Macro-Modeling Library in Simscape for MEMS Pressure Sensors Based on Energy-Flow Paradigm

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***Abstract*— A definitory feature of MEMS devices is the coupling at microscale between multiple energy domains. Reduced order macro-models (ROM) can be built using either signal flow or energy flow paradigms. While easier conceptually, reducing the device to a dynamical system defined by a set of differential equations, the so-called causal signal flow paradigm is nevertheless less portable, because the macro-models embed as well the excitation, or boundary conditions. The energy flow paradigm, on the other side, is an acausal modeling methodology that reduces the physical system to an equivalent generalized, Spice-like, network, based on across-through (power complementary) variables. The circuit-like representation makes it easier to interface with read-out or actuation electronics systems, for complete microsystems design, simulation and optimization. Besides existing standard behavioral modeling languages like VHDL-AMS and Modelica, MathWorks Simscape language has evolved in the past years to the point where it offers an alternative for the implementation of advanced MEMS macro-models; its seamless interface with Matlab and Simulink offers a strong framework for developing complex hybrid microsystems models, that mix Simulink (signal flow) models with Simscape (energy flow) ones. We have built a generic Simscape library for parameterized MEMS pressure sensors for two distinct categories, piezoresistive and capacitive sensing. The behavior of Simscape macro-models (e.g. sensitivity and frequency response) was validated against models of the same devices extracted in Coventor MEMS+ (a dedicated software for the design and simulation of MEMS devices) and finite element analyses (FEA).**

Keywords— MEMS, pressure sensors, reduced order modeling, energy flow modeling, Simscape, MEMS+.Introduction

I. INTRODUCTION

Modelling is a key step for the design and simulation of microsystems. The multiphysics interactions in these systems lead to complex nonlinear dynamics, requiring lengthy FEA simulations. A reduced order macro-model or ROM, on the other side, allows to have a sufficient level of abstraction, to significantly reduce simulation time and to enable a direct interface with the electronic subsystems, for system level optimization [1][2]. Numerous works have already shown the importance of model order reduction to get fast and accurate representations of the behavior of MEMS devices [3][4].

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The energy flow paradigm is an elegant way to implement such a ROM that allows to build a library of independent components, Spice-like, that can be used directly to interface with electronic circuits simulations [5]. Simscape toolbox from MathWorks allows to create such components, complementary to the information flow approach used by Simulink [6].

In this paper, we are focusing on building accurate Simscape library elements that can be used as equivalent circuit components modelling capacitive or piezoresisitive MEMS pressure sensors.

II. METHOD

A. Membrane mechanics

In this study, we are considering a pressure sensor based on a polyimide circular membrane. As seen in fig. 1 and fig. 2, the membrane consists of a polyimide layer on the top of which is deposited a thin aluminum layer as top-electrode, for capacitive sensing, or thin polysilicon resistors, in radial direction, for piezoresistive sensing. In both cases, the substrate (together

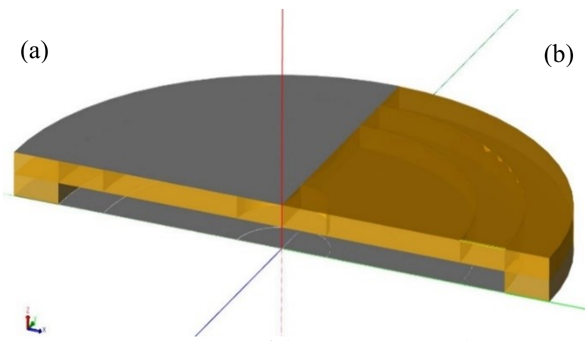

Figure 1 - 3D cross section of the capacitive (a) and piezoresistive (b) pressure sensors

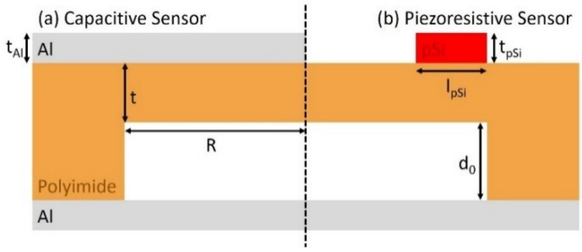

Figure 2 - Cross-sections of the capacitive (a) and piezoresistive (b) pressure sensors

with the bottom-electrode on it) is assumed rigid. Tables (1)-(4) present the materials properties, the geometrical dimensions, the parameters and the variables used to model the sensors.

From a mechanical macro-model perspective, we are reducing the dynamic to one equivalent mechanical degree of freedom (DOF) mass-spring system that has to show good agreement in terms of both static response and first modal behavior. Assuming the hypothesis of small deflection and using the mode-shape-expansion method [7], we chose a shape-function as a solution of the Kirchhoff-Love circular plate equation with clamped edge under a static uniform load [8]:

$$y(r,\theta)=\frac{pR^4}{64D}\left(1-\left(\frac{r}{R}\right)^2\right)^2 \tag{1}$$

The first-mode resonant frequency, ω_0, and the effective stiffness, k_{eff}, are computed from the equality of the maximum potential and kinetic energies, respectively W_{p_max} and W_{k_max}, of the membrane for a harmonic vibration at the resonant frequency, using the assumed mode shape [7]. These energies are obtained by integration of the shape function, and are expressed as (2) and (3).

$$W_{p\_max}[J]=\frac{\pi p^2R^6}{64D}\frac{1}{6} \tag{2}$$

$$W_{k\_max}[J]=\frac{\pi\rho t\omega_0^2p^2R^{10}}{64^2D^2}\frac{1}{10} \tag{3}$$

Resonant frequency, effective stiffness and effective displacement are then computed thanks to equations (4)-(6):

$$W_{p\_max}=W_{k\_max}\ \Rightarrow \omega_0^2=\frac{64\pi D}{mR^2}\frac{10}{6} \tag{4}$$

with $m=\rho t\pi R^2$, the total mass of the membrane

$$\omega_0^2=\frac{k_{eff}}{m}\Rightarrow k_{eff}=\frac{64\pi D}{R^2}\frac{10}{6} \tag{5}$$

$$y_{eff}=\frac{F}{k_{eff}}\Rightarrow y_{eff}=\frac{pR^4}{64D}\frac{6}{10} \tag{6}$$

with $F=p\pi R^2$, the total force applied on the membrane.

Consequently, the mechanical ROM of the sensors are implemented as mass-spring systems, using y_{eff}, m and k_{eff} parameters.

Table 1- Materials properties

Parameters	Symbols	Values	Units
Polyimide Young modulus	E_k	2.76	GPa
Polyimide Poisson's ratio	υ_k	0.34	-
Polyimide density	ρ_k	1.42	g.cm^{-3}
Polyimide relative permittivity	ε_k	3	-
Aluminum Young modulus	E_{Al}	77	GPa
Aluminum Poisson's ratio	υ_{Al}	0.3	-
Aluminum density	ρ_{Al}	2.3	g.cm^{-3}
Polysilicon Young modulus	E_{pSi}	130	GPa
Polysilicon Poisson's ratio	υ_{pSi}	0.28	-
Polysilicon density	ρ_{pSi}	2.33	g.cm^{-3}
Polysilicon conductivity	σ_{pSi}	1500	S.m^{-1}
Polysilicon longitudinal piezoresistive coefficient	π_{pSi}	300	TPa^{-1}

Table 2 - Geometrical Dimensions

Parameters	Symbols	Values	Units
Membrane thickness	t	10	µm
Membrane radius	R	250	µm
Cavity depth	d_0	40	µm
Aluminum layer thickness	t_{Al}	200	nm
Polysilicon thickness	t_{pSi}	100	nm
Polysilicon length	l_{pSi}	25	µm
Polysilicon width	w_{pSi}	20	µm

Table 3 - Useful parameters

Parameters	Symbols	Expression	Units
Flexural rigidity	D	$Et^3/(12(1-\nu^2))$	Pa.m³
Membrane area	A	πR^2	m²
Initial capacitance	C_0	$\varepsilon_0 A/d_0$	F
Initial resistance	R_0	$l_{pSi}/(w_{pSi}t_{pSi}\sigma_{pSi})$	Ω

Table 4 - Variables

Variables	Symbols	Units
Voltage	V	V
Current	I	A
Velocity	v	m/s
Force	F	N
Pressure	p	Pa
Flux	ϕ	m³/s

B. Energy Flow Paradigm in Simscape

The energy-flow paradigm attempts to represent dynamic systems using the concepts of generalized voltage (across variable) and generalized current (through variable), or in a dual representation, *flow* and *effort* variables [5]. Each energy domain has its own complementary variables that model the power flow between components: current and voltage, force and velocity, torque and angular rate, etc.

This study relies on the interaction between three energy domains: electrical, mechanical and fluidic. More specifically, our model of pressure sensor in Simscape contains two boundary elements (to model the interactions between fluidic/mechanical and mechanical/electrical domains), and one element defining the mechanical properties as seen in fig. 3.

1) *The fluidic/mechanical block*

This element models the lossless energy transfer from the fluid environment to mechanical variables and that is common to capacitive and piezoresistive sensors. This interface is represented by a two-fluidic plus two-mechanical ports element in Simscape (fig. 3) and models the energy conversion according to the relation (7):

$$\phi \cdot p = F_f \cdot v \tag{7}$$

where ϕ is the fluid flux going from port A to port B, p is the difference of pressure between ports A and B, F_f is the mechanical force going from port R to C and v is the difference of velocity between port C and R. This relation is implemented in Simscape using equations (8):

$$\begin{cases} F_f = p\pi R^2 \\ v = \phi/\pi R^2 \end{cases} \tag{8}$$

2) *The mechanical properties block*

This element models the mechanical behavior of the system that acts like a one DOF mass-spring system of mass m and stiffness k_{eff}, with two mechanical ports: one linked to the mechanical reference of the system and the other linked to the unique mechanical DOF of the system, y_{eff}, as seen on fig. 4.

For the capacitive sensor, the mechanical impact of the aluminum is modelled as a displacement of the neutral plane, t_n, and changes of effective flexural rigidity, D_{eff}, and effective density, ρ_{eff} , of the membrane. Using the approximation of the beam theory, and considering the aluminum thickness small regarding to polyimide, the position of the neutral plane is computed thanks to equation (9),

$$\int_0^t D(y-t_n)dy + D_{Al}(t-t_n)t_{Al} = 0 \Rightarrow t_n = \frac{D\frac{t^2}{2}+D_{Al}t_{Al}t}{Dt+D_{Al}t_{Al}} \tag{9}$$

and D_{eff} is computed by equation (10)

$$\int_{-t_n}^{t-t_n} Dy^2dy + D_{Al}(t-t_n)^{2t_{Al}} = \mathrm{M} = \int_{-\frac{t}{2}}^{\frac{t}{2}} D_{eff}y^2dy$$

$$\Rightarrow D_{eff\_C} = \frac{D[(t-t_n)^3+t_n^3]+3D_{Al}(t-t_n)^2t_{Al}}{[(t-t_n)^3+(-t_n)^3]} \tag{10}$$

The effective density is computed by adding the contribution of the aluminum layer as shown in (11).

$$\rho_{eff} = \frac{\rho t+\rho_{Al}t_{Al}}{t} \tag{11}$$

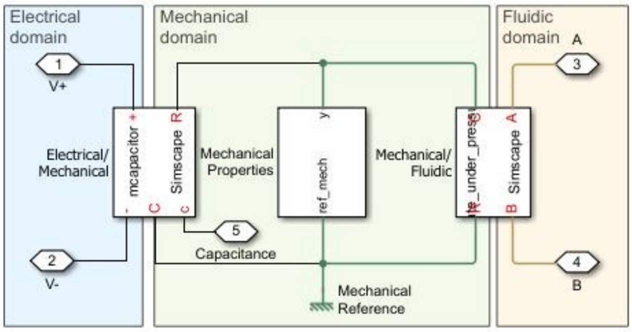

Figure 3 – Generic Simscape system

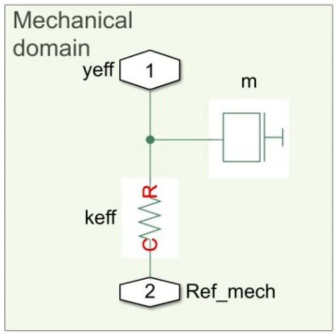

Figure 4 - Mechanical properties block

The effective density and flexural rigidity are then used to model the mass and the effective stiffness of the membrane using equations (5) and (6).

For the piezoresistive sensor, the membrane is mainly made of polyimide and the influence of the piezoresistors, which are considered as small, on the effective density and the flexural rigidity of the membrane is negligible.

3) *The mechanical/electrical block*

This element models the electro-mechanical coupling of the system. It is represented by an element with two mechanical ports, with mechanical force, F_e , going from R to C and difference of velocity, $v = dy_{eff}/dt$, between R and C, and two electrical ports, with current, I, going from + to - and difference of voltage, U, between port + and – (fig. 3).

a) For the capacitive sensor,

the energy transfer between mechanical and electrical domains is modelled as a conservative equation between the electrical and mechanical energies that are injected into the element and stored as electrostatic energy inside the capacitor $C(y_{eff})$. This is summarized by equation (12):

$$UI + F_e v = C(y_{eff})U\frac{dU}{dt} + \frac{U^2}{2}\frac{dC(y_{eff})}{dy_{eff}}v \tag{12}$$

where $C(y_{eff})$ and $dC(y_{eff})/dy_{eff}$ are obtained by integration of the shape function, assuming that the electric field between the plates is oriented in z-direction. By knowing the expression of the current inside a capacitor, this block is implemented in Simscape thanks to the equations (13):

$$\begin{aligned}
&\frac{dy_{eff}}{dt} = v \\
&C(y_{eff}) = C_0 \operatorname{arctanh}\left(\sqrt{\frac{10y_{eff}}{6d_0}}\right) \Big/ \sqrt{\frac{10y_{eff}}{6d_0}} \\
&\frac{dC(y_{eff})}{dy_{eff}} = \frac{1}{2y_{eff}}\left(\frac{C_0}{1-\frac{10y_{eff}}{6d_0}} - C(y_{eff})\right) \\
&\frac{dU}{dt} = \left(I - U\frac{dC(y_{eff})}{dy_{eff}}v\right)/C(y_{eff}) \\
&F_e = -\frac{U^2}{2}\frac{dC(y_{eff})}{dy_{eff}}
\end{aligned} \qquad (13)$$

where C_0 is the initial capacitance without any pressure applied. To complete the model of the capacitive sensor, we are taking care of the effect of the polyimide by adding a capacitor in series with the port + with a capacitance of $C_p = \varepsilon_p \frac{\pi R^2}{t}$.

b) *For the piezoresistive sensor,*

the dynamic is easier to model because there is no electrostatic bi-directional electromechanical coupling. The influence of the deflection of the membrane on the resistors is implemented using signals as a linear resistance relative variation forced by a measured effective deflection through stress appearing in the resistors (fig. 5). The value of the radial stress in a homogenous membrane at a distance r from the center can be expressed as [8]:

$$\sigma(y_{eff}, r) = \frac{3}{4}\frac{y_{eff}k_{eff}}{\pi t^2}\left((1+\upsilon) - (3+\upsilon)\frac{r^2}{R^2}\right) \qquad (14)$$

Using the beam theory approximation, and considering that the stress is decreasing from the side to the center of the membrane, the stress in the polysilicon is expressed as relation (15)

$$\sigma_{pSi} = \frac{D_{pSi}}{D_{eff_p}}\frac{3}{8}\frac{y_{eff}k_{eff}}{\pi t^2}\left(2 - (3+\upsilon)\left(\frac{l_{pSi}}{R}\right) + \frac{(3+\upsilon)}{3}\left(\frac{l_{pSi}}{R}\right)^2\right) \qquad (15)$$

The new resistance is then computed thanks to eq. (16)

$$R_{pSi}(\sigma_{pSi}) = R_{pSi}(0)(1 + \pi_{pSi}\sigma_{pSi}) \qquad (16)$$

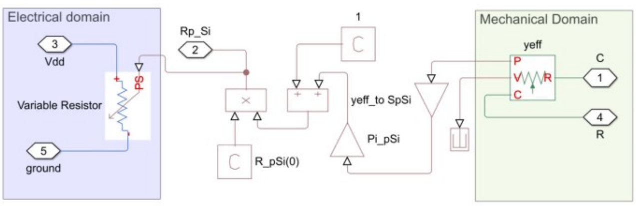

Figure 5 - Piezoresistivive transduction

III. COVENTOR MEMS+ COMPARISON

Comparisons between Simscape models and ROMs computed by MEMS+ and imported in Simulink are done in fig. 6-9. For capacitive sensing, with an applied voltage of 1V between the electrodes, we observe that the static sensitivity error stays under 7%, while the resonant frequency is evaluated in 2% of error. For the piezoresistive sensor, the static sensitivity error is less than 2%. Therefore, our models are providing fair results for both static and dynamic behavior (considering only the first resonant mode).

For the capacitive sensor, our model does not fully consider the complex electromechanical interaction due to the non-homogeneous electrostatic force that will further affect the shape of the deflection, but, in these simulations, we have assumed low levels of biasing voltages. We also assumed that the thickness of the top aluminum thin film was negligible relative to that of the membrane.

For the piezoresistive sensor, we assumed that the resistors were thin and small enough to avoid any impact on the

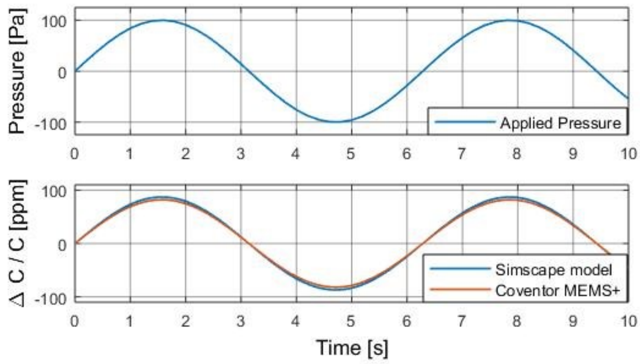

Figure 7 - Capacitive sensing: Comparison between evaluations of sensitivity

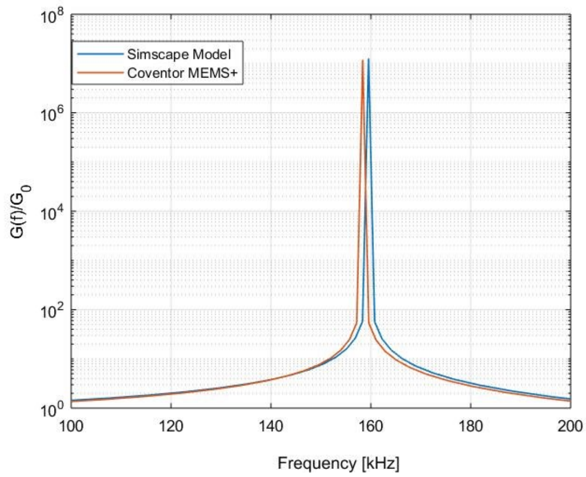

Figure 6 - Capacitive sensing: Comparison between evaluations of the first mode frequency

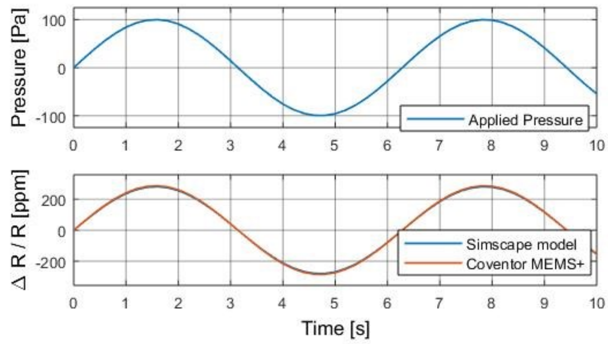

Figure 8 - Piezoresistive sensing: Comparison between evaluations of sensitivity

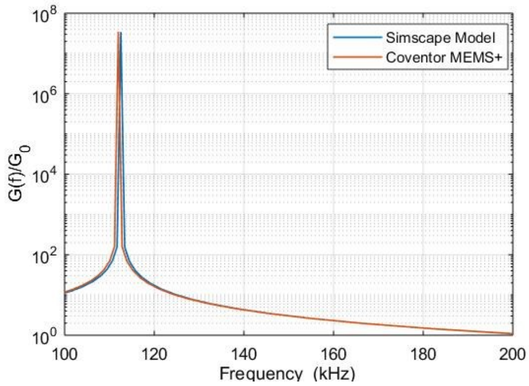

Figure 9 - Piezoresistive sensing: Comparison between evaluations of the first mode frequency

deflection, and the effect of the deflection on the resistance variation is slightly overestimated.

Comparison between Simscape and MEMS+ models time of computation was made running 100 simulations for each model, using 10 seconds as simulation stop time and variable-steps solver with maximum steps of 0.1 milliseconds (ms). In the cases of compiled models, and using the fast restart option of Simulink, the Simscape model showed shorter time of computation, 1047.3 ms, than the MEMS+ model, 1966.8 ms. The strong interest of Simscape came when multiple simulations with sweeps in parameters was performed: in this case, almost no additional time was needed with Simscape due to the change of parameters, 1051.6ms, while MEMS+ needed to build a new ROM what led to a 66094.9 ms time of computation.

IV. Conclusion

Using the energy-flow paradigm, we have built two new Simscape macro-model elements for capacitive and piezoresistive pressure sensors. Assumptions and equations used to build the models have been presented and discussed. The accuracy of our models in terms of sensitivity and first resonant mode frequency has been validated using Coventor MEMS+ simulation software.

These two new elements can directly be interfaced with circuit readout schematics for overall system level design. It has been shown that Simscape models show their real interest in term of time of computation when sweeps of parameters are needed, making Simscape a powerful tool for optimal sensing circuit design.

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