

Turbulence Energy Models in Shallow Sea Oceanography

Alan M. Davies, Patrick J. Luyten, and Eric Deleersnijder

Abstract

In this paper a brief overview, using references to published papers for detail, of the development of three dimensional shallow sea models using parameterizations of subgrid scale turbulent motion based upon simple eddy viscosity closure, or the use of turbulence energy models is presented.

The variety of physical problems in shallow sea oceanography addressed by these models is illustrated by considering a range of problems from bottom turbulent boundary layers due to wind waves and tides, to problems involving deepening thermoclines resulting from wind forcing. A series of calculations using single point models in the vertical is used to illustrate physical and numerical problems associated with these models, and the degree of vertical resolution required to obtain solutions of a prescribed accuracy and hence guarantee the quality of the model. Some comments are made concerning the nature and accuracy of the data sets that have to be collected to rigorously validate these models.

Introduction

In classic oceanographic modelling the primary aim has been to simulate the large scale long wave features such as, for example in shallow sea oceanography, changes in tidal height. Although a tidal model using a relatively coarse finite difference grid in the horizontal can resolve the large scale long wave tidal motion, and the forcing producing this motion, there is an energy cascade to small scale turbulent processes due to frictional effects which dissipate energy at the sea bed. Obviously due to computational constraints it is not possible to use a sufficiently fine grid to resolve the smallest scales of motion.

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and in any practical calculation the finite difference grid or other numerical approaches, imposes a spectral cut-off below which waves are not resolved and the influence of small scale motion, if it is important, must be parameterized. In many oceanographic problems the correct formulation of this parameterization is important in determining the accuracy of the large scale flow. For example in a shallow sea tidal model in which the turbulent processes in the near bed region are not resolved, the energy loss is usually parameterized in terms of a drag law, with the drag coefficient having a significant influence upon energy loss and hence tidal elevations. If the physics of the bed region is resolved in such a model, then the bed stress is usually determined from the product of the bed velocity shear and eddy viscosity value, with the latter, a parameterization of the sub-grid scale turbulent motions in a similar manner to a drag coefficient. Since in a pressure driven flow field such as the tides, the stress exerted at rigid boundaries and within the water column is important in determining the variations in the flow field, then the correct parameterization of the turbulent processes, particularly in high shear regions has a significant influence on the larger scale flow fields.

The need to parameterize these small scale motions has lead to the introduction of drag coefficients at well defined interfaces e.g. the sea bed [Bowden, 1978; Grant and Madsen, 1986] and sea surface, with internal stress being parameterized using an eddy viscosity concept to represent small scale motion. Since the magnitude of the eddy viscosity has a large influence upon the flow fields, and similarly eddy diffusivity influences temperature and salinity and thereby density fields, then the appropriate parameterization of these terms is critical in determining the accuracy of the larger scale processes that can be computed by oceanographic models, and hence the quality and true predictability of the models.

A large range of methods exist in the oceanographic literature for determining these coefficients. The simplest is to specify a value which remains constant with position and time. The more complex approach is to derive the eddy viscosity from the mixing length and turbulent energy intensity, which are determined by prognostic equations accounting for the production, dissipation, advection and diffusion of these quantities (a q^2 - q^2 model, [Blumberg and Mellor, 1987]) or prognostic equation for turbulence energy and its dissipation rate (k - ϵ model, [Rodi, 1984, 1987; Luyten et al, 1994]).

In this paper we present a brief overview (using references to the literature for detail) of the range of turbulence models commonly used in shallow sea oceanography. (The interested reader is referred to the paper of the [ASCE, 1988] Task Force for a comprehensive review of turbulence models). We will illustrate the numerical solution of these equations using time dependent single point models in the vertical for a range of physical processes, placing the emphasis on the causes of numerical instability, or physically unrealistic ripples in the vertical, (again using references to the literature for detail). The importance of numerical resolution, particularly in high shear layers in maintaining accuracy will be emphasised. By this means the subject will be introduced to someone new to the field, numerical problems will be illustrated, and the idealised

problems can be used to test various numerical approaches. Also the range of problems here will be used to suggest measurements that are useful in validating this range of turbulence energy models.

The Governing Equations

Hydrodynamic Equations

The three dimensional hydrodynamic equations with the hydrostatic assumption and Boussinesq approximation in Cartesian coordinates are given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (1)$$

$$\frac{\partial u}{\partial t} - \gamma v = \frac{-1}{\rho_0} \frac{\partial P}{\partial x} + \frac{\partial}{\partial z} \left(K_M \frac{\partial u}{\partial z} \right) + A_x + D_x \quad (2)$$

$$\frac{\partial v}{\partial t} + \gamma u = \frac{-1}{\rho_0} \frac{\partial P}{\partial y} + \frac{\partial}{\partial z} \left(K_M \frac{\partial v}{\partial z} \right) + A_y + D_y \quad (3)$$

$$\rho g = - \frac{\partial P}{\partial z} \quad (4)$$

In these equations x , y , z are the eastward, northward and vertical (upward pointing from sea bed) axis, with u , v and w the corresponding velocities, and γ the Coriolis parameter. The pressure is denoted by P , with ρ_0 the reference density, ρ the in situ density, g the gravitational acceleration, and K_M the coefficient which parameterizes the vertical diffusion of turbulent momentum, often referred to as eddy viscosity. The terms A_x and A_y denote the x and y components of horizontal advection of u and v , with D_x and D_y the corresponding horizontal diffusion terms. The formulation of full hydrodynamic equations in both Cartesian [Blumberg and Mellor, 1987] and Spherical coordinates [Davies, 1986a] can be found in the literature, and as we will be principally concerned here with single point models in the vertical in which these terms are zero, they will not be discussed further.

The equation describing the time variation of density, is given by

$$\frac{\partial \rho}{\partial t} = \frac{\partial}{\partial z} \left(K_H \frac{\partial \rho}{\partial z} \right) + A_p + D_p \quad (5)$$

with K_H the coefficient which parameterizes the vertical diffusion of density, often referred to as the vertical diffusivity. The terms A_p and D_p denote the horizontal advection and diffusion of density which are set to zero in the single point model, described later.

Two Equation Turbulence Models

A range of turbulence energy models exist in the literature, and have been applied to a number of oceanographic problems. The most widely used model, termed a k - ϵ model [Rodi, 1984; Cheng and Smith, 1990], involves two prognostic equations, one for turbulence energy k , and a second for its dissipation rate ϵ . An alternative approach termed a q^2 - $q^2\ell$ model also involves two prognostic equations one for $q^2=2k$, in essence the turbulence energy and one for the term $q^2\ell$, giving a formulation for the mixing length ℓ . Here we initially consider the k - ϵ form of the prognostic equations for the turbulence energy and dissipation rate.

The k - ϵ Model

Considering initially the prognostic equation for turbulence energy k , this takes the form,

$$\frac{\partial k}{\partial t} = \frac{\partial}{\partial z} \left(\frac{K_M}{\sigma_k} \frac{\partial k}{\partial z} \right) + P + G - \epsilon + A_k + D_k \quad (6)$$

where the first term represents the vertical diffusion of turbulence energy, the second term, given by

$$P = K_M M^2, \text{ with } M^2 = \left[\left(\frac{\partial u}{\partial z} \right)^2 + \left(\frac{\partial v}{\partial z} \right)^2 \right] \quad (7)$$

where P is the shear production of turbulence energy, with M the shear frequency.

The term,

$$G = -K_H N^2, \text{ with } N^2 = \frac{\partial b}{\partial z} \text{ and } b = \frac{-g}{\rho_o} (\rho - \rho_o) \quad (8)$$

is the rate of conversion of turbulence energy into potential energy which appears as the dissipation of turbulence energy by buoyancy, with N^2 the Brunt-Vaisala frequency, b is

the buoyancy term, and ε the turbulent energy dissipation rate. The terms A_k and D_k representing the horizontal advection and diffusion of turbulence energy.

The equation for turbulence energy dissipation takes the form

$$\frac{\partial \varepsilon}{\partial t} = \frac{\partial}{\partial z} \left(\frac{K_M}{\sigma_\varepsilon} \frac{\partial \varepsilon}{\partial z} \right) + C_{1\varepsilon} \frac{\varepsilon}{k} (P + C_{3\varepsilon} G) - C_{2\varepsilon} \frac{\varepsilon^2}{k} + A_\varepsilon + D_\varepsilon \quad (9)$$

where the various terms are defined previously, and $C_{1\varepsilon}$, $C_{2\varepsilon}$, σ_k , σ_ε are specified coefficients, although $C_{3\varepsilon}$ depends upon whether the vertical stratification is unstable or stable in which case $C_{3\varepsilon} = 1$ or $C_{3\varepsilon} = 0.2$ [Rodi, 1987]. Following the work of Rodi, [1984, 1987] and Luyten et al, [1994] the diffusion coefficients can be expressed as

$$K_M = S_u k^2 / \varepsilon, \quad K_H = S_b k^2 / \varepsilon \quad (10)$$

with the stability functions S_u , S_b given as algebraic functions of N^2 , k and ε .

The q^2 - $q^2\ell$ Model

An alternative approach to the k - ε model is based upon the work of Mellor and Yamada, [1974, 1982] who derived in a systematic way a hierarchy of turbulence energy models. Improvements to the theory are considered by Galperin et al, [1988] and Galperin et al, [1989]. In this group of models, the one that has been applied most extensively in oceanography [e.g. Blumberg and Mellor, 1987; Rosati and Miyakoda, 1988] and corresponds to the k - ε model, in that it involves prognostic equations for $q^2 = 2k$, and $q^2\ell$ is the so called q^2 - $q^2\ell$ model. In this approach the equation for turbulence energy takes the form

$$\frac{\partial q^2}{\partial t} = \frac{\partial}{\partial z} \left(K_q \frac{\partial q^2}{\partial z} \right) + 2P + 2G - \frac{2q^3}{B_1\ell} + A_q + D_q \quad (11)$$

where

$$K_q = q\ell S_q, \quad K_M = q\ell S_M, \quad K_H = q\ell S_H \quad (12)$$

with P and G as defined before.

The prognostic equation for the mixing length takes the form,

$$\frac{\partial}{\partial t}(q^2\ell) = \frac{\partial}{\partial z}\left(K_v \frac{\partial}{\partial z}(q^2\ell)\right) + \ell E_1(P+G) - \frac{q^3 W}{B_1} + A_1 + D_1 \quad (13)$$

with W a wall proximity function, and E_1, B_1 constants. Details of the form of W and the stability functions S_H and S_M can be found in Blumberg and Mellor [1987], and Galperin et al., [1988] and will not be repeated here.

One Equation Turbulence Models

An alternative to applying two prognostic equations is to use an expression for the mixing length, either in the form of a diagnostic equation involving the turbulence energy or a trigonometric expression. This form has been used extensively in shallow sea oceanography for a range of problems from wind waves [Davies, A.G. et al, 1988; Davies and Jones, 1991] to longer wave tidal problems [Davies and Jones, 1990], and because of its popularity we will briefly consider its relation to equation (6). In the single equation approach, ϵ in (9) is given by

$$\epsilon = S_1 k^{3/2}/\ell \text{ with } K_M = S_0 \ell k^{1/2} \quad (14)$$

where for homogeneous flow, $S_0 = S^{1/4}$, $S_1 = S_0^3$ and $S = 0.046$ [A.G. Davies 1986].

In this type of model the mixing length ℓ is often computed from a turbulent energy dependent mixing length ℓ_0 and a weighting function $D(z)$, using

$$\ell = D(z) \ell_0 \quad (15)$$

with ℓ_0 taking a number of forms, the most common of which is given by [Blackadar, 1962]

$$\ell_0 = \frac{Kz}{1 + Kz/\ell_m} \text{ with } \ell_m = \gamma_0 \frac{\int_{bed}^{surface} k^{1/2} z dz}{\int_{bed}^{surface} k^{1/2} dz} \quad (16)$$

where γ_0 is a constant in the range 0.1 to 0.4. Alternative expressions for the mixing length have been given by Vager and Kagan [1969] and Johns [1978].

These simpler expressions give mixing lengths that increase with height above the sea bed. However laboratory experiments of Nezu and Rodi [1986] suggest that eddy viscosity should decrease towards the sea surface, and to allow for the damping of turbulence near the sea surface, Celik and Rodi [1984] suggested the damping function $D(z)$ in equation (15) to reduce the mixing length to near zero at the surface. A suitable simple algebraic form of this damping function in homogeneous situations has been given

by Davies, A.G., [1989]. The application of such a damping function gives a near parabolic mixing length. In stratified regions the damping function should be of a form which also reduces the mixing length within the pycnocline, see below.

Instead of making the mixing length depend upon the turbulence energy intensity it can be specified algebraically using a simple expression such as that proposed by Yalin [1963, 1972]. An alternative formulation in which the position of maximum mixing length need not coincide with mid-water and is closer to that occurring in tidal flows [Wolf, 1980] is to express ℓ in terms of a combination of surface and bed mixing length [Johns and Xing, 1993; Xing, 1992; Davies and Xing, 1994].

In the case of stratified flows the mixing length formulations such as (16) or those described above take account of the effects of stratification by their dependence on the turbulence energy. These effects are included more directly by multiplying the mixing length, valid for homogeneous flows with a correction factor, dependent on stratification, giving

$$l = l_0(z)F_s \quad (17)$$

where F_s is smaller than one for a stable and larger than one for an unstable stratification. The factor F_s is usually expressed in terms of the Richardson number $Ri = N^2/M^2$. Based upon measurements in stratified atmospheric boundary layers [Businger et al, 1971; Geernaert, 1990] the following forms have been proposed, either

$$F_s = 1 - \alpha Ri \quad (18)$$

for stable stratification ($Ri > 0$) and

$$F_s = (1 - \beta Ri)^{1/4} \quad (19)$$

for unstable stratification ($Ri < 0$) with $\alpha \approx 5-10$ and $\beta \sim 14$. The expression (19) is also known as the KEYPS formula [Panofsky, 1963]. An alternative approach, advocated by Nihoul and Djenidi [1987], consists in expressing F_s in terms of the flux Richardson number $R_f = -G/P$ (see equation 6) instead of Ri . A suitable form appears to be $F_s = 1 - R_f$.

Zero Equation Model

Neglecting the horizontal advection together with the horizontal and vertical diffusion terms, and assuming a steady state then the production and dissipation terms in eqtn (6) balance, giving

$$K_M M^2 - K_H N^2 = \epsilon \quad (20)$$

and an equation for q^2 of the form

$$q^4 + q^2 l^2 (C_1 M^2 + C_2 N^2) + l^4 N^2 (C_3 M^2 + C_4 N^2) = 0 \quad (21)$$

where the coefficients C_i can be expressed in terms of the empirical parameters A_1, A_2, B_1, B_2, C_1 . Prescribing l using one of the forms described earlier, q^2 is then obtained as the largest root of equation (21). The stabilising effect of stratification on turbulence can be seen by letting $q^2 = 0$ in equation (21). This gives $Ri = Ri_c = 0.195$ in good agreement with the upper limiting value 0.25, necessary for the maintenance of shear flow instability (the Miles-Howard theorem) and obtained from ocean measurements [Kundu and Beardsley, 1991]. This method, originally due to Mellor and Yamada, [1974, 1982] can also be applied to the k - ϵ model described earlier, in which case a second degree equation is derived for the quantity ϵ^2/k^2 and the critical Richardson number Ri_c takes the value 0.28.

Specified Eddy Viscosity Model

The simplest type of model which has been used quite successfully in a number of shallow sea simulations is to specify the value of eddy viscosity and its profile through the vertical. Although these models do not contain all the sophistication of the turbulence closure models and may therefore be quite limited in the accuracy of the solution, they do give significant insight into the role of eddy viscosity and how its magnitude and variation influences the flow field.

In stratified conditions, the influence of stable stratification is to suppress the eddy viscosity within the pycnocline, and various formulations exist in the literature to represent this, for example the classic formula of Munk and Anderson [1948]. Allowing for a suppression of turbulence within the pycnocline, Davies, [1986b] showed that as the upper turbulence mixed layer became shallower, or the difference in eddy viscosity across the pycnocline increased then the angle between the surface current and the wind increased towards ninety degrees. The influence of a reduction in eddy viscosity due to stratification effects upon tidal current profiles has been examined [Howarth, 1994; Maas and Van Haren, 1987] using simple eddy viscosity profiles. Such models were able to reproduce the observed change in amplitude and phase of the tidal current across the thermocline. In these models the eddy viscosity was constant in both space and time, however from a scale analysis the eddy viscosity should depend upon length and velocity scale. Such a formulation was used by Bowden et al. [1959], Bowden [1978] in shallow seas with the water depth h taken as the length scale, and $|\bar{u}|$, the magnitude of the depth mean current, the velocity scale.

Such a parameterization has been used extensively in three dimensional shallow sea tidal models where the boundary layer thickness is restricted by the water depth. However in deeper water regions where the boundary layer Δ is only a small fraction of the water column then the length scale is probably determined by Δ . Details of these various forms have been given in [Davies and Aldridge, 1993] and used very successfully by Davies and Furnes, [1980]; Davies, [1986] to simulate the M_2 and M_4 tides over the continental shelf.

The solution of the hydrodynamic equations using simple flow related eddy viscosities of this form can be readily accomplished using a functional approach in the vertical [Davies 1987]. A comparison of observed and computed tidal currents in homogeneous regions [Davies and Jones, 1990; Davies and Gerritsen, 1994; Davies and Xing, 1994] determined with turbulence energy models and flow related eddy viscosity models using a functional approach in the vertical, shows that there is no significant difference between the various models. When a no-slip condition is applied at the sea bed, then as we will show later, a transformed grid with high resolution is required at the bed in order to resolve the high shear bed layer. In the functional model this is adequately resolved [Davies, 1993] due to the continuous nature of the functions.

Surface and Bed Boundary Conditions and Numerical Solution

Boundary Conditions

Since in the latter part of this paper we will be concerned with a single point model in the vertical, we will restrict our discussion here to surface and bed boundary conditions, and a brief indication of the various approaches used to discretize the equations in the vertical. In a three dimensional model, besides these boundary conditions it is necessary to specify those along land and open boundaries. In the case of a limited area model the formulation of open boundary conditions is particularly important, and although outside the scope of this paper, reviews of the range of open boundary conditions that exist in the literature can be found for example in Roed and Cooper [1987]; Martinsen and Engedahl, [1987], Chapman, [1985]; Greatbatch and Otterson, [1991].

In the calculations described subsequently we will be concerned with wind driven flow, or pressure gradient flow due to oscillatory pressure gradients at either wind wave or tidal period. For wind driven flow the surface stress balances the external wind stress components τ_{xx} , τ_{xy} (total stress τ_x), thus

$$-\rho K_M \frac{\partial u}{\partial z} = \tau_{xx}, \quad -\rho K_M \frac{\partial v}{\partial z} = \tau_{xy}. \quad (22)$$

with

$$q^2 \ell = 0, \quad \frac{q^2}{2} = k = \tau_s / \sqrt{c_p}, \quad \epsilon = \tau_s^{3/2} / (K z_s) = (k c_p^{1/2})^{3/2} / (K z_s)$$

with $c_p^{3/4} = 0.17$ [Luyten et al., 1994] an empirical coefficient, and in a numerical model z_s is often taken as the distance of the uppermost ϵ grid point from the sea surface.

The boundary condition for the vertical diffusion of density takes the form [Nihoul, 1984],

$$\left[K_H \frac{\partial \rho}{\partial z} \right]_{\text{surface}} = \frac{\rho_0 \beta_T}{C_p} \phi_{s,H} + \beta_s \phi_{s,w} \quad (23)$$

where β_T is thermal expansion coefficient, C_p is the specific heat of sea water, $\phi_{s,H}$ is the surface heat flux (positive upwards), $\phi_{s,w}$ is the surface water (mass) flux (evaporation-precipitation).

In the case of tidal flow the surface wind stress is zero, and the vertical derivative of k or q^2 is zero, with the mixing length tending also to zero, although a non-zero mixing length is possible [Kundu, 1980] thus

$$\frac{\partial k}{\partial z} = 0, \quad q^2 \ell = 0, \quad \epsilon = \frac{(k c_p^{1/2})^{3/2}}{(K z_s)} \quad (24)$$

At the sea bed, assuming a slip condition, then the bed stress is given by,

$$K_M \frac{\partial u}{\partial z} = k U_h (U_h^2 + V_h^2)^{1/2}, \quad K_M \frac{\partial v}{\partial z} = k V_h (U_h^2 + V_h^2)^{1/2} \quad (25)$$

with U_h, V_h the components of the bed current at some reference height z , above the sea bed, and k is a drag coefficient related to this reference height and the bed roughness. This quadratic friction law is a parameterization of the frictional retardation at the sea bed by turbulent processes. Also at the seabed, a zero derivative boundary condition for density is applied, with similar boundary conditions for $q, q^2 \ell$ and ϵ as those used at the sea surface, when a slip condition is applied at the bed. In shallow water when wind waves are present, enhanced turbulence at the sea bed due to the waves (see later), effectively increases the drag coefficient by the process of wave-current interaction [Christoffersen and Jonsson, 1985; Grant and Madsen, 1979, 1986]. Using a single point turbulence energy model of the form described previously, running with a time step sufficiently small to resolve the wind waves (a time step of the order of a fraction of a second), Davies et al [1988], showed that the enhanced turbulence due to wind waves had an influence upon the mean flow. In a full three dimensional model where the time step is much larger than the wave period (time step of order 180 secs, wave period usually

below 15 secs) this effect has to be parameterized by increasing the bed roughness z_0 [Signell et al, 1990; Davies and Lawrence, 1994a,b]. Calculations using a full three dimensional model of the eastern Irish Sea (grid resolution 1 km) with wave-current interaction included, have shown that enhanced turbulence due to the wind waves can modify wind driven bottom currents in near coastal regions [Davies and Lawrence, 1994a], with wind and wind wave turbulence in shallow regions during major wind events having an influence upon tidal currents; both at the fundamental frequency [Davies and Lawrence, 1994b] and the higher harmonic [Davies and Lawrence, 1994c]. Calculations such as these have clearly shown that the correct parameterisation of a range of processes through a drag coefficient k , is important in determining the large scale flow fields.

Numerical Solution

The traditional approach in the horizontal has been to apply a uniform finite difference grid, however an alternative approach is the use of boundary fitted coordinates in which the equations are transformed [Spaulding, 1984] to give in physical space a much finer grid in the near coastal region. (Although this topic is outside the scope of the present paper it is illustrated below for a single dimension). In recent years the finite element method with its ability to vary element sizes and hence improve local resolution [e.g. Leutich et al, 1994; Westerink et al, 1994; Lynch and Naimie, 1993; Foreman et al, 1993] is a particularly useful method.

Discretization in the vertical is usually accomplished using a staggered or non-staggered grid, although in the case of a parameterisation involving a constant or flow dependent eddy viscosity a functional approach has been used [Davies, 1987]. Details of the discretization of the turbulence energy equations, on the staggered or non-staggered grid, and the nature of the grid have been given in Davies and Jones [1991] and will not be repeated here. For problems in which the profile of eddy viscosity is constant in time, although its magnitude may change, then a functional approach in the vertical in which the functions are eigenfunctions of the eddy viscosity profile has been very successful. In the case of a no-slip condition at the sea bed in which the eddy viscosity reduces to a low value at the bed, then a high shear near bed region exists. In a functional model, using eigenfunctions in the vertical, then this high shear region is easily resolved due to the fact that each function is highly sheared in the near bed region with shear increasing with mode number, and as near bed eddy viscosity is reduced. Increased resolution using the finite difference approach in the vertical can be achieved by using an irregular grid, chosen in an arbitrary manner or using a method which varies the grid in a geometric manner, but still retains second order accuracy (e.g. a Kappa grid [Noye, 1984; Davies, 1991]). An alternative approach which is particularly simple when a no-slip condition is applied at the sea bed is to transform the equations from z coordinates onto a new coordinate s , using either a logarithmic transformation, or log-linear transformation in which the grid can be refined in the near bed region, [see Davies, 1991 for details]. By transforming the equations by these means, they can be solved using a regular finite

difference grid in transformed space which corresponds to a compressed grid in physical space. In order to use a fine grid in the vertical without restricting the time step to a computationally prohibitive small value an implicit time integration method such as the Crank-Nicolson approach has been used [e.g. Davies, 1991].

Naturally in all numerical calculations the prime aim is to achieve an accurate solution at a minimum computational cost. However in trying to achieve this it is essential to ensure that the physical phenomena in the model are correctly represented and that any improvement in the physics is not negated by inaccuracies such as under-resolution. In the next section we will use a number of simple models in the vertical to illustrate these problems.

Numerical Calculations

Oscillatory Flow at Tidal or Wind Wave Period with Eddy Viscosity Closure

In an initial series of calculations we will consider only a single point model in the vertical with rotational effects neglected, namely eqtn (2) with $A_x = D_x = 0$ and $\gamma = 0$, with sinusoidal forcing at a single frequency ω , thus

$$-\frac{1}{\rho_0} \frac{\partial P}{\partial x} = \omega F_x \cos(\omega t + g_x), \quad -\frac{1}{\rho_0} \frac{\partial P}{\partial y} = \omega F_y \cos(\omega t + g_y) \quad (26)$$

with F_x and F_y the amplitude, and g_x, g_y the phase of the external forcing. A zero surface stress boundary condition is used with zero velocity at the sea bed. In all calculations eddy viscosity was constant in the upper part of the water column at a value of $K_t = 0.1 \text{ m}^2 \text{ s}^{-1}$ decreasing in the near bed layer over a distance of $0.2h$ to a value of $K_b = 0.0001 \text{ m}^2 \text{ s}^{-1}$. Motion was driven by a unit pressure forcing ($F_x = 1.0, F_y = 0.0$) of 12hr period (denoted by period D_2) only in the x direction. Since motion was started from rest it was necessary to integrate forward in time for the order of two tidal periods before the influence of the initial conditions was removed. Once a periodic condition was reached, a Fourier analysis was performed to obtain the amplitude and phase at the various periods.

Amplitudes and phases of the current at various heights z above the sea bed computed using both the logarithmic and log-linear transforms for a number of grid boxes in the vertical and functions have been performed by Davies [1991]. These calculations showed that the currents computed with both the finite difference and modal approach are very similar, although the rate of convergence of the functional model is faster, with the log-linear grid having some minor advantages over the logarithmic grid.

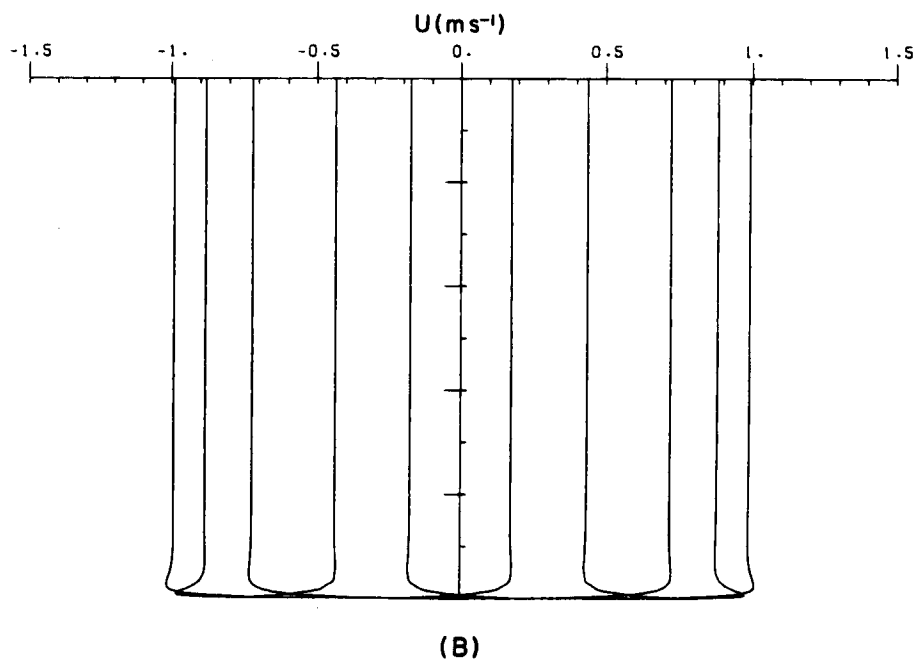
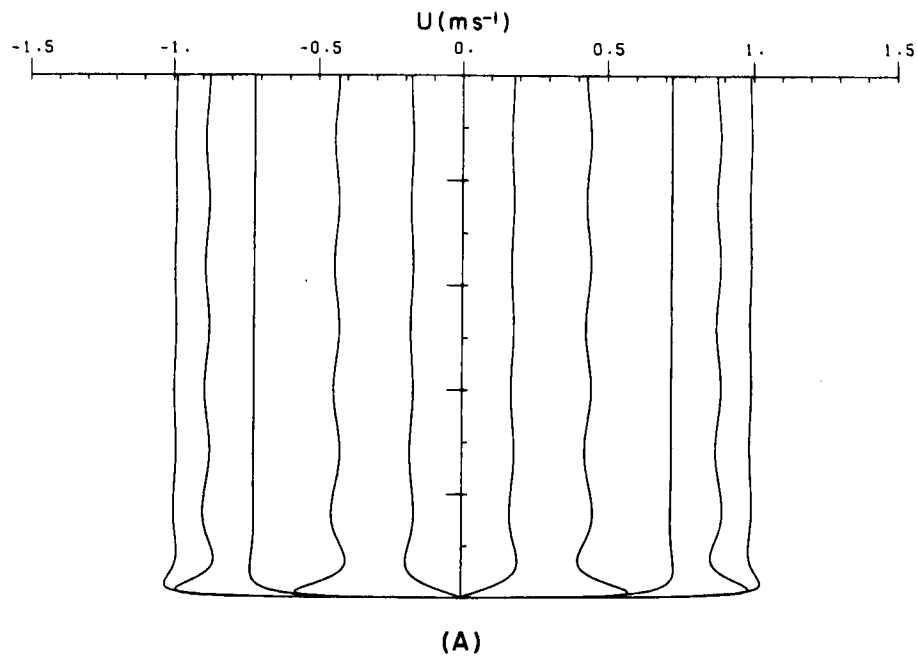


Figure 1. Profiles at intervals of 0.8s for a wind wave of $T=8\text{s}$; (A) computed with 10 modes, (B) computed with 30 modes.

Current profiles at hourly intervals over a tidal cycle showed a high shear near bed region in which the profile is logarithmic. The accurate resolution of this region is essential if the near bed currents or bed stresses are to be accurately reproduced and is the reason for using a logarithmic type transformation in the vertical with the grid box model, or a functional approach with functions designed to reproduce this high shear layer. The effect of not resolving or under-resolving this region is discussed next in connection with flow at the wind wave period, where the boundary layer thickness is much thinner and the shear much larger.

The period of wind waves is typically of the order of seconds rather than hours, with boundary layer thicknesses of the order of 10cm. Wind wave current profiles at 0.8s intervals induced by a wind wave of period $T = 8$ second in a water depth of $h = 1$ m computed with $K_0 = 0.0001\text{m}^2\text{s}^{-1}$ and $K_1 = 0.0050\text{m}^2\text{s}^{-1}$ (typical eddy viscosity values for wind waves) determined using 10 modes and 30 modes in the vertical are presented in Fig 1. This figure clearly shows a very high shear near bed layer of the order of a few centimetres, above which the flow reaches its invicid value of 1.0m s^{-1} . For the case in which an expansion of ten functions was used in the vertical it is clear that physically unrealistic oscillations occur above the bottom boundary layer, associated with the inability of the functions to resolve the high shear layer (a Gibbs type phenomenon). However as the number of functions is increased the spurious oscillations diminish and physically more realistic profiles are evident. Calculations using a transformed grid also showed that an accurate solution could only be obtained using a log. linear transformation with the order of 40 grid boxes i.e. a grid with very high resolution in the bottom few centimetres.

In these calculations eddy viscosity did not vary with time. However in a turbulence energy model, in which the turbulence energy intensity and hence eddy viscosity depend very much upon the shear production terms which are largest near the bed, then an accurate determination of the shear is essential if the shear production of turbulence is to be correctly determined.

Profiles of velocity and shear stress on a log transformed grid over the same wind wave period ($T = 8\text{s}$), induced with unit forcing but this time in a water depth $h = 10\text{m}$ computed with a turbulence energy model with the Blackadar [1962] mixing length and $\gamma_0 = 0.4$ showed similar features to those computed with the functional model. The order of 100 grid boxes on a transformed grid was used in this calculation in order to accurately resolve the shear production term. Calculations using a range of γ_0 values from 0.1 to 0.4, (an acceptable variation in γ_0) showed that the maximum bed stress was sensitive to the value of γ_0 used in the mixing length formulation. This sensitivity of bed stress to parameterisation of mixing length is of physical importance particularly in sediment transport problems since in most cases the sediment does not move until a bed shear stress value is exceeded.

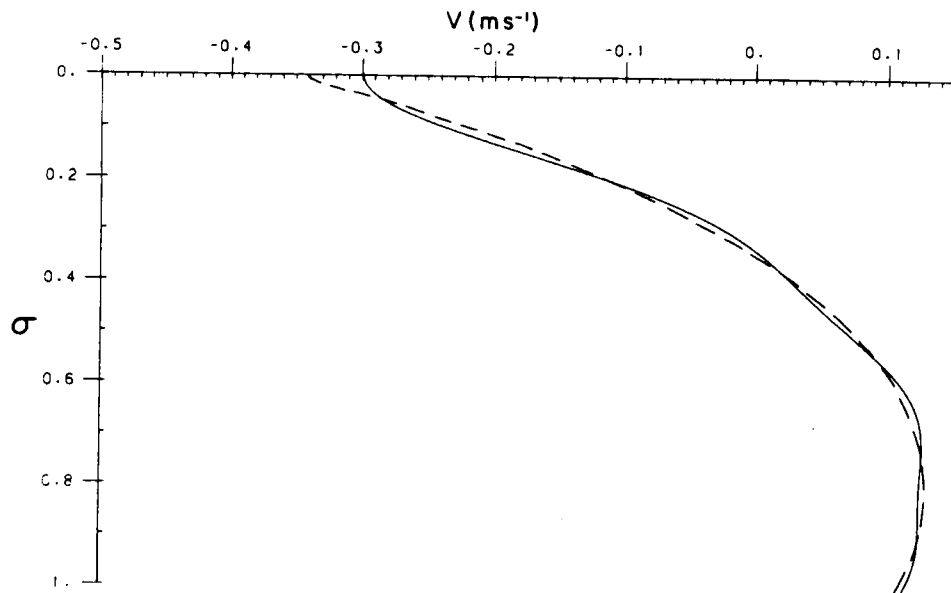


Figure 2. The v component of velocity (the component in the wind direction), computed using an expansion of 6 eigenfunctions (solid line) and a similar expansion but with an additional "enhancement" function (dashed line).

In this series of calculations the emphasis has been placed on shear in the bottom boundary layer, produced by frictional retardation at the sea bed upon pressure induced flows. In the case of wind induced flow in deep water, then shear is induced primarily in the surface layer as a result of the wind stress. Since the surface stress must balance the externally applied stress, then as surface eddy viscosity is reduced, shear in the near surface layer increases. In order to resolve this shear in a grid box model it is necessary to use either an irregular grid, with a finer grid in the surface layer [Davies and Stephens, 1983] or a log type transformation in the near surface layer.

In a functional model, an expansion of Chebyshev polynomials [Davies and Owen, 1979] has optimal convergence properties in the near surface layer, whereas a set of eigenfunctions chosen to satisfy a zero derivative surface boundary condition converges very slowly. The rate of convergence and hence the accuracy of the eigenfunction method has recently been improved by using a mixed basis set [Davies, 1992] in which an additional function having high shear in the surface layer is included.

Wind induced current profiles computed with an eddy viscosity value of $0.0130\text{m}^2\text{s}^{-1}$ and an expansion of just six functions (Figure 2, solid line) do not exhibit a high shear surface layer and spurious ripples (a Gibbs type phenomenon) occur below the surface layer. However the near surface shear layer is resolved and the spurious ripples are removed (Fig 2, dashed line) when an additional term, "an enhancement function", is added to the

eigenfunction expansion. This additional term, effectively accounts for the high shear surface layer, leaving a smoother solution which can be readily approximated with the eigenfunction expansion. A detailed discussion of this is outside the scope of the present paper but can be found in Davies [1991, 1992]. In a turbulence energy model in which the surface mixing length is assumed to be small i.e. of comparable magnitude to the bottom mixing length then the eddy viscosity in the surface layer will be small, and an accurate solution will only be possible if a fine near surface grid is employed.

In these calculations we have been concerned with either periodic pressure driven flow or that arising from wind forcing at the sea surface. In nature these forcings often occur together, giving with a flow dependent viscosity some interesting non-linear interactions, and situations where shear layers may be under-resolved by the numerical methods used in the calculation. An example of this occurs when tidal and wind forcing are considered in combination. Considering the single point model in the vertical in a water depth of 100m with a no-slip condition at the sea bed and eddy viscosity varying as previously with $h_1 = 0.2h$, and rotational effects removed. By taking $F_x = F_y$ as unity and $g_x = 0.0$ with $g_y = 90^\circ$ and forcing at $30^\circ/\text{hour}$ (a diurnal period D_2) a circular tidal current ellipse is produced with the term $(\bar{u}^2 + \bar{v}^2)^{1/2}$ constant in time. With a simple closure model in which eddy viscosity is related to the current magnitude viscosity will also be constant. In the case of a time invariant eddy viscosity and a no slip bottom boundary condition, the single point model is linear and only the D_2 harmonic is present. However when a wind stress of 1 Nm^{-2} is added in the u direction, the total current varies significantly in time with periods where the current magnitude is zero and hence the eddy viscosity is zero. At times of zero or small eddy viscosity, the wind induced shear must approach infinity and cannot be accurately resolved on the finite difference grid. A consequence of this is that in a turbulence energy model, the shear production term may be underestimated at certain times unless a fine grid is used. In the eddy viscosity model the significant time variation in the viscosity term is a major source of non-linearity which leads to the generation of higher harmonics. (An explanation of this is outside the scope of this paper but is given in Davies and Lawrence, [1994c]).

Stratification Effects

The previous series of calculations have been concerned with bed and sea surface high shear layers in a homogeneous water column. However in stable stratified conditions, buoyancy effects can suppress turbulence within the pycnocline, which can result in internal shear layers that must be resolved.

To examine the effect of stratification upon the accuracy of computed tidal currents determined with expansion methods, we again consider the single point model with a pressure forcing of 12hr period and $F_x = 0.2$, $F_y = 0.0$, $g_x = 0.0$, $g_y = 0.0$ and $\gamma = 0.00012 \text{ s}^{-1}$. A no slip condition was applied at the sea bed. A viscosity profile was used corresponding to a reduction of viscosity within the thermocline and close to the sea bed.

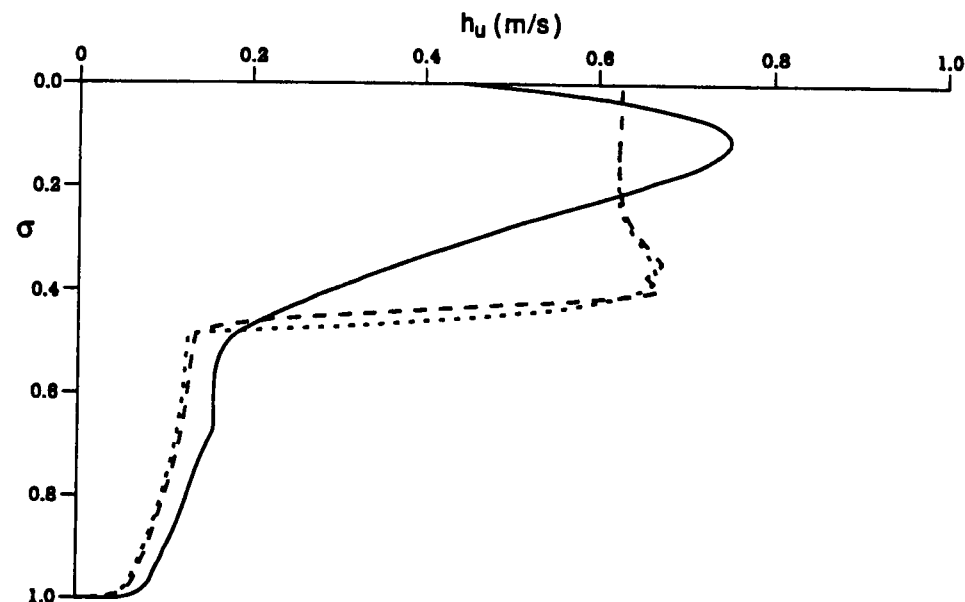


Figure 3. Profiles of tidal amplitudes h_u (m s^{-1}), computed with a coarse knot distribution (Solution 1 —), a finer knot distribution (Solution 2 - - - -), and a very fine knot distribution (Solution 3).

The stratification is specified in a diagnostic manner in this calculation and details are given in Davies [1993b].

This viscosity profile corresponds to a very intense pycnocline which effectively suppresses tidal turbulence in the upper part of the water column, with a region of significant tidal turbulence occurring at about mid-depth. As such it provides a rigorous test of a model's ability to resolve the high shear layer which occurs in the pycnocline region. Since turbulence above the pycnocline is small, the computed surface current amplitudes and phases should correspond to their inviscid values of $h_u = 0.626 \text{ m s}^{-1}$, $g_u = 270^\circ$, $h_v = 0.517 \text{ m s}^{-1}$ with $g_v = 180^\circ$, giving a test of the models accuracy in the surface region.

Profiles of the amplitude of the u -component of current computed using an expansion of B-splines [Davies, 1987], based on a coarse set of knots (solution 1), on a finer set of knots (solution 2) and a very fine set of knots (solution 3) are shown in Fig 3. The nature of the spline functions is that they are piecewise polynomials, and resolution can be enhanced locally by refining the local knot distribution. A detailed discussion is beyond the scope of this paper but is given in Davies [1987].

It is evident from this figure that as the resolution within the pycnocline is increased, (solutions 2 and 3), the spurious ripples particularly in the phase, associated with solution 1 are reduced, and a solution showing little vertical variation in the near surface layer.

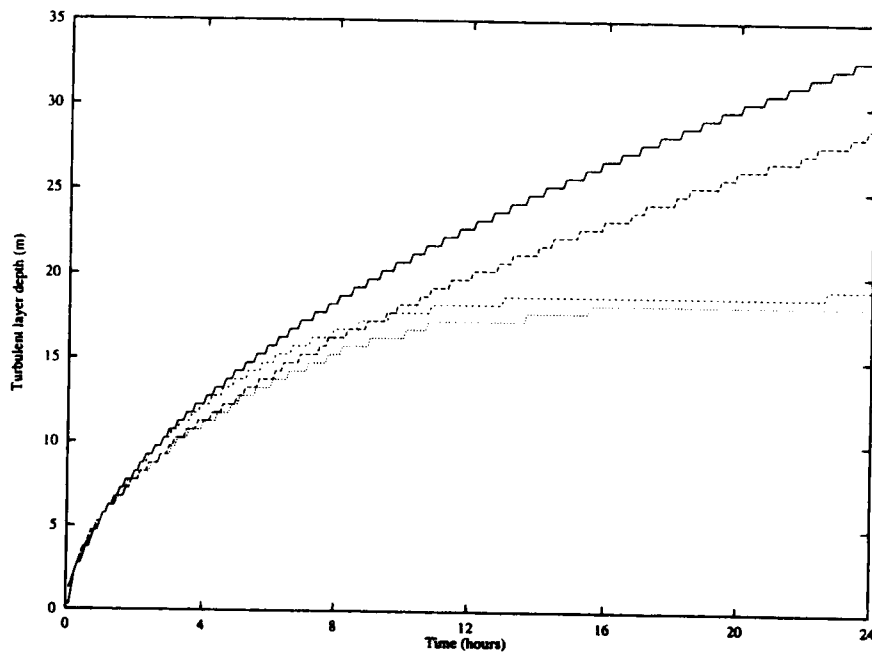


Figure 4. Evolution in time of the turbulent layer depth with $\gamma = 0$ (KE2 solid, KE0 large dashes) and $\gamma = 0.0001\text{s}^{-1}$ (KE2 small dashes, KE0 dots).

with a region of rapid shear in the pycnocline, and in the near bed region is evident. Current profiles, and large phase changes across the pycnocline, of the form shown here are in close agreement with observed tidal profiles, although the intensity of the pycnocline is larger than that normally found in field measurements.

In a last test case we consider the problem of the wind induced entrainment of a turbulent layer into a stratified fluid situated below, namely a density evolving problem. For simplicity we assume a constant wind stress and an initially linear density profile. The parameters which govern the problem, are the surface friction velocity u_* and the initial buoyancy frequency N_0 . The models, considered in the simulations, are the k - ϵ model (KE2) described previously, using two transport equations for turbulent variables, and its local equilibrium equivalent (KE0) without transport equations using the Blackadar mixing length. The sink terms in the k - and ϵ -equation are evaluated using a quasi-implicit numerical scheme [Patankar, 1980; Baumert and Radach, 1992]. The x -axis is chosen along the applied wind stress. The models are run using $u_* = 0.01\text{ m/s}$, $N_0 = 0.01\text{ s}^{-1}$ and with or without rotation ($\gamma = 0.0001\text{s}^{-1}$). A depth of $h = 100\text{m}$ is chosen to eliminate the influence of the bottom boundary layer. Time step and vertical resolution are chosen as $\Delta t = 60\text{s}$ and $\Delta z = 0.5\text{m}$.

The time evolution of the turbulent layer depth is shown in Fig. 4 for the four cases. It can be seen that entrainment is slowed down by the rotation after ~ 0.5 the inertial period.

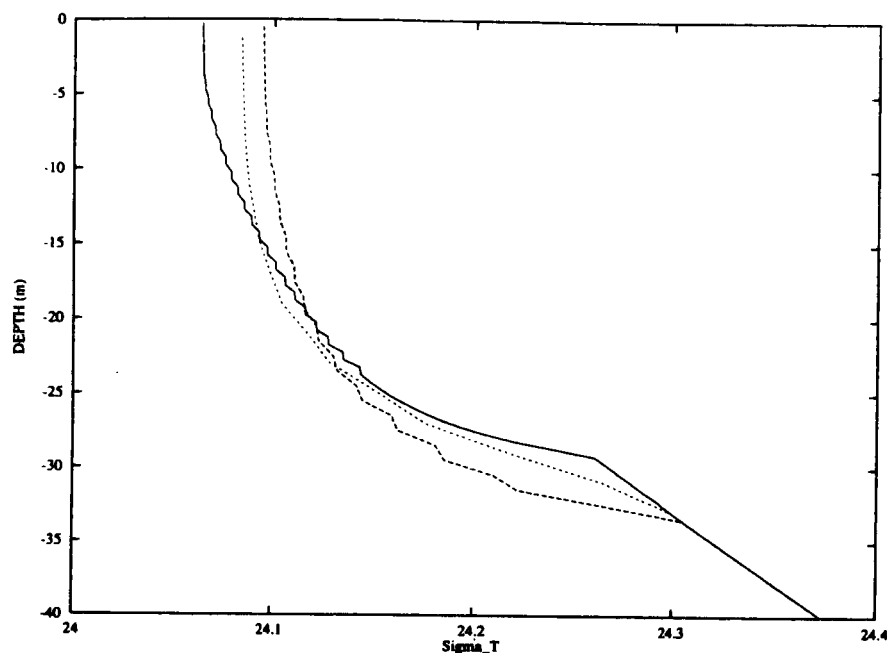


Figure 5. Profiles of σ_T using the KE0 model and $\Delta z = 0.5\text{m}$ (solid), 1m (large dashes) and 2m (small dashes) without rotation.

In that case an almost constant layer depth of 18-19m is reached after one inertial period in good agreement with the value of 17m derived from theoretical analysis [Pollard et al. 1973]. In the absence of rotation the curves grow like $t^{1/2}$ in agreement with the theory and experimental data [e.g. Kundu, 1981]. It can also be seen that lower layer depths are predicted by the KE2 model. The effect of rotation on the density and current profiles have also been examined using the KE2 model. Without rotation the curves of density and current smoothly join their interior values at the edge of the turbulent layer. Complete mixing only takes place within a surface layer, having an extent of $\sim 40\%$ of the turbulent layer depth. If rotation is present, turbulent mixing occurs throughout almost the whole layer, and produces a pycnocline at the layer bottom with sharp gradients in density and current. This is related to the previous observation that rotation inhibits entrainment so that the turbulence generated by the surface stress mixes the layer more efficiently. According to the theoretical analysis and numerical experiments of Kundu [1980, 1981] the solutions of the momentum and density equations without Coriolis acceleration can be cast, at least initially, into a self-similar form. This appeared to be valid until $t \approx 500N^{-1} \sim 14\text{hr}$ at which time sharpening occurs at the interface even in the absence of rotation. A self-similar structure was clearly visible in the calculations.

Fig. 5 represents the influence of the vertical grid spacing on the numerical solution for the KE0 model. When Δz is decreased, a "jiggling" pattern is seen in the middle of the layer where stratification starts to increase. They are also observed in the current profiles

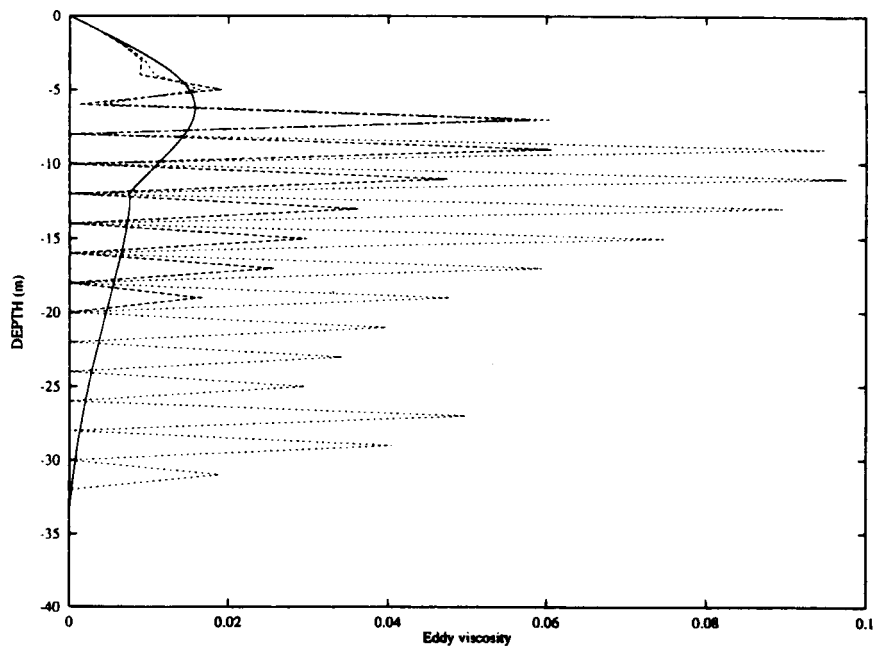


Figure 6. Profiles of eddy viscosity using the KE2 model at $t = 24\text{hr}$ (solid) and the KE0 model, $\Delta z = 0.5\text{m}$ at $t = 12\text{hr}$ (large dashes) and $t = 24\text{hr}$ (small dashes) with $\gamma = 0$.

(Luyten *et al.* 1994). A similar phenomenon occurs by increasing the time step. Its source can be inferred from Fig. 6 which shows the profiles of eddy viscosity at two times ($t = 12\text{hr}$, 24hr) for the KE0 model and at $t = 24\text{hr}$ for the KE2 model. A strong "jittering" is observed which starts to grow at the point where the viscosity starts to decrease, and which increases in time. A similar problem is reported by Frey [1991] using the Mellor-Yamada local equilibrium scheme. In both types of simulations [Frey, 1991; Luyten *et al.* 1994] it appeared that the "jittering" disappears by choosing Δz sufficiently large or Δt sufficiently small. Numerical experiments appear to support a numerical stability criterion of the form

$$\frac{\Delta t}{\Delta z^2} K_{M,\max} \leq \alpha_c \quad (27)$$

with $\alpha_c = 0.4$ or 2 depending on the type of equilibrium closure scheme. With this value one obtains a criterion close to the one valid for explicit algorithms.

Additional numerical calculations with the KE2 and a one equation model which use a transport equation for turbulence energy, showed that the results are practically insensitive to the choice of the vertical grid spacing and the time step, while no "jittering" is produced. This indicates that the problem is related to the neglect of the time derivative

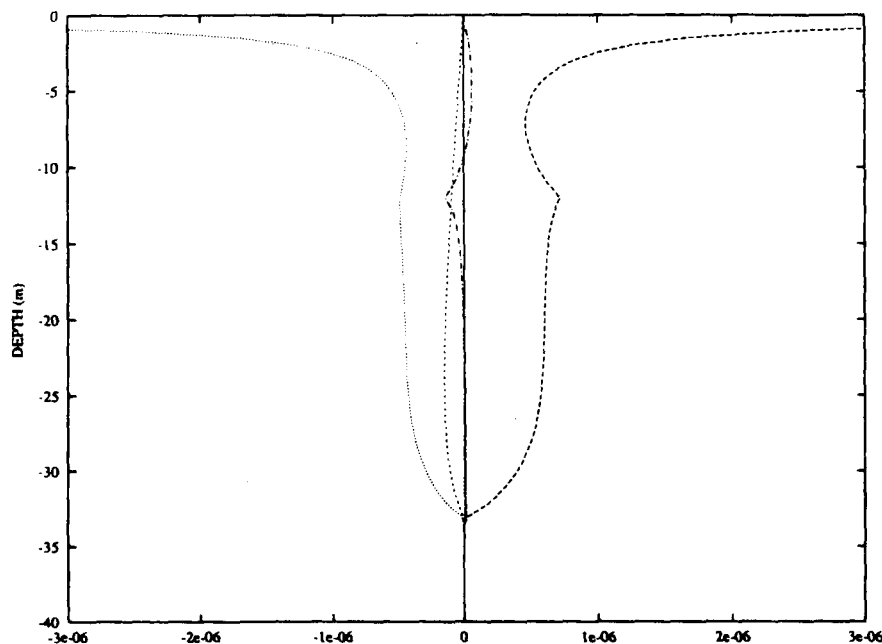


Figure 7. Kinetic energy budget using the KE2 model, $\gamma = 0$ and $\Delta z = 0.5\text{m}$ at $t = 24\text{h}$: shear production (dashes), buoyant production (small dashes), viscous dissipation (dots), diffusive transport (dash-dots) and time derivative (solid).

or diffusion term in the k -equation. The magnitude of the different terms in this equation are compared in Fig. 7. The balance is primarily achieved by the shear production and dissipation term and to a smaller extent by the buoyancy production term. The time derivative remains always negligibly small. This applies also for the diffusive transport except near the high shear zone close to the surface where a downward transfer of turbulence energy takes place from the region where shear production dominates dissipation to the layer below where $P < \epsilon$. The most probable cause of the instability must therefore be related to the absence of the diffusion term which, although small, may efficiently smooth out oscillations induced by the numerical scheme.

Concluding Remarks

In this paper we have given a brief review (relying on references to published papers for detail) of the range of turbulence energy models that have been used in oceanographic models. At present the most complex turbulence energy closure methods commonly used in shallow seas are the q^2 - $q^2\ell$ or k - ϵ models which involve two prognostic equations, one for turbulence energy, and a second for either the mixing length (q^2 - $q^2\ell$ model) or turbulence energy dissipation rate (k - ϵ). Since these models involve two prognostic

equations, both containing a number of non-linear terms, they are computationally quite expensive in full three dimensional calculations. Also since the vertical diffusion coefficients, in particular eddy viscosity, computed with these models can vary with space and time, giving rise when eddy viscosity is low to highly sheared regions which must be accurately resolved in the model, also means that a fine vertical grid is required. As we have shown with the simple point models the main problem with poor resolution in the vertical is that the high shear boundary layers are unresolved. This can give rise to spurious grid scale oscillations in the current, in the region of the boundary layer which can then act as an artificial shear production term for turbulence energy. Also if the high shear boundary layer is artificially smoothed due to poor grid resolution then the turbulence production term can be underestimated. A fine grid in the vertical is certainly necessary to ensure that the additional physics due to the use of higher order turbulence closure schemes is not spoiled by artificial smoothing or spurious grid scale oscillations due to coarse grids or poor numerics. Naturally fine grids and high order turbulence schemes will give rise to significant computational effort in a three dimensional calculation. As described in detail an alternative to two equations is to use a prognostic equation for turbulence, with the mixing length determined from the turbulence or specified in an algebraic manner. By this means the computational cost is reduced and for many problems such a system is sufficiently accurate.

An alternative formulation for eddy viscosity which does not involve any turbulence energy calculation as we have shown is to determine the viscosity from the flow field, and a characteristic length scale. This has been very successful in determining tidal current profiles in homogeneous situations where by choosing the length scale as the thickness of the bottom boundary layer, (limited to the water depth in shallow regions), results of comparable accuracy to those computed using turbulence closure models have been obtained at a fraction of the computational cost [Davies and Xing, 1994]. Also for tidal flows in homogeneous regions, comprehensive data sets exist which can be used to test the models and determine any differences between them. For stratified conditions, where the turbulence energy models may have advantages, or give significantly different results from simple flow dependent viscosity formulations modified by a Richardson number, comprehensive data sets do not exist to provide a rigorous test of the various models. In particular data sets which contain details of the high frequency component of the wind, and the internal wave fields which are important in determining mixing processes are difficult to obtain.

In theory by using a sophisticated turbulence energy model containing a full range of processes within a three dimensional hydrodynamic model, then the model should be able to accurately predict flow fields and mixing rates over a large range of different physical environments. The major difficulty is however the validity of the turbulence models (the majority of which are based on laboratory experiments) in large scale geophysical flows where for example the detailed nature of the sea bed is unknown; the influence of wind waves and wave breaking upon the surface mixing length requires investigation together with the role of internal waves and internal wave breaking. However despite a range of

physical problems in using turbulence energy models in shallow sea problems they should be fundamentally better than the use of a simple eddy viscosity, provided the equations are solved accurately.

Hopefully the review of the literature presented here should enable the reader who is new to the topic to glean an overview of the field, with the simple point calculations giving some insight into the physical and numerical problems, and provide test cases for code development, besides stimulating ideas in the scientific community as to how we rigorously validate this new generation of models.

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