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Essays on Frictions in the Labour Market and Macroeconomics

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*A mio papà e mio fratello.
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Introduction

In the last two decades, the search and matching theory has probably become the most adopted theoretical framework to understand unemployment and other important features of the labour market. As it has been observed:

For a long time, the basic obstacle was simply to think of a market where, even in equilibrium, there were some unsatisfied sellers - there was unemployment. The basic answer was given in the early 1970s, in a set of contributions edited by Phelps (1970): One should think of the labor market as a decentralized market, in which there were workers looking for jobs, and firms looking for workers. In such a market, there would always be, even in equilibrium, both some unemployment and some vacancies.

O. Blanchard (2000)

Although the volume of Phelps (1970) already contained some of the basic intuitions of the search and matching theory, it is only in the 1980s, with the work of Diamond, Mortensen and Pissarides, that the current analysis of trading frictions in the labour market is formalized. In its simpler formulation, it is assumed that the trading process in the labour market is a costly activity

that requires time. Firms post job vacancies, unemployed workers search for a job. To produce the unique consumption good, a firm-worker pair must be formed. Firms (job-seekers) find a job-seeker (firm) at an endogenous rate that depends on the level of unemployment and the level of vacancies opened in the market, while, at an exogenous rate, the firm-worker pair is destroyed. The labour force is fixed, whereas the number of vacancies posted in the market is determined by a free-entry zero profit condition.

Several reasons can explain the success of this kind of models: they describe in a relatively simple way the functioning of not competitive markets (where there is no Walrasian auctioneer and the price loses part of its allocative role), they are analytically tractable, they have some intuitive comparative static analysis, and, finally, they are consistent with many empirical facts. These characteristics make also clear why, as a recent survey of Yashiv (2006b) documents, many macroeconomic issues are nowadays studied by assuming a matching technology in the labour market.

The present thesis follows this avenue and it can be broadly divided in two parts. The first three chapters analyse the interdependences between the labour market and the product market. The final chapter assesses the empirical consistency of matching models.

On the interdependence between product and labour markets

The relationship between the efficiency of the product market and labour markets outcomes has become, especially in Europe, an hotly debated topic. Recent studies (see OECD, 2006) suggest that the poor performance of most labour markets in Continental Europe (for instance in France, Italy, and Germany) can be explained by the normative and institutional rigidities that tie their respective markets for goods. To investigate such a issue in a context of labour market frictions, some of the assumptions of a standard matching model must be abandoned. Indeed, in a Mortensen-Pissarides (1999) framework, the goods sector is modeled in a very naive form: the market is perfectly competitive and an infinitely elastic demand for goods is also assumed, so that the equilibrium price is not affected by the supply.

In the first chapter of this thesis, I abandon the assumption of infinite elasticity of demand and I introduce a two-tier productive scheme. Such set-up is not new in the macro and labour literature. However, to the best of my knowledge, nobody has studied its analytical properties. I first find the conditions under which a positive (i.e. with a level of employment and market production greater than zero) equilibrium exists. These conditions, that concern the limit properties of the marginal productivities in the final good production function, are not always fulfilled: Under a Cobb-Douglas production technology, for instance, the unique equilibrium features full unemployment and no market production.

I then inspect the mechanism at work in such a set-up, in which the demand is no longer infinitely elastic. In the state-of-the-art matching, the impact of any policy aimed at increasing employment in one sector (such as hiring subsidies or a reduction in pay-roll taxed for low-skilled workers) is overestimated in that sector and underestimated in the other sectors of the economy. The reason is that, in the sector the active policy is targeted on, the incentive for a firm to create more job-vacancies is partially offset by the decrease in revenues caused by a reduction in the market price. On the other hand, the demand for goods produced in other sectors depends positively on the quantity sold in the sector where the policy has been introduced. An increase in demand raises employment in these sectors.

In the second chapter (written together with Bruno Van der Linden) this framework is further generalized to deal with some labor market policies ('LMPs') extensively adopted in developed economies. The aim is to show how a generalization of the Mortensen-Pissarides setting can be used as an evaluation instrument. We focus on the interactions between employment subsidies and other LMPs. First, we introduce a two-tier benefit system (a stylized representation of many unemployment schemes). Second, we add a short-duration active labor market programs (counseling, job clubs, among others) that enhance the matching effectiveness of the participants. A simulation exercise is then performed in order to evaluate, in this enlarged framework, the impact of some active labour market policies targeted on low-skilled workers. For instance, we consider an employment subsidy on the low-skilled amounting to an ex ante reduction of 12 % of their wage cost. With an elasticity of

substitution between low and high-skilled sector close to 1, the state-of-the-art matching model overestimates the impact on low-skilled employment by 5 % and underestimates the impact on high-skilled employment by 0.7 %. The differences are more important in terms of job-search effort and utility levels. Altogether, this leads to very different normative conclusions. The optimal low-skilled employment subsidy (i.e. the one maximizing net output) is 63 % larger in the state-of-the-art matching model.

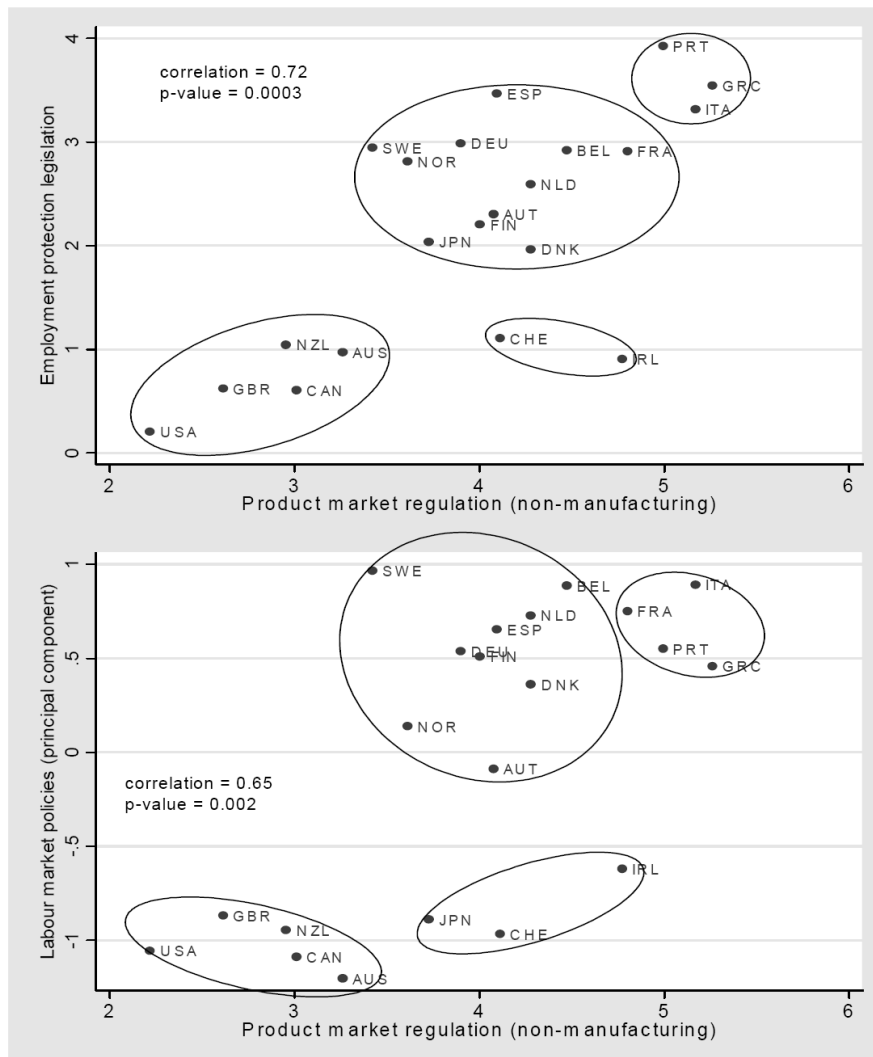


Figure 1: Labour and product market policies in OECD countries (average 1981-2002). Source: Nicoletti and Scarpetta (2005)

In the third chapter, I concentrate on the relationship between product market competition and labour market outcomes. In recent years, such a issue has gained a growing interest among scholars and policy-makers, especially in Europe. The attention has been mainly focused on the role of product market reforms as useful tool to tackle unemployment. In a recent paper, for instance, Nicoletti and Scarpetta (2005) first show the positive correlation between the level of product market regulation (synthesized by an index going from 0 to 6, with 6 capturing the most intensive level of regulation), the strictness of employment protection legislation, and the amount of labour market policies in OECD countries (see Figure 1). Then, they find a positive causal relationship between product market deregulation and the employment rate. Their estimates also suggest that the negative impact of such product market rigidities on employment are much costlier, the more regulated is the labour market. Therefore, product market reforms should induce larger gains in term of employment in countries whose labour market is more rigid.

The main question to which this chapter tries to answer is whether fiercer product market competition leads to higher employment and higher real wages. I construct a general equilibrium model with Cournot competition in the intermediate goods sector and matching frictions in the labour market. In equilibrium, the degree of competition (and, in turn, the level of employment, real wages and hours worked) is not the same among the intermediate markets but is endogenously distributed. I then show that lower entry costs or a reduction in workers' bargaining power raise aggregate employment but have an ambiguous effect on the real wage. Employment increases because with fiercer competition, firms' mark-up decreases, and this in turn raises the labour demand. Yet, two conflicting effects are at work on the real wage. Fiercer competition on the product market makes the consumption goods cheaper, but it also reduces the total rents that employees can attain in the negotiation for the wage. The first effect tends to raise the real wage, the second one tends to lower it. The final impact remains therefore unclear.

The third chapter makes also a normative contribution. The long run free-entry equilibrium leads to an inefficiently high level of employment, if workers' bargaining power is not strong enough. This result depends on two sources of externalities. Any firm deciding to enter the market lowers both

the probability for other firms to fill their vacancy and, by making the market more competitive, their (expected) profits. These two effects are not taken into account by the single firm, so that entry is more desirable to the entrant than it is to society. To limit the incentives firms have to enter the market, worker's bargaining power must be high enough, so that employees capture a large fraction of the total rent.

Matching Models and the Data

In the fourth chapter, I survey a relatively recent literature that scrutinizes the empirical consistency of matching models. Shimer (2005a) has argued that a text-book matching model is unable to explain the cyclical variation of unemployment and vacancies in the U.S. economy. In particular, the job-finding rate results 12 times more volatile in the data than in the model, whereas the standard deviation of job vacancies in the data is 10 times larger than in the model. Costain and Reiter (2006) have found the existence of a trade-off in the model's performance: Any attempt to change the calibrated values in order to amend such business cycle inability jeopardizes the model's predictions about the impact of unemployment benefits on the hazard rate.

In presenting the literature originated by these findings, I distinguish three different avenues that have been followed to correct the model: change in wage formation, change in the calibration, changes in the model specification. Papers belonging to the first avenue aim to reduce the responsiveness of wages to changes in productivity, and so to amplify the fluctuations in vacancies and unemployment. A second approach is pursued by Hagedorn and Manovskii (2006). According to them, the model is unable to mimic the business cycle behaviour of unemployment and vacancies because of an erroneous calibration of two key parameters: the instantaneous utility of being unemployed (composed by the utility of leisure and the level of the unemployment benefits) and workers' bargaining power. With a higher calibrated value for the utility of unemployment and a lower bargaining power for workers, the model succeeds in replicating the observed business cycle fluctuations. Finally, a third group of papers argues that a standard matching framework contains some simplified assumptions (for instance about the labour market flows in or out

of employment or the absence of capital) that inevitably jeopardize the quantitative consistency of the model. Enriching the basic set-up with some facts of the life features, such as turnover costs or on-the-job search, would bridge the gap between data and the model. I compare the results obtained by the three different approaches. In the light of the quantitative results obtained both at business cycle and at a policy analysis level, I conclude that the third route is the most successful.

CHAPTER **1** **Complementarities and Substitutabilities in Matching Models**

1.1 Introduction

The equilibrium matching model has become the standard framework of analysis for aggregate labour markets. In a standard Pissarides (2000) framework, the matching process between one firm and one worker is not instantaneous, because of some generic frictions present in the labour market. The product market, on the contrary, is perfectly competitive and firms face an infinitely elastic demand, so that an increase in supply does not affect the equilibrium price (see for instance Mortensen and Pissarides, 1999).

In order to introduce a more realistic framework for the goods market, some scholars (for instance Joseph, Pierrard, and Sneessens, 2004, Pierrard, 2005, or Cahuc and Zylberberg, 2004, at pages 618-622) have abandoned the assumption of an infinite elasticity of demand and considered a two-tier productive scheme. Different types of workers (usually, low-skilled and high-skilled ones) are hired in intermediate good sectors. Such goods, together with capital, are sold to a final representative firm that produces the unique consumption

good¹. Papers of this kind make often use of numerical simulations and scant attention is paid to analytical properties.

This chapter takes a different stand. I consider a simplified framework in which there are only two intermediate sectors, the capital market is ignored, and I look for the conditions under which a (unique) steady-state equilibrium exists. I show that these conditions concern the homogeneity of the final good function and the limit properties of its derivatives (i.e. the marginal productivities).

More precisely, constant returns to scale in the final good function are a sufficient condition to ensure that, if an equilibrium exists, then it is unique. Moreover, depending on the conditions imposed on the marginal productivities, two different types of equilibrium exist. If the marginal productivity is positive as the other input tends to zero, then there is a unique equilibrium production in which production occurs in the intermediate sectors and there is a positive level of employment. On the contrary, if the marginal productivity is equal to zero as the other input tends to zero, there exists only one equilibrium with no market production and full unemployment. Such a result is particularly interesting because a Cobb-Douglas technology belongs to the second, zero production, case. The papers quoted above either consider a CES production function (Cahuc and Zylberberg, 2004) or, in the case of Cobb-Douglas technology, they contain other theoretical features that make impossible to check the uniqueness of a “zero production” equilibrium. Nevertheless, the conclusions of this chapter should make scholars cautious about the introduction of a Cobb-Douglas function in models of this kind.

I then perform some comparative statics to highlight the differences between this framework and the infinitely elastic set-up. When the demand for goods is no longer infinitely elastic, new complementarities and substitutabilities arise in addition to that generated by the matching technology. Every additional vacancy created in one sector decreases firms’ revenues in that sector and enhances firms’ revenues in the other one. These effects come through the price of the intermediate goods. A standard matching framework does not take these effects into account, so it overestimates the impact of a shock or policy inter-

¹Acemoglu (2001) has also constructed a similar model, but with one decisive difference: workers are identical ex ante and can be employed in high-paid or low-paid jobs.

vention in the sector where such shock occurs and, at the same time, it ignores the effects that emerge in other sectors of the economy.

Finally, I consider the welfare problem. If the social planner has no distributional concerns, the Hosios (1990) condition applied to each labour markets ensures the efficiency of the decentralized economy. This makes sense as search externalities are the only departure from the competitive framework. So, the social planner selects the same level of employment he would choose in the infinitely elastic set-up. This is no longer true if distributional issues are taken into account, that is if the social planner wants to favour one sector of the economy. In that case, the Hosios condition is no longer sufficient to implement the optimal solution.

1.2 The Model

1.2.1 Production Technology

Assume an economy with one final good (the numeraire), two intermediate goods sectors and two types m and n of infinitely-lived and risk-neutral workers. This analysis can be carried out even with more intermediate sectors. Here, I consider only two for simplicity. The goods markets are perfectly competitive. Each producer of an intermediate good hires only one type of worker. Moreover, every m -skilled employee produces one unit of the intermediate good m and every n -skilled employee produces one unit of the intermediate good n . Let E_m (respectively, E_n) denote both the amount of the m intermediate good (respectively, the n intermediate good) used to produce the final good and the number of workers employed in the m -th (resp. n -th) sector. The final good production function is homogeneous of degree one and written as:

$$Y = F(E_m, E_n), \text{ with } \frac{\partial F}{\partial E_i} > 0 \text{ and } \frac{\partial^2 F}{\partial E_i^2} < 0, i \in \{m, n\}. \quad (1.1)$$

Furthermore, the two inputs are p-substitutes ($0 < \frac{\partial^2 F}{\partial E_m \partial E_n} < +\infty$)². Let p_i denote the real price of the intermediate good i . Cost minimization in the final sector leads to $p_i = \partial F(E_m, E_n) / \partial E_i$, with $i \in \{m, n\}$. The price of each

²I also assume Inada conditions: $\lim_{E_i \rightarrow 0} \frac{\partial F}{\partial E_i} = +\infty$ and $\lim_{E_i \rightarrow +\infty} \frac{\partial F}{\partial E_i} = 0$.

intermediate good depends negatively on the number of workers employed in that sector and positively on the number of workers employed in the other sector.

1.2.2 Search Technology

The model is Markovian and developed in steady state. Time is continuous. Each type of worker can be either unemployed or be employed in his sector. The labour market is perfectly segmented. That is, every m -type worker can be hired only by firms in the m sector and the same holds for n -type workers. Due to various imperfections, the matching process is not instantaneous. The matching function is by assumption identical in both intermediate sectors and it is written respectively $M_i = m(U_i, V_i)$, with U_i being the number of unemployed people and V_i the number of job vacancies in sector i . The function $m(., .)$ is assumed to be increasing, concave and homogeneous of degree 1. Search intensity is exogenous and normalized to 1. Due to the constant return to scale in the matching process, the model can be developed in terms of tightness indicator, namely $\theta_i \equiv \frac{V_i}{U_i}$. The rate at which vacant jobs become filled is $q(\theta_i) = M_i/V_i = m(\frac{1}{\theta_i}, 1)$, $q'(\theta_i) < 0$. Every unemployed worker moves into employment according to a Poisson process with rate $\alpha(\theta_i) \equiv M_i/U_i = \theta_i q(\theta_i)$, with $\alpha'(\theta_i) > 0$ ³. The elasticity of the probability q_i of filling a vacancy with respect to tightness θ_i is denoted by $\eta(\theta_i) \equiv -\frac{dq(\theta_i)}{d\theta_i} \frac{\theta_i}{q(\theta_i)}$. At an exogenous rate ϕ_i a match is destroyed.

In steady state, the stocks of individuals in each position are constant. With an exogenous size of the labor force, L_i , the employment rate $e_i = E_i/L_i$ in steady state is given by:

$$e_i = \frac{\alpha(\theta_i)}{\phi_i + \alpha(\theta_i)}, \quad i \in \{m, n\}. \quad (1.2)$$

1.2.3 Preferences and job creation

Individuals are risk-neutral and I ignore the capital market. Let r be the discount rate common to all agents. In steady state, the expected lifetime income

³Moreover, $\lim_{\theta_i \rightarrow 0} q(\theta_i) = +\infty$ and $\lim_{\theta_i \rightarrow +\infty} \alpha(\theta_i) = +\infty$.

for an unemployed worker is:

$$rV_{U,i} = b_i + \alpha(\theta_i)(V_{E,i} - V_{U,i}). \quad (1.3)$$

Being unemployed is similar to holding an asset that pays a dividend of b_i , the value of home production, and at a rate $\alpha(\theta_i)$ it is transformed in employment. In this case, the worker obtains $V_{E,i}$, the asset value of being employed, and he loses $V_{U,i}$. Similarly, the steady state discounted present value of employment can be written as:

$$rV_{E,i} = w_i + \phi_i(V_{U,i} - V_{E,i}), \quad (1.4)$$

where w_i is the wage bargained in the i -th intermediate sector.

On the other side of the market, let $\Pi_{E,i}$ denote the firm's discounted expected return from an occupied job if the firm produces the i th intermediate good, namely it hires workers endowed with skill i . The discounted expected return of vacant job is $\Pi_{V,i}$. I denote k_i the cost of posting a vacancy and of selecting applicants. For $i \in \{m, n\}$, the discounted expected returns satisfy the following conditions:

$$r\Pi_{E,i} = p_i - w_i + \phi_i(\Pi_{V,i} - \Pi_{E,i}), \quad (1.5)$$

$$r\Pi_{V,i} = -k_i + q(\theta_i)(\Pi_{E,i} - \Pi_{V,i}). \quad (1.6)$$

In equilibrium, firms open vacancies as long as they yield a positive expected return. Therefore, the equilibrium condition $\Pi_{V,i} = 0$, combined with (1.5) and (1.6), yields the following vacancy-supply curve for each j :

$$w_i = VS_i(\theta_i, \theta_j) \equiv p_i - (r + \phi_i) \frac{k_i}{q(\theta_i)}. \quad (1.7)$$

We can easily see that the vacancy-supply curve represents a decreasing relationship between the net wage and labor market tightness, θ_i . Note also that (1.7) depends on tightness indicators of both sectors, θ_i and θ_j , through the price of the intermediate good p_i . So, with a zero-profit condition holding, an increase in θ_i implies a lower wage w_i for two reasons. First, because higher labor market tightness raises the expected cost of filling a vacancy, $k_i/q(\theta_i)$. Second, because higher labor market tightness in sector i enhances employment through the steady state equation (1.2). More people employed in one intermediate sector lowers the marginal productivity of that intermediate good.

So i -firms' revenues are reduced (p_i goes down). Moreover, an increase in labor market tightness in the other sector, θ_j , raises the marginal productivity of the intermediate good i , via Equation (1.2) and the condition imposed in (1.1) about the cross derivative.

1.2.4 Wage formation

I assume that wages are bargained in both sectors of the economy according to the axiomatic Nash solution. If β denotes the bargaining power of the worker ($0 < \beta < 1$), the solution to the game can be written as $V_{E,i} - V_{U,i} = \beta(V_{E,i} - V_{U,i} + \Pi_{E,i})$. This property, the Bellman equations (1.4) and (1.3) and the free-entry condition ($\Pi_{E,i} = k_i/q(\theta_i)$) lead then to the following “wage-setting curve”:

$$w_i = WS_i \equiv \beta_i(p_i + \theta_i k_i) + (1 - \beta_i)b_i. \quad (1.8)$$

For each skill, the wage-setting curve $w_i = WS_i$ cannot be shown to be always upward sloping in (θ_i, w_i) space. The reason is that, as θ_i increases, $k_i\theta_i$ obviously rises, but p_i decreases. Nevertheless, I can equate the RHS of the vacancy-supply curve (1.7) and the RHS of the wage-setting curve curve (1.8) and consider an implicit equation for θ_i , namely $\mathbb{G}_i = 0$:

$$\begin{aligned} \mathbb{G}_i(\theta_i, \theta_j) &\equiv VS_i - WS_j = \\ (1 - \beta_i)p_i - k_i \left(\frac{r + \phi_i}{q(\theta_i)} + \beta_i \theta_i \right) - (1 - \beta_i)b_i &= 0 \end{aligned} \quad (1.9)$$

Note that $\mathbb{G}_i = 0$, the equilibrium condition in labor market i , depends on θ_j only through the marginal productivity p_i . Differentiating $\mathbb{G}_i$ with respect to θ_i , I obtain:

$$\frac{d\mathbb{G}_i}{d\theta_i} = A_i + B_i, \quad (1.10)$$

with

$$A_i \equiv \left[k_i(r + \phi_i) \frac{q'(\theta_i)}{q(\theta_i)^2} - \beta_i \right] \quad \text{and} \quad B_i \equiv \left[(1 - \beta_i) \frac{\partial p_i}{\partial E_i} \frac{\partial E_i}{\partial \theta_i} \right],$$

$i \in \{m, n\}$. Such derivative is negative because A_i and B_i are negative. Moreover, note that $\lim_{\theta_i \rightarrow 0} \mathbb{G}_i(\theta_i, \theta_j) = +\infty$ and $\lim_{\theta_i \rightarrow +\infty} \mathbb{G}_i(\theta_i, \theta_j) = -\infty$ because of the Inada conditions imposed both in the final good production function and in the vacancy entry rate. So, since $\mathbb{G}_i$ is continuous and always decreasing in θ_i , it exists a θ_i such that $\mathbb{G}_i = 0, \forall \theta_j$.

Notice also that A_i is the part of the derivative in (1.10) that stems from the search technology, whereas B_i is the part that stems from the production technology. Finally, I differentiate $\mathbb{G}_i$ with respect to θ_j :

$$\frac{d\mathbb{G}_i}{d\theta_j} \equiv C_{i,j} = (1 - \beta_i) \frac{\partial p_i}{\partial E_j} \frac{\partial E_j}{\partial \theta_j} > 0. \quad (1.11)$$

The term is positive because the intermediate goods are p-substitutes.

1.3 Equilibrium

The steady-state equilibrium values of tightness in both sectors are characterized by a system of two equations:

$$\begin{cases} G_n(\theta_n, \theta_m) = 0 \\ G_m(\theta_m, \theta_n) = 0 \end{cases} \quad (1.12)$$

where every function $\mathbb{G}_i$ represents the equilibrium condition in the i -th labor market. The novelty with respect to the standard matching model hinges on the link between the two intermediate sectors: each labor market depends on tightness of the other one through the price (the marginal productivity) of the intermediate good. Depending on the properties of p_n and p_m , two types of equilibria are possible in this framework: One with positive levels of tightness, the other with no production and no employment. Proposition 1 and Proposition 2 present the results.

1.3.1 Existence and uniqueness of a positive equilibrium

Proposition 1

1. *EXISTENCE. If on both markets $\lim_{E_j \rightarrow 0} p_i > 0$, there is at least one positive steady-state equilibrium in tightness levels.*
2. *UNIQUENESS. If $\lim_{E_j \rightarrow 0} p_i > 0$ on both markets and $F(E_m, E_n)$ exhibits constant returns to scale, the equilibrium is unique.*

Proof. I divide the proof in four steps. The first three steps prove point 1 of the Proposition. The last step proves point 2.

1. $\mathbb{G}_n = 0$ and $\mathbb{G}_m = 0$ define monotonously increasing relationships in (θ_n, θ_m) space.

Think of $\mathbb{G}_n = 0$ and $\mathbb{G}_m = 0$ as two functions in (θ_n, θ_m) space. Using (1.10) and (1.11) and applying the implicit function theorem, I get:

$$\left. \frac{d\theta_i}{d\theta_j} \right|_{\mathbb{G}_i=0} = - \frac{\partial \mathbb{G}_i / \partial \theta_j}{\partial \mathbb{G}_i / \partial \theta_i} = - \frac{C_{i,j}}{A_i + B_i} > 0. \quad (1.13)$$

with $i \in \{m, n\}$, $i \neq j$. I denote $\theta_m = g_m(\theta_n)$ the explicit function of $\mathbb{G}_m(\theta_m, \theta_n) = 0$, and $\theta_m = g_n(\theta_n)$ the explicit function of $\mathbb{G}_n(\theta_n, \theta_m) = 0$ in the (θ_n, θ_m) space. Let also $\theta_n = g_m^{-1}(\theta_m)$ and $\theta_n = g_n^{-1}(\theta_m)$ denote their inverse functions.

Finally, I define $H(\theta_n) \equiv g_m(\theta_n) - g_n(\theta_n)$. If $H(\theta_n)$ crosses the horizontal axis, the existence of (at least) one equilibrium is proved. If $H(\theta_n)$ is also a monotonic function, the equilibrium is unique.

2. $H(\theta_n)$ is positive as θ_n tends to a positive number χ_n .

Since $\lim_{\theta_i \rightarrow 0} E_i = 0$ and $\lim_{E_i \rightarrow 0} p_j > 0$ ($i \in \{m, n\}$) then $\lim_{\theta_n \rightarrow 0} g_m(\theta_n) = \chi_m$ and $\lim_{\theta_m \rightarrow 0} g_n^{-1}(\theta_m) = \chi_n$, with χ_m and χ_n being positive finite numbers. The function $\theta_m = g_m(\theta_n)$ is therefore positive for any $\theta_n > 0$. Hence, $\lim_{\theta_n \rightarrow \chi_n} H(\theta_n) \equiv g_m(\theta_n) - g_n(\theta_n) > 0$.

3. $H(\theta_n)$ is negative as θ_n tends to a positive number Ψ_n .

Applying L'Hospital rule to equation (1.2), I get that $\lim_{\theta_i \rightarrow +\infty} E_i = L_i$. Since L_i is a positive finite number, p_j is also positive and finite as $E_i = L_i$. So, $\lim_{\theta_n \rightarrow +\infty} g_m(\theta_n) = \Psi_m$, with Ψ_m being a positive finite number. Moreover, $\lim_{\theta_m \rightarrow +\infty} g_n^{-1}(\theta_m) = \Psi_n$, with Ψ_n being a positive finite number. Then, $\lim_{\theta_n \rightarrow \Psi_n} H(\theta_n) \equiv g_m(\theta_n) - g_n(\theta_n) = -\infty$. From step 2 and 3, one concludes that the function H crosses the horizontal axis at least once. The existence of (at least) an equilibrium is proved.

4. $H(\theta_n)$ is a decreasing function.

A sufficient condition for uniqueness of the equilibrium is $H'(\theta_n) = g'_m(\theta_n) - g'_n(\theta_n) < 0$. This implies:

$$\left. \frac{d\theta_m}{d\theta_n} \right|_{\mathbb{G}_n=0} > \left. \frac{d\theta_m}{d\theta_n} \right|_{\mathbb{G}_m=0} \quad \forall \theta_n. \quad (1.14)$$

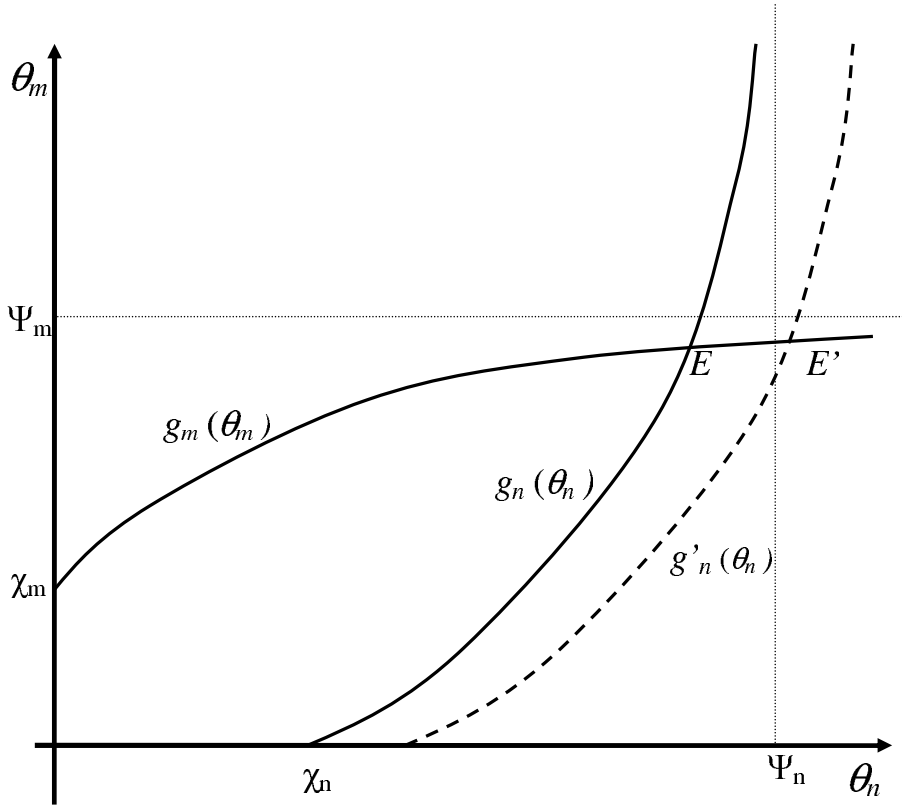


Figure 1.1: Existence and uniqueness of the steady-state equilibrium.

From (1.13), one derives:

$$\left. \frac{d\theta_m}{d\theta_n} \right|_{\mathbb{G}_n=0} = - \frac{B_n + A_n}{C_{n,m}} \quad (1.15)$$

$$\left. \frac{d\theta_m}{d\theta_n} \right|_{\mathbb{G}_m=0} = - \frac{C_{m,n}}{B_m + A_m} \quad (1.16)$$

I multiply the numerator of (1.15) by the denominator of (1.16) and the numerator of (1.16) with the denominator of (1.15). I get four positive terms in the LHS and only one positive term in the RHS. For (1.14) to hold, the four positive terms on the LHS must be greater than the term on the RHS. One of the term on the LHS is:

$$B_m B_n = (1 - \beta_m)(1 - \beta_n) \frac{\partial p_n}{\partial E_n} \frac{\partial E_n}{\partial \theta_n} \frac{\partial p_m}{\partial E_m} \frac{\partial E_m}{\partial \theta_m} \quad (1.17)$$

The positive term on the RHS is:

$$C_{n,m} C_{m,n} = (1 - \beta_m)(1 - \beta_n) \frac{\partial p_n}{\partial E_m} \frac{\partial E_m}{\partial \theta_m} \frac{\partial p_m}{\partial E_n} \frac{\partial E_n}{\partial \theta_n}. \quad (1.18)$$

Expressions (1.17) and (1.18) are equal because of the Euler's formula for linear homogeneous functions, that is $\frac{\partial^2 F}{\partial^2 Q_n} \frac{\partial^2 F}{\partial^2 Q_m} = \left(\frac{\partial^2 F}{\partial Q_n \partial Q_m} \right)^2$. Then, inequality (1.14) is verified. An equilibrium exists and it is unique. Figure 1.1 illustrates it. ■

1.3.2 Equilibrium with zero production

Proposition 2

1. *EXISTENCE.* If on both markets $\lim_{E_j \rightarrow 0} p_i = 0$, there is one steady-state equilibrium with $\theta_n = \theta_m = 0$.
2. *UNIQUENESS.* If $\lim_{E_j \rightarrow 0} p_i = 0$ on both markets and $F(E_m, E_n)$ exhibits constant returns to scale, the unique steady-state equilibrium is such that $\theta_n = \theta_m = 0$.

Proof. Notice that in the previous proof I used the condition $\lim_{E_j \rightarrow 0} p_i > 0$ only in Step 2. So Step 1, Step 3, and Step 4 hold even if $\lim_{E_j \rightarrow 0} p_i = 0$. So, to prove Proposition 2, I only need to re-consider Step 2 of the previous proof.

$H(\theta_n) = 0$ as θ_n tends to 0.

If $\lim_{E_i \rightarrow 0} p_j = 0$ ($i \in \{m, n\}$), firms do not get profits by entering market j . Hence, no vacancies are posted in sector j and $\theta_j = 0$. So $\lim_{\theta_n \rightarrow 0} g_m(\theta_n) = 0$ and $\lim_{\theta_m \rightarrow 0} g_n^{-1}(\theta_m) = 0$. From Step 1 of the previous proof, $g_n(\theta_n)$ (and, consequently, the inverse $g_n^{-1}(\theta_m)$) are monotonically increasing functions. So if $\lim_{\theta_m \rightarrow 0} g_n^{-1}(\theta_m) = 0$, then also $\lim_{\theta_n \rightarrow 0} g_n(\theta_n) = 0$. Therefore, $\lim_{\theta_n \rightarrow 0} H(\theta_n) \equiv g_m(\theta_n) - g_n(\theta_n) = 0$. This proves point 1 of Proposition 2.

$H(\theta_n)$ is a decreasing function.

From Step 4 of the previous proof, $H(\theta_n)$ is a monotonically decreasing function, provided that $F(E_m, E_n)$ is CRS. So $H(\theta_n)$ starts from the origin and for any $\theta_n > 0$ it is always negative. This means that the unique equilibrium is such that $\theta_n = \theta_m = 0$. Figure 1.2 illustrates it. ■

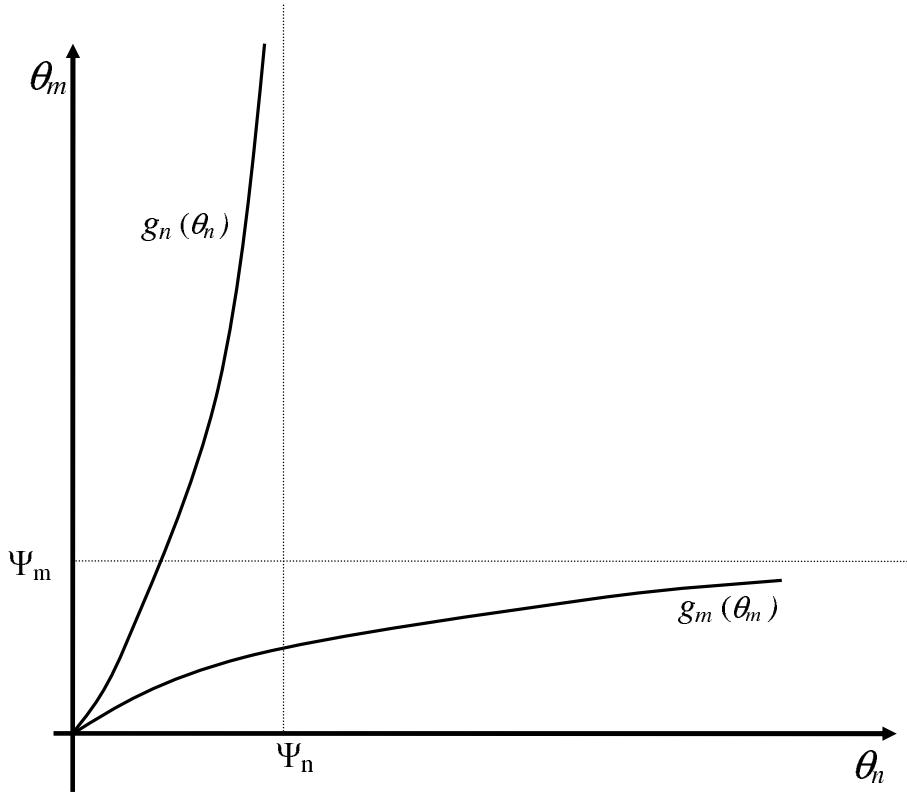


Figure 1.2: Equilibrium with no production.

The results of Proposition 1 and 2 can be summarized as follows. Constant returns to scale in the final good production function are a sufficient condition to ensure the uniqueness of an equilibrium, provided it exists. Depending on the conditions imposed on p_n and p_m , two types of equilibria in tightness levels are possible. If on both markets $\lim_{E_j \rightarrow 0} p_i > 0$, then θ_m and θ_n are positive. If, on the contrary, $\lim_{E_j \rightarrow 0} p_i = 0$ in both markets, then θ_m and θ_n are both equal to zero. In this equilibrium, no firm enter the intermediate markets, workers are all unemployed ($E_n = E_m = 0$), and there is no market production ($Y = 0$).

It is worthwhile to notice that a Cobb-Douglas function $Y = aE_n^\gamma E_m^{1-\gamma}$ fulfills the conditions of Proposition 2, because in this case the equilibrium prices are $p_n = \gamma Y/E_n$ and $p_m = (1 - \gamma)Y/E_m$.

Therefore, the results of Proposition 2 should be seen as a warning about the use of a standard Cobb-Douglas technology in models of this kind, especially

if they are aimed at policy evaluation analysis.

1.4 Complementarities and substitutabilities: Inspecting the Mechanism

The model of this chapter is not new in the search-matching literature. However, to the best of my knowledge, this chapter is the first one that studies the analytical effects that are at work in such framework. These effects crucially depend on the search and production technologies presented in Section 1.2. Therefore, I divide my analysis in two parts.

1.4.1 Production technology

The presence of a demand for goods not infinitely elastic makes productive complementarity *between sectors* and substitutability *within sectors* coexist in the economy. Consider the equilibrium system (1.12). When firms open new vacancies in sector i , labor market tightness θ_i increases and the productivity p_j in the other intermediate sector, j , increases too. This effect occurs because the two intermediate inputs are p-substitutes. Since in equilibrium the price of the intermediate good is equal to its marginal productivity, firms in sector j will get higher revenues and, to restore the zero profit condition, new vacancies in this sector will be opened too. So a productivity complementarity arise between the sectors. In terms of tightness, such complementarity is equivalent to Inequality (1.13) and it can be seen in Figure 1.1, where both curves are upward sloping. However, this is not the only effect present in the economy. When a new vacancy is opened in sector i , labor market tightness θ_i and, in turn, the level of employment E_i increase too. That has a negative impact on the price of the intermediate good p_i . Lower price of the intermediate good i reduces the expected revenues of a filled vacancy in sector i , so less vacancies will be opened in i . In other words, the increase in the activity of a firm in sector i induces other firms to create less vacancies in sector i . Productive substitutability is present in the economy; the term B_i in (1.10) captures such effect in the model.

1.4.2 Matching technology

The matching technology also affects the decentralized equilibrium outcome. As Cooper (1999) observes, any search-matching model gives rise to strategic complementarity and substitutability⁴. Strategic substitutability is present because a firm deciding to post a new vacancy lowers the probability that other vacancies can be filled and, therefore, it induces other firms to decrease their activity (that is, less vacancies are created). Such strategic substitutability must be distinguished from that we discussed in the previous subsection. There, both complementarity and substitutability are generated through a price mechanism and therefore they cannot be defined as strategic. In the goods market, agents take the price as given and adjust their behaviour according to that. On the contrary, in the labour market, the Walrasian auctioneer is replaced by a matching function and the wage is deprived by part of its allocative role (see Hosios, 1990). In this context, I can adopt the term strategic substitutability because the action of one agent negatively affects the best response of the other agents not through a price but through an aggregate matching technology. In this model, productive substitutability is represented by term B_i , strategic substitutability by A_i .

In a standard matching model, only the strategic substitutability is present: A marginal increase in the number of vacancies lowers the probability that they can be filled. In models with endogenous productivity, on the contrary, every vacancy created in sector i decreases both the probability that vacancies in i are filled and the productivity of the i -th intermediate good: search substitutability and productivity substitutability arise⁵.

⁴In his book, Cooper analyzes only the Diamond (1982) model of search and unemployment, but his reasoning can be extended to all models with a matching technology.

⁵If in our matching framework the labor force participation rate was endogenous, the symmetry between search and production technology would be perfect. Another strategic substitutability would arise, because an agent deciding to enter the labour market would negatively affect the maximizing behaviour of other agents outside the labour force. Moreover, also strategic complementarity between the sides of the market would arise: a new vacancy posted would induce agents to enter the labour market and the other way round.

1.4.3 Comparative statics and the multiplier

To better understand the policy implications of such complementarities and substitutabilities, I here analyze what happens when a policy parameter changes. Suppose for instance a reduction in the value of home production in sector n , b_n . I consider first the standard setup where the demand for intermediate goods is infinitely elastic. Then, I study this model and compare the results. A reduction in b_n affects equation $\mathbb{G}_n = 0$: a lower value of home production reduces workers' outside option in the Nash bargain and the wage w_n decreases. Hence, firms will be induced to post more vacancies in the n sector and θ_n will go up. This is the standard mechanism at work in simple matching models: when goods demand is infinitely elastic, it does not exist any link between the two intermediate sectors n and m and both the complementarity and the substitutability generated by the production function are equal to zero. Only the strategic substitutability that comes from the matching technology is present. Differentiating $\mathbb{G}_n = 0$ with respect to θ_n and b_n and applying the implicit function theorem, I obtain:

$$\left. \frac{d\theta_n}{db_n} \right|_{\mathbb{G}_n^{\bar{p}}=0} = -\frac{1-\beta_n}{A_n} < 0, \quad (1.19)$$

where the index $\bar{p}$ indicates that I am dealing with the case of infinitely elastic demand where the increase in supply does not affect the price. As expected, such derivative is negative: a decrease in the value of home production in n raises labor market tightness θ_n and so E_n .

Consider now the hypothesis of not completely elastic demand. In this case, differentiating system (1.12) with respect to θ_n , θ_m and b_n and again applying the implicit function theorem leads to:

$$\frac{d\theta_n}{db_n} = -\frac{\det \begin{bmatrix} \partial \mathbb{G}_n / \partial b_n & \partial \mathbb{G}_n / \partial \theta_m \\ 0 & \partial \mathbb{G}_m / \partial \theta_m \end{bmatrix}}{\det \begin{bmatrix} \partial \mathbb{G}_n / \partial \theta_n & \partial \mathbb{G}_n / \partial \theta_m \\ \partial \mathbb{G}_m / \partial \theta_n & \partial \mathbb{G}_m / \partial \theta_m \end{bmatrix}}. \quad (1.20)$$

Dividing the denominator and the numerator by $\frac{\partial \mathbb{G}_n}{\partial \theta_n} \frac{\partial \mathbb{G}_m}{\partial \theta_n}$ and using (1.13), I get the following expression:

$$\frac{d\theta_n}{db_n} = -\frac{\partial \mathbb{G}_n / \partial b_n}{\partial \mathbb{G}_n / \partial \theta_n} \cdot \frac{1}{1 - \left. \frac{d\theta_m}{d\theta_n} \right|_{\mathbb{G}_m=0} \cdot \left. \frac{d\theta_n}{d\theta_m} \right|_{\mathbb{G}_n=0}} < 0. \quad (1.21)$$

The first term of the product on the RHS of (1.21) is negative. It represents the effects that arise *within* the intermediate sector n when b_n marginally changes. Note that this first term is *greater* than (1.19), because $-\partial\mathbb{G}_n/\partial b_n = -\partial\mathbb{G}_n^{\bar{p}}/\partial b_n = 1 - \beta_n$, and at the denominator $\partial\mathbb{G}_n/\partial\theta_n$ is lower than $\mathbb{G}_n^{\bar{p}}/\partial\theta_n$ ⁶. The reason lies on the fact that in $\partial\mathbb{G}_n/\partial\theta_n$ there is also the component B_i that captures the productive substitutability effect: an increase in θ_n not only reduces the probability of vacancies in n to be filled, but also lowers the productivity p_n . Therefore, the first term in (1.21) is greater (namely, less negative) than (1.19) because of the productive substitutability effect within sector n . This effect tends to reduce the positive impact on θ_n caused by a decrease in the value of home production. On the other hand, the second term on the RHS of (1.21) captures the effects that intervene between the two sectors when b_n marginally changes. Both derivatives at the denominator are positive (see equation (1.13)): They represent the complementarities existing between sector n and sector m . The denominator is also positive and lower than one⁷. Therefore, the second term in (1.21) is greater than one. It is a multiplier, because it amplifies the effects of a shock on a single sector (in this case a change in b_n) through the induced response of the other sector: a lower value of home production in n increase the number of vacancies V_n and so θ_n ; this enhances productivity p_m and, consequently, V_m and θ_m . In turn, a higher θ_m will increase θ_n even more. Of course, for the existence of a multiplier, it is necessary that complementarities are present in both sectors: the increase in θ_m caused by a lower b_n must in turn affect θ_n in order to have a multiplicative impact in n sector. We can see graphically the effects of a decrease in b_n looking at Figure 1.1: the curve $\mathbb{G}_n = 0$ shifts to the right and in the new equilibrium point E' both θ_m and θ_n are higher⁸.

⁶Note that

$$\frac{d\mathbb{G}_i}{d\theta_i} = A_i + B_i < \frac{d\mathbb{G}_i^{\bar{p}}}{d\theta_i} = A_i < 0, \quad \text{with } i \in \{m, n\}.$$

⁷To see this, note that showing that the denominator in the second term of (1.21) is greater than zero is equivalent to proving inequality (1.14).

⁸Under the hypothesis of zero complementarities, the equation $\mathbb{G}_n = 0$ would be a vertical line and $\mathbb{G}_n = 0$ a horizontal line in (θ_n, θ_m) space. A decrease in b_n would shift $\mathbb{G}_n = 0$ to the right and the new equilibrium point would present a higher θ_n but the same value of θ_m .

So the presence of productive substitutability in the first term of (1.21) tends to mitigate the impact of a shock on tightness and employment, whereas productive complementarity in the second term tends to enhance it. To see which of two effects is stronger, I compute the derivatives in (1.20) and, after some algebra, I obtain:

$$\frac{d\theta_n}{db_n} = \frac{1 - \beta_n}{A_n} \cdot \frac{A_m + B_m}{A_m + B_n \frac{A_m}{A_n} + B_m}. \quad (1.22)$$

The first ratio on the RHS of (1.20) is equal to (1.19). The second ratio is positive and less than one because all the terms at the denominator and at the numerator are negative. Then:

$$\left. \frac{d\theta_n}{db_n} \right|_{\mathbb{G}_n^{\bar{p}}=0} < \frac{d\theta_n}{db_n}$$

Productive substitutability outweighs productive complementarity. So the effect of a reduction b_n on θ_n and employment E_n is lower in a model with endogenous than in a standard textbook matching model. There is an intuitive interpretation for that. Complementarities influence E_n only indirectly, via an increase in employment in the other sector, E_m . On the contrary, the productive substitutability effect influences employment E_n directly: when the value of home production b_n is reduced, less firms will enter the n -th sector and post vacancies V_n than in a standard matching model, because in this framework an increase in V_n produces not only a reduction in the probability $q(\theta_n)$ of filling a vacancy (strategic substitutability), but also a lower price p_n (productive substitutability). To sum up:

Proposition 3 *Removing the assumption of a infinitely elastic demand for goods in a Pissarides model yields the following results:*

1. *Productive complementarity: it affects the level of tightness and employment in the other sector.*
2. *Productive substitutability is stronger than complementarity: in the original sector, employment and labour market tightness change less than in a standard matching model*

Proof. Point 2 can be easily checked by applying implicit function theorem and Cramer's rule to compute $d\theta_m/db_n$. Otherwise, one can conclude about the

negative sign of $d\theta_m/d b_n$ without going through the algebraic passages; since policy in sector n affects employment in m only through p_m and p_m depends positively on E_n , any change in sector n that raises (lowers) E_n will also raise (lower) p_m and so E_m . In Figure 1.1, when the curve $\mathbb{G}_n = 0$ shifts to the right, both θ_n and θ_m increase. ■

1.5 Welfare analysis

I now look at the welfare properties of the model presented in the previous sections. I consider the simplest setup: the economy is in steady state and the discount rate r tends to 0. In Appendix 1.6 it is shown that for a social planner *without distributional concerns* maximizing social output is equivalent to maximizing the production of the consumption good and the value of home production for the unemployed workers net of the cost of posting job vacancies in both sectors. Since in this economy the only departure from the Walrasian framework is the presence of search externalities in the labour market, the Hosios (1990) condition applied both in sector n and in sector m is sufficient to ensure the efficiency of a two sectors matching model. Nevertheless, it is interesting to disentangle the effects arising from one sector and those arising in the other; so, since for linear homogeneous functions $Y = \frac{\partial F}{\partial E_n} E_n + \frac{\partial F}{\partial E_m} E_m$, I write the social planner's maximization problem as:

$$\begin{aligned} \max_{\theta_n, \theta_m} \quad & \sum_{i \in \{m, n\}} p_i E_i + b_i (L_i - E_i) - k_i V_i \\ \text{s.t.} \quad & E_i = \frac{\alpha(\theta_i)}{\phi_i + \alpha(\theta_i)} L_i, \quad i \in \{m, n\}. \end{aligned} \quad (1.23)$$

The following Proposition summarizes the results:

Proposition 4 *Consider the system (1.12) with $r \rightarrow 0$. Under the Hosios conditions, $\beta_i = \eta(\theta_i)$ for $i \in \{m, n\}$, the decentralized equilibrium is efficient.*

Proof. Knowing that $V_i = \theta_i(L_i - E_i)$ with $i \in \{m, n\}$, the F.O.C.s of problem (1.23) take the following form:

$$(p_i - b_i + k_i \theta_i) \frac{dE_i}{d\theta_i} - k_i (L_i - E_i) + E_i \frac{\partial p_i}{\partial E_i} \frac{\partial E_i}{\partial \theta_i} + E_j \frac{\partial p_j}{\partial E_i} \frac{\partial E_i}{\partial \theta_i} = 0, \quad (1.24)$$

with $i, j \in \{m, n\}$, $i \neq j$. The third term in (1.24) represents the marginal productivity loss suffered by the i th sector when θ_i increases marginally. The fourth term is the marginal productivity gain that firms in the j th sector obtain if θ_i increases. The last two terms of (1.24) can be written as:

$$E_i \frac{\partial p_i}{\partial E_i} \frac{\partial E_i}{\partial \theta_i} + E_j \frac{\partial p_j}{\partial E_i} \frac{\partial E_i}{\partial \theta_i} = \frac{\partial E_i}{\partial \theta_i} \left(E_i \frac{\partial p_i}{\partial E_i} + E_j \frac{\partial p_j}{\partial E_i} \right) = 0. \quad (1.25)$$

with $i, j \in \{m, n\}$, $i \neq j$. This is true for every constant return to scale production function, for which $\frac{\partial^2 F}{\partial^2 E_m} E_m + \frac{\partial^2 F}{\partial E_n \partial E_m} E_n = 0$.

Using (1.25) and knowing that $\alpha'(\theta_i) = q(\theta_i)[1 - \eta(\theta_i)]$, the F.O.C. (1.24) becomes:

$$q(\theta_i)[1 - \eta(\theta_i)](p_i - b_i) = k_i[\alpha(\theta_i)\eta(\theta_i) + \phi_i]. \quad i, j \in \{m, n\}, i \neq j.$$

With $\beta_i = \eta(\theta_i)$ (and $r \rightarrow 0$), this equation is equivalent to (1.9). ■

The hypothesis about a social planner that does not care about distributional issues must be taken into account. In fact, the productivity effects that are present in this model do not influence the efficient value of labor market tightness that a social planner would choose: if p_n and p_m were exogenous (i.e. if the only link between the two intermediate sector disappeared), then a social planner without distributive concern would select exactly the same values of θ_m and θ_n . Yet, introducing a decreasing demand affects the distribution of the resources between the sectors. Put in other terms, the four productivity effects present in the FOCs (1.24) can be viewed as a mechanism that shifts resources from one sector to the other. The net marginal amount of resources that accrues to sector m by means of productive complementarity and substitutability is:

$$E_m \left(\frac{\partial p_m}{\partial E_m} \frac{\partial E_m}{\partial \theta_m} + \frac{\partial p_m}{\partial E_n} \frac{\partial E_n}{\partial \theta_n} \right), \quad (1.26)$$

while the net marginal amount of resources going to sector n is equal to:

$$E_n \left(\frac{\partial p_n}{\partial E_m} \frac{\partial E_m}{\partial \theta_m} + \frac{\partial p_n}{\partial E_n} \frac{\partial E_n}{\partial \theta_n} \right). \quad (1.27)$$

As I showed above, the sum of (1.26) and (1.27) is equal to zero; so expressions in (1.26) and (1.27) either have the same absolute value with opposite sign or

are both equal to zero. If the latter is true, we have no transfer of resources between sectors. Otherwise, with (1.26) positive (negative), resources go from sector n to sector m (from sector m to sector n). A social planner that prefers one sector to the other does not select the same values of θ_m and θ_n he would have chosen in the case of infinitely elastic demand.

1.6 Conclusions

In this chapter I have studied the analytical properties of a standard matching model with a two-tier productive scheme.

I have shown the conditions under which a unique positive equilibrium exists. Such conditions are not always satisfied: imposing a standard Cobb Douglas technology, an equilibrium still exists, but useless for policy evaluation analysis, since it features full unemployment and no market production.

I have also shown that in this kind of set-up, complementarities arise between sectors and substitutabilities arise within sectors. The formers emerge because an increase in the number of vacancies posted in one sector raises also the number of vacancies posted in the other one (given the assumption of p-substitutes inputs), and this in turn enhances employment. Substitutabilities emerge because a new vacancy posted in one sector decreases both the price in that sector and the probability for another vacancy to be filled. The effects on employment are the following: in the sector where the shock occurred, employment changes less than in the standard matching framework. In the other(s) sector(s), employment also varies, and in the same direction as in the sector where the shock occurred.

1.7 Appendix: Derivation of the steady-state value of the social output

At time t , the social output can be written as

$$\mathbb{W}(t) = \int_t^{+\infty} e^{-r(T-t)} W(T) dT$$

From which, we have $W(t) = r\mathbb{W}(t) - \dot{\mathbb{W}}(t)$. Writing $\sum$ for $\sum_{i \in \{n,m\}}$ and taking all parameters as fixed, I assume that:

$$\mathbb{W}(t) = \mathbb{P}(t) + \sum [E_i(t)V_{E,i}(t) + U_i(t)V_{U,i}(t) + \Pi_i(t)]$$

in which,

$$\begin{aligned} \mathbb{P}(t) &= \int_t^{+\infty} e^{-r(T-t)} [F(E_n(T), E_m(T)) - p_n(T)E_n(T) - p_m(T)E_m(T)] dT \\ \Pi_i(t) &= \int_t^{+\infty} e^{-r(T-t)} [E_i(T)(p_i(T) - w_i(T)) - k_i V_i(T)] dT \\ rV_{E,i}(t) &= w_i(t) + \phi_i(V_{U,i}(t) - V_{E,i}(t)) + \dot{V}_{E,i}(t), \\ rV_{U,i}(t) &= b_i + \alpha(\theta_i(t))(V_{E,i}(t) - V_{U,i}(t)) + \dot{V}_{U,i}(t). \end{aligned}$$

in which V_i denotes the number of vacancies in sector i . From the last expressions,

$$\begin{aligned} W(t) &= r\mathbb{W}(t) - \dot{\mathbb{W}}(t) \\ &= r\mathbb{P}(t) - \dot{\mathbb{P}}(t) + \sum [E_i(t)(rV_{E,i}(t) - \dot{V}_{E,i}(t))] - \sum \dot{E}_i(t)V_{E,i}(t) \\ &\quad + \sum U_i(t)(rV_{U,i}(t) - \dot{V}_{U,i}(t)) - \sum \dot{U}_i(t)V_{U,i}(t) \\ &\quad + \sum E_i(t)(r\Pi_i(t) - \dot{\Pi}_i(t)) \end{aligned}$$

$$\begin{aligned} W(t) &= r\mathbb{W}(t) - \dot{\mathbb{W}}(t) \\ &= F(E_n(t), E_m(t)) - p_n(t)E_n(t) - p_m(t)E_m(t) \\ &\quad + \sum [E_i(t)(w_i(t) + \phi_i(V_{U,i}(t) - V_{E,i}(t)))] \\ &\quad + \sum U_i(t)(w_i(t) + \alpha(\theta_i(t))(V_{E,i}(t) - V_{U,i}(t))) \\ &\quad - \sum \dot{E}_i(t)V_{E,i}(t) - \sum \dot{U}_i(t)V_{U,i}(t) \\ &\quad + \sum E_i(t)(p_i(t) - w_i(t)) - k_i V_i(t). \end{aligned}$$

Simplifying,

$$\begin{aligned}
W(t) &= r\mathbb{W}(t) - \dot{\mathbb{W}}(t) \\
&= F(E_n(t), E_m(t)) \\
&\quad \sum [E_i(t)(\phi_i(V_{U,i}(t) - V_{E,i}(t)))] \\
&\quad + \sum U_i(t)(b_i + \alpha(\theta_i(t))(V_{E,i}(t) - V_{U,i}(t))) \\
&\quad - \sum \dot{E}_i(t)V_{E,i}(t) - \sum \dot{U}_i(t)V_{U,i}(t) \\
&\quad - \sum k_i V_i(t)
\end{aligned}$$

If the labour force is exogenous, $L_i = E_i + U_i$. Hence, $\dot{E}_i(t) = -\dot{U}_i(t)$. Plugging this in the last expression, a term cancels out, namely:

$$(\alpha(\theta_i(t))U_i(t) - \phi_i E_i(t) + \dot{U}_i(t))(V_{E,i}(t) - V_{U,i}(t)) = 0$$

Finally,

$$\begin{aligned}
W(t) &= F(E_n(t), E_m(t)) + \sum U_i(t)b_i \\
&\quad - \sum k_i V_i(t)
\end{aligned}$$

where both E_i and V_i have to be related to tightness. In sum,

$$\mathbb{W}(t) = \int_t^{+\infty} e^{-r(T-t)} W(T) dT$$

with $W(t)$ as defined above. After a dynamic adjustment, $W(t)$ will reach a steady state. If $r \rightarrow 0$ and since I integrate over $[t, \pm\infty)$, I can neglect the adjustment path and consider that a planner should maximize $W(t)$ where t denotes the steady state.

Employment Subsidies and Substitutable Skills

This chapter is the result of a joint work with Bruno Van der Linden, Department of Economics, Université catholique de Louvain.

2.1 Introduction

To boost employment among the relatively low-paid, several countries have introduced employment subsidies, in-work benefits or cut in payroll taxes. In frictional labor markets, these fiscal instruments change the quasi-rents that accrue to employers and workers who have matched. This induces various effects on firms' and workers' decisions. Developing a comprehensive view of these effects is essential to evaluate whether these fiscal instruments can alleviate the unemployment problem. The equilibrium matching theory is admittedly a powerful setting for such an evaluation. Davidson and Woodbury (1993) develop a matching model with different types of workers where the total number of jobs is given. At the other extreme, in Mortensen and Pissarides (2003), firms supply their optimal amount of a final good facing a infinitely

elastic demand. Mortensen and Pissarides (2003) are conscious that the second assumption, henceforth the “*M-P assumption*”, remains quite restrictive. They write: “One way to introduce a middle ground would be to assume that worker productivity depends on employment levels” (p. 72). The present chapter develops this idea.

This chapter makes a second contribution. Fiscal reforms do not take place in isolation. They interact with other existing policies. Some papers have looked at interactions with employment protection (see e.g. Mortensen and Pissarides, 2003). Such chapter looks at the interplay between the profile of unemployment insurance (respectively, short-duration active programs) and fiscal reforms targeted on low-skilled workers.

We consider an economy with a final consumption good produced with two substitutable intermediate goods under constant returns to scale.¹ Each of them is produced with a single input, namely labor of a given skill. The marginal product of labor is constant. An additional vacancy accessible to one type of worker increases employment and the quantity of the corresponding intermediate good. This decreases its productivity in the production of the final good and raises the productivity of the other intermediate good. These changes in productivity modify the marginal value of labor and hence the quasi-rents that accrue to employers and workers in all the economy. The decision to open new vacancies and the effort to search for a job are therefore affected, too. These new interactions complement the standard matching externalities (“caused by the congestion that searching firms and workers cause for each other during trade”, Pissarides, 2000, p.8). We show that the steady-state equilibrium of this economy is unique.

This framework is then further generalized to deal with some labor market policies (‘LMPs’) extensively used in OECD economies. Our aim is to show how a generalization of the M-P setting can be used to look at the interactions between employment subsidies and other LMPs. First, we introduce a two-tier benefit system (a stylized representation of many unemployment schemes). As many authors (see e.g. Fredriksson and Holmlund, 2001, or Albrecht, van den Berg, and Vroman, 2005), we assume that the fall from the ‘high’ to the ‘low’

¹Acemoglu (2001), Joseph, Pierrard, and Sneessens (2004) and Pierrard (2005) use a similar setting.

benefit occurs at a Poisson rate. Compared to a flat rate, time-varying unemployment benefits have different effects on job-search and on the wage bargain (Cahuc and Lehmann, 2000, Fredriksson and Holmlund, 2001, and Coles and Masters, 2006). Second, we add short-duration active labor market programs (counseling, job clubs, among others) that enhance the matching effectiveness of the participants. They influence job-search intensity (see Van der Linden, 2005) and wage formation (see Holmlund and Lindén, 1993). However, by assumption, this kind of active program does *not* modify workers' productivity. More generally, the model takes the distribution of skills as given. On the role of wage subsidies on human capital, see Heckman, Lochner, and Cossa (2002) and Blundell, Costas Dias, and Meghir (2003).

A natural question is to what extent these extensions to the Mortensen and Pissarides model lead to different properties. To answer this question, as M-P, we introduce a tax-subsidy schedule $a + \tau \cdot w$, where w is the net wage rate and τ is a positive proportional tax rate. If a is negative, it can be interpreted as a lump-sum employment subsidy (the interpretation retained below), in-work benefit or cut in payroll taxes. Through bargaining, the employment subsidy is partly used to raise net wages and partly to raise employment. In the case of a lump-sum employment subsidy targeted on the low-skilled workers, we show that the M-P model overestimates (respectively, underestimates) its effect on low-skilled (respectively, high-skilled) employment.

We develop a simulation exercise to provide order of magnitudes of the various effects. Contrary to what is often done, we do not contrast highly stylized European and Nord-American economies. Instead, we calibrate and then simulate the model for a specific country plagued with a large low-skill problem and an important tax wedge (Belgium).² We consider an employment subsidy on the low-skilled amounting to an ex ante reduction of 12% of their wage cost.

²Compared to individuals holding a higher education degree, the unemployment rate of those with at most a lower-secondary education level is about three times higher. As other countries of Western Europe, Belgium extensively uses reductions of employers' social security contributions targeted on low-skilled workers. Reductions of social security contributions amounted to 1.2% of GDP in 2004. In addition, a tax reform has been introduced in Belgium in 2001. Despite a small individualized income tax credit at the bottom of the income distribution, the effects of the reform in terms of increase in disposable income appear to be strongest for the middle to upper class.

With an elasticity of substitution between skilled and unskilled labor close to 1, the M-P model overestimates the impact on low-skilled employment by 5% and underestimates the impact on high-skilled employment by 0.7%. The differences are more important in terms of job-search effort and utility levels. Altogether, this leads to very different normative conclusions. The optimal low-skilled employment subsidy (i.e. the one maximizing net output) is 63% larger in the M-P model.

Economists are nowadays more and more conscious that labor market reforms should be comprehensive. Theoretical analyses of complementarities can be found in Coe and Snower (1997), Orszag and Snower (1998), Chapter 4 of OECD (2003) and Boone and van Ours (2004). Dealing with imperfectly substitutable skills and endogenous search allows to extend this literature. Empirical analyses, such as Layard, Nickell, and Jackman (1991) and Belot and van Ours (2004), conclude that particular combinations of labor market institutions and policies can be responsible of good or bad performances on the labor market. These analyses are however constrained by the availability of data. Some features such as the profile of unemployment benefits can at best be proxied by some aggregate indicators. Simulations of an equilibrium matching framework seem to be preferable. To the best of our knowledge, the literature has paid scant attention to the complementarities between employment subsidies on the one hand and the time-profile of unemployment benefits and active labor market policies on the other hand. Our simulations show that the efficiency of employment subsidies can be reinforced by reforms to active and passive labor market policies.

This chapter is organized as follows. Section 2.2 introduces the model. A streamlined version is first presented and then generalized to deal with active and passive programs. Section 2.3 provides some descriptive information about the structure of taxes on earnings and about Belgium. Section 2.4 explains how the model has been calibrated and validated. Section 2.5 presents simulation results and Section 2.6 concludes the chapter.

2.2 The framework

2.2.1 A model where worker productivity depends on employment levels

Consider a continuous-time model with a continuum of infinitely-lived and risk-neutral workers who have perfect foresight. Each firm is made of a single (filled or vacant) job. There are frictions on the labor market. Other markets are instead frictionless (perfect competition prevails). Assume two skill groups (high-skilled indexed by h and low-skilled indexed by l) and skill-specific technologies. Let y_n denote the fixed marginal products of labor ($y_l < y_h$) and E_n the employment levels ($n \in \{l, h\}$). Total output is a convex combination of $E_l y_l$ and $E_h y_h$. The interpretation is the following. A single final good (the numeraire) is produced with two intermediate goods. Let Q_l (respectively, Q_h) denote the amount of the low-skilled intermediate good (respectively, the high-skilled intermediate good). Keeping y_n constant, we have $Q_n = E_n \cdot y_n$. The final good production function is homogeneous of degree one. Total output is now given by $F(Q_l, Q_h)$, with :

$$\frac{\partial F}{\partial Q_n} > 0, \lim_{Q_n \rightarrow 0} \frac{\partial F}{\partial Q_n} + \infty, \lim_{Q_n \rightarrow +\infty} \frac{\partial F}{\partial Q_n} = 0 \text{ and } \frac{\partial^2 F}{\partial Q_n^2} < 0, n \in \{h, l\}. \quad (2.1)$$

The two inputs are p-substitutes ($0 < \frac{\partial^2 F}{\partial Q_l \partial Q_h} < +\infty$). Compared to the M-P assumption, there are two differences. First, the elasticity of substitution between the two skills can take any positive value. The higher the elasticity of substitution, the closer we are to the M-P assumption.³ Second, the marginal value of labor now varies with the number of workers in *both* sectors. Let p_n denote the real price of the intermediate good n . Profit maximization in the final good firm implies that

$$p_n = \frac{\partial F(E_l y_l, E_h y_h)}{\partial E_n y_n}, n \in \{h, l\}. \quad (2.2)$$

The marginal value product of labor depends negatively on the number of workers employed in the sector (since $\partial p_n / \partial E_n < 0$)⁴ and positively on the number of workers employed in the other sector ($\partial p_n / \partial E_m > 0, n \neq m$).

The other assumptions are standard. Workers are able to direct their search. The flow of hires, M_n is a function of the number of vacancies, V_n and the

³The elasticity of substitution between skilled and unskilled labor lies between 1 and 2 (Cahuc and Zylberberg, 2004, p. 211).

⁴A similar property could also be achieved with “large” firms and diminishing returns to labor. However, we here avoid the complex intra-firm bargaining issues (see Stole and Zwiebel, 1996, and Cahuc and Wasmer, 2001).

number of job-seekers measured in efficiency units, $s_n \cdot U_n$, where s_n designates the job-search effort of the U_n unemployed. L_n denotes the size of the labor force. The matching function is written $M_n = m(s_n \cdot U_n, V_n)$. The function $m(\cdot, \cdot)$ is assumed to be increasing, concave and homogeneous of degree 1. Tightness is measured in efficiency units, namely $\theta_n \equiv V_n / (s_n U_n)$ or equivalently, after division by the exogenous and constant labor force L_n , $\theta_n \equiv v_n / (s_n u_n)$. The rate at which vacant jobs become filled is $q(\theta_n) \equiv M_n / V_n$, $q'(\theta_n) < 0$. A job-seeker moves into employment according to a Poisson process with rate $s_n \cdot \alpha(\theta_n) \equiv s_n \cdot \theta_n \cdot q(\theta_n)$, with $\alpha'(\theta_n) > 0$. Moreover, it is assumed that $\lim_{\theta_n \rightarrow 0} q(\theta_n) = +\infty$ and $\lim_{\theta_n \rightarrow 0} \alpha(\theta_n) = 0$.

The model is developed in steady state and in continuous time. The equality between separations (occurring at an exogenous rate ϕ_n) and entries lead to an increasing relationship between employment on the one hand, search and tightness on the other

$$E_n = \mathbb{E}(\theta_n, s_n) \equiv \frac{s_n \alpha(\theta_n)}{\phi_n + s_n \alpha(\theta_n)} L_n, \quad n \in \{h, l\} \quad (2.3)$$

Substituting this expression in (2.2) yields $p_n = p_n(\theta_l, s_l, \theta_h, s_h)$. Individuals have no access to capital markets. Let r be the discount rate common to all agents. For a worker endowed with skill n , the discounted present value in employment, $V_{E,n}$ verifies :

$$rV_{E,n} = w_n + \phi_n(V_{U,n} - V_{E,n}), \quad n \in \{h, l\} \quad (2.4)$$

where w_n is the net wage in the n th intermediate sector (working time is normalized to 1) and $V_{U,n}$ represents the discounted expected lifetime income of an unemployed. We assume that the instantaneous utility in unemployment is equal to the level of unemployment benefits (proportional to the net wages) net of the cost of job-search $d(s_n)$ (with $d(0) = 0$, $d' > 0$ and $d'' > 0$). Denoting the replacement ratio by ρ_n , $V_{U,n}$ verifies the following Bellman equation:

$$rV_{U,n} = \max_{s_n} \{ \rho_n w_n - d(s_n) + s_n \alpha(\theta_n)(V_{E,n} - V_{U,n}) \}, \quad n \in \{h, l\}. \quad (2.5)$$

At each point in time, the unemployed chooses the best level of job-search taking tightness and the net intertemporal gain as given. The first-order (and sufficient) condition balances the marginal cost of search and the corresponding marginal gain:

$$d'(s_n) = \alpha(\theta_n)(V_{E,n} - V_{U,n}), \quad n \in \{h, l\}. \quad (2.6)$$

Let $\Pi_{E,n}$ denote the firm's discounted expected return from an occupied job if the firm produces the n th intermediate good (and recruits workers endowed with skill n). For simplicity, taxation is linear. Let $a_n + \tau_n w_n$ be the amount of taxes paid if the net wage is w_n ($\tau_n \geq 0$). According to its sign, a_n is an employer tax or subsidy. It does not matter which side of the market pays or receives a_n . So, the latter can also be interpreted as a lump-sum in-work tax or subsidy. Each filled vacancy yields y_n units of output times the price p_n of the intermediate good. The discounted expected return of a vacant job in sector n is denoted by $\Pi_{V,n}$. Let k_n be the flow cost of posting a vacancy. The discounted expected returns satisfy the following conditions:

$$r\Pi_{E,n} = p_n \cdot y_n - a_n - (1 + \tau_n)w_n + \phi_n(\Pi_{V,n} - \Pi_{E,n}), \quad n \in \{h, l\} \quad (2.7)$$

$$r\Pi_{V,n} = -k_n + q(\theta_n)(\Pi_{E,n} - \Pi_{V,n}), \quad n \in \{h, l\}. \quad (2.8)$$

There is free entry of vacancies. In equilibrium, $\Pi_{V,n}$ then equals 0 in each sector. From (2.7) and (2.8), the demand side of the market can be summarized by the following “vacancy-supply curve” relating the wage and tightness on the labor market for skill n :

$$w_n = VS_n(\theta_l, s_l, \theta_h, s_h) \equiv \frac{p_n(\theta_l, s_l, \theta_h, s_h) y_n - a_n - (r + \phi_n)(k_n/q(\theta_n))}{1 + \tau_n}, \quad n \in \{h, l\}, \quad (2.9)$$

with $\partial VS_n/\partial \theta_n < 0$. Higher tax parameters have a negative effect on the ‘feasible’ wage $w_n \forall (\theta_l, s_l, \theta_h, s_h)$.

When a worker and an employer form a match, the surplus $V_{E,n} - V_{U,n} + \Pi_{E,n}$ is shared through bargaining.⁵ Under Nash bargaining, if β_n denotes the exogenous bargaining power of the type- n worker ($0 < \beta_n < 1$), the solution to the game can be written as

$$(1 - \beta_n)(1 + \tau_n)(V_{E,n} - V_{U,n}) = \beta_n \Pi_{E,n}, \quad n \in \{h, l\} \quad (2.10)$$

This property, the Bellman equations (2.4) and (2.5) and the free-entry condi-

⁵In some countries, like France, the wage of the low-skilled is not bargained over but equal to the legal minimum wage. The latter is periodically adjusted upwards to keep relative wages approximately constant. We have verified that the qualitative properties of the employment subsidy remain when w_h is negotiated and $w_l = a w_h$, where a is an exogenous parameter ($0 < a < 1$).

tion ($\Pi_{E,n} = k/q(\theta_n)$) lead then to the following “wage-setting curve”:

$$w_n = WS_n(\theta_n, s_n) \equiv \frac{1}{1 - \rho_n} \left[\frac{\beta_n}{1 - \beta_n} \frac{k_n}{1 + \tau_n} \left(s_n \theta_n + \frac{r + \phi_n}{q(\theta_n)} \right) - d(s_n) \right], n \in \{h, l\}, \quad (2.11)$$

with $\partial WS_n / \partial \theta_n > 0$. A rise in τ_n has a negative effect on the bargained wage $\forall(\theta_n, s_n)$. However, the effect is less than proportional since the instantaneous income in unemployment contains an untaxed component $-d(s_n)$.

Under free entry and taking (2.10) into account, the optimality condition (2.6) becomes:

$$d'(s_n) = \frac{\beta_n}{1 - \beta_n} \frac{k_n}{1 + \tau_n} \theta_n, \quad n \in \{h, l\}. \quad (2.12)$$

This defines an implicit increasing relationship between s_n and θ_n . Conditional on tightness, a rise in the tax rate τ_n lowers the equilibrium return of search and hence search effort. From (2.12), it is obvious that a marginal change in job-search effort does not shift the wage-setting curve.

The relationship $VS_n - WS_n = 0$ can be written as:

$$\begin{aligned} \mathbb{G}_n \equiv & (1 - \rho_n)(1 - \beta_n)\{p_n y_n - a_n\} - (1 - (1 - \beta_n)\rho_n) \frac{(r + \phi_n)k_n}{q(\theta_n)} \\ & - \beta_n s_n k_n \theta_n + (1 - \beta_n)(1 + \tau_n) d(s_n) = 0, \quad n \in \{h, l\} \end{aligned} \quad (2.13)$$

in which $p_n = p_n(\theta_l, s_l, \theta_h, s_h)$. Taking the implicit relationship (2.12) between search effort and tightness into account, differentiating $\mathbb{G}_n$ yields $\forall n$:

$$\frac{\partial \mathbb{G}_n}{\partial \theta_n} = A_n + B_n < 0 \quad (2.14)$$

$$\frac{\partial \mathbb{G}_n}{\partial \theta_m} = C_{n,m} > 0 \quad (2.15)$$

in which

$$A_n = (1 - (1 - \beta_n)\rho_n) \frac{(r + \phi_n)k_n}{q(\theta_n)^2} q'(\theta_n) - \beta_n s_n k_n < 0, \quad (2.16)$$

$$B_n = (1 - \rho_n)(1 - \beta_n) \frac{\partial p_n}{\partial y_n E_n} y_n \left[\frac{\partial \mathbb{E}_n}{\partial \theta_n} + \frac{\partial \mathbb{E}_n}{\partial s_n} \frac{\partial s_n}{\partial \theta_n} \right] < 0, \quad (2.17)$$

$$C_{n,m} = (1 - \rho_n)(1 - \beta_n) \frac{\partial p_n}{\partial y_m E_m} y_m \left[\frac{\partial \mathbb{E}_m}{\partial \theta_m} + \frac{\partial \mathbb{E}_m}{\partial s_m} \frac{\partial s_m}{\partial \theta_m} \right] > 0. \quad (2.18)$$

In these expressions, $A_n < 0$ is the effect found in the standard matching model. A higher tightness raises the exit rate out of unemployment (pushing bargained wages upwards) and increases the expected duration needed to

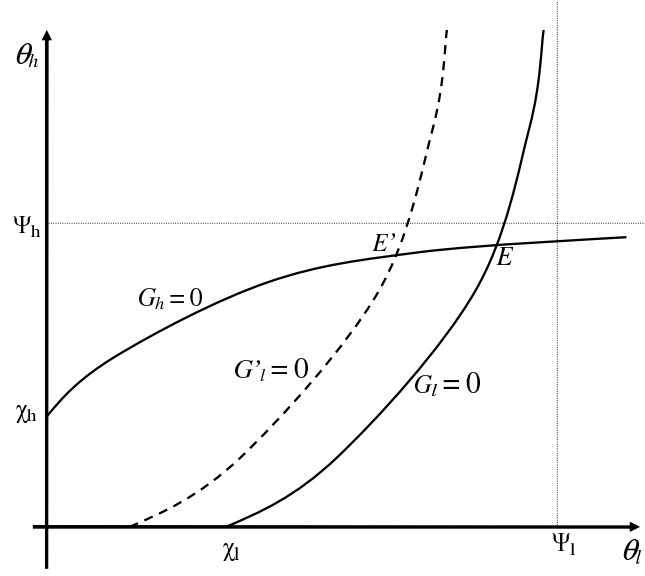


Figure 2.1: The unique steady-state equilibrium.

fill a vacancy (reducing the wage that firms can afford under free entry). B_n is a new negative term that captures the effects of a higher tightness in sector n on employment in this sector and hence on the price of the corresponding intermediate good.⁶ As the labor market becomes more tight, employment increases. In addition, a higher job finding rate raises search effort which in turn raises employment. These combined positive effects on employment lower the marginal product of the corresponding intermediate good in the production of the final good. Hence, the equilibrium price p_n shrinks and this depresses the creation of vacancies. Finally, $C_{n,m}$ captures a positive cross effect. Increasing tightness in sector m raises employment in this sector. As the two intermediate goods are substitutes, the marginal product of the other intermediate good increases and this eventually stimulates the opening of vacancies in the other sector (n).

In steady-state, the equilibrium pair(s) (θ_l, θ_h) verify the system of equations $\mathbb{G}_l = \mathbb{G}_h = 0$. Each of these equalities define an increasing implicit relationship between θ_l and θ_h . It is therefore far from obvious that an equilibrium exists and is unique. This property is shown in Appendix 2.6. Figure 2.1 illustrates

⁶The partial derivative of the price $\partial p_n / \partial y_n E_n$ is computed from (2.2) and is negative. $\partial \mathbb{E}_n / \partial \theta_n$ and $\partial \mathbb{E}_n / \partial s_n$ are computed from (2.3) and are positive. Finally, $\partial s_n / \partial \theta_n$ is computed from (2.12) and is positive.

the equilibrium. Knowing the levels of tightness, the values of employment and net wages follow immediately from (2.3), (2.12) and (2.11).

We now look at the partial⁷ effects of changes in the lump-sum employment tax/subsidy, a_n , and in the tax rate, τ_n . We only consider the case $n = l$ and focus on equilibrium tightness. It should be clear that the comparative static properties are derived for an equilibrium (θ_l, θ_h) that would be the same under the M-P assumption and in our setting.

Consider a marginal change in a_l . Under the M-P assumption, the real price of the two intermediate goods being constant, the effect on tightness would be:

$$\frac{d\theta_l}{da_l} = - \left(\frac{dG_l}{da_l} \right) / \left(\frac{dG_l}{d\theta_l} \right) = (1 - \rho_l)(1 - \beta_l)/A_l < 0 \quad (2.19)$$

$$\frac{d\theta_h}{da_l} = 0 \quad (2.20)$$

Taking the endogeneity of prices into account, one gets after some manipulation:

$$\frac{d\theta_l}{da_l} = \mu_l (1 - \rho_l)(1 - \beta_l)/A_l < 0, \quad (2.21)$$

$$\frac{d\theta_h}{da_l} = - \frac{(1 - \rho_l)(1 - \beta_l)C_{h,l}}{(A_l + B_l)(A_h + B_h) - C_{l,h}C_{h,l}} < 0, \quad (2.22)$$

where, exploiting Euler's formula for linear homogeneous function,⁸ one has:

$$\mu_l = \frac{A_l(A_h + B_h)}{(A_l + B_l)(A_h + B_h) - C_{l,h}C_{h,l}} = \frac{A_l(A_h + B_h)}{A_l(A_h + B_h) + B_l A_h} < 1.$$

Figure 2.1 illustrates these effects (see the interrupted line). So, compared to the case where the real prices of the two intermediate goods are taken as constant, $d\theta_l/da_l$ is less negative. Two opposite effects are present. First, if the employment tax is augmented in a given sector, say l , there is at given prices (p_l, p_h) a reduction in tightness and hence in employment in this sector. Less employment implies a rise in the marginal product of workers and this leads to a higher price for the corresponding intermediate good Q_l . More vacancies

⁷At this stage, the budget of the State is ignored. When the budget of the State is binding, the uniqueness of equilibrium is not always guaranteed (see Rocheteau, 1999). Appendix 2.6 presents sufficient conditions for a unique equilibrium.

⁸Namely, $\frac{\partial^2 F}{\partial^2 Q_n} \frac{\partial^2 F}{\partial^2 Q_m} = \left[\frac{\partial^2 F}{\partial Q_m \partial Q_n} \right]^2$.

are therefore posted. This attenuates the initial drop in employment. Second, less employment in sector l , where the employment tax is augmented, implies a lower marginal product of the other intermediate good Q_h . Less vacancies are therefore created in sector h . And this in turn depresses job creation in sector l . One easily sees that this chain of effects creates a multiplicative effect which tends to amplify the initial decline in tightness θ_l . Since $0 < \mu_l < 1$, the first effect dominates.

This discussion implies that an employment subsidy in the low-skill sector will rise its tightness level less than under the M-P assumption. The induced effect of the employment subsidy is moreover positive in the skilled sector (while it is zero under the M-P assumption). The quantitative importance of these differences will be studied in Section 2.4.

It should also be noticed that contrary to the standard M-P model (in which marginal variations in search effort do not affect tightness in equilibrium), endogeneizing search effort matters here. For, marginal changes in search effort affect the level of employment (see (2.3)) and hence the prices of the intermediate goods. The equilibrium levels of tightness are therefore eventually modified. Consequently, endogeneizing search effort changes the impacts of the tax parameters a_n and τ_n in our model.

Effects of the tax rate

Consider a marginal change in the tax parameter τ_l . Totally differentiating $\mathbb{G}_l = \mathbb{G}_h = 0$ with respect to θ_l, θ_h and τ_l , it can be verified that the sign of the variation in both θ_l and θ_h is the one of $\partial \mathbb{G}_l / \partial \tau_l$. From (2.13), the latter is:

$$\frac{\partial \mathbb{G}_l}{\partial \tau_l} = (1 - \beta_l) \left[y_l(1 - \rho_l) \frac{\partial p_l}{\partial y_l E_l} \frac{\partial E_l}{\partial s_l} \frac{\partial s_l}{\partial \tau_l} + d(s_l) \right] > 0 \quad (2.23)$$

in which $\partial s_l / \partial \tau_l < 0$. Two mechanisms are at work. First, rising the tax rate τ_l reduces search effort and hence employment in Sector l . This raises the equilibrium price for the intermediate good sold by this sector. So, the first product between brackets in (2.23) is nonnegative. The second mechanism is not new (see Holmlund, 2002). Search activities entail an untaxed cost, namely $d(s_n)$. Therefore, despite constant replacement ratios, bargained wages adjust more than proportionally when the tax rate rises (see (2.11)). Now, the feasible wage, (2.9), adjusts proportionally. These adjustments are not compatible in equilibrium. A rise in tightness θ_l is needed to restore equilibrium.

2.2.2 Generalizing the model to encompass other LMPs

Employment subsidies do not take place in isolation. They are typically introduced in labor markets where so-called active and passive LMPs are also present. Building upon Van der Linden (2005), we now show how the framework of the previous subsection can be further extended to evaluate the interactions between these policies in a general equilibrium setting. The model will also be generalized to deal with participation decisions and an aggregate budget constraint of the State. Some hypotheses will be chosen with Continental Europe in mind.

In accordance with institutions in many OECD countries, a two-tiered benefit system is assumed to prevail. An insured unemployed whose ‘high’ benefits have expired enters a state where (s)he indefinitely can benefit from a lower unemployment (assistance) benefit. High benefits expire at an exogenous rate $\pi_n > 0$. For jobless individuals, three states are identified: Insured unemployment with high benefits (U_n), insured unemployment with low benefits (X_n) and participation (T_n) in a short-duration active labor market policy (ALMP) organized by the Public Employment Services (PES). We have in mind counseling programs, job clubs or very brief training schemes. By assumption, these policies do not change the productivity of the participants.⁹ These upper-case symbols will designate both the states and the number of individuals occupying them in steady state. The corresponding intertemporal discounted values will be denoted by $V_{U,n}$, $V_{X,n}$ and $V_{T,n}$. Figure 2.2 displays the various states and the flows in this economy. A growing literature shows that duration dependence is largely spurious in Continental Europe (see van den Berg and van Ours, 1999, Machin and Manning, 1999, Rosholm, 2001, Dejemeppe, 2005). True duration dependence is therefore assumed to be a negligible phenomenon in this economy.

Let $s_{U,n}$, $s_{X,n}$ and $s_{T,n}$ denote search intensities in the various states. A unique *exogenous* matching effectiveness parameter c_n will be associated to states U_n and X_n . For ALMP participants, this parameter can be different and will be

⁹It should be stressed that this chapter does not deal with (long-duration training) schemes that intend to enhance skills (see Albrecht, van den Berg, and Vroman, 2006, and Boone and van Ours, 2004) or to enlarge the set of occupations that are accessible (see Masters, 2000).

denoted $c_{T,n}$. It is assumed that $c_{T,n} > c_n > 0$.¹⁰ So, in the matching function $m(S_n, V_n)$, $S_n \equiv c_n (s_{U,n} U_n + s_{X,n} X_n) + c_{T,n} s_{T,n} T_n$ and tightness is defined as $\theta_n \equiv V_n/S_n$.

The unemployed receive an offer to take part to the ALMP at an exogenous rate $\gamma_n \geq 0$.¹¹ The unemployed have then to decide whether they enter the program (right away) or not. Two cases will be distinguished. First, this offer is not used to verify the availability of the unemployed. Then, the intertemporal value of those who receive an offer in state U_n (resp., X_n) is $\bar{V}_{U,n} = \max(V_{T,n}; V_{U,n})$ (resp., $\bar{V}_{X,n} = \max(V_{T,n}; V_{X,n})$). Second, this offer is used to monitor the unemployed. Someone in state U_n can be sanctioned if (s)he refuses to take part to the program. Let the sanction be an immediate entry in state X_n . Then, $\bar{V}_{U,n} = \max(V_{T,n}; V_{X,n})$. $\bar{V}_{X,n}$ remains unchanged. Finally, to capture the idea that the ALMP can be unsuccessful, it is assumed that the program fails at an exogenous rate $\lambda_n \geq 0$.

In steady state, equalities between entries and exits in each state ($U_n, X_n, \dots$) determine the level of employment E_n for each skill n . E_n increases θ_n and search effort levels $S_n \equiv (s_{U,n}, s_{X,n}, s_{T,n})$ (for details, see Appendix 2.7).

If the wage negotiation took place at the individual level, the (entry) wage would be different according to the state of origin. Having Continental Europe in mind, as Cahuc and Lehmann (2000), we assume instead that the wage in sector n is bargained over by incumbent employees on behalf of all workers of this sector. The fall-back position of these “insiders” is the intertemporal value of an unemployed entering state U_n , $V_{U,n}$. Then, the skill-specific wage is unique. The discounted value of holding a job still verifies (2.4). We keep the hypothesis of constant replacement ratios and assume the following very plausible ranking: $1 > \rho_{T,n} \geq \rho_{U,n} > \rho_{X,n} > 0$. Let $v_{\iota,n} \equiv \rho_{\iota,n} \cdot w_n - d(s_{\iota,n}) \geq$

¹⁰The ALMP can intrinsically improve the effectiveness of search effort. Alternatively, the PES can for instance give priority to participants to ALMP, in particular in the case of a closed treatment of job offers. This refers to the case where the PES select those who are suitable for vacancies in their register.

¹¹Conditioning the access to an ALMP on the level of unemployment benefits would be considered as discriminatory. So, this possibility is ruled out here. As it is observed in several countries, participation to active programs is a sufficient condition to become eligible to high benefits again. Relaxing this assumption would substantially complicate the mathematical expressions used below and in the appendixes without adding much insight.

0, $\iota \in \{U, X, T\}$. For jobless people endowed with skill n , the intertemporal values solve the following equations:

$$rV_{U,n} = \max_{s_{U,n}} \{v_{U,n} + c_n s_{U,n} \alpha(\theta_n)(V_{E,n} - V_{U,n}) + \gamma_n(\bar{V}_{U,n} - V_{U,n}) + \pi_n(V_{X,n} - V_{U,n})\}, \quad (2.24)$$

$$rV_{X,n} = \max_{s_{X,n}} \{v_{X,n} + c_n s_{X,n} \alpha(\theta_n)(V_{E,n} - V_{X,n}) + \gamma_n(\bar{V}_{X,n} - V_{X,n})\}, \quad (2.25)$$

$$rV_{T,n} = \max_{s_{T,n}} \{v_{T,n} + c_{T,n} s_{T,n} \alpha(\theta_n)(V_{E,n} - V_{T,n}) + \lambda_n(V_{U,n} - V_{T,n})\}. \quad (2.26)$$

Under the assumptions made so far and if $\lambda_n > \phi_n$,¹² Appendices A and B of Cardullo and Van der Linden (2006) show that the intertemporal values can always be ranked ($V_{E,n} > V_{T,n} > V_{U,n} > V_{X,n}$). Consequently, all the unemployed choose to take part to the program and $\bar{V}_{U,n} = \bar{V}_{X,n} = V_{T,n}$. The optimal levels of search effort s_n solve first-order conditions that are similar to (2.6). They are stated in Appendix 2.7. They imply that $s_{X,n} > s_{U,n}$ because the unemployed in the second tier gain more from searching ($V_{E,n} - V_{X,n} > V_{E,n} - V_{U,n}$). On the contrary, $s_{T,n}$ and $s_{U,n}$ cannot be ranked. The treated are induced to search harder because search effort is more efficient ($c_{T,n} > c_n$). However, when search is successful, the net gain is lower for the treated: $V_{E,n} - V_{T,n} < V_{E,n} - V_{U,n}$.

Job creation is modeled in the same way as in Section 2.2.1. Thus, the vacancy-supply curve (2.9) remains unchanged. Since the expression relating $V_{U,n}$ to the endogenous variables and the parameters is much more complex than in Section 2.2.1, the “wage-setting curve” is more involved, too (see (2.47) in Appendix 2.7). However, the properties found earlier remain. The net wage w_n is an increasing function of tightness θ_n . Marginal changes in job-search effort do not shift the wage-setting curve. The equations that characterize search effort levels in equilibrium are much more complex than (2.12). They are stated in Appendix 2.7. It remains true that search effort increases with tightness and decreases with the tax rate τ_n .

Eliminating the net wage from the wage-setting and the vacancy-supply curves yields a system of equations $\mathbb{G}_l = \mathbb{G}_h = 0$. As in Section 2.2.1, each of these equalities define an increasing implicit relationship between θ_l and θ_h . It can

¹²In Continental Europe, it is quite natural to assume that the expected length of an employment spell (taking all types of contracts into account) is longer than the expected duration of the short-duration ALMP.

easily be seen that the employment tax a_n and the tax rate τ_n play *qualitatively* the same role as in Section 2.2.1.

The equilibrium effects of the parameters characterizing the unemployment insurance system and the ALMP have already been developed under the M-P assumption.¹³ Here, we briefly summarize these effects and then explain how the comparative statics changes in our more flexible setting. We focus on two major parameters only: π_n and γ_n .

In a two-tiered unemployment benefit scheme, Van der Linden (2003b) shows that a marginal increase in the rate (π_n) at which jobless workers flow from the high to the low benefit levels raises tightness (via a decrease in the fall-back position of the workers) but it has an ambiguous impact on employment. Three different mechanisms are at work: 1) since $s_{X,n} > s_{U,n}$, a positive direct effect (i.e. conditional on θ_n); 2) A positive indirect effect via the increase in tightness; 3) A negative “entitlement effect” (see Mortensen, 1977).¹⁴

In our more flexible setting, not only the effect on employment but also the one on tightness are ambiguous in sector n . The above-mentioned direct effect and the entitlement effect influence E_n in opposite ways. If the entitlement effect is dominated by the other one,¹⁵ the rise in employment lowers the price of the intermediate good. So, in a (θ_n, w_n) space, the vacancy-supply curve shifts downwards. This move and the downward shift of the wage-setting curve explain why the net effect on θ_n is now ambiguous. In addition, the variation of E_n has an induced effect on tightness in the other sector. This mechanism has already been explained in Section 2.2.1. If the entitlement effect is dominated by the other one, a rise in π_n would unambiguously increase equilibrium tightness in the other sector.¹⁶ Recall that this cross-effect does not appear under the M-P assumption.

Van der Linden (2005) shows that a marginal rise in the rate of entry into the ALMP, γ_n , has a clear-cut negative impact on tightness but an unclear net

¹³See Van der Linden (2003b), Van der Linden (2003a), and Van der Linden (2005).

¹⁴The gain of a successful search activity also depends on the utility if the new job is lost. This gain is negatively affected by π_n .

¹⁵That is, if $\frac{\partial E_n}{\partial \pi_n} + \sum_{i \in U, X, T} \frac{\partial E_n}{\partial s_{i,n}} \frac{\partial s_{i,n}}{\partial \pi_n} > 0$. Although there is no formal proof, this sounds plausible because the entitlement effect is a delayed effect if the worker returns in unemployment.

¹⁶The proof which makes use of Euler’s formula is available upon request.

effect on employment.¹⁷ Tightness decreases because increasing γ_n has a wage-push effect (since $V_{T,n} > V_{U,n}$) and no effect on the vacancy-supply curve under the M-P assumption. Employment is influenced by γ_n via three effects: 1) a positive direct effect if, as we assume for the rest of this section, the “matching effectiveness” $c_{T,n} \cdot s_{T,n}$ is sufficiently higher than $c_n \cdot s_{X,n}$ ¹⁸; 2) A negative effect via tightness and, in turn, search effort; 3) A direct negative effect on search effort in state X_n and a positive direct effect on search effort in state T_n .

Now, in our more flexible setting, not only the effect of employment but also the one on tightness are in general ambiguous because, under the M-P assumption, the net effect of γ_n on E_n is ambiguous (see Appendix 2.8). However, if this effect is nonnegative, Appendix 2.8 shows that $d\theta_l/d\gamma_l < 0$ and $d\theta_h/d\gamma_h < 0$ in our more flexible setting.

The model of Section 2.2.1 is in addition extended to deal with the extensive margin (participation decisions). Furthermore, the government budget constraint is added to close the model. Participation is modeled as in Pissarides (2000). Inactive people have an arbitrage condition: Staying inactive or entering state X_n .¹⁹ Let $\mathcal{P}_n$ be the exogenous size of the working age population ($\mathcal{P} \equiv \sum_n \mathcal{P}_n$). Let $[V_{1,n}, V_{2,n}]$ be the finite support of the distribution of intertemporal utility levels in inactivity, $V_{I,n}$. With a uniform distribution, the participation rate is simply defined as:

$$p_n \equiv \frac{L_n}{\mathcal{P}_n} = \frac{V_{X,n} - V_{1,n}}{V_{2,n} - V_{1,n}}. \quad (2.27)$$

The budget of the State scaled by $\mathcal{P}$ can be written as follows:

$$O + \sum_n (\rho_{U,n} u_n + \rho_{X,n} x_n + (\rho_{T,n} + C) t_n) p_n \mathcal{P}_n = \sum_n (a_n + \tau_n w_n) e_n p_n \mathcal{P}_n, \quad (2.28)$$

where $u_n = U_n/L_n$, $x_n = X_n/L_n$, $t_n = T_n/L_n$, $e_n = E_n/L_n$, C is the average cost of the program and O is an exogenous level of net public expenses. The

¹⁷The same kind of reasoning holds in case of a decline in the failure rate, λ_n .

¹⁸The matching effectiveness of those in the first tier (U_n) can be raised either by sending them into the active program or by letting them enter the second tier (X_n). Independently of the induced effects, the active program cannot raise employment if the former channel is less effective than the latter (i.e. if $c_{T,n} \cdot s_{T,n} < c_n \cdot s_{X,n}$). See Appendix 2.8 for details.

¹⁹Alternatively, they could enter uninsured unemployment. In many countries, they would have access to a minimum income guarantee. The latter is in a way or another related to the lowest level of UBs.

introduction of a binding budget condition does not allow to get clearcut comparative static results. We therefore turn now to a simulation exercise. The latter will introduce a normative criterion. With risk-neutral agents and in the absence of a concern for redistribution, we consider a benevolent planner who at any moment t maximizes

$$\int_t^{+\infty} e^{-r(\tilde{t}-t)} W(\tilde{t}) d\tilde{t}, \quad (2.29)$$

where $W(\tilde{t})$ is the sum of the instantaneous income of the individuals (weighted by their numbers) and of profits made by the final and the intermediate firms. Under the budget constraint (2.28) and the assumption that the discount rate tends to zero, this benevolent planner actually maximizes net output in steady state:

$$F(E_l y_l, E_h y_h) - \sum_n (U_n d(s_{U,n}) + X_n d(s_{X,n}) + T_n d(s_{T,n})) - \sum_n k_n V_n. \quad (2.30)$$

2.3 Calibration, validation and extensions

We take the month as unit of time. Data refer mostly to 1997 where the stocks of people in the various states were fairly stable in Belgium. It should be stressed at the outset that we do not have access to individual data about (non-)participants to LMPs nor to a pilot-study. Long-term unemployment is a major problem in Belgium. During the last thirty years or so, more than 50% of the stock has typically been unemployed for more than a year. The median duration in the stock amounts to about 2 years. In Belgium, negative duration dependence is very strong but Cockx and Dejemeppe (2005) have shown that it is largely spurious in the South. On data covering the period 1995 - 2004, Heylen and Bollens (2005) find positive duration dependence for men (nearly no dependence for women) in the North of the country. Their result for men is in accordance with our theoretical model. The level of skill (understood as education) is one of the key individual characteristics that affect the hiring rate. In each region of the country and for each gender, the unemployment rate of the less-educated (at most a lower-secondary degree) is for many years two to four times higher than the one of those with post-secondary education. Due to statistical availability, only two levels of skill are distinguished. The low-skill

population is assumed to hold at most a lower-secondary degree. The low-skilled represent 34% of the active population, 30% of salaried employment and 64% of the stock of unemployed. Table 1 presents the calibrated values and the rates of people in the various states. The low-skilled total unemployment rate is about 20% against 6.5% for the skilled workers. The ratio between salaried employment and the active population, e , and the participation rate, p , are much lower for the low-skilled.

As far as the lowest wages are concerned, European countries can be broadly split in two groups: Those with a legal minimum wage fixed by the government and those without it. In the latter case, collective agreements determine the lowest wages. Belgium is an exception. There is a legal minimum gross monthly income that is negotiated by social partners. Moreover, minimum monthly or hourly wages are negotiated at the sectorial level and are typically above the legal minimum income. The theoretical model where all wages are bargained over is therefore appropriate.

To calibrate the model, we first exploit relationships derived from the model (equilibrium of flows in steady-state, the wage-setting curves, the optimality conditions). We also make use of various surveys,²⁰ published statistics,²¹ other statistics collected for the purpose of this study, and results from the literature. A sensitivity analysis is conducted on some parameters.

We take $a_h = a_l = 0$. Data on wage costs and net wages are used to fix the tax rate (including social security contributions) τ_n . These are high ($\tau_l = 1$, $\tau_h = 1.17$). The main features of an unemployment benefit (UB) system are the eligibility and end-of-entitlement rules and the time profile of UBs. Administrative data indicate that less than 2.5% of the unemployed do not receive unemployment benefits (UBs) in Belgium. In 1998, less than 2% of the unemployed have lost their entitlement (after a very long spell of unemployment). So, the model of the previous section is a good approximation of the Belgian UB system. As far as the profile of UBs is concerned, there is first a period of one year where UBs stay constant. With the month as unit of time, π_n is therefore set equal to 0.083. For about two-third of the insured unemployed, the level of ben-

²⁰Simoens, Denys, and Denolf (1998) Denolf, Denys, and Simoens (1999) and Delmotte, Van Hootegem, and Dejonckheere (2001).

²¹Published by national and regional PES in Belgium and by Eurostat (2002a) and Eurostat (2002b).

efits decreases afterwards. For the others, UBs are flat. The time-profile of skill-specific UBs is an average of the various profiles computed from administrative data. The net²² replacement ratios are displayed in Table 1. At the end of the nineties, many beneficiaries of active programs participated (often simultaneously) to a combination of three interventions (Vos, Struyven, and Bollens, 2000): Individual advice and guidance,²³ job-search assistance (such as job clubs, tips on finding jobs and writing a successful resume) and short-duration vocational training.²⁴ Due to constraints on data, those policies are taken as an aggregate and henceforth called ‘counseling programs’.

As many other papers, let us assume the following Cobb-Douglas matching function (see Petrongolo and Pissarides, 2001): $m(S_n, V_n) \equiv m_0 S_n^{0.5} V_n^{0.5}$. Parameter m_0 is a scaling factor for the various c_i ’s and for k_n .²⁵ The discount rate is fixed at 0.004 (5% on an annual basis). Annual reports of the PES allow to fix parameters ϕ_n , λ_n and γ_n (see Table 1).

The expected duration of a vacancy (2.5 month) and the share of the low-skilled in the total number of recruitments (0.38) is used to calibrate the θ ’s. The aggregate production function is a C.E.S. Due to a lack of appropriate time-series for Belgium, we use a French study (Biscourp and Gianella, 2001) to fix the elasticity of substitution to 1.1. The “vacancy-supply curves” (2.9) are then used to calibrate the k ’s. The unobserved vacancy costs can be interpreted as a black-box capturing search, screening and training costs incurred by firms to recruit workers. More generally, they implicitly also include all other set-up costs incurred in order to create the job. This explains why the calibrated values of the cost k_n are often large in the literature when the capital stock is ignored.²⁶ The marginal products $p_n \cdot y_n$ are chosen so as to produce sensible values for the ratio of the share of the wage bill in output.

²²The unemployed who live only on UBs do not pay income taxes in Belgium.

²³“Plan d’accompagnement des chômeurs” i.e. a small number of meetings with a member of the PES during a period of four months.

²⁴According to annual reports of the regional PES, there exist very short programs mixing counseling and short-lived training that lasted about 100 hours on average.

²⁵Assuming that $m_0 = 0.5$ yields reasonable values. A sensitivity analysis has been conducted. We consider an alternative matching function, inspired by the results of Cockx and Dejemeppe (2002), namely $m_0 S_n^{0.4} V_n^{0.6}$. Unreported simulation results show that the effects of changes in the tax wedge are similar.

²⁶See also Fredriksson and Holmlund (2001) and the references cited in their footnote 20.

We assume an iso-elastic cost of job-search $d(s) = \psi_n \cdot s^{\xi_n} / \xi_n$, with $\psi_n > 0$ and $\xi_n >$

1. The products $c_\iota s_\iota$, $\iota = \{T, n\}, \{X, n\}, \{U, n\}, n \in \{l, h\}$ can be computed from the flow equilibrium conditions. Conditional on these products, the calibration then fixes the c_ι 's, the s_ι 's, ξ_n , and the bargaining power of the workers β_n . This part of the calibration is based on Equations (2.47), (2.48), (2.49) and (2.50) in Appendix 2.7. Raising parameters ψ_n induces a proportional increase in $c_{T,n}$ and c_n and a proportional reduction in all search-effort levels without affecting the other parameters. So, we adopt the following normalization: $\psi_l = \psi_h = e^2$. From Table 1, an increase in γ_n has a direct positive effect on employment. Skilled workers search more intensively. As expected, they have higher matching effectiveness parameters.

The bargaining power of the skilled workers happens to verify the Hosios condition. In an economy without taxes, the calibrated bargaining power would lead to an inefficiently high level of unemployment for the low-skilled.²⁷

Following Immervoll, Kleven, Thurstup Kreiner, and Saez (2004), the elasticity of the participation rate p_n with respect to w_n is fixed to 0.4 for the low-skilled and 0.2 for the high-skilled. These assumptions and the participation rates allow to calibrate the boundaries $V_{1,n}$ and $V_{2,n}$ in (2.27). Finally, from data in Eurostat (2002a, 2002b), the average cost C is about 130 Euro per worker and per month (net of transfers to beneficiaries of the program).

To check the validity of this calibration, we look at two properties of the model that were not used during the calibration and about which some data are available. In 1997, the average stock of vacancies registered by the PES amounted to 24,500. With a market share of the PES in the range $[0.4, 0.5]$, the calibrated stock of vacancies (53,000) is an acceptable order of magnitude. With the calibrated parameters, the expected duration of an unemployment spell amounts to 11 months for the skilled and 31 months for the low-skilled. Weighted by the share of each skill in the inflow into unemployment, the mean duration would then be equal to 19 months, a result that is in line with the computations of Dejemeppe (2005).²⁸

²⁷One can wonder why β_l is somewhat higher than β_h in Table 1. Union density is one of the determinants of the bargaining power. It turns out that union density is more important among blue-collar workers than among white-collar ones.

²⁸From her analysis of unemployment dynamics in Belgium, the average unemployment duration all along the period 1987 - 1992 was close to 2 years in the South of Belgium and to

The computed wage elasticity of salaried employment (job-search effort remaining fixed) amounts to low but reasonable values, namely - 0.54 for low-skilled workers and - 0.33 for skilled ones. We also consider later an elasticity of 2 instead of 1.1. After a new calibration, the two elasticities of labor demand become respectively equal to - 0.65 and - 0.34. Finally, the elasticity of unemployment duration with respect to the level of UBs (tightness remaining fixed) is equal to 0.39 for the high-skilled and 0.26 for the low-skilled. The latter elasticities are relatively low but plausible.²⁹

2.4 Simulation results

In this section, we illustrate by how much the effects of an employment subsidy on the low-skilled, a_l , change when the M-P assumption is replaced by our more flexible setting. Then, we look for the optimal level employment subsidy. Finally, we consider the interactions between an employment subsidy and other LMPs. When the budget constraint is binding in the simulation exercises, *both tax rates τ_n are adjusted proportionally to fulfill (2.28)*.

Table 2 considers an employment subsidy $a_l = -300$ Euro/month, i.e. 12% of the calibrated wage cost. It seems to us important to measure the differences between the two models independently of a specific assumption about how the budget of the State is balanced. So, at this stage, the tax rates τ_n remain at their calibrated values. Comparing the case where the marginal values of labor are fixed (the M-P assumption) to the one where they vary lead to relatively small differences in tightness but large ones in search effort. Assuming fixed marginal values of labor leads to overestimate the level of employment E_l by about 5% and to underestimate E_h by 0.7% (see the two first lines of Table 2). Taking into account the differences in terms of the net wages, the over- and underestimations are more substantial in the case of intertemporal discounted values. As expected, the differences shrink when the elasticity of substitution, σ , increases. The magnitude of the over- and underestimations vary with the size of the employment subsidy. This is illustrated by Figure 2.3 that plots several indicators as a function of a_l . In this figure, both tax rates τ_n ($n \in \{l, h\}$)

1.5 years in the North.

²⁹According to Meyer (2002), an elasticity of 0.5 is a standard order of magnitude.

are adjusted to balance the budget of the State. This figure only displays search effort levels and the intertemporal discounted values in a single state (namely, $U_n, n \in \{l, h\}$) because the profiles in other states are very similar. The profile of τ_h is by construction the same as the one of τ_l and is therefore not displayed.

		θ	E	w	s_U	s_X	$r V_E$	$r V_U$	$r V_X$
$\sigma = 1.1$	l	+1.95	+4.97	+3.90	+11.43	+11.43	+5.15	+6.10	+6.13
	h	-0.43	-0.66	-1.39	-2.40	-2.44	-1.64	-1.86	-1.87
$\sigma = 2$	l	+1.25	+3.18	+2.53	+7.22	+7.22	+3.33	+3.93	+3.95
	h	-0.28	-0.43	-0.91	-1.58	-1.61	-1.08	-1.22	-1.23

Table 2. Properties when the marginal values of labor are fixed (the M-P assumption) and when they vary: The case of an employment subsidy $a_l = -300$ Euro/month when the budget of the State (2.28) is ignored. The elasticity of substitution of the C.E.S., $\sigma = 1.1$ or 2. Relative differences in %.

From now on, $\sigma = 1.1$. Next, we look for the optimal employment subsidy taking the other parameters of the model unchanged. This task can be divided in two steps. First, the choice of the eligible population. Second, conditional on this choice, the level of the employment subsidy a_n . All simulations made lead to one first conclusion: Targeting the employment subsidy on the low-skilled is the best thing to do. To illustrate this assertion, consider the following comparison. An employment subsidy scheme ($a_l = -300, a_h = 0$) Euro/month is compared to a scheme that has the same cost ex ante (wages and employment being fixed) and a structure similar to current practices in Belgium, namely ($a_l = -110, a_h = -81$).³⁰ Taking the budget constraint (2.28) into account, the first scheme clearly performs much better (see Table 3).³¹ So, from now on, we put a_h to zero and focus on a_l only.

³⁰Section 3 of Cardullo and Van der Linden (2006) provides more information about the Belgian policy.

³¹It is not obvious that even the high skilled prefer the first scheme to the second one. For, the latter would a priori raise their bargained wage and their employment rate more than the first scheme. However, the lower global effect of the second scheme on (un)employment leads to higher tax rates $\tau_n, \forall n \in \{l, h\}$. And this effect turns out to outweigh the others. Unreported simulation results indicate that the advantage of targeting employment subsidies on low-skilled workers remains when σ increases.

%	E_l	E_h	w_l	w_h	$r V_{E,l}$	$r V_{E,h}$	$r V_{U,l}, r V_{X,l}$	$r V_{U,h}, r V_{X,h}$
(1)	+11.6	+1.4	+8.8	+3.4	+11.6	+3.8	+13.7	+4.2
(2)	+5.0	+1.2	+2.6	+2.7	+3.5	+3.2	+4.2	+3.7

Table 3. Comparing the properties of two employment subsidy schemes: ($a_l = -300$, $a_h = 0$) Euro/month (line 1) and ($a_l = -110$, $a_h = -81$) Euro/month (line 2) when the budget of the State (2.28) is binding. Relative differences in %.

Let us now look at the optimal value of a_l . The criterion of optimality, Y , is net output in steady state (2.30) scaled by the size of the population.³² Y reaches a maximum when a_l is close to -1490 Euro/month (see Figure 2.3). We also compute net output assuming fixed marginal values of labor and find a maximum that lies out of the range of values considered in Figure 2.3, namely close to $a_l = -2430$ Euro/month. So, the normative conclusion appears to be very different whether marginal values of labor are assumed to be fixed or not. The optimal employment subsidy looks extremely large compared to the calibrated value of the net wage (1229 Euro/month). Through bargaining, the employment subsidy is to some extent appropriated by the low-skilled workers. When $a_l = -1490$, the net monthly wage amounts to 1677 Euro (+36%). Thanks to a cut in unemployment (the low-skilled unemployment rate is halved), the rise in the tax rate needed to finance the subsidy (11%) is not huge. Altogether, the optimal low-skilled wage cost is equal to 2053 Euro (16% lower than the calibrated wage cost, 2258 Euro). The optimal total amount of taxes (income taxes and social security contributions) paid on low-skilled work equals 376 Euro (i.e. 70% lower than without the employment subsidy). By comparison with current policies, consider a low-skilled single person without children. The corresponding average total amount of taxes is about 850 Euro/month in 2005. In the case of a couple with two children and a single (low-skilled) wage, the average total amount of taxes is about 620 Euro/month in 2005. So, the gap between current and optimal levels of taxes on low-skilled workers is still large.

Adopting a political economy perspective, it is however doubtful that the optimal value of a_l would be implemented. For, the skilled workers, who represent

³²Since our calibration uses a discount rate of 0.004, a fully rigorous analysis would require to look also at the adjustment path towards the steady-state values.

two-third of the active population, first benefit from the employment subsidy but start losing below $a_l \approx -600$ Euro (See Figure 2.3). The relationship between the tax rates τ_n and a_l is U-shaped. For sufficiently small values of a_l , the employment subsidy is so effective that the tax rates τ_n can be slightly reduced. Above an employment subsidy ≈ 370 Euro/month, the tax rates τ_n start rising. So, on the one hand there is the favorable effects of a_l on tightness θ_h explained in Section 2.2.1 and on the other the rise in the tax rate τ_h eventually reduces the net wage and the employment level of the high-skilled.³³ Their intertemporal discounted income starts shrinking, too.

The previous simulation exercise illustrates that “countries can engineer a reduction of unemployment *without* a sacrifice of low-end pay and a rise in low-end pay *without* a sacrifice of employment” (Phelps, 2003, p. 11). We now consider interactions between fiscal instruments and other LMPs and raise the question: Could reforms to LMPs improve the effectiveness of employment subsidies? Public expenditures on LMPs represented 3.75% of GDP in Belgium in 2003. Is it possible to engineer reforms to LMPs that reduce public expenditures but reinforce the effects of an employment subsidy and are welfare improving (at least in steady state)? To answer that question, we consider a reform that induces steeper time-profile of UBs (a rise in parameter π_n) and another that lowers the rate of entry into the active program, γ_n . Below, we consider an employment subsidy $a_l = -300$ Euro/month. According to Figure 2.3, such a subsidy improves the intertemporal utility of all individuals. Moreover, with $a_l = -300$, the ex ante cost of the subsidy equals 1% of GDP, i.e. the total amount of reductions in employers’ payroll taxes in Belgium in 2003. Given the huge public debt of this country, keeping total (ex ante) expenses constant looks reasonable.

At given tax levels, Section 2.2.2 indicated that a rise in π_n would have a clear-cut negative effect on net wages and ambiguous effects on tightness and employment. If this reform allows to reduce the tax wedge, the effect on the wage could however be reversed. This is illustrated by the following simulation. Let

³³For negative values of a_l close to zero, Figure 2.3 indicates that θ_h actually declines when a_l becomes more negative. Two effects are at work. First, as we saw in Section 2.1, conditional on $\tau_n, n \in \{l, h\}$, both tightness levels increase when a_l becomes more negative. Second, if the tax rates $\tau_n, n \in \{l, h\}$, decline, it can be checked that both levels of tightness decline too and conversely (see (2.23)). In the case of the high-skilled segment, the second effect outweighs the former for negative values of a_l close to zero.

us compare the status quo as far as π_n is concerned with a subsidy $a_l = -300$ and a reform that *simultaneously* introduces the subsidy $a_l = -300$ and a higher rate of loss of high benefits (namely, $\pi_n = 1/3$, $\forall n \in \{l, h\}$). When $a_l = -300$, total net output, Y , is 2% higher when $\pi_n = 1/3$: 2137 Euro instead of 2095. The tax rates τ_n can be reduced (for instance, τ_h equals 1.07 when $\pi_n = 1/3$ versus 1.13 when $\pi_n = 1/12$). Without this tax reduction, the low-skilled, whatever their current position, would be worse-off when $\pi_n = 1/3$, while the high-skilled would be better-off. Compared to $(\pi_n = 1/12, a_l = -300)$, Table 4 shows that the reform $(\pi_n = 1/3, a_l = -300)$ becomes a steady-state Pareto improvement when tax rates are adjusted (downwards) to fulfill the budget constraint (2.28). Unreported simulations results show that this conclusion is robust to changes in the discount rate. To check this, we considered an annual discount rate of 20% (instead of 5%), we calibrated the model again and then simulated it. Remember however that the model assumes risk neutrality.

Van der Linden (2005) studies the effect of an active program that enhances matching effectiveness. In a framework where the marginal values of labor are fixed, his simulation exercise for Belgium leads to mixed conclusions with respect to the rate of entry into the active program. (Un)employment deteriorates with γ_l while the low-skilled intertemporal indicators of welfare are improving. In addition, the welfare of the high-skilled is negatively affected by a rise in the size of the program. Given these results, we here compare the performances of $a_l = -300$ when γ_l is left unchanged with the case where this program is abandoned. In the latter case, net output is larger (2131 Euro versus 2095 with the active program). The tax rate is reduced ($\tau_h = 1.08$ instead of 1.13). Table 5 indicates that again net wages, employment and utility levels can simultaneously increase if the active program analyzed here disappears as the employment subsidy is introduced.

These simulation exercises have illustrated the existence of reforms to LMPs that at the same time reduce public expenditures and improve the effects of an employment subsidy.

		e	w	$r V_E$	$r V_U$	$r V_X$
$\pi_n = 1/12$	l	0.647	1337	960	792	788
$\pi_n = 1/3$	l	0.659	1368	981	809	807
$\pi_n = 1/12$	h	0.754	1562	1306	1136	1130
$\pi_n = 1/3$	h	0.762	1617	1354	1178	1176

Table 4. Properties of an employment subsidy $a_l = -300$ Euro/month when the budget of the State (2.28) is balanced and the expected duration of “high” benefits, $1/\pi_n$, equals 12 or 3 months.

		e	w	$r V_E$	$r V_U$	$r V_X$
With active program	l	0.647	1337	960	792	788
Without	l	0.658	1365	981	810	805
With active program	h	0.754	1562	1306	1136	1130
Without	h	0.761	1606	1344	1169	1163

Table 5. Properties of an employment subsidy $a_l = -300$ Euro/month when the budget of the State (2.28) is balanced and the active program is either present or abandoned.

2.5 Conclusion

This chapter has shown that the equilibrium search-matching model can be enriched to become an instrument of evaluation of policies targeted on specific groups. Instead of assuming a juxtaposition of labor markets, we have modeled interactions between them. The marginal value of labor then varies with the number of workers in all sectors. This chapter shows that the model remains tractable. Several analytical conclusions can still be derived. For policy evaluations, the model has afterwards been extended to deal with institutional features and various labor market policies.

Using this framework, computational experiments have shown that employment subsidies targeted on low-skilled workers perform well. At least in countries with large tax wedges, they can simultaneously raise employment, wages and intertemporal income levels of all groups. This conclusion is in accordance with those of Drèze and Malinvaud (1994), Phelps (1997) and Mortensen and

Pissarides (2003). We have also illustrated that the efficiency of employment subsidies can be reinforced by reforms to active and passive labor market policies. We have developed an extensive sensitivity analysis which suggests that these conclusions are robust.

There are some caveats to add concerning the following limitations of our theoretical framework. First, employment subsidies influence training and schooling decisions made by individuals and firms (Blundell, Costas Dias, and Meghir, 2003). Second, employment subsidies affect job destruction rates (Mortensen and Pissarides, 2003). If they are targeted on low productivity jobs, such subsidies have a clear-cut negative effect on job destruction rates. Third, there is evidence that skilled workers supply labor on less-skilled labor markets (ladder effect) and that this phenomenon reduces the effectiveness of employment subsidies (Batyra and Sneessens, 2007). These two last features have been combined in the model of Pierrard (2005), who concludes that employment subsidies targeted on low-paid workers have substantial positive effects on employment and on welfare. Fourth, it has been argued that employment subsidies targeted on low-skilled workers lock them in low-paid jobs. A model with on-the-job search and additional skill categories could take such an effect into account. Finally, for several years now, countries such as France, Germany, The Netherlands and Belgium have played a non-cooperative game to maintain their competitiveness. Cuts in payroll taxes are among the instruments used. Our analysis has only been conducted for a single economy.

2.6 Appendix: Existence and Uniqueness of the Equilibrium

CASE 1: No balanced-budget constraint

Existence and Uniqueness If $\forall (n, m), n \neq m \lim_{E_n \rightarrow 0} p_m > 0$ and $\lim_{E_n \rightarrow +\infty} p_m < +\infty$, there is a unique steady-state equilibrium.

From (2.12), search effort s_n is an increasing function of θ_n , $s_n = S(\theta_n)$. We substitute $S(\theta_n)$ into $\mathbb{G}_n = 0$ and express $\mathbb{G}_n$ in terms of θ_n and θ_m , $n \in \{l, h\}$ only ($n \neq m$).

$\mathbb{G}_h(\theta_l, \theta_h) = 0$ implicitly yields a relationship $\theta_h = g_h(\theta_l)$ with $g'_h(\theta_l) > 0$ by (2.14) and (2.15). Similarly, $\mathbb{G}_l(\theta_l, \theta_h) = 0$ implicitly defines a solution $\theta_h = g_l(\theta_l)$ with $g'_h(\theta_l) > 0$. Let $\theta_l = g_h^{-1}(\theta_h)$ and $\theta_l = g_l^{-1}(\theta_h)$ denote their inverse functions. Then, we define $H(\theta_l) \equiv g_h(\theta_l) - g_l(\theta_l)$. If $H(\theta_l)$ crosses the horizontal axis, the existence of (at least) one equilibrium is proved (steps 1 and 2 below). If $H(\theta_l)$ is in addition a monotonic function, the equilibrium is unique (step 3 below).

FIRST STEP: $H(\theta_l)$ is positive as θ_l tends to 0.

As $\lim_{\theta_n \rightarrow 0} E_n = 0$ and $\lim_{E_n \rightarrow 0} p_m > 0$ ($n \in \{l, h\}$), then $\lim_{\theta_l \rightarrow 0} g_h(\theta_l) = \chi_h > 0$ and $\lim_{\theta_h \rightarrow 0} g_l^{-1}(\theta_h) = \chi_l > 0$ (see Figure 2.1). Since $g'_l(\theta_l) > 0$, then $\lim_{\theta_l \rightarrow 0} g_l(\theta_l) < 0$. So, $\lim_{\theta_l \rightarrow 0} H(\theta_l) \equiv g_h(\theta_l) - g_l(\theta_l) > 0$.

SECOND STEP: $H(\theta_l)$ is negative for some positive value of θ_l .

Since $\lim_{E_l \rightarrow +\infty} p_h < \infty$, we get $\lim_{\theta_l \rightarrow +\infty} g_h(\theta_l) = \Psi_h > 0$ and $\lim_{\theta_h \rightarrow +\infty} g_l^{-1}(\theta_h) = \Psi_l > 0$ (see Figure 2.1). So, $\lim_{\theta_l \rightarrow \Psi_l} g_l(\theta_l) = +\infty$ and $\lim_{\theta_l \rightarrow \Psi_l} H(\theta_l) \equiv g_h(\theta_l) - g_l(\theta_l) = -\infty$.

THIRD STEP: $H(\theta_l)$ is a decreasing function.

$H'(\theta_l) = g'_h(\theta_l) - g'_l(\theta_l) < 0$ or equivalently:

$$\left. \frac{d\theta_h}{d\theta_l} \right|_{\mathbb{G}_l=0} > \left. \frac{d\theta_h}{d\theta_l} \right|_{\mathbb{G}_h=0} \quad \forall \theta_l. \quad (2.31)$$

From (2.14) and (2.15), this condition can be written as:

$$(A_l + B_l)(A_h + B_h) > C_{l,h} C_{h,l} \quad (2.32)$$

From (2.17), (2.18) and the Euler's formula for linear homogeneous functions, it can be checked that $B_l B_h = C_{l,h} C_{h,l}$. So, remembering (2.16) and (2.17), inequality (2.31) is verified.

CASE 2: Sector specific balanced-budget conditions.

Assume a specific budget constraint for each sector n ($n \in \{l, h\}$):

$$B_n(\theta_n, \tau_n) \equiv (a_n + \tau_n w_n) E_n - \rho_n w_n (L_n - E_n) = 0, \quad (2.33)$$

where $w_n = WS(\theta_n)$ and $E_n = \mathbb{E}_n(\theta_n, S(\theta_n))$. Then:

$$\frac{dB_n}{d\theta_n} = (a_n + \tau_n w_n + \rho_n w_n) \left[\frac{\partial \mathbb{E}_n}{\partial \theta_n} + \frac{\partial \mathbb{E}_n}{\partial s_n} \frac{\partial s_n}{\partial \theta_n} \right] + [\tau_n E_n - \rho_n (L_n - E_n)] \frac{dW S_n}{d\theta_n}$$

Such derivative is positive if $a_n + \tau_n w_n + \rho_n w_n > 0$ and $\tau_n E_n - \rho_n (L_n - E_n) > 0$, or one is equal to zero and the other is positive. The first condition seems plausible as long as a_n is not too negative. The second condition is fulfilled if $a_n < 0$. Moreover,

$$\frac{dB_n}{d\tau_n} = w_n E_n + [a_n + \tau_n w_n + \rho_n w_n] \frac{\partial \mathbb{E}_n}{\partial s_n} \frac{\partial s_n}{\partial \tau_n} + [\tau_n E_n - \rho_n (L_n - E_n)] \frac{dW S_n}{d\tau_n}.$$

The first term, representing the direct effect of an increase in τ_n on the budget of the State, is positive. The last two terms represent the induced effects of a higher τ_n on the budget. They are negative under the assumptions just made on the terms inside the square brackets. The sign of the derivative is therefore ambiguous. If we assume that the direct effect is stronger than the induced ones, then, applying the implicit function theorem $\left. \frac{d\theta_n}{d\tau_n} \right|_{\mathbb{E}_n=0} > 0$. Hence, the budget constraint (2.33) defines a downward relationship $\tau_n = T(\theta_n)$. Then, we can show the following result:

Existence and Uniqueness *If $\forall (n, m), n \neq m, \left. \frac{d\theta_n}{d\tau_n} \right|_{\mathbb{E}_n=0} > 0, \lim_{E_n \rightarrow 0} p_m > 0$, and $\lim_{E_n \rightarrow +\infty} p_m < +\infty$, there is a unique steady-state equilibrium in tightness levels.*

Let us substitute $s_n = S_n(\theta_n)$ and $\tau_n = T(\theta_n)$ in $\mathbb{G}_n = 0$. Then, instead of (2.14), we have:

$$\frac{dG_n}{d\theta_n} = A_n + B_n + D_n < 0.$$

where $A_n + B_n < 0$ (recall (2.16) and (2.17)) and $D_n = (1 - \beta_n)d(s_n)\frac{dT_n}{d\theta_n} < 0$. It is now easy to check that Steps 1 and 2 are the same as in the previous case. So at least an equilibrium exists. For the uniqueness, we still have to prove Inequality (2.31). When the budget of the state is introduced, this inequality becomes:

$$\left. \frac{d\theta_h}{d\theta_l} \right|_{\mathbb{G}_l=0} = -\frac{B_l + A_l + D_l}{C_{l,h}} > -\frac{C_{h,l}}{A_h + B_h + D_h} = \left. \frac{d\theta_h}{d\theta_l} \right|_{\mathbb{G}_h=0}. \quad (2.34)$$

The proof of this condition is as in Step 3. So, an equilibrium exists and is unique.

2.7 Appendix: Precise specification of various equations

The steady-state relationship defining the employment level E_n is:

$$E_n = \mathcal{E}(\theta_n, \mathbb{S}_n) \equiv L_n [(c_{T,n} s_{T,n} \alpha(\theta_n) + \lambda_n) (c_n s_{U,n} \alpha(\theta_n) [c_n s_{X,n} \alpha(\theta_n) + \gamma_n] + \pi_n c_n s_{X,n} \alpha(\theta_n)) + \gamma_n c_{T,n} s_{T,n} \alpha(\theta_n) [\pi_n + c_n s_{X,n} \alpha(\theta_n) + \gamma_n]] \Delta_{0,n}^{-1}, \quad (2.35)$$

where,

$$\begin{aligned} \Delta_{0,n} &\equiv [c_{T,n} s_{T,n} \alpha(\theta_n) + \lambda_n] ([c_n s_{U,n} \alpha(\theta_n) + \phi_n] [c_n s_{X,n} \alpha(\theta_n) + \gamma_n] \\ &\quad + \pi_n [c_n s_{X,n} \alpha(\theta_n) + \phi_n]) + \gamma_n [c_{T,n} s_{T,n} \alpha(\theta_n) + \phi_n] [\pi_n + c_n s_{X,n} \alpha(\theta_n) + \gamma_n]. \end{aligned} \quad (2.36)$$

Let

$$\begin{aligned} \Delta_{1,n} &\equiv (r + c_n s_{X,n} \alpha(\theta_n) + \gamma_n) [(r + c_n s_{U,n} \alpha(\theta_n) + \phi_n) [r + c_{T,n} s_{T,n} \alpha(\theta_n) + \lambda_n] \\ &\quad + \gamma_n [r + c_{T,n} s_{T,n} \alpha(\theta_n) + \phi_n]] + \pi_n [(r + c_{T,n} s_{T,n} \alpha(\theta_n) + \lambda_n) [r + c_n s_{X,n} \alpha(\theta_n) + \phi_n] \\ &\quad + \gamma_n [r + c_{T,n} s_{T,n} \alpha(\theta_n) + \phi_n]], \\ \Delta_{2,n} &\equiv r + \pi_n + c_n s_{X,n} \alpha(\theta_n) + \gamma_n, \\ \Delta_{3,n} &\equiv r + c_{T,n} s_{T,n} \alpha(\theta_n) + \lambda_n + \gamma_n. \end{aligned} \quad (2.37)$$

Let $\delta_{ET,n} \equiv w_n - v_{T,n} > 0$ and $\delta_{\ell',n} \equiv v_{\ell,n} - v_{\ell',n}$, $\ell, \ell' \in \{U, X, T\}$, $\ell \neq \ell'$. The following differences can be derived from Equations (2.4), (2.24), (2.26) and

(2.25):

$$V_{E,n} - V_{U,n} = [(r + c_{T,n}s_{T,n}\alpha(\theta_n) + \lambda_n) [(r + c_n s_{X,n}\alpha(\theta_n) + \gamma_n) (w_n - v_{U,n}) + \pi_n(w_n - v_{X,n})] + \gamma_n (r + \pi_n + c_n s_{X,n}\alpha(\theta_n) + \gamma_n) w_n - v_{T,n}] \Delta_{1,n}^{-1}, \quad (2.38)$$

$$V_{U,n} - V_{X,n} = [\delta_{UX,n} + c_n(s_{U,n} - s_{X,n})\alpha(\theta_n)(V_{E,n} - V_{U,n})] \Delta_{2,n}^{-1}, \quad (2.39)$$

$$V_{T,n} - V_{U,n} = [(r + c_n s_{X,n}\alpha(\theta_n) + \gamma_n) (\delta_{TU,n} + (c_{T,n}s_{T,n} - c_n s_{U,n})\alpha(\theta_n))(V_{E,n} - V_{U,n}) + \pi_n (\delta_{TX,n} + (c_{T,n}s_{T,n} - c_n s_{X,n})\alpha(\theta_n))(V_{E,n} - V_{U,n})] [\Delta_{2,n}\Delta_{3,n}]^{-1}, \quad (2.40)$$

$$V_{E,n} - V_{T,n} = [\delta_{ET,n} + (\lambda_n - \phi_n)(V_{T,n} - V_{U,n})] \Delta_{4,n}^{-1}. \quad (2.41)$$

So, if $\phi_n < \lambda_n$, one has $V_{E,n} > V_{T,n}$.

Search-effort levels verify the following (sufficient) conditions:

$$d'(s_{U,n}) = c_n \alpha(\theta_n)(V_{E,n} - V_{U,n}), \quad (2.42)$$

$$d'(s_{X,n}) = c_n \alpha(\theta_n)(V_{E,n} - V_{X,n}), \quad (2.43)$$

$$d'(s_{T,n}) = c_{T,n} \alpha(\theta_n)(V_{E,n} - V_{T,n}). \quad (2.44)$$

Under free-entry, the Nash bargain over wages leads to:

$$V_{E,n} - V_{U,n} = \mathcal{V}(\theta_n) \equiv \frac{\beta_n}{1 - \beta_n} \frac{k_n}{q(\theta_n)} \frac{1}{1 + \tau_n}, \quad \frac{\partial \mathcal{V}}{\partial \theta_n} > 0. \quad (2.45)$$

Equation (2.4) can be used to replace $V_{E,n} - V_{U,n}$ by $(w_n - rV_{U,n})/(r + \phi_n)$. So,

$$w_n = rV_{U,n} + (r + \phi_n)\mathcal{V}(\theta_n). \quad (2.46)$$

Finally, one has to replace $rV_{U,n}$ in the previous equality. This task is more complex because the number of possible positions on the labor market is larger than in Section 2.2.1. It leads to the following explicit function for the wage:

$$w_n = WS(\theta_n, \mathbb{S}_n) \equiv \frac{\sum_{\iota} \Omega_{\iota,n} (z_n - d(s_{\iota,n}) + c_{\iota,n} s_{\iota,n} \alpha(\theta_n) \mathcal{V}(\theta_n)) + (r + \phi_n) \mathcal{V}(\theta_n)}{1 - \sum_{\iota} \Omega_{\iota,n} \rho_{\iota,n}}, \quad (2.47)$$

with $\iota \in \{U, X, T\}$ and

$$\Omega_{U,n} \equiv [r + c_{T,n}s_{T,n}\alpha(\theta_n) + \lambda_n] [r + \gamma_n + c_n s_{X,n}\alpha(\theta_n)] / \Delta_{5,n}$$

$$\Omega_{T,n} \equiv \gamma_n [r + \gamma_n + c_n s_{X,n}\alpha(\theta_n) + \pi_n] / \Delta_{5,n}$$

$$\Omega_{X,n} \equiv \pi_n [r + c_{T,n}s_{T,n}\alpha(\theta_n) + \lambda_n] / \Delta_{5,n}$$

$$\Delta_{5,n} \equiv [r + c_{T,n}s_{T,n}\alpha(\theta_n) + \lambda_n + \gamma_n] [r + \gamma_n + c_n s_{X,n}\alpha(\theta_n) + \pi_n]$$

and $\Omega_{U,n} + \Omega_{T,n} + \Omega_{X,n} = 1$. As in the model of Section 2.2.1, it can be shown that this curve is not affected by marginal changes in search effort levels.

In a symmetric equilibrium, Expression (2.45) can be substituted for $V_{E,n} - V_{U,n}$ in the first-order conditions (2.42), (2.43) and (2.44) in which $V_{U,n} - V_{X,n}$ has first been replaced by (2.39) and $V_{T,n} - V_{U,n}$ by (2.40). After some manipulation, this leads for each n to:

$$\Sigma_U(\theta_n, s_{U,n}) \equiv d'(s_{U,n}) - c_n \alpha(\theta_n) \mathcal{V}(\theta_n) = 0, \quad (2.48)$$

$$\Sigma_X(\theta_n, s_{U,n}, s_{X,n}) = 0 \quad (2.49)$$

$$\begin{aligned} \text{with } \Sigma_X &\equiv \Delta_{2,n} d'(s_{X,n}) - c_n \alpha(\theta_n) [\delta_{UX,n} + (\Delta_{2,n} + c_n [s_{U,n} - s_{X,n}] \alpha(\theta_n)) \mathcal{V}(\theta_n)], \\ \Sigma_T(\theta_n, s_{U,n}, s_{X,n}, s_{T,n}) &= 0 \end{aligned} \quad (2.50)$$

$$\begin{aligned} \text{with } \Sigma_T &\equiv \Delta_{2,n} \Delta_{3,n} d'(s_{T,n}) - c_{T,n} \alpha(\theta_n) [(\Delta_{2,n} \Delta_{3,n} - [r + c_n s_{X,n} \alpha(\theta_n) + \gamma_n] \\ &\quad [c_{T,n} s_{T,n} - c_n s_{U,n}] \alpha(\theta_n) - \pi_n [c_{T,n} s_{T,n} - c_n s_{X,n}] \alpha(\theta_n)) \mathcal{V}(\theta_n) \\ &\quad - (r + c_n s_{X,n} \alpha(\theta_n) + \gamma_n) \delta_{TU,n} - \pi_n \delta_{TX,n}]. \end{aligned}$$

Totally differentiating equations (2.48), (2.49) and (2.50), it can be checked that $\frac{\partial \Sigma_{\iota}}{\partial s_{\iota'}} = 0 \ \forall \iota, \iota' \in \{\{T, n\}, \{X, n\}, \{U, n\}\}, \iota \neq \iota'$. Moreover, the levels of search effort of type- n workers increase with tightness θ_n and decrease with the tax rate τ_n .

Expression (2.43) implies that $s_{X,n}$ increases with the gain $V_{E,n} - V_{X,n} = V_{E,n} - V_{U,n} + V_{U,n} - V_{X,n} = \mathcal{V}(\theta_n) + V_{U,n} - V_{X,n}$. As $V_{T,n} > V_{U,n} > V_{X,n}$, those in state X_n gain more from the ALMP than those in U_n . Therefore, Van der Linden (2005) shows that $V_{U,n} - V_{X,n}$ shrinks with γ_n and so does $s_{X,n}$ (conditional on θ_n). From (2.44), $s_{T,n}$ increases with the gain $V_{E,n} - V_{T,n} = \mathcal{V}(\theta_n) - (V_{T,n} - V_{U,n})$. Van der Linden (2005) shows that $V_{T,n} - V_{U,n}$ shrinks with γ_n (conditional on θ_n). And so, the direct effect of γ_n on $s_{T,n}$ is positive.

2.8 Appendix: Comparative static analysis

The equilibrium effect of a marginal change in γ_l can be measured by differentiating the following system where (θ_l, θ_h) are the endogenous variables and γ_l is the parameter of interest here:

$$\begin{aligned} VS_l(\theta_l, \theta_h \mid \gamma_l) - WS_l(\theta_l \mid \gamma_l) &= 0 \\ VS_h(\theta_l, \theta_h \mid \gamma_l) - WS_h(\theta_h) &= 0 \end{aligned} \quad (2.51)$$

In these equations, $VS_n(\theta_n, \theta_m | \gamma_n)$ is defined by (2.9) where

$$p_n = \frac{\partial F(\mathcal{E}(\theta_l, \mathbb{S}_l)y_l, \mathcal{E}(\theta_h, \mathbb{S}_h)y_h)}{\partial E_n y_n},$$

$\mathbb{S}_n$ is a function of θ_n and, if $n = l$, of γ_l (see (2.48) to (2.50)).³⁴ The function $\mathcal{E}(\theta_l, \mathbb{S}_l)$ is influenced by γ_l (see (2.35)). In System (2.51), the WS_n functions are defined by (2.47).³⁵ As can be seen from this definition, γ_l influences WS_l conditional on θ_l .

Differentiating with respect to θ_l , θ_h and γ_l leads to:

$$\frac{d\theta_l}{d\gamma_l} = - \frac{\det \begin{bmatrix} \frac{\partial VS_l}{\partial \gamma_l} - \frac{\partial WS_l}{\partial \gamma_l} & \frac{\partial VS_l}{\partial \theta_h} \\ \frac{\partial VS_h}{\partial \gamma_l} & \frac{\partial VS_h}{\partial \theta_h} - \frac{\partial WS_h}{\partial \theta_h} \end{bmatrix}}{\det \begin{bmatrix} \frac{\partial VS_l}{\partial \theta_l} - \frac{\partial WS_l}{\partial \theta_l} & \frac{\partial VS_l}{\partial \theta_h} \\ \frac{\partial VS_h}{\partial \theta_l} & \frac{\partial VS_h}{\partial \theta_h} - \frac{\partial WS_h}{\partial \theta_h} \end{bmatrix}} < 0. \quad (2.52)$$

Consider first the matrix at the numerator. We know that $\frac{\partial WS_l}{\partial \gamma_l} > 0$. Moreover,

$$\frac{\partial VS_l}{\partial \gamma_l} = \frac{y_l}{1 + \tau_{1,l}} \frac{\partial p_l}{\partial (E_l y_l)} \left[\frac{\partial \mathcal{E}(\theta_l, \mathbb{S}_l)}{\partial \gamma_l} + \sum_{i \in U, X, T} \frac{\partial \mathcal{E}(\theta_l, \mathbb{S}_l)}{\partial s_{i,l}} \frac{\partial s_{i,l}}{\partial \gamma_l} \right]. \quad (2.53)$$

Looking at equation (2.35) we have that $\frac{\partial E_l}{\partial \gamma_l} > 0$ if $c_{T,l} s_{T,l}$ is “sufficiently larger” than $c_{l} s_{U,l}$ and $c_{l} s_{X,l}$.³⁶ However, the direct effects of γ_l on $s_{U,l}$ is nil, on $s_{X,l}$ is negative and on $s_{T,l}$ is positive (see the end of Appendix 2.7). So, in (2.53), the sign of the sum between brackets is ambiguous. Therefore, it can be checked that the sign of $d\theta_l/d\gamma_l$ is in general ambiguous, too.

³⁴Looking at these equations, it turns out that conditional on tightness, $s_{U,n}$ is not affected by γ_n .

³⁵In which again $\mathbb{S}_n$ is a function of θ_n but this does not matter since marginal changes in search effort do not shift the wage-setting curve.

³⁶

$$\begin{aligned} \frac{\partial E_n}{\partial \gamma_n} &= \frac{\phi_n(c_{T,n} s_{T,n} \alpha(\theta_n) + \lambda_n)}{\Delta_{0,n}^2} \left[\pi_n(c_{T,n} s_{T,n} \alpha(\theta_n) + \lambda_n + \gamma_n) c_n(s_{U,n} - s_{X,n}) \alpha(\theta_n) \right. \\ &\quad + (\pi_n + c_n s_{X,n} \alpha(\theta_n) + \gamma_n) \left((c_n s_{X,n} \alpha(\theta_n) + \gamma_n)(c_{T,n} s_{T,n} - c_n s_{U,n}) \alpha(\theta_n) \right. \\ &\quad \left. \left. + \pi_n(c_{T,n} s_{T,n} - c_n s_{X,n}) \alpha(\theta_n) \right) \right] \\ &\geq 0 \quad \text{if } c_{T,n} s_{T,n} \text{ is sufficiently larger than } c_n s_{U,n} \text{ and } c_n s_{X,n} \\ &< 0 \quad \text{if } c_{T,n} s_{T,n} < c_n s_{U,n} \text{ and } c_{T,n} s_{T,n} < c_n s_{X,n} \end{aligned}$$

Let us however assume that the expression between brackets in (2.53) is non-negative. Then, $\frac{\partial V S_l}{\partial \gamma_l}$ is nonpositive since $\frac{\partial p_l}{\partial E_l} < 0$ and so $\frac{\partial V S_l}{\partial \gamma_l} - \frac{\partial W S_l}{\partial \gamma_l} < 0$. Moreover then, $\frac{\partial V S_h}{\partial \gamma_l}$ ³⁷ is nonnegative since $\frac{\partial p_h}{\partial E_l} > 0$. It can easily be checked that $\frac{\partial V S_l}{\partial \theta_h} > 0$ and $\frac{\partial V S_h}{\partial \theta_h} - \frac{\partial W S_h}{\partial \theta_h} < 0$. Since, using again Euler's formula, we can prove that

$$\frac{\partial V S_l}{\partial \gamma_l} \frac{\partial V S_h}{\partial \theta_h} > \frac{\partial V S_h}{\partial \gamma_l} \frac{\partial V S_l}{\partial \theta_h},$$

the numerator in (2.52) is then positive, too. So does the denominator. Therefore, we conclude that $\frac{d\theta_l}{d\gamma_l}$ is negative if the direct effect of γ_l on E_l (i.e. the expression between brackets in (2.53)) is nonnegative.

To check the sign of $\frac{d\theta_h}{d\gamma_l}$ we follow the same procedure. The numerator is equal to:

$$\underbrace{\left(\frac{\partial V S_l}{\partial \theta_l} - \frac{\partial W S_l}{\partial \theta_l} \right)}_{-} \underbrace{\frac{\partial V S_h}{\partial \gamma_l}}_{+ \text{ or } 0} - \underbrace{\frac{\partial V S_h}{\partial \theta_l}}_{+} \underbrace{\left(\frac{\partial V S_l}{\partial \gamma_l} - \frac{\partial W S_l}{\partial \gamma_l} \right)}_{-}. \quad (2.54)$$

We are not able to sign the determinant at the numerator. So, the marginal effect of γ_l on θ_h is ambiguous.

³⁷ $\frac{\partial V S_h}{\partial \gamma_l} = \frac{y_h}{1+\tau_{1,h}} \frac{\partial p_h}{\partial E_l} \left[\frac{\partial \mathcal{E}(\theta_l, S_l)}{\partial \gamma_l} + \sum_{i \in U, X, T} \frac{\partial \mathcal{E}(\theta_l, S_l)}{\partial s_{i,l}} \frac{\partial s_{i,l}}{\partial \gamma_l} \right]$.

2.9 Appendix: Figures and Tables

Parameters	l	h
ϕ	0.009	0.006
$p \cdot y$ (Euro/month)	3300	4200
k	18211	41442
$\rho_U = b_U/w$	0.55	0.55
$\rho_X = b_X/w$	0.43	0.40
τ	1.00	1.17
γ	0.006	0.02
λ	0.1	0.1
π	0.083	0.083
ψ	7.4	7.4
ξ	1.142	1.139
β	0.56	0.50
c	0.17	0.31
c_T	0.19	0.34
Endogenous var.		
u	0.056	0.031
x	0.139	0.027
t	0.008	0.006
e	0.61	0.75
p	0.54	0.72
$s_U (c s_U \alpha(\theta))$	0.16 (0.03)	0.39 (0.08)
$s_X (c s_X \alpha(\theta))$	0.20 (0.04)	0.49 (0.10)
$s_T (c_T s_T \alpha(\theta))$	0.26 (0.06)	0.55 (0.12)
θ	2.22	0.83
$V/(U + X + T)$	0.09	0.14
w (Euro/month)	1229	1512

Table 1. Calibration: Parameters and levels of endogenous variables in steady state.

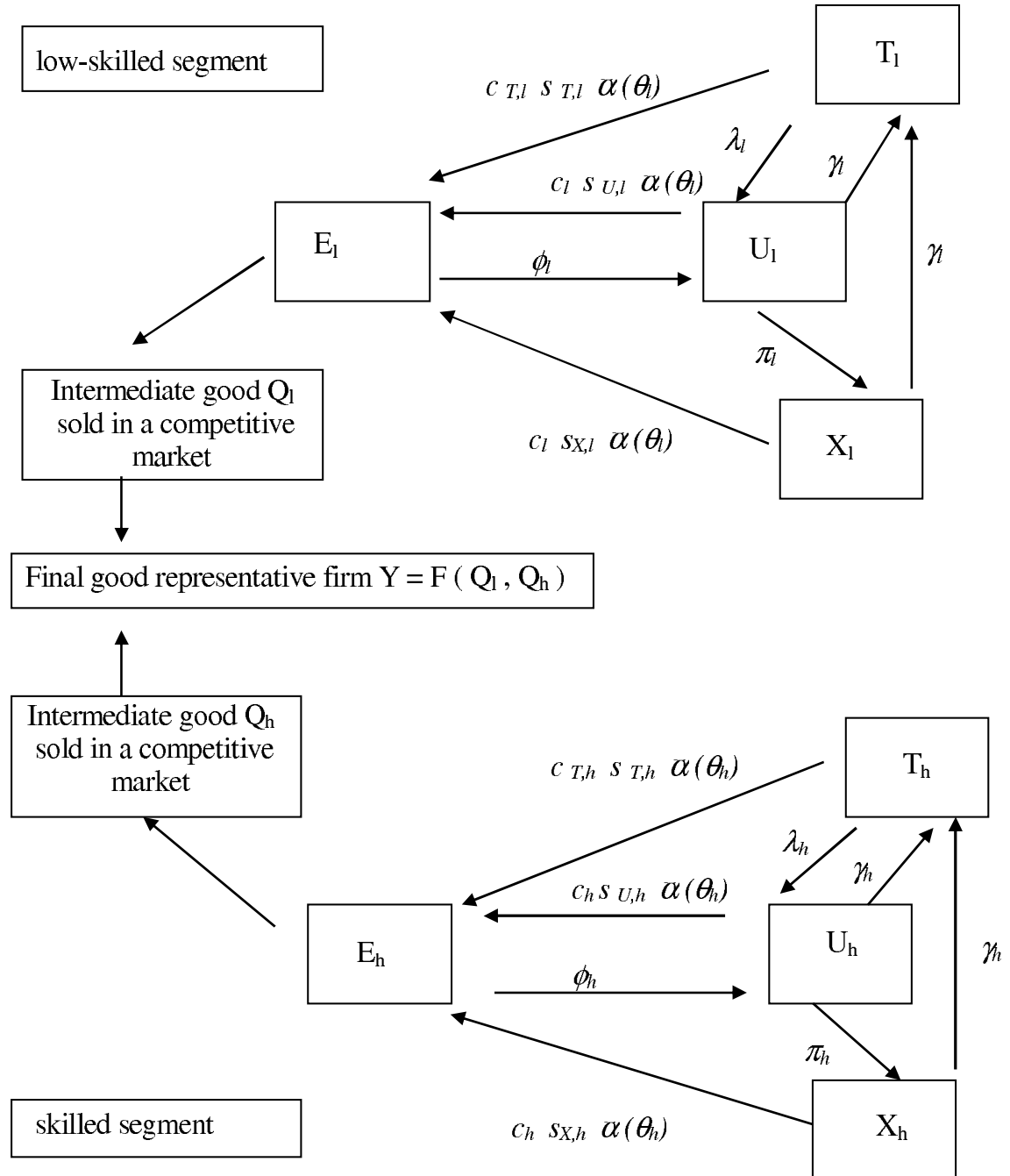


Figure 2.2: Labor market flows.

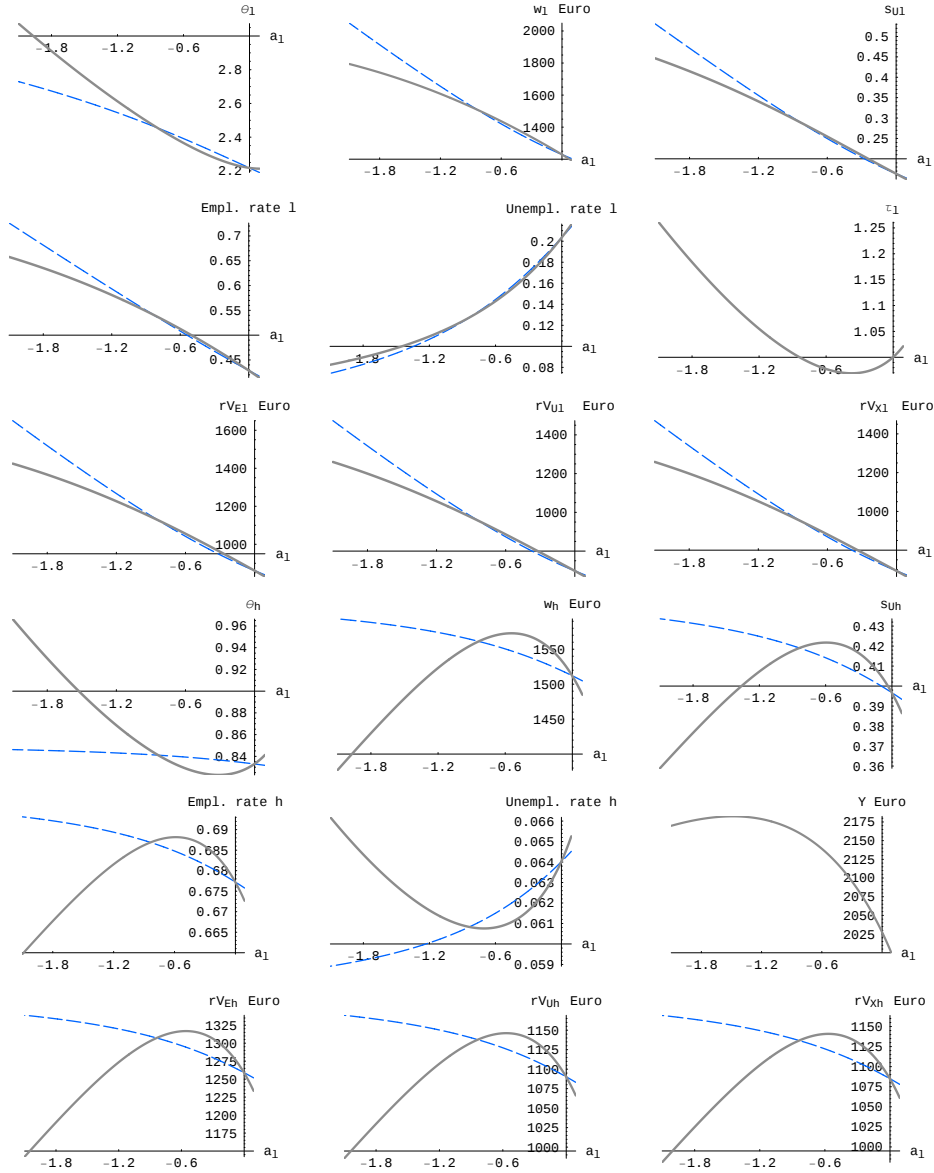


Figure 2.3: Various indicators as a function of a_l in 1000 Euro. Interrupted (respectively, thick) lines: When the marginal values of labor are fixed (resp., when they vary). The budget constraint (2.28) is binding.

Product Market Competition and Labour Market Performance

3.1 Introduction

The interactions between product market (de)regulation and labour market performance have been the objective of many empirical and theoretical papers in recent years. Does tougher competition in the goods market increase the level of employment in the labour market? According to most of the literature, the answer seems to be a qualified yes. At a theoretical level, more agents competing in the product market implies a lower mark-up that can be chosen by each single firm and a larger aggregate quantity produced in equilibrium. This in turn raises labour demand, for any given level of wages. Such a theoretical prediction seems to be confirmed by recent empirical studies. For instance, according to the OECD (2006), liberalization in goods market is one decisive factor that helps to explain why some countries (Ireland, Austria, Scandinavia, and the Netherlands) experience high employment rates even if their labour markets remain very regulated¹.

¹In this avenue, the most recent analyzes are conducted by Nicoletti and Scarpetta (2005) and Griffith, Harrison, and Macartney (2007), while a detailed survey is in Schiantarelli (2005). Considering a panel of some OECD countries over the past two decades, Nicoletti and Scarpetta reach two important conclusions. First, regulations that curb competition and entry have

Less attention, however, has been devoted to the welfare implications of product market (de)regulation on the labour market. The objective of this chapter is twofold. First, I analyze the effects of tougher competition in the goods market on employment, wages and hours worked when the labour market presents frictions and efficient bargaining is assumed between workers and firms. Second, turning to the normative analysis, I wonder what is the optimal level of competition and employment in such an economy.

To perform this task, I construct a general equilibrium model with Cournot competition in the goods sector and matching frictions *à la* Pissarides (2000) in the labour market. The choice of Cournot competition is made for two reasons. First, differently from other papers (for instance Blanchard and Giavazzi, 2003 and Ebell and Haefke, 2006), I am considering a framework in which the number of firms producing in a market varies in equilibrium according to a stochastic process, so that any firm's strategy depends not only on the actual level of competition, but also on the probability that new competitors will enter the market. The properties of the Cournot equilibrium as the number of players varies are well-known (see Frank, 1965), and it seems therefore an appropriate choice for this kind of analysis. Second, this chapter focuses on the long run free-entry equilibrium, in which Cournot models are not subject to the critiques sometimes addressed to other settings (for instance, free-entry in a monopolistic competition set-up is modeled as a change in the elasticity of substitution in the utility function, a parameter that should remain fixed).

I consider an economy with a finite and exogenous number of intermediate sectors, each of them composed by a constant labour force, and only one final consumption good. In the final good sector perfect competition is assumed, whereas firms compete *à la* Cournot in the intermediate sectors. To produce in the intermediate market, any firm needs to find a worker by posting a vacancy in the labour market, and then negotiate with him the wage and the amount of hours worked. Considering hours worked in firms' production function is coherent with the empirical evidence recently emphasized by Hall (2006). He

substantially reduced the employment rates in OECD countries over the past two decades. Second, the negative impact of such product market rigidities on employment is much costlier, the more regulated is the labour market. Therefore, product market reforms should induce larger gains in terms of employment in countries whose labour market is more rigid.

shows that in the U.S. economy over the past 60 years hours per worker can account for more than half of cyclical variations in total hours of work. Keeping the assumption of one firm-one job present in standard matching models, in a generic sector the level of employment, and consequently the degree of competition, can vary between a monopoly, (only one firm is active and only one worker is employed) and L (the maximum possible level of competition and full employment). The creation and the destruction of jobs in each market follow a continuous-time Markov Chain with a discrete number of states. The probability that one more job is created in one sector is endogenous and depends on the level of unemployment and the number of vacancies posted in that sector. In addition, at a certain exogenous rate, a new intermediate product, replacing an existing one, is invented in the economy and all the jobs present in the “old” sector are destroyed. A free-entry condition imposes that firms post vacancies as long as they earn positive expected profits. At the equilibrium, the level of competition (i.e. the number of firms competing), the real wage, and the amount of hours worked is not the same among the intermediate markets but is endogenously distributed.

The main contributions of this chapter are the following. I show that a reduction in entry costs or in workers’ bargaining power raises employment, but it has ambiguous effects on the average real wage in the economy and on employees’ utility. Previous papers (see for instance Blanchard and Giavazzi, 2003) have concluded that in the long run lower entry costs in the product market have a positive impact both on employment and on the real wage, while a reduction in workers’ bargaining power leads to more employment and leaves real wages unaffected². So, why are the conclusions on the real wage different? In this chapter, two opposite effects are at work on the average real wage. A reduction in the cost of opening a vacancy, by increasing employment and the total amount of the final consumption good produced, decreases its price and therefore raises real wages for any given level of competition in the intermediate markets (income effect). Yet, at the new equilibrium, there is a higher fraction of workers employed in sectors with fiercer competition, where real

²Spector (2004) gets a different result in a context of fixed capital in the production function. The reduction in workers’ rents may offset the reduction in the consumption good price, so that the final effect on the real wage is negative.

wages are lower (distribution effect). Blanchard and Giavazzi consider only symmetric equilibria (in which any intermediate goods has the same price), and the long-run equilibrium condition requires that profits per worker are equal to a fixed cost of entry. These two features of their model imply that the long-run real wage is a decreasing function of the cost of entry only³. Then, a decrease in this parameter raises both employment and the real wage.

The second contribution of the present chapter is a normative one. I show that a free-entry equilibrium in which workers' bargaining power is not strong enough, may deliver an inefficiently high level of employment and competition. Under the standard hypothesis of a Cobb-Douglas matching technology, imposing that the worker's bargaining power is equal to the elasticity of the matching technology leads to an excess of employment and competition. In other terms, the Hosios (1990) condition does not ensure the efficiency of the decentralized equilibrium. This is quite obvious, for the present model presents several departures from a standard matching framework: the law of motion of employment, the bargaining problem, the imperfect competition in the product market.

This result depends on two sources of externalities. Any firm deciding to enter the market lowers both the probability for other firms to fill their vacancy and, by making the market more competitive, their (expected) profits. These two effects are not taken into account by the single firm, so that entry is more desirable to the entrant than it is to society. To limit the incentives firms have to enter the market, worker's bargaining power must be high enough, so that employees capture a large fraction of the total rent.

In a similar model, Ebell and Haefke (2006) reach the opposite conclusion: If the Hosios condition holds, the level of employment is inefficiently low. The source of such conflicting results mainly depends on the different welfare functions considered. In the present chapter, the social planner's problem consists in choosing the optimal quantity produced by the single intermediate firm, and the optimal number of intermediate firms that must compete in each sec-

³In a symmetric equilibrium, the relative price (i.e the price of the good produced over the consumption price index) is equal to 1. The real wage is given by the difference between the relative price and the profit of the firm per worker. Hence, in the long run, the real wage is equal to 1 minus the fixed entry cost.

tor. Such results are then compared with the free-entry long run equilibrium. In Ebell and Haefke's paper, the social planner has to select the quantity produced by a single firm, but not the number of firms that can be active in the market. This choice is then compared with the short-run decentralized equilibrium, where free-entry is not allowed. In such a case, monopolistic competition induces each firm to produce less than the optimal level, in order to secure a higher mark-up. So, firms hire less workers than in a competitive optimal framework⁴.

It must be also stressed that such excess of entry result is in line with the conclusion exposed by Mankiw and Whinston (1986) and Suzumura and Kiyono (1987) about free entry and social inefficiency. Mankiw and Whinston prove that imperfect competition models with an homogeneous good and a fixed cost of entry deliver an inefficiently high level of competition, exactly because of the "business stealing" effect explained above.

A numerical simulation is finally conducted on the basis of Belgian data. The aim of such exercise is simply to see how the gap between the *laissez faire* and the optimal employment rates can be reduced. A policy aimed at lowering the cost of opening a vacancy does not better the performance of the decentralized economy. Instead, increasing workers' bargaining power allows to bridge the gap between the optimal and the *laissez faire* outcomes.

The rest of the chapter is organized as follows. Section 3.2 presents the model. Section 3.3 characterizes the decentralized equilibrium. Section 3.4 studies the policy implications, while Section 3.5 analyzes the welfare problem. Section 3.6 shows the quantitative results obtained. Finally, section 3.7 concludes.

⁴Actually, in Ebell and Haefke's framework, there is also a hiring externality - opposite in sign. Since the wage is proportional to the marginal revenues, that are decreasing in a monopolistic set-up, firms will be induced to hire more than the optimal level in order to reduce the wage paid to all the workers. Such strategic behaviour has been first studied by Stole and Zwiebel (1996) and extended to matching models by Cahuc and Wasmer (2001). In their model, Ebell and Haefke show that the first, monopolistic effect prevails and firms hire less than in a competitive framework, unless workers' bargaining power is extremely high.

3.2 The model

3.2.1 Preferences and technology

I consider an economy with one final consumption good and a large number I of intermediate goods. The final good market is perfectly competitive, whereas Cournot competition is assumed within each intermediate sector. The final good production function takes a CES form:

$$Y = \left[\sum_{i=1}^I Q_i^{\frac{s-1}{s}} \right]^{\frac{s}{s-1}} \quad (3.1)$$

in which Q_i is the amount of intermediate good i used by the production process of the final good and $s > 1$ to ensure decreasing marginal productivity. Cost minimization in the final good sector leads to the inverse demand for each intermediate good i :

$$p(Q_i) \equiv \frac{P_i}{P} = \left(\frac{Q_i}{Y} \right)^{-\frac{1}{s}} \quad (3.2)$$

with $P \equiv \left[\sum_{i=1}^I P_i^{1-s} \right]^{\frac{1}{1-s}}$

P is the price index. Parameter s is the elasticity of the demand for good i .

Time is continuous. In each intermediate sector there are L infinitely-lived and risk-neutral workers; they can be employed only in that industry, so there are I perfectly segmented labour markets. Each firm is made of a (filled or vacant) job. The I labour markets present some unexplained frictions that make the trading process between firms and workers not instantaneous. Therefore, to produce and compete in one sector, a firm has to post a job vacancy, meet a worker and bargain with him about the wage and the number of hours worked. The intermediate firm production function is identical in each sector and is given by l_i , where l_i is the amount of hours worked supplied by the employee in sector i , and $0 \leq l_i \leq 1$. The total amount of good i produced at time t is equal to $Q_{i,t} = \sum_j l_{i,j}(t)$, the subscript j denoting a generic firm operating in sector i at time t .

Workers have homogeneous instantaneous utility functions, denoted by $v_i l_i + \phi(l_i)$, with v_i being the hourly real wage and $\phi(l_i)$ the disutility of work. For

simplicity, I assume an iso-elastic function $\phi(l_i) = z - l_i^\epsilon/\epsilon$, $\epsilon > 1$. When unemployed, the worker enjoys an instantaneous utility z , the value of devoting all your time to leisure.

3.2.2 The Stochastic Environment

The creation and destruction of jobs in each intermediate market i follows a continuous time Markov chain that takes values in the set $L = \{0, 1, 2, \dots, L\}$. I assume that in small interval of time dt at most one firm can enter in a sector. So, if x_i is the number of firms active in sector i , the probability that one more firm enters is given by $M_{x_i} dt$, while the probability that more than one firm enter is equal to zero. The rate M_{x_i} positively depends on V_{x_i} , the number of job-vacancies, and $L - x_i$ the number of unemployed workers in sector i . So, $M_{x_i} = m(V_{x_i}, L - x_i)$, with $m(\cdot, \cdot)$ being identical in every sector, homogeneous of degree one, and increasing in both arguments. M_x is a sort of black box, capturing the presence of frictions in the labour market.

Moreover, with a probability δdt a new intermediate product is invented in the economy, making one existing good obsolete. All the jobs in the “old” intermediate sector are destroyed and massive layoffs occur. To keep the model as simple as possible, I also assume that all the L workers of the sector destroyed start searching for a job in the new one. Such hypothesis about a sector-specific destruction rate wants to be an (admittedly simplified) approximation of a product life-cycle. The economy is subject to a “creative destruction” force that allows the creation of new products but makes the existing ones obsolete. Indeed, as stressed in many marketing studies, the final stage of a product life-cycle does not necessarily take the form of a slow decline in time⁵. Sometimes, the rise of new goods (often but not always technologically more advanced) makes the decline more steady or even transform it in a “collapse”⁶.

⁵Consider for instance the analysis about “disruptive innovation” pioneered by Christensen (1997).

⁶In a standard matching model, the destruction rate is job-specific, meaning that every match faces a probability of being destroyed. I consider a sector-specific destruction rate for simplicity. A job-specific separation rate would make the asset price equations even more difficult to manage with, since every firm would have to consider both the probability that the sector evolves by one unit and the probability that it decreases by one unit.

The Markov chain just described can be represented by the following σ -matrix $\Sigma \equiv (\sigma_{x,y}, x, y \in [1, 2, \dots, L])$:

$$\begin{aligned} \sigma_{x,x+1} &= M_x, & \sigma_{x,y} &= 0, & y - x > 1, \\ \sigma_{x,x} &= -(M_x + \delta), & \sigma_{x,0} &= \delta \\ \sigma_{0,1} &= M_1, & \sigma_{0,0} &= -M_1. \end{aligned} \quad (3.3)$$

Following Karlin and Tavaré (1982) and Van Doorn and Zeifman (2005), I refer to a process of this type as a birth process with killing, with M_x and δ respectively being the birth (i.e. the creation of one more job) and the killing (i.e. the destruction of all the jobs in the sector) rate. Let define the level of tightness in the labor market as $\theta_{x_i} \equiv \frac{V_{x_i}}{L-x_i}$. By the constant returns to scale assumption, the rate at which a single firm fills its vacancy when x firms are already active in market i can be defined as $q(\theta_{x_i}) \equiv M_{x_i}/V_{x_i}$ and the rate at which a single worker finds a job is given by $M_{x_i}/(L-x_i) = \theta_{x_i}q(\theta_{x_i})$. I also define $\eta \equiv \frac{d(1/q(\theta))}{d\theta} \cdot \theta q(\theta)$, the elasticity of the expected duration of filling a vacancy with respect to tightness.

Notice that in a text-book Pissarides model, a unique labour market is assumed and the measure $M = m(V, U)$ represents the *number* of matches produced at each moment in the aggregate economy. The law of motion of employment is therefore given by $dE/dt = M_t - E_t\delta$. In this chapter, on the contrary, I consider a large number of small and distinct labour markets and $M_{x_i} = m(V_{x_i}, L-x_i)$ represents the *rate* at which a new match is created in a generic labour market i . This setting is preferable to the standard one, since I study the dynamic behaviour of firms subject to Cournot competition. Any firm computing its optimal strategy has to consider both the number of competitors present in the market and the rate at which new players will enter. Such a stochastic process, where the number of possible entrants in each intermediate market cannot be greater than one in a small interval of time dt , allows to model firms' dynamic decisions, while keeping the model as tractable as possible⁷.

⁷Usually, equilibrium matching models are adopted to study the behaviour of aggregate labour markets. However, this does not mean in principle that search frictions should be negligible if the number of potential traders in the market is small. Indeed, the assumption of constant returns to scale for matching functions implies that the magnitude of frictions (trade

Intermediate sectors are identical *ex-ante*, having the same number of workers L , and the same matching and production technology. So I can remove the subscript i . Let $\pi_{x,t}$ be the probability that a time t there are x active firms in a generic intermediate market. Then:

$$\begin{aligned}\pi_{x,t+dt} &= [1 - \delta dt - M_x dt] \cdot \pi_{x,t} + M_{x-1} dt \cdot \pi_{x-1,t} \quad \forall x \in [1, 2, \dots, L], \\ \pi_{0,t+dt} &= [1 - M_0 dt] \cdot \pi_{0,t} + \delta dt \cdot \sum_{x=1}^L \pi_{x,t}.\end{aligned}$$

One can look for a steady-state probability distribution, where $\pi_{x,t+dt} = \pi_{x,t}$, $\forall t$. Expressing π_x in terms of π_{x-1} and knowing that $\sum_{x=1}^L \pi_x = 1 - \pi_0$ yields:

$$\begin{aligned}\pi_x &= \frac{M_{x-1}}{M_x + \delta} \cdot \pi_{x-1} \quad \text{with } x \in [1, 2, \dots, L], \\ \pi_0 &= \frac{\delta}{\delta + M_0}.\end{aligned}$$

Finally, solving backwards, one obtains:

$$\pi_x = \prod_{n=0}^{x-1} \frac{M_n}{M_{n+1} + \delta} \cdot \pi_0 = \frac{\delta}{M_x + \delta} \cdot \prod_{n=0}^{x-1} \frac{M_n}{M_n + \delta} \quad (3.4)$$

The probability π_x that in one intermediate sector x firms compete in the market depends on L , δ and the endogenous rates $M_n = (L - n)\theta_n q(\theta_n) \quad \forall n \in [0, 1, 2, \dots, x]$.

If I is sufficiently large, I can apply the law of large numbers and define the aggregate level of employment

$$E = \sum_{x=0}^L x \cdot \pi_x \cdot I. \quad (3.5)$$

Of course, the level of unemployment is given by: $U = \sum_{x=0}^L (L - x) \cdot \pi_x \cdot I$.

Lemma 1 *The level of employment E is increasing in M_x , $\forall x \in [0, 1, 2, \dots, L-1]$. More in general, $\sum_{x=0}^L g(x) \cdot \pi_x \cdot I$ is increasing (decreasing) in M_x for any function $g(\cdot)$ increasing (decreasing) in x .*

costs, asymmetry of information, geographical distances) in the economy does not depend on the number of people searching for a job or firms opening a vacancy.

It easy to check that $\frac{d\pi_x}{dM_x} < 0$, $\frac{d\pi_x}{dM_n} > 0$ if $n \in [0, 1, 2, \dots, x-1]$, and $\frac{d\pi_x}{dM_n} = 0$ if $n \in [x+1, x+2, \dots, L]$. Hence, differentiating (3.5), one gets:

$$\frac{dE}{dM_x} = x \cdot \frac{d\pi_x}{dM_x} + \sum_{n=x+1}^L \frac{d\pi_n}{dM_x} \cdot n$$

The first term at the RHS is negative, while the sum is composed by positive terms. Since $\sum_{n=0}^L \pi_n = 1$, then

$$\begin{aligned} -\frac{d\pi_x}{dM_x} &= \sum_{n=x+1}^L \frac{d\pi_n}{dM_x} \iff \\ -\frac{d\pi_x}{dM_x} \cdot x &= x \cdot \sum_{n=x+1}^L \frac{d\pi_n}{dM_x} \iff \\ -\frac{d\pi_x}{dM_x} \cdot x &< \sum_{n=x+1}^L \frac{d\pi_n}{dM_x} \cdot n \end{aligned}$$

The last inequality implies that $\frac{dE}{dM_x} > 0, \forall x \in [0, 1, \dots, L-1]$. It is easy to verify that the same result applies if x is replaced by an increasing transformation of x . Hence, $\sum_{x=0}^L g(x) \cdot \pi_x \cdot I$ is increasing (decreasing) in M_x for any function $g(\cdot)$ increasing (decreasing) in x . ■

3.2.3 Asset price equations

At each moment, the timing of decisions is by assumption the following:

1. Intermediate firms enter the market by posting vacancies. This costs a fixed amount h per unit of time. Jobless workers search for a job.
2. At a certain endogenous rate, a firm meets a worker and both the wage and the number of hours worked are bargained over.
3. If an agreement is reached, production occurs in the intermediate-goods sector. Intermediate firms compete *à la* Cournot to sell their goods to the final sector. Total surplus is shared between the worker and the firm.
4. At some endogenous rates $M_x, x \in [0, 1, \dots, L-1]$, a new competitor enters the intermediate market. Wages and hours worked are modified accordingly. At an exogenous rate δ each intermediate product is replaced

by a “new” one. All the jobs in the “old” sector are destroyed. A worker employed in a sector destroyed enter unemployment and start searching for a job in the new one.

Let r be the discount rate common to all agents. The expected lifetime income for an unemployed worker in a sector with x competitors, $W_U(x)$ solves the following equation:

$$\begin{aligned} rW_U(x) = & z + \theta_x q(\theta_x) [W_E(x+1) - W_U(x)] \\ & + (L - x - 1) \theta_x q(\theta_x) [W_U(x+1) - W_U(x)] + \delta [W_U(0) - W_U(x)], \end{aligned} \quad (3.6)$$

with $x \in [0, 1, \dots, L-1]$. Being unemployed when the level of employment is equal to x is like holding an asset that pays you a dividend of z and at a rate $\theta_x q(\theta_x)$ it can be transformed into employment (hence, $x+1$ jobs are active in that market). In addition, the value of the asset can also change because at a rate $(L-x-1) \theta_x q(\theta_x)$ some other unemployed worker can find a job. In that case, the value of being unemployed shifts from $W_U(x)$ to $W_U(x+1)$. Finally, at a rate δ that sector can become obsolete in the economy. All the workers employed there lose their job and start their unemployment spell in the new sector. The capital gain will be equal to $W_U(0) - W_U(x)$.

Consider the probability that another worker but you is hired and so employment increases by one unit. This event is taken into account by every agent, for one more firm in the market changes the quantity produced (and so the price) in the Nash equilibrium of the Cournot game. In a standard matching model, on the contrary, firms and workers are price takers in the product market and such price variation is ignored by the single agent computing his expected lifetime income.

Similarly, the asset price equation for a worker employed in a sector with x competitors is equal to:

$$\begin{aligned} rW_E(x) = & v_x l_x + z - \frac{l_x^\epsilon}{\epsilon} + \delta [W_U(0) - W_E(x)] \\ & + (L - x) \theta_x q(\theta_x) [W_E(x+1) - W_E(x)], \end{aligned} \quad (3.7)$$

with $x \in [1, 2, \dots, L]$.

On the other side of the market, the Bellman equation for a job vacancy is then given by:

$$\begin{aligned} rJ_V(x) = & -h + q(\theta_x) [J_E(x+1) - J_V(x)] \\ & + [M_x - q(\theta_x)] [J_V(x+1) - J_V(x)] + \delta [J_V(0) - J_V(x)], \end{aligned} \quad (3.8)$$

with $x \in [0, L-1]$. Similarly, the value of an active firm with $x-1$ competitors takes the following form:

$$\begin{aligned} rJ_E(x) = & p(Q_x) l_x - v_x l_x + \delta [J_V(0) - J_E(x)] \\ & + (L-x)\theta_x q(\theta_x) [J_E(x+1) - J_E(x)], \end{aligned} \quad (3.9)$$

with $x \in [1, 2, \dots, L]$. Function $p(Q_x)$ is expressed in (3.2) and represents the real price of the intermediate good when x firms are competing in the market. $J_V(0)$ is the value of a vacancy when the sector is destroyed. I define $p'(Q_x) \equiv \partial p(Q_x) / \partial l_x$.

3.3 Equilibrium

3.3.1 Bargaining

Firms and workers bargain over wages and hours worked. I assume continuous renegotiation, meaning that every employer-employee pair renegotiates the level of the wage and the numbers of hours worked every time a change in the demand occurs because a new competitor enters the market⁸. An axiomatic Nash solution is considered. I impose that the threat points for workers and firms in the Nash program are not their options outside the match (respectively, W_U and J_V), but their utilities of remaining together and producing nothing. I make this choice for two reasons.

First, in this framework with imperfect competition in the goods market, adopting the values of remaining together without producing as threat points seems

⁸Assuming continuous renegotiation seems more “rational” than imposing that the wage and the hours worked remain constant whatever the conditions in the goods market. If this was the case, then, for instance, a worker would receive a really high wage even when the product market is very competitive only because he was hired when there was a monopoly. In other terms, wages and hours worked should instantaneously adjust to changes in firms’ profits.

a more convenient and realistic choice⁹. Instantaneous renegotiation implies that each firm-worker pair bargains wages and hours worked every time a new job is formed in that market. In other words, wages and hours worked are bargained not only by workers (respectively, firms) that have just ended their unemployment (resp. vacancy) spell, but also by incumbents that have to change their strategy in the Cournot game. It seems more appropriate, especially for such workers and firms, to assume that in the case of failure of an agreement they decide not to leave. One can think for instance that workers are on strike and nothing is produced.

The second reason is tractability. Assuming, as in a standard Pissarides model, that the threats points are the outside options does not rule out the existence of an equilibrium, but makes the model less tractable (details are available on request). I assume therefore that the threat points for an employee and an employer when the negotiation fails are respectively given by:

$$r\bar{W}_E = z + \delta [W_U(0) - \bar{W}_E] \quad (3.10)$$

$$r\bar{J}_E = -\delta [\bar{J}_E - J_V(0)] \quad (3.11)$$

If an agreement is not concluded, the worker remains unemployed, he does not receive any wage and enjoys an instantaneous utility of z . The firm does not produce and does not pay the wage. Still, at a rate δ that sector becomes unproductive.

I define $w \equiv v \cdot l$, the total real wage received by the employee, and solve the Nash maximization problem with respect to $\{w, l\}$ instead of $\{v, l\}$:

$$\begin{aligned} w_x, l_x &= \operatorname{argmax} [W_E(x) - \bar{W}_E]^\beta [J_E(x) - \bar{J}_E]^{1-\beta} \\ \text{s.t.} & \\ W_E(x) &> W_U(x-1) \\ J_E(x) &> J_V(x-1) \quad \text{with } x \in [1, 2, \dots, L]. \end{aligned} \quad (3.12)$$

$J_V(x-1)$ represents the expected discounted value of a vacancy when $x-1$ firms compete in the market. In Appendix 3.8, I show that the solution of

⁹This kind of bargaining game has been introduced by Rosen (1997) and Hall and Milgrom (2006). The arguments they advance in favour of such new setting are convincing: in reality, workers (or unions) and firms negotiate without seriously considering either permanent resignations or discharging employees as an option. A disagreement over wages and hours worked usually implies a delay in the production, strikes, not massive lay-offs or quits.

(3.12) coincides with the equilibrium of an extensive form game with workers and firms alternating each other in making offers in the limit case in which parties have only one instant to make their bargain. The constraints imposed in the maximization imply that the worker (the firm) always has the possibility to abandon the negotiation and become unemployed (an idle vacancy) if this choice makes him (it) better off. I assume, as Rosen (1997) and Hall and Milgrom (2006) do, that such constraints are not binding: no player has an incentive to quit the negotiation and this holds for any value of x .

Computing the F.O.C.s yields :

$$\beta [J_E(x) - \bar{J}_E] = (1 - \beta) [W_E(x) - \bar{W}_E].$$

$$\beta \frac{l_x^{\epsilon-1}}{W_E(x) - \bar{W}_E} = (1 - \beta) \frac{p'(Q_x) l_x + p(Q_x)}{J_E(x) - \bar{J}_E},$$

$\forall x \in [1, 2, \dots, L]$. By using equations (3.7), (3.9), (3.11) and (3.10), I get the following equilibrium equations of wages and hours worked :

$$w_x = \beta p(Q_x) l_x + (1 - \beta) \frac{l_x^\epsilon}{\epsilon} \quad (3.13)$$

$$\begin{aligned} l_x^{\epsilon-1} &= [p'(Q_x) l_x + p(Q_x)] \\ &= p(Q_x) \left[1 - \frac{1}{x \cdot s} \right] \end{aligned} \quad (3.14)$$

$\forall x \in [1, 2, \dots, L]$. The second line in (3.14) is obtained by using equation (3.2). For every x , equations (3.13) and (3.14) define the equilibrium values of l_x and w_x . Equation (3.13) has a straightforward interpretation. The wage is a weighted average of the total revenues obtained in the intermediate sector ($p(Q_x) l_x$) and the opportunity cost of employment in terms of hours worked ($z - \phi(l_x) = l_x^\epsilon / \epsilon$). The weights are given by the bargaining power of workers and firms, β and $1 - \beta$. If the worker has no bargaining power, he receives an instantaneous utility from being employed exactly equal to z . On the other hand, when $\beta = 1$, the firm has no bargaining power and all the profits earned in the market accrue to the employee. As we will see, in this limit case, firms cannot recoup the cost h and no firm will post a vacancy.

Equation (3.14) looks very similar to a standard solution of a x -players Cournot game. I restrict the attention only to symmetric equilibria, where all firm-

worker pairs in each sector produce exactly the same quantity l_x . Each worker-firm pair maximizes its surplus, given the optimal strategy of the other players. In equilibrium, the marginal revenue of a firm must be equal to the marginal utility of leisure for a worker¹⁰. Notice also that, from a single firm's viewpoint, Y is given. This is due to the fact that, since I is large, a single firm's decision has an impact only within each sector but does not affect the price index P and quantity Y .

Properties of wages and hours worked

I consider now the properties of wages and hours worked as competition increases in a generic sector, while the rest of the economy is considered as given. In other terms, there is no income effect, and the final good production Y is constant.

From (3.13), the derivative of w with respect to l is:

$$\frac{dw_x}{dl_x} = \beta [p'(Q_x)l_x + p(Q_x)] + (1 - \beta)l_x^{\epsilon-1} = l_x^{\epsilon-1} > 0.$$

In l_x/w_x space, equation (3.14) is a vertical line, whereas (3.13) is a monotonically increasing function (see Figure 3.1). Moreover, some standard properties of Cournot models are fulfilled: As the number of competitors x increases, the quantity produced by a single firm, l_x , decreases, whereas the aggregate quantity Q_x increases¹¹. So the total wage decreases as x increases:

$$w_{x+1} - w_x = \beta[p(Q_{x+1})l_{x+1} - p(Q_x)l_x] + \frac{1 - \beta}{\epsilon} [l_{x+1}^\epsilon - l_x^\epsilon] < 0, \quad (3.15)$$

People employed in more competitive sectors get lower wages but more leisure time. If I ignore for simplicity the integer problem, the former effect outweighs

¹⁰Differentiating the first line of equation (3.14) with respect to l_x , I obtain:

$$[p''(Q_x)l_x + 2p'(Q_x)] - (\epsilon - 1)l_x^{\epsilon-2}.$$

A sufficient condition for this equation to be negative is $[p''(Q_x)l_x + 2p'(Q_x)] < 0$. Computing the derivatives, this implies $\frac{1+s}{s} < 2x$, that is always true for any $x \geq 1$ and $s > 1$.

¹¹The necessary assumptions to prove such properties are satisfied (demand twice differentiable and tending to 0 for Q_x sufficiently large, cost function increasing and twice differentiable, profit function strictly concave). For the complete proof, I refer to Frank (1965).

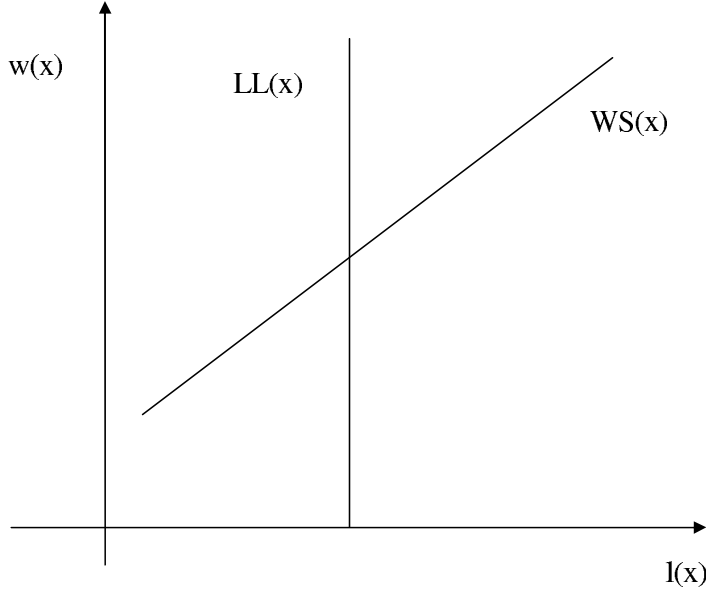


Figure 3.1: Equilibrium equations for wage and hours worked

the latter: Using (3.13), the instantaneous utility of an employed worker, $w + \phi(l_x)$, is decreasing in x :

$$\frac{d [w_x + \phi(l_x)]}{dx} = \beta \left\{ \frac{dp(Q_x)}{dQ_x} \frac{dQ_x}{dx} l_x + \frac{dl_x}{dx} [p(Q_x) - l_x^{\epsilon-1}] \right\} < 0.$$

The first term inside the braces is negative because Q_x is increasing in x ; the second term is also negative since l_x decreases in x , while the expression inside the square brackets is positive by (3.14).

3.3.2 Free-entry in vacancy creation

To close the model and find the equilibrium values of θ_x , I introduce a free-entry condition in vacancy creation. Firms enter one intermediate market as long as the expected return of posting a vacancy is non negative. This means that:

$$rJ_V(x) = 0 \quad \forall x \in [0, 1, ..L-1] \quad (3.16)$$

The expected discounted value of a job when $x+1$ agents are active in a market must be equal to the expected cost of filling a vacancy:

$$J_E(x+1) = \frac{h}{q(\theta_x)} \quad \forall x \in [0, 1, 2, \dots, L-1] \quad (3.17)$$

Finally, using (3.9), (3.16), and (3.17) one gets:

$$\frac{h}{q(\theta_{x-1})} = \frac{p(Q_x)l_x - w_x + (L-x)\theta_x h}{r + \delta + (L-x)\theta_x q(\theta_x)} \quad \forall x \in [1, 2, \dots, L]. \quad (3.18)$$

The LHS represents the expected duration of filling a vacancy when $x-1$ firms are already active in the market. On the RHS, the expected profit is made of two terms: profits attained when the firm has $x-1$ competitors (that is $p(Q_x)l_x - w_x$) and all the profits that can be earned with at least x competitors, weighted by the rate M_x (since $(L-x)\theta_x h = M_x \frac{h}{q(\theta_x)} = M_x \cdot J_E(x+1)$). The equations in (3.18) represent a system of L unknown variables, $[\theta_0, \theta_1, \dots, \theta_{L-1}]$. Note that for $x = L$ we have:

$$\frac{h}{q(\theta_{L-1})} = \frac{p(Q_L)l_L - w_L}{r + \delta} \quad (3.19)$$

Labour market tightness θ_{L-1} does not depend on other values of θ . The endogenous variables l_L and w_L are uniquely defined by the F.O.C.s (3.13) and (3.14) evaluated at $x = L$. I can therefore solve the system in (3.18) “backward”, starting from θ_{L-1} and going back to $\theta_{L-2}, \theta_{L-3}, \dots, \theta_0$.

Properties of labour market tightness

I am interested in knowing how the equilibrium value of tightness θ_x changes with x . The following lemma summarizes the results:

Lemma 2 $\theta_x < \theta_{x-1}, \forall x \in [1, 2, \dots, L-1]$. Hence, $M_x < M_{x-1} \forall x \in [1, 2, \dots, L-1]$.

Proof. See Appendix 3.9. ■

Lemma 2 states that the number of vacancies posted decrease as competition gets tougher. This makes sense, since a more competitive product market squeezes firms' profits, dampening the incentives in vacancy creation. Such

negative effect on the supply side of the labour market outweighs the reduction in the number of unemployed workers as x goes up, so that $\theta_x \equiv V_x/(L-x)$ is decreasing in x . Equation (3.16) implies that expected discounted profits are equal to zero for any given level of competition in the goods market. A trade-off arises: in less competitive markets firms can attain higher revenues but stand in a longer queue to fill their vacancies.

3.3.3 Equilibrium

Definition 1 *A long-run general equilibrium is defined as a vector $[l_x, w_x, \theta_{x-1}, P_x] \forall x \in [1, 2, \dots, L]$, a probability distribution $[\pi_0, \pi_1, \pi_2, \dots, \pi_L]$, and a value Y of the final good satisfying:*

1. *the F.O.C.s (3.13) and (3.14) of the bargaining problem, $\forall x \in [1, 2, \dots, L]$.*
2. *The zero profit condition (3.18), $\forall x \in [1, 2, \dots, L]$.*
3. *The steady-state distribution (3.4).*
4. *The conditions in the final good sector (3.1) and (3.2).*

The F.O.C.s (3.13), (3.14), and the demand function (3.2) determine the values of w_x and l_x as a function of $Y \forall x$. Then, substituting the equilibrium values of w_x and l_x in the system (3.18), I can express the elements of the vector $[\theta_0, \theta_1, \dots, \theta_{L-1}]$ in terms of Y . In turn, using (3.4), I also determine the probabilities $[\pi_0, \pi_1, \dots, \pi_L]$ as a function of Y . Finally, equilibrium in the final good sector implies: $Y = \left[\sum_{i=1}^I (Q_i(Y))^{\frac{s-1}{s}} \right]^{\frac{s}{s-1}}$. Using (3.14), this equality is equivalent to:

$$Y = Y^{\frac{1}{1+s(\epsilon-1)}} \cdot A,$$

$$\text{with } A \equiv \left\{ \sum_{i=1}^I x_i^{\frac{s-1}{s}} \cdot \left[\frac{1}{x_i} \left(\frac{x_i s}{x_i s - 1} \right)^s \right]^{\frac{s-1}{s[1+s(\epsilon-1)]}} \right\}^{\frac{s}{s-1}}$$

It is easy to see that this equilibrium has two solutions for Y , one equal to zero and the other positive. As $Y = 0$, nothing is produced in the intermediate sectors, all workers are unemployed and the probability distribution collapses to a mass point $x = 0$. Henceforth, I will concentrate on the positive equilibrium.

3.4 Competition in Products and Labour Markets

I now assess the impact on average employment, real wage and workers' utility of a change in β and h in every sector of the economy. To simplify the analysis, I assume henceforth a Cobb-Douglas matching function, $M_x = a(L - x)^\eta \cdot V_x^{1-\eta}$, with $\eta = 0.5$, in line with the findings of Petrongolo and Pissarides (2001).

3.4.1 Effects on Employment

The results are summarized in the following Proposition:

Proposition 1 *A decrease in workers' bargaining power β or in the cost of opening a vacancy h raises the aggregate level of employment, E .*

Proof.

In Appendix 3.10, I show that θ_x is decreasing in β and h , $\forall x \in [0, 1, \dots, L - 1]$. Hence, a lower β or h raises θ_x and, in turn, $M_x = (L - x)\theta_x q(\theta_x)$. From Lemma 1, the average level of employment, E , goes up. ■

A lower bargaining power for workers reduces the wage and raises firms' expected profits. So, more competitors will enter the labour market by posting a vacancy. This in turn augments the employment. A similar effect occurs by lowering the cost of opening a vacancy h . Proposition 1 is in line with the empirical findings of Nicoletti and Scarpetta and Griffith, Harrison, and Macartney (2007), and with the theoretical conclusions obtained by Blanchard and Giavazzi (2003) and Ebell and Haefke (2006).

3.4.2 Effects on the real wage and on workers' utility

A decrease in the cost of opening a vacancy.

From Proposition 1, a decrease in h augments $M_x \forall x$. M_x raises the final good Y and, in turn, this has an impact on the real wage and the hours worked. To analyze the effect of M_x on the production of the consumption good, notice

that Y can be written as:

$$Y = \left[\sum_{i=1}^I Q_i^{\frac{s-1}{s}} \right]^{\frac{s}{s-1}} = \left[\sum_{x=0}^L Q_x^{\frac{s-1}{s}} \pi_x I \right]^{\frac{s}{s-1}}$$

Recall that the intermediate quantity Q_x is increasing in x . Then, from Lemma 1, a higher M_x raises the amount of the final good, Y . From (3.2), a higher Y enhances $p(Q_x)$. In turn, this has a positive impact on hours worked: the RHS of the F.O.C. (3.14) shifts upwards, so hours worked go up. As a consequence, the real wage also increases:

$$\frac{dw_x}{dY} = \beta \frac{dp(Q_x)}{dY} l_x + \frac{dl_x}{dY} [\beta (p(Q_x) + p'(Q_x) l_x) + (1 - \beta) l_x^{\epsilon-1}] > 0. \quad \forall x$$

Knowing that the instantaneous utility of workers $w_x + \phi(l_x) = \beta \left(p(Q_x) l_x - \frac{l_x^\epsilon}{\epsilon} \right) + z$, we have:

$$\frac{d[w_x + \phi(l_x)]}{dY} = \beta \frac{dp(Q_x)}{dY} l_x + \beta \frac{dl_x}{dY} [p(Q_x) + p'(Q_x) l_x - l_x^{\epsilon-1}].$$

Such derivative is positive because $p(Q_x)$ and l_x are increasing in x , while the term inside the square brackets is zero for the F.O.C. (3.14). So, reducing the cost of opening a vacancy raises both the real wage and worker's instantaneous utility for any given level of competition x .

Consider now the impact of h on the average real wage,

$$\bar{w} \equiv \frac{1}{E} \sum_{x=1}^L w_x x \pi_x I.$$

Two opposite effects are at work. On one hand, a higher Y raises $\bar{w}$, because w_x is increasing in Y for any x . On the other hand, Lemma 1 cannot be used to assess the impact of a higher M_x on $\bar{w}$, because we do not know if $w_x \cdot x$ is an increasing or decreasing function of x . Since a reduction in the vacancy cost raises both M_x and Y , the final effect on the average real wage cannot be signed. In other terms, a lower h enhances the real wage for any given level of competition x , but, by raising tightness, it also changes the distribution $[\pi_0, \pi_1, \pi_2, \dots, \pi_L]$. It may be possible that at the new equilibrium, in which the average level of employment is higher, workers are more likely to be in sectors with fiercer competition, where the real wages are lower. The former, income,

effect pushes $\bar{w}$ up, whereas the latter, distribution, effect lowers it. Of course, the same reasoning applies to the average workers' utility.

A decrease in workers' bargaining power.

It is easy to verify that a reduction in workers' bargaining power β also augments the final good produced Y and hours worked l_x , for any given x . However, the effect on the real wage w_x is now ambiguous. The reason is that w_x is positively affected by Y and l_x , as in the case of a reduction in h , but it is also decreasing in β . A lower bargaining power for the workers implies a smaller fraction of the surplus originated by the match and, in turn, a lower wage. That means:

$$\frac{dw_x}{d\beta} = \frac{\partial w_x}{\partial \beta} + \frac{\partial w_x}{\partial Y} \cdot \sum_{x=0}^{L-1} \frac{\partial Y}{\partial M_x} \frac{\partial M_x}{\partial \beta}.$$

The first term is positive, while the second one is negative, for a higher β lowers M_x . As consumers, workers benefit of the decrease in β , since a larger amount of the final is produced and consumed. Yet, the employees receive a lower fraction of the surplus. The final effect of β on w_x cannot be signed. *A fortiori*, I cannot assess the impact of a reduction in workers' bargaining power on the average real wage.

Differently from Proposition 1, the conclusions of this sub-section contrast with those obtained by Blanchard and Giavazzi (2003)¹². The reason of these competing results is twofold: the (a)symmetry of the long-run equilibrium, and the different zero-profits conditions imposed. Blanchard and Giavazzi focus on a symmetric equilibrium, in which all the intermediate firms set the same price. So, the relative price $p(Q_i)$ is equal to 1. Further, in the free-entry equilibrium, profits per worker (i.e. $p(Q) - w$) are equal to a fixed cost of entry. It is then clear that the real wage w increases as the fixed cost decreases, whereas it is unaffected by changes in β . In the present chapter, on the contrary, the equilibrium is asymmetric (in the sense that the quantity and price is not the same among the intermediate sectors), and the real wage in the long-run is still a function of β .

¹²Ebell and Haefke (2006) do not analyse the long-run effect of competition on the real wages.

3.5 Optimality

In the decentralized economy, there are two departures from the competitive framework, namely frictions in the labour market and imperfect competition in the goods market. Finding the optimal level of product market competition and the optimal employment level and comparing both with the *laissez faire* outcomes is not therefore an obvious task. I consider a centralized economy in which a social planner has to choose the optimal number of vacancies and hours worked in any sector. Notice that the interactions *between* intermediate sectors come only from the final good production function. By the constant returns to scale assumption, the latter can be written as $Y = \sum_{i=1}^I p(Q_i)Q_i$. Moreover, from Euler's formula, $-\frac{dp(Q_i)}{dQ_i} \frac{dQ_i}{dl_i} Q_i = \sum_{i \neq j=1}^I \frac{dp(Q_j)}{dQ_j} \frac{dQ_j}{dl_i} Q_j$. Hence, any effect arising between intermediate sectors (the RHS of the equation above) disappears and I can study the social planner problem focusing only on what happens *within* a generic intermediate sector. Following Shimer (2004b), the welfare function can be expressed for any given x in the following recursive form:

$$\begin{aligned} r \Omega_x = & \max_{\theta_x, l_x} p(Q_x)Q_x + x \left(z - \frac{l_x^\epsilon}{\epsilon} \right) + (L - x)z - h(L - x)\theta_x \\ & + (L - x) a \theta_x^{1-\eta} [\Omega_{x+1} - \Omega_x] + \delta [\Omega_0 - \Omega_x] \end{aligned} \quad (3.20)$$

$$\text{s.t. } Q_x = x \cdot l_x. \quad \forall x \in [0, 1, 2, \dots, L]$$

When x firms are active in a generic intermediate sector, the social planner has to maximize intermediate firms' revenues, the utility of leisure of workers, net to the cost of opening a vacancy. Moreover, at a rate $M_x = (L - x) a \theta_x^{1-\eta}$ the level of employment increases by one unit, causing a change of the surplus from Ω_x to Ω_{x+1} , and at a rate δ the sector is destroyed and another one is instantaneously created. The constraint in (3.20) reminds that, differently from the *laissez faire* economy, the social planner considers *ex ante* a symmetric solution, in which every firm uses the same amount of hours worked. Of course at $x = L$, the sector is in full employment and the social planner has only to choose the amount of hours worked. The solutions (θ°, l°) s to problem (3.20) verify the following F.O.Cs:

$$(1 - \eta) a (\theta_x^\circ)^{-\eta} \cdot [\Omega_{x+1} - \Omega_x] = h \quad (3.21)$$

$$p(Q_x^\circ) = (l_x^\circ)^{\epsilon-1} \quad (3.22)$$

The intuition of the above equations is the following. At the social optimum, the cost of marginal increase in θ_x , h , must be equal to the marginal gain, given by $(d\theta_x q(\theta_x)/d\theta_x) [\Omega_{x+1} - \Omega_x] = (1 - \eta) a(\theta_x^\circ) [\Omega_{x+1} - \Omega_x]$. Moreover, the optimal level of hours worked l_x° is such that the increase in production must be equal to the opportunity cost in terms of leisure.

By the Euler's formula, $\frac{dp(Q_x)}{dQ_x} \frac{dQ_x}{dl_x} Q_x$ cancels out with the sum of the derivatives of the prices in the other sector with respect to l_x . Comparing (3.14) with (3.22) one obtains $l_x^\circ > l_x^* \forall x$, the superscript $*$ denoting henceforth the decentralized equilibrium values of the endogenous variables. This inequality holds since l_x^ϵ is increasing in l and $p(Q_x)$ is always greater than $p(Q_x) + p'(Q_x)l_x$. So the level of hours worked in equilibrium is always inefficiently low. Notice also that equation (3.22) would coincide with the outcome of a worker-firm negotiation, were the good market perfectly competitive.

Denote $S_x \equiv p(Q_x)l_x - l_x^\epsilon/\epsilon$. Using (3.21) and (3.22) and subtracting the optimal solution Ω_x from Ω_{x+1} yields:

$$\frac{(r + \delta)h}{a(1 - \eta)} (\theta_x^\circ)^\eta = (x+1) S_{x+1}^\circ - x S_x^\circ + \frac{\eta}{1 - \eta} h [(L - x - 1)\theta_{x+1}^\circ - (L - x)\theta_x^\circ].$$

After some algebra, one gets:

$$\frac{r + \delta}{a} (\theta_x^\circ)^\eta + \eta(L - x) \theta_x^\circ = \frac{1 - \eta}{h} [(x+1) S_{x+1}^\circ - x S_x^\circ] + \eta(L - x - 1)\theta_{x+1}^\circ. \quad (3.23)$$

A comparison of (3.23) with the free-entry equilibrium condition (3.18) delivers the following result:

Proposition 2 *If $\beta \leq \eta$, in the decentralized equilibrium the aggregate level of employment is inefficiently high $\forall x$.*

Proof. I first consider the case in which in the decentralized equilibrium $\beta = \eta$ and I prove that the level of employment is inefficiently high. Then, I show that this results holds *a fortiori* if $\beta < \eta$. Using the wage equation (3.13) and imposing $\eta = \beta$, the decentralized equilibrium condition (3.18) can be

written as:

$$\begin{aligned} & \frac{r + \delta}{a} (\theta_x^*)^\eta + (L - x - 1) (\theta_{x+1}^*)^{1-\eta} (\theta_x^*)^\eta - (1 - \eta)(L - x - 1) \theta_{x+1}^* \\ &= \frac{1 - \eta}{h} S_{x+1}^* + \eta(L - x - 1) \theta_{x+1}^*. \end{aligned} \quad (3.24)$$

I proceed now in three steps. First, I show that, for all x , S_{x+1}^* is always larger than $(x + 1) S_{x+1}^\circ - x S_x^\circ$. Then I show that $\theta_{L-1}^* > \theta_{L-1}^\circ$. Finally, I prove that $\theta_x^* > \theta_x^\circ, \forall x$.

STEP 1 : $S_{x+1}^* > (x + 1) \cdot S_{x+1}^\circ - x \cdot S_x^\circ, \forall x \in [1, 2, \dots, L]$.

For the proof, see Appendix 3.11.

STEP 2 : $\theta_{L-1}^* > \theta_{L-1}^\circ$.

When $x = L - 1$, equations (3.23) and (3.24) respectively become:

$$\begin{aligned} & \frac{r + \delta}{a} (\theta_{L-1}^\circ)^\eta + \eta \theta_{L-1}^\circ = \frac{1 - \eta}{h} [L \cdot S_L^\circ - (L - 1) \cdot S_{L-1}^\circ] \\ & \frac{r + \delta}{a} (\theta_{L-1}^*)^\eta = \frac{1 - \eta}{h} S_L^*. \end{aligned}$$

From Step 1, the RHS in the decentralized equilibrium equation is larger than the RHS in the welfare equation. Then, looking at the LHS, $\theta_{L-1}^* > \theta_{L-1}^\circ$.

STEP 3 : $\theta_x^* > \theta_x^\circ \forall x \in [0, 1, 2, \dots, L - 2]$.

Having shown that $\theta_{L-1}^* > \theta_{L-1}^\circ$ I can proceed backward and consider the case $x = L - 2$. It is then clear that the RHS in (3.24) is larger than the RHS in (3.23), because of the inequality proved in Step 1 and because $\eta(L - x - 1) \theta_{L-1}^* > \eta(L - x - 1) \theta_{L-1}^\circ$ by Step 2. So, the LHS in (3.24) is larger than the LHS in (3.23). Consider now the LHS in (3.24). If:

$$\begin{aligned} & \eta(L - x) \theta_x^* \geq \\ & (L - x - 1) (\theta_{x+1}^*)^{1-\eta} (\theta_x^*)^\eta - (1 - \eta)(L - x - 1) \theta_{x+1}^* \quad \forall x \end{aligned} \quad (3.25)$$

then,

$$\frac{r + \delta}{a} (\theta_x^*)^\eta + \eta(L - x) \theta_x^* > \frac{r + \delta}{a} (\theta_x^\circ)^\eta + \eta(L - x) \theta_x^\circ, \quad \forall x \quad (3.26)$$

since the LHS in (3.26) is larger than the RHS in (3.24), that in turn is larger than the RHS in (3.26). But (3.25) implies:

$$\left(\frac{\theta_x^*}{\theta_{x+1}^*} \right)^\eta - \eta \frac{L - x}{L - x - 1} \cdot \frac{\theta_x^*}{\theta_{x+1}^*} - 1 + \eta \leq 0.$$

Such inequality is always verified, provided that $\theta_x^* > \theta_{x+1}^*$ ¹³. Then, with (3.26) being always true, $\theta_x^* > \theta_x^\circ \forall x \in [0, 1, 2, \dots, L-1]$. Finally, from Lemma 1, a higher θ_x implies a higher level of employment.

If $\beta < \eta$, the inequality in STEP 1 holds *a fortiori*, so the level of employment is also too high..

■

Cournot competition leads to an inefficiently low level of hours worked, as each firm tends to produce a quantity l_x smaller than the optimal one in order to keep the market price higher.

Three features explain why with $\beta \leq \eta$ the optimal level of employment and competition is lower than *laissez faire* one: Search frictions in the labour market, imperfect competition in the product market, and the rent sharing rule *à la* Hall and Milgrom (2006).

The presence of frictions in the labour market makes search externalities emerge, as any firm deciding to post a vacancy fails to consider both the decrease in other firm's vacancy-filling probability and the increase in workers' job-finding probability. Moreover, any firm deciding to enter or not the market also fails to consider the reduction in other firms' profits caused by the increase in competition. Thus, if employers' bargaining power is not low enough, entry is more desirable to the single firm than it is to the social planner, that takes the reduction in incumbent firms's profits into account.

On top of that, the rent-sharing rule I imposed also leads to an excessive level of tightness if worker's bargaining power is not high enough. Indeed, in Appendix 3.12, I show that, even if the product market was perfectly competitive, still the level of employment would be inefficiently high with $\beta \leq \eta$. The reason is the following. In the bargaining process (3.12), workers do not use the opportunity cost of employment rW_U as threat point. Thus, the wage equation (3.13) is not affected by tightness. *Ceteris paribus*, firms post more vacancies than under a standard Pissarides (2000) bargaining rule in which the fall-back position is W_U , since an increase in tightness does not push the wage up, squeezing firms' profits.

Hence, there are too few unemployed workers. Since in the *laissez faire* econ-

¹³When $\theta_x^* = \theta_{x+1}^*$, the LHS is negative. Moreover, the function is decreasing in $\theta_x^*/\theta_{x+1}^*$ when $\theta_x^* > \theta_{x+1}^*$, $\forall 0 < \eta \leq 1$.

omy the extent of substitution of these two inputs depends on workers' bargaining power, a strong β is needed to limit vacancy posting.

This excess of entry result is in line with the findings of Mankiw and Whinston (1986) and Suzumura and Kiyono (1987). Both papers prove that imperfect competition models in which firms can enter the market paying a fixed cost deliver an inefficiently high level of competition. Indeed, this chapter can be framed in the same environment: It assumes an imperfectly competitive good market where firms can enter only by involving in a costly search in the labour market. What for Mankiw and Whinston is a fixed cost, in this chapter corresponds to the expected cost of filling a job vacancy, $h/q(\theta_x)$. Were the labour market perfectly competitive (i.e. a spot market in which entry has no cost), an infinite number of firms would enter and produce, ensuring perfect competition even in the goods market. The social optimum would then coincide with the decentralized outcome.

Simulation results (presented in the next section) try to quantify the order of magnitude in terms of employment of such excess of entry inefficiency.

3.6 Quantitative Results

3.6.1 Calibration

I take the month as unit of time. Data refer to the 1997-1998 period where the stocks were fairly stable in Belgium. To calibrate the model, I make use of various surveys¹⁴, published statistics¹⁵, the quantitative results obtained in the second chapter, and results found in the literature. Table 3.1 presents the results. As in the previous sections, I assume the following Cobb-Douglas matching function $M_x \equiv a(L - x)^\eta V_x^\eta$. The elasticity η is imposed equal to 0.5, the value mostly adopted in the literature (see Petrongolo and Pissarides, 2001). In the second chapter, the calibrated value for workers' bargaining power β is 0.5 for the high-skilled sector and 0.56 for the low-skilled one. I set it equal to 0.5. Making use of the zero profit condition in vacancy creation,

¹⁴Simoens, Denys, and Denolf (1998), Denolf, Denys, and Simoens (1999) and Delmotte, Van Hootegem, and Dejonckheere (2001).

¹⁵Published by national and regional PES in Belgium and by Eurostat (2002a) and Eurostat (2002b).

I calibrate the cost of opening a vacancy h so that the expected duration of unemployment is in line with the findings of Dejemeppe (2005).¹⁶ Parameter a is a scaling factor for h and it is set equal to 0.12, so that the expected duration of filling a vacancy is around 3 months. The discount rate is fixed at 0.004 (5% on an annual basis). The number of workers in each intermediate sector is set equal to 20 in order to have a sufficiently large degrees of product competition. The elasticity of the demand in the intermediate sectors, s , is set equal to 5, in order to have an average wage in the economy of 1235 Euros (a value in accordance with the results obtained in the second chapter). I assume that hours worked l are in an interval between 0 and 2. Workers' utility of leisure is given by $2^\epsilon - l^\epsilon$. The parameter ϵ is set equal to 4, so that on average employees devote to market work around 41 % of their time¹⁷. A sensitivity analysis is conducted on these parameters.

In absence of precise estimations about the sector specific destruction rate δ , a value of 0.005 is taken.

3.6.2 Simulation Results

Figures 3.4 and 3.5 show that labour market tightness θ_x is decreasing in x while the steady-state distribution π_x is an increasing function both in the *laissez faire* economy and in the centralized one.

The simulation results are summarized in Table 3.2 and Table 3.3. I first evaluate the impact of a decrease in the cost of opening a vacancy h on the average values of the following variables: the wage, the rate of employment ($e = E/L$), the share of hours worked, and the volume of work, defined as the total number of hours worked in the economy over their total potential amount, $H \equiv \sum_{x=0}^L l_x \pi_x / L \cdot 2$. The first column of Table 3.2 shows the main result: the employment rate in the free-entry equilibrium is higher than the optimal one, the difference being around 11 %. In terms of volume of work H such discrepancy is much lower, around 4 %. The other columns of Table 3.2 show the effects of a decrease in the cost of opening a vacancy h . Such

¹⁶From her analysis of unemployment dynamics in Belgium, the average unemployment duration in 1992 was equal to 2 years in the South of Belgium and to 1.5 years in the North.

¹⁷The average wage and the average number of hours worked are defined respectively as: $(1/E) \cdot \sum_{x=1}^L w_x x \pi_x I$ and $(1/E) \cdot \sum_{x=1}^L l_x x \pi_x I$.

a reduction has almost no impact both on the wage and on the share of time spent working, whereas it slightly raises the employment rate and the volume of work. The discrepancy between the optimal and the decentralized employment level remains fairly stable. A reduction by 20% of the vacancy cost is needed in order to shorten the employment gap only by 1 %. Intervening on the vacancy cost is ineffective for the following reason. A lower h decreases the externality of one more vacancy created, but at the same time induces more firms to post vacancies. In other terms, the negative externality a single firm creates when entering the market has a lower cost for the society, but there are more firms that generate such externality in the new equilibrium. The first effect tends to reduce the gap between the optimal and the *laissez faire* outcomes, the second tends to widen it.

In Table 3.3, I consider the effects of a change in workers' bargaining power. Keeping the assumption of a matching function elasticity $\eta = 0.5$, I wonder for which value of β the welfare inefficiency can be close to 0. Differently from h , the parameter β does not appear in the welfare function, since the social planner cares only about the total surplus and not about its distribution between workers and firms. So, a higher β , by squeezing firms' profits and making entry less attractive, could (partially) offset the excess of entry inefficiency. Indeed, with $\beta = 0.7$, the difference between optimal and decentralized volume of work is around 2 %.

3.6.3 Sensitivity analysis

A sensitivity analysis is conducted on some parameters of the model. Tables 3.4 and 3.5 list the results. In Table 3.4, I consider a change in the elasticity of demand s , as well as in the workers' utility parameter ϵ . Such variations do not change the main conclusions of the original model, that is a difference around 11 percent between the optimal and the decentralized employment rate and a difference of 3 percent in terms of total hours worked .

In Table 3.5, I consider different values for the matching elasticity η . The level of wages and the amount of hours worked barely change, since these variables are chosen via the bargaining process and β is kept equal to 0.5. Employment increases with η . In the present simulation, $0 < \theta_x < 1$, for all x . Hence, by equation (3.18), a higher η lowers the expected duration of filling a vacancy

($1/q(\theta_x) = a^{-1} \theta_x^\eta$) but raises the factor at which future profits are discounted ($\theta_x q(\theta_x) = a \theta_x^{1-\eta}$). The first effect is stronger: more vacancies are created, raising the employment rate. Keeping workers' bargaining power equal to 0.5, the employment inefficiency gap decreases with η . This is because even the social planner, when η goes up, selects more vacancies for any given level of $L - x$. Such increase is slightly larger than in the *laissez faire* equilibrium. In the last row of Table 3.5, I compute for any η the value of β such that the difference between the decentralized and the optimal total numbers of hours worked is less than 1 %. Since the inefficiency gap decreases with η , a lower β/η ratio is needed to be close to the optimum. With $\eta = 0.5$, β must be equal to 0.75; with $\eta = 0.7$, β must be set to 0.8, around 14% more.

So, as far as the value of 0.5 can be considered a good proxy of the elasticity in the matching technology, β should be at least 50 per cent larger of η to set to zero the inefficiency gap.

3.7 Conclusions

In this chapter, the two-way relationship between product market competition and labour market performance has been studied both from a positive and from a normative viewpoint. As far the positive analysis is concerned, it is shown that a lower cost of opening a vacancy or a reduction in workers' bargaining power raise aggregate employment, but has ambiguous effect on the average real wage and on employee's instantaneous utility. Turning to the welfare analysis, however, the conclusion reached is that the decentralized economy may lead to an excessive level of employment and competition. A "business stealing" effect is at work in such framework: Any single firm deciding to enter the market fails to consider the reduction both in other firms' expected profits and in their probability of finding a worker. Simulation results predicts that, in order to be close to the optimal level of employment, workers' bargaining power must be larger than the elasticity η in the matching function. If the latter is imposed to be 0.5, then β must be around 0.75.

Some caveats must be advanced about the model specification. Imposing perfectly segmented labour markets is undoubtedly a major restriction. Workers are locked in their sector unless a new product is invented. Allowing workers

to search across sectors would be a more realistic extension. Finally, it would be also interesting to study the dynamic evolution of the model and not focusing only on the steady state distribution. All these extensions are left for future research.

3.8 Appendix: The bargaining game

The bargaining process I pursue is very close to Hall and Milgrom (2006); their model, in turn, is an adapted version of Binmore, Rubinstein, and Wolinsky (1986). The maximization problem in (3.12) can be seen as a limit case of an extensive form bargaining game of offers and counter-offers. More precisely, consider a bargaining process that takes place over time and where firms and workers alternate in making proposals about the wage and the numbers of hours worked. After a proposal of the counterpart, a player has three options. He can abandon the bargaining (and so get an utility of either J_V or W_U , the outside options of the employer and the employee), disagree and make a counter-offer, accept the offer. Binmore et al. (1986) show that the subgame perfect equilibria of two bargaining games beginning with a proposal either by the employer or the worker are unique. So the value of rejecting an offer and continuing to bargain is uniquely defined.

When the worker (respectively, the firm) decides to reject the other player's offer and make a counter-proposal, he receives a utility flow equal to z (resp. to zero), his utility of leisure. I also introduce an hazard rate, s , that the agreement is no longer convenient. In this case, the firm-worker pair is broken. Then, the pair starts a new negotiation. The expected discounted values for an employer and an employee in the case the production opportunity disappears and x firms are active, are given respectively by $\bar{W}_E$ and $\bar{J}_E$ (equations (3.11) and (3.10)). Consider a negotiation over the wage¹⁸. The time period separating one offer from the next one is τ . Since the value of rejecting an offer and continuing to bargain is uniquely defined¹⁹, the worker's equilibrium strategy is to accept any offer that makes him at least as well-off than both continuing the bargaining and abandoning it. There exists, therefore, a lowest wage w' that makes the worker indifferent between such options and, symmetrically, there exists a highest wage w'' that makes the firm indifferent. It is then clear that the optimal strategy for a worker is to offer always w'' and for a firm to

¹⁸The case of a negotiation over wages and hours worked is similar.

¹⁹For the proof, I refer to Binmore et al. (1986).

offer always w' . The equations governing the equilibrium are the following:

$$\begin{aligned} W_E(x, w') &= \max \{ W_U(x-1), z\tau + e^{-r\tau} [(1 - e^{-s\tau}) \bar{W}_E + e^{-s\tau} W_E(x, w'')] \} \\ J_E(x, w'') &= \max \{ J_V(x-1), e^{-r\tau} [(1 - e^{-s\tau}) \bar{J}_E + e^{-s\tau} J_E(x, w')] \} \end{aligned} \quad (3.27)$$

I assume, as Hall and Milgrom (2006), and Rosen (1997), that neither workers nor firms have an incentive to abandon the negotiation. In other terms, the constraints in (3.12) are never binding. Therefore, the system (3.27) becomes:

$$\begin{aligned} W_E(x, w') &= z\tau + e^{-r\tau} (1 - e^{-s\tau}) \bar{W}_E + e^{-(r+s)\tau} W_E(x, w'') \\ J_E(x, w'') &= e^{-r\tau} (1 - e^{-s\tau}) \bar{J}_E + e^{-(r+s)\tau} J_E(x, w') \end{aligned} \quad (3.28)$$

In equilibrium, $w' = w'' = w$. So, $W_E(x, w') = W_E(x, w'') = W_E(x)$ and $J_E(x, w'') = J_E(x, w') = J_E(x)$. Moreover, letting τ , the period separating one offer from the next, approach 0, I get:

$$(W_E(x) - J_E(x)) = \frac{z}{r+s} + \frac{s}{r+s} (\bar{J}_E - \bar{W}_E) \quad (3.29)$$

This equation is very similar to equation (17) in Hall and Milgrom (2006). If I assume $s \rightarrow +\infty$, that is the parties have only an instant to make their bargain, the surplus sharing rule will become:

$$W_E(x) - \bar{W}_E = J_E(x) - \bar{J}_E. \quad (3.30)$$

It coincides with the F.O.C. for w_x of the maximization problem in (3.12) when $\beta = 0.5$.²⁰ The threats points for an employer and employee are given respectively by $\bar{J}_E(x)$ and $\bar{W}_E(x)$. Using equations (3.9) and (3.7), I get:

$$\begin{aligned} (1 - \beta) \left[\frac{w_x^* + \phi(l_x^*) + \delta W_U(0) + (L - x)\theta_x q(\theta_x) W_E(x+1)}{r + \delta + (L - x)\theta_x q(\theta_x)} - \bar{W}_E \right] = \\ \beta \left[\frac{p(Q_x^*) l_x^* - w_x^* + \delta J_V(0) + (L - x)\theta_x q(\theta_x) J_E(x+1)}{r + \delta + (L - x)\theta_x q(\theta_x)} - \bar{J}_E \right]. \end{aligned} \quad (3.31)$$

Finally, using (3.10) and (3.11), I obtain:

$$w_x^* = \beta p_x l_x^* + (1 - \beta) \frac{(l_x^*)^\epsilon}{\epsilon}.$$

²⁰ Assuming a probability β that Nature selects the worker as first mover in the game yields the generalized Nash solution.

3.9 Appendix: Proof of Lemma 2

Let denote for simplicity $R_x \equiv p(Q_x)l_x - w_x \forall x \in [1, 2, \dots, L]$ and recall that R_x is decreasing in x (firms' revenues decrease with competition). Knowing by (3.19) that $r + \delta = \frac{R_L q(\theta_{L-1})}{h}$, equation (3.18) can be written as:

$$\frac{1}{q(\theta_{x-1})} = \frac{R_x + h(L-x)\theta_x}{R_L q(\theta_{L-1}) + h(L-x)\theta_x q(\theta_x)}.$$

Multiplying both sides by $q(\theta_x)$, one gets:

$$\frac{q(\theta_x)}{q(\theta_{x-1})} = \frac{R_x q(\theta_x) + h(L-x)\theta_x q(\theta_x)}{R_L q(\theta_{L-1}) + h(L-x)\theta_x q(\theta_x)} \quad \forall x \in [1, \dots, L]. \quad (3.32)$$

Consider the case $x = L - 1$. Equation (3.32) evaluated at $x = L - 1$ implies that $q(\theta_{L-1}) > q(\theta_{L-2})$ if and only if $R_{L-1} > R_L$. This is always the case, since firms' revenues R_x decrease with competition.

Now consider the case $x = L - 2$. Again, equation (3.32) evaluated at $x = L - 2$ implies that $q(\theta_{L-2}) > q(\theta_{L-3})$ if and only if $R_{L-2} q(\theta_{L-2}) > R_L q(\theta_{L-1}) = h(r + \delta)$. This yields:

$$\begin{aligned} \frac{R_{L-2}}{r + \delta} &> \frac{h}{q(\theta_{L-2})} = \frac{R_{L-1} + h\theta_{L-1}}{r + \delta + \theta_{L-1}q(\theta_{L-1})} \iff \\ (r + \delta) R_{L-1} + h(r + \delta) \theta_{L-1} &< (r + \delta) R_{L-2} + R_{L-2} \theta_{L-1} q(\theta_{L-1}) \end{aligned}$$

Since $R_{L-2} > R_{L-1}$, a sufficient condition for the last inequality to hold is:

$$\begin{aligned} h(r + \delta) &< R_{L-2} q(\theta_{L-1}) \iff \\ \frac{h}{q(\theta_{L-1})} &< \frac{R_{L-2}}{r + \delta} \iff \\ \frac{R_L}{r + \delta} &< \frac{R_{L-2}}{r + \delta} \end{aligned}$$

The last inequality is always verified since R_x is decreasing in x . So $q(\theta_{L-2}) > q(\theta_{L-3})$ holds.

With $x = L - 3$, by (3.32), one gets that $q(\theta_{L-3}) > q(\theta_{L-4})$ if and only if $R_{L-3} q(\theta_{L-3}) > R_L q(\theta_{L-1}) = h(r + \delta)$. Following the same steps, one gets:

$$\begin{aligned} \frac{R_{L-3}}{r + \delta} &> \frac{h}{q(\theta_{L-3})} = \frac{R_{L-2} + 2h\theta_{L-2}}{r + \delta + 2\theta_{L-2}q(\theta_{L-2})} \iff \\ (r + \delta) R_{L-2} + 2h(r + \delta) \theta_{L-2} &< (r + \delta) R_{L-3} + R_{L-3} 2\theta_{L-2} q(\theta_{L-2}) \end{aligned}$$

A sufficient condition for the last inequality to hold is:

$$\begin{aligned} h(r + \delta) &< R_{L-3} q(\theta_{L-2}) \iff \\ \frac{h}{q(\theta_{L-2})} &< \frac{R_{L-3}}{r + \delta} \end{aligned}$$

The last inequality always holds, since we have just shown that $\frac{h}{q(\theta_{L-2})} < \frac{R_{L-2}}{r + \delta}$ and $R_{L-3} > R_{L-2}$. Therefore $q(\theta_{L-3}) > q(\theta_{L-4})$.

The same steps can be undertaken for any other value of x . So, $\theta_x < \theta_{x-1}$, $\forall x \in [1, ..L]$.

3.10 Appendix: Proof of Proposition 1

- Comparative statics on β

Consider a Cobb-Douglas matching function: $M_x = a(L - x)^\eta V_x^{1-\eta}$ with $\eta = 0.5$ and recall that $R_x \equiv p(Q_x)l_x - w_x = (1 - \beta) \left[p(Q_x)l_x - \frac{l_x}{\epsilon} \right]$. Equation (3.18) then becomes:

$$\theta_{x-1} = \left[\frac{R_x + h(L - x)\theta_x}{a \cdot h \cdot (r + \delta + M_x)} \right]^{\frac{1}{\eta}}. \quad (3.33)$$

To show that $\frac{d\theta_{x-1}}{d\beta} < 0 \quad \forall x \in [1, 2, \dots, L]$, I undertake the following steps:

1. STEP : I show that $\frac{d\theta_{L-1}}{d\beta} < 0$.
2. STEP : I show that $\theta_{L-1} = -(1 - \beta)(1 - \eta)\frac{d\theta_{L-1}}{d\beta}$.
3. STEP: I show that $\frac{d\theta_{L-2}}{d\beta} < 0$ if $\theta_{L-1} \geq -(1 - \beta)(1 - \eta)\frac{d\theta_{L-1}}{d\beta}$.
4. STEP: I show that $\frac{d\theta_{L-3}}{d\beta} < 0$ if $\theta_{L-2} \geq -(1 - \beta)(1 - \eta)\frac{d\theta_{L-2}}{d\beta}$ and, in turn, a sufficient condition for such inequality is $\theta_{L-1} \geq -(1 - \beta)(1 - \eta)\frac{d\theta_{L-1}}{d\beta}$.
5. STEP: More in general, $\frac{d\theta_{x-1}}{d\beta} < 0$ if $\theta_x \geq -(1 - \beta)(1 - \eta)\frac{d\theta_x}{d\beta}$ and, in turn, a sufficient condition for such inequality is $\theta_{x+1} \geq -(1 - \beta)(1 - \eta)\frac{d\theta_{x+1}}{d\beta}$.

From the last step, moving backward, I get $\frac{d\theta_{x-1}}{d\beta} < 0 \quad \forall x \in [1, 2, \dots, L]$.

1 STEP:

From (3.33) evaluated at $x = L$, one gets:

$$\frac{d\theta_{L-1}}{d\beta} = \frac{1}{\eta} \left[\frac{R_L}{a \cdot h \cdot (r + \delta)} \right]^{\frac{1}{\eta}-1} \cdot \frac{1}{a \cdot h \cdot (r + \delta)} \cdot \frac{dR_L}{d\beta} < 0$$

The derivative is negative because $\frac{dR_x}{d\beta} = -\frac{R_x}{1-\beta} < 0, \forall x \in [1, 2, \dots, L]$.

2 STEP:

The equality comes directly by imposing $\eta = 0.5$ and using $\frac{dR_L}{d\beta} = -R_L/(1-\beta)$.

3 STEP:

Differentiating (3.33), one gets:

$$\frac{d\theta_{x-1}}{d\beta} = \frac{\theta_x^{1-\eta}}{\eta} \cdot \frac{\Gamma_x}{a \cdot h \cdot (r + \delta + M_x)^2}, \quad (3.34)$$

with

$$\Gamma_x \equiv \left[\frac{dR_x}{d\beta} + h(L-x) \frac{d\theta_x}{d\beta} \right] (r + \delta + M_x) - \frac{dM_x}{d\beta} [R_x + h(L-x)\theta_x].$$

The sign of the derivative is equal to the sign of Γ_x . In addition:

$$\begin{aligned} \frac{dM_x}{d\beta} &= (L-x)(1-\eta)q(\theta_x) \frac{d\theta_x}{d\beta}. \\ -\frac{dM_x}{d\beta} h(L-x)\theta_x &= h(L-x)^2(1-\eta)\theta_x q(\theta_x) \frac{d\theta_x}{d\beta}. \\ \frac{d\theta_x}{d\beta} M_x \cdot h(L-x) &= h(L-x)^2\theta_x q(\theta_x) \frac{d\theta_x}{d\beta}. \\ \frac{dR_x}{d\beta} M_x &= -\frac{R_x}{1-\beta} (L-x)\theta_x q(\theta_x) \\ -\frac{dM_x}{d\beta} R_x &= -R_x(L-x)(1-\eta)q(\theta_x) \frac{d\theta_x}{d\beta}. \end{aligned}$$

Substituting these equations in Γ_x , one gets that:

$$\text{if } \frac{d\theta_x}{d\beta} < 0 \text{ and } \theta_x \geq -(1-\beta)(1-\eta) \frac{d\theta_x}{d\beta}, \text{ then } \frac{d\theta_{x-1}}{d\beta} < 0. \quad (3.35)$$

From Step 1, we know that $\frac{d\theta_{L-1}}{d\beta} < 0$; from Step 2, $\theta_{L-1} = -(1-\beta)(1-\eta) \frac{d\theta_{L-1}}{d\beta}$. So $\frac{d\theta_{L-2}}{d\beta} < 0$.

4 and 5 STEP:

The procedure underlying Step 3 and Step 4 is the following.

From (3.35), $\frac{d\theta_{L-3}}{d\beta} < 0$ if $\theta_{L-2} \geq -(1-\beta)(1-\eta)\frac{d\theta_{L-2}}{d\beta}$ and $\frac{d\theta_{L-2}}{d\beta} < 0$. The latter condition has been proved in the previous step, so only the former has to be shown.

Once proved that $\frac{d\theta_{L-3}}{d\beta} < 0$, the only condition needed to verify that $\frac{d\theta_{L-4}}{d\beta} < 0$ is $\theta_{L-3} \geq -(1-\beta)(1-\eta)\frac{d\theta_{L-3}}{d\beta}$. In turn, once proved that $\frac{d\theta_{L-4}}{d\beta} < 0$, to show that θ_{L-5} is decreasing in β , one only needs to prove that $\theta_{L-4} \geq -(1-\beta)(1-\eta)\frac{d\theta_{L-4}}{d\beta}$, and so on.

In general, $\frac{d\theta_{x-1}}{d\beta} < 0$ if $\theta_x \geq -(1-\beta)(1-\eta)\frac{d\theta_x}{d\beta} \forall x \in [0, 1, \dots, L-1]$. Comparing (3.33) and (3.34) with $\eta = 0.5$, one gets that $\theta_x \geq -(1-\beta)(1-\eta)\frac{d\theta_x}{d\beta}$ if

$$-(1-\beta) \cdot \Gamma_{x+1} \leq [R_{x+1} + h(L-x-1)\theta_{x+1}] \cdot (r + \delta + M_{x+1}) \quad (3.36)$$

Since:

$$-(1-\beta)\frac{dR_{x+1}}{d\beta}(r + \delta + M_{x+1}) = R_{x+1}(r + \delta + M_{x+1}),$$

the inequality (3.36) can be written in the following way:

$$\begin{aligned} -(1-\beta) \left\{ h(L-x-1)\frac{d\theta_{x+1}}{d\beta}(r + \delta + M_{x+1}) - \frac{dM_{x+1}}{d\beta}[R_{x+1} + h(L-x-1)\theta_{x+1}] \right\} \\ \leq h(L-x-1)\theta_{x+1}(r + \delta + M_{x+1}) \end{aligned} \quad (3.37)$$

So, for $\frac{d\theta_{L-3}}{d\beta} < 0$ to be verified, it is sufficient to show that (3.37) holds at $x+1 = L-1$. From Step 2, I know that:

$$-(1-\beta)(1-\eta)h\frac{d\theta_{L-1}}{d\beta}(r + \delta + M_{L-1}) \leq h\theta_{L-1}(r + \delta + M_{L-1}). \quad (3.38)$$

So, if the LHS of (3.37) evaluated at $x+1 = L-1$ is not greater than the LHS of (3.38), then inequality (3.37) is verified and $\frac{d\theta_{L-3}}{d\beta} < 0$. Dividing the LHS of (3.37) by $(r + \delta + M_{L-1})$ and doing some algebra yields:

$$-(r + \delta + M_{L-1})\frac{d\theta_{L-1}}{d\beta}h(1-\beta)\eta \leq \frac{d\theta_{L-1}}{d\beta}(1-\eta)(1-\beta)[R_{L-1} + h\theta_{L-1}]$$

Simplifying and imposing $\eta = 0.5$:

$$\frac{h}{q(\theta_{L-1})} \leq \frac{R_{L-1} + h\theta_{L-1}}{r + \delta + M_{L-1}} = \frac{h}{q(\theta_{L-2})}$$

This inequality is always verified since, from Lemma 2, $\theta_x < \theta_{x-1}, \forall x$. Therefore, $\theta_{L-2} \geq -(1-\beta)(1-\eta)\frac{d\theta_{L-2}}{d\beta}$ and, consequently, $\frac{d\theta_{L-3}}{d\beta} < 0$. To show that $\frac{d\theta_{L-4}}{d\beta} < 0$, one must undertake the same passages: from Step 3, θ_{L-4} is decreasing in β if $\theta_{L-3} \geq -(1-\beta)(1-\eta)\frac{d\theta_{L-3}}{d\beta}$. In turn, this is equivalent to prove inequality (3.37) evaluated at $x+1 = L-2$. Using the fact that $\theta_{L-2} \geq -(1-\beta)(1-\eta)\frac{d\theta_{L-2}}{d\beta}$, inequality (3.37) holds even at $x+1 = L-2$. Proceeding backward, we get that $\frac{d\theta_x}{d\beta} < 0 \quad \forall x \in [0, 1, 2, \dots, L-1]$.

- *Comparative statics on h*

The procedure is the same as in the comparative statics for β . I show that:

1. $d\theta_{L-1}/dh$ is negative and that $\theta_{L-1} = h(1-\eta) \cdot d\theta_{L-1}/dh$.
2. $d\theta_{x-1}/dh$ is negative if $\theta_x \geq h(1-\eta) \cdot (d\theta_x/dh)$ that, in turn, holds if $\theta_{x+1} \geq h(1-\eta) \cdot (d\theta_{x+1}/dh)$.

Consider point 1:

$$\frac{d\theta_{L-1}}{dh} = -\frac{\theta_{L-1}}{h\eta} < 0.$$

Hence, $\theta_{L-1} = h(1-\eta) \cdot d\theta_{L-1}/dh$. To show point 2, consider the following derivative:

$$\frac{d\theta_{x-1}}{dh} = a\theta_{x-1}^{1-\eta} \cdot \frac{T_x}{\eta[h(r+\delta+M_x)]^2}, \quad (3.39)$$

with

$$T_x \equiv \left[(L-x)\theta_x + h(L-x)\theta_x \frac{d\theta_x}{dh} \right] h(r+\delta+M_x) - \left[(r+\delta+M_x) + h \frac{dM_x}{dh} \right] [R_x + h(L-x)\theta_x].$$

The sign of (3.39) is equal to the sign of T_x . After some algebra, one gets that $T_x < 0$ if $\theta_x \geq -h(1-\eta)\frac{d\theta_x}{dh}$. In turn, by (3.33) and (3.39), a sufficient condition for such inequality to hold is:

$$-h \cdot T_{x+1} \leq [R_{x+1} + h(L-x-1)\theta_{x+1}] \cdot (r+\delta+M_{x+1}). \quad (3.40)$$

So, to show that θ_{L-2} is decreasing in h , I have to show that (3.40) holds at $x+1 = L-1$. Since $\theta_{L-1} \geq h(1-\eta)\frac{d\theta_{L-1}}{dh}$, (3.40) is verified if

$$-h^3(1-\eta)\frac{d\theta_{x+1}}{dh}a(L-x-1)(r+\delta+M_x) \geq -h \cdot T_{x+1},$$

evaluated at $x + 1 = L - 1$. After some algebra, such inequality is equivalent to:

$$h(1-\eta)(r+\delta+M_{x+1}) + R_{x+1}(1-\eta)q(\theta_{x+1}) \geq h(r+\delta+M) - h(L-x-1)(1-\eta)\theta_{x+1}q(\theta_{x+1}),$$

evaluated at $x + 1 = L - 1$. Dividing by $1 - \eta = 0.5$, and simplifying one gets:

$$R_{x+1}q(\theta_{x+1}) \geq h(r + \delta) = R_{L-1}q(\theta_{L-1})$$

Such inequality is always verified, for the LHS of (3.32) is greater than one, $\forall x$. Hence, $\theta_{L-2} \geq -h(1-\eta)\frac{d\theta_{L-2}}{dh}$ and θ_{L-3} is decreasing in h . The same steps undertaken for θ_{L-4} , θ_{L-5} , ..., θ_0 .

3.11 Appendix: Details of the proof of Proposition

2

The inequality $S_{x+1}^* > (x+1) \cdot S_{x+1}^\circ - x \cdot S_x^\circ$ is equivalent to:

$$p(Q_{x+1}^*)l_{x+1}^* - \frac{(l_{x+1}^*)^\epsilon}{\epsilon} > (x+1) \left[p(Q_{x+1}^\circ)l_{x+1}^\circ - \frac{(l_{x+1}^\circ)^\epsilon}{\epsilon} \right] - x \left[p(Q_x^\circ)l_x^\circ - \frac{(l_x^\circ)^\epsilon}{\epsilon} \right].$$

Notice that the term at the RHS can be written as:

$$p(Q_{x+1}^\circ)l_{x+1}^\circ - \frac{(l_{x+1}^\circ)^\epsilon}{\epsilon} + x \cdot \left[p(Q_{x+1}^\circ)l_{x+1}^\circ - \frac{(l_{x+1}^\circ)^\epsilon}{\epsilon} - p(Q_x^\circ)l_x^\circ + \frac{(l_x^\circ)^\epsilon}{\epsilon} \right]. \quad (3.41)$$

Consider first the term outside the square brackets. Recall from (3.22) that the optimal and the decentralized level of hours worked coincide if there is perfect competition. Moreover, revenues are always higher in a Cournot market than in a perfect competition:

$$p(Q_{x+1}^\circ)l_{x+1}^\circ - \frac{(l_{x+1}^\circ)^\epsilon}{\epsilon} < p(Q_{x+1}^*)l_{x+1}^* - \frac{(l_{x+1}^\circ)^\epsilon}{\epsilon}.$$

It is then sufficient to show that the term in (3.41) inside the square graphs is negative to prove the inequality of Step 1. But this is the case if $p(Q_x^\circ)l_x^\circ + \frac{(l_x^\circ)^\epsilon}{\epsilon}$ is decreasing in x . Ignoring for simplicity the integer problem, I get:

$$\frac{d \left[p(Q_x^\circ)l_x^\circ - \frac{(l_x^\circ)^\epsilon}{\epsilon} \right]}{dx} = \frac{dp(Q_x^\circ)}{dQ_x^\circ} l_x^\circ \frac{dQ_x^\circ}{dx} + \frac{dl_x^\circ}{dx} \left[p(Q_x^\circ) - (l_x^\circ)^{\epsilon-1} \right] < 0.$$

The term inside the square bracket is equal to zero, while the first term is negative since $p(Q_x)$ has a negative slope.

3.12 Appendix: Decentralized vs. optimal solution in the case of perfect competition

Consider the same two-tier productive scheme explained in section 3.2. The only difference is that in each intermediate sector there is a continuum of workers of measure L . Perfect competition prevails in each intermediate market. Firms and workers are price-takers. Thus, in computing their expected lifetime income, $W_E(x) = W_E$, $J_E(x) = J_E$, $J_V(x) = J_V$, and $W_U(x) = W_U$, $\forall x$. Keeping the same bargaining process (3.12), the F.O.C.s for wage and hours worked become:

$$\begin{aligned} w^* &= \beta p(Q^*) l^* + (1 - \beta) \frac{l^{*\epsilon}}{\epsilon} \\ l^{*\epsilon-1} &= p(Q^*) \end{aligned}$$

The free-entry condition $J_E = \frac{h}{q(\theta)}$ can be written as:

$$\frac{h}{q(\theta)} = \frac{p(Q^*) - w^*}{r + \delta} = \frac{(1 - \beta) \left[p(Q^*) l - \frac{l^{*\epsilon}}{\epsilon} \right]}{r + \delta}.$$

The social planner's problem is the same as in (3.20), with the only difference that x , the level of employment in a given sector, is now a continuous variable. So, $\Omega_{x+1} - \Omega_x$ is replaced by $d\Omega_x/dx$. Computing the F.O.C.s and applying the envelope theorem yields²¹:

$$\frac{h}{q(\theta)} = \frac{(1 - \eta) \left[p(Q^\circ) l^\circ - \frac{l^{\circ\epsilon}}{\epsilon} \right] - \eta h \theta}{r + \delta},$$

in which $l^\circ = l^*$ for the F.O.C. (3.22). A comparison between the *laissez faire* outcome and the social planner's one shows that the Hosios condition $\beta = \eta$ is not sufficient to decentralize the optimum. If $\beta \leq \eta$, the equilibrium level of tightness is inefficiently high. The Hosios condition *and* a tax $\tau = \eta h \theta$ levied on firms' profits are needed to ensure the efficiency in the decentralized economy.

²¹For the existence of a solution, see Shimer (2004b).

3.13 Figures and Tables

Parameters	
r	0.004
δ	0.005
ϵ	4
s	5.5
I	50
h (Euro/month)	24000
β	0.5
η	0.5
L	20
a	0.12
Endogenous var. (average)	
l (%. average)	41.0
$1/\theta q(\theta)$ (months)	19.5
$1/q(\theta)$ (months)	3.4
$\bar{w}$ (Euro/month)	1235

Table 3.1. Calibration: Parameters and levels of endogenous variables in steady state.

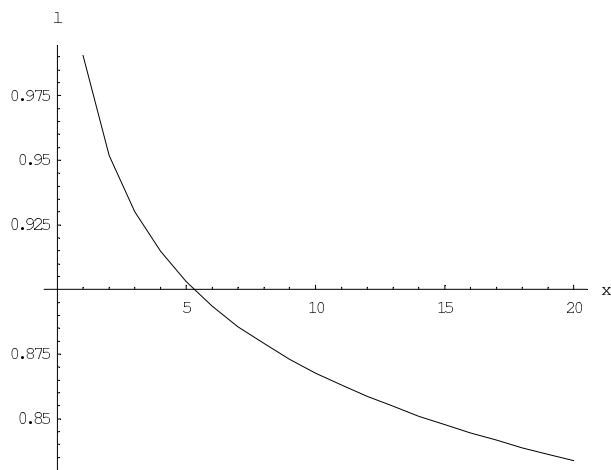


Figure 3.2: Simulation results: Hours Worked $l \in [0, 2]$.

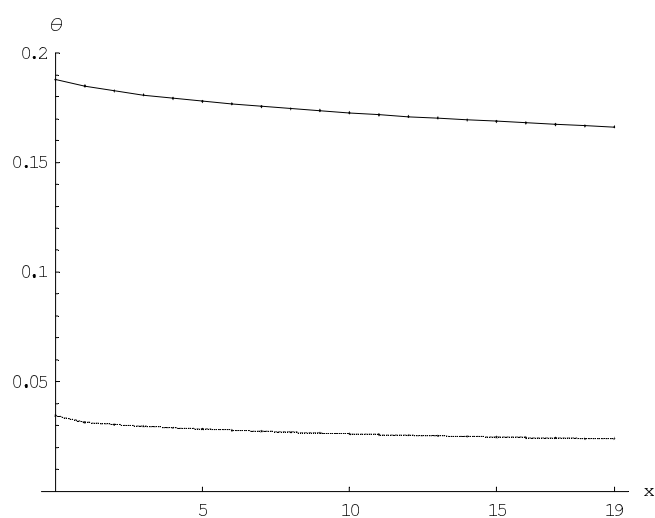


Figure 3.3: A comparison of the optimal level of labour market tightness (dotted line) with the decentralized one (continuous line).

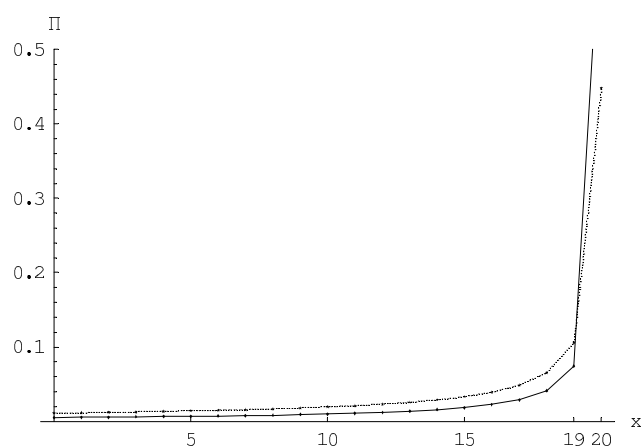


Figure 3.4: A comparison of the optimal steady state distribution (dotted line) with the decentralized one (continuous line).

Variables	h = 24000	h = 16000	h = 14000
$\bar{w}$ (euros per month)	1235	1232	1231
e^* (per cent)	90.9	92.3	92.6
Share of hours worked	41.0	41.8	41.8
H^* (per cent)	38.0	38.5	38.7
$e^* - e^\circ$	11.1	10.6	10.4
$H^* - H^\circ$	4.3	4.2	4.1

Table 3.2. Simulation Results. Variation in the cost of opening a vacancy. Superscript * denotes the free-entry equilibrium values, while superscript $^\circ$ the optimal ones.

Variables	$\beta = 0.5$	$\beta = 0.6$	$\beta = 0.7$
Employment rate e^* (per cent)	90.9	88.9	85.9
Volume of work H^* (per cent)	38.0	37.2	36.0
$e^* - e^\circ$ (per cent)	11.1	9.1	6.0
$H^* - H^\circ$ (per cent)	4.3	3.6	2.3

Table 3.3. Simulation Results. Variation in workers' bargaining power β when $\eta = 0.5$.

Parameters	Benchmark	1 $^\circ$ case	2 $^\circ$ case	3 $^\circ$ case
ϵ	4	2	4	3
s	5.5	5.5	6.5	4
Variables				
e^* (per cent)	90.9	89.5	89.2	94.2
$\bar{w}$ (euros per month)	1235	1877	1015	2404
Share of hours worked (per cent)	41	55.7	39.8	53.0
Volume of work H^* (per cent)	38.0	49.9	35.5	50.0
$H^* - H^\circ$ (per cent)	4.3	5.3	4.2	5.0
$e^* - e^\circ$ (per cent) if $\beta = \eta = 0.5$	11.1	12.4	11.1	10.7
$H^* - H^\circ$ (per cent) if $\beta = 0.7$	2.3	3.1	1.9	3.4

Table 3.4. Sensitivity analysis.

Parameter	Benchmark	1° case	2° case	3° case
η	0.5	0.4	0.6	0.7
Variables				
e^* (per cent)	90.9	86.8	93.0	94.3
w (euros per month)	1235	1244	1230	1228
Share of hours worked (per cent)	41	41.9	41.8	41.8
Volume of work H^* (per cent)	38.0	36.4	38.9	30.3
$H^* - H^\circ$ (per cent)	4.3	4.6	3.5	2.6
$e^* - e^\circ$ (per cent) if $\beta = \eta$	11.1	11.7	9.1	6.8
β/η s.t $H^* - H^\circ < 1\%$	1.5	1.62	1.3	1.14

Table 3.5. Sensitivity analysis: change in the matching elasticity η .

Matching Models Under Scrutiny: A Survey

4.1 Introduction

The search and matching model¹ is nowadays the standard workhorse adopted by macro and labour economists to study aggregate labour markets. In its simpler formulation, it is assumed that the trading process in the labour market is a costly activity that requires time. Firms post job vacancies, unemployed workers search for a job. To produce the unique consumption good, a firm-worker pair must be formed. Firms (job-seekers) find a job-seeker (firm) at an endogenous rate that depends on the level of unemployment and the level of vacancies opened in the market, while, at an exogenous rate, the firm-worker pair is destroyed. The labour force is fixed, whereas the number of vacancies posted in the market is determined by a free-entry zero profit condition.

With Merz (1995), Andolfatto (1996), and Feve and Langot (1996) the assumption of a labour market with frictions and a matching technology is introduced in a RBC framework, adding a theory of equilibrium unemployment in a model otherwise based on market clearing conditions and full employ-

¹Pioneered by Diamond, Mortensen, and Pissarides. See Mortensen and Pissarides (1999) and Pissarides (2000) for a detailed exposition.

ment. Compared to Hansen's (1985) lottery framework, these papers succeed in replicating some empirical observations of the labour market, such as the countercyclical behaviour of the labour share, the lower volatility of real wages with respect to labour productivity, and the dynamic correlation between the latter and employment.

Cole and Rogerson (1999) also study the business cycle consistency of an RBC matching model. Their conclusion is that the ability of the model to account for the business cycle facts crucially depends on the steady-state around which the model fluctuates. If the average quarterly job finding rate is around 0.3 (that means an expected duration of unemployment of 9 months), the model matches the data quite well. With a lower job finding rate, the results are much less consistent. Cole and Rogerson base their analysis on the empirical results of Davis and Haltiwanger (1992), Davis, Haltiwanger, and Schuh (1996), and Blanchard and Diamond (1989) and (1990)². Examining U.S. gross worker flows data, this bunch of research states that fluctuations in unemployment mainly depends on variations in job destruction along the cycle, whereas job creation plays a less important role.

Shimer (2005a) challenges this view. In this paper, he first measures workers' transition probabilities by using U.S. monthly statistics on the level of unemployment and not gross worker flow data. The results contradict those obtained by the previous literature: the job-finding rate is strongly procyclical, whereas the separation rate is only weakly countercyclical. So, the former, not the latter, is the variable that explains most of the fluctuations in unemployment. Then, Shimer compares these new empirical results with the theoretical model. The most important claims of his paper can be summarized as follows:

1. A textbook search and matching model is not able to explain the observed fluctuations of unemployment and vacancies in the U.S. economy in response to productivity shocks of plausible magnitude. The job-finding rate results are 12 times more volatile in the data than in the model, whereas the standard deviation of job vacancies in the data is 10 times larger than in the model.

²A more recent paper based on worker flow data is Davis, Faberman, and Haltiwanger (2006).

2. This discrepancy between data and the model stems from the wage formation assumptions. In a standard matching model, a Nash bargaining solution is introduced in order to share the total surplus between the firm and the worker. When a positive productivity shock hits the economy, wages instantaneously increase, dampening firms' vacancy creation. Thus, vacancies (and, as a consequence, the job finding rate) are much more variable in data than in the model.
3. A shock on the separation rate not only fails to replicate vacancies and unemployment fluctuations, but also predicts a counterfactual positive correlation between these two variables. A higher separation rate raises both vacancies and unemployment.

The aim of this chapter is to survey the literature originated from Shimer's findings. RBC models with search and matching in the labour market are widespread in macroeconomics. Covering all the most recent contributions in this field would require one survey for any single topic faced in such papers.³ In the present chapter I will study only the papers whose main scope is to react to Shimer's results. In doing so, I distinguish three different and in some parts opposite avenues that have been taken up by scholars.

In the first group, I list papers that agree with Shimer's first and second claim: A textbook matching model fails to replicate the U.S. business cycle features of its most important variables because wages are too responsive (both directly and via a general equilibrium effect) to changes in productivity. The most straightforward way to reconcile model and data is to modify the wage formation rules, introducing some form of wage rigidity (Shimer, 2004, and Hall, 2005), imposing imperfect information in the firm-worker negotiation (Kenan, 2006), modeling staggered wage contracts (Gertler and Trigari, 2006), or changing the fall-back positions of workers and firms in the Nash bargaining problem (Hall and Milgrom, 2006). For though these papers are very different from each other, they aim at the same goal, namely to loosen the link between the wage and average labour market tightness (i.e the vacancy/unemployment

³For instance, some dynamic stochastic models with matching frictions are more oriented on monetary issues (Gali and Blanchard, 2005 or Walsh, 2005), while others focus on the dynamic behaviour of labour force participation (Veracierto 2002).

ratio) in order to make vacancies more reactive to productivity changes. The results seem to confirm Shimer's claim: changing the wage formation assumptions, a matching model fits U.S. data better.

A second avenue is pursued by Hagedorn and Manovskii (2006). They do not agree on Shimer's first two claims. According to them, the model is unable to mimic the business cycle behaviour of unemployment and vacancies because of an erroneous calibration of two key parameters: the instantaneous utility of being unemployed (composed by the utility of leisure and the level of the unemployment benefits) and workers' bargaining power. In Shimer's paper, the former is set at a too low level, whereas the latter is too high, implying an elasticity of the wage with respect to productivity close to one, twice as large as the normal estimates. With a higher calibrated value for the utility of unemployment and a lower bargaining power for workers, the wage elasticity is more conveniently around 0.5 and the model succeeds in replicating the observed business cycle fluctuations.

Hagedorn and Manovskii's viewpoint is called in question by Mortensen and Nagypál (2006) and is in contrast with the results of Costain and Reiter (2006). Mortensen and Nagypál agree with Hagedorn and Manovskii's claim that wage rigidity or any other departure from the Nash bargaining solution plays only a marginal role in making unemployment and vacancies more volatile. For instance, they show that even completely rigid wages, set equal to the instantaneous utility in unemployment, are ineffective if the latter is fixed at too low level. Yet, according to them, even the parametrization performed by Hagedorn and Manovskii is not the right solution, since it postulates a too small and unrealistic difference between the instantaneous values of being employed and unemployed.

Costain and Reiter verify the quantitative consistency of the model not only in response to productivity and destruction rate shocks, but also to changes in the level of unemployment benefits (UB). The conclusion is that any attempt to calibrate a standard matching model in order to match the business cycle unemployment and vacancies data produces unrealistic results about the effects of an increase in UB on the unemployment rate, and *vice versa*. A parametrization of the model that explains half of the variation of unemployment over the business cycle (still unsatisfactory but that improves Shimer's results) pro-

duces an elasticity of the unemployment rate with respect to UB more than 5 times larger than in the most recent microeconomic estimations. A trade-off arises: Either the calibration makes the model more volatile to reconcile it with business cycle data, or the calibration makes it less volatile to fit policy evaluation data. Both tasks cannot be reached. Therefore, even if Hagedorn and Manovskii's calibration were more accurate than Shimer's one, still the model should be amended, since it would deliver wrong policy evaluation results. Costain and Reiter propose two possible solutions: sticky wages and embodied technological progress.

This naturally leads to a third way Shimer's findings have been confronted. This last group of papers (more or less) implicitly questions both the idea that changes in the wage formation criterion can alone better the model's performance and the idea that a wrong calibration is the source of Shimer's findings. What this third vein of the literature argues is that a standard matching framework contains some simplified assumptions (for instance about the labour market flows in or out of employment or the absence of capital) that inevitably jeopardize the quantitative consistency of the model. Enriching the basic set-up would bridge the gap between the data and the model. For instance, Silva and Toledo (2006) introduce turnover costs, while Nagypál (2005), Krause and Lubik (2006), and Mortensen and Nagypál (2006) consider on-the-job search⁴.

A separate explanation deserves Shimer's third finding, namely the fallacious response of the model to changes in the job-destruction rate. Under the usual assumption of a Cobb-Douglas matching technology, in the stochastic dynamic model a positive shock on the destruction rate raises both unemployment and vacancies, while at the comparative statics level a higher destruction rate raises unemployment but has ambiguous effects on vacancies. The dynamic stochastic results are in contrast with the observed negative correlation between unemployment and vacancies, the so-called Beveridge curve. Only Nagypál (2005) offers an answer to this problem. In Shimer's model, a positive shock on the destruction rate raises unemployment because firms discount future profits at a lower value and tend to open less vacancies. However, as unemployed workers are the only job-seekers in that model, a higher level of unemploy-

⁴Other examples are Yashiv (2006a), Garibaldi (2006), and Rotemberg (2006).

ment implies a higher number of searchers, so a single firm fills its vacancy quicker and vacancy creation is boosted. This second indirect effect is stronger than the first one: as the destruction rate rises, both vacancies and unemployment go up. Nagypál shows that this counter-factual increase in the number of vacancies disappears once we allow for on-the-job search. In this case, the number of job seekers does not coincide with the level of unemployment so that the link between higher unemployment and larger vacancy creation is broken.

Finally, in the concluding remarks, I compare the results obtained by the three different approaches. In the light of the quantitative results obtained both at business cycle and at a policy analysis level, I conclude that the third route is the most successful.

The chapter is organized as follows. Section 4.2 describes the empirical and theoretical framework. Section 4.3 presents the quantitative inconsistency of the model. Sections 4.4, 4.5, and 4.6 respectively survey the three different approaches followed to react Shimer's claims. Section 4.7 concludes.

4.2 Empirical and Theoretical Framework

4.2.1 U.S. Labour Market Facts

Table 4.1, taken from Shimer (2005a), summarizes the statistics on the economic variables of interest. In bold are the numbers on which Shimer and other scholars have concentrated more. I first present Shimer's data collection method and then, in section 4.2.2, I discuss the differences with the literature using gross worker flow data. The variables are the following: u , unemployment, v vacancies, f , the job-finding rate, s the destruction rate, and p labour productivity. Data are quarterly and refers to the period from 1951 to 2003. In order to disentangle business-cycle fluctuations from other long-run factors, all variables are taken in logs as deviations from an HP trend with smoothing parameter 10^5 .

Some papers question the choice of the unemployment level as the most appropriate indicator of job search activity. Shimer opts for it, claiming that the other indicator often advocated, the employment population ratio, appears to

		u	v	v/u	f	s	p
Standard deviation		0.190	0.202	0.382	0.118	0.075	0.020
Quarterly autocorrelation		0.936	0.940	0.941	0.908	0.733	0.878
Correlation matrix	u	1	-0.894	-0.971	-0.949	0.709	-0.408
	v	-	1	0.975	0.897	-0.684	0.364
	v /u	-	-	1	0.948	-0.715	0.396
	f	-	-	-	1	-0.574	0.396
	s	-	-	-	-	1	-0.524
	p	-	-	-	-	-	1

Table 4.1. Shimer's summary statistics. Quarterly U.S Data 1951-2003. All variables are reported in logs as deviations from an HP trend with smoothing parameter 10^5 . Source: Shimer (2005a).

be more pro-cyclical than the unemployment rate, worsening even more the performance of the model.

On the other side of the market, the Job Openings and Labor Turnover Survey (JOLTS) is the ideal source for an analysis of job vacancies. Unfortunately, it collects data only from December 2000, so Shimer has to consider a data proxy for vacancies, the Conference Board help-wanted index, measuring the number of help-wanted advertisements in 51 major newspapers. A comparison of the two measures from 2000 to 2003 shows that the second-best proxy does not differ substantially from the JOLTS. What emerges from a first inspection of data is the relatively high volatility of the level of unemployment and vacancies. The standard deviation of unemployment, for instance, is 0.19, meaning that this variable can be as much as 38 percent above or below trend. Since unemployment is counter-cyclical and vacancies are pro-cyclical, the v/u ratio is extremely pro-cyclical. Moreover, both unemployment and vacancies are very persistent variables, with an autocorrelation around 0.94.

As usual in this kind of models, Shimer assumes a constant returns to scale matching technology, $m(v, u)$, increasing and concave in both arguments, representing the measure of new jobs created as a function of the total stocks of unemployment u and vacancies v . The constant returns to scale assumption

is particularly useful, since the job-finding rate $f \equiv m(v, u)/u = m(1, \theta)$ depends only on the v/u ratio, denoted by θ , labour market tightness. Function f is increasing and concave in θ . To measure the job-finding rate, Shimer prefers not to use gross worker flow data, for the dataset is available only since 1976 and measurement and classification errors could bias his estimation. Instead, he constructs it by making use of the monthly number of unemployed people and assuming that workers neither enter nor exit the labour force⁵. In a subsequent paper, he shows that relaxing such strong assumption does not change its findings on unemployment and vacancy fluctuations (see Shimer (2005b)). He gets an average monthly hazard rate around 0.45 and a standard deviation of 0.118. As Shimer points out, the high positive correlation between f and θ is a strong argument in favour of a constant returns to scale matching technology. Once f_t and θ_t have been computed, Shimer looks at the matching function. He imposes a Cobb-Douglas functional form, $m(v, u) = \mu u^\alpha v^{1-\alpha}$. Two parameters need to be estimated, α and μ , and using his data on f_t and θ_t , he gets a value of α between 0.70 and 0.75, beyond the plausible range of 0.3 to 0.5 reported by Petrongolo and Pissarides (2001).

To compute the destruction rate, Shimer uses another labour market flows equation. Let u_{t+1}^s denote the number of people unemployed for less than one month in month $t + 1$ and e_t the level of employment at time t . When workers lose their jobs they have approximately half a month to find a new one and not being recorded as unemployed. The labour market flow equation takes the following form:

$$u_{t+1}^s = s_t e_t \left(1 - \frac{1}{2} f_t\right)$$

Making use of such formula, he gets an average monthly destruction rate of 0.034. Such variable results less volatile than the job finding rate and it presents a negative correlation with labour market tightness.

Labour productivity is computed as the ratio between real output and the number of workers in the non-farm business sector. Data show that labour

⁵More in detail, let u_t^s the number of unemployed people for less than one month in month t , and assume that unemployed workers do not exit the labour force, the labour market flow equation can be written as:

$$u_{t+1} = u_t(1 - f_t) + u_{t+1}^s$$

Solving for f_t , Shimer can compute the hazard rates from 1951 to 2003.

productivity is positive correlated with tightness and, more crucially, its standard deviation is ten times lower than that of vacancies and almost twenty times lower than standard deviation of labour market tightness. Labour productivity is fairly stable. It can depend on a composition bias, since less productive jobs are more likely to be destroyed during recessions. However, as Shimer points out, another effect goes in the opposite direction, as labour productivity is higher in more cyclical sectors of the economy (such as durable goods manufacturing). Moreover, empirical investigation about the cleansing effects of recession offers mixed conclusions.

Not Shimer (2005a), but some other papers I will survey later on, concentrate also on the volatility of the real wage with respect to productivity. Gertler and Trigari (2006) estimate a standard deviation of the real wage over the standard deviation of productivity around 0.46; for Hagedorn and Manovskii (2006), such ratio is around 0.449.

4.2.2 Job Creation vs. Job Destruction Volatility

The strong procyclicality of the job-finding rate and the weak countercyclicality of the job separation rate with respect to output contradict the conclusions reached by Blanchard and Diamond (1989) and (1990), Davis and Haltiwanger (1992) and Davis, Haltiwanger, and Schuh (1996) that consider the job destruction as the main source of unemployment fluctuations. The question can be put in these terms: Shimer argues that recessions are essentially periods where it is extremely difficult to find a job, whereas most of the previous literature identifies recessions as periods mainly characterized by high job loss rates. What can explain such opposite results?

First of all, Shimer considers the dynamic behaviour of monthly unemployment levels to measure f_t and s_t , whereas the authors quoted above analyse gross worker and job flow data. More in detail, Davis and Haltiwanger define job creation as the net employment gains at establishments that experience positive net gains in a certain period. Similarly, job destruction is defined as the net job losses at establishments experiencing negative net employment gains in a certain period. But, as Shimer (2005b) rightly observes, such definitions do not coincide with those of the job-finding and separation rates and

can therefore lead to a misinterpretation of data. The main reason is that in the job destruction estimates there are computed *both* the firings of existing employees *and* the decisions of not hiring new workers that replace quitters. Firings represent an increase in the separation rate, but the decision of not hiring represents a decrease in the job-finding rate.

In addition, Davis and Haltiwanger consider only manufacturing establishments with more than five employees. Boeri (1996) and Foote (1998) claim that these restriction deeply affect the results. For instance, Boeri argues that small firms can explain much of the cyclical variation in job creation.

Finally, these conflicting results in part depend on the different statistics measured. Much of the gross flows literature (such as Blanchard and Diamond, 1990) refer to the *levels*, that is $f_t u_t$ and $s_t e_t$, whereas Shimer's findings are about *rates*, f_t and s_t ; u_t and f_t moves in opposite direction over the cycle, so that $f_t u_t$ remain fairly stable even with a strongly procyclical job finding rate.

4.2.3 Policy Evaluation Estimates

Shimer has only focused on business cycle fluctuations. The search and matching model, however, can be quantitatively assessed also with respect to labour market micro data. For instance, one could confront the predictions of the model about the effects of unemployment benefits on re-employment probabilities with the estimations found in the literature. Consider the estimates about the impact of the UB both on unemployment duration and on the level of unemployment. Studies about the hazard rate, surveyed by Layard, Nickell, and Jackman (1991) and Atkinson and Micklewright (1991), argue that the elasticity of unemployment duration with respect to the generosity of benefits should lie between 0.1 and 1.0. For instance, for Spain, Bover, Arellano, and Bentolila (2002) have found an elasticity of 0.1 (others estimates are cited in Hornstein, Krusell and Violante, 2005). These estimates are not relevant for our analysis, as they concentrate on the partial equilibrium effects of these policies, i.e. on the labour supply decisions.

It is therefore worthwhile to look at the cross-country studies conducted by Layard, Nickell and Jackman (1991) and Layard and Nickell (1999), whose aim is to evaluate the general equilibrium effects of some labour market policies in

various countries. They consider OECD data that go back to 1960 and find a semi-elasticity of unemployment with respect to the UI benefits replacement ratio around 1.3. Costain and Reiter (2006) also run some cross-country regressions on the basis of Layard and Nickell's dataset and obtain a semi-elasticity close to 2. Baker, Glyn, Howell, and Schmitt (2003) survey other recent estimates. The results are not substantially larger than those obtained by Layard and Nickell.

4.2.4 Theoretical Framework

The model built up by Shimer is a standard matching framework with the addition of a stochastic economy-wide change in two parameters, productivity and the destruction rate, that hit the economy. I present the case of productivity shocks, the environment with a stochastic destruction rate being symmetric. All the endogenous variables that depend on the current value of productivity are henceforth denoted by the subscript p . The economy is composed by a measure L of risk-neutral (employed or unemployed) workers. Time is continuous and the discount factor is denoted by r . Unemployed workers search for a job, whereas every firm can post only one vacancy (one job - one firm assumption). All jobs are identical and every firm-worker pair produces the unique consumption good at the flow rate of p . Autocorrelated shocks affect the value of p . More precisely, the time sequence $\{p_t\}$ is a jump process with an arrival rate λ and a conditional distribution of new values represented by the c.d.f $F_p : P \times P \rightarrow [0, 1]$, P being the support of the process.

The instantaneous utility enjoyed by the unemployed worker is denoted by z , while the cost of posting a vacancy is c . The flow of new matches is denoted by $m(v, u)$ and the job finding rate is given by $f(\theta) \equiv m(v, u)/u$; the rate at which vacancies are filled is $m(v, u)/v = f(\theta)/\theta$, positive, decreasing and convex function of θ . At an exogenous rate a firm-worker pair is destroyed. The law of motion of unemployment is equal to:

$$\dot{u}_p = s(L - u_p) - f(\theta_p)u_p$$

In steady-state, we have:

$$u_p = \frac{sL}{s + f(\theta_p)}. \quad (4.1)$$

Once a worker finds a firm with a vacant job, a surplus of the match arises. It is given by the difference between the expected discounted value that the two parties will receive by forming a match and the expected discounted value they renounce by being employed. A zero profit condition is imposed on the demand side of the market. Firms open a vacancy as long as it yields positive expected profits. Thus, the expected discounted value of a vacancy unfilled is equal to zero. Using Shimer's notation, the surplus is given by $S_p \equiv J_p + W_p - U_p$, where J_p is the value of a filled vacancy, W_p is the value of being employed for a worker and U_p is the value of unemployment. Denoting by E_p the expectation operator conditional on the current state p , the Bellman equations take the following form:

$$rU_p = z + f(\theta_p)(W_p - U_p) + \lambda(E_p U_{p'} - U_p), \quad (4.2)$$

$$rW_p = w_p + s(U_p - W_p) + \lambda(E_p W_{p'} - W_p), \quad (4.3)$$

$$rJ_p = p - w_p - sJ_p + \lambda(E_p J_{p'} - J_p), \quad (4.4)$$

in which z is the instantaneous utility in unemployment and w_p is the wage. Using the definition of the total surplus, one gets:

$$rS_p = p - z - f(\theta_p)(W_p - U_p) - sS_p + \lambda(E_p S_{p'} - S_p) \quad (4.5)$$

To solve the model, two conditions are imposed. First, a rent sharing rule that determines the wage allocation. As usual in the literature, Shimer considers a Nash bargaining solution; the wage w_p is chosen in order to maximize the product $(W_p - U_p)^\beta (J_p)^{1-\beta}$. Parameter $\beta \in [0, 1]$ represents workers' bargaining power. The unique solution to this maximization problem is

$$(1 - \beta)(W_p - U_p) = \beta J_p \quad (4.6)$$

Notice that $W_p - U_p = \beta S_p$ and $J_p = (1 - \beta) S_p$. The second condition used to determine the number of vacancies created in the market and, consequently, the level of tightness is the free-entry zero-profit condition. Thus, the expected cost of filling a vacancy (given by the expected duration of finding a worker multiplied by the flow cost of keeping a vacancy opened, c) must be equal to the value of a match to the employer:

$$\frac{c\theta_p}{f(\theta_p)} = J_p \quad (4.7)$$

Mortesen and Nagypál have proved that under some conditions, an equilibrium solution defined as a vector of functions $(w_p, \theta_p, U_p, W_p, J_p, S_p)$ for any possible value of productivity p exists, is unique, and all the functions are increasing in p .⁶ For the proof, I refer to them. Moreover, a standard wage equation can be derived even in this stochastic set-up. Using (4.3), (4.4), and the F.O.C. (4.6) one gets:

$$(1 - \beta) \frac{w_p + \lambda E_p W_{p'} - (r + \lambda) U_p}{r + s + \lambda} = \beta \frac{p - w_p + \lambda E_p J_{p'}}{r + s + \lambda} \quad (4.8)$$

I substitute U_p from the RHS of (4.2), and then use (4.6) and the zero profit condition (4.7). Recalling that $(1 - \beta) (E_p W_{p'} - E_p U_{p'}) = \beta E_p J_{p'}$ yields:

$$w_p = \beta(p - rU_p) + rU_p = \beta(p + c\theta_p) + (1 - \beta)z. \quad (4.9)$$

If worker's bargaining power is equal to zero, employees simply get their instantaneous utility when unemployed, z . If β is equal to 1, then the flow value generated by the match, p , accrues entirely to the worker. The match surplus (4.5) can also be rewritten using $W_p - U_p = \beta c\theta_p / [(1 - \beta)f(\theta_p)]$:

$$\frac{c\theta_p}{f(\theta_p)} = \frac{(1 - \beta)(p - z) - \beta c\theta_p + (1 - \beta)\lambda E_p S_{p'}}{r + s + \lambda} \quad (4.10)$$

Equation (4.10) is the equilibrium condition for labour market tightness, with the other endogenous variable being the expectation of future surplus $E_p S_{p'}$. In his paper, Shimer shows that the elasticity of tightness with respect to productivity when there are no shocks, obtained by differentiating (4.10) with $\lambda = 0$, is a useful approximation of the volatility of tightness in the dynamic stochastic set-up. Indeed, Mortensen and Nagypál prove that the two outcomes coincide in the limit when the arrival rate λ is close to 0 or the change in productivity Δ is small (see Proposition 2 of their paper)⁷. Shimer sets $\lambda = 4$,

⁶The necessary conditions for the existence and the uniqueness of the equilibrium are: $f(\theta)$ increasing and concave, $\lim_{\theta \rightarrow 0} \frac{\theta}{f(\theta)} = 0$, and p' stochastically increasing in p .

⁷Shimer assumes that the initial values of net productivity $\ln(p - z)$ lie on a discrete grid, that is $\ln(p - z) \in \{-n\Delta, -(n-1)\Delta, \dots, 0, (n-1)\Delta, n\Delta\}$. When a shock arrives, the productivity moves up or down by one grid point:

$$\begin{cases} \ln(p' - z) = \ln(p - z) + \Delta & \text{with probability } \frac{1}{2} \left(1 - \frac{\ln(p-z)}{n\Delta}\right) \\ \ln(p' - z) = \ln(p - z) - \Delta & \text{with probability } \frac{1}{2} \left(1 + \frac{\ln(p-z)}{n\Delta}\right) \end{cases}$$

As $\Delta \rightarrow 0$, the expectation of a function conditional on p tends to the current value of the function. The complete proof is in Mortensen and Nagypál (2006).

a relatively large value, but also imposes Δ equal to 0.0083, so the approximation can be accepted. Mortensen and Nagypál's result is extremely useful, since it allows to compare different set-ups without the need of computing numerical simulations. The elasticity of tightness with respect to productivity when $\lambda = 0$ is equal to:

$$\eta_{\theta p} \equiv \frac{\partial \ln \theta}{\partial \ln p} = \frac{r + s + \beta f(\theta)}{\alpha(r + s) + \beta f(\theta)} \cdot \frac{p}{p - z}, \quad (4.11)$$

where $\alpha = 1 - f'(\theta)\theta/f(\theta)$ is the elasticity of the expected duration of filling a vacancy with respect to tightness, constant with a Cobb-Douglas matching function. Recalling that $f(\theta) = \mu\theta^{1-\alpha}$, the elasticity of the job-finding rate with respect to productivity is given by $\eta_{f,p} = (1 - \alpha)\eta_{\theta p}$. Using (4.1), I also get the elasticity of unemployment with respect to productivity, $\eta_{u,p} = (1 - \alpha)(1 - u)\eta_{\theta p}$.

Since I am also interested in the labour policy implications of the model and in the volatility of the wage, issues ignored by Shimer (2005a), I compute other two related elasticities. Parameter z is the sum of the level of unemployment benefits b and the value of leisure. Hence:

$$\zeta_{u,b} \equiv \frac{\partial \ln u}{\partial b} = \frac{r + s + \beta f(\theta)}{\alpha(r + s) + \beta f(\theta)} \cdot \frac{(1 - \alpha)(1 - u)}{p - z} = \frac{\eta_{u,p}}{p}. \quad (4.12)$$

I compute the semi-elasticity of u with respect to b and not the elasticity simply to follow Costain and Reiter (2006) and compare more easily the results of the model with the estimates presented in section 4.2.3.

Finally, the elasticity of the wage with respect to productivity is given by:

$$\eta_{wp} \equiv \frac{\partial \ln w}{\partial \ln p} = \left(\frac{\beta}{w/p} \right) \cdot \left[\frac{\alpha(r + s) + f(\theta)}{\alpha(r + s) + \beta f(\theta)} \right]. \quad (4.13)$$

4.3 Comparing the Model with the Data

The business cycle data to which scholars have paid more attention are the volatility of the unemployment, vacancies and the job-finding rate. The dynamic correlation between productivity and the labour market variables, as well as the autocorrelation of the vacancies, have received slightly less attention. Some papers (for instance Hagedorn and Manovskii, 2006 and Rotemberg, 2006) have also focused on the wage statistics.

Comparative statics	Model with shock on p	Data
$\eta_{\theta p} = 1.72$	$\sigma_{\theta}/\sigma_p = 1.75$	$\sigma_{\theta}/\sigma_p = 19.1$
-	$\rho_{\theta p} = 0.999$	$\rho_{\theta p} = 0.396$
$\eta_{fp} = 0.481$	$\sigma_f/\sigma_p = 0.5$	$\sigma_f/\sigma_p = 5.9$
-	$\rho_{fp} = 0.999$	$\rho_{fp} = 0.396$
Comparative Statics	Model with shock on s	Data
$\eta_{\theta s} = -0.0947$	$\sigma_{\theta}/\sigma_s = -0.08$	$\sigma_{\theta}/\sigma_s = -5.09$
-	$\rho_{uv} = 0.999$	$\rho_{uv} = -0.894$

Table 4.2. A comparison of the model with the U.S. data (period 1951-2003). Source: Shimer (2005a).

As far as the policy analysis estimates are concerned, the performance of the model is mainly evaluated looking at the (semi)elasticity of the unemployment with respect to the UBs.

4.3.1 Business Cycle Viewpoint

The productivity shocks

Shimer's calibrated values are the following: $r = 0.012$, p normalized to 1, $s = 0.10$, $f = 1.355$ (recall he obtained a monthly destruction rate of 0.034 and a monthly job finding rate of 0.45), and $\alpha = 0.72$. Moreover, Shimer considers z only as the level of unemployment benefits, ignoring the value of leisure. He set it to $z = 0.4$. Since mean labour income in the stochastic model is equal to 0.993, a value of $z = 0.4$ belongs to the upper end of the range of replacement ratios in the United States. Substituting in (4.11) these values and imposing the Hosios (1990) condition $\beta = \alpha$, that ensures the efficiency of the decentralized equilibrium, one gets:

$$\eta_{\theta p} = \frac{0.012 + 0.10 + 0.72 \cdot 1.355}{0.72 \cdot (0.012 + 0.10) + 0.72 \cdot 1.355} \cdot \frac{1}{1 - 0.4} = 1.72$$

Assuming that shocks on productivity are the only source of fluctuations in labour market tightness, one can compare such elasticity with the ratio σ_{θ}/σ_p found in data. The latter is equal to 19.1, a value eleven times larger than the former. This is the first claim advanced by Shimer in his paper: a standard

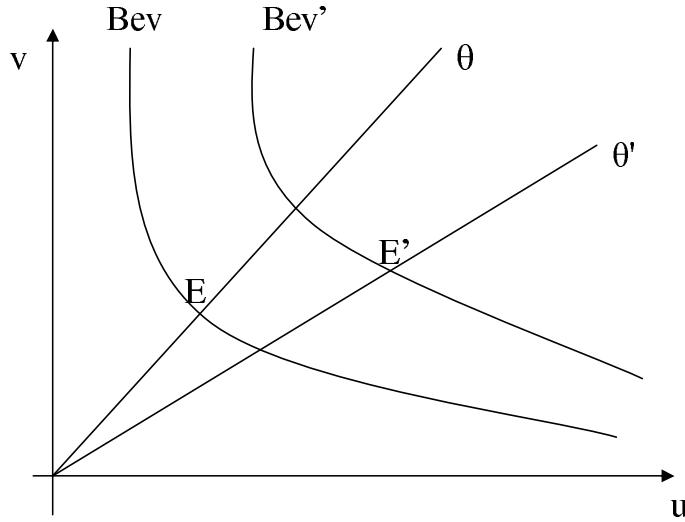


Figure 4.1: An increase in the destruction rate. The new equilibrium point is E' .

matching model can explain less than 10 per cent of the observed fluctuations in the vacancy/unemployment ratio. Analogous results are obtained comparing the elasticity of the job finding rate with respect to productivity, η_{fp} with σ_f/σ_p . The former is equal to 0.481, the latter is 5.9, twelve times larger.

Another deficiency of the model is the absence of propagation of the labour productivity shock. In the data, labour market tightness follows productivity by one year and the contemporaneous correlation between these two variables is 0.40. Simulation results, on the contrary, predicts both a correlation $\rho_{\theta p}$ and a correlation ρ_{fp} equal to 0.999: the model exhibits no propagation.

The separation rate shocks

The performance of this set-up is even worse if we consider a shock on the separation rate. As in the productivity shock case, the stochastic process chosen by Shimer allows us to consider the elasticity in the deterministic model as a good approximation of the simulation results:

$$\eta_{\theta s} \equiv \frac{\partial \ln \theta}{\partial \ln s} = \frac{s}{\alpha(r+s) + \beta f(\theta)} = -0.0947$$

In U.S. data, on the other hand, the σ_θ/σ_s ratio results equal to -5.09 . Even for the propagation of the shock, the model does not perform well: the con-

temporaneous correlation between s and θ is equal to -0.524 in data, while simulation results again predict no propagation.

The most striking drawback concerning the destruction rate, however, is that it delivers a positive correlation between unemployment and vacancies. To understand why, consider equation (4.10): a higher destruction rate lowers θ , since it makes entry less profitable for firms. A decrease in labour market tightness is depicted in the $v - u$ space by a less steep ray starting from the origin; yet, the Beveridge curve (4.1) moves to the right, so it is not possible to discern the behaviour of vacancies (see Figure 4.1). With a Cobb-Douglas matching technology, the shift of the Beveridge curve *may be* large enough to make both unemployment and vacancies increase, explaining why in the stochastic simulation it is observed a positive correlation between these two variables.

Labour productivity and separation rate shocks

In his paper, Shimer also considers a framework with both productivity and separation rate shocks but he does not obtain significant improvements. Unemployment appears to be more cyclical, but both tightness and the job finding rate are still less than 10 per cent as volatile as expected. Instead, a shock on workers' bargaining power seems to reach the best results. In this case, both vacancy and unemployment standard deviation are much larger. Yet, the lack of theoretical foundations explains why this avenue has been neglected in the subsequent literature. It is difficult to identify what should induce a recurrent change in bargaining power.

Wage share and wage volatility

Two distinctive features of Shimer's calibrated model are the large value obtained for the wage share and the high volatility of the wage. We will see later on that a high wage share, and a correspondingly low profit share, is a crucial element to replicate the business cycle fluctuations in labour market tightness. On the other hand, one decisive critique addressed to Shimer's model is the excessive volatility in the wage it implies.

To derive the wage share, I follow Hornstein, Krusell, and Violante (2005) and

divide (4.9) by p :

$$\frac{w}{p} = \beta + \beta \frac{c\theta}{p} + (1 - \beta) \frac{z}{p} \quad (4.14)$$

Then I re-write the expression for the surplus (4.5) as $\lambda = 0$, using $W - U = \beta S$:

$$S = \frac{p - z}{r + s + \beta f(\theta)} = \frac{c\theta}{(1 - \beta) f(\theta)}$$

The second equality stems from the zero profit condition. Rearranging and dividing by p , one gets:

$$\frac{c\theta}{p} = \frac{(1 - \beta) f(\theta)}{r + s + \beta f(\theta)} \left(1 - \frac{z}{p} \right) \quad (4.15)$$

Recall that $r = 0.012$ and $s = 0.10$, quite small relative to $\beta f(\theta) = 0.72 \cdot 1.35 = 0.972$. Hence, I can write:

$$\frac{c\theta}{p} \approx \frac{1 - \beta}{\beta} \left(1 - \frac{z}{p} \right)$$

Combining this expression with the equation for the wage share (4.14) yields:

$$\frac{w}{p} \approx 1.$$

The wage share is large in Shimer's calibrated model because both the job finding rate and the workers' bargaining power β (imposed equal to the elasticity α under the Hosios condition) are much greater than the job separation rate and the discount factor. Such conclusion is confirmed by substituting the calibrated values in equation (4.9): the wage w is equal to 0.973.

What about the elasticity of the wage? From (4.13), the closer is β to 1, the more correct is the approximation $\eta_{wp} \approx p/w$. So, in Shimer's calibrated model, the volatility of the wages is around 1, twice larger than the value reported at the end of Section 4.2.1.

4.3.2 Policy Evaluation Viewpoint

Now I compare the semi-elasticity of unemployment with respect to b predicted by the model with its empirical counterpart. Substituting the calibrated values chosen by Shimer in (4.12), I get $\zeta_{u,b} = 0.45$.

This value is different from 1.3, the estimate obtained by Layard and Nickell (1999)⁸. Shimer's calibrated model is not consistent with the policy evaluation estimates presented in section 4.2.3; yet, the discrepancy between the theoretical framework and its empirical counterpart is lower than for the business cycle data.

But the crucial point here is another. With the present calibration in which $p = 1$, the semi-elasticity of unemployment with respect to the unemployment benefits, $\zeta_{u,b}$ is equal to the elasticity of unemployment with respect to productivity $\eta_{u,p}$. From Table 1, $\sigma_u/\sigma_p = 9.5$. The trade-off is clear: either the parametrization is constructed to match the business cycle volatility of unemployment, or it is constructed to match the policy estimates cited above. No calibration can attain both tasks.

4.4 First Approach: Changes in the Wage Formation

Why is the matching model unable to replicate the observed fluctuations in unemployment and vacancies? Shimer argues that the main culprit is the Nash bargaining solution adopted to share the rent between the worker and the firm. He compares the outcome of the *laissez faire* economy with the social planner's solution in a centralized economy having the same preferences, technologies and endowments. Since workers' and firms' utility levels are linear in wages and the social planner maximizes total output without any distributional concerns (so that no rent sharing rule appears in the optimization problem), such a comparison sheds a light on the role of wage formation in a decentralized economy.

A higher productivity makes vacancies "cheaper" and unemployment more "expensive". In the centralized economy, the magnitude of the switch from unemployment to vacancies is simply determined by the matching technology. The higher the elasticity of substitution between the two inputs in the matching function the larger will be the magnitude of the switch from unemployment to vacancies as productivity increases.

In the decentralized economy, however, the extent of substitution between un-

⁸Actually, Layard and Nickell consider the semi-elasticity of unemployment with respect to the replacement ratio. So one would have $b = v \cdot w$ with v being the replacement ratio. However, in the present calibration, $w \approx p = 1$. So $\zeta_{u,b} \approx \zeta_{u,v}$.

employment and vacancies hinges not only on the matching technology, but also on the rent sharing rule that, by determining the amount of the total surplus that goes to the firm, affects vacancy creation.

For Shimer, adopting a Nash bargaining solution to divide the rents is similar to a Cobb-Douglas matching function in the centralized economy, meaning a moderate degree of substitutability between inputs. Getting rid of the Nash wage bargain solution, the argument goes, makes vacancies and unemployment more reactive to changes in parameters and ameliorates the quantitative performance of the model. In this section, I present different changes in the wage formation rule: completely rigid wages, sticky or staggered wages, asymmetric information or different fall-back position in the bargaining process.

4.4.1 Wage rigidity

The most straightforward way to break the link between wages and productivity is to impose wage rigidity. Shimer (2004a) performs this task. He first shows that assuming wage rigidity only in the existing matches (meaning that firms and workers negotiate the wage only the first time they match and then it never changes following subsequent shocks) does not have any effect on the vacancy/unemployment ratio. The reason is that this kind of rigidity does not affect the discounted expected profits of the firms, but simply the timing of the wage payments. Since any firm decides to post or not a vacancy only on the basis of its future expected profits, the level of tightness takes the same value as in the flexible wage set-up.

The case is different of rigid wages also in new matches. Under this hypothesis, firms and workers never bargain and take the wage as an exogenous variable. The results change dramatically. Fixing the wage to the value 0.967 (recall that $p = 1$) in order to get an average U.S. unemployment rate of 5.7 raises significantly the business cycle fluctuations in vacancies and unemployment. We can easily see that by considering the new equilibrium equation for tightness when $w = \bar{w}$ and $\lambda = 0$:

$$\frac{c\theta_p}{f(\theta_p)} = \frac{p - \bar{w}}{r + s}$$

The elasticity of tightness with respect to productivity⁹ is equal to $\eta_{\theta p} = p / [\alpha (p - \bar{w})] = 42.08$, a value more than twice larger as in the data¹⁰. The rigid wage model succeeds in generating the volatility in unemployment and vacancies. However, four caveats must be highlighted:

1. The large values for the elasticities mainly depend on Shimer's choice of $\bar{w} = 0.967$. With a wage $\bar{w} = 0.5$, for instance, $\eta_{\theta p} = 2.7$, higher than in the flexible wage set-up but still inconsistent with data. Wage rigidity is not sufficient to improve the business cycle consistency of the model. A small profit share $(p - w)/p$ is also needed, so that the percentage increase in profits is large for a given percentage increase in productivity.
2. Wage rigidity *and* a low profit share reconcile the model with the data if we focus on productivity shocks. The volatility of tightness in response to a change in the destruction rate is still too low: $\eta_{\theta s} = s / [\alpha(r + s)] = -1.24$, whereas in U.S. data $\sigma_{\theta}/\sigma_s = -5.09$.
3. With perfect wage rigidity, the level of unemployment benefits does not affect tightness and employment. The elasticities η_{fz} and η_{uz} are equal to zero. So, the price of the business cycle consistency of the model is to make it useless for a policy evaluation analysis¹¹.
4. Finally, a model of this kind incurs Barro (1977)'s well-known critique: a rigid wage solution implies that the agents are not fully rational, since they are not exploiting all the advantages of the negotiation¹².

⁹Hornstein, Krusell and Violante (2005) shows that when wages are rigid the elasticity $\eta_{\theta p}$ is not a good proxy for the relative standard deviation σ_{θ}/σ_p as in the flexible wage case. In particular, $\eta_{\theta p}$ overestimates the response of θ to a productivity shock. For simplicity, I still compute $\eta_{\theta p}$, considering it as an upper-bound for volatility.

¹⁰Shimer (2004) sets $\alpha = 0.5$. In this case, $\eta_{\theta p} = 60.6$.

¹¹Considering endogenous search effort would correct such shortcoming. However, as Merz (1995) has noticed, a model with endogenous search can predict a positive relationship between unemployment and vacancies. I will come back to this issue in section 4.6.4.

¹²Another shortcoming of the model that wage rigidity cannot amend is the lack of persistence in vacancies: their autocorrelation is 0.715, while in the data it is 0.930. However, as Shimer observes, it can depend on the choice of vacancies as a jump variable. Introducing planning lags in vacancy creation should correct this deficiency. See Fujita (2003).

Hall (2005) tries to overcome the last point, by imposing a wage norm that never lies outside the bargaining set. He considers the wage negotiation as an auction in which the worker and the firm know the counterpart's reservation value. Worker's reservation wage is rU , whereas p is the highest level of wage that an employer is willing to pay. The worker proposes a wage w_L and contemporaneously the firm proposes a wage w_H . The payoffs of the game are the following: if $w_L \leq w_H$, the match is formed and the worker receives a wage $w = \beta w_L + (1 - \beta)w_H$, with $\beta \in (0, 1)$. Otherwise, no agreement is reached. It is easy to see that any $w = w_L = w_H \in [rU, p]$ is a Nash equilibrium of this game¹³. If $\beta = 0.5$, the auction game coincides with the symmetric Nash solution, since $w^* = (rU + p)/2$ (look at equation (4.8) when $\lambda = 0$). Hall considers a different approach. Suppose an idiosyncratic random shock, ϵ , normally distributed with zero mean and standard deviation σ , that shifts the bargaining set so that it becomes $[rU + \epsilon, p + \epsilon]$. Then the current wage takes the following form:

$$\begin{cases} w_t = rU_t + \epsilon & \text{if } w_{t-1} < rU_t + \epsilon \\ w_t = p_t + \epsilon & \text{if } w_{t-1} > p_t + \epsilon \\ w_t = w_{t-1} & \text{otherwise} \end{cases}$$

Intuitively, the formula above means that the wage does not change if it remains in the bargaining set, otherwise it takes the value of the nearest boundary. Hall imposes the norm $w_t = E(w_t(\epsilon))$. The average wage at time t is a function of w_{t-1} , rU_t , and p_t ; moreover, it can be expressed as a convex combination between w_{t-1} and w^* , the symmetric Nash solution.

Hall's analysis differ from Shimer's one in that he considers a permanent price shock. Productivity jumps from 1 to $1 + \Delta$ and then remains at that level. He then compares the results of his model with "rational" wage stickiness with those obtained under the hypothesis of Nash bargain. The former succeeds in replicating the behaviour of key labour market variables in the U.S. economy. A reduction in productivity by 1 per cent produces the classical hump-shaped form for the dynamics of the unemployment rate: it starts from 5.6%, reaches the maximum value of 6.7% after seven months, and then it starts to decline.

¹³Given the strategy w_L played by the worker, the firm's best response is w_L . At the same time, given the strategy w_H played by the firm, the worker's best response is w_H . Nash equilibria of this game are all the wages $w = w_L = w_H \in [rU, p]$.

Under the assumptions of Nash bargain, on the contrary, the model is not able to generate realistic impulse responses, since all the endogenous variables (job finding rate, unemployment, vacancies) reach rapidly their new stationary levels.

4.4.2 Sticky wages

Farmer and Hollenhorst (2006) also want to assess the role of wage stickiness in amplifying unemployment and vacancies fluctuations. Their approach differs from Hall (2005) and Shimer (2004) in that they construct a fully specified DSGE model, in which households take consumption-saving decisions and search effort is endogenous. Then they compare three alternative models: 1) an efficient (i.e. with the Hosios condition satisfied) flexible wage economy, 2) a rigid wage solution, in which the real wage grows at the rate of the technological progress but is unaffected by current productivity shocks, and 3) a sticky wage economy, in which 19% of the wage is adjusted towards its optimal level in any period.

The last framework fits particularly well the U.S. data with respect to unemployment and vacancies fluctuations. Compared to the data that Farmer and Hollenhorst have collected, the sticky wage economy matches the unemployment standard deviation and slightly overshoots on vacancy standard deviation. As the authors stress, two parameters are decisive: the disutility of effort parameter and the degree of wage stickiness. The latter in particular is key in replicating both a positive (negative) correlation between output and vacancies (unemployment)¹⁴.

4.4.3 Staggered wage contracts

Staggered wage contracts constitute a middle way between perfectly rigid and flexible payments. In the simplest formulation, at an exogenous Poisson rate $1 - \varphi$ firms and workers bargain over the wage. The expected duration of a contract is therefore equal to $1/(1 - \varphi)$. The aggregate wage in the economy is $w_t = (1 - \varphi)w_t^* + \varphi w_{t-1}$, with w_t^* being the payment negotiated at time t . Gertler and Trigari (2006) pursue this approach. As we have already noticed in rigid wage models, labour market tightness is affected only by wage contracts

¹⁴See footnote 11.

on new matches, and not on the existing ones. Thus, for Gertler and Trigari's model to work, a newly hired worker has to receive the same wage paid as the other employees of the firm.

The U.S. data collected by Gertler and Trigari go from 1951 to 2005. According to their computations, $\sigma_\theta/\sigma_p = 12.10$ and $\sigma_u/\sigma_p = 5.81$. In Shimer (2005a), the former is 19.1, the latter is 9.5. Assuming three or four quarters as average length of the contract, Gertler and Trigari get results compatible with their data. The model explains at least 81% of the unemployment volatility and 89% of tightness volatility. In addition, staggered contracts mimic quite well the dynamics of the wages, a feature not investigated by Shimer. The volatility of wages with respect to productivity is around 0.46 in the data, and the model accounts for 95% of it.

However, Gertler and Trigari's framework present some shortcomings. The first point is theoretical: in their model, the expected duration of a contract is exogenous, so the reason for which the wage is renegotiated in one period and not in another is left unexplained. Moreover, as emphasized by Mortensen and Nagypál (2006), staggered contracts imply that labour market tightness increases more than in the flexible set-up after a positive shock from p to p' because only a fraction $1 - \varphi$ of the new employees bargain the wage at time t . But when the wages are finally renegotiated in accordance with the new productivity value p' , tightness decreases to a level *below* its initial response. This is at odds with data.

Finally, staggered wages worsen the policy analysis consistency of the model. We have seen that in a flexible wage set-up, the effect on unemployment of a change in the UB are underestimated. Staggered contracts loosen the link between the wage and other exogenous variables, such as p and z . The intuition is the following. Since unemployment benefits affect vacancy creation only through the wage, a weaker impact of z on w will reduce even more the elasticity of unemployment with respect to z .

4.4.4 Nominal wage rigidities

In the papers of Farmer and Gertler and Trigari the business cycle is driven only by productivity shocks and no distinction is needed between nominal and

real wage setting. On the other hand, a growing literature has been focusing on the integration of nominal price and wage stickiness in search and matching models. The aim of these papers is not only to react to Shimer's puzzle but also to have a better understanding of the inflation dynamics and monetary policy transmission (for instance recent studies are Walsh, 2005, Blanchard and Gali, 2005, Bodart *et al.*, 2006). Due to space limitations, and since this chapter wants to be a survey of papers whose main scope is to react to Shimer's critique, it is not possible to account here for the contributions made along this avenue. I only refer to the work of Bodart *et al.*, (2006), because they explicitly compare their results with those of Gertler and Trigari (2006). Their model captures quite well the volatilities of unemployment and vacancies: $\sigma_\theta/\sigma_p = 11.8$ and $\sigma_u/\sigma_p = 5.35$. The model does not perform so well as far the wage is concerned: $\sigma_w/\sigma_p = 0.29$. Bodart *et al.*, 's (2006) framework has many departures from a standard RBC model (price stickiness, wage staggering, habit formation in consumption). It is therefore difficult to understand which mechanism is the most important in getting the aforementioned results.

4.4.5 Asymmetric information

According to Shimer (2005a), introducing asymmetric information in the bargaining process can limit the responsiveness of wages to the shocks. For instance, one could imagine that a positive shock on productivity does not automatically lead to an increase in the wage because workers are not completely informed about the value of their job.

The hypothesis of introducing bargaining with asymmetric information about the counterpart's utility in a matching framework is studied by Brügemann and Moscarini (2006). The conclusion they reach, however, is that a large part of these models are not able to generate the observed fluctuations in vacancies and unemployment. Brügemann and Moscarini explain why by analysing the properties of the wage equation. When the matching between a worker and a firm gives rise to a quasi-rent, the wage can be divided in two parts: a fraction of the total rent generated by the match and the opportunity cost of employment. For instance, in the Nash solution, the former is given by $\beta(p - rU)$, and the latter by rU . The opportunity cost of employment is always procyclical,

since it positively depends on the probability of finding a job $f(\theta)$ and on the capital gain obtained becoming employed, both increasing in booms. Therefore, when wages are rigid (that is completely acyclical), the part of the rent accruing to the worker must be countercyclical. The contribution of Brügemann and Moscarini is twofold. First, they show that in many models with asymmetric information in the bargaining process, workers' rents are at most acyclical, but are never countercyclical. Second, they prove that an acyclical rent (or, using their definition, rent rigidity) is not sufficient to generate the observed business cycle fluctuations in the vacancy/unemployment ratio.

The rationale behind this second point goes as follows. Suppose first that worker's fraction of the total rent is rigid and large. A large rent implies that the capital gain from finding a job is also high. To fit data about vacancies and unemployment volatility, we know that the job finding rate must be in turn extremely volatile and procyclical. But a high job finding rate and a large capital gain during booms will enhance the opportunity cost of employment. In turn, a strongly procyclical opportunity cost of employment will produce strongly procyclical wages, dampening the incentives on vacancy creation during booms. Brügemann and Moscarini call such mechanism the feedback effect: even if workers' rents are rigid, wages increase via the opportunity cost of employment. The feedback effect is present in the Nash solution, and, more in general, in every wage setting frameworks in which worker's and firms' rents depend on p and rU only through their difference, $p - rU$.

On the other hand, if the rent accruing to the worker is low and acyclical, the same problem analyzed in the rigid wage model occurs. To replicate the observed business cycle fluctuations in unemployment and vacancies, the share of profits must be small; a lower rent going to the employee means large profits for the firm, so that a percentage increase in productivity will enhance them only by a small percentage amount. Firms will not open too many vacancies during booms. This second mechanism is called by Brügemann and Moscarini the congestion effect, because it depends on the free-entry condition that links directly the number of vacancies posted with the profits gained by the firms.

Both the congestion effect and the feedback effect limit the response of tightness to a productivity shock. To dwarf the former, the wage setting framework must exhibit countercyclical workers' rents. To dwarf the latter, profits' share must be low. In most of the models of bargaining with asymmetric informa-

tion, Brügemann and Moscarini notice that workers' rents are not countercyclical. In a take-it-or-leave-it context, for instance, firms will concede more informational rents to workers during booms when they are more eager to strike a deal. This is the case both when workers make the offer and when firms make it.

Kennan (2006) develops a model with asymmetric information about the productivity value of the job. There are two states in the economy (1, the bad state and 2, the good state) and two different values for the productivity of a job (high or low). After matched with a worker, only the firm knows if the job has a high or low productivity. The probability of drawing the high match is on the contrary common knowledge. The expectation of a surplus is higher in the good state. In the bargaining process, a "rational" dictator chooses which part makes the offer. If the firm makes the offer, it will get all the surplus and the worker will receive his opportunity cost of employment. On the other hand, if the worker makes the offer, he faces a potential trade-off: asking the low productivity surplus he would reach a sure agreement; on the contrary, proposing the high productivity surplus would imply sure acceptance by the employer only if it has been effectively realized. Kennan imposes the crucial assumption that workers always demand the low surplus. In this way, the wage is close to the low productivity level and it is not affected by small productivity changes observed only by firms. Such informational rent enjoyed by the firms translates into large fluctuations in unemployment and vacancies. To show this effect, Kennan compares the steady-state rate of unemployment both in the good and in the bad state of the economy with the respective values obtained in a model without asymmetric information (i.e. in which even the worker knows the value of the match). The informational rent moves unemployment by about 40%, even though the difference in productivity levels is only 3%: $u_1 = 5.6\%$ and $u_2 = 5.5\%$ in the case of complete information, whereas $u_1 = 7.5\%$ and $u_2 = 5.2\%$ when productivity is observed privately by the employer.

4.4.6 Breaking the link between wage and unemployment

Hall and Milgrom (2006) eliminate the feedback effect highlighted by Brügemann

and Moscarini and avoid that a higher opportunity cost of employment rU translates into higher wages. In a standard Nash wage bargain solution, the threat points are constituted by the utility for firms and workers of breaking the match and searching another counterpart. Hall and Milgrom (2006) claim that such assumption is not realistic. According to them it is not credible to assume as a threat that the other party will leave in case of disagreement. It is more realistic to imagine that firms and workers negotiate the wage knowing that a disagreement will imply a delay in the production or strikes. The theoretical background for their claim is in the non cooperative bargaining theory of Binmore, Rubinstein, and Wolinsky (1986). Hall and Milgrom assume that for workers and firms it is never optimal to abandon the bargaining process. So, the options of a worker and a firm are either accepting the counterpart's offer or rejecting it and making a counterproposal. To disagree implies a flow cost λ for firms and a flow benefit ω for workers. When the interval of time between a proposal and a counterproposal tends to zero, the unique equilibrium of the extensive form game coincides with the axiomatic Nash solution with λ/r and ω/r as threats points. The F.O.C. of this problem is $(1 - \beta)(W - \omega/r) = \beta(J + \lambda/r)$. The wage is then equal to:

$$w = \beta p + (r + s) \left[(1 - \beta) \frac{\omega}{r} + \beta \frac{\lambda}{r} \right] - (1 - \beta) s U$$

We can compare such formula with (4.9). In both equations, the wage depends on labour market tightness only via U . In a standard Pissarides (2000) model, the wage curve is increasing in the (θ, w) space because better conditions in the labour market (i.e. a higher tightness) enhance U , the worker's threat point in the Nash bargain solution, and this allows him to receive a higher wage.

Recall that $W = \frac{w + sU}{r + s}$. Therefore, in the F.O.C. $(1 - \beta)(W - \omega/r) = \beta(J + \lambda/r)$, U is present only in the equation for W . Improved conditions in the labour market raise W via U , making the agreement more convenient and the delay more expensive than before for workers, so that they are willing to accept a lower wage. Hence, in Hall and Milgrom's model, the wage is decreasing in tightness.

Furthermore, the feedback effect is no longer present in the model. A higher productivity enhances the wage only directly, but not through an increase in the opportunity cost of employment. In such a new setting, Hall and Milgrom

amend the quantitative inconsistency of the matching model. The model even overshoots on unemployment volatility: $\eta_{u,p} = 63.8$, more than sixty times larger than in Shimer's paper.

Yet, two caveats must be raised about Hall and Milgrom's set-up. First, in such a model, an increase in productivity may lower the wage, since the wage curve and the zero profit condition (4.7) are both downward sloping in the (θ, w) space¹⁵. Countercyclicity in the real wage is not observed in the data. Adding in their model an exogenous probability that the production opportunity of the match disappears during the negotiation allows Hall and Milgrom to correct this shortcoming, but at the price of a lower $\eta_{u,p}$ (it becomes equal to 7.35, with $\sigma_u/\sigma_p = 9.5$ in data).

Second, an increase in the flow benefit when unemployed, z , reduces the wage. In the (θ, w) space, the downward sloping wage setting curve shifts to the left. The effect on tightness depends on the slope of the zero profit condition function. If it is steeper than the wage setting curve, tightness is higher at the new equilibrium point. This possibility is ruled out by Hall and Milgrom that, using Shimer's calibrated values, get an elasticity $\eta_{u,z}$ close to zero. But even a nil elasticity is not consistent with the microeconomic estimates.

Like in the rigid wage case, a trade-off between business cycle and policy analysis consistency is present. With the opportunity cost of employment no longer being the threat point, wages are less affected by changes in productivity or in the level of unemployment benefits. This is an advantage for the business cycle consistency of the model, since a higher p is not dampened by an increase in w ; but it is bad for its microeconomic consistency, for a change in z has little impact on vacancy creation and unemployment merely changes.

4.4.7 Final remarks on the first approach

It is undeniable that the wage flexibility implied by the Nash bargaining solution plays an important role in weakening the volatility of unemployment and vacancies in response to a productivity shock. However, Shimer (2005a) and the subsequent literature surveyed in this section have probably put too much

¹⁵This also raises problems for the unicity of the equilibrium. Actually, Hall and Milgrom do not prove neither the existence nor the uniqueness of an equilibrium, paying attention only to the numerical properties of the model.

emphasis on that aspect. Indeed, loosening or even breaking the link between wages and productivity in the model is not *per se* sufficient to replicate business cycle facts. A low profit share is also needed. This first approach also fails to match the microeconomic estimates of the effects of UB on unemployment.

4.5 Second Approach: Changes in the Calibration

The papers we have examined so far are based on the belief that Shimer is right both when he denounces the quantitative inconsistency of a standard matching model and when he identifies the Nash wage bargain as the main culprit. Hagedorn and Manovskii (2006) take a different route. They claim that Shimer's conclusions are invalidated by a wrong calibration of two key parameters: the instantaneous utility of being unemployed, z , and the worker's bargaining power, β .

The model built up by Hagedorn and Manovskii is substantially similar to the one presented in section 4.2.4, the only difference being the introduction of capital in the production technology. Capital is added not for having further analytical insights, but in order to measure the capital cost of vacancy creation and it is sold in a competitive market.

Some calibrated values are the same as in Shimer's paper: $r = 0.012$, $f = 1.355$, and $s = 0.10$. Mean labour productivity $\bar{p} = A \cdot F_L$ is normalized to 1. Hagedorn and Manovskii's calibration method differs in computing the remaining parameters, θ , c , β , z and in the choice of the matching function. For labour market tightness, while Shimer normalizes θ to 1, Hagedorn and Manovskii compute it, since they have to measure the fraction $\frac{1-u}{v+1-u}$ of capital employed. Following Den Haan, Ramey, and Watson (2000), they consider a quarterly job filling rate $f/\theta = 2.13$, thus obtaining a value for θ of 0.634. To ensure that the probability of finding a job or filling a vacancy is always between 0 and 1, Hagedorn and Manovskii choose the matching function considered by Den Haan, Ramey and Watson: $m(u, v) = u \cdot v / (u^l + v^l)^{1/l}$, with l being the only parameter to be estimated. They then divide the flow cost of opening a vacancy in two parts: the capital cost c^K and the labour cost c^W . The capital cost is computed after some manipulations by knowing that the capital income share $F_K \cdot K/F = 1/3$ in typical estimates. They get $c^K = 0.474$. With

respect to c^W , Hagedorn and Manovskii use the 1982 Employment Opportunity Pilot Project, and the Small Business Administration survey to conclude that on average the labour cost of hiring one worker is 4.5% of quarterly wages of a new hire or 11% of labour productivity. The total flow cost of opening a vacancy is then assumed equal to $c = c^K p + c^W p^{\eta_{wp}}$.

4.5.1 The values of z and β

It remains to select z , β , and l . To pin down these three parameters, Hagedorn and Manovskii use as targets the job finding rate $f = 1.35$, the labour market tightness $\theta = 0.634$, and, more importantly, the elasticity of wages with respect to productivity $\eta_{w,p} = 0.449$. They get the following results: $z = 0.955$, $\beta = 0.052$, and $l = 0.407$. The difference between these values of β and z and those chosen by Shimer is striking. Recall that Shimer has set z equal to 0.4, arguing that it is in the upper bound of the replacement ratios in the U.S., and $\beta = \alpha = 0.72$, for the Hosios condition.

Hagedorn and Manovskii claim that such a discrepancy can be explained by focusing on two aspects: 1) In the model, the profit share $(p - w)/p$ is small and 2) the wages are moderately procyclical in the data, a fact confirmed by the value of σ_w/σ_p . Both pairs of calibrated values for β and z imply a very low profit share, but those chosen by Shimer are inconsistent with the slight procyclicality of wages observed in the data.

Let examine this point more in detail. The first fact has been already emphasized in the previous sections. A low profit share, and consequently a high labour share, is necessary (but not sufficient) to amplify the business cycle fluctuations in unemployment and vacancies. As we have noticed in section 4.3.1, Shimer obtains a large value for w/p because he sets $\beta = 0.72$, so $\beta f(\theta)$ is large relative to s and r . But we can have a high labour share also by choosing a high value for z : combining equations (4.14) and (4.15), one gets:

$$\frac{w}{p} = \frac{(r + s) \left[\beta + (1 - \beta) \frac{z}{p} \right] + \beta f(\theta)}{r + s + \beta f(\theta)} \quad (4.16)$$

The labour share is close to one if the term inside the square brackets is close to one. This can be obtained either with a large β , as Shimer does, or with a high fraction z/p . To see which of the two choices is the more appropriate, Hagedorn and Manovskii look at the volatility of the wages in the data. Recall

that Shimer's calibration, in which β is close to one, implies a $\eta_{wp} \approx 1$, twice as large as the estimates of Hagedorn and Manovskii and Gertler and Trigari. On the contrary, setting for instance $\beta = 0.15$ and keeping the calibrated values chosen by Shimer, η_{wp} equals to 0.325.

A low bargaining power for the worker and a high instantaneous utility in unemployment allow to have a low profit share and a volatility of wages consistent with the data.

4.5.2 Business cycle consistency

Hagedorn and Manovskii show that their results are not sensitive to the kind of matching technology assumed. So, keeping the usual Cobb-Douglas matching function with $\alpha = 0.72$, we get the following elasticities values:

$$\begin{cases} \eta_{\theta p} = \frac{r+s+\beta f(\theta)}{\alpha(r+s)+\beta f(\theta)} \cdot \frac{p}{p-z} = \frac{0.012+0.10+0.052 \cdot 1.355}{0.72 \cdot (0.012+0.10)+0.052 \cdot 1.355} \cdot \frac{1}{1-0.955} = 26.8 \\ \eta_{fp} = (1-\alpha) \cdot \eta_{\theta p} = 0.28 \cdot 26.8 = 7.504 \\ \eta_{\theta s} = -\frac{s}{\alpha(r+s)+\beta f(\theta)} = -\frac{0.10}{0.72 \cdot (0.012+0.10)+0.052 \cdot 1.355} = -0.66 \end{cases}$$

We can compare these values with the results in Table 4.2. Hagedorn and Manovski's calibrated model succeeds in amplifying the fluctuations in tightness and in unemployment. The reason hinges on the high value of z and the low value of β . The former implies a tiny profit share, so that a one percentage increase in productivity leads to a large percentage increase in profits, boosting vacancy creation and tightness. The latter is the coefficient of productivity and tightness in the wage equation (4.9). A low β weakens both the direct effect on the wage caused by a higher productivity and the feedback effect via the increase in the opportunity cost of employment.

However, β has only a slight impact on the elasticity of the destruction rate, whereas z does not play any role. Hagedorn and Manovskii better the performance of the model in response to a shock on s , but their result is still unsatisfactory. Indeed, $\sigma_{\theta}/\sigma_s = -5.09$ in the data, almost 8 times larger than $\eta_{\theta s}$.

4.5.3 Pros and cons of the second approach

The parametrization performed by Hagedorn and Manovskii can be questioned on two different grounds. First, with a $z = 0.955$ the gap between the utility of being employed and the utility of being unemployed becomes extremely small. Second, in their model the effects of an increase in UB on unemployment result counterfactually too high.

The first point has been emphasized by Mortensen and Nagypál (2006). The calibrated wage in Hagedorn and Manovskii's set-up is equal to 0.977, implying a difference $w - z = 0.022$. Is it realistic to think that employees work for a 2.2% surplus? Hagedorn and Manovskii list a series of reasons in favour of a high z :

- It is incorrect to identify the instantaneous utility in unemployment with the replacement ratio, for we also have to account for the utility of leisure and any other non-market activity. Moreover, as Anderson and Meyer (1997) and Vroman (2002) document, a large fraction of unemployed do not receive unemployment benefits, either because they abandoned their last job or because they did not work enough time in the same firm. Looking at the replacement ratios data in order to calibrate the value of z , as Shimer does, does not seem therefore the right approach¹⁶.
- The frustration caused by unemployment is greater the longer is the unemployment spell. But in the model the monthly job finding rate is equal to 0.45, so the probability of becoming long-term unemployed is really low. It is therefore plausible to imagine that for people remaining without a job for 2.5 months on average the utility gap is almost zero.
- Recent studies have found that the difference in consumption expenditures between employed and unemployed workers is modest.
- Even in Shimer's calibrated model the ratio $(W - U)/U$ is small, around 0.003.

The second viewpoint under which the calibration choices of Hagedorn and Manovskii can be criticized concerns the policy evaluation estimates. We can

¹⁶Note however that the fact that a large fraction of unemployed workers do not receive any insurance is an argument against a high calibrated value for z .

see it by computing the following elasticities: $\eta_{\theta z} = \eta_{\theta p} \cdot (z/p) = 25.6$; $\eta_{fz} = (1 - \alpha) \cdot \eta_{\theta z} = 7.16$ and $\eta_{uz} = (1 - u)(1 - \alpha) \cdot \eta_{\theta z} = 6.65$. These values are clearly at odds with the policy estimates presented in section 4.2.3, confirming the existence of the business cycle / policy analysis trade-off: Any attempt to better the business cycle consistency of the matching model by changing the parametrization jeopardizes its microeconomic consistency. Notice also that while the papers of the first group, that keep Shimer's calibration and limit the responsiveness of the wages to unemployment, tend to underestimate the effects of a change in z on the hazard rate, Hagedorn and Manovskii's paper, by selecting a high z , greatly overestimates them.

Hagedorn and Manovskii react even to this second point, by offering several explanations:

- The microeconomic studies cited by Costain and Reiter are not comparable with the quantitative conclusions offered by a standard matching model. The former evaluates the impact of higher UB on the search behaviour of unemployed workers, whereas the latter focuses only on the firm's decision.
- Endogeneity problems in cross-country regressions like those under exam cannot be ruled out.
- The policy estimates refer to a change in the UB, whereas the elasticities computed above concern the instantaneous utility in unemployment z .
- Finally, a more realistic model in which z is decreasing in the expected duration of unemployment or on-the-job search is allowed, would dampen the value of $\eta_{\theta z}$. The reason is that as the UB increases, wages increase as well and firms post fewer vacancies. A lower tightness in turn raises the expected duration of unemployment; but with z decreasing in $1/f(\theta)$, this implies a reduction in the wages; the elasticity $\eta_{\theta z}$ will be lower than in the standard framework. If on-the-job search is also introduced, the impact of z on the wages is weaker, since in this case only a fraction of job-seekers that negotiate the wage with firms is unemployed.

4.6 Third approach: Enriching the Standard Model

Papers following the third approach take a middle way in the dispute over Shimer's findings. Like the papers belonging to the first approach, they agree with Shimer's diagnosis about the business cycle inconsistency of a standard matching model. But, as Hagedorn and Manovskii (2006), they doubt that the Nash wage bargain is the main culprit for the quantitative failings of the model. What papers of the third approach (more or less) implicitly argue is that a text-book matching model is a useful tool to look at the qualitative effects of a policy change or a shock, but it is too stylized to be also consistent with data. Embedding in the standard setting realistic features like on-the-job search, hiring and firing costs, imperfectly competitive product markets, firms heterogeneity, makes the model more suitable for a quantitative scrutiny.

4.6.1 Turnover costs

Turnover costs are a key ingredient of modern labour markets. For the sake of simplicity, Silva and Toledo (2006) focus only on two different kinds of turnover costs: training costs that firms spend for new entrants and separation costs suffered by employers when a job is destroyed. They show that taking such costs into account and distinguishing between incumbents and entrants workers improves the quantitative performance of the model. The Bellman equations that incorporate these three additions take the following form:

$$\begin{aligned}
 rW_e &= w_e + s(U - W_e) + \iota(W_i - W_e) + \lambda(E_p W'_e - W_e), \\
 rW_i &= w_i + s(U - W_i) + \lambda(E_p W'_i - W_i), \\
 rJ_e &= p(1 - \xi) - w_e - sJ_e + \iota(J_i - J_e) + \lambda(E_p J'_e - J_e), \\
 rJ_i &= p - w_i - s(J_i + \gamma) + \lambda(E_p J'_i - J_i).
 \end{aligned} \tag{4.17}$$

Compared to the equations in (4.2), there are four differences. First, the subscripts e and i respectively denote a match with an entrant worker or a match with an incumbent worker. An entrant employee has a lower productivity, captured by the fraction $(1 - \xi) \in (0, 1)$, in which ξ is a training cost parameter. Hence, his wage w_e is different from the wage of the incumbent employee, w_i . An entrant becomes incumbent at an exogenous rate ι . Finally, when a

match with an incumbent worker dissolves, the firm has to pay a fixed separation cost, γ , that Silva and Toledo assume to be fully wasted (administrative costs may have this characteristic). The asset value equation in unemployment is given by (4.2). The usual zero profit condition on vacancy creation is imposed, together with the Nash bargain solutions to share the rents originated by matches either with incumbents or with entrants. The former is given by $(1 - \beta)(W_i - U) = \beta J_i$, the latter by $(1 - \beta)(W_e - U) = \beta J_e$. After some computations (details can be found in Silva and Toledo's Appendix), one gets the following equilibrium equation in the case $\lambda = 0$:

$$(r + s)\frac{c}{q(\theta)} + \beta c\theta = (1 - \beta)\frac{(r + s)[p(1 - \xi) - z] + \iota(p - z - \gamma s)}{r + s + \iota}$$

Differentiating such expression, we can find the elasticity value $\eta_{\theta p}$:

$$\eta_{\theta p} = \frac{p[(1 - \xi)(r + s) + \iota]}{p[(1 - \xi)(r + s) + \iota] - z(r + s + \iota)} \cdot \frac{[r + s + \beta f(\theta)]c + \frac{(1 - \beta)\iota\gamma s f(\theta)}{\theta(r + s + \iota)}}{[(r + s)\alpha + \beta f(\theta)]c}$$

Silva and Toledo consider quarterly data. Some calibrated values coincide with those considered by Shimer: $s = 0.10$, $r = 0.012$, $\alpha = 0.72$, and p normalized to 1. The job finding rate $f(\theta)$ is set equal to 1.65, slightly larger than Shimer's estimates. Silva and Toledo also compute the average duration of a vacancy equals to 17 days, so $\theta/f(\theta) = 0.29$. For the cost of keeping a vacancy open, Silva and Toledo consider the estimates on a broad range of hiring costs obtained by the Institute Saratoga (2004) and get $c/q(\theta) = 0.14$ and $c = 0.488$. Data taken from the Employer Opportunity Pilot Project, a 1982 cross-sectional firms-level survey, and from the 1992 Small Business Administration Survey allow to set $\iota = 1/4$ (meaning that an entrant takes one year to become fully productive) and $\xi = 0.31$. According to the 2004 World Bank Doing Business survey, firing costs amount to 8 weeks of weekly wages of an incumbent employee: therefore, supposing that $w_e \approx p$, $\gamma = 8/12$. Finally, following Costain and Reiter (2006) and avoiding the criticisms addressed to Hagedorn and Manovskii's (2006) setup, Silva and Toledo choose the calibrated values for β and z in order to obtain a reasonable semielasticity of unemployment duration to unemployment benefits. With $\beta = 0.34$ and $z = 0.715$, ζ_{ub} is equal to 2.0, in line with the policy evaluation estimates presented in section 4.2.3. Substituting these values in the equation for the elasticity, we get $\eta_{\theta p} = 6.62$,

almost four times larger than in Shimer's setting. Moreover, $\eta_{fp} = (1-\alpha)\eta_{\theta p} = 1.85$ and $\eta_{u,p} = 1.747$. Recall that in U.S. data, $\sigma_f/\sigma_p = 5.9$ and $\sigma_u/\sigma_p = 9.5$. At the steady-state level, Silva and Toledo improve the performance of the matching model, but still do not match the data.

Yet, in their simulation part, Silva and Toledo get $\sigma_{\theta}/\sigma_p = 20$, much larger than $\eta_{\theta p}$ and very close to its empirical counterpart. Such discrepancy between the steady state elasticity result and the numerical simulation outcome depends on the choice of Δ made by the authors. Recall that Mortensen and Nagypál (2006) prove that $\eta_{\theta p} \approx \sigma_{\theta}/\sigma_p$ if the arrival rate λ or the grid step size Δ tend to zero. In Shimer (2005a), $\lambda = 4.0$ and $\Delta = 0.0083$. In Silva and Toledo's paper, $\lambda = 0.4$ and $\Delta = 0.053$: these values are not sufficiently close to 0 for the approximation to hold. Silva and Toledo's approach improves upon the business cycle consistency of a matching model and is consistent with the microeconomic estimates presented in section 4.2.3. This is the most important difference with respect to the papers belonging to the first and the second approach.

Why do turnover costs better the performance of the model? As we have seen previously, in order to amplify the effects of a productivity shock, the profit share must be low, so that for a given percentage increase in p the percentage variation in profits is large and more vacancies must be posted to ensure the zero profit condition. The same mechanism applies in this setting. Turnover costs lower the value of a filled job, so a higher p yields a large percentage increase in profits. Tightness must change more in order to restore the free-entry equilibrium. The elasticity of tightness with respect to productivity is therefore higher.

A similar effect is present in Garibaldi (2006). He considers two types of large (multiple jobs) firms in his model: those that, having a high productivity value, react to adverse shocks by simply posting less vacancies, and those "at the margin", with a very low profit share. When a productivity shock hits the economy, this second kind of firms either declares bankruptcy, firing all the employees, or freezes its hirings, not replacing the workers who quit. The difference between the expected losses and the firing costs determines which of the two actions the firms chooses. If the expected losses are greater in absolute value than firing costs, the firm declares bankruptcy, otherwise it experiences

a “hiring freeze”. For firms “at the margin” a small change in productivity can make a great difference, and the number of vacancies they post in good times is much higher than the number of vacancies (not) posted in bad times. The volatility of such variable greatly increases. Accounting for hiring freeze and bankruptcy allows Garibaldi to explain up to 35% of the tightness volatility displayed in the data.¹⁷

4.6.2 Introducing capital

Does the introduction of capital amplify the volatility of θ and $f(\theta)$? In a previous version of their discussion paper, Mortensen and Nagypál (2006) consider a naive extension of the benchmark model in which firms have to pay a fixed flow cost k in order to produce. In equations (4.5) and (4.10) productivity p is replaced by $p - k$ and the elasticity of tightness with respect to productivity becomes:

$$\eta_{\theta p} = \frac{r + s + \beta f(\theta)}{\alpha(r + s) + \beta f(\theta)} \cdot \frac{p}{p - k - z}.$$

The higher is the cost of capital, the higher will be the elasticity $\eta_{\theta p}$. With k equal to 1/3, the standard value for the capital share, and keeping the same calibrated values of Shimer (2005), $\eta_{\theta p}$ becomes equal to 3.87. The volatility of tightness has more than doubled. The mechanism is the same as in Hagedorn and Manovskii’s calibration and in Silva and Toledo’s set-up with turnover costs. Capital costs lower profits, so that for a given percentage increase in p the percentage variation in profits is larger, and more vacancies are posted.

Yet, this result probably depends on the simplistic way in which capital market is formalized. The RBC matching models cited in this survey include a market for capital (for instance Farmer and Hollenhorst, 2006, Gertler and Trigari, 2006). Nevertheless, as the authors document, the ability of such models to replicate the U.S. labour market fluctuations does not stem from the presence or not of capital. Other papers, such as Hall and Milgrom (2006) or Silva and Toledo (2006) ignore the capital market, and they are able to offer a partial answer to Shimer’s puzzle. The least that can be said is that capital is not decisive in reconciling the model with the data.

¹⁷Recall that in Shimer (2005a)’s model σ_{θ}/σ_p is eleven times lower than in U.S. data

4.6.3 On-the-job search

Nagypál (2005), Mortensen and Nagypál (2006), and Krause and Lubik (2006) introduce on-the-job search to ameliorate the quantitative performance of the model. Employed workers can accept another job because it pays a higher wage or for non-pecuniary characteristics that make it more attractive. The first two papers consider the first reason, whereas Krause and Lubik build up a model with high and low paid jobs.

Heterogeneity in the value of the match

Nagypál (2005), Mortensen and Nagypál (2006) assume that the utility flow of a worker is given by the sum of the wage and an idiosyncratic payoff, ψ , drawn from a distribution $F(\cdot)$, where $F : [\underline{\psi}, \bar{\psi}] \rightarrow [0, 1]$ is a continuous twice differentiable, strictly increasing distribution function. Such idiosyncratic payoff represent the appeal of a job and it is known only by workers. Employed searchers therefore quit their position only to find a job with a higher ψ .

As Nagypál (2005) points out, adding on-the-job search in a standard matching setting is not sufficient to improve the quantitative properties of the model. Such framework performs even worse than Shimer's one. The reason is the following. The pool of searchers is composed by unemployed workers and employed searchers. While an unemployed worker accepts all the offers they receive, an employee quits his job only if the new one gives a higher expected utility. In other words, the former has an acceptance probability of one, the latter less than one. Of course, firms want a high acceptance rate, for this implies a low expected cost of filling a vacancy. Such acceptance rate is countercyclical, because during booms there are less unemployed workers and more employed searchers. So, vacancy creation is dampened by the lower acceptance rate that firms face after a positive productivity shock.

The argument is reversed if we assume that firms incur hiring costs, modeled as a fixed amount of resources that firms must pay once a worker is hired. The reason is explained by Nagypál (2005). Describing her model in detail goes beyond the scope of this chapter, thus I simply expose the rationale of her findings and consider a simplified version of it, taken from Mortensen and Nagypál (2006).

In a framework with hiring costs and on-the-job search, some matches have a negative pay-off for firms, for an employee can quit before the employer has recouped his initial investment. In turn, this occurs because their appealing value ψ is low and the employees are more willing to break them and find another job. By assumption, firms do not know the idiosyncratic value of the match ψ , but they realize that unemployed searchers are more likely to accept matches with a low match quality than employed searchers. This is due to a positive selection effect that shifts workers into match qualities towards the top of the distribution. So, when the hiring costs are very large, firms have a lower expected pay-off from contacting an unemployed searcher than an employed one, even if the latter has a lower acceptance rate. Such phenomenon is called by Nagypál the acceptance curse.

What does it imply for our quantitative scrutiny of the matching models? As we noticed earlier, when a positive productivity shock hits the economy, the fraction of employed searchers out of the total number of searchers increases. In a framework with hiring costs it means a lower proportion of matches with low ψ and a negative payoff. Firms are more willing to post their vacancies in response to a positive shock.

We can see this point analytically. Suppose for simplicity that all employed workers search on-the-job. The matching function is equal to $m(e + u, v) = m(1, v)$ and the job finding rate $f(v) = m(1, v)$ depends only on the number of vacancies. Market tightness is now measured in terms of vacancy and the elasticity of the job-finding rate $f(v)$ is given by $1 - \alpha = \sigma_f \rho_{fv} / \sigma_v$. From data in Table 4.1, we have $1 - \alpha = 0.524$. Define $\hat{\psi}$ as the value that makes a worker indifferent between being employed or unemployed and $G(\psi)$ as the equilibrium measure of employed workers with a quality match below ψ .¹⁸ Then the following steady state equations hold:

$$[s + f \cdot (1 - F(\psi))] \cdot G(\psi) = f \cdot (F(\psi) - F(\hat{\psi})) \cdot u \quad (4.18)$$

$$s \cdot (1 - u) = f \cdot (1 - F(\hat{\psi})) \cdot u \quad (4.19)$$

The LHS in (4.18) is the measure of workers that flow out of jobs with a match

¹⁸Recall from search on-the-job models *à la* Burdett and Mortensen (1998) that $F(\cdot)$, the ex ante distribution of ψ , is different from $G(\cdot)$, the stationary equilibrium distribution of jobs according to their match quality.

quality ψ or lower. The RHS in (4.18) is the measure of workers that flow into the pool of matches with quality ψ or lower. Equation (4.19), is the Beveridge curve in this set-up with on-the-job search. The asset value of a filled job with an idiosyncratic component ψ solves:

$$rJ(\psi) = p - w - [s + f \cdot (1 - F(\psi))] J(\psi)$$

Following Mortensen and Nagypál (2006), I consider that the wage equation is simply given by $w = z + \beta(p - z)$.¹⁹ The equation above takes then the following form:

$$J(\psi) = \frac{(1 - \beta)(p - z)}{r + s + f \cdot (1 - F(\psi))}.$$

The probability that a job with idiosyncratic value $\psi > \hat{\psi}$ is accepted is given by:

$$A(\psi) = u + G(\psi) = \frac{s}{s + f \cdot (1 - F(\psi))},$$

where the second equality is obtained using (4.18) and (4.19). Hence, we can express the free-entry zero-profit condition in the following form:

$$\frac{cv}{f(v)} = \int_{\hat{\psi}}^{\bar{\psi}} A(\psi) [J(\psi) - H] dF(\psi)$$

The LHS of the equation above represents the expected cost of filling a vacancy (recall that the measure of labour market tightness is now given by v , the number of vacancies). The term in the RHS represents firms' expected profits: the sum over ψ of the difference between the expected value of a filled job and the hiring cost, H , multiplied by the acceptance rate $A(\psi)$. Integrating by parts, and assuming for simplicity that r tends to zero and $\hat{\psi} = \underline{\psi}$, Mortensen and Nagypál get the following equilibrium equation (computations can be found

¹⁹Menzio (2005) shows that in an alternating-offers bargaining game with asymmetric information the limit wage solution as the time between offers tends to zero is independent of the worker's type ψ . Specifically, the equilibrium wage results equal to $w = \beta p + (1 - \beta)(z - \hat{\psi})$, irrespective of the idiosyncratic value of the match. This strong result, used by Nagypál, can by itself amplify the fluctuations in labour market tightness. The reason is the same presented in section 4.4.6 discussing Hall and Milgrom's model. In the wage equation above there is no opportunity cost of employment. Hence, the general equilibrium effect that attenuates vacancy creation, by raising w via rU after a positive shock on p , is not present.

in section 6.2 of their paper):

$$\frac{cv}{f(v)} = \frac{(1-\beta)(p-z)}{s+f(v)} - \frac{s}{f(v)} \ln\left(\frac{s+f}{s}\right) H \quad (4.20)$$

Increasing the job-finding rate has an ambiguous effect on the RHS of (4.20): the first term decreases because firms discount at a higher rate future profits. The second term increases because of the acceptance curse effect stressed previously. Notice that with $H = 0$ the RHS would be decreasing in $f(v)$. Mortensen and Nagypál then compute the elasticity of the job finding rate in response to a positive shock on productivity and a negative one on the destruction rate, in order to amplify the effects in terms of vacancy creation. If hiring costs H are imposed to be 2 or 3 times the quarterly profit flow $p - w$ this implies a σ_f/σ_p respectively of 3.086 and 7.168. Such numbers are close to the value of 5.9 found by Shimer in the data.

Shock on the destruction rate

The model constructed by Nagypál and Mortensen succeeds therefore in replicating the fluctuations of the job-finding rate. But they also rule out the counterfactual increase in vacancies after a positive shock on the destruction rate. As we have seen in section 4.3.1, without on-the-job search, a higher destruction rate encourages firms to post more vacancies because it raises the number of unemployed people, that are the only searching workers in the economy. In a framework with on-the-job search, an increase in s has a smaller impact on the total number of searching workers, composed by unemployed and employed people. The incentive for firms to post more vacancies as s goes up is therefore weakened. In the special case we are considering, where all the employees search on the job, vacancies always decrease in response to a positive shock on the destruction rate. We can see that in Figure 4.2. The zero profit condition (4.20) does not depend on unemployment and it is a horizontal line in the (u, v) space, whereas the Beveridge curve (4.19) is decreasing. A higher s shifts the former towards the origin and the latter to the right. In the new equilibrium, both unemployment and vacancies unambiguously decrease.

Wage heterogeneity

Krause and Lubik (2006) also introduce on-the-job search in order to improve

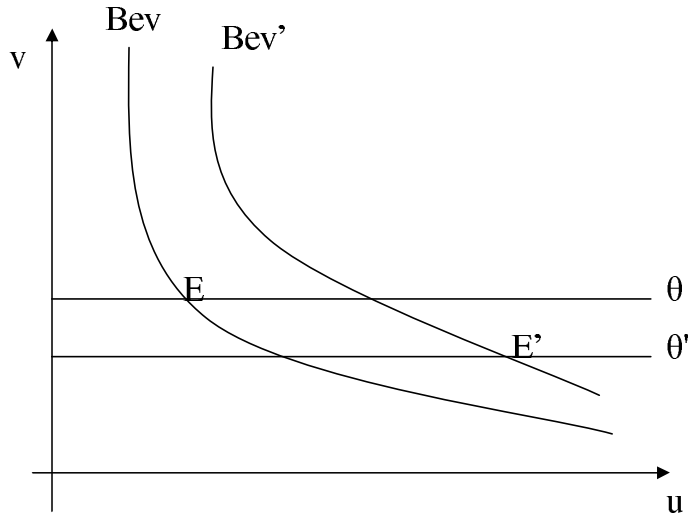


Figure 4.2: An increase in the destruction rate when all workers search for a job. The new equilibrium point is E' .

the ability of the model to match the data. Differently from Mortensen and Nagypál, on-the-job search is motivated by wage heterogeneity. The model that Krause and Lubik build up is similar to Acemoglu (2001). There are two intermediate good sectors in the economy, denoted by g and b . The two intermediate goods are combined together to produce the unique final consumption good. Sectors g and b differ in the cost of opening a vacancy: $c_g > c_b$. The free-entry zero profit conditions and the Nash wage bargain solutions imply: $J_i = c_i \theta_i / f(\theta_i) = \frac{1-\beta}{\beta} (W_i - U_i)$, $i \in \{b, g\}$. Unemployed workers direct their search toward one of the two sectors. The asset value equations for an unemployed worker are:

$$rU_i = z + f(\theta_i) [W_i - U_i]$$

with $i \in \{b, g\}$. An arbitrage condition leads to $U_g = U_b = U$. This implies that $f(\theta_i)(1 - \beta)(W_i - U) = \beta \theta_i c_i$ must be equal across sectors. Since $c_g > c_b$, then $\theta_g < \theta_b$. Sector g has a lower tightness, a lower employment, a higher productivity, (because of decreasing marginal returns on input in the final good production function), and a higher wage. The employees that work in the low-paid sector search on-the-job. The matching technologies for the two sectors are respectively, $m_g = \mu(u_g + \varsigma e_b)^\alpha v_g^{1-\alpha}$ and $m_b = \mu u_b^\alpha v_b^{1-\alpha}$,

where ς represents the search effort devoted by low-paid employed workers. Defining $\theta_g = v_g/(u_g + \varsigma e_b)$, the job finding rate in g is then given by $f(\theta_g) = m_g/(u_g + \varsigma e_b) = \mu\theta_g^{1-\alpha}$. Their search intensity ς is endogenous: The higher the value of being employed in a high-paid sector, the more intensive will be the low-paid workers' search effort. This can be seen formally considering the Bellman equation for workers in sector b :

$$rW_b = \max_{\varsigma} \{ w_b - k(\varsigma) + \varsigma f(\theta_g) [W_g - W_b] + s[U - W_b] \}$$

The function $k(\varsigma)$ is increasing and convex. The F.O.C. of such maximization problem takes the following form:

$$\begin{aligned} k'(\varsigma) &= f(\theta_g) \cdot [W_g - W_b] \\ &= f(\theta_g) \cdot [W_g - U - (W_b - U)] \\ &= f(\theta_g) \cdot \frac{\beta}{1-\beta} [J_g - J_b] \\ &= f(\theta_g) \cdot \frac{\beta}{1-\beta} \left[\frac{c_g \theta_g}{f(\theta_g)} - \frac{c_b \theta_b}{f(\theta_b)} \right] \end{aligned}$$

With the Cobb-Douglas matching function the last equality becomes:

$$k'(\varsigma) = \frac{\beta}{1-\beta} c_g \theta_g \left[1 - \left(\frac{c_b}{c_g} \right)^{1-\alpha} \right] \quad (4.21)$$

When a positive productivity shock hits the intermediate sectors of the economy, firms post more vacancies both in the high-paid and in the low-paid sector. By equation (4.21), a higher θ_g raises search effort that, in turn, tend to decrease θ_g (recall that $\theta_g = v_g/(u_g + \varsigma e_b)$ in this set-up). Such feedback effect lowers the expected duration of filling a vacancy $1/f(\theta_g)$, so even more vacancies are posted in sector g . Vacancy creation and employees' search effort are strategic complements: An increase in v_g triggers search effort that, in turn, raises v_g even more. The process ends because of the convexity assumption about the search cost. The final result is a larger amplification in vacancy posting in response to a productivity shock. A complementarity between sectors also arises. If search effort of low-paid employees goes up, congestion effect in the matching technology will make more difficult for unemployed workers to find a high-paid job. Then, they will direct their search toward the low-paid sector. This in turn will boost vacancy creation in that sector.

4.6.4 Endogenous search effort?

Since the major determinant of the amplifications in vacancies and unemployment in Krause and Lubik's model is search effort, one could ask why the need of introducing on-the-job search. Is it possible to better the performance of the matching model simply by inserting endogenous search activities on the workers' side? Costain and Reiter (2006) provide the answer. A model with endogenous search effort but without on-the-job search succeeds in making vacancies and unemployment more cyclical. Yet, the steady-state negative relationship between unemployment and vacancies (the Beveridge curve) disappears. When endogenous search effort is accounted for, the steady-state level of unemployment becomes:

$$u = \frac{sL}{s + \varsigma f(\theta)},$$

where θ is now defined as $v/\varsigma u$. A decrease in productivity lowers both vacancies and search effort. The Beveridge curve shifts to the right. As in the case of a shock on the separation rate (see Figure 4.2), the new equilibrium point may present higher unemployment and a higher level of vacancies. The reason is that with a lower search effort, more vacancies must be posted for the equality of labour market flows to hold. This is not the case in Krause and Lubik's model, in which the intensity of the search activity is constant for the unemployed workers. The Beveridge curve is not directly affected by search effort.

Endogenous search effort also deteriorates the policy analysis consistency of the model. An increase in the level of UB makes a job less attractive for unemployed workers. Search effort decreases. Unemployment goes up not only because firms, paying higher wages, post less vacancies, but also because workers search for a job with lower intensity. As Costain and Reiter document, the elasticity η_{uz} results much higher than the estimates suggest.

4.6.5 Cohort-specific productivity shocks

Shimer assumes that the productivity shock hits all the matches in the economy. Costain and Reiter (2006) argue that considering a shock that affects only new matches and does not change the productivity of the old ones is not only

empirically grounded but also helps to solve the puzzle raised by Shimer. For the empirical evidence, they refer to several studies showing that wages of new hires are more procyclical than the incumbents' ones and that it is more likely to find jobs with high quality in booms than in recessions.

From a theoretical point of view, a shock that affects only new matches implies that the productivity value of a job remains constant until separation. More firms will decide to post a vacancy when the shock is positive. The volatility of the vacancies, unemployment, and the job-finding probability is higher. Setting the instantaneous utility in unemployment z equal to 0.7 (in order to target the semi-elasticity $\zeta_{u,b} = 2$), Costain and Reiter get a ratio $\sigma_u/\sigma_p = 9.66$ and a ratio $\sigma_f/\sigma_p = 11.32$, even higher than in the data.

Notice that such mechanism is very similar to that occurring in a rigid wage model. Wage rigidity or productivity rigidity, in the sense that their value remain unchanged during the entire lifetime of the match, makes firms more reactive to the shocks. Yet, differently from the rigid wage case, a model with cohort-specific technology shock does not worsen the policy analysis consistency of the model. In Costain and Reiter's framework, wages are bargained over so the negative impact of z on tightness does not disappear as in the rigid wage hypothesis.

The only serious drawback of the model lies on the excessive volatility of wages. The ratio $\sigma_w/\sigma_p = 3.64$, seven times larger than in the data. As stressed in section 4.4.5, under the Nash bargaining rule (4.6), the wage increases in booms not only via productivity but also because it positively depends on the opportunity cost of employment, that is procyclical. So, wages respond strongly to output fluctuations even though the productivity of the match remain constant over time. This does not appear however a major problem. Assuming another surplus sharing rule in which rU affects less the wage equation would soften the excessive volatility of the wages.

4.6.6 Other sources of shocks

A VAR approach

Yashiv (2005) and (2006a) performs an exercise close to that endeavored by Shimer (2005a). To investigate to which extent a matching model can mimic U.S. business cycle facts, he considers a reduced-form VAR of the actual data

to specify the driving shocks. He assumes that three variables (the rate of productivity growth, the separation rate, and the interest rate) follow a first-order VAR. The coefficient matrix of such system of three equations and the variance-covariance matrix of the disturbances are computed by a reduced-form estimates of the data. Yashiv reaches a conclusion completely different from Shimer's one. His model captures the persistence, the volatility and some co-movements of the main labour market variable in the data. Such results hold even considering alternative measure of the pools of searching workers²⁰.

The reason of such performance are essentially two. First, he considers convex costs of posting a vacancy. He abandons the assumption one firm - one job so any employer can post more than one vacancy. Convex costs make vacancy creation more sluggish. This enhances the persistence of vacancies (in the case of linear costs, $\rho_v = 0.001$, while with convex costs $\rho_v = 0.961$, a value very close to the data). Yet, convex costs tend to reduce the volatility of the vacancies.

Second, the stochastic properties of the separation rate are crucial in raising the volatility of all the variables of interest. To show that, Yashiv performs a counterfactual analysis. In the coefficient matrix of the VAR, the separation rate is highly persistent, with a $\rho_s = 0.92$. He sets it to 0.10. The persistence of the variables is reduced by a 50% (for instance ρ_v falls from 0.961 to 0.287, while ρ_u goes from 0.983 to 0.763), but, more importantly, the standard deviations are more than 20 times lower than in the benchmark case $\rho_s = 0.92$. For instance, the standard deviation of unemployment σ_u , that in the benchmark case is equal to 0.183, drops to 0.004, while σ_u falls from 0.037 to 0.002. Why does the separation rate plays such an important role in amplifying fluctuations? Yashiv rightly argues that the separation rate is key in evaluating the expected discounted value of a match. Nevertheless, it is not clear why in Yashiv's VAR approach it is so essential in engendering the correct volatility, while Shimer's setting with contemporaneous shocks both in p and in s does not provide satisfactory results.

²⁰He considers four different measure for the pool of searching workers: 1) the official number of unemployed workers u , 2) u and people out of the labour force that explicitly say to want a job, 3) people of the second group and 15% of the working age population outside the labour force, 4) people of the second group and 30% of the working age population outside the labour force, and 5) all the working age population not employed.

Market power

Most of the papers I have surveyed so far focus on unemployment and vacancy fluctuations. Rotemberg (2006) wants to dampen the excessive procyclicality of real wages exhibited by the model. He builds up a matching model with large firms (i.e. any firm hires more than one worker) in which he introduces three decisive features: concave vacancy costs, imperfect competition in the product market, and diminishing returns of labour.

Concave costs induce firms to post more vacancies in booms, so amplifying the fluctuations of such variable. Imperfect competition allows him to modify the source of the shocks. While Shimer constructs a stochastic model in which technological shock hits the economy, Rotemberg considers a change in firms' market power. The key is that in the former set-up the marginal productivity of labour and, in turn, the wage increase. In the latter, firms react to fiercer competition by producing more, and labour productivity goes down. So, workers have to moderate their wage demand, while employment increases²¹.

Rotemberg's main objective is to match data on wage volatility. As far as this variable is concerned, his model is successful: the standard deviation of the real wage relative to output is 0.56 in Rotemberg's set-up, very close to its empirical counterpart. Even the other statistics are not so far from the values found in the data. The model overshoots on vacancies volatility, whereas it captures one third of the employment fluctuations and half of unemployment volatility. However, as Trigari (2006) documents in a detailed comment, the success of the model depends less on the imperfect competition, diminishing returns of labour, or concave vacancy costs, than on the high value assigned to the instantaneous utility in unemployment. Rotemberg sets $z/w = 0.9$, close to Hagedorn and Manovskii' parametrization. Again, the performance of the model stems more from the low profit share than from other theoretical features. Moreover, the change in the demand elasticity that is needed to fit the data on employment results too large. This strand of research is therefore

²¹Indeed, firms may hire more than at the optimum for any given level of competition. In an imperfect competition framework, firms' marginal revenues are decreasing. The wage is a constant fraction of the such marginal revenues. So, hiring one more worker reduces the wage bill. On intra-firm wage bargaining within a matching framework see Cahuc and Wasmer (2001) and Ebell and Haefke (2006)

still in progress. Rotemberg suggests for instance to embed such framework in a more complex environment with multiple shocks.

4.7 Concluding Remarks

Shimer (2005a) and Costain and Reiter (2006) have called in question the quantitative consistency of standard search and matching models. Considering the relative short period time elapsed from the publications of these papers, many scholars have reacted to their findings, and with competing approaches. In this chapter, I have distinguished three main routes, that Table 4.3 summarizes. Unfortunately, it is not possible to adopt a unique criterion in evaluating all the papers I have surveyed: Some of them ignore policy analysis issues, others do not report statistics on the real wage, some others present their results in terms of impulse response functions and not in terms of ratios of standard deviations. Table 4 therefore can give only an idea of the results presented in the previous sections.

However, I may draw some general conclusions. Modifying the wage formation rules, if accompanied with a high calibrated wage share, betters the business cycle performance of the model, but tends to underestimate the effects of changes in the level of unemployment benefits on unemployment. Imposing both a high calibrated value for the utility in unemployment and a low one for worker's bargaining power makes a standard matching model consistent with U.S. business cycle fluctuations, but it greatly overestimates the impact of UB on unemployment.

The third route, that consists in adding some unquestionable characteristics of modern labour markets in a standard matching framework, appears to me the most effective. With turnovers costs the model can account both for business cycle fluctuations and for policy analysis long-run estimates. Cohort-specific shocks on productivity fit data quite well and are empirically plausible. The combination of hiring costs and on-the-job search better the performance of the matching model also in response to shocks on the destruction-rate.

Nevertheless, some questions remain open.

Is it possible to reconcile the gross worker flow data approach with Shimer's

PAPERS	Volatility u and v	Volatility w	$\zeta_{u,b}$
Shimer (2005a)	inconsistent	twice larger	inconsistent
First Approach			
Shimer (2004a)	matched	inconsistent	inconsistent
Hall (2005)	matched	n.c.	n.c.
Farmer and Hollenhorst (2006)	matched	not matched	n.c.
Gertler and Trigari (2006)	almost matched	matched	low
Kennan (2006)	improved	n.c.	low
Hall and Milgrom (2006)	improved	inconsistent	low
Second Approach			
Hagedorn and Manovskii (2006)	matched	matched	inconsistent
Third Approach			
Silva and Toledo (2006)	improved	not matched	matched
Garibaldi (2006)	improved	n.c.	n.c.
Mortensen and Nagypál(2006)	matched	n.c.	n.c.
Krause and Lubik (2006)	matched	not matched	n.c.
Costain and Reiter (2006)	almost matched	not matched	matched
Rotemberg (2006)	almost matched	matched	n.c.
Yashiv (2006)	matched	n.c.	n.c.

Table 4.3. Results of the models compared with the data. $\zeta_{u,b}$ is the semi-elasticity of unemployment with respect to the unemployment benefits; n.c. = not considered.

one, based on unemployment data?²² To which extent is job destruction important in explaining unemployment fluctuations? Can a standard matching model match business cycle facts in other countries, in Europe for instance? The macroeconomic performance of matching models is far from being an exhausted research area.

²²A recent paper by Yashiv (2006c) goes in this direction.



Conclusions

This thesis can be seen as an attempt to study some relevant issues related to the labour market within a search and matching framework. The conclusions reached in the previous chapters suggest that such an approach is useful both for a policy evaluation analysis and for a better comprehension of the business cycle fluctuations.

Inspecting the relationship between product and labour market when the latter is not perfectly competitive but presents trading frictions leads to non trivial conclusions in terms of employment and real wages. Moreover, compared to a Walrasian set-up, a search and matching model matches some important labour market facts, as the countercyclicality of the labour share or the dynamic correlation between labour productivity and employment; however, in its simplest version, it is unable to mimic the behaviour of unemployment and the job-finding rate.

As usual, the attempt of clarifying some problems gives rise to other questions and topics that may well constitute part of a future research agenda. I here list four points that, according to me, deserve a further investigation.

First, it should be better understood the role played by risk-aversion and precautionary savings in labour supply decisions and unemployment. In the

models presented in chapter 1, 2, and 3, workers' instantaneous utility is linear and households consume all the disposable income. Tractability reasons impose such restrictive conditions. Yet, a recent paper by Flodén (2006) gets interesting analytical results, even if in a perfectly competitive context; in this sense, it may represent a good starting point.

A second point is related to wage dispersion. In chapter 3, because of the imperfect competition in the product market, wages are endogenously distributed. The shape of such a distribution mainly depends on the exogenous (and difficult to calibrate) sector-specific destruction rate and does not present the standard features of a wage distribution (unimodal, skewed, with a Paretian right tail, etc...). One natural question is whether removing the assumption of a sector specific destruction rate may improve the results.

Third, one should test the theoretical conclusions reached in the third chapter. Nicoletti and Scarpetta (2005) and Griffith, Harrison, and Macartney (2007) estimate the impact of deregulation in the goods market on employment. It would be interesting to evaluate also the effects on real wages and hours worked. Fourth, the empirical consistency of matching models should be tested not only on the basis of U.S. data but also looking at other countries.

The search and matching approach has undoubtedly improved our comprehension of the labour market and the four open issues I have just mentioned are only an example of the vitality of this research area.



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