General equilibrium models of monopolistic competition: CRRA versus CARA*

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Abstract

We analyze a class of 'large group' Chamberlinian monopolistic competition models using multiplicatively quasi-separable (MQS) and additively quasi-separable (AQS) functions. We first prove that the MQS and AQS functions are equivalent to the 'constant relative risk aversion' (CRRA) and 'constant absolute risk aversion' (CARA) classes of functions, respectively. Whereas both approaches allow for closed-form solutions, only the AQS functions yield profit-maximizing prices that decrease in the mass of competing firms. We then characterize the equilibrium mass of firms in both cases.

Keywords: monopolistic competition; general equilibrium; price equilibrium; additive quasi-separability; multiplicative quasi-separability

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1 Introduction

Within a quarter century, Dixit and Stiglitz's (1977) model of monopolistic competition has become a major analytical framework in many economic fields such as international trade, monetary economics, economic growth, economic development, and economic geography.[1] One of the reasons explaining the success of the 'constant elasticity of substitution' (CES) model is its analytical tractability, which allows us to explore the issues of imperfect competition and increasing returns in general equilibrium. This advantage, however, comes at a cost. Indeed, the large group approximation of the elasticity of demand leads to profit-maximizing prices with a constant mark-up over marginal cost. Such a feature may be a drawback in fields of economics where *pro-competitive effects*, i.e., profit-maximizing prices are decreasing in the mass of competing firms, are important for explaining various behaviors, both from a positive and a normative point of view.

The main objective of this paper is to develop a 'large group' Chamberlinian monopolistic competition model that allows for closed-form solutions and pro-competitive effects. To achieve this, we introduce multiplicatively quasi-separable (MQS) and additively quasi-separable (AQS) classes of functions. We prove that MQS and AQS functions are equivalent to 'constant relative risk aversion' (CRRA) and 'constant absolute risk aversion' (CARA) functions, respectively.[2] Whereas both the MQS and AQS approach yield closed-form solutions, only the AQS approach can generate pro-competitive effects.

To the best of our knowledge, there exists so far no 'large group' Chamberlinian monopolistic competition model yielding closed-form solutions and pro-competitive effects in a general equilibrium framework.[3] Although Yang

[1] Well-known contributions using the Dixit-Stiglitz model include: Krugman (1980) and Helpman and Krugman (1985) for international trade; Blanchard and Kiyotaki (1987) for monetary economics; Romer (1990) and Grossman and Helpman (1991) for innovation and growth; the special issue of the *Journal of Development Economics*, vol. 49(1), 1996; Krugman (1991) and Fujita *et al.* (1999) for economic geography. See also Matsuyama (1995) and Brakman and Heijdra (2004) for recent surveys of developments drawing on this framework.

[2] This result may be related to the specifications in Blanchard and Fisher (1989, Ch. 2) who assume that instantaneous utility functions are of the CRRA and CARA types in intertemporal optimization models. Whereas they focus on the instantaneous elasticity of substitution in the absence of product differentiation, we focus on the elasticity of demand when products are differentiated.

[3] Ottaviano *et al.* (2002) develop a continuum model of monopolistic competition with pro-competitive effects, but their quasi-linear specification gives their framework a partial equilibrium flavor. Galí (1995) develops a CES general equilibrium model with pro-

and Heijdra (1993) and d'Aspremont *et al.* (1996) have shown that pro-competitive effects can be restored in the CES model when the impact of firms' decisions on market aggregates is taken into account, doing so *requires that the number of firms be relatively small.* This violates the 'large group' assumption that individual agents are negligible and cannot influence market aggregates (see Chamberlin, 1933; Hart, 1985; Dixit and Stiglitz, 1993; Matsuyama, 1995).

A natural way of modeling 'large group' monopolistic competition is by assuming that there is a continuum of horizontally differentiated varieties. The advantages of this assumption are that firms are negligible and that there is no integer problem. However, the continuum assumption is incompatible with the existence of pro-competitive effects in the CES model. In this paper, we show that the continuum approach and pro-competitive effects can be reconciled in general equilibrium by using CARA instead of CES preferences.

The remainder of the paper is organized as follows. Section 2 presents the consumer problem, introduces the concept of quasi-separability and characterizes the MQS and AQS classes of functions. It is of interest to note that the results we derive in this section are general and hold regardless of market structure. Sections 3 and 4 then focus on monopolistic competition and derive the price equilibrium and the equilibrium mass of firms, respectively. Section 5 reinterprets the consumer problem in terms of the producer problem with differentiated intermediate inputs. Finally, Section 6 concludes.

2 Quasi-separable preferences

In this section, we introduce two concepts of quasi-separability and derive closed-form solutions for the consumer demands.[4] We then discuss, in the next section, when the profit-maximizing prices display pro-competitive effects given these demands.

Consider an economy with a single consumption good, which is provided as a continuum of horizontally differentiated varieties. The utility maximization problem of a representative consumer can then be expressed as

competitive effects by assuming that the elasticity of substitution depends on the mass of firms.

[4]Let us emphasize from the beginning that the concept of quasi-separability that we use in this paper is, to the best of our knowledge, not related to the concepts of homothetic separability or additive separability in the sense of Blackorby *et al.* (1970).

follows:

$$\max_{q(i),\, i\in[0,N]} \quad U \equiv \int_0^N u\big(q(i)\big)\mathrm{d}i$$

$$\text{s.t.} \quad \int_0^N p(i)q(i)\mathrm{d}i = E$$

where $E > 0$ stands for expenditure, $p(i) > 0$ and $q(i) \geq 0$ stand for the price and the quantity of variety i respectively. We henceforth assume that $u : \mathbb{R}^+ \to \mathbb{R}^+$ is a twice continuously differentiable function satisfying $u' > 0$ and $u'' < 0$, which ensures that u' has a continuously differentiable inverse. Letting λ be the Lagrange multiplier, the Lagrangian for the maximization problem is given by:

$$\mathcal{L}(q,\lambda) = \int_0^N u\big(q(i)\big)\mathrm{d}i + \lambda \left[E - \int_0^N p(i)q(i)\mathrm{d}i\right]$$

The first-order conditions for an interior solution can then be expressed as follows:

$$\frac{\partial \mathcal{L}}{\partial q(i)} = u'\big(q(i)\big) - \lambda p(i) = 0, \qquad \forall i \in [0,N] \tag{1}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = E - \int_0^N p(i)q(i)\mathrm{d}i = 0. \tag{2}$$

Let $\varphi \equiv (u')^{-1}$, which is a continuously differentiable function. Taking the ratio of (1) with respect to i and j, and rearranging the resulting expression, we obtain

$$\varphi^{-1}\big(q(i)\big) = \varphi^{-1}\big(q(j)\big)\frac{p(i)}{p(j)} \qquad \forall i,j. \tag{3}$$

We can then apply φ to both sides of (3) to get

$$q(i) = \varphi\left[\varphi^{-1}(q(j))\frac{p(i)}{p(j)}\right] \qquad \forall i,j. \tag{4}$$

As can be seen from (4), we cannot generally solve for the demand functions q without imposing some further restrictions on φ. We thus limit ourselves to the following two cases:

$$\varphi(xy) = \varphi(x) \times f(y) \tag{5}$$

$$\varphi(xy) = \varphi(x) + f(y) \tag{6}$$

where f is assumed to be a continuously differentiable and strictly decreasing function. A function satisfying (5) will be called *multiplicatively quasi-separable*, whereas a function satisfying (6) will be called *additively quasi-separable*.[5] Note that a MQS function must satisfy $f(1) = 1$, whereas an AQS function must satisfy $f(1) = 0$, thus imposing further restrictions on the functional forms.

Let us begin by characterizing the classes of MQS and AQS functions with the following representation theorems.

Theorem 1 (MQS representation) *A continuously differentiable function $\varphi \equiv (u')^{-1} : \mathbb{R}^+ \to \mathbb{R}^+$ is MQS if and only if it is of the CRRA type, i.e., $\varphi(x) = \alpha x^\beta$. Then, the functions u and f are also of the CRRA type.*

Proof. (i) Let us first show that a MQS function is of the CRRA type. Assume that φ is MQS. By assumption on u, φ is continuously differentiable. Let $a > 0$ and $b > 0$ be arbitrarily given. Differentiating (5), we have:

$$\varphi'(ab)b = \varphi'(a)f(b) \qquad \text{and} \qquad \varphi'(ab)a = \varphi(a)f'(b) \qquad \forall a > 0, b > 0.$$

Since $\varphi' < 0$, $a > 0$ and $b > 0$, it follows that φ is strictly positive, so that all expressions are non-zero. We can therefore take the ratio to get:

$$\frac{b}{a} = \frac{\varphi'(a)f(b)}{\varphi(a)f'(b)} \qquad \Rightarrow \qquad \frac{\varphi'(a)}{\varphi(a)}a = \frac{f'(b)}{f(b)}b \qquad \forall a > 0, b > 0. \tag{7}$$

Keeping a fixed, we then see that the elasticity of f must be constant, whereas keeping b fixed we see that the elasticity of φ must be constant. Therefore, $a\varphi'(a)/\varphi(a) = c$, where c is a constant. Integrating this expression, we get $\ln(\varphi(a)) = c \ln a + k$, where k is the constant of integration. Thus, $\varphi(a) = \mathrm{e}^{c \ln a}\mathrm{e}^k = a^c \mathrm{e}^k$. It then follows that the function u must be of the CRRA type. Given (7), f must also be of the CRRA type.

(ii) Let us next show that a CRRA function is MQS. Assume that u is of the CRRA form, i.e., $u(a) = \kappa a^\rho$ with $0 < \rho < 1$. It is readily verified that

$$(u')^{-1}(a) = \varphi(a) = \left(\frac{a}{\kappa\rho}\right)^{1/(\rho-1)},$$

which implies that u must have the MQS property, because

$$\varphi(ab) = \left(\frac{a}{\kappa\rho}\right)^{1/(\rho-1)} b^{1/(\rho-1)} = \varphi(a) \times f(b), \qquad f(b) = b^{1/(\rho-1)}. \tag{8}$$

[5] In the special case where $f \equiv \varphi$, we say that the function is *multiplicatively separable* (resp. *additively separable*).

This proves our claim. ■

Theorem 2 (AQS representation) *A continuously differentiable function* $\varphi \equiv (u')^{-1} : \mathbb{R}^+ \to \mathbb{R}^+$ *is AQS if and only if it is of the logarithmic type, i.e.,* $\varphi(x) = \alpha \ln x + \beta$*. Then, the function* u *is of the CARA type.*

Proof. (i) Let us first show that an AQS function is of the logarithmic type. Assume that φ is AQS. By assumption on u, φ is continuously differentiable. Let $a > 0$ and $b > 0$ be arbitrarily given. Differentiating (6), we have:

$$\varphi'(ab)b = \varphi'(a) \qquad \text{and} \qquad \varphi'(ab)a = f'(b) \qquad \forall a > 0, b > 0.$$

Since $\varphi' < 0$, $a > 0$ and $b > 0$, all expressions are non-zero. We can therefore take the ratio to get

$$\frac{b}{a} = \frac{\varphi'(a)}{f'(b)} \qquad \Rightarrow \qquad f'(b)b = \varphi'(a)a \qquad \forall a > 0, b > 0.$$

Keeping a fixed, we then see that $f'(b)b$ must be constant, whereas keeping a fixed we see that $\varphi'(a)a$ must be constant. We thus have $\varphi'(a)a = c$, where c is a constant. This yields by integration $\varphi(a) = c \ln a + k$, where k is the constant of integration. It is then readily verified that $u'(a) = \mathrm{e}^{a/c}\mathrm{e}^{-k}$, which proves that u must be of the CARA type.

(ii) Let us next show that a CARA function is AQS. Assume that u takes the CARA form

$$u(a) = k - \kappa \mathrm{e}^{-\alpha a}, \tag{9}$$

where $\alpha > 0$ and $k \geq \kappa > 0$. Note that $u(0) = k - \kappa \geq 0$, $u' > 0$ and $u'' < 0$. It is readily verified that

$$u'(a) = \alpha\kappa \mathrm{e}^{-\alpha a}, \qquad (u')^{-1}(a) = \varphi(a) = \frac{1}{\alpha} \ln\left(\frac{\alpha\kappa}{a}\right),$$

which shows that φ is AQS because

$$\varphi(ab) = \varphi(a) + f(b), \qquad f(b) = \frac{1}{\alpha} \ln\left(\frac{1}{b}\right). \tag{10}$$

This proves our claim. ■

Note finally that, as required by our assumption, f is a decreasing function as shown by (8) and (10).

We now derive closed-form solutions for the demand functions in the MQS and the AQS cases.

(i) MQS: Assume that φ is MQS, so that (4) reduces to:

$$q(i) = q(j) f\left[\frac{p(i)}{p(j)}\right] \qquad \forall i, j.$$

Dividing by f, multiplying both sides by $p(j)$, integrating over j and using the budget constraint yields the following demand functions:

$$q(i) = E\left[\int_0^N \frac{p(j)}{f\big(p(i)/p(j)\big)}\mathrm{d}j\right]^{-1}. \tag{11}$$

Note that (11) is increasing in E. Because f is decreasing, (11) is also decreasing in $p(i)$. Note finally that since $f(1) = 1$, $q(i) = E/Np$ holds for symmetric prices.

(ii) AQS: Assume that φ is AQS, so that (4) reduces to:

$$q(i) = q(j) + f\left[\frac{p(i)}{p(j)}\right] \qquad \forall i, j.$$

This can be solved for $q(i)$ by multiplying both sides by $p(j)$, integrating over j, and using the budget constraint, which yields the following demand functions:

$$q(i) = \frac{E + \displaystyle\int_0^N f\big(p(i)/p(j)\big)p(j)\mathrm{d}j}{\displaystyle\int_0^N p(j)\mathrm{d}j}. \tag{12}$$

Note that (12) is increasing in E. Because f is decreasing, (12) is also decreasing in $p(i)$. Note finally that since $f(1) = 0$, $q(i) = E/Np$ holds for symmetric prices.

3 The price equilibrium

We next derive the price equilibrium and discuss its properties when φ is either MQS or AQS. To keep the analysis simple, we assume that labor is the only factor of production. In order to produce q units of any variety, $l = mq + F$ units of labor are required, where F and m stand for the fixed and marginal input requirements, respectively.

Assuming that there are L consumers in the economy, the profit of firm i is as follows:

$$\Pi(i) = Lq(i)\big[p(i) - mw\big] - Fw, \tag{13}$$

where $q(i)$ is given by (11) or (12), depending on whether φ is MQS or AQS. We also assume that the labor market is perfectly competitive, so that each firm takes the wage rate w as given.

(i) MQS: Assume that φ is MQS. Plugging (11) into (13), it is readily verified that, given the continuum assumption, the first-order condition for profit maximization with respect to $p(i)$ can be expressed as follows:

$$[p(i) - mw]\int_0^N \frac{f'\big(p(i)/p(j)\big)}{\big[f\left(p(i)/p(j)\right)\big]^2}\mathrm{d}j + \int_0^N \frac{p(j)}{f\left(p(i)/p(j)\right)}\mathrm{d}j = 0. \tag{14}$$

A price distribution p satisfying (14) for all $i \in [0, N]$ will be called a *MQS price equilibrium.*

The Representation Theorem 1 establishes that $u\big(q(i)\big) = \kappa q(i)^\rho$, where we set $\kappa = 1$ without loss of generality. As shown by expression (8),

$$f\left[\frac{p(i)}{p(j)}\right] = \left[\frac{p(i)}{p(j)}\right]^{\frac{1}{\rho-1}}$$

holds in the MQS case. Inserting this expression into the first-order conditions (14), we obtain

$$\frac{\rho}{\rho-1}\left[p(i) - \frac{mw}{\rho}\right]p(i)^{1/(1-\rho)}\int_0^N p(j)^{\rho/(\rho-1)}\mathrm{d}j = 0, \tag{15}$$

which is equivalent to $p(i) = mw/\rho$ for all $i \in [0, N]$. This shows that *the price set by firm i does not depend on the prices set by the other firms.* Hence, the price equilibrium is symmetric and unique and has a constant multiplicative mark-up over marginal cost:

$$p^{\mathrm{MQS}} = \frac{mw}{\rho}\,\cdot \tag{16}$$

We have established the following result:

Proposition 1 *When φ is MQS, the price equilibrium is symmetric and unique. The profit-maximizing price is increasing in marginal cost, and independent of both expenditure and the mass of competing firms. The mark-up rate is constant and positive regardless of industry size.*

(ii) AQS: Assume now that φ is AQS. Plugging (12) into (13), it is readily verified that, given the continuum assumption, the first-order condition for profit maximization with respect to $p(i)$ is given by

$$E+\int_0^N f\left(\frac{p(i)}{p(j)}\right)p(j)\mathrm{d}j+[p(i)-mw]\int_0^N f'\left(\frac{p(i)}{p(j)}\right)\mathrm{d}j=0. \tag{17}$$

A price distribution p satisfying (17) for all $i\in[0,N]$ will be called an *AQS price equilibrium.*

The Representation Theorem 2 establishes that $u\big(q(i)\big)=k-\kappa\mathrm{e}^{-\alpha q(i)}$, where $\alpha>0$ is a parameter related to the concavity of u. Note that $U=\int_0^N u\big(q(i)\big)\mathrm{d}i$ displays 'love for variety'. To see this, let $\overline{Q}\equiv Nq$ stand for a fixed level of total consumption when the consumer spreads his expenditure equally over all varieties. We then have

$$U=N\left(k-\kappa\mathrm{e}^{-\alpha\frac{\overline{Q}}{N}}\right)\qquad\text{so that}\qquad\frac{\partial U}{\partial N}=k-\kappa\mathrm{e}^{-\alpha\frac{\overline{Q}}{N}}\left(1+\alpha\frac{\overline{Q}}{N}\right)\geq 0.$$

To obtain the last inequality, let $x\equiv\alpha\overline{Q}/N$ and define $g(x)=\kappa e^{-x}(1+x)$. Clearly, $g(0)=\kappa\geq 0$ and $g'(x)<0$ for all $x>0$, which implies the result.

In what follows, we set $k=\kappa=1$ without loss of generality, so that

$$u\big(q(i)\big)=1-\mathrm{e}^{-\alpha q(i)}. \tag{18}$$

As shown by expression (10)

$$f\left[\frac{p(i)}{p(j)}\right]=-\frac{1}{\alpha}\ln\left[\frac{p(i)}{p(j)}\right]. \tag{19}$$

Inserting this expression into (17) and letting $P\equiv\int_0^N p(j)\mathrm{d}j$, the demand functions can be rewritten as follows:

$$q(i)\;=\;\frac{E}{P}-\frac{1}{\alpha}\left[\ln\left(\frac{p(i)}{P}\right)+\mathcal{S}(p)\right], \tag{20}$$

where

$$\mathcal{S}(p)=-\int_0^N\ln\left(\frac{p(j)}{P}\right)\frac{p(j)}{P}\mathrm{d}j$$

stands for Shannon's (1948) entropy. It is readily verified that the own-price elasticity of demand of firm j is equal to

$$\eta(j)\equiv-\frac{p(j)}{q(j)}\frac{\partial q(j)}{\partial p(j)}=\frac{1}{\alpha q(j)}, \tag{21}$$

which is monotonically decreasing in $q(j)$.

Using f as defined by (10), the first-order conditions (17) for profit maximization can be expressed as follows:

$$\int_0^N \ln\left(\frac{p(i)}{p(j)}\right)p(j)\mathrm{d}j + [p(i) - mw]\int_0^N \frac{p(j)}{p(i)}\mathrm{d}j = \alpha E. \tag{22}$$

It is of interest to note that although no individual firm has an impact on the market aggregates, the market aggregates have an impact on each individual firm. Hence, there is 'weak strategic interdependence' in the sense that to determine its own price, each firm has to take the price aggregates into account. This is a major departure from the MQS case in which, as shown above, each firm can determine its equilibrium price without knowing the price aggregates.

We can then establish the following result.

Proposition 2 *The AQS price equilibrium is symmetric and unique.*

Proof. Let us first prove that the AQS price equilibrium is symmetric. Assume that p is an AQS price equilibrium satisfying $\overline{p} = \max_i\ p(i) > \underline{p} = \min_i\ p(i)$. By definition of an equilibrium,

$$\int_0^N \ln\left(\frac{\overline{p}}{p(j)}\right)p(j)\mathrm{d}j + [\overline{p} - mw]\int_0^N \frac{p(j)}{\overline{p}}\mathrm{d}j \quad = \quad \alpha E, \tag{23}$$

$$\int_0^N \ln\left(\frac{\underline{p}}{p(j)}\right)p(j)\mathrm{d}j + [\underline{p} - mw]\int_0^N \frac{p(j)}{\underline{p}}\mathrm{d}j \quad = \quad \alpha E \tag{24}$$

must hold. Subtracting (24) from (23) and rearranging, we get

$$\left[\ln\left(\frac{\overline{p}}{\underline{p}}\right) + \frac{mw(\overline{p} - \underline{p})}{\overline{p}\ \underline{p}}\right]\int_0^N p(j)\mathrm{d}j = 0,$$

which cannot hold because $\overline{p} > \underline{p}$. Hence, every AQS price equilibrium must satisfy $\overline{p} = \underline{p}$, which implies that it must be symmetric. Evaluating (22) at a symmetric price yields

$$p^{\mathrm{AQS}} = mw + \frac{\alpha E}{N}, \tag{25}$$

which is the unique solution. ■

Expression (25) shows that the profit-maximizing price features pro-competitive effects. More precisely, it is decreasing with respect to the mass of firms N.[6] Using (21) one can verify that the elasticity of demand in the symmetric equilibrium is given by

$$\eta = 1 + mw\frac{N}{\alpha E},$$

which is increasing with respect to the mass of firms. Since η goes to infinity as N goes to infinity, prices converge to marginal costs at a rate $1/N$, as they usually do in models of oligopolistic competition à la Bertrand and Cournot (see Vives, 1985).

Let us summarize these results as follows:

Proposition 3 *When φ is AQS, the profit-maximizing price is increasing in expenditure, increasing in marginal cost, and decreasing in the mass of competing firms. It converges to marginal cost at a rate of $1/N$.*

4 The equilibrium mass of firms

So far, we have shown that the price equilibrium in both the MQS and AQS cases is symmetric and unique. We now determine the equilibrium mass of firms which must satisfy the following two conditions: (i) profits are zero due to free entry and exit, i.e., $\Pi(i) = Lq(i)[p(i) - cw] - Fw = 0$ for all i; and (ii) the labor market clears, i.e., $\int_0^N [cLq(i) + F]\mathrm{d}i = L$.

Given the fact that the price equilibrium is symmetric and unique, these two conditions are equivalent in a static economy where the expenditure E is equal to the wage rate w. Thus, we can use either of them to solve for the equilibrium mass of firms.

(i) MQS: In equilibrium, we have $q(i) = w/Np$ for all i. Using the MQS equilibrium price (16) and the zero profit condition, we obtain the following equilibrium mass of firms:

$$N^{\text{MQS}} = \frac{L(1-\rho)}{F} > 0. \tag{26}$$

(ii) AQS: In equilibrium, we have $q(i) = w/Np$ for all i. Using the AQS equilibrium price (25) and the zero profit condition, we obtain the following

[6] It is of interest to note that equation (25) closely resembles the expression in Salop (1979, p. 147), except for the role of consumer expenditure and wage.

equilibrium mass of firms:[7]

$$N^{\mathrm{AQS}} = \frac{\sqrt{4\alpha mFL + (\alpha F)^2} - \alpha F}{2mF} > 0. \tag{27}$$

In what follows, we explore the difference between N^{MQS} and N^{AQS}.

5 Intermediate inputs

An alternative way of interpreting the Dixit-Stiglitz model is in terms of production in the presence of intermediate inputs (see Ethier, 1982; Romer, 1990; Grossman and Helpman, 1991). In this section we reinterpret the AQS model in a similar way. In place of the consumption problem developed previously, we now assume that a final good is produced by a continuum of horizontally differentiated intermediate inputs.

We assume that there are L final good producers. Given an output level Q and the mass N of available inputs, each final producer chooses quantities of the intermediate inputs so as to minimize her production costs:

$$\min_{q(i),\, i\in[0,N]} \int_0^N p(i)q(i)\mathrm{d}i$$

$$\text{s.t.} \quad \int_0^N [1-\mathrm{e}^{-\alpha q(i)}]\mathrm{d}i = Q$$

where $p(i) > 0$ and $q(i) \geq 0$ stand for the price and the quantity of intermediate input i respectively. In what follows, we assume that $Q < N(1-\mathrm{e}^{-1})$. As shown below, this inequality makes sure that the profit-maximizing prices in the intermediate sector are positive. It is also sufficient for the cost minimization problem to be well defined.

Letting λ be the Lagrange multiplier and denoting the Lagrangian by $\mathcal{L}$, the first-order conditions for an interior solution are as follows:

$$\frac{\partial \mathcal{L}}{\partial q(i)} = p(i) - \lambda\alpha \mathrm{e}^{-\alpha q(i)} = 0, \qquad \forall i \in [0,N] \tag{28}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = Q - \int_0^N [1-\mathrm{e}^{-\alpha q(i)}]\mathrm{d}i = 0. \tag{29}$$

[7] It is readily verified that the other root, although positive, yields a price below marginal cost and must, therefore, be ruled out.

Taking the ratio of (28) with respect to i and j and rearranging terms, we obtain $-\mathrm{e}^{-\alpha q(i)} = -\mathrm{e}^{-\alpha q(j)}p(i)/p(j)$. Adding 1 to both sides, integrating over i and using the constraint (29), we get

$$Q = N - \frac{\mathrm{e}^{-\alpha q(j)}}{p(j)}\int_0^N p(i)\mathrm{d}i,$$

which can be solved for the demand functions of each final good producer

$$q(j) = \frac{1}{\alpha}\ln\left[\frac{\int_0^N p(i)\mathrm{d}i}{(N-Q)p(j)}\right]. \tag{30}$$

The elasticity of demand for each intermediate input is again given by (21).

We assume that the production technology in the monopolistically competitive intermediate sector is the same as that in Section 3. Each firm maximizes its profit (13) with respect to own price, given the demand function (30). This yields the following first-order conditions:

$$-\frac{p(j)-mw}{p(j)} - \ln\left[\frac{(N-Q)p(j)}{\int_0^N p(i)\mathrm{d}i}\right] = 0 \qquad \forall j \in [0,N]. \tag{31}$$

A price equilibrium is a distribution p such that (31) holds for all $j \in [0,N]$. Using the same argument as in the proof of Proposition 2, it can be readily shown that the price equilibrium is symmetric and unique:

$$p^* = \frac{mw}{1+\ln\left(\frac{N-Q}{N}\right)} > mw. \tag{32}$$

As in the AQS case, the profit-maximizing price is decreasing in N and increasing in w. Note also that it is increasing in Q, which reflects the fact that for any given value of N, the elasticity of demand decreases as final production expands. Finally, it can be verified that $\lim_{N\to\infty} p^* = mw$, thus showing that the profit-maximizing price goes to marginal cost when the mass of intermediate firms becomes arbitrarily large. Hence, we have shown in this section that the AQS model can be readily extended to the case of production with differentiated intermediate inputs. As in the consumption problem, the price equilibrium displays pro-competitive effects.

6 Concluding remarks

We have identified two classes of monopolistic competition models, depending on whether preferences are MQS or AQS. In Section 2 we proved that the MQS functions are equivalent to the CRRA functions, whereas the AQS functions are equivalent to the CARA functions. As shown in Section 3, the price equilibria associated with these two classes display very different properties: the profit-maximizing price is decreasing in the mass of firms in the AQS case, whereas there is no such pro-competitive effect in the MQS case. We then solved for the equilibrium mass of varieties in Section 4. Section 5 finally reinterpreted the consumption model as a production model featuring horizontally differentiated intermediate inputs. We believe that the CARA framework constitutes an alternative to the CES and CRRA models of monopolistic competition and can be used to analyze general equilibrium issues in which pro-competitive effects play a role.

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