CHAPTER 2:

THE ORIGINS OF NUMERICAL COGNITION

Introduction

As presented in the previous chapter, numerical processing involves the observation of strong recurrent behavioural effects. Moreover, the robustness of these effects led to the formulation of a strong metaphor in the literature on numerical cognition, according to which numbers are represented by numerical magnitudes on a mental number line. Recent theoretical proposals suggest that individuals' experience with numbers and numerosities has an impact on their numerical representations and abilities. But before the acquisition of experience with numbers (for example via the development of counting abilities), what are the hypotheses on how the semantic numerical representation emerges in the human brain? Two opposite perspectives have been proposed regarding the origins of numerical representations and skills. According to the nativist hypothesis, human infants, adults and animals are equipped with an innate "number-sense", which constitutes the foundations for higher numerical abilities in humans. In contrast, according to the empiricist hypothesis, infants are born with a "brain without numbers", but they have general non-numerical competences, such as a pre-attentive individuation process for keeping track of visible objects, which constitute the foundations for later arithmetical cognition development. Noteworthy, the empiricist hypothesis stresses the role of one sensory modality (i.e., vision) in the elaboration of the numerical representations and abilities.

In the present chapter, the two opposite hypotheses on the origins of the numerical representations and abilities will be reviewed. Furthermore, we will see that both perspectives show some limitations, and that the debate on the question of the innateness of a numerical sense remains clearly open. But more importantly, this debate puts light on the fact that neither the nativist nor the empiricist hypothesis can completely account for the development of the numerical system in humans and that both hypotheses should probably be considered. Indeed, even if a large set of data support the postulate of an innate "number-sense", it might be relevant, in the future, to take into consideration and to further investigate the role of the environment and of the input on the elaboration of numerical cognition.

1. The nativist hypothesis: the "number-sense"

Several authors in the literature on numerical cognition agree on the postulate that animals, young infants and adult humans possess a biologically determined, domain-specific representation of number, usually called the ""number-sense"" (Butterworth, 2005; Dehaene, 1997; Dehaene et al., 1998; Gallistel & Gelman, 1992, 2000). According to these authors, animals, human infants and adults are equipped with an innate ""number-sense"", an abstract and approximate sense of numbers that allows understanding, approximating and manipulating numerical quantities whatever their modality (e.g., vision, audition), format (e.g., dots, objects) or modes (sequential, simultaneous) of presentation. This innate ability to process numerical magnitudes is assumed to be implemented on cerebral networks, located in the inferior parietal cortex, that have evolved specifically for the purpose of representing basic arithmetic knowledge. Moreover the "number-sense" is suggested to constitute a foundation for the development of higher-level arithmetic in humans, to guide the acquisition of verbal counting and to give a framework for the development of arithmetic reasoning (Barth et al., 2006; Butterworth, 2005; Dehaene, 1997; Gallistel & Gelman, 1992, 2000).

According to Dehaene (1997), four lines of evidence support the postulate of the existence of the "number-sense": 1) the presence of evolutionary precursors of arithmetic in animals; 2) the early emergence of arithmetic competence in infants independently of other abilities such as language; 3) an homology between animals, human infants and adults abilities for number processing; 4) the existence of a dedicated cerebral substrate to process numerical magnitudes.

1.1. Numerical competence in animals

On the study of numerical cognition in animals, various animal species (rats, pigeons, raccoons, dolphins, monkeys, etc) demonstrate numerical discrimination abilities (see for review, Gallistel & Gelman, 2000) in the visual (e.g., Brannon & Terrace, 1998, 2000; Emmerton, Lohman, & Nieman, 1997) and in the auditory modality (e.g., Fernandes & Church, 1982; Hauser, Dehaene, Dehaene-Lambertz, & Patalano, 2002). Simple arithmetic competences, such as adding and subtracting, have also been found for example in rhesus monkeys when they have to observe or choose between arrays with small amount of food objects (Hauser & Carey, 2003). Moreover, in macaque monkeys, a neural correlate of Weber's law was observed and numerosity-selective neurons were identified by recording the neuronal activity in their prefrontal and parietal cortex, (Nieder & Miller, 2003, 2004a, 2004b). These numerosity-selective neurons were tuned (they showed maximum activity) to specific numbers of items presented on visual displays. The results also demonstrated that all numerosity-selective neurons formed a bank of overlapping numerosity filters. When macaques were trained in numerical match-to-sample tasks with sets of one to seven visual objects, preferred population of cells appears to be activated by each numerosity with tuning curves fitted by a logarithmic function. In consequence, the monkeys' neuronal tuning appears to obey Weber's law and then to account for the basic distance and size effects: the neurons become less precisely tuned as the preferred numerosity increases (size effect), the neural filter functions overlap heavily in the discrimination of adjacent numerosities (distance effect).

1.2. Numerical competence in infants

In the developmental literature, several studies using the habituation paradigm[1], demonstrated the existence of numerical discrimination abilities in preverbal infants on small numerosities (up to three or four) in the visual modality with stimuli presented in the

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[1] This procedure includes two experimental phases: the habituation and the test phases. During the habituation phase, a fixed numerosity (e.g., 2 dots on a visual array) is presented repeatedly to the infant until his/her interest (measured, for example, by his/her looking time or his/her suction rate) decreases for this numerosity, indicating that he/she is habituated to it. Then in the test phase, a new numerosity (e.g., 3 dots on a visual array) is presented to the infant. If the infant is able to discriminate between the two numerosities presented, then he/she should regain interest to the new numerosity, by showing for example increased looking time, indicating dishabituation.

simultaneous (Antell & Keating, 1983; Starkey & Cooper, 1980) and the sequential modes (puppet jumps) (Wynn, 1996), as well as in the auditory modality (Bijeljac-Babic, Bertoncini, & Mehler, 1993). There is also evidence for cross-modal numerosity matching competences in infants. Starkey, Spelke and Gelman (1983, 1990) demonstrated that 6-8-month-old infants are able to detect numerical correspondence between sets of entities presented in the visual (photographs of two or three objects) and the auditory (two or three drumbeats) modalities: infants looked longer at the visual display with a different number of objects than the number of drumbeats heard (but see Mix, Huttenlocher, & Levine, 1996; Moore, Benenson, Reznick, Peterson, & Kagan, 1987). More recently, using a cross-modal transfer task from touch to vision, Féron, Gentaz & Streri (2006) showed that 5-month-old infants are able to detect numerical correspondence between sets of objects presented in the tactile (two or three objects presented one by one in the infant's right hand) and visual modalities (visual displays with 2 or 3 objects presented either simultaneously or sequentially). By the introduction of the violation of expectation paradigm (see Figure 7), Wynn (1992) demonstrated that babies have simple arithmetic competence on small numerosities: they can anticipate the result of small additions and subtractions as they react to numerically impossible events such as 1 + 1 = 1 and 2 - 1 = 2. Wynn's (1992) findings were replicated and extended by Hobayashi and colleagues (2004), who demonstrated that 5-month-old infants recognize basic arithmetic operations across sensory-modalities: infants recognize that "1object + 1 tone = 2 objects" rather than 3, and that "1 object + 2 tones = 3 objects" rather than 2. These findings indicate that arithmetic capacities based on the processing of numerical quantities emerges early in humans. This conclusion was recently supported by Barth and colleagues' (2006) results in older children, showing that 5-year-old kids without training in arithmetic successfully performed approximate arithmetic on large sets of elements prior to mathematical instruction.

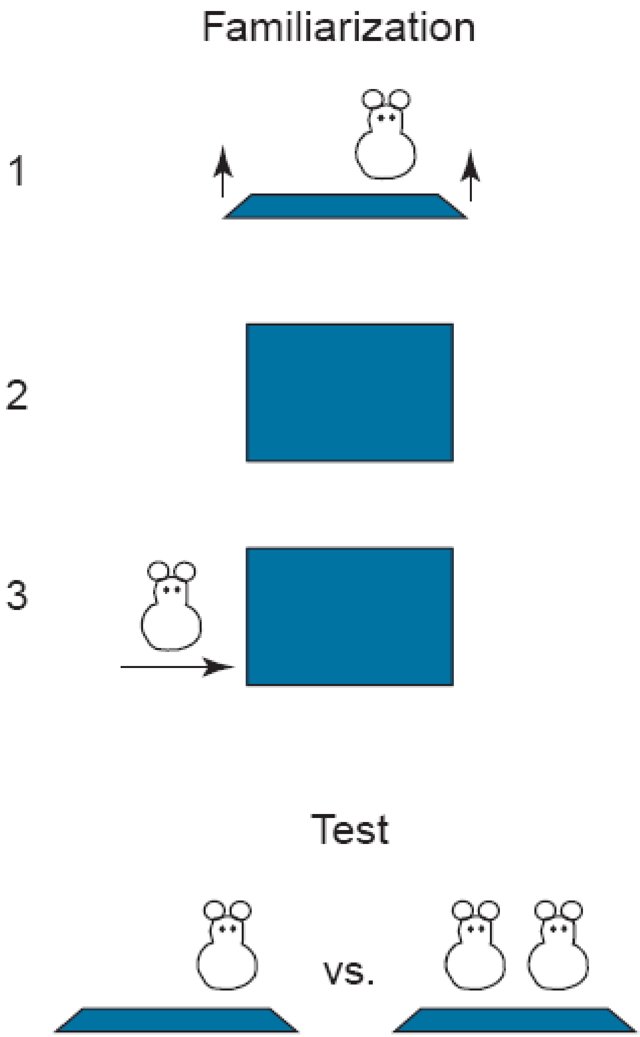

Figure 7: The violation of expectation paradigm: illustration of the "1+1" operation condition used in Wynn (1992). During the familiarization phase, infants see a doll placed in a display stage (1). Then a screen rotates upwards to temporarily hide the doll from view (2). A hand then enters the display stage with another identical doll, which is placed behind the screen along with the first doll (3). Then the hand exits the stage empty. In the test phase, the screen comes down to reveal either the possible and correct outcome (2 dolls) or an impossible and incorrect outcome (1 doll). The same experimental procedure is used in the "2-1" operation condition. Infants show significant increased looking times to the impossible and unexpected outcomes of the operations they witness (1 doll in the "1+1" operation condition, and 2 dolls in the "2-1" operation condition) *(reproduced from Feigenson, Dehaene, & Spelke, 2004)*.

1.3. Homology between animals, human infants and adults

An homology appears between animal and human number processing, as like humans, animals seem to represent numbers by analogue magnitudes obeying Weber's law: the greater the numerosities, the more imprecise their processing. Indeed, the distance (see Figure 8) and the size effects (see Figure 9) have been repeatedly observed in animals (e.g., Hauser, Tsao, Garcia, & Spelke, 2003; Meck & Church, 1983; Nieder & Miller, 2003, 2004a, 2004b; Platt & Johnson, 1971). For example in Brannon and Terrace's (2000) study, when rhesus macaques had to use number as a discriminative cue to obtain food rewards, they discriminated large differences (2 vs. 9) more readily than small differences (2 vs. 3; i.e., distance effect), and small numbers more readily than large numbers (2 vs. 3 as opposed to 8 vs. 9; i.e., size effect).

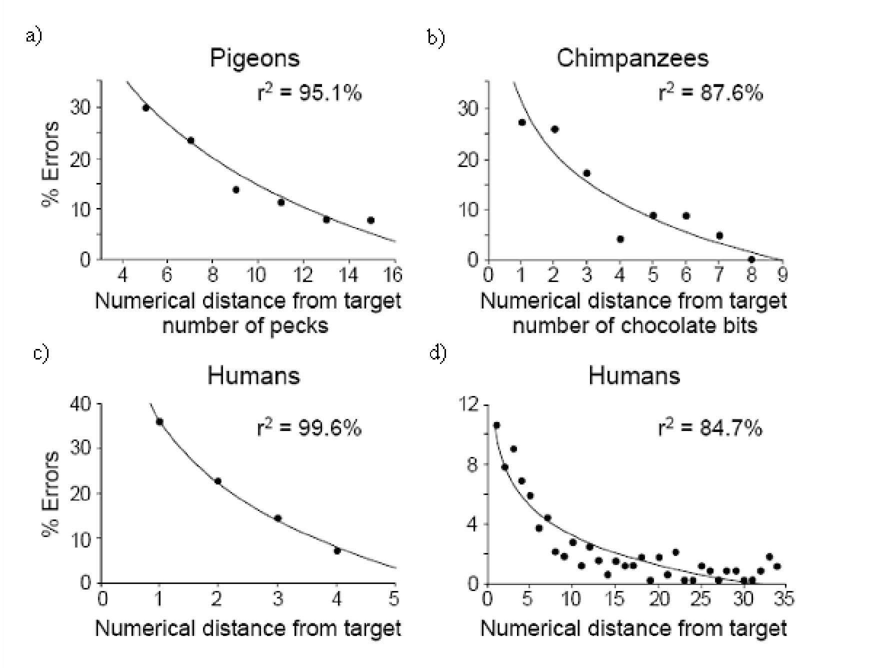

Figure 8: The distance effect in all species. Error rates decrease as an approximately logarithmic function of the numerical distance between the numbers to compare: a) in pigeons having to compare their numbers of pecks to a fixed standard of 50; b) in chimpanzees having to select the larger of 2 small numbers of chocolate bits; c) in humans having to decide whether a pattern contained exactly 12 dots or d) having to compare 2-digit Arabic numerals to the standard 65 *(reproduced from Dehaene et al., 1998)*.

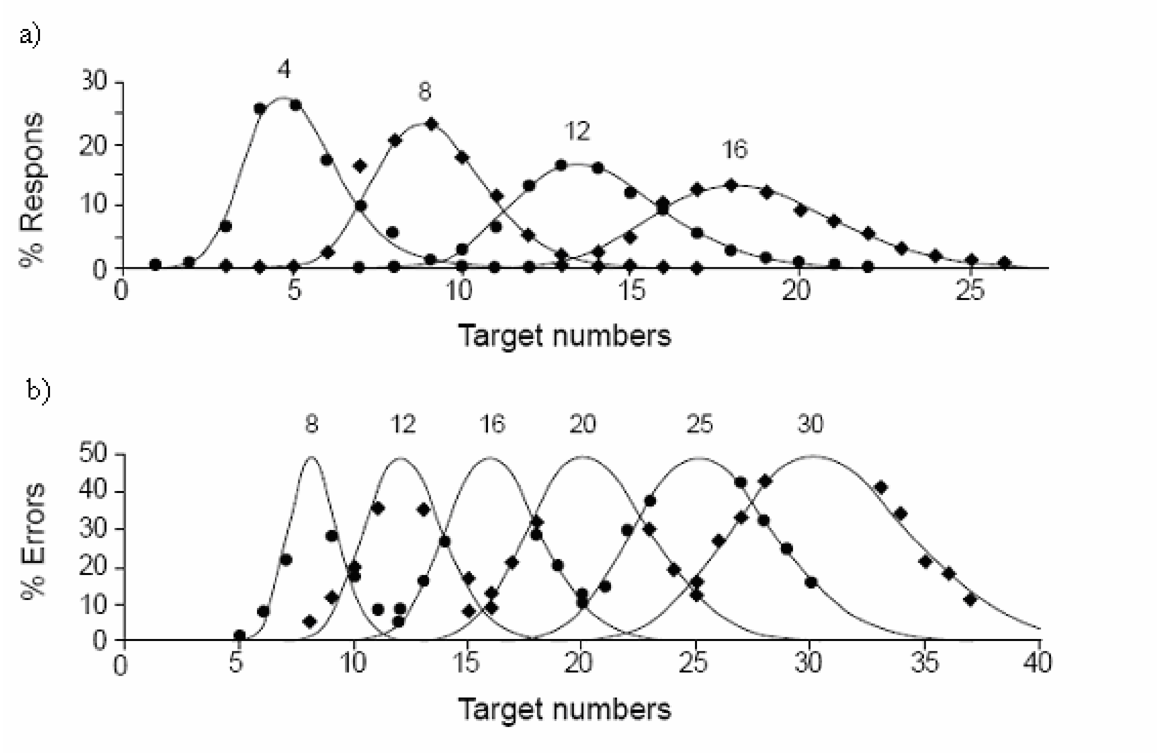

Figure 9: The size effect in all species. Numerical performance decreases as the numbers to process get larger, resulting in an increasing dispersion of responses (Weber's law): a) in rats having to compare the numbers of lever presses to a fixed standard of 4, 8, 12 or 16; b) in humans having to compare the numerosity of dot patterns to a fixed standard of 8, 12, 16, 20 or 30. In both cases, an increasing dispersion of the distribution of responses indicates the presence of a size effect (Weber's law) *(reproduced from Dehaene et al., 1998)*[2].

Homologies between animals' and humans' numerical processing were confirmed by psychophysics and neuro-imaging data. With the fMRI adaptation paradigm, Piazza and colleagues (2004) found a similar neural coding scheme in humans' numerical processing to the one observed in animals (Piazza, Izard, Pinel, Le Bihan, & Dehaene, 2004). The experimental paradigm of this study consisted firstly in the repeated presentation during an fMRI experimental block of a specific numerosity. The aim of this repeated presentation was to habituate (i.e., decrease of neuronal discharge) the putative population of numerosity-selective neurons in the brain that were tuned to this specific numerosity. Once

[2] Both graphs illustrating animals' and humans' size effect appear dissimilar visually, but this is only due to scaling difference related to divergence in tasks demands.

the numerosity-selective neuronal population habituated, the presentation of a deviant numerosity led to a recovery of fMRI responses, with an amount of activation inversely related to the distance between the "habituated" numerosity and the deviant numerosity (see Figure 10). Piazza et al.'s (2004) results indicated that both behavioural performance and brain activation followed Weber's law and that a selective response to numerosity change occurred in the intraparietal sulci. In consequence, these findings demonstrate the existence of a functional and an anatomical parallel between the human and monkey codes for number: 1) at a functional level, humans, like animals, encode approximate numbers on a compressed internal scale; 2) at an anatomical level, the intraparietal site coding for numerosity in humans is compatible with the one observed in macaque monkeys.

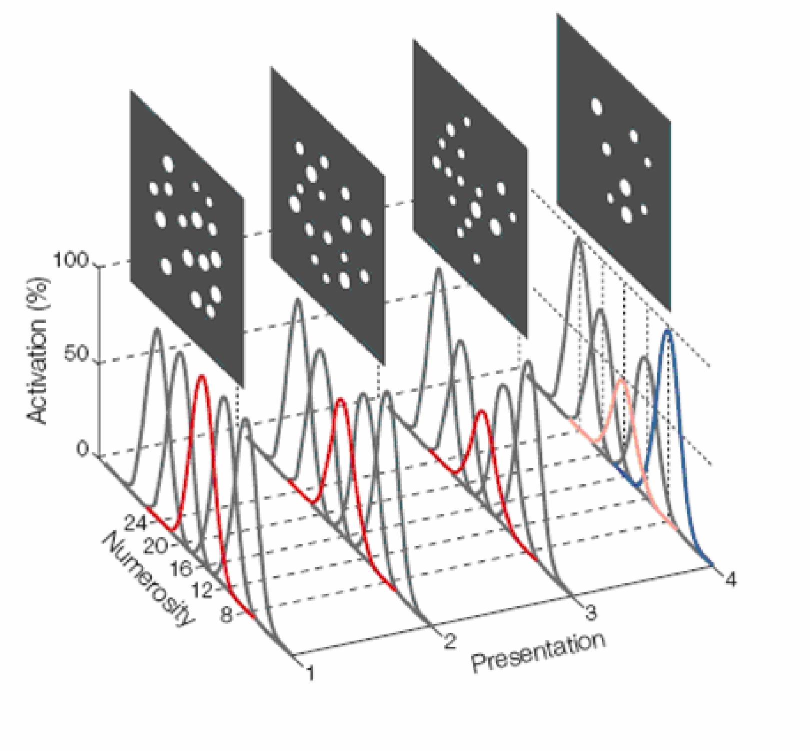

Figure 10: The fMRI adaptation paradigm: a fixed numerosity is presented repeatedly on visual displays, involving the population of numerosity-selective neurons in the brain that are tuned to this specific number of dots to become habituated (decreasing of its discharge). Then the presentation of a deviant numerosity is presented, resulting in a recovery of fMRI responses *(reproduced from Nieder, 2005)*.

In infants, recent studies showed that they present discrimination abilities on large numerosities, when they differ by a ratio ½ in the visual modality (Lipton & Spelke, 2003;

Xu, 2003; Xu & Spelke, 2000). This finding clearly illustrates that homologies exist between human adults, infants and children, as it demonstrates that their numerical magnitude processing also obeys Weber's law (Huntley-Fenner & Cannon, 2000; Temple & Posner, 1998).

1.4. Cerebral substrate of the "number-sense"

A large set of neuropsychological and brain-imaging data with human adults support the idea that the intraparietal cortex is crucial in numerical cognition. Therefore, according to several authors (Dehaene, 1997; Dehaene et al., 1998; Gallistel & Gelman, 1992, 2000), this cerebral region constitutes the cerebral basis for the "number-sense".

Neuropsychological data showed that patients with brain lesions of the inferior parietal cortex demonstrate selective impairment in the processing of numerical magnitudes (Cipolotti, Butterworth, & Denes, 1991; Dehaene & Cohen, 1997; Gerstmann, 1940). Moreover, a large set of brain-imaging studies gives further support to the important role of the intraparietal cortex in numerical processing by showing its systematic activation during numerical magnitude representation and manipulation (e.g., Dehaene, 1996; Dehaene, Spelke, Stanescu, et al., 1999; Fias et al., 2003; Piazza et al., 2004) (see Figure 11). The duration and the intensity of the signal in the inferior parietal region also appear to be sensitive to the numerical size of the target numbers to process and to the numerical distance between them, reflecting then the neuronal distance and size effects (Pinel, Dehaene, Rivière, & Le Bihan, 2001; Pinel, Le Clec'H, van de Moortele, et al., 1999). In addition, these last brain-imaging findings are reinforced by the observation in human adults, that their neural coding scheme in the parietal cortex obeys Weber's law (Piazza et al., 2004).

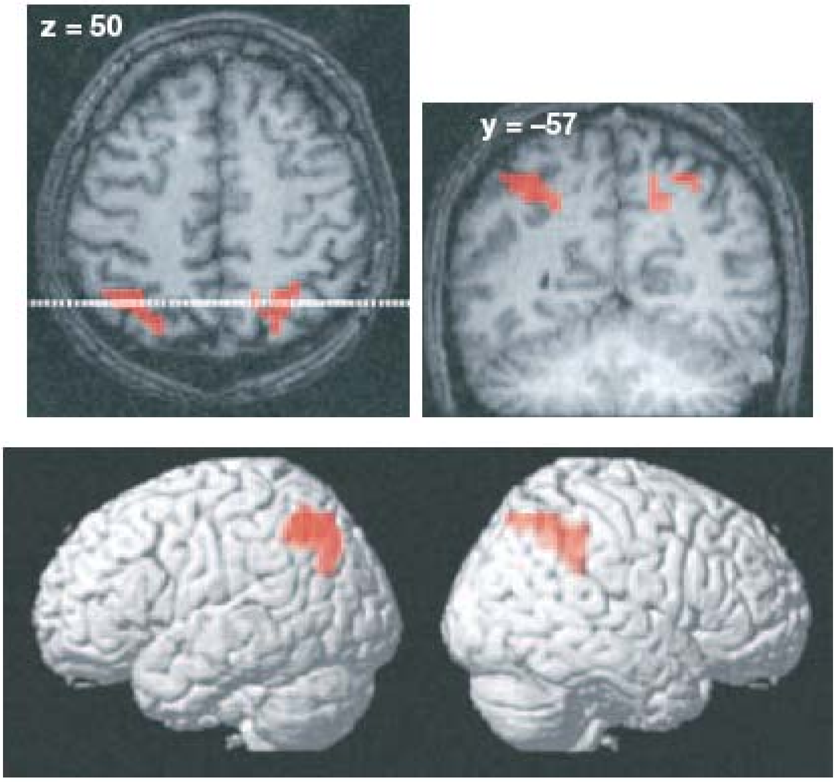

Figure 11: Involvement of the intraparietal cortex in numerical processing: the coloured areas in the axial (top left) and coronal (top right) sections and on the surface image (bottom) indicate the involvement of the intraparietal sulci activated when human brain responds to numerosity changes in Piazza et al. (2004). z and y values give ventral-dorsal and posterior-anterior Talairach coordinates *(reproduced from Nieder, 2005)*.

Taken together these data demonstrate that numerosities are processed by mental magnitudes obeying Weber's law in human adults, but also in both non-verbal animals and pre-verbal human infants. This reinforces the postulate of the existence of an innate "number-sense", as numerical processing appears to present phylogenetic and ontogenetic continuity (Butterworth, 2005; Dehaene et al., 1998; Dehaene, 1997, 2001; Gallistel & Gelman, 1992, 2000). This set of data also indicates that numerical processing and reasoning do not depend on language, as it has been postulated by some authors like Carey

(2001)[3] (Gelman & Gallistel, 2004; Gelman & Butterworth, 2005). Moreover, this last postulate was reinforced recently by the observation in some indigenous human cultures, that lack number words or have a restricted concept of verbal counting, of a non-verbal representations of numerical magnitudes obeying Weber's law, like in subjects with a fluent, well-developed verbal counting system (Gordon, 2004; Pica, Lemer, Izard, & Dehaene, 2004). In Gordon's (2004) study, individuals from a tribe in South America (i.e., the Piraha), who have a very simple number word system with only three words to designate "very small set sizes", "somewhat larger set sizes" and "sets of many items", were submitted to a matching task of the number of items placed before them with an equal number of objects. Their results demonstrate an imprecise capacity to enumerate the target numbers, with decreasing precision as these numbers became higher (Gordon, 2004). Similar findings were observed by Pica et al. (2004) with individuals from a tribe in Amazonia (i.e., the Munduruku), who have number words up to five. These participants still show ability to compare and approximately add large sets of items beyond their naming range (Pica et al., 2004). Therefore it seems that even without linguistic number concepts, humans possess a non-verbal quantification system that allows approximate numerical treatment (Gelman & Gallistel, 2004; Nieder, 2005), in the same way as non-human animals and pre-verbal infants do.

2. The empiricist hypothesis: the object-file model

According to the empiricist hypothesis, usually called the object-file model, non-numerical general competences, rather than innate numerical abilities, account for the early quantitative sensitivity observed in infants and constitute the foundations for later acquired numerical and arithmetical knowledge.

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[3] According to Carrey (2001), language is necessary for the development of numerical concepts. In other words, the ability to develop numerical concepts depends on our ability to use language. However, the large set of data in numerical cognition in infants, young children, animals and in Amazonian tribes shows that even if the language with a system of number words can be useful in learning to count, to calculate, in keeping track and communicating, it is not obligatory for the possession of numerical representations and skills (Gelman & Butterworth, 2005).

First of all, Trick and Pylyshyn (1994) postulated a theory of numerical enumeration based on vision to explain how preverbal infants, children and adults are able to discriminate between collections containing up to four items. This phenomenon of numerical discrimination is commonly named subitizing and refers to a rapid, effortless and accurate numerical enumeration process of small sets of entities. According to Trick and Pylyshyn (1994), the subitizing corresponds to a parallel pre-attentive visual process of spatial individuation, quite independent of any numerical knowledge. This individuation process allows keeping track of a number of visible objects by parallel assignment of reference token, called FINST (Fingers of Instantiation), to each distinct feature cluster in a visual scene. The FINSTs act as pointer variables, which individuate each item pointed and its spatial location. As the assignment of FINSTs occurs at a pre-attentive level, the number of FINSTs that can be allocated is limited up to four. Therefore, the subitizing process corresponds to a pre-numeric individuation process with up to four FINSTs attributed to each item presented in a visual display. When the visual set to enumerate exceeds 4 items, then numerical and attentional abilities are needed as counting or estimation are then necessary. In view of the infants' performance in numerical discrimination tasks on small numerosities with the habituation paradigm (e.g., Antell & Keating, 1983), Trick and Pylyshyn (1994) assumes that the subitizing is available early in life and constitutes the foundations for the development of later numerical skills. It may even be the part of enumeration process we share with animals.

Following Trick & Pylyshyn's (1994) suggestion that the subitizing is the result of an individuation rather than a numerical process, a non-numerical account was proposed for infants' behaviour on quantification tasks (Simon, 1997). According to this author, the results observed in infants on small numerosities (discrimination and arithmetical abilities) (Antell & Keating, 1983; Starkey & Cooper, 1980; Strauss & Curtis, 1981; Wynn, 1992) reflect a simple same/different discrimination rather than a numerical discrimination, thanks to several minimal general competences. These minimal non-specific competences allowing straightforward discrimination between small sets of entities consist in:

- memory and comparison of perceived entities: infants have the ability to remember items from a previously viewed collection and to compare these remembered items to those presented in a currently visible collection by the execution of a one to one mapping process;

- limited individuation and discrimination: infants have the ability to make discrimination between collections of up to four items thanks to a limited individuation process as the one described by Trick and Pylyshyn (1994);
- abstract encoding/representation: infants have the ability to encode and represent distinct entities at an abstract level, by prioritizing spatiotemporal over identity information;
- physical reasoning: infants have the ability to interpret object existence and movements in line with simple physical laws (e.g., one object cannot be at two different places at a same time).

Like Trick and Pylyshyn (1994), Simon (1997) postulates that these simple perceptual discrimination abilities account for performance reflecting a subitizing-like process in infants and constitute the foundations for more complex numerical representations and abilities acquired during later development. Moreover, these general competences seem to be used throughout the lifespan for the processing of small sets of entities, as the subitizing is still observed in adults.

From then on, Simon (1999) reinforced his position by assuming that infants' numerical relevant competences lie in their object representation and individuation abilities depending on the cerebral regions dedicated to visuo-spatial processing. In his theoretical enlargement, Simon (1999) put stress on the role of vision and the visual areas in the brain in the development of numerical representations and abilities. The non-numerical competences, and more specifically the ability to individuate up to four items which is postulated to constitute the foundation of mathematical thinking, rely on cerebral regions devoted to vision: the occipital cortex. Therefore, Simon (1999) assumes that the link from simple individuation to mathematical thinking, so the link from occipital to parietal cortex, might occur with the development of the counting skills: when they learn to count, children use their fingers; and by the use of this counting support, they might develop the association between numbers and the parietal lobes.

Therefore, according to the two empiricist theoretical proposals reviewed here, the role of vision in the elaboration of human numerical cognition is stressed by the description of the subitizing as a visual perceptual pre-attentive process, which provides the foundations for the development of numerical representations and abilities.

3. The nativist and empiricist hypotheses limitations

Two opposite perspectives have been made to account for the development of numerical cognition in humans. But which one could be considered as more relevant compared to the other? At first glance, the nativist hypothesis seems to be better supported than the empiricist hypothesis, as it is based on a large set of data and on different claims. But actually, neither the cognitive nativist nor the empiricist hypothesis appears to be completely viable, as both of them present some limitations which are presented here and which may suggest that both hypotheses should be taken into consideration.

3.1. The nativist hypothesis limitations

The nativist hypothesis is largely based on habituation and calculation studies having demonstrated that infants are sensitive to quantity (Antell & Keating, 1983; Starkey & Cooper, 1980; Starkey et al., 1990; Strauss & Curtis, 1981; Wynn, 1992; Xu, 2003; Xu & Spelke, 2000). For many years, it was assumed that this sensitivity is based on numerical discrimination abilities. However, according to several authors, none of these previous studies provide unequivocal evidence that infants' performance reflect the existence of numerical representations and abilities (Clearfield & Mix, 2001; Mix, Huttenlocher, & Levine, 2002; Rousselle, Palmers, & Noël, 2004). Indeed, the majority of the studies on numerical competences in infants lacked of control for perceptual cues that co-vary generally with numerosity (contour length, area, brightness, rhythmic pattern, etc.) and on which infants could have based their judgement to solve the tasks[4]. Moreover, when strong

[4] Xu and Spelke (2000) and Xu (2003) tried to control for perceptual continuous dimensions, such as total areas, but not sufficiently according to Mix et al. (2002) and Rousselle et al. (2004) (i.e., lack of control for contour length in Xu & Spelke (2000); area and contour length not controlled simultaneously in Xu (2003)).

control for perceptual variables is applied, infants dishabituated to a change in the continuous dimension, but not to a change in the number of items presented. For example, in their study, Clearfield and Mix (1999) applied a strict control for the contour of the visual items. Infants were habituated to slides of 2 objects. Then in the test phase, in one condition, the number of items presented differed from the number of items in the habituation phase, but not the amount of contour of the stimuli: a slide with 3 objects was presented, but with the same amount of contour as the slide of 2 objects in the habituation set. In this condition, the infants did not show dishabituation. In another condition, the amount of contour of the stimuli changed, but not the number of stimuli: 2 objects were presented but having more contours than the 2 objects in the habituation set (see Figure 12). In this condition, the infants showed dishabituation. The authors concluded that infants actually respond in numerical tasks on the amount of continuous variables, such as contour, rather than on the number of items presented. These findings have been replicated (Clearfield & Mix, 2001; Feigenson, Carey, & Spelke, 2002) and even in an explicit comparison task with 3-year-old children (Rousselle, Palmers, & Noel, 2004).

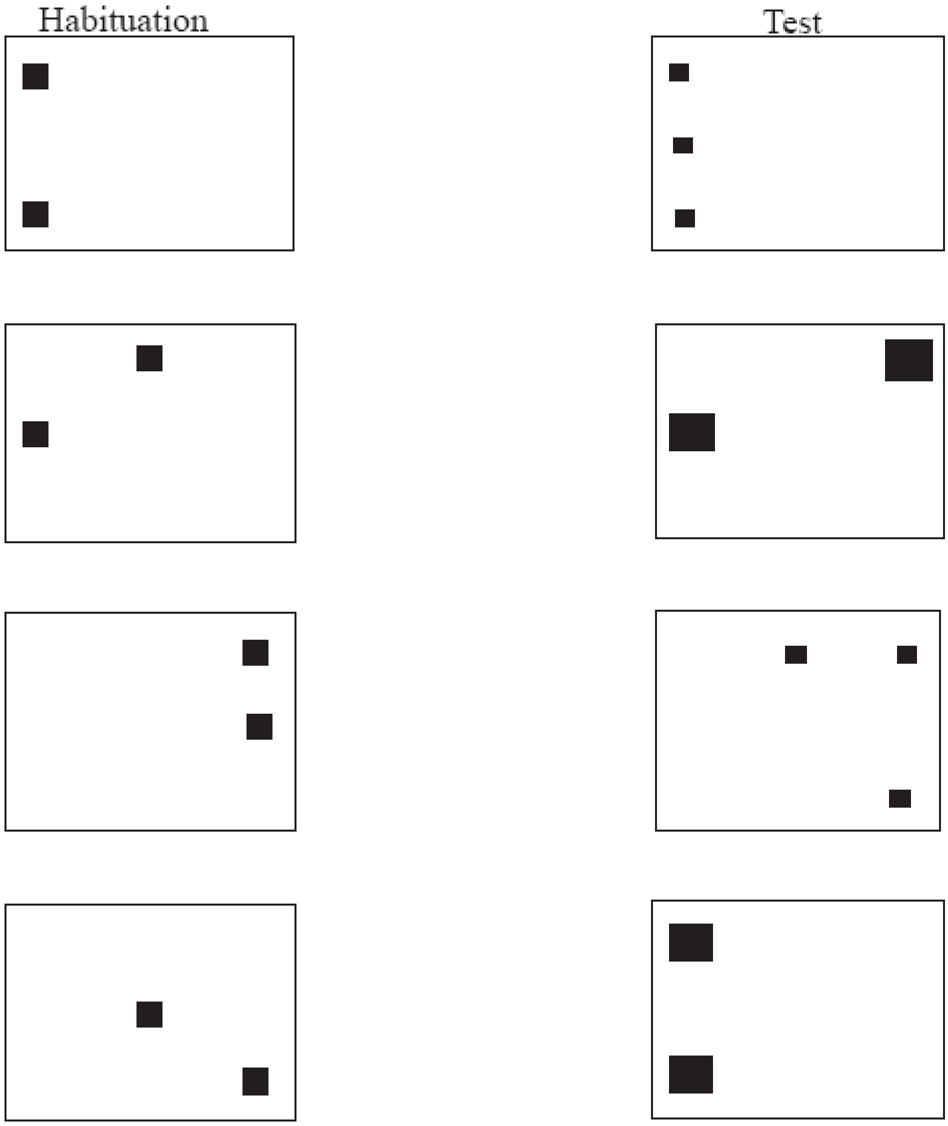

Figure 12: Examples of the stimuli used in Clearfield and Mix (1999). In the first and the third pairs of stimuli, the number of items presented in a set varies in the test phase compared to the habituation phase, while the contour length is constant. For these pairs of stimuli, the infants did not show dishabituation. In the second and the fourth pairs of stimuli, the contour length of the items presented in a set varies in the test phase compared to the habituation phase, while the number of items is constant. For these pairs of stimuli, the infants did show dishabituation.

Following these sets of results, another non-numerical empiricist hypothesis was postulated in the literature on the development of numerical representations and skills: the "undifferentiated amount hypothesis" (Mix et al., 2002). According to this hypothesis, infants' abilities are based on an analogue mechanism that does not code numerosity per se, but all types of continuous perceptual variables: for simultaneous visual sets of entities, features such as contour length, brightness and density; for sequential visual or auditory sets, features such as rhythmic pattern, rate, duration, etc. In other words, infants perceive quantities in terms of amount rather than number. This quantification process is assumed to be initially non-specific, undifferentiated and inherently approximate. In consequence, the ability to represent spatial extent and temporal cues in infancy would be sufficient to account for the current findings in the developmental literature (Mix et al., 2002; Rousselle et al., 2004). Only with the development of a minimal cardinal knowledge and mastery of counting, children will acquire pure numerical representations and abilities[5].

Taken together these data and new considerations on infants' numerical abilities raise questions on the postulate of an innate "number-sense" in humans, with ontogenetic but also phylogenetic origins (as the undifferentiated amount hypothesis could also be relevant for animals).

3.2. The empiricist hypothesis limitations

According to the object-file model (Simon, 1997, 1999; Trick & Pylyshyn, 1994), infants' performance merely reflect the capacity to keep track of up to 4 visual objects through representation of object files (i.e., subitizing), rather than a genuine representation of numerosity. However, this model encounters some difficulty to account for infants' performance on sets of events (Wynn, 1996) and in other modalities, such as the auditory modality (Bijeljac-Babic et al., 1993). Trick and Pylyshyn (1994) recognized these limitations of the FINST hypothesis. And Simon (1997) pointed out that, even if the non-

[5] According to Butterworth (2005) and Féron et al. (2006), studies with control of perceptual variables showing that infants show perceptual discrimination, but no numerical discrimination abilities, only indicate that the results observed in many previous experiments in which there was a confound between the numerosity and the perceptual variables of the stimuli were probably accounted by sensitivity to continuous variables. But these studies do not give evidence that infants are not able to form numerical representations. Therefore, there is no reason to exclude the idea that infants have brain systems for processing both types of information, which normally correlate in the environment.

numerical hypothesis had been developed to account for infants' performance on sets of objects in the visual modality, it could be extended to the sequential mode and other sensory modalities, by postulating, for example, that the general abilities to remember and to compare memory units in a one-to-one comparison process enable infants to determine the correspondences between collections encoded from different modalities (Starkey et al., 1990) and to discriminate the numerosity of events (Wynn, 1996). Nevertheless, in the enlargement of his theory, Simon (1999) definitely focused on the importance of vision and the visual cerebral areas in the development of numerical representations and abilities. However, according to Von Glasersfeld (1982), there is no reason to believe that the subitizing is restricted to the labelling of items presented simultaneously in the visual modality, as it could also be applied to sequential stimuli of any modality by the recognition of rhythmic patterns. Moreover, recently, Riggs and colleagues (2006) demonstrated that the subitizing can also occur in the tactile modality. These authors used in their study an apparatus to simultaneously stimulate the fingertips of both hands. Participants were asked to name the number of stimulated fingertips as quickly and as accurately as possible. A subitizing effect was clearly observed, as the average naming time for correct response was quicker for 1 to 3 fingers than for 4 to 6 fingers (270 ms/item vs. 627 ms/item). Therefore, these findings show that the subitizing is not limited to visual perception but extends to tactile perception, leading the authors to postulate that the subitizing may occur as the result of a general perceptual mechanism for individuating a small number of objects, rather than a visual mechanism.

In addition, the empiricist hypothesis cannot account for infants' abilities to discriminate large sets of entities (e.g., Xu & Spelke, 2000), unless it assumes the existence of a second number-specific mechanism as postulated by Feigenson et al. (2002). Actually, Feigenson et al. (2002) tried to gather in their proposal the empiricist and the nativist points of view by postulating that infants rely on multiple mechanisms to discriminate between sets of entities: 1) when the number of items to discriminate is small (up to four) and composed of visual objects, infants use a non-numerical individuation process (e.g., Simon, 1997, 1999); 2) when the number of items to process exceeds the limits of the parallel individuation process, or when the stimuli are not objects (sounds, events, etc), infants use a analogue-magnitude mechanism (e.g., Gallistel & Gelman, 1992).

Moreover, the object-file model shows also limitations if we consider animal data in the literature. On several occasions, animals demonstrated basic numerical competences, presenting a homology with human numerical abilities (i.e., obedience to Weber's law). These animal data cannot be accounted by the object-file model, as according to Simon (1999), a genuine numerical representation is not available to non-linguistic organisms (i.e., animals and infants) and the elaboration of numerical representations and skills only results from a long and slow process of learning and education.

Conclusion

To sum up, two opposite hypotheses have been made on the emergence and the development of numerical cognition.

Firstly, the nativist hypothesis assumes that we are equipped with an innate "number-sense" implemented on dedicated cerebral networks, and constituting the foundation for higher-level arithmetical abilities in humans. This postulate is based on a large set of data in animals, infants, neuropsychology and neuro-imaging. However, recently, some researchers questioned the number-specific nature of the quantification process observed in infants by showing that when continuous perceptual variables are controlled, infants do not demonstrate any more numerical discrimination abilities. To date, the debate regarding the nature of the quantification process observed in infants is still open. Nevertheless, the recent studies supporting the "undifferentiated amount hypothesis" indicate that the nativist hypothesis, which appeared evident at first glance, needs perhaps to be reconsidered and nuanced.

Secondly, the empiricist hypothesis assumes that numerical cognition foundations lie on general non-numerical competences, and particularly on a pre-attentive individuation process, usually known as subitizing, that allows keeping track of visual objects. This process would account for the infants' ability to discriminate between sets of up to 3 or 4 items. An important particularity of this theory is that it assumes importance to the role of vision in the elaboration of numerical representations and skills. But, this hypothesis also appears to present several limitations. Indeed, it cannot account for several data indicating infants' ability to discriminate large sets of entities or stimuli presented in other modality than vision. Moreover, it has been recently demonstrated that the ability to individuate a small number of objects is not restricted to the visual modality, but extends to the tactile modality.

To our point of view, both hypotheses have to be taken into consideration in order to account for humans' numerical cognition. The great set of data collected in animals, infants, neuropsychology and neuro-imaging support the assumption that our numerical representations and abilities have phylogenetic and ontogenetic origins. However, the empiricist hypothesis should also be considered as it is the first one to put light on the role of the environment, and especially the role of one specific input modality (i.e., vision), in the elaboration of the numerical system. As several authors recognized, environmental input is vital to humans' brain development and behaviour that rely obviously on sensory experience (Bavelier & Neville, 2002; De Volder, Toyama, Kimura, et al., 2001; Kujala, Alho, & Näätänen, 2000; Newcombe, 2002; Renier, Laloyaux, Collignon, et al., 2005). Furthermore, we have seen that recent theoretical proposals (e.g., Siegler & Opfer, 2003; Verguts et al., 2005) suggest that experience with numbers could have an impact on our numerical abilities. Now, the experience individuals have developed with numbers is necessarily related to the input modalities from which numbers are accessed. Currently, the role of the input modalities, and more particularly the role of vision, has been examined in the development of spatial cognition (see Collignon, Renier, Bruyer, et al., 2006; Thinus-Blanc & Gaunet, 1997), as the visual modality was also considered to be critical in the emergence of the spatial representations and abilities (e.g., Von Senden, 1960). Moreover, a strong link exists between numbers and space, notably as the semantic numerical representation itself has been spatially defined as a mental number line oriented from left to right. Therefore, the question remains: what is the real role of the input modality, and more precisely of vision, in the elaboration of human numerical representations and skills?