = 102 \text{ kN} \\ &F_d(0, t_0) = 5{,}000 \text{ kN} \\ &F_d(0, t_{s,1}) = 1{,}000 \text{ kN} \\ &F_d(0, t_{s,2}) = 4{,}901 \text{ kN} \end{aligned} \tag{21}$$

and the estimates and errors made by both Case Methods are:

$$\begin{aligned} \text{Original Case: } R_s &= 4{,}123 \text{ kN} \qquad \text{Relative error: } 106\% \\ \text{Modified Case: } R_s &= 2{,}183 \text{ kN} \qquad \text{Relative error: } 9\% \end{aligned} \tag{22}$$

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NOTATION

The following symbols are used in this paper:

$\imath$, $\imath_k$ = impedance ratio [-];

$\Lambda(t)$ = imposed downward wave at pile head [MN];

ρ = pile density [kg/m^3];

A, A_0 = pile cross section, pile cross section at head [cm^2];

c = wave propagation speed [m/s];

E = pile Young modulus [GPa];

$F(z,t)$ = pile internal force at coordinate z and time t [MN];

$F_d(z,t)$, $F_u(z,t)$ = downward, upward pile force wave at coordinate z and time t [MN];

F^i, F^r, F^t = incident, reflected and transmitted force waves (can be upward or downward) [MN];

I, I_0, I_k = pile impedance, pile impedance at head, pile impedance after the k-th impedance change [MNs/m];

j_c = Case damping factor [-];

L = pile length [m];

R = total resistance (static and damping) [MN];

R_d = damping (dynamic) resistance [MN];

R_s = static resistance (=SRD while driving) [MN];

R_{shaft} = shaft resistance [MN];

R_{base} = base resistance [MN];

SRD = static resistance to driving [MN];

t = time [s];

t_0 = time corresponding to $\Lambda(t)$ peak [s];

t_s, $t_{s,k}$ = time at which originates the wave that crosses the (k-th) impedance change at the same time the upward wave $\Lambda(t_0)$ reflected from the base does [s];

u = pile displacement [m];

$v(z,t)$ = pile velocity at coordinate z and time t [m/s];

$v_d(z,t)$, $v_u(z,t)$ = downward, upward pile velocity wave at coordinate z and time t [m/s];

z = vertical coordinate [m]; and

z_s, $z_{s,k}$ = vertical coordinate of the (k-th) impedance change [m].

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TABLE 1. Specifications of the pile and driving system model

Parameter Name	Symbol	Value	Unit
Modulus of elasticity	E	200	GPa
Density	ρ	7,850	kg/m^3
Cross section (at head)	A_0	1,000	cm^2
Length	L	200	m
Impedance (at head)	I_0	3.96	MNs/m
Impedance ratio	$\imath$	0.5	-
Echo time	$2L/c$	79.25	ms
Max. downward force (at head)	$\Lambda(t_0)$	5	MN

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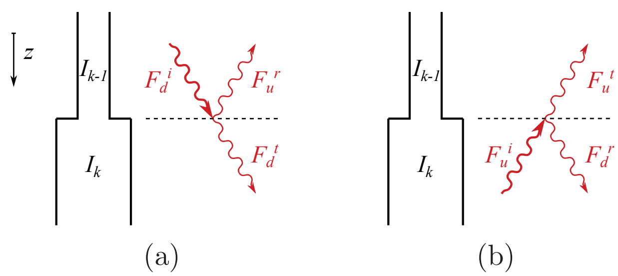

FIG. 1. Incident (i), reflected (r), and transmitted (t) internal force waves about an impedance change for (a) downward; and (b) upward incident waves

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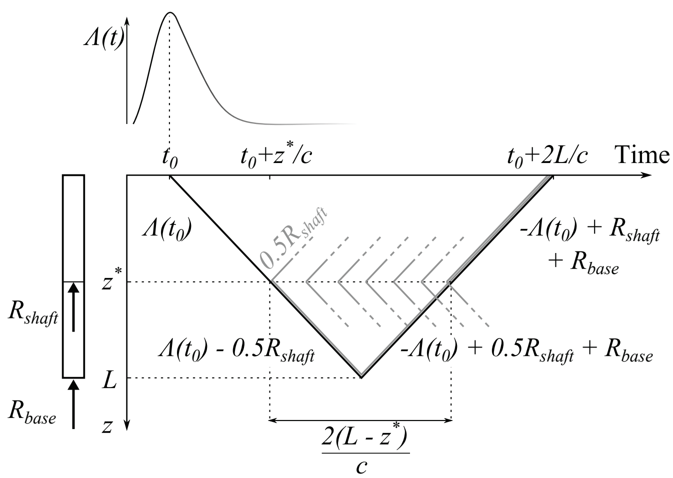

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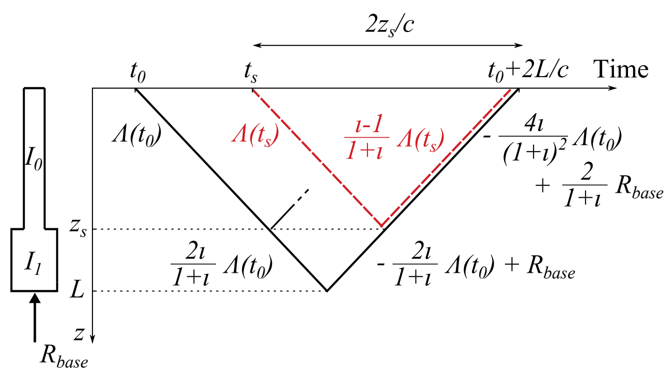

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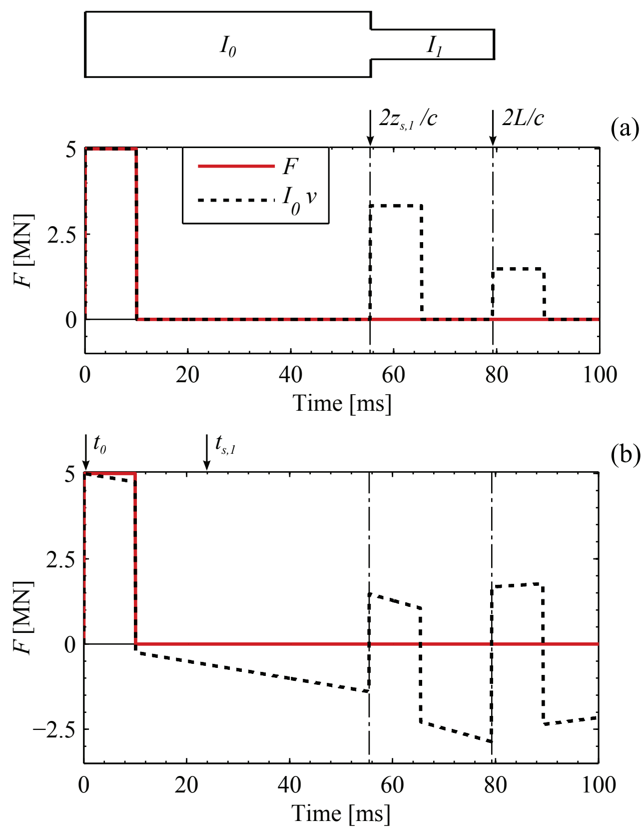

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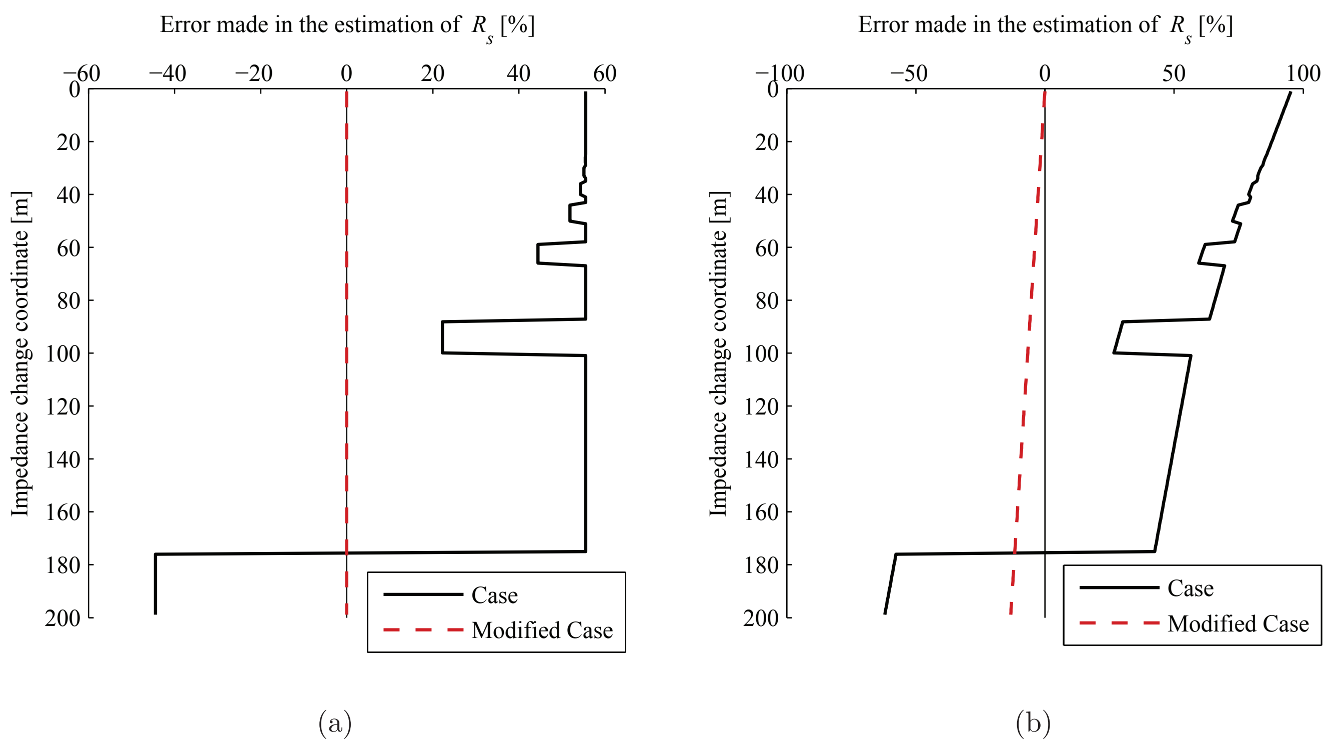

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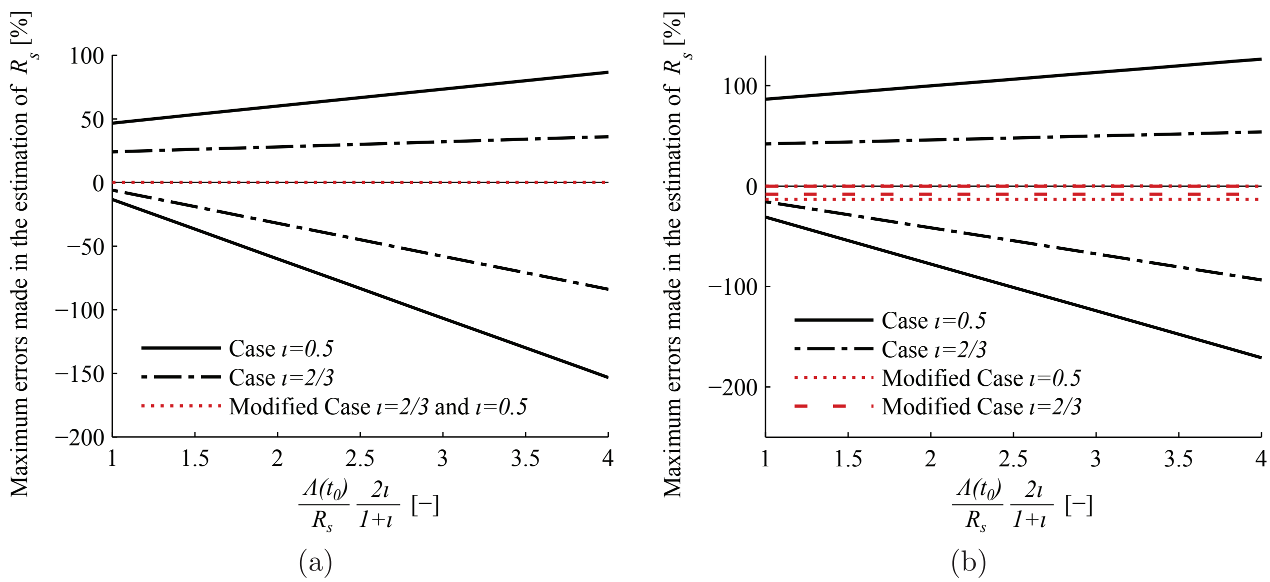

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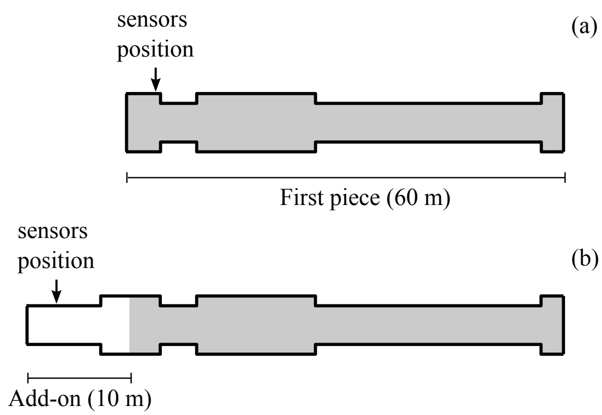

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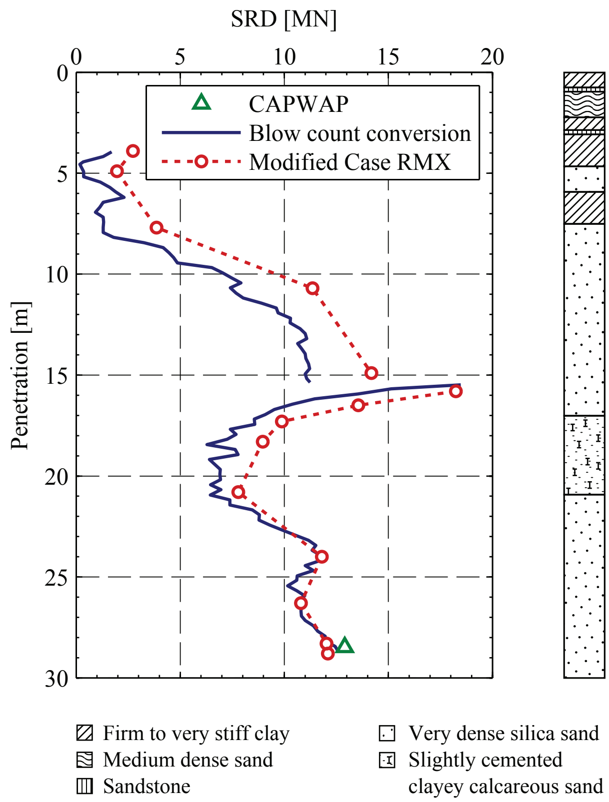

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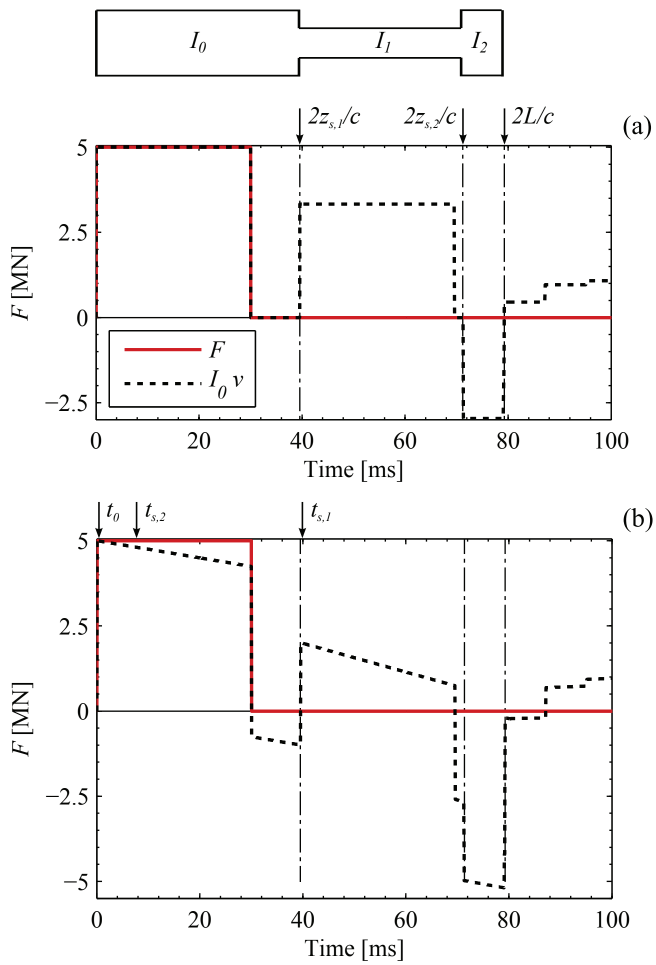

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