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**CORE**

DISCUSSION PAPER

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**Voting on pensions: sex and marriage**

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**Abstract**

Existing political economy models of pensions focus on age and productivity. In this paper we incorporate two additional individual characteristics: sex and marital status. We ignore the role of age, by assuming that people vote at the start of their life, and characterize the preferred rate of taxation that finances a Beveridgean pension scheme when individuals differ in wage, sex and marital status. We allow for two types of couples: one-breadwinner and two-breadwinner couples. Marriage pools both wage and longevity differences between men and women. Hence singles tend to have more extreme preferred tax rates than couples. We show that the majority voting outcome depends on the relative number of one-breadwinner couples and on the size of derived pension rights.

**Keywords:** social security, differential longevity, majority voting, individualization of pension rights.

**JEL Classification:** D72, D78, H55

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# 1 Introduction

Most pension schemes provide benefits that are longevity-invariant and sometimes contribution-invariant. Given that men have a shorter life expectancy than women and earn, and thus contribute, more, it is clear that such pension systems are to their detriment. So why do men consistently agree with pension schemes that penalize them? The first answer one can offer is that women outnumber men and can impose their views. Another possible reason is that, with flat-rate benefits, low-income men may support such schemes if earnings differences dominate longevity differences. Yet the best alternative explanation might be that, in a society where a majority of men and women are married, longevity and earnings differences are pooled within the couple and this makes any sex war irrelevant.

Women live longer than men and they earn less than men on average. For instance, in France, it is estimated that women life expectancy at 60 is 20% higher than that of a man and that the pay gap is around 20%. At the same time, there is evidence that low-income people, men and women, have lower longevity than high-income people. This has led to studies that show that social security schemes that look redistributive, but provide longevity-invariant benefits, are in fact not so redistributive (see e.g. Coronado et al., 2000, Liebman, 2001, and Bommier et al., 2006). For example, Bommier et al. (2006) estimated that the redistribution in the French public pension system is reduced by up to 50% because it is longevity-invariant.

Social Security redistribution by marital status is also surprisingly large. For instance, Galasso (2002) showed that one-earner couples get the highest internal return from the Social Security, followed by two-earner couples with 70/30 earnings split; returns are equal for two-earner couples with a 50/50 earnings split and single women, while single men are the most disadvantaged. The difference in returns observed between singles and married couples, either one-earner or two-earner, can be explained by the so-called “derived pension rights”. Several countries, like France, provide the surviving spouse (more often the woman) with a survivor benefit, while some other countries provide one-earner couples with a higher replacement rate than the one applied to single men; some countries, like Belgium or Japan, provide both types of derived benefits.<sup>1</sup> The marital status and the generosity of the system towards the non-working spouse is then likely to play an important role in the support for a pension system.

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<sup>1</sup>For example, in Belgium, the supplementary pension is evaluated to 1/4 of the working spouse pension. As shown in Gruber and Wise (1999), derived pension rights may take very different forms depending on the country.

A number of political economy papers have attempted to explain existing pension systems using majority voting models.<sup>2</sup> In his seminal contribution, Browning (1975) focused on age differences and showed that, if the old favour generous pensions and the young prefer private savings, the decisive voter is the median age one. More recent models include wage differences alongside age differences. In such a framework, Casamatta et al. (2000) show that the pension system is chosen by a majority made of rich and poor workers who collude against a coalition of retirees and middle class workers: this is the so-called *ends against the middle* outcome.

In this paper we concentrate on two additional individual characteristics: sex and marital status. We characterize the pension scheme that is chosen by majority voting in a society where men live shorter and earn more than women, and most men and women are married. Assuming that retirement consumption is financed by the returns of private savings and a Beveridgean pension benefit, we explore several issues: i) the effect of longevity and wage gender gaps on the chosen tax rate, ii) the effect of an increase in the number of married couples on the size of the pension system, iii) the effect of an increase in the relative number of one-breadwinner (versus two-breadwinner) couples on the pension system, and iv) the effect of the individualization of pension rights (equivalently, the reduction of derived pension rights) on the size of the chosen tax rate.

These issues are certainly relevant for prevailing pension systems and, surprisingly, have hardly been addressed in the literature.<sup>3</sup> In particular, the generosity of the system towards non-working spouses may play an important role in the political support of one-breadwinner couples towards existing pension systems.<sup>4</sup> This is a timely topic to address. Indeed, women are increasingly participating in the labour force and pension systems are increasingly individualized. For instance, more and more countries are abandoning the “derived pension rights model” and adopting instead the so-called “adult worker model”. For example, Denmark has suppressed survivor benefits and Germany has moved towards a “family splitting” system, which also provides a compensation for interrupted careers (for example, a pension credit per child).<sup>5</sup> One can thus expect that this dual evolution (i.e. increased labour participation by women and individualization of pension rights) will have some incidence on the size of the pension system.

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<sup>2</sup>For good surveys, see Galasso and Profeta (2002) and de Walque (2005).

<sup>3</sup>See however Borck (2007) and Leroux (2008) who have introduced longevity differentials in political economy models of social security.

<sup>4</sup>For a good survey on the role of derived pension rights on old-age income security of women in OECD countries, see Choi (2006).

<sup>5</sup>On this, see Choi (2006), Veil (2007) and Bonnet and Geraci (2009).

The setting we adopt is standard. People live for two periods, the second one being of variable length. They work in the first period and retire in the second one. The retirement consumption depends on the amount of private savings but also on the pension benefit, which is chosen through voting. To keep the analysis tractable, we make a number of simplifying assumptions like, for instance, a quasi-linear utility function, no liquidity constraints and certain length of life. Individuals vote at the beginning of their life. All men have the same longevity, which is lower than that of women. Men and women have the same productivity, but the wage of women is only a fixed fraction of that of men. Later in the paper, we discuss the implications of assuming a continuous productivity distribution instead. We also assume positive assortative mating (i.e. men marry women who have the same underlying productivity but earn a given fraction of their wage).<sup>6</sup> Finally, the pension system is Beveridgean so that pension benefits and payroll tax rates are uniform.

In this framework, lower productivity and higher longevity individuals benefit from the existence of a pension scheme. Thus, single women, who have lower wages and longer lives, will be in favour of a pension scheme while single men, who have higher wages and shorter lives, will be against it. We then explore the role of couples. If the couple comprises two breadwinners, gender differences in wages and longevity are neutralized so that the couple gets a zero net benefit from the pension system. In this case, they are indifferent between public pensions and private savings as a mean of smoothing consumption between periods. However, because labour supply is endogenous in our setting, a pension system creates labour supply distortions so that they end up preferring a zero tax rate. On the contrary, one-breadwinner couples do not neutralize gender differences and they will be in favour of a generous pension system when derived pension rights are important and sufficient to outweigh the husband's net contribution to the pension system. Thus, the support for a pension system will depend both on the number of one-breadwinner couples and on the generosity of the pension system towards them. We further extend our model to allow for a continuous productivity distribution. Our results are robust to this new specification. The only difference is that now, for each type of household, some individuals (the ones at the bottom of the productivity distribution) are always in favour of a pension system due to the amount of income redistribution they obtain.

Finally, our model predicts that the recent trend towards the individualization of pension

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<sup>6</sup>The papers of Mare (1991), Pencavel (1998) and Qian (1998) find strong evidence of positive assortative mating with respect to education. Education can be regarded as a good proxy for income.

rights should lead to reduced payroll tax levels. On the contrary, our model does not give clear conclusions regarding the effect of an increase in the number of two-breadwinner couples, which may imply an increase or decrease in the preferred tax rate depending on the generosity of the system.

The rest of the paper is organized as follows. Section 2 presents a standard political economy model where individuals differ in wages and longevity. Section 3 introduces gender and marriage. In Section 4 we allow for productivity differences and in the last Section we discuss the assumptions made in our model and some possible extensions.

## 2 The basic model

We assume that individuals live for two periods. They work in the first period and retire in the second one. Each individual of type  $i$  is characterized by a pair  $(w_i, \pi_i)$ , where  $w_i$  is the labour productivity in the first period and  $\pi_i$  is the length of the second period of life.<sup>7</sup>

The intertemporal utility function of any individual of type  $i$  is quasi-linear (linear in the first-period consumption) and is represented by

$$u_i(c_i, d_i, l_i) = c_i - v(l_i) + \pi_i u(d_i),$$

where  $c_i$  and  $d_i$  denote the first- and second-period consumptions, respectively, and  $l_i$  is labour supply. Second-period utility function  $u(\cdot)$  is such that  $u'(\cdot) > 0$  and  $u''(\cdot) < 0$ . For simplicity, we assume that the disutility of labour  $v(l_i)$  is quadratic and equal to  $l_i^2/2$ . Individuals work, contribute to the pension system, consume and save in the first period. In the second period, they retire and receive a pension benefit  $p$ . We also assume a perfect annuity market and a zero interest rate so that the return on savings is simply  $1/\pi_i$ . First- and second-period consumptions can then be written as

$$\begin{aligned} c_i &= (1 - \tau) w_i l_i - s_i, \\ d_i &= \frac{s_i}{\pi_i} + p, \end{aligned}$$

where  $\tau \in [0, 1]$  is the payroll tax rate and  $s_i$  is the amount of savings.

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<sup>7</sup>It would be possible, but more complicated, to allow for uncertain mortality,  $\pi_i$  being then the probability of surviving the second period. We believe that it would not modify substantially our conclusions.

Throughout the paper, we assume away liquidity constraints so that  $s_i$  can be positive as well as negative. The problem of type  $i$ 's individual consists in solving

$$\begin{aligned} \max_{l_i, c_i} \quad & c_i - l_i^2/2 + \pi_i u(d_i) \\ \text{s.t.} \quad & \begin{cases} c_i = (1 - \tau) w_i l_i - s_i \\ d_i = \frac{s_i}{\pi_i} + p \end{cases} \end{aligned}$$

From the first-order conditions, we obtain:

$$\begin{aligned} l_i^* &= (1 - \tau) w_i, \\ u'(d_i^*) &= 1. \end{aligned}$$

As to the pension benefit, we assume that individuals contribute to the pension system during the first period of their life and receive a flat pension benefit in the second period (i.e. the retirement period).<sup>8</sup> Thus a feasible pension system must satisfy the budget constraint

$$p \sum n_i \pi_i \leq \sum \tau n_i w_i l_i^*,$$

where  $n_i$  denotes the relative number of individuals of type  $i$ . Note that here  $p$  is an annual pension benefit, which implies that a person that lives longer gets more in total than a person that has a shorter life. Under the assumption of perfect budget balance, the expression for the pension benefit is

$$p(\tau) = \tau \frac{(1 - \tau) E[w^2]}{\bar{\pi}}, \quad (1)$$

where  $E[w^2]$  is the average square productivity.<sup>9</sup> Every individual contributes an amount that is proportional to his labour income and receives a uniform pension benefit during a retirement period of unequal length  $\pi_i$ . Such a pension system redistributes resources from high-productivity to low-productivity individuals and from short-lived to long-lived individuals.<sup>10</sup>

The indirect utility function of an individual of type  $i$  is then

$$V^i(\tau) = \frac{(1 - \tau)^2 w_i^2}{2} - s_i^* + \pi_i u\left(\frac{s_i^*}{\pi_i} + p(\tau)\right), \quad (2)$$

where the star stands for the optimal level. The preferred tax rate of this individual is obtained by solving the following program:

$$\max_{\tau \in [0,1]} V^i(\tau).$$

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<sup>8</sup> Assuming a pension benefit that would be partially contributive would not change the nature of the results but would complicate the analysis.

<sup>9</sup> Note that here, the terms “wage” and “productivity” are used indifferently.

<sup>10</sup> Most PAYG pension schemes exhibit such features. On this topic see, for example, Coronado et al. (2000), Liebman (2001) and Bommier et al. (2006).

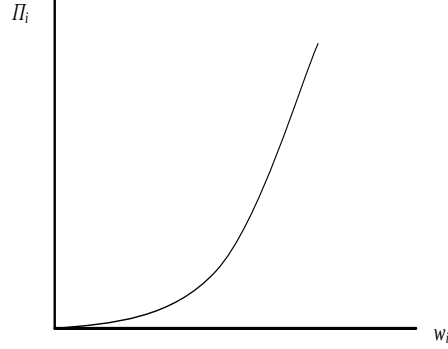


Figure 1: Separating locus of types for and against a positive payroll tax

In Appendix A we show that the solution to this problem is

$$\tau_i^* = \begin{cases} 0 & \text{for } \frac{w_i^2}{E[w^2]} \geq \frac{\pi_i}{\bar{\pi}}, \\ \frac{\frac{\pi_i}{\bar{\pi}} E[w^2] - w_i^2}{\frac{2\pi_i}{\bar{\pi}} E[w^2] - w_i^2} & \text{otherwise.} \end{cases} \quad (3)$$

The preferred tax rate of an individual depends on the level of redistribution he expects to get from the pension system. He may benefit from the pension system either because of a longer life and / or of a lower productivity than the average. Hence, the preferred tax rate of any individual will be zero if he has characteristics such that  $w_i^2/\pi_i \geq E[w^2]/\bar{\pi}$ . It is clear that the lower the wage rate and the higher the longevity the more likely an individual will be in favour of the pension scheme. The equality  $w_i^2/\pi_i = E[w^2]/\bar{\pi}$  gives the separating locus of types for and against a positive payroll tax and, thus, a public pension scheme. In Figure 1, we represent this function in the plane  $(w_i, \pi_i)$ .

To the left of the curve, the  $(w_i, \pi_i)$ -types are in favour of a positive tax; to the right, they are against. It is also worth noticing that, when positive, the most preferred tax rate decreases with  $w_i$  and increases with  $\pi_i$ .

For someone with a zero wage, the most preferred tax rate is equal to 1/2 and not 1. This is due to the efficiency cost of taxation: 1/2 is the tax that provides the maximum revenue (i.e. the peak of the Laffer curve).

Majority voting raises some technical problems when there are two characteristics. This will be addressed below by assuming a particular relationship between the characteristics  $w_i$  and  $\pi_i$ . For the time being, we assume that all individuals have the same longevity  $\pi_i = \bar{\pi}$  and that the wage rate has a standard density function with median wage below average wage:  $\bar{w} \geq w_m$ . We know from Jensen inequality that  $\sqrt{E[w^2]} > \bar{\pi}$ . Given that in the relevant range of  $w$ , the most preferred tax rate decreases with  $w$ , the Condorcet winner is the tax rate preferred by the individuals with median wage.

### 3 Model with unique productivity level

We now assume that individuals differ in gender. We consider first a society consisting only of singles, and introduce later the possibility of marriage. We also allow couples to comprise either one breadwinner or two breadwinners.

We assume that there is a mass 1 of men as well as of women. These are characterized by a pair  $(\pi, w)$  for men and a pair  $(\pi_f, w_f)$  for women such that

$$\begin{aligned}\pi_f &= \beta\pi, \\ w_f &= \alpha w,\end{aligned}$$

with  $\alpha \leq 1$  and  $\beta \geq 1$ .<sup>11</sup> In other words, we posit that women have a longer life than men but also obtain a lower wage. Note that, in this section, we assume a unique productivity level, which implies a wage  $w$  for men and a different lower wage  $\alpha w$  for women.<sup>12</sup> In this case, the pension benefit (1) is now equal to

$$p(\tau) = \tau(1 - \tau) \frac{(1 + \alpha^2) w^2}{(1 + \beta) \pi}. \quad (4)$$

with  $E[w^2] = (1 + \alpha^2) w^2/2$  and  $\bar{\pi} = (1 + \beta) \pi/2$ .<sup>13</sup>

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<sup>11</sup>For simplicity, we restrict attention to the most realistic case where  $\alpha \leq 1$  and  $\beta \geq 1$  but the analysis could be extended to  $\alpha > 1$  and  $\beta < 1$ . We do not explore in this paper why women have a lower wage. We simply account for the empirical fact that women, on average, have a lower wage than men.

<sup>12</sup>In Section 4, we relax this assumption to allow for a continuous productivity distribution.

<sup>13</sup>Note that here, productivity and wage are not equivalent. For men, the two concepts coincide but for women, they differ by a term  $\alpha$ . Thus,  $E[w^2]$  represents the average square wage.

### 3.1 The political equilibrium in a society of singles

Under our assumptions of different wage and longevity for different genders, using (3), we have that the preferred tax rates for men and women are, respectively,

$$\begin{aligned}\tau_m^* &= 0 \quad \text{since} \quad \frac{w^2}{E[w^2]} = \frac{2}{1+\alpha^2} \geq \frac{\pi}{\bar{\pi}} = \frac{2}{1+\beta}, \\ \tau_f^* &= \frac{\frac{\beta\pi}{\bar{\pi}}E[w^2] - \alpha^2w^2}{\frac{2\beta\pi}{\bar{\pi}}E[w^2] - \alpha^2w^2} \quad \text{since} \quad \frac{\alpha^2w^2}{E[w^2]} = \frac{2\alpha^2}{1+\alpha^2} \leq \frac{\beta\pi}{\bar{\pi}} = \frac{2\beta}{1+\beta}.\end{aligned}$$

A man, who has lower longevity and higher productivity than the average, always prefers a zero tax rate since he is a net contributor to the pension system. On the contrary, a woman always gets a net benefit from the pension system and votes for a positive tax rate. The political equilibrium corresponds to the preferred tax rate of the median individual. Hence, if the number of women was slightly higher than the number of men, the political outcome in a society composed by singles only would be the preferred tax rate of women:  $\tau^* = \tau_f^*$ .

### 3.2 The political equilibrium in a society with both singles and couples

We now study the decisions made by a couple. In this paper we adopt the unitary model of the household (i.e. a model where the household has only one set of preferences).<sup>14</sup> Under such a specification, spouses play cooperatively and share their resources over their life-cycle. A two-breadwinner couple thus solves the following problem:<sup>15</sup>

$$\begin{aligned}\max_{c, d, l_m, l_f} \quad & 2c - l_f^2/2 - l_m^2/2 + (\pi_f + \pi_m)u(d) \\ \text{s.t.} \quad & (wl_m + w_f l_f)(1 - \tau) + (\pi_f + \pi)p \geq 2c + (\pi_f + \pi)d\end{aligned}\tag{A}$$

where  $d$  represents the individual level of (annual) consumption in the second period for each member of the couple. The labour supply of the husband and the wife are, respectively,  $l_m^* = w(1 - \tau)$  and  $l_f^* = \alpha w(1 - \tau)$ . Note that, under our assumptions, these are independent of their marital status (i.e. whether they belong to a couple or are single). Hence, the labour supply

<sup>14</sup>A number of alternatives have been recently suggested, ranging from bargaining to non-cooperative models. Our choice is mainly guided by the concern for simplicity.

<sup>15</sup>Note that we do not model endogenous marriage, i.e. the fact that some single individuals may gain from forming a couple. Given the quasilinearity of utilities, the utility of a couple is the same as the sum of utility of a single man and a single woman with the same productivity. From a laissez faire perspective, marriage creates a welfare gain for the woman and a loss for the man, which is reduced when there exists a pension system. Endogenizing marriage is highly relevant but it is outside the scope of this paper and is left for future work.

of a woman is always lower than that of a man. This implies that her total contribution to the pension scheme,  $\alpha w\tau$ , is also lower while she receives a higher total pension benefit,  $\beta\pi p \geq \pi p$ .

Substituting for  $l_m^*$  and  $l_f^*$  and  $p(\tau)$ , we obtain the couple's indirect utility function

$$V^{c2}(\tau) = \frac{(1-\tau)^2}{2} (1+\alpha^2) w^2 - (1+\beta) \pi d^* + (1+\beta) \pi p(\tau) + (1+\beta) \pi u(d^*), \quad (5)$$

where the superscript  $c2$  stands for a couple with two breadwinners and  $d^*$  is the optimal level of second period consumption.<sup>16</sup> Note that the equation of the pension benefit (4) is not modified by the introduction of two-breadwinner couples since both members contribute to, and benefit from, the pension scheme in the same way as if they were singles. Differentiating this indirect utility function with respect to the tax rate  $\tau$ , it is straightforward to show that the preferred tax rate of a two-breadwinner couple is always nil,  $\tau_{c2}^* = 0$ . Note that, if labour supply were exogenous, the couple would be indifferent between any level of taxation (it would obtain the same return from savings as from the pension scheme). When labour supply is endogenous, the preferred tax rate is zero since, in this case, the pension scheme introduces distortions on the labour supply (i.e. the individual return from the pension scheme is smaller than the return from private savings).

We now consider the political equilibrium and assume that a fraction  $\varphi$  of men and women are married. The preferred tax rates for single women and men remain the same, since the existence of couples does not modify the expression of the pension benefit, so that  $\tau_m^* = 0$  and  $\tau_f^* > 0$  (as shown before) while  $\tau_{c2}^* = 0$ . With equal number of women and men, as soon as some of them are married there is a majority of individuals who favour a zero tax rate and the political outcome will be  $\tau^* = 0$ .<sup>17</sup>

### 3.3 Introducing one-breadwinner couples

#### 3.3.1 The modified model

Let us now assume that society consists of four different categories of households: single men, single women, couples with two breadwinners and couples with one breadwinner. As in the previous sections, there is still an equal fraction  $(1-\varphi)$  of single males and of single females and a fraction  $\varphi$  of couples, so that a number  $2\varphi$  of individuals live in couple. But we now assume

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<sup>16</sup>Note that  $d^*$  is independent of income changes.

<sup>17</sup>If the labour supply were exogenous, the couple would be indifferent between any level of taxation and we would then have had exactly the reverse result, i.e. a maximum tax rate,  $\tau^* = \tau_f^* = 1$  (under the assumption of a slightly higher number of women than of men).

that, among these couples, a fraction  $\mu$  is composed of two breadwinners, while a fraction  $(1 - \mu)$  of couples consists of only one breadwinner.<sup>18</sup>

This breadwinner is always the husband.<sup>19</sup> His wife may be entitled to a pension benefit in the retirement period even if she did not personally contribute to the pension scheme. These benefits, sometimes called derived pension rights, consist of a small supplementary pension plus a survival pension. We thus assume that she receives a fraction  $\gamma \in [0, 1]$  of the full annual pension benefit  $p(\tau)$  during the second period of her life of length  $\pi_f$ .<sup>20</sup> If  $\gamma = 0$  the spouse receives nothing in the second period while if  $\gamma = 1$  she gets a full pension. Whatever the value of  $\gamma$ , annual consumption is the same for both spouses.

Let us first explore the problem of a one-breadwinner couple, which is slightly different from the two-breadwinner one (i.e. problem A):

$$\begin{aligned} \max_{c, d, l_m} \quad & 2c - l_m^2/2 + (\pi_f + \pi_m) u(d) \\ \text{s.t.} \quad & w l_m (1 - \tau) + (\gamma \pi_f + \pi) p(\tau) \geq 2c + (\pi_f + \pi) d \end{aligned} \quad (B)$$

Only the man supplies labour, with  $l_m^* = w(1 - \tau)$ . Substituting for  $l_m^*$  and  $\pi_f$ , the indirect utility function is equal to

$$V^{c1}(\tau) = \frac{w^2(1 - \tau)^2}{2} + (1 + \gamma\beta) \pi p(\tau) - (\beta + 1) \pi d^* + (\beta + 1) \pi u(d^*), \quad (6)$$

where the superscript  $c1$  stands for a couple with one breadwinner.

The expression for  $p(\tau)$  is now modified due to the existence of one-breadwinner couples. To see this clearly, we rewrite the budget constraint as

$$p[\pi + \pi_f(1 - \varphi + \varphi\mu + \gamma\varphi(1 - \mu))] \leq w l_m^* \tau + w_f l_f^* (1 - \varphi + \varphi\mu).$$

On the left hand side we have total benefits distributed: every working individual (men or women) receives a pension  $p$  and a fraction  $\varphi(1 - \mu)$  of non-working women receive  $\gamma p$ . On the right hand side we have total contributions. In this model men supply labour, whatever their marital status, but only women who are single or belong to two-breadwinner couples supply

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<sup>18</sup>Note that Section 3.1 is equivalent to assuming  $\varphi = 0$ , while Section 3.2 corresponds to  $\varphi \in [0, 1]$  and  $\mu = 1$ .

<sup>19</sup>Up to now, the total lifetime income was  $Y = w^2(1 + \alpha^2)$  normalizing the size of the population to 2. With a fraction  $\varphi(1 - \mu)$  of the population ending up in one breadwinner couples, this amount decreases to  $Y' = w^2(1 + \alpha^2)[1 - \varphi(1 - \mu)] + w^2\varphi(1 - \mu) = w^2[1 + \alpha^2(1 - \varphi(1 - \mu))]$ .

<sup>20</sup>The parameter  $\gamma$  may account either for a survivor benefit or for the higher replacement rate provided to a one-earner couple than to a single individual. As mentioned in the introduction, such features are observed in many countries with a public pension scheme.

labour and thus pay contributions. Using the optimal labour supplies, total contributions are equal to  $(1 - \tau) \tau [1 + \alpha^2 (1 - \varphi + \varphi \mu)] w^2$ . Substituting for  $\pi_f = \beta \pi$  on the left hand side and using the budget balance condition, we obtain a new expression for the flat-rate pension benefit:

$$p(\tau) = \frac{(1 - \tau) \tau [1 + \alpha^2 (1 - \varphi + \varphi \mu)]}{\pi [1 + \beta (1 - \varphi + \varphi \mu + \gamma \varphi (1 - \mu))]} w^2.$$

Note that if we assume a society of only singles ( $\varphi = 0$ ), or a society of two-breadwinner couples ( $\varphi = 1$  and  $\mu = 1$ ),  $p(\tau)$  is equal to (4) as before. In these cases, we recover the solutions obtained in the previous subsections.

For the following sections, and in order to simplify notation, we define here the function  $\chi(\alpha, \beta, \varphi, \mu, \gamma)$ :

$$\chi(\alpha, \beta, \varphi, \mu, \gamma) = \frac{[1 + \alpha^2 (1 - \varphi + \varphi \mu)]}{[1 + \beta (1 - \varphi + \varphi \mu + \gamma \varphi (1 - \mu))]}.$$
 (7)

The “modified” pension benefit can be rewritten as

$$p(\tau) = \frac{(1 - \tau) \tau}{\pi} \chi(\alpha, \beta, \varphi, \mu, \gamma) w^2.$$
 (8)

### 3.3.2 Preferred tax rates and the political equilibrium

Individuals preferred tax rates are obtained by solving

$$\max_{\tau \in [0,1]} V^i(\tau),$$

where  $i$  accounts for  $m$  (single male),  $f$  (single female),  $c2$  (two-breadwinner couples) and  $c1$  (one-breadwinner couples). For one-breadwinner couples, the indirect utility function is (6) while, for the other households, indirect utility functions remain the same and given by (2) and (5); only the expression for  $p(\tau)$  is modified and given by (8). In Appendix B, we derive the solution for each type of individual and show that the preferred tax rates are equal to:

$$\tau_m^* = \tau_{c2}^* = 0$$
 (9)

$$\tau_f^* = \begin{cases} 0 & \text{if } \gamma > \hat{\gamma}_f \equiv \frac{\beta - \alpha^2}{\beta \alpha^2} \frac{1}{\varphi (1 - \mu)} \\ \frac{\beta \chi(\alpha, \beta, \varphi, \mu, \gamma) - \alpha^2}{2\beta \chi(\alpha, \beta, \varphi, \mu, \gamma) - \alpha^2} & \text{if } \gamma < \hat{\gamma}_f \end{cases}$$
 (10)

$$\tau_{c1}^* = \begin{cases} 0 & \text{if } \gamma < \hat{\gamma}_{c1} \equiv \frac{\beta - \alpha^2}{\beta} \frac{1 - \varphi (1 - \mu)}{1 + \alpha^2 [1 - \varphi (1 - \mu)] + \varphi (1 - \mu)} \\ \frac{(1 + \beta \gamma) \chi(\alpha, \beta, \varphi, \mu, \gamma) - 1}{2(1 + \beta \gamma) \chi(\alpha, \beta, \varphi, \mu, \gamma) - 1} & \text{if } \gamma > \hat{\gamma}_{c1} \end{cases}$$
 (11)

where  $\hat{\gamma}_f$  and  $\hat{\gamma}_{c1}$  are the threshold levels of  $\gamma$ . Above  $\hat{\gamma}_f$ , single women prefer a zero tax and below  $\hat{\gamma}_{c1}$ , one-breadwinner couples also prefer a zero tax. As before, the preferred tax rates of single men and two-breadwinner couples are zero. Their preference for a zero tax rate is here reinforced by the fact that the pension system now also redistributes towards one-breadwinner couples. For single women and one-breadwinner couples, the level of preferred tax rate depends on the value of  $\gamma$  (i.e. the level of generosity of the pension scheme towards one-breadwinner couples). Indeed, single women prefer a strictly positive tax rate only when the system is not too generous towards the non-working spouse, since more redistribution to the latter is always to the detriment of single women (they get less from the pension scheme). For one-breadwinner couples, we obtain the opposite: they will prefer a strictly positive tax rate if the scheme is sufficiently redistributive towards them. If  $\gamma \rightarrow 0$ , the man in the couple contributes to a system that is not favourable to him, since he has higher productivity and lower longevity. In this specific case, the couple obtains almost no survivor benefit compensation and they vote for a zero tax rate. On the contrary, if  $\gamma$  is high, the husband's net contribution to the pension scheme can be compensated by the benefit received by his non-working spouse.

We now characterize the political equilibrium level of the tax rate. It depends on the generosity of the system towards one-breadwinner couples (i.e. on the level of  $\gamma$ ). In Appendix B, we show that  $\hat{\gamma}_{c1} < \hat{\gamma}_f$ . We obtain three possible cases:

- If  $\gamma < \hat{\gamma}_{c1} < \hat{\gamma}_f$ , preferred tax rates are  $\tau_m^* = \tau_{c2}^* = \tau_{c1}^* = 0$  and  $\tau_f^* > 0$ . In this case, a majority of individuals prefers a zero tax rate:  $\tau^* = 0$ .
- If  $\hat{\gamma}_{c1} < \gamma < \hat{\gamma}_f$ ,  $\tau_m^* = \tau_{c2}^* = 0$  and  $\tau_{c1}^* > 0$ ,  $\tau_f^* > 0$ . If  $\mu < 0.5$ , the equilibrium tax rate should be positive. In Appendix B, we show that:

$$\tau_f^* \begin{cases} < \tau_{c1}^* \text{ iff } \gamma > \frac{\beta - \alpha^2}{\beta \alpha^2} \equiv \hat{\gamma} \\ > \tau_{c1}^* \text{ iff } \gamma < \hat{\gamma} \end{cases}$$

If  $\hat{\gamma}_{c1} < \gamma < \hat{\gamma}$  (i.e.  $\gamma$  is not too high) the chosen tax rate is likely to be the one preferred by one-breadwinner couples. On the contrary, if  $\hat{\gamma} < \gamma < \hat{\gamma}_f$ , the chosen tax rate is the one preferred by single women. This corresponds to the traditional case of couples with male breadwinners and non-working housewives who benefit from generous derived pension rights.

- Finally, if  $\hat{\gamma}_{c1} < \hat{\gamma}_f < \gamma$ , only one-breadwinner couples vote for a positive tax rate,  $\tau_{c1}^* > 0$ , while the other categories vote for a zero tax rate so that the political equilibrium is most likely  $\tau^* = 0$  (except if the number of one-breadwinner couples forms the majority).

To sum up, the existence of one-earner couples (and their relative number) as well as the generosity of the system towards them, through the level of the parameter  $\gamma$ , crucially influence the level of tax rate chosen by majority voting. To obtain a positive tax one needs an intermediate value of  $\gamma$ , not too high to keep the support of single women, not too low to keep the support of one-breadwinner couples.

In the next section, we extend our model to take into account differences in productivity, not only between genders but also across individuals in general.

## 4 Model with continuous productivity distribution

In this section we keep the assumption that longevity can take only two values (i.e.  $\pi$  and  $\pi_f$  for men and women, respectively). In contrast, we now assume that  $w$  is uniformly distributed, with support  $[0, 1]$ . The average and the median productivity are then identical and equal to  $\bar{w} = w_m = 1/2$ . Thus,  $w_f (= \alpha w)$  is distributed over  $[0, \alpha]$  with density  $1/\alpha$ .<sup>21</sup> Finally, we assume positive assortative mating: when a man and a woman form a couple they do so with someone with the same underlying productivity, which means wages  $w$  and  $\alpha w$  for man and woman, respectively.<sup>22</sup>

With the results obtained above in mind, we expect that now every individual at the bottom of the wage distribution will be in favour of a pension system, since they will benefit from the inherent redistribution. Thus, independently of their marital status and gender, a fraction of individuals (in each type of household) will vote for a positive tax rate. In contrast, some individuals, those at the top of the wage distribution, will be against a pension system. To illustrate our point let us use the histogram representation given on Figure 2.

We have two sets of histograms, one for each case, that represent the proportion of members of each of our 4 categories who support the equilibrium tax rate and even a higher one. On the vertical axis we have the frequencies or, following our assumption on the density of  $w$ , the

<sup>21</sup>We could alternatively assume a right-skewed distribution, but this would complicate our model without providing additional insights.

<sup>22</sup>Note that independently of their (low or high) productivity, some women do not work, which is due to exogenous reasons or preferences, which we do not model here. This certainly would be an interesting extension.

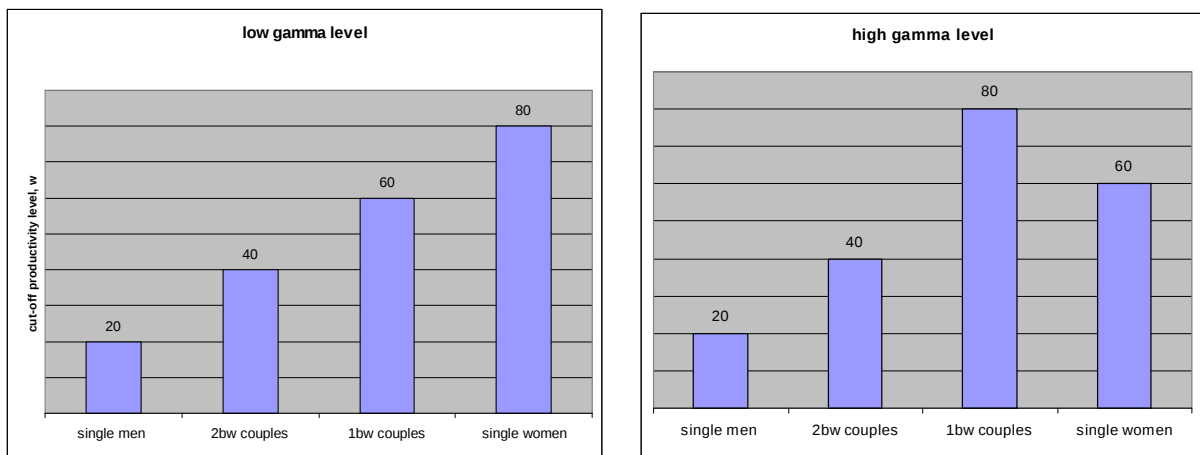


Figure 2: Percentage of households in favor of the equilibrium tax or a higher one

productivity involved. Below we provide numerical examples of equilibrium tax rates that are shown to depend on the relative size of each category. For the sake of simplicity, we assume that the four categories have the same size. Not surprisingly the frequency bar of the single men is the lowest, implying that their cut-off productivity is also the lowest. The two-breadwinner couples follow them. Depending on the case, namely depending on the value of  $\gamma$ , the category with the highest bar and thus with the highest cut-off wage is either that of single women or that of one-breadwinner couples. Given that half the population supports a tax rate equal or superior to the equilibrium rate, the sum of these bars is equal to one.<sup>23</sup> Yet, as Figure 2 shows, depending on the household category they belong to, more or less individuals will be against it.

#### 4.1 Preferred tax rates

When  $w$  follows a uniform distribution, the pension benefit becomes

$$p(\tau) = \frac{1}{3} \frac{\tau(1-\tau)}{\pi} \chi(\alpha, \beta, \varphi, \mu, \gamma),$$

<sup>23</sup>In the first panel, 20% of single men, 40% of two-breadwinner couples, 60% of one-breadwinner couples and 80% of single women are in favor of a tax equal or higher than the equilibrium tax. Given that each category comprises the same number of voters, this gives half of the population.

where  $1/3$  corresponds to the average square productivity. In Appendix C, we show that individuals preferred tax rates are now:

$$\tau_f^*(w) = \begin{cases} 0 & \text{if } \frac{\alpha^2 w^2}{1/3} > \beta \chi(\alpha, \beta, \varphi, \mu, \gamma) \\ \frac{\frac{\beta}{3} \chi(\alpha, \beta, \varphi, \mu, \gamma) - \alpha^2 w^2}{2 \frac{\beta}{3} \chi(\alpha, \beta, \varphi, \mu, \gamma) - \alpha^2 w^2} & \text{otherwise} \end{cases} \quad (12)$$

$$\tau_m^*(w) = \begin{cases} 0 & \text{if } \frac{w^2}{1/3} > \chi(\alpha, \beta, \varphi, \mu, \gamma) \\ \frac{\frac{\chi(\alpha, \beta, \varphi, \mu, \gamma)}{3} - w^2}{2 \frac{\chi(\alpha, \beta, \varphi, \mu, \gamma)}{3} - w^2} & \text{otherwise} \end{cases} \quad (13)$$

$$\tau_{c2}^*(w) = \begin{cases} 0 & \text{if } \frac{w^2}{1/3} > \frac{(1+\beta)}{(1+\alpha^2)} \chi(\alpha, \beta, \varphi, \mu, \gamma) \\ \frac{\frac{\chi(\alpha, \beta, \varphi, \mu, \gamma)}{3} (1+\beta) - (1+\alpha^2) w^2}{\frac{2\chi(\alpha, \beta, \varphi, \mu, \gamma)}{3} (1+\beta) - (1+\alpha^2) w^2} & \text{otherwise} \end{cases} \quad (14)$$

$$\tau_{c1}^*(w) = \begin{cases} 0 & \text{if } \frac{w^2}{1/3} > (1+\gamma\beta) \chi(\alpha, \beta, \varphi, \mu, \gamma) \\ \frac{\frac{\chi(\alpha, \beta, \varphi, \mu, \gamma)}{3} (1+\gamma\beta) - w^2}{\frac{2\chi(\alpha, \beta, \varphi, \mu, \gamma)}{3} (1+\gamma\beta) - w^2} & \text{otherwise} \end{cases} \quad (15)$$

where the  $w$  relates to the productivity (and not the wage) of the individuals belonging to the different groups  $m, f, c2, c1$ .

## 4.2 Political equilibrium

In order to characterize the political equilibrium, we manipulate expressions (12) to (15) so as to obtain the wage rate as a function of the most preferred tax rate, instead of the other way round. With the uniform distribution assumption, this also represents the number of individuals who prefer this tax rate (or a greater one) over any other lower tax rate. This yields:

$$\begin{aligned}
w_m(\tau) &= \sqrt{\frac{1-2\tau}{1-\tau} \frac{\chi(\alpha, \beta, \varphi, \mu, \gamma)}{3}} \\
w_f(\tau) &= \sqrt{\frac{\beta}{\alpha^2}} w_m(\tau) \\
w_{c2}(\tau) &= \sqrt{\frac{1+\beta}{1+\alpha^2}} w_m(\tau) \\
w_{c1}(\tau) &= \sqrt{1+\gamma\beta} w_m(\tau)
\end{aligned}$$

It is straightforward to show that  $w_m(\tau) < w_{c2}(\tau) < w_f(\tau)$  and that  $w_{c2}(\tau) < w_{c1}(\tau)$ . However, whether  $w_{c1}(\tau) \leq w_f(\tau)$  depends on the value of  $\gamma$ . Indeed, if  $\gamma < \hat{\gamma}$  (as defined in the previous section),  $w_{c1}(\tau) < w_f(\tau)$ : that is, if the system is not very generous towards the non-working spouse, the number of single women supporting a specific tax rate  $\tau$  is higher than the number of individuals belonging to one-breadwinner couples that supports the tax rate  $\tau$ . On the contrary, if  $\gamma$  is high enough, the opposite happens. These two cases are depicted in Figure 2.

We now turn to the characterization of the equilibrium payroll tax rate under majority voting. The equilibrium tax rate is defined such that at least one half of the population prefers this tax rate (or a higher one) to any other lower tax rate. The voting equilibrium tax rate,  $\tau^*$  is then such that the number of individuals with higher wage (and thus who would prefer a lower tax level) represents exactly one half of the total population:

$$(1-\varphi) \left[ w_m(\tau^*) + \frac{w_f(\tau^*)}{\alpha} \right] + 2\varphi\mu w_{c2}(\tau^*) + 2\varphi(1-\mu) w_{c1}(\tau^*) \geq 1, \quad (16)$$

where a mass 1 of individuals corresponds to one half of the population. Solving the above equation (see Appendix C), we obtain that:

$$\begin{aligned}
\tau^* &= \frac{1-3/\Omega(\alpha, \beta, \varphi, \mu, \gamma)^2 \chi(\alpha, \beta, \varphi, \mu, \gamma)}{2-3/\Omega(\alpha, \beta, \varphi, \mu, \gamma)^2 \chi(\alpha, \beta, \varphi, \mu, \gamma)} \\
&\text{with } \Omega(\alpha, \beta, \varphi, \mu, \gamma) = (1-\varphi) \left( 1 + \frac{\sqrt{\beta}}{\alpha^2} \right) + 2\varphi \left( \mu \sqrt{\frac{1+\beta}{1+\alpha^2}} + (1-\mu) \sqrt{1+\gamma\beta} \right).
\end{aligned}$$

We now illustrate this formula with a numerical example. We take as given  $\alpha = 0.8$ ,  $\beta = 1.2$  and  $\varphi = 0.6$  and we focus on the incidence of a variation in the number of two-breadwinner couples ( $\mu$ ) and in the generosity of the pension system ( $\gamma$ ) on the equilibrium tax rate. So doing, we focus on the two phenomenom already mentioned: increasing labor participation of women and the individualization of pension systems. The results are reported in the Table 1.

| $\gamma \rightarrow$<br>$\mu \downarrow$ | 0      | 0.2    | 0.3     | 0.4    | 0.5    | 0.6    | 0.7    | 0.8    | 1      |
|------------------------------------------|--------|--------|---------|--------|--------|--------|--------|--------|--------|
| 0                                        | 0.2440 | 0.2527 | 0.25568 | 0.2580 | 0.2599 | 0.2613 | 0.2625 | 0.2634 | 0.2645 |
| 0.1                                      | 0.2437 | 0.2526 | 0.25571 | 0.2582 | 0.2602 | 0.2618 | 0.2631 | 0.2641 | 0.2654 |
| 0.2                                      | 0.2438 | 0.2526 | 0.25571 | 0.2583 | 0.2603 | 0.2620 | 0.2633 | 0.2644 | 0.2659 |
| 0.3                                      | 0.2443 | 0.2527 | 0.25570 | 0.2582 | 0.2602 | 0.2619 | 0.2633 | 0.2644 | 0.2659 |
| 0.4                                      | 0.2451 | 0.2528 | 0.25567 | 0.2580 | 0.260  | 0.2616 | 0.2629 | 0.2640 | 0.2657 |
| 0.5                                      | 0.2463 | 0.2531 | 0.25563 | 0.2578 | 0.260  | 0.2611 | 0.2623 | 0.2634 | 0.2649 |
| 0.6                                      | 0.2477 | 0.2534 | 0.25558 | 0.2574 | 0.2590 | 0.2603 | 0.2614 | 0.2624 | 0.2638 |
| 0.7                                      | 0.2493 | 0.2538 | 0.25552 | 0.2570 | 0.2583 | 0.2594 | 0.2603 | 0.2611 | 0.2623 |
| 0.8                                      | 0.2511 | 0.2542 | 0.25545 | 0.2565 | 0.2574 | 0.2582 | 0.2589 | 0.2594 | 0.2603 |
| 0.9                                      | 0.2531 | 0.2547 | 0.25536 | 0.2559 | 0.2564 | 0.2568 | 0.2572 | 0.2575 | 0.2580 |
| 1                                        | 0.255  | 0.255  | 0.25527 | 0.255  | 0.255  | 0.255  | 0.255  | 0.255  | 0.255  |

Table 1: Tax rate levels as a function of gamma and mu

For any level of  $\mu < 1$ , the equilibrium tax rate is increasing in  $\gamma$  (i.e. in the generosity towards one-breadwinner couples), which results from the increasing support of the one-breadwinner couples. In contrast, the variation of the tax rate with  $\mu$  (i.e. the number of two-breadwinner couples) is ambiguous. For instance, for very low levels of  $\gamma$  ( $< 0.2$ ), the tax rate is first decreasing and then increasing in  $\mu$ , while for  $\gamma \geq 0.3$ , the tax rate is first increasing and then decreasing in  $\mu$ . Indeed, when  $\mu$  increases, two forces are playing in opposite directions. On the one hand, the increase in the number of two-breadwinner couples who prefer a lower tax pushes the equilibrium tax rate downward but on the other hand it generates more resources in the economy and increases the tax base (see footnote 19) which pushes the tax up. In our model, depending on the level of the parameter  $\gamma$ , one or the other effect may dominate and the tax rate will increase or decrease with the number of two-breadwinner couples. This effect of  $\mu$  on the tax rate is confirmed by the tables below.

Hence, regarding the recent trend towards the individualization of pension rights together with the increase in the number of two-breadwinner couples, our model fails to provide clear-cut predictions when both move simultaneously, since the effects of changes in  $\gamma$  and  $\mu$  on the equilibrium tax rate may go in opposite directions.

In the next tables, we study how the equilibrium tax rate varies when the family structure changes (i.e. when the proportion of couples,  $\varphi$ , and of two-breadwinner couples,  $\mu$ , vary). For this purpose, we first posit the level of  $\gamma$  to be equal to 0.25 (for instance, in Belgium, the supplementary pension obtained by a one-breadwinner couple is equal to 1/4 of the working

spouse pension) and then to 0.75. Our results are reported in Tables 2 and 3, respectively.

| $\varphi \rightarrow$<br>$\mu \downarrow$ | 0     | 0.2    | 0.3    | 0.4    | 0.5    | 0.6    | 0.7    | 0.8    | 0.9    | 1      |
|-------------------------------------------|-------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 0                                         | 0.312 | 0.2950 | 0.2859 | 0.2761 | 0.2656 | 0.2543 | 0.2422 | 0.2291 | 0.2151 | 0.2    |
| 0.2                                       | 0.312 | 0.2951 | 0.2860 | 0.2761 | 0.2656 | 0.2542 | 0.2419 | 0.2287 | 0.2143 | 0.1986 |
| 0.4                                       | 0.312 | 0.2952 | 0.2861 | 0.2763 | 0.2657 | 0.2543 | 0.2420 | 0.2286 | 0.2140 | 0.1981 |
| 0.5                                       | 0.312 | 0.2952 | 0.2862 | 0.2763 | 0.2658 | 0.2544 | 0.2421 | 0.2287 | 0.2141 | 0.1982 |
| 0.6                                       | 0.312 | 0.2953 | 0.2862 | 0.2764 | 0.2659 | 0.2545 | 0.2422 | 0.2288 | 0.2146 | 0.1983 |
| 0.8                                       | 0.312 | 0.2953 | 0.2863 | 0.2766 | 0.2662 | 0.2549 | 0.2426 | 0.2293 | 0.2148 | 0.1990 |
| 1                                         | 0.312 | 0.2954 | 0.2865 | 0.2768 | 0.2665 | 0.2553 | 0.2431 | 0.2300 | 0.2157 | 0.2    |

Table 2: Tax rate levels for gamma= 0.25

| $\varphi \rightarrow$<br>$\mu \downarrow$ | 0     | 0.2    | 0.3    | 0.4    | 0.5    | 0.6    | 0.7    | 0.8    | 0.9    | 1     |
|-------------------------------------------|-------|--------|--------|--------|--------|--------|--------|--------|--------|-------|
| 0                                         | 0.312 | 0.2989 | 0.2915 | 0.2831 | 0.2737 | 0.2629 | 0.2506 | 0.2364 | 0.2197 | 0.2   |
| 0.2                                       | 0.312 | 0.2984 | 0.2909 | 0.2828 | 0.2738 | 0.2639 | 0.2529 | 0.2407 | 0.2269 | 0.211 |
| 0.4                                       | 0.312 | 0.2978 | 0.2902 | 0.2819 | 0.2731 | 0.2635 | 0.2531 | 0.2419 | 0.2295 | 0.216 |
| 0.5                                       | 0.312 | 0.2975 | 0.2897 | 0.2814 | 0.2724 | 0.2629 | 0.2525 | 0.2414 | 0.2293 | 0.216 |
| 0.6                                       | 0.312 | 0.2971 | 0.2892 | 0.2807 | 0.2716 | 0.2619 | 0.2515 | 0.2403 | 0.2283 | 0.215 |
| 0.8                                       | 0.312 | 0.2963 | 0.2879 | 0.2790 | 0.2694 | 0.2592 | 0.2482 | 0.2364 | 0.2236 | 0.210 |
| 1                                         | 0.312 | 0.2954 | 0.2865 | 0.2768 | 0.2665 | 0.2553 | 0.2431 | 0.2300 | 0.2157 | 0.2   |

Table 3: Tax rate levels for gamma= 0.75

Before analyzing these tables, let us observe the following:

- As we mentioned earlier, when  $\mu$  increases, for given  $\varphi$  and  $\gamma$ , there are two effects: an increase in the overall tax base and a decrease in the number of couples who are in favour of a pension system (i.e. the one-breadwinner couples). These two effects act in opposite directions. The second effect is sharper when  $\gamma$  is high.
- When  $\varphi$  increases, for given  $\mu$  and  $\gamma$ , there are three effects: an increase in the number of one-breadwinner couples, an increase in the number of two-breadwinner couples, but a decrease in the overall tax base. The first effect leads to a higher tax but the last two to a lower tax.

Let us now check whether these priors are confirmed in the above numerical example. Concerning the first, we do not obtain a monotonic profile, as it was the case with Table 1. This

is not surprising given the two countervailing forces at stake. Turning to our second prior, we find that the equilibrium tax rate always decreases with the number of couples, which implies that the two negative effects noted above dominate the third positive one in the example. Note however that the decline in the equilibrium tax rate is smaller when the generosity towards one-breadwinner couples,  $\gamma$ , is high.

## 5 Conclusion

In this paper, we have introduced two individuals' characteristics - gender and marital status differences - that are not generally taken into account in political economy models of Social Security. Our analysis is one of the first to shed light on the importance of considering the couple as a distinct economic agent in order to explain the size of a pension system.<sup>24</sup> As opposed to standard political economy models, which only consider single agents, we distinguish between single individuals, male or female, and couples. We also distinguish between one-breadwinner and two-breadwinner couples and account for the existence of derived pension rights.

We show that when there are only two wage levels - one for men and a lower one for women - single men and two-breadwinner couples are always against the pension system. On the contrary, one-breadwinner couples and single women may be in favour of it depending on the size of derived pension rights (i.e. the amount of redistribution one-breadwinner couples get from the pension system). On the one hand, women benefit from the pension system because they have lower wages and higher longevity; on the other hand, one-breadwinner couples benefit from redistribution through derived pension rights. Thus, if derived pension rights are high, the pension system is favourable to one-breadwinner couples, but this is to the detriment of single women, who may end up voting for a zero tax (if the pensions rights are too high). We also generalize our analysis by assuming a uniform distribution of productivity. We find that the equilibrium payroll tax should decrease when the system becomes less generous towards one-breadwinner couples. On the contrary, the effect of changes in the relative number of two-breadwinner couples is found to be ambiguous.

Clearly, our paper shows that the marital status and the composition of households influence

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<sup>24</sup>The closest paper to ours is Recoules (2009), which focuses on the political support for family-friendly policies (i.e. policies aimed at child-rearing) in a society constituted only of couples and in which there is gender discrimination in the labor market. The degree of discrimination is likely to influence the size of government spending, under some conditions.

the political support for the pension system. Moreover, another important observation of our paper is that, while two-breadwinner couples neutralize gender differences in longevity and wage, one-breadwinner couples do not. If the pension system is generous towards the non-working spouses, these couples will push for large pension benefits. In that respect, it is worth noticing the current trend in many countries: the progressive decline in couples with only one breadwinner and the individualization of pensions systems, which implies less generosity towards non-working spouses. According to our model, an increasing individualisation of the pension rights leads to a lower level of payroll taxation while the increasing number of two-breadwinner couples yields mitigated results.<sup>25</sup>

In our paper we make a number of simplifying assumptions: no liquidity constraints, labour supply invariant to the marital status, zero interest rate, actuarially fair annuity, no widowers, quadratic disutility of labour, quasi-linear utility, uniform density of wages, assortative mating and no overlapping generations. We do not think that the qualitative results would change if these assumptions were relaxed; at the same time, it is clear that the analysis would be more complicated.

Finally, the type of pension system considered in this paper is the Beveridgean one: namely, a pension system wherein the annual pension is invariant to contributions and longevity. This does not mean that everyone has the same resources upon retirement. As public pensions are supplemented by private savings, it includes traditional savings but also all forms of defined contribution private pensions. An alternative specification would be the traditional Bismarckian system (i.e. a pension system in which benefits are related to earnings but independent of longevity). A further possibility would be a pension system in which pension benefits are related to both longevity and contributions. Such a system would be akin to private savings. The most realistic specification would probably include a mix of Beveridge and Bismarck with benefits partially related to contributions, with survival pension for spouses without or negligible pension rights, and minimum pensions. Unfortunately, adopting such a realistic specification would significantly complicate the analysis.

Our model could still be extended in several directions. First, we could investigate the implications of adjusting the couples' pension benefits for scale economies. Second, we have

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<sup>25</sup>There is a conjecture that the increasing participation of women in the labor market is accompanied by a decline in the level of wages. If this were the case the wealth effect would disappear and we would then expect that the increase in the number of two-breadwinner couples would lead to an unambiguous decrease in the equilibrium tax.

only considered differences in longevity between men and women but we have not accounted for the empirical fact that men have, on average, longer life expectancy when married than when single. Our conjecture is that, taking this second feature into account, would reinforce our results. The support for the pension system should increase as then, not only a married man would gain from the benefit received by his wife but also he would get benefit from a pension for a longer period than if he had been single.

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## A Preferred tax rate

We solve the following program:

$$\max_{\tau \in [0,1]} V^i(\tau) = \frac{(1-\tau)^2 w_i^2}{2} - s_i^* + \pi_i u \left( \frac{s_i^*}{\pi_i} + \tau \frac{(1-\tau) E[w^2]}{\bar{\pi}} \right).$$

Differentiating  $V^i(\tau)$  with respect to  $\tau$ , we obtain

$$\frac{\partial V^i(\tau)}{\partial \tau} = -(1-\tau) w_i^2 + \pi_i u'(d_i^*) \frac{E[w^2]}{\bar{\pi}} (1-2\tau)$$

with  $u'(d_i^*) = 1$ . Evaluating this expression at  $\tau = 0$ , we find that any individual with  $w_i^2/E[w^2] \geq \pi_i/\bar{\pi}$  always prefers a zero tax rate. For those with  $w_i^2/E[w^2] < \pi_i/\bar{\pi}$ , the solution is interior and the preferred tax rate is equal to (3).

## B Model with unique productivity level

Indirect utility functions of the four categories of household are

$$\begin{aligned} V^f(\tau) &= \frac{(1-\tau)^2 \alpha^2 w^2}{2} - s^* + \pi \beta u \left( \frac{s^*}{\pi \beta} + p(\tau) \right), \\ V^m(\tau) &= \frac{(1-\tau)^2 w^2}{2} - s_i^* + \pi u \left( \frac{s^*}{\pi} + p(\tau) \right), \\ V^{c1}(\tau) &= \frac{w^2 (1-\tau)^2}{2} + (1+\gamma\beta) \pi p(\tau) - (\beta+1) \pi d^* + (\beta+1) \pi u(d^*), \\ V^{c2}(\tau) &= \frac{(1-\tau)^2}{2} (1+\alpha^2) w^2 - (1+\beta) \pi d^* + (1+\beta) \pi p(\tau) + (1+\beta) \pi u(d^*), \end{aligned}$$

with  $p(\tau)$  given by (8) and  $E[w^2] = w^2$ . Preferred tax rates are such that

$$\frac{\partial V^f(\tau)}{\partial \tau} = -(1-\tau) \alpha^2 w^2 + \pi \beta u'(d) \frac{dp(\tau)}{d\tau}, \quad (17)$$

$$\frac{\partial V^m(\tau)}{\partial \tau} = -(1-\tau) w^2 + \pi u'(d) \frac{dp(\tau)}{d\tau}, \quad (18)$$

$$\frac{\partial V^{c1}(\tau)}{\partial \tau} = -(1-\tau) w^2 + (1+\gamma\beta) \pi \frac{dp(\tau)}{d\tau}, \quad (19)$$

$$\frac{\partial V^{c2}(\tau)}{\partial \tau} = -(1-\tau) (1+\alpha^2) w^2 + (1+\beta) \pi \frac{dp(\tau)}{d\tau}, \quad (20)$$

where  $u'(d^*) = 1$ , from first-order conditions of the individual's problem, and where

$$\frac{dp}{d\tau} = \frac{(1-2\tau)}{\pi} \chi(\alpha, \beta, \varphi, \mu, \gamma) w^2$$

with  $\chi(\alpha, \beta, \varphi, \mu, \gamma)$  defined by (7). Evaluating  $\partial V^i(\tau) / \partial \tau$  at  $\tau = 0$ , we find that, for male and two-breadwinner couples,  $\partial V^f(\tau) / \partial \tau < 0$  and  $\partial V^{c2}(\tau) / \partial \tau < 0$  so that their preferred tax rate is always zero (equation 9). For the other groups,

$$\begin{aligned} \left. \frac{\partial V^f(\tau)}{\partial \tau} \right|_{\tau=0} &< 0 \text{ iff } \frac{\beta - \alpha^2}{\beta \alpha^2} \frac{1}{\varphi(1 - \mu)} < \gamma, \\ \left. \frac{\partial V^{c1}(\tau)}{\partial \tau} \right|_{\tau=0} &< 0 \text{ iff } \gamma > \frac{\beta - \alpha^2}{\beta} \frac{1 - \varphi(1 - \mu)}{1 + \alpha^2 [1 - \varphi(1 - \mu)] + \varphi(1 - \mu)}, \end{aligned}$$

which imply the following threshold levels of  $\gamma$  for women and one-breadwinner, respectively:

$$\begin{aligned} \hat{\gamma}_f &= \frac{\beta - \alpha^2}{\beta \alpha^2} \frac{1}{\varphi(1 - \mu)}, \\ \hat{\gamma}_{c1} &= \frac{\beta - \alpha^2}{\beta} \frac{1 - \varphi(1 - \mu)}{1 + \alpha^2 [1 - \varphi(1 - \mu)] + \varphi(1 - \mu)}, \end{aligned}$$

such that for  $\gamma < \hat{\gamma}_f$ , the preferred tax rate of a single woman is always positive (resp. if  $\gamma > \hat{\gamma}_f$ , her preferred tax rate is null) and for  $\gamma < \hat{\gamma}_{c1}$ , the preferred tax rate of a one-breadwinner couple is always positive (resp. if  $\gamma > \hat{\gamma}_{c1}$ , the preferred tax rate is null). Thus, for  $\gamma < \hat{\gamma}_f$ , the preferred tax rate level of women is strictly positive and solves the following equality:

$$-(1 - \tau) \alpha^2 w^2 + \beta (1 - 2\tau) \chi(\alpha, \beta, \varphi, \mu, \gamma) w^2 = 0.$$

This yields (10). We use the same procedure for one-breadwinner couples when  $\hat{\gamma}_{c1} < \gamma$ . The solution is interior and solves

$$-(1 - \tau) w^2 + (1 + \gamma \beta) (1 - 2\tau) \chi(\alpha, \beta, \varphi, \mu, \gamma) w^2 = 0,$$

which yields (11).

We also show that  $\hat{\gamma}_{c1} < \hat{\gamma}_f$  as

$$\frac{1 - \varphi(1 - \mu)}{1 + \alpha^2 [1 - \varphi(1 - \mu)] + \varphi(1 - \mu)} < \frac{1}{\alpha^2 \varphi(1 - \mu)} \Leftrightarrow -\alpha^2 [1 - \varphi(1 - \mu)]^2 < 1 + \varphi(1 - \mu)$$

which is always verified given that the LHS is negative and the RHS is positive.

Finally, we compare (10) and (11) and show that  $\tau_f^* > \tau_{c1}^*$  if and only if

$$\gamma < \frac{\beta - \alpha^2}{\beta \alpha^2} \equiv \hat{\gamma}.$$

It is straightforward to show that  $\hat{\gamma} \in [\hat{\gamma}_{c1}, \hat{\gamma}_f]$ .

## C Model with continuous productivity distribution

### C.1 Preferred tax rates

Substituting for

$$\frac{dp(\tau)}{d\tau} = \frac{1}{3} \frac{(1-2\tau)}{\pi} \chi(\alpha, \beta, \varphi, \mu, \gamma)$$

into (17), (18), (19) and (20), we obtain that

$$\left. \frac{\partial V^f(\tau)}{\partial \tau} \right|_{\tau=0} < 0 \text{ if } \frac{\alpha^2 w^2}{1/3} > \beta \chi(\alpha, \beta, \varphi, \mu, \gamma).$$

Then,  $\tau_f^* = 0$ . On the contrary, if  $3\alpha^2 w^2 < \beta \chi(\alpha, \beta, \varphi, \mu, \gamma)$ , the preferred tax rate is positive and such that  $\partial V^f(\tau_f^*) / \partial \tau = 0$ . In this case,

$$\tau_f^* = \frac{\frac{\beta}{3} \chi(\alpha, \beta, \varphi, \mu, \gamma) - \alpha^2 w^2}{2 \frac{\beta}{3} \chi(\alpha, \beta, \varphi, \mu, \gamma) - \alpha^2 w^2}.$$

Using the same procedure for single men, we have that  $\partial V^m(\tau) / \partial \tau|_{\tau=0} < 0$  if  $3w^2 > \chi(\alpha, \beta, \varphi, \mu, \gamma)$  so that in this case,  $\tau_m^* = 0$ , while for  $3w^2 < \chi(\alpha, \beta, \varphi, \mu, \gamma)$ ,  $\tau_m^* \in [0, 1]$  and is equal to (13). For two-breadwinner couples,  $\partial V^{c2}(\tau) / \partial \tau|_{\tau=0} < 0$  if  $3w^2 > (1 + \beta) \chi(\alpha, \beta, \varphi, \mu, \gamma) / (1 + \alpha^2)$ ; otherwise, it is equal to (14). For one-breadwinner couples,  $\partial V^{c1}(\tau) / \partial \tau|_{\tau=0} < 0$  if  $3w^2 > (1 + \gamma\beta) \chi(\alpha, \beta, \varphi, \mu, \gamma)$ ; otherwise the solution is interior and equal to (15).

### C.2 Equilibrium tax rate

Substituting the expressions of  $w_m(\tau^*)$ ,  $w_f(\tau^*)$ ,  $w_{c2}(\tau^*)$  and  $w_{c1}(\tau^*)$  into (16) we get

$$(1 - \varphi) \left[ w^m(\tau^*) + \frac{\sqrt{\beta}}{\alpha^2} w_m(\tau^*) \right] + 2\varphi\mu\sqrt{\frac{1+\beta}{1+\alpha^2}} w_m(\tau^*) + 2\varphi(1 - \mu) \sqrt{1 + \gamma\beta} w_m(\tau^*) = 1,$$

$$w^m(\tau^*) \left[ (1 - \varphi) \left( 1 + \frac{\sqrt{\beta}}{\alpha^2} \right) + 2\varphi\mu\sqrt{\frac{1+\beta}{1+\alpha^2}} + 2\varphi(1 - \mu) \sqrt{1 + \gamma\beta} \right] = 1,$$

$$\sqrt{\frac{1-2\tau}{1-\tau}} \frac{\chi(\alpha, \beta, \varphi, \mu, \gamma)}{3} = \frac{1}{\Omega(\alpha, \beta, \varphi, \mu, \gamma)},$$

$$\text{where } \Omega(\alpha, \beta, \varphi, \mu, \gamma) = (1 - \varphi) \left( 1 + \frac{\sqrt{\beta}}{\alpha^2} \right) + 2\varphi \left( \mu\sqrt{\frac{1+\beta}{1+\alpha^2}} + (1 - \mu) \sqrt{1 + \gamma\beta} \right).$$

Rearranging terms, we obtain

$$\tau^* = \frac{1 - 3/\Omega(\alpha, \beta, \varphi, \mu, \gamma)^2 \chi(\alpha, \beta, \varphi, \mu, \gamma)}{2 - 3/\Omega(\alpha, \beta, \varphi, \mu, \gamma)^2 \chi(\alpha, \beta, \varphi, \mu, \gamma)}.$$

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