16 STATE-DEPENDENT UTILITY AND DECISION THEORY

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Contents

1 Technical Summary

Section 1 gives a technical summary of the paper. Section 2 gives a more extensive and more intuitive summary. Readers may skip either (or both!) at first reading. Readers having skipped Section 1 may still find it useful as a final summary.

(i) In games against nature with state-dependent consequence domains, the primitives are a set S of states s, a set Π of outcomes π, a set G of games g, mappings from S to Π, and a preference relation $\precsim$ on G. Conditional preferences given a state are assumed well defined, but are allowed to be state dependent. Using the framework of Anscombe and Aumann (1963) where Π is a mixture set, one only needs to relax their assumption of monotonicity into an assumption of well-defined conditional preferences given any state. One then obtains a representation theorem in terms of S linear functions on outcomes $v(\cdot)$, defined up to a common scale factor and S arbitrary origins, such that

$$f \precsim g \text{ iff} \sum_s v_s(f(s)) \leq \sum_s v_s(g(s)). \tag{1.1}$$

Given an arbitrary probability σ on S, one may rewrite the functions $v_s(\cdot)$ as $\sigma_s u_s(\cdot)$, where $u_s(\cdot) := \frac{v_s(\cdot)}{\sigma_s}$.

Thus, there exists an expected state-dependent utility representation, but the subjective probabilities are not identified from observable choices among games; correlatively, the relative units of scale and origins (ranges) of the state-dependent utilities are not identified (Section 4).

(ii) Relating the origins (levels) of the state-dependent utilities would call for observing choices among different probabilities on S for a given game g. In a context where the agent chooses among probabilities as well as state distributions of outcomes, the identification of relative units of scale is also achieved, and so is that of subjective (variable) probabilities. This is the subject matter of a theory of games with moral hazard, where unobserved strategies enable the agent to modify the probabilities of the states, see Drèze (1961, 1987).

The main complication introduced by contexts of moral hazard is: The assumption of "reversal of order" (for a lottery over games, it is immaterial preference-wise that the lottery be drawn before or after observing the state) fails; this is because the drawing of the lottery provides information which is useful to choose the best strategy. That assumption is naturally relaxed into the weaker "non-negative value of information": In a single agent context, more information cannot hurt. The theory of Anscombe-Aumann can then be extended, leading to a *generalised* representation theorem asserting the existence of a closed convex set O of probabilities σ on S, and S state-dependent utilities $u_s(\cdot)$ such that

$$f \precsim g \text{ iff} \max_{\sigma \in O} \sum_s \sigma_s u_s(f(s)) \leq \max_{\sigma \in O} \sum_s \sigma_s u_s(g(s)). \tag{1.2}$$

The set O is uniquely identified, and so are the units and origins of the utilities, if and only if O is full-dimensional, i.e. if and only if the agent believes that (s)he can influence the probability of every state—a remote possibility. Otherwise, the identification is only partial (Section 6).

(iii) When identification is incomplete, additional information equivalent to that supplied by choices among both probabilities and games must be brought in. Two constructions seek that information in hypothetical preferences among games that specify alternative assumed probabilities.

The first, due to Karni and Schmeidler (1981), was developed for games against nature; it relies on hypothetical preferences among all possible pairs (g, τ) where g is a game in G and τ is an "assumed" probability on S. These preferences are assumed to be well defined (which entails a strong test of internal consistency) and consistent with the conditional preferences derived from the observed preferences among games. There is a unique (fully identified) state-dependent expected utility representing simultaneously the actual and hypothetical preferences (Section 5).

The second, called "Conditional Expected Utility Theory" (CEUT), is due to Fishburn (1964, 1973), Pfanzagl (1968) and Luce and Krantz (1971). In a streamlined version meant to be combined with the theory of games with moral hazard, CEUT relies on hypothetical preferences among outcomes conditional on states; these are again assumed well defined, also for comparisons across different states, and compatible with the conditional preferences given a state derived from observed preferences. There is a unique (fully identified) *generalized* state-dependent expected utility representation applicable simultaneously to the hypothetical and actual preferences (Section 7).

Risk aversion with state-dependent preferences is the subject of Section 8. This is followed by applications to life insurance and the value of safety (Section 9) and by a brief general conclusion. A simplified proof of the main Theorem 6.8 is given in the appendix. Section 2 is an non-formal alternative to the present summary and Section 3 introduces the formal framework. Section 6 may be read before Section 5 at no loss of continuity.

2 Introduction, Retrospect and Preview

2.1 Retrospect: Theory

The representation of uncertainty through a set of alternative, mutually exclusive "states of the world", or "states of the environment", made its way into economic theorizing on May 13, 1952 at a symposium on "Foundations and Applications of the Theory of Risk Bearing" held in Paris at the initiative of Maurice Allais [see Centre National de la Recherche Scientifique (1953)]. On that day, Arrow (1953) presented a paper entitled "Le rôle des valeurs boursières pour la répartition la meilleure des risques", developing the framework adopted by Debreu in Chapter 7 of *Theory of Value* (1959); and Savage (1953) presented a paper entitled "Une axiomatisation du comportement raisonnable face à l'incertitude", a preview of the theory expounded in his *Foundations of Statistics* (1954). Both the Arrow-Debreu theory of general equilibrium with uncertainty, and the Savage theory of decision in games against nature, have held a center-stage status ever since. Interestingly, the Arrow-Debreu formulation relies on state-dependent preferences, whereas Savage postulates state-independent preferences. The less general formulation is the price paid for a more specific conclusion, namely subjectively expected utility.

The link between the two theories, and the motivation for the state-independence assumption in decision theory, are easily brought out. A timeless context simplifies the exposition. The event tree in Chapter 7 of *Theory of Value* then collapses to a finite set S of alternative states s. Consumption conditional on state s is a vector $x_s \in R^L$. In an alternative interpretation, used by Anscombe and Aumann (1963), there exists for each state s a set of L outcomes over which lotteries are defined by probability vectors $x_s \in R^L$. In either case, preferences are defined over vectors $x = (x_1, ..., x_s) \in R^{SL}$, and are assumed complete and continuous, so that they can be represented by a utility $U(x)$, defined up to monotone increasing transformations.[1] Under an assumption labeled "weak separability" in consumer demand theory[2], there exists a separable representation of the form

$$U(x) = f(v_1(x_1), ..., v_S(x_S)). \tag{2.1}$$

[1]In general equilibrium theory, preferences are also assumed convex, implying that $U(\cdot)$ is quasi-concave. Convexity of preferences is related to risk aversion in Guesnerie and de Montbrial (1974, Section V2).

[2]The decision-theoretic counterpart is given in Section 4 below.

Under a stronger assumption, labeled "additive separability" in consumer demand theory[3],

$$U(x) = \sum_s v_s(x_s). \tag{2.2}$$

For $\sigma \in R^S$ a strictly positive probability vector, there exists an associated expected utility representation

$$U(x) = \sum_s \sigma_s u_s(x_s) \tag{2.3}$$

where for all s $u_s(x_s) = v_s(x_s)/\sigma_s$. Such a representation always exists, as an algebraic identity, without additional assumptions. But σ is exogenous, whereas a specific goal of decision theory is to elicit endogenously a unique (subjective) probability. State-independent preferences permit such an elicitation.

State-independent preferences obtain when, for all $s \in S$, for all $a, b \in R^L$ and $\bar{x}_t (t \neq s) \in R^{(S-1)L}$, the bundle ($x_s = a, x_t = \bar{x}_t$ for all $t \neq s$) is preferred to the bundle ($x_s = b, x_t = \bar{x}_t \; \forall t \neq s$) if and only if the SL-dimensional bundle $(a, a, ..., a)$ is preferred to the bundle $(b, b, ..., b)$. If preferences are both additively separable and state independent, the functions $v_s(\cdot)$ in the additive representation can be written as $\alpha_s u(\cdot) + \beta_s$, with $\alpha_s \geq 0, \sum_s \alpha_s = 1, \sum_s \beta_s = 0$; thus,

$$U(x) = \sum_s \alpha_s u(x_s) \tag{2.4}$$

where α is a probability vector. Most importantly, α is the unique probability vector such that preferences admit a state-independent expected utility representation. Hence the probability vector is no longer exogenous: It is implied (uniquely) by the assumptions on preferences, **and** the selection of the unique[4] representation in terms of a state-independent expected utility.

The foregoing illustrates the important property that a representation in terms of a state-independent expected utility exists if and only if the S functions $v_s(\cdot)$ in (2.2) are affine transformations of each other. Whether or not that property holds can also be ascertained by eliciting cardinally the functions $v_s(\cdot)$ from conditional preferences among vectors like x_s, x'_s; that elicitation is possible when the vectors x_s define lotteries. When the property fails, the analysis must allow for state-independent preferences, as in (2.2)–(2.3).

It is also readily verified that the uniqueness of the representation as in (2.3) is obtained under either one of two conditions: (i) A probability vector

[3]Additive separability of preferences does not rule out non-linear transformations of $U(\cdot)$, but the ensuing representations are no longer additive.

[4]Up to positive affine transformations.

σ is exogenously given; (ii) the origins and units of scale, or the ranges, of the functions $u_s(\cdot)$ are exogenously given.[5]

2.2 Retrospect: Applications and Moral Hazard

Substantive application of the state-dependent expected utility model appeared in the economic literature around 1960 in relation to a specific event, namely the decision-maker's survival. That event is of methodological interest, because it illustrates vividly the impracticality of Savage's attempt at reducing indirectly the case of state-dependent preferences to that of state-independent preferences. Indeed, Savage (1954) builds his theory on preferences among state distributions of "consequences", where "a consequence is anything that may happen to a person" (loc. cit. p. 13). Thus, "being alive and poor", "being dead having bequeathed a substantial estate", etc. might describe succinctly alternative consequences. Savage assumes that *every* consequence can be associated with *every* event. For instance, he assumes that the consequence "being alive and poor" can be associated with the event "being dead". His followers have not followed him along that route[6] and prefer to face squarely the complication of state-dependent preferences.

The early applications relied on exogenous ("objective") probabilities. Write $s = 0$ for the state of death, $s = 1$ for the state of life, σ_s for the respective probabilities, w_s for wealth in state s, and $u_s(w_s)$ for the associated "utility". A parsimonious expression for expected utility, congruent with (2.3) is

$$U(w_0, w_1) = \sigma_0 u_0(w_0) + \sigma_1 u_1(w_1). \tag{2.5}$$

That parsimonious formulation proved useful to analyse two problems.

(i) *Life Insurance*. Let the agent have initial wealth levels (w_0, w_1); an insurance company offers a policy paying an indemnity $k(y)$ in case of death against a premium y, $y \geq 0$. What insurance coverage should the agent buy? Answer:

$$y \in \operatorname*{argmax}_{y \geq 0} \sigma_0 u_0(w_0 + k(y) - y) + \sigma_1 u_1(w_1 - y). \tag{2.6}$$

One can then analyse how the solution varies with the function $k(y)$, with (w_0, w_1), with properties of the functions u_0 and u_1, and so on.

(ii) *Safety Outlays*. Let the same agent (uninsured) be offered access to a safety program whereby his probability of survival can be raised to

[5]See Remark 4.11 for a more precise statement of (ii).

[6]Savage's defence is stated concisely in a letter to Robert Aumann, see Savage (1971).

$1-\sigma_0(x)$ at a cost x, with $\sigma_0(x) \leq \sigma_0(x')$ whenever $x \geq x'$. What should the agent spend on this program? Answer:

$$x \in \underset{x \geq 0}{\operatorname{argmax}} \, \sigma_0(x) u_0(w_0 - x) + [1 - \sigma_0(x)] u_1(w_1 - x). \tag{2.7}$$

One can then analyse how the solution varies with (w_0, w_1), with properties of the function $\sigma_0(x)$, with properties of u_0 and u_1, and so on.

The life-insurance problem is a standard example of choice among "acts" and fits squarely into the framework of either general equilibrium theory or decision theory as sketched in Section 2.1. On the other hand, the safety-outlays problem does not fit into that common framework, which covers "games against nature" but not situations where the "likehood" of an event is affected by the agent's decisions – situations labeled "moral hazard" in the insurance literature.

It is interesting to write down the (first-order) conditions for these two problems, when the relevant functions are differentiable, and the non-negativity constraints are not binding. We write k' for $\frac{dk}{dy}$, u'_s for $\frac{du_s(w_s)}{dw_s}$, and σ'_0 for $\frac{d\sigma_0}{dx}$. The conditions are:

$$(i^*) \qquad \frac{(1-\sigma_1)u'_0}{\sigma_1 u'_1 + (1-\sigma_1)u'_0} = \frac{1}{k'};$$

$$(ii^*) \qquad \sigma'_0 = \frac{\sigma_0 u'_0 + (1-\sigma_0)u'_1}{u_0(w_0) - u_1(w_1)}.$$

Condition (i^*) is unaffected, identically in (w_0, w_1), if σ_1 is replaced by $\lambda\sigma_1 \in (0,1)$ and simultaneously $u_1(w_1)$ is replaced by $\frac{1}{\lambda}u_1(w_1), u_0(w_0)$ is replaced by $\frac{1-\sigma_1}{1-\lambda\sigma_1}\, u_0(w_0)$. This is the identification problem of subjective probabilities under state-dependent preferences: Choices among realistic alternatives in games against nature (like insurance policies) do not permit separate identification of subjective probabilities and the units of scale of the utility functions associated with alternative states; only their product can be meaningfully elicited from observable choices among alternative prospects (among "acts" in the Savage terminology). Also condition (i^*) is unaffected if distinct constant terms are added to u_0 and u_1.

Look now at the first-order condition (ii^*) for the safety-outlays problem. If we replace σ_0 by $\lambda\sigma_0$ and rescale u_0, u_1 as above, we now obtain

$$\lambda\sigma'_0 \neq \frac{\sigma_0 u'_0 + (1-\sigma_0)u'_1}{\frac{1}{\lambda}u_0(w_0) - \frac{1-\sigma_0}{1-\lambda\sigma_0}\, u_1(w_1)}.$$

Therefore, if products like $\sigma_0 u_0(\cdot)$ have been elicited, say from conditional preferences given state 0, separate identification of σ_0 and u_0 can be attempted from observed behavior in games with moral hazard. Also the denominator in (ii^*) would change if distinct constant terms were added to u_0 and u_1. The identification of state-dependent utilities is complete. Games with moral hazard thus belong naturally in a theory aiming at elicitation *from observed behavior* of subjective probabilities and state-dependent utilities.

2.3 One-Person Games with Moral Hazard

The context of games with moral hazard is of necessity more complex than that of games against nature, and this calls for a substantive extension of the theory; such an extension is reviewed in Section 6 for a specific framework; namely a framework where the variations in the "likelihood" of alternative states result from *unobserved* strategies[7] of the agent (rather than say, from observable safety outlays). This (more difficult) framework leads, under minimal behavioral assumptions, to the elicitation from observable decisions, not of a single probability vector σ, but of a closed convex set O of probability vectors on S such that

$$U(x) = \max_{\sigma \in O} \sum_s \sigma_s u_s(x_s). \tag{2.8}$$

When O is a singleton, (2.8) reduces to (2.4). Otherwise, (2.8) extends the theory of games against nature on two scores:

(i) It covers games with moral hazard, extending naturally the criterion of expected utility maximisation to the choice of a strategy (of a probability vector) $\sigma \in O$;

(ii) it provides a partial solution to the problem of separate identification of subjective probabilities and state-dependent utilities.

To understand (ii), consider the case where the probability of each state can be modified through some strategy available to the agent (the set O has a non-empty interior relative to the unit simplex of R^S). If an act $\bar{x}$ is such that its expected utility is invariant to the choice of a probability $\sigma \in O$, then it must be the case that $u_s(\bar{x}_s) = u_t(\bar{x}_t)$ for all $s, t \in S$; i.e. x is a "constant-utility" act. *Constant-utility acts play the same role under state-dependent preferences that constant acts play under state-independent preferences.*[8] Specifically, if two constant-utility acts have been identified, both the units of scale and the origins of the conditional utility functions $u_s(\cdot)$ and $u_t(\cdot)$ are uniquely determined. Hence, the identification problem is solved (see last paragraph of Section 2.1).

[7]The fact that strategies are not observed justifies the terminology "games with moral hazard".

[8]Constant games assign the same consequence to every state.

How does one identify constant-utility acts? The property that all available strategies are equally attractive for such acts can be elicited by withholding information about which act is the relevant one. A simple example should make the principle clear. Carmina Burana will be performed tonight in the agent's town. He is eager to attend, even though tickets sell for \$ 60. Write $s = 1$ for "agent attends concert", $s = 2$ for "agent does not attend concert", and w_s for wealth in state s. We would like to elicit a \$ amount y such that $u_1(w_1 - 60) = u_2(w_2 + y)$. To that end, define an act whereby the agent receives a prize of \$ z if $s = 2$ occurs. It stands to reason that for small z, the agent will prefer to attend the concert and forego the prize, whereas for large z he will forego the concert and collect the prize. There should thus exist an amount y such that the agent will attend the concert if $z > y$, will forego the concert if $z < y$, and will be indifferent if $z = y$. *Let then the toss of a coin decide whether the prize is equal to 0 or to z*, and consider two alternative information sequences:

(i) The coin is tossed today (before the concert);

(ii) the coin is tossed tomorrow (after the concert).

The agent should be indifferent between (i) and (ii) whenever $z \leq y$, planning to attend the concert anyhow; but should strictly prefer (ii) over (i) when $z > y$, planning to attend if the coin toss delivers a price of 0, not to attend if the price is $z > y$. In this way, the amount y is revealed from observable choices, the origins and units of scale of u_1 and u_2 can be ascertained and the identification problem is solved.

The generality of this approach is established in Section 6. Its realm of application remains limited, of course, by the extent to which states are subject to moral hazard. For states lying entirely outside the control of the agent, like the weather or macroeconomic realisations, this approach is of no use.

2.4 Motivation and Organisation

The two illustrations given in Section 2.2 invite an obvious (trivial) conclusion. Observing behavior in a class of decision situations and interpreting the observations within a suitable theoretical framework should permit elicitation (identification) of the parameters *needed to predict behavior in similar situations—neither more, nor less.* (By "similar situations", we mean decision problems with the same logical structure—say, games against nature or games with the same strategy set.)

Why then be concerned with the identification of subjective probabilities in a context of games against nature? Some authors declare themselves unconcerned;[9] Yet, there are two good reasons to be concerned.

- To go beyond prediction. In problems of medical decision (like whether or not to perform a risky operation), one may wish to evaluate alternatives in terms of the patient's state-dependent utility and the doctor's subjective probabilities. Hence the need for separate elicitation.

- "To police one's own decisions for consistency and, when possible, to make complicated decisions depend on simpler ones".[10] This is the "introspective" use of decision theory, where the subjectively expected utility theorem is particularly helpful in separating utility and probability considerations and in bringing probability calculus to bear on decision problems.

Savage himself was somewhat ambivalent, in that he stressed both the introspective use of the theory[11] and the "great importance that preferences, and indifference, ... be determined at least in principle by decisions between acts and not by response to introspective questions".[12] The important difference is that "acts" carry material consequences for the agent, whereas introspective questions (or hypothetical preferences) do not.

We are now in a position to explain the organisation of our paper. We shall deal successively with games against nature, then with games involving moral hazard. A theoretical framework suitable for both pursuits is introduced in Section 3. The theory of games against nature, leading to the representation (2.2), is reviewed in Section 4. We then discuss in Section 5 suggestions for exogenous calibration of probabilities or utility scales, on the basis of hypothetical preferences. That section can be read indifferently before or after Section 6. In Section 6, we turn to games with moral hazard and explain how they provide a partial answer to the identification problem. We then review in Section 7 suggestions for completing exogenously the partial identification obtained in Section 6. Risk aversion with state-dependent preferences is the subject of Section 8. Section 9 reviews briefly the application of the model to life insurance and safety outlays (value of life). A general conclusion is formulated in Section 10. An appendix supplies a proof of the main theorem in Section 6.

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[9] See, e.g. Rubin (1987).

[10] Cf. Savage (1954, p. 20).

[11] Cf. Savage (1954, pp. 19–21).

[12] Cf. Savage (1954, p. 17).

Throughout, we make no attempt at comprehensiveness of either results or references,[13] and concentrate on basic issues. Except for the appendix, no proofs are given.

3 A General Framework

As a rule we denote a set with a capital roman letter, an algebra of sets with the corresponding script capital letter.

The set of **states** of the world is denoted by S, a finite set; the set of all subsets of S is, according to our convention, $\mathcal{S}$. We generally follow the approach of Anscombe and Aumann, as in Drèze (1987).

There is a finite set of **prizes** denoted by P. They play the role of what Savage calls **consequences** in the *Foundations*. Π is the set of probability measures on P. For any $\alpha \in [0,1]$, and any pair $\pi, \rho \in \Pi$ we write $\alpha\pi + (1-\alpha)\rho$ simply as $\pi\alpha\rho$.

A **game** is a mapping from S to Π. So games replace the acts of the framework of Savage.[14] The set of games is denoted by G; so $g(s) \in \Pi$ is the prize mixture associated by the game g with the state of the world s. For any set $A \subset S$ and any game g, g_A denotes the restriction of g to A. A game g' such that $g'(s) = \pi, g'(t) = g(t)\ \forall t \in S \setminus s$ will be denoted $(\pi, g_{S\setminus s})$. A **constant game** simply assigns the same prize mixture to every state.

A simple probability measure γ on the set G is called a **lottery**, with the set of lotteries denoted by Γ. So given a lottery γ a random device chooses a game g according to γ. Of course the timing of information is essential here; in particular we have two essentially different situations if the true state is observed by the decision maker before or after the lottery has been drawn and the game determined. This difference becomes essential because we want to allow for moral hazard in the decision problem: **Moral hazard** denotes here any situation in which the decision maker can influence the course of events, *i.e.* he can choose from some set of probabilities over the state space.

Consider a lottery γ. If the draw of the *lottery* is postponed until after the state is revealed, then we have a naturally defined *game* which for each state gives a probability mixture on prizes equal to the average over games weighted by γ. Formally we have:

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[13] A systematic treatment of hypothetical preferences in games against nature, with application to life insurance, is given in Karni (1985).

[14] The use of the word "games" is motivated by the context with moral hazard of Sections 6 and 7 and Section 9.2. It is used throughout for unity. In Sections 4 and 5, games are an exact equivalent of acts.

DEFINITION 3.1 The game g_γ corresponding to the lottery γ is defined by

$$g_\gamma(s) = \sum_G \gamma(g)g(s)$$

for any $s \in S$.

The primitive **preference order** of the decision maker is assumed to be defined over lotteries, and is denoted by $\precsim$. It induces in a natural way an order on G, Π, and P, respectively, as follows. Let δ_x denote the probability with mass concentrated at x. Then for $f, g \in G$ we say $f \precsim g$ if and only if $\delta_f \precsim \delta_g$. Then for $\mu, \nu \in \Pi$ we say $\mu \precsim \nu$ if and only if for the two constant games f and g giving μ and ν respectively one has $f \precsim g$. Finally for $p, q \in P$ we say $p \precsim q$ if and only if $\delta_p \precsim \delta_q$. When $\gamma \precsim \gamma'$ and $\gamma' \precsim \gamma$ we say γ is indifferent to γ' and write $\gamma \sim \gamma'$.

We maintain throughout without reminder a standard set of assumptions on the preference order:[15]

ASSUMPTION 3.2 (Weak Order) The preference order $\precsim$ is a weak order; that is:

(i) For all $\gamma, \gamma' \in \Gamma$, either $\gamma \precsim \gamma'$ or $\gamma' \precsim \gamma$;

(ii) for all $\gamma, \gamma', \gamma'' \in \Gamma$, if $\gamma \precsim \gamma'$ and $\gamma' \precsim \gamma''$, then $\gamma \precsim \gamma''$.

ASSUMPTION 3.3 (Independence) For all $\gamma, \gamma', \gamma'' \in \Gamma$ and for all $\alpha \in (0, 1]$, $\gamma\alpha\gamma'' \precsim \gamma'\alpha\gamma''$ if and only if $\gamma \precsim \gamma'$.

ASSUMPTION 3.4 (Continuity) For all $\gamma, \gamma', \gamma'' \in \Gamma$, if $\gamma \prec \gamma'$ then there is an $\alpha \in (0, 1]$ such that $\gamma\alpha\gamma'' \prec \gamma'$.

The following classical theorem establishes the existence of a utility function V defined on the set of games.[16]

THEOREM 3.5 Assume Weak Order (3.2), Independence (3.3), and Continuity (3.4); then there exists a real valued function V defined on G, which is unique up to positive affine transformations, such that $\gamma \precsim \gamma'$ if and only if

$$\sum_G \gamma(g)V(g) \le \sum_G \gamma'(g)V(g). \tag{3.1}$$

[15] Cf. von Neumann and Morgenstern (1944).

[16] Cf. e.g. Fishburn (1970).

4 Games Against Nature

The following definitions and assumptions are standard.[17]

DEFINITION 4.1 (Conditional Preferences) For all $s \in S$, and all $\pi, \rho \in \Pi$, we say that

(i) $\pi \prec \rho$ given s if there exists a $g \in G$, such that $(\pi, g_{S\setminus s}) \prec (\rho, g_{S\setminus s})$;

(ii) $\pi \sim \rho$ given s if and only if neither $\pi \prec \rho$ given s nor $\rho \prec \pi$ given s.

DEFINITION 4.2 (Null State) A state s is null if and only if for all $\pi, \rho \in \Pi$ one has $\pi \sim \rho$ given s.

ASSUMPTION 4.3 (Conditional Preferences) For all $s \in S$, for all $\pi, \rho \in \Pi$, if $\pi \prec \rho$ given s then it is not true that $\rho \prec \pi$ given s.

The next assumption simply requires that the problem we are discussing is not trivial.

ASSUMPTION 4.4 (Non-Degeneracy) It is not true that $f \precsim g$ for all $f, g \in G$.

LEMMA 4.5 Assume Non-Degeneracy (4.4) and Conditional Preferences (4.3); then there exist S real-valued linear functions v_s defined on Π, each of them unique up to positive affine transformations, and a real-valued function F from R^S to R, such that for all $g \in G$

$$V(g) = F(v_s(g(s))_{s=1,\dots,S}). \tag{4.1}$$

The next assumption rules out the possibility of moral hazard: It is indifferent to know the outcome of a lottery before or after the true state is observed.

ASSUMPTION 4.6 (Reversal of Order) Every lottery is indifferent to the corresponding game; that is for all $\gamma \in \Gamma$, $\gamma \sim g_\gamma$.

THEOREM 4.7 (Additive Utility) Assume Non-Degeneracy (4.4), Conditional Preferences (4.3) and Reversal of Order (4.6); then there exist S real-valued linear functions v_s defined on Π such that, for all $g \in G$

$$V(g) = \sum_S v_s(g(s)). \tag{4.2}$$

DEFINITION 4.8 (Cardinal Unit-Comparable Transformation) Let $\{v_s(\cdot)\}$ and $\{w_s(\cdot)\}$ be two sets of functions from Π to $R, s = 1, ..., S$; if there exist a positive constant $c > 0$ and S real numbers d_s, such that for all $s = 1, \cdots, S$,

[17]Almost—Definition 4.1 and Assumption 4.3 are usually stated in terms of *events* rather than *states*; an explanation for our (weaker) option is given in Remark 6.13.

$w_s(\cdot) = cv_s(\cdot) + d_s$, then the sets of functions $\{v_s(\cdot)\}$ and $\{w_s(\cdot)\}$ are said to be related by a cardinal unit-comparable transformation.

THEOREM 4.9 Assume the sets of functions $\{v_s(\cdot)\}$ and $\{w_s(\cdot)\}$, $s = 1, ..., S$, both verify (4.2); then these two sets of functions are related by a cardinal unit-comparable transformation.

The identification problem corresponds to the possibility of expressing each function $v_s(\cdot)$ as a product $\sigma_s u_s(\cdot)$, with $u_s(\cdot) \equiv \frac{v_s(\cdot)}{\sigma_s}$, for arbitrary σ (a probability on S).

REMARK 4.10 The functions $v_s(\cdot)$ defined on Π are linear, because elements of Π are probability vectors (lotteries) on the finite set P of prizes. In many applications, the set P is an interval of R (e.g. wealth levels), and the functions $v(\cdot)$ are defined directly on P, up to a positive affine transformation (cardinally).

REMARK 4.11 In Lemma 4.5 and Theorem 4.9, two nested levels of determinacy appear. In equation (4.1) each function $v_s(\cdot)$ could be replaced by $c_s v_s(\cdot) + d_s, c_s > 0$, with the function F suitably adjusted. In equation (4.2), each function $v_s(\cdot)$ could be replaced by $cv_s(\cdot) + d_s, c > 0$. A third level of determinacy would only allow replacing $v_s(\cdot)$ by $cv_s(\cdot) + d,\ c > 0$. The three types of allowable transformations are labeled "state-dependent positive affine transformations", "cardinal unit-comparable transformations"[18] and "positive affine transformations" respectively.

For a given set P, each function $v_s(\cdot)$ is defined on a compact domain (the set Π of lotteries on P) and has range in R – say $\mathcal{R}_s$, for $v_s(\cdot)$. These ranges are specific to the domain defined by P. Extensions of P may result in extensions of $\mathcal{R}_s,\ s = 1, ..., S$.

When state-dependent affine transformations are allowed, the ranges $\mathcal{R}_s$ are arbitrary (undefined). When only cardinal unit-comparable transformations are allowed, each range $\mathcal{R}_s$ may be translated by a constant d_s; in addition, the S ranges may be extended ($c > 1$) or contracted ($c < 1$) in the same proportion c. We then say, informally, that the (relative) origins of the functions $v_s(\cdot)$ are undefined, but their (relative) units of scale are uniquely defined. When only affine transformation are allowed, both (relative) origins and (relative) units of scale are uniquely defined.

In later developments, these definitions will be applied to utilities $u_s(\cdot)$ related to the functions $v_s(\cdot)$ by probabilities σ, with $\sigma_s u_s(\cdot) \equiv v_s(\cdot)$ for all s.

[18]This term is used by Karni and others (1983).

5 Hypothetical Preferences

Observed preferences among games against nature (acts) cannot take us further than Theorem 4.7, thus leaving subjective probabilities as well as relative origins and units of scale of state-dependent utilities unidentified. As indicated in Section 2.4, identification is not required to predict further choices among acts, but it may be essential to other pursuits. To go beyond the conclusion of Theorem 4.7, other kinds of information must be brought it.

In what follows "the agent" is either the investigator herself exploring introspectively her beliefs and preferences, or another person whose beliefs and preferences are of interest to the investigator.

Asking the agent outright what are her subjective probabilities, or how her utility scales relate across states, is a natural next step. This approach is of little help to the agent herself, and may be unconvincing to the investigator if there is no way of checking the consistency and accuracy of the response.

A richer approach has been introduced in the literature by Karni and Schmeidler (1981),[19] hereafter KS, followed by Karni, Schmeidler and Vind (1983), hereafter KSV, and Karni (1985). These authors suggest eliciting *hypothetical* preferences, conditional on *assumed* values for the probabilities of the states. (The same idea can be applied to assumed specifications of utilities—see Remark 5.8 below.)

The basic idea is to confront the agent with hypothetical choices among games defined not only through the prizes associated with alternative states but also through "assumed" probabilities assigned to these states.[20] The hypothetical preferences are used to elicit origins and units of scale for the state-dependent utilities. Unique subjective probabilities follow. Implementation of this idea calls for three steps: (i) Defining the experiment; (ii) verifying the internal consistency of hypothetical preferences; (iii) verifying the consistency between hypothetical and actual preferences. Steps (ii) and (iii) are axiomatic.

We review the three steps, relying recurrenlty on the recent contribution by Karni and Mongin (1997). Our description of the experiment is slightly different from that of KS and KSV, but the difference is of no analytical consequence. We explain the difference in Remark 5.6 below.

(i) Let $\Delta^S = \{\tau \in R_+^S \mid \sum_s \tau_s = 1\}$ define the set of probabilities on S. The experimenter wishes to confront the agent with choices among hypothetical games $f^\tau, g^\tau, ...$, where f^τ is to be understood as "the game f on the assumption that state s has probability τ_s, $s = 1, ..., S$." Let

[19]This paper has never been published, but its content is reproduced in Karni (1985), Sections 1.6–1.8 and 1.12.

[20]For example: Assuming that states s and t were equally probable, would you rather stake a given prize on s or on t?

G^τ denote the set of games with probability $\tau \in \Delta^S$ and $\precsim^\tau$ denote the reported preferences over G^τ. These preferences are introduced as a primitive.

(ii) Assume that $\precsim^\tau$ on G^τ ($\tau \in \Delta^S$ but otherwise arbitrary) satisfies the von Neumann-Morgenstern (VNM) axioms 2.2–2.4, conditional preferences 4.3, non-degeneracy 4.4 and reversal of order 4.6, i.e. the assumptions of Theorem 4.7. Applying that theorem to $\precsim^\tau$, there exist a function V^τ, defined up to positive affine transformations, representing $\precsim^\tau$, and S functions v_s^τ defined up to a cardinal unit-comparable transformation, such that

$$V^\tau(f) = \sum_s v_s^\tau(f(s)) \doteq \sum_s \tau_s u_s^\tau(f(s)). \tag{5.1}$$

Internal consistency means that, for all τ, $\tau' \in \Delta^S$, for all $f \in G$, $u_s^\tau(f(s)) = u_s^{\tau'}(f(s))$—possibly after a suitable unit-comparable transformation. For a convenient axiomatic formulation of that condition, let $\delta_s \in \Delta^S$ denote the probability with mass concentrated at s.

Then:

Assumption 5.1 (Internal Consistency of Hypothetical Preferences) For all $f \in G$, $\tau \in \Delta^S$, f^τ is indifferent to the lottery assigning probability τ_s to f^{δ_s}, $s = 1, ..., S$.

It follows from Assumption 5.1 and Theorem 4.7 that, for all $f \in G$, $\tau \in \Delta^S$:

$$V^\tau(f) = \sum_s \tau_s V^{\delta_s}(f). \tag{5.2}$$

Write G' for $\cup_{\tau \in \Delta^S} G^\tau$ and $\precsim'$ for the preference on G' induced by $\precsim^\tau$. Imposing the assumptions of Theorem 4.7 on $(G^\tau, \precsim^\tau)$ for all $\tau \in \Delta^S$ and Assumption 5.1 is equivalent to imposing the assumptions of Theorem 4.7 on $(G', \precsim')$. The implication is the following representation theorem:

Theorem 5.2 Assume 3.2–3.4, 4.3, 4.4 and 4.6 on $(G', \precsim')$; there exist S linear functions $u_s : \Pi \to R$, defined up to a *common* positive affine transformation, such that, for all $f \in G$ and $\tau \in \Delta^S$,

$$V^\tau(f) = \sum_s \tau_s u_s(f(s)) = \sum_s \tau_s V^{\delta_s}(f). \tag{5.3}$$

(iii) Consistency between hypothetical and actual preferences means that the functions $u_s(\cdot)$ of Theorem 5.2 represent, for each s, the conditional preferences derived from the unconditional preferences among games.

Assumption 5.3 (Consistency of Hypothetical and Derived Conditional Preferences) For all $s \in S,\ f,\ g \in G$, if $f^{\delta_s} \precsim^{\delta_s} g^{\delta_s}$, then it is not true that $g \prec f$ given s.

Under Assumption 5.3, there exist for each s real coefficients $\alpha_s > 0$ and β_s relating the functions $v_s(\cdot)$ of Theorem 4.7 and $u_s(\cdot)$ of Theorem 5.2, i.e. such that $v_s(\cdot) = \alpha_s u_s(\cdot) + \beta_s$.

Because $V(f)$ is defined up to a positive affine transformation, we may choose a normalisation such that $\sum_s \alpha_s = 1,\ \Sigma_s \beta_s = 0$. It follows that, for all $f \in G$,

$$V(f) = \sum_s \alpha_s u_s(f(s)), \alpha_s \in \Delta^S, \tag{5.4}$$

where α is the *unique* element in Δ^S verifying (5.4), for given functions $v_s(\cdot)$ and $u_s(\cdot)$. The vector α is thus the unique probability vector reconciling the hypothetical and actual preferences in a common subjectively expected utility representation.

Theorem 5.4 (Karni and Schmeidler) Impose the assumptions of Theorem 4.7 on $\precsim$ and on $\precsim'$, and assume 5.3. There exist a unique vector $\sigma \in \Delta^S$ and s linear functions $u_s(\cdot) : \Pi \to R$ defined up to a common positive affine transformation, such that for all $f \in G$:

$$(i) V(f) = \sum_s \sigma_s u_s(f(s))$$

$$(ii) \text{ for all } \tau \in \Delta^S,\ V^\tau(f) = \sum_s \tau_s u_s(f(s)).$$

The functions $u_s(\cdot)$ (implying the probabilities σ) are the *only ones* (up to the common affine transformation) capable of representing *simultaneously* the actual and hypothetical preferences.[21]

Remark 5.5 Still, if one allowed the actual and hypothetical preferences to be represented by *different* state-dependent utilities (different yet related by state-dependent affine transformations), the identification problem would remain unsolved.

[21] It follows from Theorem 5.4 that $\precsim$ and $\precsim^\sigma$ agree—a property called "stability" by Karni and Mongin (1997).

Thus let the "true" (whatever that means) beliefs and preferences of the agent be defined by $v_s(\cdot) = \sigma_s w_s(\cdot),\ \sigma \in \Delta^S$. Let the agent express consistent hypothetical preferences represented by $u_s(\cdot) = \phi_s w_s(\cdot) + \psi_s \doteq \frac{\phi_s}{\sigma_s}\ v_s(\cdot) + \psi_s$, so that $v_s(\cdot) = \frac{\sigma_s}{\phi_s}[u_s(\cdot) - \psi_s]$. Unless $\phi_s = \phi_t$ for all $s, t, \in S$, the vector α of (5.4) will be different from σ. Yet all the assumptions of Theorem 5.4 are verified. This is of course a far-fetched possibility. It reminds us that transferring properties elicited form *hypothetical* preferences into a particular representaion of *actual* preferences always entails on act of faith, not amenable to empirical testing (not amenable to a test with material consequences for the agent). The merit of the KS approach reviewed above lies in having reduced the margin of faith to its incompressible limit.

REMARK 5.6 As noted above, our description of the experiment is slightly different from that of KS and KSV, though we hope not to have misrepresented their construction. These authors formalise the notion "on the assumption that state s has probability τ_s" by defining hypothetical objects of choice, say $\hat{f}^\tau$, (called "acts " in KSV, "state-outcome lotteries" in KS) as follows: $\hat{f}^\tau(s) \in \{\pi^\tau | \sum_{p \in P} \pi_s^\tau(p) = \tau_s\}$. They consider the set $\hat{G}^\tau$ of these objects $\hat{f}^\tau$ and the hypothetical preferences order over $\hat{G}^\tau$ (KSV) or $\hat{G}' = \cup_{\tau \in \Delta^S} \hat{G}^\tau$ (KS). The interpretation of these objects and the intuitive meaning of preferences among them are discussed by Karni and Mongin.[22]

We have chosen to rely on the concepts and definitions of Sections 3–4, for unity of exposition. In experiments involving hypothetical preferences, it is up to the experimenter to appraise which specific description of the experiment is apt to prove most fruitful. The acid test that the notion "on the assumption that state s has probability τ_s" is well understood by the subject comes with the verification of (5.3).

REMARK 5.7 The KSV analysis differs from the KS analysis by omitting Assumption 5.1—technically, by imposing the assumptions of Theorem 4.7 on $(G^\tau, \precsim^\tau)$ for a *single* τ (with $\tau_s > 0$ for all s), rather than on $(G', \precsim')$. There is thus no guarantee that the elicitation of utilities is invariant to the assumed probabilities τ—or, for that matter that expressed hypothetical preferences take correctly into account the assumed probabilities τ. The acid test (5.3) is not available. Also, the origins of the state-dependent utilities $u_s(\cdot)$ are not identified (not uniquely related)—whereas they are in the KS analysis.

REMARK 5.8 States s that are null for $\precsim$ are states such that there do not exist f and g with $f \prec g$ given s or $g \prec f$ given s. Such a situation may arise in

[22]Cf. Karni and Mongin (1997, pp. 7 and 23).

two ways: (i) $\sigma_s = 0$; (ii) $u_s(\cdot)$ is a constant function. Whether or not $u_s(\cdot)$ is a constant function is a property of the hypothetical preferences, given any τ with $\tau_s > 0$. Thus, conditions under which $\sigma_s = 0$ are well defined, and there is no need to single out null states in the statement of Theorem 5.4. (KS proceed differently, but we do not see the need for their qualification.) Note also that a constant $u_s(\cdot)$ still has a well-defined (relative) origin.

Remark 5.9 The same idea applied to hypothetical utilities would lead the investigator to tell the agent: "Assume that you were advising a principal who wishes to maximise expected profit; would you rather advise that principal to stake a prize on the occurrence of state s, or on a roulette spin with probability of success ϕ?" Assume that a probability ϕ is found for which the hypothetical bets are declared indifferent. The investigator could then conclude that the agent's subjective probability for state s is equal to ϕ. The probabilities so elicited could be compared with those obtained from the KS approach, and consistency between the two could be axiomatised. Again, there would remain to validate the conclusion through variations of the hypothetical experiment.

Remark 5.10 The KS construction reveals that hypothetical preferences can be put to a strong consistency test, defined by Assumptions 5.1 and 5.3. If an agent has passed that test successfully in an extensive experiment, are there grounds to view the implied probability σ with suspicion? We would be hard put to propose a solid ground, Remark 5.5 notwithstanding.

The question has been given an interesting twist by Karni and Mongin.[23] Suppose that an agent's choices among acts admit a state-independent expected utility representation (2.4), with probability vector α. Let the agent express hypothetical preferences, leading to the state-dependent expected utility representation (5.4) with probabilities $\alpha' \neq \alpha$. If you regard the agent as an expert (e.g. a medical doctor), and wish to rely on his probability judgements, would you trust α or α'? The case (presented by Karni and Mongin) for relying on α', which is based on more information, seems well taken, especially when the hypothesis of state-dependent preferences is plausible (as might be the case for states differing with regard to the success of an operation performed by that same doctor).[24] Conversely, if hypothetical preferences revealed that utility depends upon whether a roulette bet was won on red or black, suspicion would unavoidably creep in.

[23] Cf. Karni and Mongin (1997, Section 4.2).

[24] This example might suggest to some that the "farfetched" possibility mentioned in Remark 5.5 might occasionally prove genuine...

The methodological stand of Karni and Mongin can also be applied to an alternative suggestion of Karni (1993), who proposes eliciting subjective probabilities under state-dependent preferences from a representation in terms of utilities with a state-independent *range*. That representation would be dominated by the KS elicitation on the very ground given in the previous paragraph. (State-independent ranges is an exogenous normalisation, no more, and generally less compelling than state-independent preferences.) With that conclusion, it is hard to disagree.

6 Games with Moral Hazard

As argued in the introduction, the presence of moral hazard is signalled by violation of the "Reversal of order" assumption. When the agent responds to prizes also by adjusting, through unobserved strategies, the probabilities of the states,[25] then the agent will value early information about the state-distribution of prizes. A lottery (drawn before the state is observed) will be preferred to the corresponding game (which entails the same marginal probabilities, but with the lottery drawn after the state obtains). In a single agent decision problem, information cannot hurt. So a natural weakening of the reversal of order assumption is obtained by giving a non-negative value to information: The knowledge of the outcome of the lottery may in some cases help in choosing a better strategy. Formally:

ASSUMPTION 6.1 (Value of Information) Every lottery is preferred or indifferent to the corresponding game; that is for all $\gamma \in \Gamma$, $g_\gamma \precsim \gamma$.

It may happen that for two particular games f and h there is no advantage to knowing in advance the outcome of the special lottery *which involves only these two games*. We make this idea formal in:

DEFINITION 6.2 (Equipotence) Two games f and h are equipotent, written fEh, if and only if for every $\alpha \in [0,1]$ the lottery among them is indifferent to the corresponding game, that is: fEh if and only if $f\alpha h \sim g_{f\alpha h}$, for all $\alpha \in [0,1]$.

The relation defined by E is reflexive and symmetric, but not transitive. Intuitively, two games are equipotent if there exists a strategy simultaneously optimal for both. So sets of equipotent games cannot be defined as equivalence classes. This makes the following definition necessary:

[25] This response is in the spirit of "consequentialism", as defined in Hammond (1999).

Definition 6.3 (Equipotent Set) A subset G' of G is called an equipotent set if every lottery among elements in G' is indifferent to the corresponding game.

The lotteries of interest in the Definition 6.3 are not limited to pairwise lotteries.

A special case will help to understand the concept of equipotence, namely the case in which one of the two games, f say, is a constant-utility game. Consider the game corresponding to a lottery between f and g alone. In contemplating the choice of a strategy for that game, the decision maker can ignore f, because if f will obtain his action (strategy) will have no relevant effect; and so he can choose the best action *as if* g were the relevant game. In this case the information about the outcome of the lottery is irrelevant and the two games are equipotent. Notice that in this example the properties of g are irrelevant; the two games are equipotent because f is a constant-utility game. This singles out a special class of games, which are equipotent to any other game.

Definition 6.4 (Omnipotent Games) The game f is omnipotent if and only if it is equipotent to any other game; *i.e.*, iff fEh for every $h \in G$.

Omnipotent games play an important role in the sequel. Accordingly, we strengthen the non-degeneracy Assumption 4.4 into:

Assumption 6.5 (Existence of Omnipotent Games) There exist at least two omnipotent games, g_0 and g_1 with $g_0 \prec g_1$ and, for every s not null, $g_0 \prec [g_1(s), g_0(S \setminus s)]$.

The added complication introduced by moral hazard, namely by the possibility that $f\alpha h \prec g_{f\alpha h}$, is that the assumptions of independence and continuity of preferences among *lotteries* do *not* carry over automatically to the corresponding *games*. (Reversal of order entailed that property in Section 4.) Continuity, being a local property, is not a problem. It simply needs to be listed as a separate assumption.[26]

Assumption 6.6 (Continuity of Conditional Preference) For all $\pi, \rho, \psi \in \Pi$, for all $s \in S$, if $\pi \prec \rho$ given s, there exists $\alpha \in [0,1)$ such that $\pi\alpha\psi \prec \rho$ given s and $\pi \prec \rho\alpha\psi$ given s.

[26]Drèze (1987) enunciates our Assumption 6.6 in the text, then adds that it is sufficient to formalise that assumption for games of the form $[g_1(s)\alpha\pi, g_0(S \setminus s)]$. However, the general formulation is used implicitly in the proof of Corollary 8.1 there. We correct that oversight in our formulation. The logical content is equivalent.

Independence, however, is a global property. [It is imposed in Assumption 3.4 for every γ'' and $\alpha \in (0,1)$.] The saving grace is that the assumption carries over to equipotent sets. Do such sets exist? An additional, weak and intuitively plausible, assumption stipulates how a specific equipotent set can be associated with every non-null state, on the basis of the omnipotent games g_0 and g_1 of Assumption 6.5.

Consider the omnipotent game g_0, for which (reasoning intuitively) all unobserved strategies are equally good. Suppose now that g_0 is modified on s alone to replace there $g_0(s)$ by a preferred outcome, like $g_1(s)$. It stands to reason that a best strategy for $[g_1(s), g_0(S \setminus s)]$ should maximise the probability of s, over the set of achievable probabilities. And such a property should hold not only for $[g_1(s), g_0(S \setminus s)]$ but also for any game $[\pi, g_0(S \setminus s)]$ such that $g_o(s) \prec \pi$ given s, i.e. such that $g_O \prec [\pi, g_0(S \setminus s)]$.

We introduce this requirement as our final assumption, borrowing the name given to it in Drèze (1987, p. 56).[27]

Assumption 6.7 (Independence of Conditional Preference) For every non-null state s, the set $G_0(s) = \{g_h \in G | g_h(S \setminus s) = g_0(S \setminus s), g_0 \prec g_h\}$ is an equipotent set.

The extension to games with moral hazard of the representation Theorem 4.7 for games against nature can now be stated.[28]

Theorem 6.8 Assume Conditional Preferences (4.3), Value of Information(6.1), Existence of Omnipotent Games (6.5), Continuity (6.6) and Independence of Conditional Preference (6.7), then there exists a closed convex set $O \subseteq \Delta^S$ and S real-valued linear functions u_s defined on Π such that, for all $g \in G$

$$V(g) = \max_{\sigma \in O} \sum_s \sigma_s u_s(g(s)). \tag{6.1}$$

This is the extension to moral hazard of the state-dependent subjectively expected utility theorem, announced in the introduction—see (2.6). We label it "generalised subjectively expected utility theorem".

Clearly, if the set O had a non-empty interior relative to Δ^S (i.e. were full dimensional), then an omnipotent game g_i would have the property that $u_s(g_i(s)) = u_t(g_i(t))$ for all $s, t \in S$. In that case, the identification problem would be fully resolved; the set O would be uniquely identified, and the

[27] The name "independence" reflects the fact that Assumption 6.7 is equivalent to: For all $f, g, h \in G_0(s)$, for all $\alpha \in (0,1]$, $g_{f\alpha g} \precsim g_{h\alpha g}$ iff $f \precsim h$.

[28] This is Theorem 8.1 in Drèze (1987, p. 61).

state-dependent utilities $u_s(\cdot)$ would be defined up to a *common* positive affine transformation. Otherwise, the identification could only be partial; in particular, if O where a singleton, we would be back to games against nature, with no identification of subjective probabilities. The general case is intermediate, corresponding to partial control over the probabilities of the states and partial identification.

In order to spell out the partial identification properties associated with Theorem 6.8, we need two definitions.

DEFINITION 6.9 (Optimal Strategies) For every game $g \in G$, the set of optimal unobserved strategies is reflected in the set of probabilities $\sigma^*(g)$ defined by

$$\sigma^*(g) = \{\sigma \in O : V(g) = \sum_s \sigma_s u_s(g(s))\} \tag{6.2}$$

where O and $\{u_s(\cdot)\}$ verify Theorem 6.8.

Definition 6.10 (Linearly Independent Games) Assume that the probabilities $\{\sigma^1, ..., \sigma^m\}$ are linearly independent; if the games $\{g_1, ..., g_m\}$ are such that $\sigma^*(g_i) = \{\sigma^i\}$, $i = 1, ..., m$, then $\{g_1, ..., g_m\}$ are linearly independent games.

The statement of the uniqueness properties associated with Theorem 6.8 is of necessity somewhat intricate. It is given by the following theorem.[29]

THEOREM 6.11 (Uniqueness) Under the assumptions of Theorem 6.8, let there be k states, $k \leq S$, which are non-null, and let $S^k \doteq \{1, ..., k\}$ be the set of such states.

(i) If there are k linearly independent games, then the set O of Theorem 6.8 is unique on S^k, and for each $s \in S^k, u_s(\cdot)$ is unique up to the same affine transformation as V;

(ii) if there exist at most $\ell < k$ linearly independent games, then O and $\{u_s(\cdot)\}$ are defined on S^k and $S^k \times \Pi$ up to a set of joint transformations which take $(\sigma_s)_{s=1}^k$ into $(\sigma_s c_s)_{s=1}^k$, and $u_s(\cdot)$ into $\frac{1}{c_s}(u_s(\cdot) + d_s)$, where the $(c_s)_{s=1}^k$ are positive numbers, lying in a $(k-\ell)$-dimensional subspace of R^k, and $\sum_{s=1}^k (\sigma_s c^s) = 1, \sum_{s=1}^k (\sigma_s d_s) = 0$. Also, for each $s \in S^k$, $u_s(\cdot)$ remains subject to the same positive affine transformations as V.

The statement of (ii) in Theorem 6.11 is far from transparent, when $\ell > 1$.[30] It defines a restricted set of admissible state-dependent positive affine transfor-

[29]This is Theorem 8.2 in Drèze (1987, p. 62).

[30]When $\ell = 1$, Theorem 6.11 reduces to Theorem 4.9.

mations, restricted to lie in a $(k-\ell)$-dimensional space; where k is the number of non-null states, and ℓ is the maximal number of linearly independent elements of O (intuitively: of linearly independent strategies). To aid intuition, we invite the reader to consider the following problem: Characterise the set of affine functions which are constant over a simplex of dimensionality $\ell \in R^k, \ell < k$. One formulation of the answer is given by (ii) in Theorem 6.11. We expand and illustrate that analogy in Remark 6.12, and offer two additional remarks (about conditional preferences and null states).

REMARK 6.12 (Partial Identification and Omnipotent Games) As was remarked following Theorem 6.8, when the set O is full-dimensional, so that identification is complete, then a game g is omnipotent if *and only if* it is a constant-utility game; i.e. iff $u_s(g_s) = u_t(g_t)$ for all s, t. The two omnipotent games g_0 and g_1 of Assumption 6.5 may thus be used for normalisation by setting $V(g_0) = u_s(g_0(s)) = 0$, $V(g_1) = u_s(g_1(s)) = 1$, for all s. Accordingly, in such a case, the ranges of the S functions $u_s(\cdot)$ overlap (have a common intersection with non-empty interior).

The suggested normalisation underlies the elicitation of the set O in the proof of Theorem 6.8 (see Appendix, Definition A.3). When *only* constant-utility games are equipotent, then the elicitation is invariant to the arbitrary selection of omnipotent games used for normalisation. By contrast, when the set O has lower than full dimensionality, the set of omnipotent games is broader, and includes games violating the constant-utility property. In the limit, when O is a singleton (absence of moral hazard), every game is omnipotent; the selection of the two games g_0 and g_1 is arbitrary, and the elicited probabilities are likewise arbitrary.

A simple example illustrates the pitfalls surrounding the elicitation of subjective probabilities in games with moral hazard. There are three states, $S = \{1, 2, 3\}$, and the attainable probabilities (known to the agent but not to the investigator) are defined by $O = \{p \in \Delta^3 | p_1 \leq \frac{1}{2}\}$. The set O, with extreme points $\{A, B, C, D\}$ is depicted in Figure 6.1.[31] We study the implications of alternative ranges for the state-dependent utility functions.[32]

(i) Let $u_1(\cdot) \in [1, 1.5]$, $u_2(\cdot) \in [.5, 1]$, $u^3 \in [0, .5]$.[33]

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[31]For instance, the agent is to participate in an archery contest against two opponents, and state i corresponds to achieving the i-th rank in the contest, $i = 1, 2, 3$. The agent believes that she can rank second with certainty, first with probability .5. She can underperform at will.

[32]Reminder (of Remark 4.11): The utility ranges depend upon the set of prizes (P).

[33]The utility levels associated with ranks 1, 2 and 3 are 1, .5 and 0 respectively. Money prizes staked on the states raise utility by up to .5 in each state.

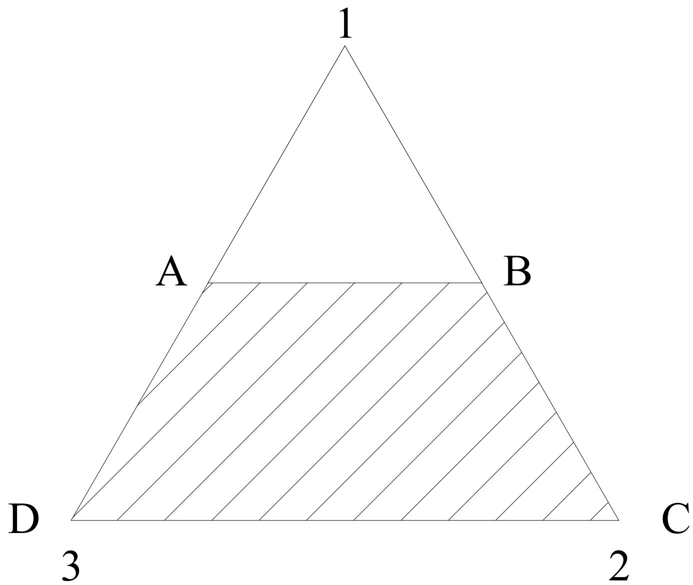

Figure 6.1 Attainable probabilities, example 6.12.

The probability vector $(.5, .5, 0) \in O$, namely point B, is then simultaneously optimal for *all* games, which are omnipotent (pairwise equipotent).

This situation is not distinguished from a game against nature. The probabilities and utility scales are not identified.

(ii) Let $u_1(\cdot) \in [1, 1.5]$, $u_2(\cdot) \in [.5, 1]$, $u_3 \in [.1, .6]$.

The probability $(.5, 0, .5)$, or point A, is optimal for games f such that $u_1(f(1)) > u_3(f(3)) \geq u_2(f(2))$; the probability $(.5, .5, 0)$ or point B is optimal for games g such that $u_1(g(1)) \geq u_2(g(2)) \geq u_3(g3))$; and both A and B are optimal for games h such that $u_1(h(1)) > u_2(h(2)) = u_3(h(3)) \in [.5, .6]$. Such games h are omnipotent, but there exist no triplets of linearly independent games—only pairs. So the identification is incomplete. The elicitation as per Theorems 6.8 and 6.11 can reveal that $\sigma_2 = 0$ at A, $\sigma_3 = 0$ at B and σ_1 has the same value at A and B—but that value cannot be ascertained uniquely.

(iii) Let $u_1(\cdot) \in [1, 1.75]$, $u_2(\cdot) \in [.5, 1.25]$, $u_3 \in [.1, .75]$.

The probability $(.5, 0, .5)$ or point A is optimal for games f such that $u_1(f(1)) > u_3(f(3)) \geq u_2(f(2))$; the probability $(.5, .5, 0)$ or point B is optimal for games g

such that $u_1(g(1)) \geq u_2(g(2)) \geq u_3(g(3))$; the probability $(0,1,0)$ or point C is optimal for games h such that $u_2(h(2)) \geq u_1(h(1)) > u_3(h(3))$. *There exist no omnipotent games,* Assumption 6.5 is violated, and the theory of this section is not applicable (though it could be extended, at major technical cost).[34]

(iv) Let $u_1(\cdot) \in [1, 2.25]$, $u_2(\cdot) \in [.5, 1.75]$, $u_3 \in [0, 1.25]$.

There exist games g such that $u_1(g(1)) = u_2(g(2)) = u_3(g(3)) \in [1, 1.25]$, i.e. constant-utility games. Such games, and only these are omnipotent. The identification of subjective probabilities, as well as units of scale and origins of state-dependent utilities, is complete.

Remark 6.13 (Conditional Preferences) Conditional preferences were defined above (Definition 4.1) given a *state,* not an event—in contrast to standard practice. There is a good reason for that choice. *In situations involving moral hazard, conditional preferences given an event need not be well defined.* This is most easily seen through a simple example. Let $S = \{s_1, s_2, s_3, s_4\}$ and let the attainable probabilities be defined by $p_1 + p_2 \leq \frac{1}{2}, p_3 = p_1, p_4 = p_2$.[35] Consider then four games with payoff as described in Table 6.1.

	s_1	s_2	s_3	s_4
f	0	50	50	30
g	50	0	50	30
f'	0	50	30	100
g'	50	0	30	100

Table 6.1 Payoff matrix for Example 6.13

One would not be surprised to observe the pairwise orderings: $g \succ f$, $f' \succ g'$, in violation of the "conditional preferences given *events* are well-defined" assumption (sure-thing principle).[36]

As explained in Section 7.2, conditional preferences given any event are generally not well defined unless the set of attainable probabilities O is either a singleton or the full simplex Δ^S. The results of Section 7.1 require only that conditional preferences be well defined given any state, as per Definition 4.1.

[34] See Remark 7.12 below.

[35] For instance, the event $\{s_1, s_3\}$ might correspond to "agent attends concert" (as in the example of Section 2.3) and the event $\{s_1, s_2\}$ to "it rains tonight", with a probability of rain equal to .5.

[36] The rationale is that "agent would attend concert" if the relevant choice is between f and g, but "would not attend" if the choice is between f' and g'.

The formulation in terms of states rather than events is a serious impediment to the applicability of the theory; see however Remark 7.10.

REMARK 6.14 (Partial Identification and Null States) As explained in Remark 5.8, a state s can be null because $\sigma_s \equiv 0$, or because $u_s(\cdot)$ is a constant function. In games against nature, there is no obsversable distinction between the two cases. In games with moral hazard, a state s such that $u_s(\cdot)$ is constant but σ_s is affected by the agent's strategy still matters to choices, provided the constant utility level $u_s(\cdot)$ be sometimes above and sometimes below the expected utility level on $S \setminus s$. Such must be the case, in particular, when the set O of Theorem 6.8 is full-dimensional, revealing that the agent chooses to increase or decrease σ_s in response to the prizes associated to $S \setminus s$; in that case, σ_s is identified, because O is unique, and $u_s(\cdot)$ is uniquely related to $u_t(\cdot), t \in S \setminus s$. Conversely, if σ_s is constant on O, then O is not full-dimensional, identification is partial, and in particular σ_s is not identified—not even to the point of ascertaining whether $\sigma_s = 0$ or $\sigma_s > 0$.

7 Conditional Expected Utility

7.1 Representation Theorem

Observed preferences among games with moral hazard cannot take us further than Theorems 6.8 and 6.11, resulting in partial identification of subjective probabilities as well as relative origins and units of scale of state-dependent utilities. To go beyond these conclusions, one must again resort to hypothetical preferences. The KS approach outlined in Section 5 could be extended to games with moral hazard, at some technical cost. A natural extension would call for eliciting hypothetical preferences among games conditionally on arbitrary (convex, closed) *sets* of attainable probabilities. Indeed, preferences expressed conditionally on given probabilities cannot bring out the expected-utility maximising property of the choice of a probability from a set. There is however an earlier model of hypothetical preferences designed to accommodate games with moral hazard, namely the "conditional expected utility" (CEUT) model of Fishburn (1964, 1973), Pfanzagl (1967, 1968) and Luce and Krantz (1971). In the course of preparing this chapter for the *Handbook,* we were led to explore the relationship of CEUT to the theory of games with moral hazard presented in Section 6. This section accordingly incorporates results established in Drèze and Rustichini (1999).

Events, subsets of S, are denoted $A, B, \ldots$. CEUT rests upon a *primitive* notion of preferences among *conditional games* $f_A, g_B, \ldots$ defined as mappings from the events $A, B, \ldots$ into $\times_{s\in A}\Pi(s)$, $\times_{s\in B}\Pi(s)$. This primitive relation $f_A \precsim^c g_B$ is assumed to be a weak order. The compatibility of that *primitive* order with *derived* conditional preferences, as per Definition 4.1, is part of the axiom system.

Under moral hazard, the primitive relation $f_A \precsim^c g_B$ is subject to verification through observed choices when the events A and B are under the full control of the agent, i.e. when there exist "strategies" resulting in the certainty of A, resp. B. When that is not the case, $f_A \precsim^c g_B$ is a hypothetical preference. Such hypothetical preferences are identical to those elicited by Karni and Schmeidler (1981), where the assumed probabilities specify a probability equal to one for A in the case of f_A, alternatively for B in the case of g_B. When the event A consists of the single state s, the conditional game f_s is the same as the hypothetical game f^{δ_s} of Section 5.

When the events A and B are under partial control of the agent, there exist "strategies" affecting the probabilities of the events A and B, but these probabilities must lie in a set O which is a proper subset of Δ^S. As explained in Remark 6.13, assuming $f_A \precsim^c g_B$ to be a weak order is then problematic—although $f_s \precsim^c g_t$, $s, t \in S$, could still be assumed to be a weak order.

Accordingly, we restrict attention to *elementary conditional games* f_s, g_t, elements of a set ECG; and simple lotteries over these, elements of a set $EC\Gamma$. We still denote such lotteries by $\gamma, \gamma', ...$ The following result is but another interpretation of the von Neumann-Morgenstern representation theorem.

THEOREM 7.1 Let the preference relation on $EC\Gamma$ satisfy Weak Order (3.2), Independence (3.3) and Continuity (3.4); then there exists a real-valued function V^c defined on ECG up to a positive affine transformation such that, for all lotteries $\gamma, \gamma' \in EC\Gamma$, $\gamma \precsim \gamma'$ if and only if

$$\sum_{g \in G,\ s \in S} \gamma(g_s) V^c(g_s) \leq \sum_{g \in G,\ s \in S} \gamma'(g_s) V^c(g_s). \tag{7.1}$$

For future reference, we denote by $\mathcal{R}_s$, the range of the function $V^c(g_s)$, $s \in S$, and by $\mathcal{R}$ the product of the sets $\mathcal{R}_s$.

It follows from Theorem 7.1 that $f_s \precsim^c g_t$ if and only if $V^c(f_s) \leq V^c(g_t)$. We shall use the two notations interechangeably.

The consistency of the primitive and derived definitions of conditional preference is captured by the next assumption, identical to Assumption 5.3.

ASSUMPTION 7.2 (Consistency of Elementary and Derived Conditional Preferences) For every $s \in S$, $f, g \in G$, if $f_s \precsim^c g_s$, then it is not true that $g \prec f$ given s.

We may now use the numerical (cardinal) representation of primitive elementary conditional preferences to relax the Assumption 6.5 that there exist omnipotent games—an assumption whose restrictive nature was explained in Remark 6.12. That assumption was mainly used to construct equipotent sets in Assumption 6.7. As an alternative, we shall consider games with *linearly related* conditional values, for short linearly related games; that is games (f, g) such that for some real numbers $\alpha > 0$ and β, $V^c(f_s) = \alpha V^c(g_s) + \beta$ for all $s \in S$. The relevance of such games is easily understood: If $\sigma \in O$ is such that $\sum_s \sigma_s V^c(f_s) \geq \sum_s \sigma'_s V^c(f_s)$ for all $\sigma' \in O$, then clearly $\sum_s \sigma_s V^c(g_s) \geq \sum_s \sigma'_s V^c(g_s)$ for all $\sigma' \in O$. Thus, the elements of O corresponding to optimal strategies for f also correspond to optimal strategies for g. Hence, f and g should be equipotent. This defines an alternative approach to identifying equipotent games, which proves adequate to elicit subjective probabilities and utility ranges. The existence of linearly related games follows from assuming that the ranges $\mathcal{R}_s$ have a non-empty interior (no conditional indifference). This amounts to excluding the existence of states s for which $V^c(f_s)$ is a constant function. The more general case is amenable to a related though less transparent analysis.

The following assumptions and result are from Drèze and Rustichini (1999).

Assumption 7.3 (No Conditional Indifference) There exist two games f and g such that $V^c(f_s) < V^c(g_s)$, for all $s \in S$.

Assumption 7.4 (Linearly Related Games are Equipotent) Let f and g be such that there exist real numbers $\alpha \geq 0$ and β with $V^c(f_s) = \alpha V^c(g_s) + \beta$, for all $s \in S$; then:

(i) $V(f) = \alpha V(g) + \beta$;

(ii) fEg.

Note that $V(f), V(g)$ in property (i) represent unconditional preferences, as per Theorem 3.5. Assumption 7.4 thus imposes a weak requirement of consistency between actual and hypothetical preferences. Also, if $V^c(f_s) = V^c(f_t)$ for all $s, t \in S$, then f satisfies (i) for any g with $\alpha = 0$, $\beta = V(f)$; hence f is omnipotent, as befits a constant-utility game.

Theorem 7.5 Assume Weak Order (3.2), Independence (3.3) and Continuity (3.4) on Γ and on $EC\Gamma$; Conditional Preference (4.3), No Conditional Indifference (7.3) and Consistency of Elementary Conditional Preferences (7.2); Value of Information (6.1) and Linearly Related Games are Equipotent (7.4); then there exists a closed convex set $O_c \subseteq \Delta^S$ of probabilities on S such that, for all $f \in G$,

$$V(f) = \max_{\sigma \in O_c} \sum_s \sigma_s V^c(f_s). \tag{7.2}$$

Furthermore, there exists a unique minimal set O with these properties. If there exists a point $v \in$ int $\mathcal{R}$ such that $v_s = v_t$ for every $s, t \in S$, then the set O_c is itself unique.

When there exist omnipotent games, the functions $V^c(f_s)$ of Theorem 7.5 and $u_s(f(s))$ of Theorem 6.8 are identical (under a suitable common positive affine transformation of the latter, if needed).

Comparing with Theorems 6.8 and 6.11, the additional property is the uniqueness of the minimal set O.[37] Therefore, the subjective probabilities which the agent believes (s)he can attain are fully identified, leading to identification of the relative origins and units of scale of the state-dependent utilities. Of course, the full identification results from the acceptance of hypothetical conditional preferences as a primitive.

7.2 *Extensions and Remarks*

As an easy extension of Theorem 7.5, one obtains a necessary condition for general conditional preferences (i.e. preferences conditional on events, not only on states) to be well defined.

ASSUMPTION 7.6 (General Conditional Preferences are Well Defined) For every $B \in S$, for every $f, g \in G$, if $f_B \prec^c g_B$, then it is not the case that $g_B \prec^c f_B$.

THEOREM 7.7 (Drèze and Rustichini) Under the assumptions of Theorem 7.5, if general conditional preferences are well defined, then the set O of Theorem 7.5 is either $O = \Delta^S$ or $O = \{\sigma\}$.

Using the representation in Theorem 7.5, the condition is also sufficient, under the same assumptions.

Theorem 7.7 is important to an understanding of the earlier work on conditional expected utility, which proceeds from the assumption that the preference relation among conditional games $f_A, g_B, ...,$ is a weak order. Clearly, that requirement includes Assumption 7.6, hence either full control over events ($O = \Delta^S$), or no control at all ($O = \{\sigma\}$).

This feature is generally quite restrictive. It should be noted, however, that authors like Luce and Krantz (1971) were interested in modeling a situation where there exists a partition M of S into events $\{A1, ..., A_m\}$ that are, at least partially, under the control of the agent; the determination of the state within

[37]In some but not all cases, the minimal set can be enlarged with irrelevant (never chosen) elements; see for instance case (i) of the example in Remark 6.12.

any element A_k of the partition is left to nature. Even in that case, full control is a rather special case, so we do not reproduce the technical analysis of these authors.[38]

Under full control over events (elements of the partition M, or states in Section 7.1), one would expect the agent to choose, given game f, the *event* for which the utility under that game is maximal. Take then any two disjoint events A and B (for instance A_i and A_j in the partition M, with $i \neq j$), and suppose that $f_A \precsim^c g_B$. The agent should then be indifferent between g_B and the game $h_{A\cup B}$ which is equal to f on A and to g on B; the simple reason being that, faced with $h_{A\cup B}$, the agent can "choose" B for sure. This suggests a simple way of axiomatising full control of events.

ASSUMPTION 7.8 (Full Control of Events) Let $A, B \in \mathcal{S}, \mathcal{A} \cap \mathcal{B} = \emptyset$. If $f_A \precsim^c g_B$ and $h = f$ on A, $h = g$ on B, then $h_{A\cup B} \sim^c g_B$.

This is a modified version of Assumption 3 in Luce and Krantz (1971), which assumes $f_A \sim^c g_B$. It leads to an immediate representation theorem.

THEOREM 7.9 Assume Weak Order (3.2), Independence (3.3) and Continuity of Preferences (3.4) among conditional games, and Full Control of Events (7.8). Then

$$V(f_B) = \max_{s\in B} V^c(f_s) = \max_{s\in\Delta^B} \sum_s \sigma_s V^c(f_s). \tag{7.3}$$

REMARK 7.10 Drèze and Rustichini (1999, Section 7) show how the existence and definition of a partition M such that "$\precsim$ given A_k" is well defined for every $A_k \in M$ can be elicited from preferences, instead of being assumed as a primitive (the route followed by earlier CEUT authors). This mitigates the seriousness of the problem raised in the last sentence of Remark 6.13.

REMARK 7.11 The CEUT approach as developed in Section 7.1 may be viewed as an application of the KS approach, where the assumed probabilities $\tau \in \Delta^s$ are restricted to the form $\delta_s, s = 1, ..., S$. Assumption 7.2 is the exact counterpart of Assumption 5.3. Assumption 7.4 is not needed in the case of games against nature, where it follows from (5.4). Thus these two ways of incorporating hypothetical preferences are fully compatible. In particular, the stronger tests of consistency of hypothetical preferences (defined in Section 5) could be added to the CEUT approach, thereby strengthening its reliability.

REMARK 7.12 Complementing the analysis of observed preferences with that of hypothetical elementary conditional preferences, as per Section 7.1, entails two

[38] We did so in Rustichini and Drèze (1994).

contributions: (i) It leads to full identification of subjective probabilities and utility scales; (ii) it frees the analysis from the Assumption 6.5 (Existence of Omnipotent Games), the restrictive nature of which was illustrated in Theorem 6.11.

Resorting to linearly related games, rather than to omnipotent games, as a basis for axiomatising equipotence is a conceivable extension of the theory in Drèze (1987) and in Section 6. Carrying out that program entails technical difficulties, that we have not explored in detail. But the intuitive principle is reasonably straightforward.

Lemma 4.5 above only assumes non-degeneracy and conditional preferences, two assumptions clearly applicable to games with moral hazard. The lemma asserts the existence of S functions $v_s(g(s))$, each defined up to a positive affine transformation, which represent conditional preferences cardinally.[39] Writing $c_s > 0, d_s$ for the coefficients of a general state-dependent affine transformation, replace each function $v_s(\cdot)$, arbitrarily normalised, by $c_s v_s(\cdot) + d_s$. Now, Assumption 7.4 (linearly related games are equipotent) uses Theorem 7.1, whereby the S-tuples of constants $(c_s, d_s)_{s=1,\dots,S}$ are related uniquely (up to a common affine transformation), on the basis of hypothetical elementary conditional preferences. The assumption states that linearly related conditional values implies equipotence.

Without relying upon hypothetical preferences, one could *search for* constants $(\hat{c}_s, \hat{d}_s)_{s=1,\dots,S}$ such that the following property holds:

Property 7.13 (Linearly Related Values Entail Equipotence) The real vectors $(\hat{c}, \hat{d}) \in R_+^S \times R^S$ are such that, if $\hat{c}_s v_s(g(s)) + \hat{d}_s = \alpha[\hat{c}_s v_s f(s)) + \hat{d}_s] + \beta$ for all $s \in S$, with $v_s(\cdot)_{s=1,\dots,S}$ verifying (6.1) and $(\alpha, \beta) \in R \times R$ verifying $V(g) = \alpha V(f) + \beta$, then fEg (f and g are equipotent).

When all states are subject to at least partial control by the agents, vectors $(\hat{c}, \hat{d})$ with that property should be defined *uniquely* (up to a common affine transformation), solving the identification problem. More generally, the extent of identification would be comparable to that asserted in Theorem 6.11.

A suitable axiomatisation of Property 7.13, and a proof of Theorem 6.8 under the weakened assumptions, are beyond the scope of the present paper.

[39] Theorem 6.8 asserts that the function F of Lemma 4.5 has the specific form appearing in (6.1).

8 Risk Aversion

8.1 State-Independent Preferences, or Single Commodity

For a univariate von Neumann-Morgenstern utility function $u(y)$, with derivatives $u_y, u_{yy}, ...$, Arrow (1965) and Pratt (1964) have defined the absolute risk-aversion function $R_A(y) = -\frac{u_{yy}}{u_y}$ and the relative risk-aversion function $R_R(y) = -y\frac{u_{yy}}{u_y}$ which have been the subject of an extensive literature. $R_A(y)$ admits of a natural interpretation as "twice the risk premium per unit of variance for infinitesimal risks". For stochastic y with expectation $\bar{y}$, the risk premium P_R is defined implicitly by $u(\bar{y} - P_R) = Eu(y)$. For small risks

$$P_R \simeq .5\sigma_y^2 R_A(\bar{y}). \tag{8.1}$$

$R_R(y)$ admits of a similar interpretation for proportional risks. For a compendium of properties and further results, see Eeckhoudt and Gollier (1992).

Let the argument y of utility be a level of income or wealth, spent on a single consumer good c acquired at a price q, with $cq = y$. If the function $v(c)$ represents cardinal consumption preferences, then $u(y) = u(y|q)$ is an indirect representation of $v(c)$, valid for fixed q. More generally, $u(y,q) = v(\frac{y}{q})$ is an indirect representation of $v(c)$. See Chapter 4 of Volume I of this *Handbook* for a general discussion of indirect utility functions.

Treating $v(c)$ as the natural representation of primitive preferences, one obtains for the derived representation $u(y,q)$ that $u_y(y,q) = \frac{1}{q}v_c$, $u_{yy}(y,q) = \frac{1}{q^2}v_{cc}$, $R_A(y,q) = \frac{R_A(c)}{q}$, $R_R(y,q) = R_R(c)$. One implication of these formulae is that $R_R(y,q)$ is invariant under equiproportionate variations of y and q, but $R_A(y,q)$ is not. For instance, if y and q are both doubled, at unchanged c, $R_A(y,q)$ is halved. This is because σ_y^2 is implicitly kept fixed in (8.1), and $\sigma_c^2 = \frac{\sigma_y^2}{q^2}$.[40]

8.2 State-Dependent Preferences, or Many Commodities

The theory of risk aversion has been extended to "many commodities", starting with Stiglitz (1969), followed by Deschamps (1973), then Kihlstrom and Mirman (1974), Hanoch (1977), Karni (1979) and others; see Biswas (1997) for additional references.[41] That extension covers the case of state-dependent preferences, as already suggested in Section 2.1.

[40] Thus, $P_{R_y} = .5\sigma_y^2 R_A(y) = .5q^2\sigma_c^2\left(\frac{R_A(c)}{q}\right) = qP_{R_c}$ as desired.

[41] A particular case of "many commodities" concerns intertemporal consumption decisions; see Leland (1968), Sandmo (1970, 1974), Mirman (1971) or Drèze and Modigliani (1966, 1972) for early contributions.

8.2.1 Commodity Risks

There are two ways of extending the theory of risk aversion to many commodities. The first looks at risks affecting the consumption of *specific* commodities, at unchanged consumption levels for the remaining ones. Let x_{-i} denote the $n-1$ dimensional vector $(x_j)_{j\neq i}$ and write $u(x)=u(x_{-i},x_i)$. Fix $x_{-i}=\bar{x}_{-i}$ and consider the distribution function $F_i(x_i)$ with mean $\bar{x}_i$. One then defines the commodity-specific risk premium P_{R_i} by

$$u(\bar{x}_{-i},\bar{x}_i-P_{R_i})=E_{F_i}u(\bar{x}_{-i},x_i). \tag{8.2}$$

For infinitesimal risks, denoting partial derivatives by subscripts,

$$P_{R_i}\simeq .5\sigma^2_{x_i}(-\frac{u_{ii}}{u_i})\doteq .5\sigma^2_{x_i}R_A(\bar{x}_i|\bar{x}_{-i}). \tag{8.3}$$

The notation recognises that $R_A(\bar{x}_i)$ depends on $\bar{x}_{-i}$.[42] When $u(\cdot)$ is additively separable, for instance when $u(x)=\sum_s\sigma_su_s(x_s)$, that dependence vanishes.

Kihlstrom and Mirman (1974) or Biswas (1983) have suggested formulae for aggregating (averaging) commodity-specific measures of risk aversion into an overall scalar measure.

The most transparent scalar measure is obtained when the n commodities are subject to perfectly correlated risks. In particular, let $Eu=\sum_s\sigma_su_s(y_s)$, where y_s denotes income or wealth in state s. Consider the (infinitesimal) risky prospect η with mean 0 taking y_s into $y_s+\eta$. Then:

$$\begin{aligned} Eu &= \sum_s\sigma_sE_\eta u_s(y_s+\eta)\simeq\sum_s\sigma_su_s(y_s)+\frac{\sigma^2_\eta}{2}\sum_s\sigma_su''_s(y_s)\\ &=\sum_s\sigma_su_s(y_s-P_R)\simeq\sum_s\sigma_su_s(y_s)-P_R\sum_s\sigma_su'_s(y_s), \end{aligned} \tag{8.4}$$

$$P_R\simeq\frac{\sigma^2_\eta}{2}\left(-\frac{\sum_s\sigma_su''_s}{\sum_s\sigma_su'_s}\right)=\frac{\sigma^2_\eta}{2}\cdot\left(-\frac{Eu''}{Eu'}\right)=\frac{\sigma^2_\eta}{2}\frac{\sum_s\sigma_su'_sR_A(y_s)}{\sum_s\sigma_su'_s}. \tag{8.5}$$

One may then define $R_A:=-\frac{Eu''}{Eu'}$, a weighted average of $R_A(y_s)$ with marginal utility weights.

Turning to relative risk aversion, let $y_s=w_s+y$, where for instance w_s is labour income or human wealth in state s and y is property income or nonhuman wealth. Let y be subject to a proportional risk ξ with mean 1 taking

[42] In an elementary application to intertemporal consumption, let $u(c_1,c_2)$ represent preferences among vectors of present and future consumption. $R_A(c_2|\bar{c}_1)$ is the relevant risk aversion measure for risks affecting future consumption at given levels of present consumption, i.e. "delayed risks".

y_s into $w_s + \xi y$. Then

$$\begin{aligned} Eu &= \sum_s \sigma_s E_\xi u_s(w_s + \xi y) \simeq \sum_s \sigma_s u_s(w_s + y) + \frac{\sigma_\xi^2 y^2}{2} \sum_s \sigma_s u_s''(w_s + y) \\ &= \sum_s \sigma_s u_s(w_s + P_R y) \simeq \sum_s \sigma_s u_s(w_s + y) + (P_R - 1)y \sum_s \sigma_s u'(w_s + y), \end{aligned} \tag{8.6}$$

$$\begin{aligned} 1 - P_R = \frac{\sigma_\xi^2}{2} y \left(- \frac{\sum_s \sigma_s u_s''(w_s + y)}{\sum_s \sigma_s u'(w_s + y)} \right) &= \frac{\sigma_\xi^2}{2} \left(- \frac{yEu''}{Eu'} \right) \\ &= \frac{\sigma_\xi^2}{2} \frac{\sum_s \sigma_s u_s' R_R(y_s)}{\sum_s \sigma_s u_s'}. \end{aligned} \tag{8.7}$$

One may then define $R_R(y) := -y \frac{Eu''}{Eu'}$.

8.2.2 *Income Risks*

An alternative approach considers situations where the vector x is chosen by the agent under a budget constraint, say $\sum_i q_i x_i \leq z$. In that case, $x = x(z, q) \in \operatorname{argmax}_x \{u(x) | \sum_i q_i x_i \leq z\}$. The interest then focusses on risks affecting z.

When $x = x(z, q)$, the indirect uitility function $v(z, q)$ is defined by

$$v(z, q) = u(x(z, q)) = \max_x \{u(x) | \sum_i q_i x_i \leq z\}. \tag{8.8}$$

Under differentiability assumptions, the measures of absolute, respectively relative risk aversion for z are obtained from the indirect utility function as

$$R_A(z|q) = \frac{-v_{zz}}{v_z}, \quad R_R(z|q) = z\ R_A(z|q). \tag{8.9}$$

In a decision-theoretic context, this second approach assumes that incomes y_s in states $s = 1, ..., S$ can be traded *ex ante* on insurance markets at unit premia q_s subject to the budget constraint $\sum_s q_s y_s = z$. The measures of risk aversion in (8.9) are then applicable.[43] They call for eliciting the indirect utility function $v(z, q)$. Under the (very special) structural assumption of existence of a full set of set of insurance markets, the function $v(z|q)$ is a representation of choices among lotteries over z and can thus be elicited *directly.* No knowledge

[43] Biswas (1983) rejects that definition of $R_A(y|q)$ on the ground R_A is not invariant to equiproportionate changes in y and q. That objection strikes us as misplaced, for the reason stated in the third paragraph of Section 8.1.

of q is required. (It is imperative that these lotteries be drawn before the true state is observed, thus allowing the agent to adjust her insurance purchases to the realised value of x.) If instead the function $u(x) = \sum_s \sigma_s u_s(x_s)$ were elicited, knowledge of the insurance prices q would be needed to recover $v(z, q)$ from $u(x)$. The additive separability of $u(x)$ should prove helpful for that operation. We are not aware that this topic has been investigated.

When insurance prices (i.e. markets) are not available, one may resort to *hypothetical* preferences under *assumed* insurance prices. Karni (1983a, 1983b, 1985) has suggested measuring risk aversion under the assumption of "fair insurance", i.e. insurance prices $q_s = \sigma_s$. The foregoing discussion is then applicable under the hypothetical preferences represented by $v(z, \sigma)$, subject to the qualifications introduced in Section 5.[44] Of course the rationale of the second approach is unclear, when insurance markets are incomplete. We do not pursue that far-reaching issue here.

9 Applications: Life Insurance and Value of Life

Applications of decision theory with state-dependent preferences deal in particular with problems involving states of life and death. Several other problems are *formally* equivalent to those involving life and death; for instance, those involving two alternative states of health, as in Zweifel and Breyer (1997, Section 6.3), or the loss of an irreplaceable object, as in Cook and Graham (1978). Other problems involve many states, as in multiperiod life insurance problems,[45] or multiperiod safety problems,[46] health insurance or health care problems with a continuum of states,[47] job safety problems with a continuum of states,[48] and some others. These problems are of necessity more complicated. We restrict ourselves to static problems with two states (life and death), which illustrate neatly the role and implications of state-dependent preferences. (The growing literature on health economics lies outside the scope of this paper.)

We follow the literature in assuming that the decision-maker maximises expected state-dependent utility with *given* probabilities and utilities—however elicited.

[44] The analytical developments in Karni's work do not start from the indirect function, but are presented in terms of "reference sets"; the results are reconciled *ex post* with those implied via the indirect utility function approach. See Karni (1985) for a systematic discussion.

[45] Cf. Yaari (1964), Karni and Zilcha (1985) or Karni (1985, Chapter 4).

[46] Cf. Bergström (1982).

[47] Cf. Grossman (1972), Phelps (1973), and additional references in Zweifel and Breyer (1997).

[48] Cf. Moore and Viscusi (1990).

9.1 *Life Insurance*

9.1.1 *Statics*

The simplest possible life-insurance problem was stated in Section 2.2, equation (2.6), as:

$$\max_{y \geq 0} \sigma_0 u_0(w_0 + k(y) - y) + (1 - \sigma_0) u_1(w_1 - y) \tag{9.1}$$

with first and second order conditions

$$y\left[\sigma_0 u_0' \cdot (k' - 1) - (1 - \sigma_0) u_1'\right] \leq 0 \tag{9.2}$$

$$\sigma_0 u_0'' \cdot (k' - 1)^2 + (1 - \sigma_0) u_1'' < 0. \tag{9.3}$$

Under fair insurance, $k' = \frac{1}{\sigma_0}$ and condition (9.2) simplifies to $u_0' = u_1'$. More generally, $k' < \frac{1}{\sigma_0}$, with $k' > 1$ for otherwise $y \equiv 0$. If $k' = \frac{\alpha}{\sigma_0}, \alpha < 1$, then (9.2) reduces to

$$y\left[u_0' \cdot (\alpha - \sigma_0) - (1 - \sigma_0) u_1'\right] \leq 0. \tag{9.4}$$

It follows that $u'(w_0) > u'(w_1)$ is necessary for $y > 0$, and $u'(w_0) > u'(w_1)$. $\frac{1-\sigma_0}{\sigma_0(k'-1)}$ is sufficient. The incentive for life insurance comes from a higher marginal utility of wealth in case of death than in case of life—whether due to loss of human wealth or to state-dependent preferences.

Our next remark is more interesting. It concerns the relationship between life-insurance purchases and risk aversion. It is well known that, under state independent preferences ($u_0(z) = u_1(z)$ identically in z), the amount y devoted to insurance is positive iff $w_0 < w_1$; when y is positive, a concave transformation of the utility function, i.e. a transformation that increases risk aversion, cannot result in a decrease of y. We show by means of an example that a similar property may fail, under state-dependent utility.[49]

PROPOSITION 9.1 It is **not** true that a concave monotone transformation of $u_0(\cdot)$ and $u_1(\cdot)$, i.e. a transformation increasing simultaneously $R_A^0(\cdot)$ and $R_A^1(\cdot)$, never decreases the insurance premium y solving problem (9.1).

PROOF We provide a counterexample. Let

$$u_0(z) = \frac{-z_0^{-\lambda\gamma}}{\lambda\gamma}, u_1(z_1) = \frac{-z_1^{\gamma}}{\gamma}, \lambda > 0.$$

[49] This proposition finds its inspiration in Karni (1985, Section 4.5), which treats a more complex model.

Increasing γ is a concave transformation, which simultaneously increases $\frac{-u_0''}{u_0'} = \frac{\lambda\gamma+1}{z_0}$ and $\frac{-u_1''}{u_1'} = \frac{\gamma+1}{z_1}$.

To simplify, let $w_0 = 0, w_1 = 1$ and $k = \frac{y}{\sigma_0}$ (fair insurance). (By continuity, the result also holds for $k = \frac{y}{\sigma_0}\alpha, \alpha < 1$.) The first-order condition (9.2), with $y > 0$, becomes

$$F = (1-\sigma_0)\left[\frac{1-\sigma_0}{\sigma_0}y\right]^{-\lambda\gamma-1} - (1-\sigma_0)(1-y)^{-\gamma-1} = 0, \quad (9.5)$$

$$\frac{1}{1-\sigma_0}\frac{\partial F}{\partial\gamma} = \left[\frac{1-\sigma_0}{\sigma_0}y\right]^{-\lambda\gamma-1}\left[-\lambda\ln\frac{1-\sigma_0}{\sigma_0}y\right] + (1-y)^{-\gamma-1}\ln(1-y). \quad (9.6)$$

Using (9.5) twice, (9.6) becomes successively

$$\begin{aligned}\frac{1}{1-\sigma_0}\frac{\partial F}{\partial\gamma} &= (1-y)^{-\gamma-1}\left[\ln(1-y) - \lambda\ln\frac{1-\sigma_0}{\sigma_0}y\right]\\ &= (1-y)^{-\gamma-1}\ln(1-y)\cdot\frac{(1-\lambda)}{(\lambda\gamma+1)}. \quad (9.7)\end{aligned}$$

The sign of $\frac{\partial F}{\partial\gamma}$ is the sign of $\lambda - 1$, which is indeterminate, since λ is a free parameter. Since $\frac{\partial F}{\partial y} < 0$ (verifying the second-order condition), the sign of $\frac{\partial y}{\partial\gamma}$ at the solution is also the sign of $\frac{\partial F}{\partial\gamma}$, i.e. of $\lambda - 1$. In particular, if $u_0(\cdot)$ is less concave (less risk averse) than $u_1(\cdot)$, i.e. $\lambda < 1$, then an increase in γ (in risk aversion) will reduce y. ∎

The logic of the proposition is suggested by the last sentence in the proof. When $u_0(\cdot)$ is less risk averse than $u_1(\cdot)$, a proportional increase in the concavity of both functions affects u_1 more significantly, reducing the willingness to pay for insurance.

Some qualitative properties of the solution to problem (9.1) are listed in Dehez and Drèze (1982, Section 2).

9.1.2 Dynamics

More specific theoretical predictions, and empirical research about life insurance come to grips with the complications introduced by the obvious fact that most life-insurance contracts are multiperiod contracts; some are lifetime contracts (the indemnity is paid at the time of death, whenever it occurs), others are fixed duration contracts (the policy expires at a fixed date if death has not occured prior to that date), possibly combined with an annuity thereafter. The prevalence of long-term contracts is explained, among other factors, by the fact that a long-term contract provides insurance against the risk of becoming a high risk; see Villeneuve (1998, Section 3.1).

The seminal paper on long-term life insurance is Yaari (1964), which also introduced the notion that life insurance serves the ancillary role of collateral for borrowing against human wealth. His and subsequent models address jointly the issues of savings and portfolio selection, with explicit attention to life insurance and annuities as elements of the portfolio. These considerations go well beyond the scope of the present paper. We refer readers to the recent survey by Villeneuve (1998) on "Life insurance" and to the 99 references given there.

9.2 Value of Life

9.2.1 Theory

Insurance problems fall under the theory of games against nature, so long as the state-probabilities are given. Problems involving safety outlays, motivated by the desire to reduce the probabilities of unfavourable states, fall under the theory of games with moral hazard. Early interest in these problems arose in relation to the provision of public safety, for instance in road transportation.[50] These authors considered as a first approximation a "value of life" reflecting expected future earnings. That approach was soon replaced by an expected utility approach to individual preferences and decisions, leading naturally to view public safety as a public good. The well-developed theory of public goods[51] could then be relied upon to analyse public decisions. We limit ourselves here to the individual level.

The expected utility approach, better known today as the "willingness-to-pay" approach to the value of safety, is an application of decision theory with state-dependent preferences. It was introduced in Drèze (1962), then independently in Schelling (1968) and Mishan (1971), followed by many others; see Jones-Lee (1982) for an account of early developments,[52] and Viscusi (1993) for a more recent empirical survey.

The problem of optimal spending on safety was posed in Section 2.2, equation (2.7), as:

$$\max_{x\geq 0} \sigma_0(x)u_0(w_0 - x) + (1 - \sigma_0(x))u_1(w_1 - x) \tag{9.8}$$

[50] Cf. Abraham and Thédié (1960).

[51] Cf. e.g. Milleron (1972) or any modern text on public economics.

[52] In the introduction to that volume, Jones-Lee writes: "It is probably fair to say that by the late 1970's the willingness-to-pay approach had acquired the status of the "conventional methodology" as far as academic economists were concerned"(p. viii).

with first and second order conditions

$$\sigma_0'(u_0 - u_1) - \sigma_0 u_0' - (1-\sigma_0)u_1' = 0, \tag{9.9}$$

$$2\sigma_0'(u_1' - u_0') + \sigma_0 u_0'' + (1-\sigma_0)u_1'' + \sigma_0''(u_0 - u_1) \leq 0. \tag{9.10}$$

Condition (9.9) yields a simple expression for the "marginal willingness to pay for safety", ϕ, namely

$$\phi = \frac{-dx}{d\sigma_0}|_{Eu} = \frac{-1}{\sigma_0'} = \frac{u_1 - u_0}{Eu'}. \tag{9.11}$$

The motivation to "buy safety" comes from the higher *level* of utility in case of life than in case of death. Proper assessment of the relative origins of u_0 and u_1 is thus central to this problem.

The interpretation of ϕ deserves attention. By definition, $-\frac{dx}{d\sigma_0}$ is the willingness to pay for a decrease in the probability of death, *per unit of* σ_0, *as evaluated for an infinitesimal decrease of* σ_0. Thus, if ϕ were to remain constant as incremental safety is bought at incremental expenses x, then $\phi\sigma_0$ (respectively ϕ) would measure the amount which the decision maker would be willing to pay to eliminate a probability of death σ_0 (respectively 1).[53]

Of course, ϕ is not apt to remain constant under incremental expenses on safety: These reduce disposable incomes $w_. - x$, so that Eu' increases. On this score, ϕ would fall as x increases. But there are other effects: The numerator is adjusted by $u_0' - u_1'$; increased safety expenses reduce σ_0, so that Eu' is adjusted by the difference $u_0' - u_1'$. One way to capture these effects is the following proposition, easily verified by implicit differentiation.

PROPOSITION 9.2 Let $\delta > 0$ and $x(\delta)$ be defined by

$$\sigma_0 u_0(w_0) + (1-\sigma_0)u_1(w_1) = (\sigma_0 - \delta)u_0(w_0 - x(\delta)) + (1 - \sigma_0 + \delta)u_1(w_1 - x(\delta)).$$

The increasing function $x(\delta)$ is concave[54] iff

$$2(u_0' - u_1') + \frac{dx}{d\delta}Eu'' = 2(u_0' - u_1') + \frac{u_1 - u_0}{Eu'}Eu'' < 0. \tag{9.12}$$

[53]The problem is typically considered in contexts where σ_0 has the interpretation either of a one-shot hazard (like contemplating a dangerous trip) or of a probability per unit of time (like the probability of meeting death on the road within a year). The context has implications for the definition of w_0 and w_1.

[54]A concave function $x(\delta)$ corresponds to a downward-sloping demand curve for safety.

The second term is negative under risk aversion. The first term is zero when the decision maker has access to life insurance at fair odds—in which case the strict concavity of $x(\delta)$ is established. More generally, the first term is positive under optimal insurance at less than fair odds, so that (9.12) holds only for sufficiently high absolute risk aversion $R_A = -\frac{Eu''}{Eu'}$. For instance, let z denote non-human wealth, a component of both w_0 and w_1 and let as before $\alpha \leq 1$ denote the ratio of actual to fair insurance indemnities. Under the plausible assumption that $u_1(w_1 - \frac{1-\alpha}{\alpha-\sigma_0}z) > u_0$, a coefficient of relative risk aversion $R_R(z) = -z\frac{Eu''}{Eu'}$ greater than or equal to 2 would validate (9.12).[55]

The interpretation of ϕ suggested above has led to the question whether its value exceeds human wealth (the financial loss associated with death). For an optimally insured person, the answers—as given in Bergström (1978) or Dehez and Drèze (1982)—comes in three statements. In each case, the relevant concept is "human wealth net of insurance benefits" i.e. $w_1 - w_0 - \frac{\alpha}{\sigma_0}y$.

(i) $\phi \geq \frac{\alpha(1-\sigma_0)}{\alpha-\sigma_0}(w_1 - w_0 - \frac{\alpha}{\sigma_0}y)$;

(ii) if the terms of the insurance policy are adjusted to the changes in σ_0 (α unchanged), then

$$\phi \geq \frac{\alpha(1-\sigma_0)}{\alpha-\sigma_0}(w_1 - w_0 - \frac{\alpha}{\sigma_0}y) + \frac{y}{\sigma_0};$$

(iii) if the safety outlay is paid *ex post* in case of life only, then

$$\phi \geq \frac{1}{1-\sigma_0}(w_1 - w_0\frac{\alpha}{\sigma_0}y).$$

Interestingly, the lower bound in (i) which is precisely equal to "human wealth net of insurance benefits" when $\alpha = 1$ (fair insurance), is a decreasing function of α: The higher the loading factor in the insurance contract, the greater is the willingness to pay for safety. This is due to the fact that the financial loss associated with death increases when its coverage through insurance diminishes.

9.2.2 Empirics

The empirical literature on the value of life and the demand for safety is vast and varied; see Viscusi (1993) for a survey of some 80 contributions. The bulk

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[55]Let $u_0' = \frac{1-\sigma_0}{\alpha-\sigma_0}u_1'$ as per Section 9.1 and write (9.12) as $2(u_0' - u_1') = 2\frac{1-\alpha}{\alpha-\sigma_0}u_1' \leq \frac{u_1-u_0}{z}R_R(z)$, $u_0 \leq u_1 - \frac{2}{R_R(z)}u_1'\frac{1-\alpha}{\alpha-\sigma_0}z$. By concavity of u_1, $u_1 - u_1'x > u_1(w_1 - x)$, so that $u_0 \leq u_1(w_1 - \frac{1-\alpha}{\alpha-\sigma_0}z) < u_1(w_1 - \frac{2}{R_R(z)}\frac{1-\alpha}{\alpha-\sigma_0}z)$ validates (9.12) provided $R_R(z) \geq 2$.

of these studies rest on two kinds of data: market data on compensating wage differentials (compensating for risk differentials),[56] which correspond to observable choices; and survey data labeled "contingent valuation", which correspond to hypothetical preferences.[57] Among the authors most active on these two fronts, the names of Viscusi and Jones-Lee respectively deserve mention.

The main common weakness of the two approaches is the diversity of the resulting evaluations. In addition, contingent valuation results display inconsistencies—another weakness, though the possibility of detecting inconsistencies (as stressed in Section 5) is a strength. Thus, Kniesner and Leeth (1991) note: "Policy makers are reluctant to use estimated compensating wage differentials for health risks in designing programs to reduce environmental hazards or encourage workplace safety because the estimates vary widely". And Beattie *et al* (1998, abstract) conclude to "...serious doubt on the reliability and validity of willingness-to-pay based monetary values of safety estimated using conventional contingent valuation procedures."

We illustrate the first point from the data collected in Table 2 of Viscusi (1993) concerning 25 estimates of value of life from labour market studies. Table 9.1 provides a summary of these data, grouped by the measure of risk (annual probability of fatal accident) reported in the study.[58]

It is noteworthy that low values of life (around 1 million dollars) are concentrated in relatively risky occupations (annual fatality rates $\geq 1/1000$), and stabilise (around 5 millions dollars) thereafter. This reflects both self-selection (workers with lower values for safety accept riskier jobs) and a downward-sloping demand curve for safety (as argued earlier in Section 9.2). This remark adds to the credibility of the data.

Beyond self-selection, many factors affect the willingness-to-pay for safety; family composition, age and wealth (both human and non-human) are obvious examples. It should not come as a surprise that estimates of ϕ vary across samples.

Estimates of values of life based on contingent valuation complement those based on wage differentials, first by providing independent checks, second by possibly providing more information about the characteristics of respondents (like age)[59] and third by eliciting responses from groups of persons not collecting

[56]The classic reference on this topic is Thaler and Rosen (1976).

[57]A few studies concern commodity purchases, from seat belts to smoke detectors or even real estate (in districts with varying degrees of air pollution) and new automobiles (with varying safety records), see Viscusi (1993, Section 5).

[58]Annual fatality rates for broad industry classes reported in Viscusi (1993, Table 3) range from 2/100.000 for services and 3/100.000 for trade to 24/100.000 for construction and 35/100.000 for mining.

[59]See e.g. Jones-Lee *et al.* (1985) or Johannesson and Johansson (1996).

wage differentials. This is in the same spirit (though with a different bearing) as complementing observed choices with elicitation of hypothetical preferences (Sections 5 and 7 above).

Risk	Number of studies	Value of Life ($ million)[a]			Average income level ($ million)[a]	
		Median	Mean	Range	mean	Range
NA	4	10	10	7.2/13.5	NA	NA
$\geq 1/1000$	5	.9	1	.6/1.6	25	21–27
$= 1/10000$	8	5	6	2.8/10.3	21.4	11.3/28.7
$< 1/10000$	7	5	5	1.1/7.6	26.2	19.4/35
Structural approach[b]	2		12	7.8/16.2	19.2	

a: All values are in 1990 dollars.
b: Simultaneous equations estimation.

Table 9.1 Data Summary on Value of Life. Source: Viscusi (1993, Table 2).

A number of contingent valuation surveys have been conducted. Viscusi (1993, Table 6) reports results from six studies, with highly variable estimates of the value of life ranging from 1 to 15 million 1990 \$. A careful and well-documented survey is described in Jones-Lee *et al.* (1985), yielding a mean value (for a sample of over 1100 interviews) of 1.5 million 1982 £, but a median value of .8 million only. (It is not surprising that the mean should exceed the median, given the skewness of wealth distributions; but this raises an aggregation problem for public decisions). The main deficiency in the results is the insensitivity of reported willingness-to-pay figures to the assumed reductions in fatality probabilities.[60] A more recent, but more limited survey reported in Beattie *et al.* (1998) confirmed this problem. Some authors, like Krupnick and Cropper (1992) find that responses to risk-risk tradeoffs are more stable than responses to risk-income tradeoffs. Also, several authors, including Viscusi (1993), Beattie *et al.* (1998, p. 7) or Lanoie *et al.* (1995), report substantial variations linked to the contexts of assumed risk reductions (workplace, road, aviation, fires, cancer,...).

[60]Thus, 42% of respondents gave identical answers to questions involving risk reductions from 8/100.000 to either 4/100.000 or 1/100.000; see Jones-Lee *et al.* (1985, p. 67).

The conclusion seems to be that both wage differentials and contingent valuation suggest an *order of magnitude*—say "a few million $" - for the value of a statistical life saved. It is helpful to know that the order of magnitude is there, and not five times lower of five time higher. But little more can be claimed at this time.[61]

10 Conclusion

In this paper, we have presented the axiomatic theory of decision under state-dependent preferences, both for contexts of games against nature and for contexts of one-person games with moral hazard. The axioms are straightforward and compelling. The only problematic one, Existence of Omnipotent Games (6.5), is a structural condition on the environment, not on behaviour; it is amenable to relaxation.

The axioms lead to representation theorems *consistent with* the maximisation of subjectively expected state-dependent utility, or its generalisation to contexts of moral hazard. But separate identification of subjective probabilities and state-dependent utilities is possible only to the extent that the realisation of the corresponding states can be influenced by the decision maker. That identification problem limits the direct applicability of the theory. In games against nature, it is not possible to elicit from *observed* choices the subjective probabilities of experts or the (dis)utility to consumers of specific states.

We have presented in Sections 5 and 7 systematic approaches to achieving identification on the basis of hypothetical preferences. Testing the consistency of these, both internally and vis-à-vis observed choices, is an important ingredient of these approaches.

In general equilibrium economic theory, preferences are allowed to be state dependent, and need not admit an expected utility representation. Specific applications often rest on more structure.

Expected state-dependent utility has been used by many authors as a natural *starting point* in formulating, then analysing, models of decision or allocation in diverse areas of practical relevance, like life or health insurance, safety provision and—increasingly—the provision of health services. Without the axiomatic decision theory, these models would lack theoretical foundations. The representation theorems are thus of genuine theoretical significance; that may also be their prime role.

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[61] An interesting curiosum is due to Broder (1990), who reports that a firm's shareholder equity value on the stock market may fall (temporarily) by as much as 50 millions $ following an accident involving the firm's workplace or products. The fall in equity value is unrelated to the victim's own value of life, but helps explain why some firms promote safety more than others.

In these applications, the identification problem is ignored. In some cases, exogenous "objective" probabilities are used. In other cases, qualitative properties are derived, assuming existence of subjective probabilities that need not be elicited explicitly. In either case, one may wonder whether the representation theorems do indeed provide adequate theoretical foundations, the identification problem notwithstanding. We are not aware of applications whose theoretical underpinning is in jeopardy due to the identification problem. Rather, that problem restricts the range of possible applications, as noted above. Still it is important that researchers developing applications be aware of the identification problem and satisfy themselves as well as their readers (or referees) that their models rest on adequate theoretical foundations.

Appendix

In this appendix, we reproduce a complete proof of Theorem 6.8, using a slightly modified definition of conditional preferences. According to Definition 4.1, $\pi \prec \rho$ given s iff there exists a $g \in G$ such that $(\pi, g_{S\setminus s}) \prec (\rho, g_{S\setminus s})$.

A slight weakening of that definition is:

DEFINITION 4.1' (Weak Conditional Preferences) For all $s \in S$ and $\pi, \rho \in \Pi$, we may say that

(i) $\pi \prec \rho$ given s if and only if there exist an $\alpha \in [0,1]$ and a $g \in G$ such that $(g(s)\alpha\pi, g_{S\setminus s}) \prec (g(s)\alpha\rho, g_{S\setminus s})$;

(ii) $\pi \sim \rho$ given s if and only if neither $\pi \prec \rho$ given s nor $\rho \prec \pi$ given s.

If the property of Independence (see Assumption 3.3) held for conditional preferences, Definitions 4.1 and 4.1' would be equivalent. Independence states precisely that:

$$\pi \prec \rho \text{ given } s \text{ iff } \forall \alpha \in (0,1], \forall g(s) \in \Pi, g(s)\alpha\pi \prec g(s)\alpha\rho \text{ given } s. \qquad (10.1)$$

It was noted in motivating Assumption 6.7 that independence for preferences among lotteries does not imply independence for preferences among the corresponding games. Therefore, Definition 4.1' covers cases not covered by Definition 4.1, and Assumption 4.3 (Conditional Preferences are Free of Inconsistencies) is stronger when applied to Definition 4.1' than when applied to Definition 4.1.

The logical distinction turns out to be of no consequence, however, because Assumption 6.7 (together with the other assumptions) is sufficient to restore the equivalence of the two definitions—without, however, implying the Full Independence Property (10.1) for *all* α and $g(s)$. This equivalence is validated by Lemma 8.2 in Drèze (1987, p. 57); but the proof of Lemma 8.2 is very tedious.

(One referee labeled it "impossible to read"; Drèze agrees and confesses that looking at page 59 invariably makes him dizzy). The motivation for replacing Definition 4.1 by Definition 4.1' is precisely to free the reader from the need to swallow the proof of Lemma 8.2 in Drèze (1987).

When Assumption 4.3 is applied to Definition 4.1' of Conditional Preferences, we label it Assumption 4.3' (Strong Conditional Preferences).

Using the two omnipotent games g_0 and g_1 of Assumption 6.5, we introduce the following (innocuous)

NORMALISATION A.1 $V(g_0) = 0$, $V(g_1) = 1$.

The elicitation of the set O of attainable probabilities in Theorem 6.8 is based on treating g_0 and g_1 as constant-utility games. As explained in Remark 6.12, that procedure yields a set O which is invariant to the arbitrary selection of g_0 and $g_1 \succ g_0$ from the set of omnipotent games, if and only if that set O turns out to be full-dimensional (to have a non-empty interior relative to Δ^S); otherwise, the extent of indeterminacy is as stipulated in Theorem 6.11.

We now give the proof, which rests on two lemmata (corresponding to Lemma 8.2 and Corollary 8.1 in Drèze (1987)). In the following, we assume without explicit reminder Strong Conditional Preferences (4.3'), Value of Information (6.1), Existence of Omnipotent Games (6.5), Independence (6.7) and Continuity (6.6) of Conditional Preferences.

LEMMA A.2 $\pi \precsim \rho$ given s iff, for some $\alpha \in [0,1)$, $[g_1(s)\alpha\pi, g_0(S \setminus s)] \precsim [g_1(s)\alpha\rho, g_0(S \setminus s)]$.

PROOF Write $g_{\alpha\pi}$ for $[g_1(s)\alpha\pi, g_0(S\setminus s)]$ and $g_{\alpha\rho}$ for $[g_1(s)\alpha\rho, g_0(S\setminus s)]$. We only need to show that $\pi \prec \rho$ given s and $g_{\alpha\pi} \sim g_{\alpha\rho}$ cannot both be true. Indeed, by Definition 4.1' and Assumption 4.3', $\pi \sim \rho$ given s implies $g_{\alpha\pi} \sim g_{\alpha\rho}$ and $g_{\alpha\pi} \prec g_{\alpha\rho}$ implies $\pi \prec \rho$ given s, whereas $\pi \prec \rho$ given s and $g_{\alpha\pi} \succ g_{\alpha\rho}$ cannot both be true.

Assume instead that $\pi \prec \rho$ given s and $g_{\alpha\pi} \sim g_{\alpha\rho}$ are both true. Then, there exist $f \in G$ and $\alpha' \in [0,1]$ such that $[f(s)\alpha'\pi, f(S \setminus s)] \prec [f(s)\alpha'\rho, f(S \setminus s)]$. Write ψ_π for $f(s)\alpha'\pi \in \Pi$, ψ_ρ for $f(s)\alpha'\rho \in \Pi$. By Assumptions 6.5 and 6.6 there exists $\beta \in [0,1]$ such that $[g_1(s)\beta\psi_\pi,\, g_0(S \setminus s)] \succ g_0$, $[g_1(s)\beta\psi_\rho,\, g_0(S \setminus s)] \succ g_0$. By Assumption 6.7, it follows that $[g_1(s)\beta\psi_\pi, g_0(S \setminus s)] \sim [g_1(s)\beta\psi_\rho, g_0(S \setminus s)]$; indeed, independence holds over $G_0(s)$. Let $\psi \in \Pi$ verify $g_{\alpha\psi} \succ g_{\alpha\rho}$. Using Assumption 6.6 there exists β' such that $[\psi_\pi\beta'\psi, f(S \setminus s)] \prec [\psi_\rho, f(S \setminus s)]$, so that $\psi_\pi\beta'\psi \prec \psi_\rho$ given s. On the other hand, $[g_1(s)\beta\psi_\pi, g_0(S \setminus s)] \succ [g_1(s)\beta\psi_\rho, g_0(S \setminus s)]$, so that $\psi_\pi\beta'\psi \succ \psi_\rho$ given s. This contradicts Assumption 4.3'. The lemma is thus proved. ∎

We now define "state-dependent utilities" $u_s(\cdot)$ as follows:

DEFINITION A.3 (State-Dependent Utilities) For all $s \in S$, for all $\pi \in \Pi$, let

$$u_s(\pi) = \begin{cases} 0 & \text{if } s \text{ is null;} \\ \left[\frac{V[g_1(s)\alpha\pi, g_0(S\setminus s)]}{V[g_1(s), g_0(S\setminus s)]} - \alpha\right]\frac{1}{1-\alpha} & \text{for } \alpha \text{ such that } [g_1(s)\alpha\pi, g_0(S\setminus s)] \\ & \succ g_0, \text{ if } s \text{ is not null.} \end{cases}$$

The function u_s is thereby uniquely defined, under normalisation A.1.

Indeed, $V[g_1(s), g_0(S \setminus s)] > 0$ by Assumption 6.5, Theorem 3.5 and $V(g_0) = 0$. To show that $u_s(\pi)$ is independent of α, let $\alpha' \neq \alpha$ be such that $V[g_1(s)\alpha'\pi, g_0(S \setminus s)] \succ g_0$. W.l.o.g., let $\alpha' > \alpha$, $\alpha' = \alpha\lambda + 1 - \lambda$ for some $\lambda \in (0, 1)$. By Assumption 6.7, $V[g_1(s)\alpha'\pi, g_0(S \setminus s)] = \lambda V[g_1(s)\alpha\pi, g_0(S \setminus s)] + (1 - \lambda)V[g_1(s), g_0(S \setminus s)]$ so that

$$\begin{aligned} &\left[\frac{V[g_1(s)\alpha'\pi, g_0(S \setminus s)]}{V[g_1(s), g_0(S \setminus s)]} - \alpha'\right]\frac{1}{1-\alpha'} \\ &= \left[\lambda\frac{V[g_1(s)\alpha\pi, g_0(S \setminus s)]}{V[g_1(s), g_0(S \setminus s)]} + 1 - \lambda - \alpha'\right]\frac{1}{\lambda(1-\alpha)} = u_s(\pi). \end{aligned}$$

The definition implies that $u_s(g_0(s)) = 0, u_s(g_1(s)) = 1$ for all $s \in S, s$ not null.

LEMMA A.4 If $u_s(f(s)) \leq u_s(g(s))$ for all $s \in S$, then $f \precsim g$.

PROOF For each $s = 1, ..., S$, define a game h_s by

$$h_s(t) = \begin{cases} g(t) & \forall t = 1, ..., s \\ f(t) & \forall t = s + 1, ..., S. \end{cases}$$

By Lemma A.2, $h_s \precsim h_{s+1}$; so $f \precsim h_1, ..., \precsim h_S = g$. ∎

PROOF OF THEOREM 6.8 For a game $f \in G$, denote by $u(f)$ the vector of state-dependent utilities $u(f) = [u_1(f(1)), ..., u_S(f(S))]$. The function $F : \mathfrak{R} = \times_{s=1,...,S} \mathfrak{R}_s \to R$, defined by $F(x) = V(f)$ for any f such that $u(f) = x$, is well defined. By Lemma A.4, if $u_s(f(s)) = u_s(g(s))$ for all s, then $f \sim g$; so if f' is any other game such that $u(f') = x$, then $V(f) = V(f')$.

From the Assumption 6.1 (Value of Information), the function F is convex. From the Definition A.3 and Assumption 6.5 (Existence of Omnipotent Games), F is homogeneous of degree one; indeed, g_0 is omnipotent with $u(g_0) = 0 \in R^S$; hence, for all $g \in G$, for all $\alpha \in [0, 1], \alpha F(u(g)) + (1-\alpha)\ F(u(g_0)) = \alpha F(u(g)) = F(\alpha u(g) + (1 - \alpha)u(g_0)) = F(\alpha u(g))$. Also F is continuous.

Extend F to a function from R^S to $R \cup +\infty$ by first extending its domain of definition, by homogeneity of degree one, to the smallest convex cone containing the set $\mathcal{R}$; and then extend its domain to the entire space by setting F equal to $+\infty$ on the complement of this cone. Note that F so extended is still homogenous of degree one and convex. Therefore, by Corollary 13.2.1 in Rockafellar (1970) it is the support function of a uniquely determined closed and convex set

$$O^* = \{\sigma \in R^S | \forall u \in \mathcal{R}, \sigma.u \leq F(u)\}.$$

Note that $F(u) = \sup_{\sigma \in O^*} \sigma.u$. Define $O = O^* \cap \Delta^S$. We claim that for all $f \in G$ there is a $\sigma \in O$ such that $F(u(f)) = \sigma.u(f)$. The proof of our claim consists of two steps.

(i) Let ι denote the vector $(1, ..., 1) \in R^S$. For the game g_1, with $u(g_1) = \iota, F(u(g_1)) = \sup_{\sigma \in O^*} \sigma.\iota = 1$, so that $\sigma.\iota \leq 1$ for all $\sigma \in O^*$.

For the game $[g_1(s), g_0(S \setminus s)]$, with $u[g_1(s), g_0(S \setminus s)] = \delta_s$, $F[u(g_1(s), g_0(S \setminus s)] = \sup_{\sigma \in O^*} \sigma.\delta_s \leq 1$, so that $\sigma_s \leq 1$ for all $\sigma \in O^*$, for all s.

For the game $[g_0(s), g_1(S \setminus s)]$ with $u[g_0(s), g_1(S \setminus s)) = \iota - \delta_s$, $F[u(g_0(s), g_1(S \setminus s)] = \sup_{\sigma \in O^*} \sigma.(\iota - \delta_s) = 1 - \inf_{\sigma \in O^*} \sigma_s \leq 1$, so that $\sigma_s \geq 0$ for all $\sigma \in O^*$, for all s. Accordingly, the closed set O^* is contained in $\{\sigma \in R^S | 1 \geq \sigma_s \geq 0 \text{ for all } s, \sigma.\iota \leq 1\}$. Hence O^* is compact and, for all $u \in \mathcal{R}$, a bounded set, $\sup_{\sigma \in O^*} \sigma.u = \max_{\sigma \in O^*} \sigma.u$.

(ii) For an arbitrary game f, consider the lotteries $f\alpha g_1, \alpha \in (0, 1)$ and the corresponding games $g_{f\alpha g_1}$, for which $V(g_{f\alpha g_1}) = F[\alpha u(f) + (1 - \alpha)u(g_1)] = \max_{\sigma \in O^*} \sigma.[\alpha u(f) + (1 - \alpha)\iota]$. Denote by $\sigma^*_{f\alpha g_1}$ an element of O^* at which the maximum is attained. Because g_1 is omnipotent, $g_{f\alpha g_1} \sim f\alpha g_1$, so that

$$\begin{aligned} V(g_{f\alpha g_1}) = \alpha V(f) + (1 - \alpha)V(g_1) &= \alpha \max_{\sigma \in O^*} \sigma.u(f) + (1 - \alpha) \\ &= \alpha \sigma^*_{f\alpha g_1}.u(f) + (1 - \alpha)\sigma^*_{f\alpha g_1}.\iota. \end{aligned}$$

Because $\max_{\sigma \in O^*} \sigma.u(f) \geq \sigma^*_{f\alpha g_1}.u(f)$ and $1 \geq \sigma^*_{f\alpha g_1}.\iota$, it must be the case that $\max_{\sigma \in O^*} \sigma.u(f) = \sigma^*_{f\alpha g_1}.u(f)$ *and* $\sigma^*_{f\alpha g_1}.\iota = 1$. Thus there exists $\sigma^*_{f\alpha g_1}$ in O^* with $\sigma^*_{f\alpha g_1}.\iota = 1$ such that $V(f) = F(u(f)) = \sigma^*_{f\alpha g_1}.u(f)$. That is, for all $f \in G$, there exists $\sigma \in O$ with $V(f) = \sigma.u(f)$. ∎

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