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ENDOGENOUS QUALIFICATIONS AND FIRMS' AGGLOMERATION

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Abstract

This paper analyzes firms' location when workers endogenously choose to qualify for professional skills but when they remain uncertain about the potential match between their personal abilities and/or affinities and the firms' specific production tasks. By qualifying in a region where firms agglomerate, workers benefit from higher prospects of good match. At the equilibrium, we show that firms may locate in a single cluster, symmetric clusters or even asymmetric clusters. Comparative statics with respect to product market demand and labor supply parameters are provided.

Keywords: qualifications, firms' location

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1 Introduction

The nature of firm agglomeration into industrial clusters is the subject of numerous studies. It is often argued that when firms locate together, they benefit from costs reductions through the effects of economies of agglomeration. According to Marshall's theories of industry agglomeration, agglomeration in a cluster firstly allows firms to save transport costs on both final products and intermediate goods. Second, agglomeration facilitates industrial spill-over. Finally, it induces workers to acquire the qualifications that are needed by the industry. Whereas the first two explanations of agglomeration have been under extensive study (see e.g. Henderson (1988), Krugman (1991), Matsuyama (1995), Ottaviano and Puga (1998), Feldman and Audretsch (1999) and Fujita, Krugman and Venables (1999)), theoretical issues about qualification acquisition remain relatively uncovered.

Empirically, Dumais, Ellison and Glaeser (2002) show that in the U.S., the adequacy of regional labor supply is a more important predictor of new plant location than the proximity of firms to consumers and suppliers. In Hanson (2001), the evidence of 'localized human capital externalities' is presented as one of the most robust results in the estimation of the nature of agglomeration effects. Hence, there is a need to explore the relationship between plant location and qualification.

The theoretical literature on industry agglomeration and labor qualification is divided in two views. On the one hand, some authors argue that agglomeration is grounded on human capital externalities. Accordingly, workers acquire human capital at a lower cost in a region with large human capital. For instance, Glaeser (1999) and Desmet (2000) postulate that increasing returns to scale in the acquisition of human capital at the regional or urban level stem from more effective and more numerous interactions in denser areas. Also, in dynamic contexts, it is often assumed that within a district, the human capital is acquired at a lower cost by one generation when the human capital accumulated by previous generations is large. Hence human capital externalities between generations are viewed as important elements in the city formation and industry agglomeration (see e.g. Stokey (1991), Redding (1996)).

The second view does not exclude the first. Accordingly, agglomeration of economic activity can be grounded on matching processes in labor markets. Indeed, matches between workers and firms present a circular causality: the workers' probability to find a good match is higher in locations with more firms whereas the firms' probability to find a good match is higher in locations with more workers. Here, the complementarity between workers

and employers provides the rationale for economic clustering.¹ This idea follows a well established literature in which input complementarities foster concentration of economic activities (see Matsuyama (1995) for a review).

In a first approach, the location effects of labor market matching is studied under the assumption of exogenous and diversified qualifications and job requirements (see e.g. Helsley and Strange (1990), Abdel-Rahman and Wang (1995, 1997), Brueckner, Thisse and Zenou (2002), Hamilton, Thisse and Zenou (2000)). Perfect complementarity between workers' qualifications and employers' needs can be obtained through costly training of workers. In this approach, the assumption of imperfect labor markets ensures that the benefits of a better match are shared between firms and workers. Hence, it is in the interests of firms and workers to agglomerate. Another important assumption in these models is the workers' mobility: workers can find a job in a location that is different from their initial location. However, in the absence of migrations between regions, workers cannot move towards the region with the better matches: the agglomeration process breaks down. Although workers' mobility is a relevant assumption for the analysis of economic activity in a small or medium geographical area such as a city, it is questionable in a larger geographical area. Hence, this first approach hardly justifies economic agglomeration at the regional level in the European socio-economic context where language, cultural and institutional differences hinder worker mobility (see Decressin and Fatás (1995)).

We follow a second approach, stressing the role of endogenous and industry specific qualification and introducing a role for uncertainty in the labor market matching. Using this approach we relate labor market matching to economic agglomeration even in the absence of labor mobility and/or imperfections in the labor market, which, to our knowledge, has not been covered in the literature. In the present model, workers choose to educate themselves for industry-specific skills that are required in a modern industry. The type and level of industry specific skills obviously vary according to the industry under study. Thus, skills can be acquired either in technical colleges, or through apprenticeship and professional training, or in under-graduate and post-graduate university programs. Whereas education qualifies workers to the production tasks in the industry, workers remain uncertain about the match between the production tasks and their idiosyncratic abilities and/or affinities. Indeed, although each firm asks for industry-specific skills, it also asks for firm-specific abilities and/or affinities that cannot be acquired

¹Davidson, Martin and Matusz (1991) study the industry specialization when firms need to match two complementary types of workers.

through education. The specialization in specific tasks, the personal affinity with a job, the relationship with the firm's manager, the adherence to the firm's culture and the work environment are factors that affect each worker personal productivity.

By allowing movements in the supply of labor, endogenous formation of human capital and uncertainty in the labor matching replace the assumption about the mobility of workers that is made in the above mentioned literature. Relocation of firms has indeed two opposite effects. In the region where firms agglomerate, relocation of firms firstly increases the labor demand and pushes wages up. Secondly, it raises each worker's probability to find a job that fits his/her abilities and/or affinities, which increases the worker's motivation to qualify. The labor supply increases in that region and wages fall. In the deserted region, these two effects take place in the reverse directions. Therefore, when a group of firms relocate from one region to the other, the direction of the changes in the wage differential between regions is no longer as clear. At the equilibrium, the two effects can be balanced such that each region ends up with asymmetric populations of firms and qualified workers. One region might attract the largest part of the industry, which induces the population of that region to acquire the right qualification for that industry because of better matches. Since few firms locate in the other region, the matching process is bad in that region: the qualification for that industry remains poorly attractive and people prefer to work in alternative sectors. Thus, the complementarity between job requirements and the qualification and affinities of workers plays a crucial role in firms' location.

In equilibrium, firms may agglomerate in a single cluster, or they may spread across regions. In the latter case, firms can spread evenly (symmetric clusters) or unevenly across regions (asymmetric clusters). Because firms can freely re-locate, wages of workers with good matches are equal across regions. Workers with bad matches also benefit from the same wage in all regions. In asymmetric clusters, many qualified workers have a good match and a high wage in the region where firms agglomerate, whereas most workers from the other region do not benefit from a good match and must be content with low wages. Since more workers are qualified in the region where firms cluster, the mean wage is higher there. This result confirms the literature that shows that mean wages are larger in the region where agglomeration takes place (see Henderson (1988), Ciccone and Hall (1996) and Möller and Haas (2002)) and is consistent with wages being equalized between the two regions.

We build on Ottaviano, Tabuchi and Thisse (2002) who present a model of a modern industry in which firms sell differentiated products to customers

located in several regions. The analytical tractability of the model allows to identify all equilibria, including those with asymmetric clusters. We also derive the comparative statics with respect to all economic parameters. In contrast to most of the literature on labor matching and economic clustering, we are able to study the impact of the characteristics of the matching process, product demand and wages in the alternative sector on the firms' location and on the regional workers' level of qualification. We ultimately study the welfare implications of the equilibria. We show that when product demand is not too large, there is too much dispersion from the point of view of all groups of agents, i.e. workers, consumers and firms' owners. Indeed, when more firms are present in a region, the improvement of the matching process overcomes the disadvantage caused by the use of workers in one region only. In contrast, when product demand is large, the welfare implications are less clear-cut and there may exist conflicts of interest between groups of agents. For instance, the symmetric equilibrium can be preferred by consumers and firms owners but not by workers. The reverse set of preferences is also possible.

The paper is organized as follows. Section 2 presents the model and Section 3 studies the location equilibria. Section 4 provides some comparative statics on the location equilibria. Section 5 studies the welfare of workers, consumers and firms' owners. A conclusion follows.

2 The Model

In this paper we build on the Ottaviano, Tabuchi and Thisse (2002) model of imperfect competition with differentiated products. We consider a model with a continuum $[0, 1]$ of differentiated varieties of a commodity. Each variety i of the commodity is produced by a single firm. This continuity assumption allows a characterization of the firms' location choices in a simpler way than models with a finite number of firms.

2.1 Consumers

The representative consumer uses a quantity $q(i)$ of variety $i \in [0, 1]$ and a quantity q_0 of the numéraire. She is endowed with $\bar{q}_0 > 0$ units of the numéraire. The initial endowment $\bar{q}_0$ is supposed to be large enough for the optimal consumption of the numéraire to be strictly positive at the market

outcome. The budget constraint can be written as:

$$\int_0^1 p(i)q(i)di + q_0 = \bar{q}_0$$

where $p(i)$ is the price of variety i .

The representative consumer's utility function is quadratic:

$$U(q_0; q(\cdot)) = \alpha \int_0^1 q(i)di - \frac{\beta - \delta}{2} \int_0^1 q(i)^2 di - \frac{\delta}{2} \int_0^1 \int_0^1 q(i)q(j)diddj + q_0 \quad (1)$$

where $\alpha > 0$ and $\beta > \delta > 0$. In this expression, α measures the intensity of preferences for the differentiated product with respect to the numéraire. The condition $\beta > \delta$ implies that the representative consumer prefers a dispersed consumption of varieties.

Solving the budget constraint for the numéraire consumption, plugging the corresponding expression into (1) and solving the first order conditions with respect to $q(i)$ yields

$$\alpha - (\beta - \delta)q(i) - \delta \int_0^1 q(j)dj = p(i), \quad i \in [0, 1]$$

The demand function for variety $i \in [0, 1]$ can be written as

$$q(i) = a - bp(i) + d \int_0^1 [p(j) - p(i)]dj \quad (2)$$

where $a \equiv \alpha/\beta$, $b \equiv 1/\beta$ and $d \equiv \delta/\beta(\beta - \delta)$.

Parameter a measures the size of the product market. Parameter b gives the link between individual and industry demand. When b increases, consumers are more sensitive to the output price. Parameter d reflects the degree of product differentiation between varieties. When varieties are perfect substitute, $d \rightarrow \infty$, whereas, when they are independent, $d = 0$, and firm i has monopoly power on variety i .

2.2 Workers

Decressin and Fatás (1995), Bentolila (1997) and Faini, Galli, Gennari and Rossi (1997) show that migration between European countries or within each European country is quite low. Mobility is hindered not only between regions of distinct cultural and language characteristics, but also within such regions because of the high propensity to house ownership in Europe and

because of the presence of working partners (see Henley (1998) and Oswald, Gardner and Pierre (2001)). Hence, in accordance with this stylized fact of European labor markets, we assume that workers are immobile in the sense that they cannot move between regions.

We assume that there are two regions A and B and we focus on agglomeration patterns of an industry that requires acquisition of specific skills. In any region, workers who do not acquire the industry-specific skills are employed in a traditional sector that offers a wage v . Workers who invest in education qualify for a job in the industry but face some uncertainty. Although each firm asks for the industry-specific skills, it also asks for firm-specific abilities and/or affinities that cannot be acquired through education. The specialization in specific tasks, the personal affinity with a job, the relationship with the firm's manager, the adherence to the firm's culture and the work environment are factors that affect each worker's personal productivity. In the sequel, we firstly present a stylized model for this matching process. This model offers a function for the probability of a good job match that is simple and analytically tractable. We finally present the qualification choice that workers face before they apply to a qualified job and hence before the matching uncertainty is realized. We derive the workers' effort to qualify for professional skills as well as their labor supplies.

Consider first each worker after qualification. For simplicity, we assume that qualified workers are able to contemplate all firms in their own region $K \in \{A, B\}$. If a qualified worker is lucky, he finds the firm that fits his abilities and/or affinities in his own region. The worker works at a high productivity, say 1 efficiency unit, and gets the wage w_K . If the qualified worker is unlucky, there exists no firm that fits his abilities and/or affinities in his region. The worker may still find a job in the industry at a lower productivity, say θ efficiency unit ($\theta < 1$). We assume that both types of workers are perfect substitutes. In this case, the unlucky worker receives the wage θw_K . Finally, qualified workers work in the traditional sector at wage v when there is no industry in the region or when their wage is too low, i.e. when $\theta w_K < v$.

Qualified workers are more likely to find the jobs with high productivity when more firms locate in their region. Let each set of abilities and/or affinities be indexed by $s \in [0, I]$. Because abilities and/or affinities are also specific to the firms and their industry, a worker can be informed about his set of abilities and/or affinities only after qualification. A same set of abilities and/or affinities s can be shared by many workers. For tractability, we assume that, in every region, workers are uniformly distributed across the sets of abilities and/or affinities. As will be explained in Section 2.3, we

also assume that each firm produces one variety of the commodity. Each firm is then indexed according to the variety $i \in [0, 1]$. Since the distribution of variety is uniform, the distribution of firms is also uniform. We say that worker with abilities and/or affinities s finds the job that fits his abilities and/or affinities in the firm i if and only if $s = i$. If the number of firms in region K is N_K , the probability that a worker s finds the job with the high productivity 1 in his/her region, is $g(N_K) = N_K/I$. The probability of getting a job at productivity θ is $1 - g(N_K) = 1 - N_K/I$. Under these assumptions, the parameter I represents the amount of uncertainty in the job match. A larger I yields a lower probability of a good match, $g(N_K)$. Obviously, I must be greater than $N_A + N_B = 1$ to keep the probability $g(N_K)$ lower than 1.²

Consider now workers before qualification choice. Investing in industry-specific education is costly. The effort $e \geq 0$ to acquire the needed skills varies across people in the region. For analytical tractability, we assume that each worker's utility function is linear in income and effort:

$$V(w, e) = w - v - e,$$

where the utility of workers who do not qualify for the industry-specific skills ($e = 0$) and who work in the traditional sector ($w = v$) is normalized to zero. Workers choose to qualify when their expected utility from qualification is larger than the utility of working in the traditional sector ($V(w, e) \geq 0$).

When there is no industry in the region, or when the wage of high-productivity workers in the industry is lower than the wage in the traditional sector, all workers find a job in the traditional sector and get the wage v . Thus, workers have negative expected utility levels if they qualify:

$$-e \leq 0, \text{ if } N_K = 0 \text{ or } w_K \leq v.$$

When the wage of low-productivity workers (θw_K) is larger than v , any worker who qualifies gets a job in the industry. Thus, his expected utility is

$$(w_K - v)g(N_K) + (\theta w_K - v)(1 - g(N_K)) - e, \text{ if } N_K > 0 \text{ and } \theta w_K > v.$$

When the wage of low-productivity workers is equal to v , a worker who qualifies but who does not find a firm that fits his abilities and/or affinities,

²These assumptions can be altered. In this case, more complicated formula for the probability $g(N_K)$ will be derived. In general, $g(N_K)$ needs only to be increasing in N_K and equal to zero at $N_K = 0$ for the model to be valid.

gets a wage v in the traditional sector *or* in the industry. His expected utility is

$$(v/\theta - v)g(N_K) - e, \text{ if } N_K > 0 \text{ and } \theta w_K = v.$$

Finally, when the wage of low-productivity workers is lower than v , a worker who qualifies but who does not find a firm that fits his abilities and/or affinities, gets a wage v *only* in the traditional sector. Thus, his expected utility is

$$(w_K - v)g(N_K) - e, \text{ if } N_K > 0 \text{ and } \theta v < \theta w_K \leq v.$$

As a result, the effort of the last worker who qualifies in region K is given by

$$\tilde{e}_K = \begin{cases} 0 & \text{if } N_K = 0 \text{ or } w_K \leq v, \\ \tilde{e}_K^1 & \text{if } N_K > 0 \text{ and } \theta w_K > v, \\ \tilde{e}_K^2 & \text{if } N_K > 0 \text{ and } \theta w_K = v, \\ \tilde{e}_K^3 & \text{if } N_K > 0 \text{ and } \theta v < \theta w_K < v, \end{cases}$$

where

$$\begin{aligned} \tilde{e}_K^1 &= w_K [g(N_K) + \theta(1 - g(N_K))] - v, \\ \tilde{e}_K^2 &= v \frac{1 - \theta}{\theta} g(N_K) \\ \tilde{e}_K^3 &= (w_K - v) g(N_K). \end{aligned} \tag{3}$$

In this paper, we assume that the effort needed for qualification is uniformly distributed and independent of the set of abilities and/or affinities of workers. We denote $F(e) = e$ the number of workers who need an effort that is lower than e to qualify. Hence, the number of workers who qualify in region K is $F(\tilde{e}_K)$.

The number of efficiency units supplied in region K is equal to the number of efficiency units offered by the sum of qualified workers who find a job that fits their abilities and those who do not. The labor supply in the industry is either given by all workers or only by the lucky. When there is no industry or when the wage is not attractive enough, the labor supply falls to zero:

$$L_K^S = 0 \text{ if } N_K = 0 \text{ or } w_K \leq v.$$

When the wage in the industry is high enough to attract the low-productivity workers, the labor supply includes high- and low-productivity workers:

$$L_K^{S1} = \tilde{e}_K^1 [g(N_K) + \theta(1 - g(N_K))] \text{ if } N_K > 0 \text{ and } \theta w_K > v. \tag{4}$$

When the wage of low-productivity workers in the industry is just equal to v , these workers are indifferent between getting a job in the industry and in the traditional sector. Low-productivity workers supply at most $\tilde{e}_K^2 \theta (1 - g(N_K))$ efficiency units. Hence, the total labor supply is given by

$$L_K^{S2} \in [\tilde{e}_K^2 g(N_K), \tilde{e}_K^2 (g(N_K) + \theta(1 - g(N_K)))] \text{ if } N_K > 0 \text{ and } \theta w_K = v.$$

Finally, when the wage of low productivity workers is too low, the labor supply is

$$L_K^{S3} = \tilde{e}_K^3 g(N_K) \text{ if } N_K > 0 \text{ and } \theta v < \theta w_K < v.$$

2.3 Firms

The production of one unit of variety i requires the use of one efficiency unit of labor. Since in region K , lucky workers earn a wage w_K for a unit productivity whereas unlucky workers receive a wage θw_K for a productivity θ , the marginal cost is equal to w_K regardless of the type of worker who is employed.

We study here the process of competition between firms for a given spatial distribution (N_A, N_B) of firms. Since there is a continuum $[0, 1]$ of a differentiated variety i of a commodity and since each firm produces one variety, $N_A + N_B = 1$. In the continuous interval $[0, 1]$, each firm has a zero mass and has no direct impact on the market (see Ottaviano, Tabuchi and Thisse (2002)). Hence, firms neglect the impact of their price decision over the regional price indices. In addition, the demand for the variety produced by firm i has a finite elasticity. However, in the determination of its own price, each firm accounts for the price index of each region. As a consequence, the market solution is given by a Nash equilibrium with a continuum of players in which prices are interdependent: each firm neglects its impact on the market but is aware that the market as a whole has a non-negligible impact on its behavior.

The absence of transport costs implies that firms do not discriminate consumers by setting different prices across regions (see Belleflamme, Picard and Thisse (2000) for the role of transport costs in a similar model). The price of variety i is the same in both regions. We adopt the following notation for the price index in region K :

$$P_K \equiv \int_{i \in K} p_K(i) di.$$

By (2), demands for firm $i \in K$ are given by:

$$q_K(i) = a - (b + d)p_K(i) + d(P_K + P_L) \text{ where } K \neq L, L \in \{A, B\}.$$

Firm i sells in both regions and its profit is equal to:

$$\Pi_K(i) = 2q_K(i)[p_K(i) - w_K] \quad (5)$$

where w_K is the marginal cost in region K .

We first differentiate (5) with respect to prices $p_K(i)$ to obtain the first order conditions for a representative firm i . We then integrate the corresponding expressions across firms i located in K , to obtain the following equation:

$$[2(b+d) - dN_K]P_K - dN_K P_L = N_K[a + (b+d)w_K] \text{ where } K \neq L \quad (6)$$

Since profit functions are concave in own price and varieties are symmetric, solving the system of equations (6) for $K \in \{A, B\}$ yields the equilibrium prices and quantities:

$$p_K = \frac{w_K}{2} + \frac{2a + d(N_A w_A + N_B w_B)}{2(2b + d)} \quad (7)$$

$$q_K = (b + d)(p_K - w_K) \quad (8)$$

Insofar as cost conditions are different between regions ($w_A \neq w_B$), the prices and quantities of varieties produced in region A may differ from those produced in region B . Using (7) and (8), the labor demand (in efficiency units) in region K is equal to

$$L_K^D = 2q_K N_K = 2(b + d) \left[-\frac{w_K}{2} + \frac{2a + d(N_A w_A + N_B w_B)}{2(2b + d)} \right] N_K \quad \forall K. \quad (9)$$

The profits of any firm located in region K are thus

$$\Pi_K = 2(p_K - w_K)q_K = \begin{cases} 2(b + d)(p_K - w_K)^2 & \text{if } p_K > w_K \\ 0 & \text{otherwise.} \end{cases}$$

Our objective is to highlight regional asymmetries in the context of structurally identical regions. We assume that whatever the number of firms in one region, production would generate some surplus in that region, that is, the highest valuation of any variety (a/b) is always greater than marginal cost:

$$\frac{a}{b} > w_K \quad \forall K \text{ and } \forall N_K \in [0, 1] \quad (A1)$$

This assumption simplifies the presentation by avoiding binding constraints on quantities, i.e. $q_K = 0$. Hence, in the rest of the paper, $q_K > 0$ and, therefore $p_K > c_K$.

If the profits of firms located in region A are larger than those of firms located in region B , firms relocate from region B to region A , and vice versa. Therefore, the difference in profitability between the two regions is the key indicator that determines the location of firms. Using (7), the difference in profitability between regions A and B may be written as

$$\begin{aligned}\Delta\Pi &\equiv \Pi_A - \Pi_B \\ &= 2(b+d) [(p_A - w_A)^2 - (p_B - w_B)^2] \\ &= (b+d) [w_B - w_A] [(p_A - w_A) + (p_B - w_B)].\end{aligned}\tag{10}$$

Note that under A1, the last bracket in this expression is strictly positive. From these expressions, it is clear that the difference in profitability between regions depends on the wages w_A and w_B . Thus labor markets play a major role in the firms' location choices.

3 Location and Labor Market Equilibria

In this model two forces drive the firms' location decisions and the workers' decisions with respect to their education. First, private firms are attracted by low wages. Thus they choose to locate in the region with the lowest wages. Second, workers are more likely to invest in education if they live in a region with more firms. They are indeed more likely to find a job that fits their abilities and / or affinities in such a region. In this section, we show that firms may locate in one cluster, symmetric clusters or even asymmetric clusters. Relocation of firms has indeed two effects on the demand and supply of labor. Labor demand increases in the region where firms relocate. This tends to push wages up in that region. However, labor supply increases in that region because the larger set of firms improves the likelihood of a good job match. This tends to push wages down. Therefore, the net effect of a relocation of firms on wages is unknown.

To analyze the location and labor market equilibria, we first define location equilibrium and stability, we then derive the interior equilibria where firms do not agglomerate in a single cluster. We finally derive the corner equilibria where firms agglomerate in a single cluster.

3.1 Definitions

The labor market is in equilibrium when the wage w_K equalizes the labor supply and the labor demand in each region K : $L_K^S = L_K^D \ \forall K \in \{A, B\}$. Before proceeding to the characterization of the location equilibrium, it is

convenient to take advantage of the symmetry of the problem by setting $\Delta N \equiv N_A - N_B$. Thus, $N_A = (1/2)(1 + \Delta N)$ and $N_B = (1/2)(1 - \Delta N)$. A location equilibrium is defined as follows:

Definition 1 *A location equilibrium is such that (i) no locational deviation by a single firm is strictly profitable and (ii) the labor markets clear in both regions.*

Hence, there must be no incentives for firms to relocate. If profits of firms located in one region are larger than those of firms located in the other region, firms will move to the former region until the *profit differential* $\Delta\Pi(\Delta N)$ between the regions falls to zero or until all firms are located in that region. A *location equilibrium* is interior if $\Delta\Pi(\Delta N) = 0$ at $\Delta N \in (-1, 1)$. By (10), this occurs when $w_A = w_B$. A *location equilibrium* arises at corner points $N_A = 0$ when $\Delta\Pi(-1) \leq 0$, and $N_A = 1$ when $\Delta\Pi(1) \geq 0$.

Definition 2 *A location equilibrium ΔN^* is stable if, in the neighborhood of ΔN^* , no locational deviation by a group of firms (with non zero mass) is strictly more profitable.*

For interior solutions ΔN^* where $\Delta\Pi(\Delta N^*) = 0$, stability requires that $\Pi_A - \Pi_B$ decreases (resp. increases) if a group of firm moves from B to A (resp. A to B). That is, the slope of $\Delta\Pi(\Delta N)$ must be negative in the neighborhood of the equilibrium. At the corner point $N_A = 0$, the location equilibrium is stable either if $\Delta\Pi(-1) < 0$ or if $\Delta\Pi(-1) = 0$ and the slope of $\Delta\Pi(\Delta N)$ is negative in the neighborhood of $\Delta N = -1$. At the corner point $N_A = 1$, the location equilibrium is stable either if $\Delta\Pi(1) > 0$ or if $\Delta\Pi(1) = 0$ and the slope of $\Delta\Pi(\Delta N)$ is negative in the neighborhood of $\Delta N = 1$.

Several kinds of equilibria may arise in this setting. Either all firms agglomerate in one region (corner solution) or they spread across regions (interior solution) in a way that equalizes profits. In the latter case, firms can spread evenly ($\Delta N = 0$) or unevenly across regions. We start by characterizing the interior stable location equilibria; we then study the corner location equilibria.

3.2 Interior Location Equilibria

First, suppose that the wages w_A and w_B are larger than v/θ . The labor market equilibrium requires that $L_K^{S1} = L_K^D \forall K \in \{A, B\}$. Solving these two

conditions for w_A and w_B and plugging the results in the profit differential defined by (10) (see Appendix 1), we get

$$\Delta\Pi = K_1(\Delta N)(w_B - w_A) = -K_2(\Delta N) * \Delta N [(\Delta N)^2 - X],$$

where $K_1(\Delta N)$ and $K_2(\Delta N)$ are strictly positive and where

$$X \equiv \frac{2(a/v)I(b+d)[(1-\theta)^2 - (2\theta I)^2] + (1-\theta)(2\theta I + (1-\theta))^2(2b+d) + 8\theta I^3 b(b+d)}{(1-\theta)^2[2(a/v)I(b+d) + (1-\theta)(2b+d)]}. \quad (11)$$

Interior location equilibria are given by $\Delta N = 0$ if $X \leq 0$ and by $\Delta N \in \{0, \pm\sqrt{X}\}$ if $0 < X < 1$. These equilibria are stable when $\Delta\Pi$ decreases with ΔN . Therefore interior stable location equilibria are $\Delta N = 0$ if $X \leq 0$ and $\Delta N = \pm\sqrt{X}$ if $0 < X < 1$.

Note that X decreases with a/v (see Appendix 3). It takes values in $[0, 1)$ when $a/v \in (\hat{b}, \bar{b}]$ where $\hat{b}$ is the value of a/v that yields $X = 1$ and $\bar{b}$ is the value of a/v that yields $X = 0$. This gives the following proposition:

Proposition 1 *Stable interior equilibria with wages strictly greater than v/θ exist for $a/v > \hat{b}$. (i) For $a/v \in (\hat{b}, \bar{b})$ firms locate in partially asymmetric clusters; the stable interior location equilibria are $\Delta N = \pm\sqrt{X} \in (0, 1)$. (ii) For $a/v > \bar{b}$ firms locate in symmetric clusters: $\Delta N = 0$.*

Figure 1 illustrates the Proposition. Stable interior equilibria are represented by the bold curves on the right of $\hat{b}$. In case of asymmetric clusters, wages of workers with a good (or with a bad) match are equal in the two regions: $w_A = w_B$ and $\theta w_A = \theta w_B$. However, more workers have a good match in the region where firms agglomerate. Hence, the mean (or expected) wage ($w_K g(N_K) + \theta w_K [1 - g(N_K)]$) is higher in that region. In other words, the inter-regional mean wage differences that are presented in e.g. Henderson (1988), Ciccone and Hall (1996) and Möller and Haas (2002), are consistent with wages of each type of workers being equalized between the two regions.

INSERT FIGURE 1 HERE

Second, suppose that the wages $w_A = w_B$ are strictly lower than v/θ . The labor market equilibrium requires that $L_K^{S3} = L_K^D \forall K \in \{A, B\}$. Solving these two conditions for w_A and w_B and plugging the results in the profit differential defined by (10) (see Appendix 1) gives

$$\Delta\Pi = K_1(\Delta N)(w_B - w_A) = K_3(\Delta N) * \Delta N,$$

where $K_3(\Delta N)$ is strictly positive. The interior equilibrium at $\Delta N = 0$ is therefore not stable.

Finally, suppose that the wages $w_A = w_B$ are equal to v/θ . The labor market equilibrium requires that $L_K^{S2} = L_K^D \forall K \in \{A, B\}$. That is, $L_K^D \in [\tilde{e}_K^2 g(N_K), \tilde{e}_K^2 (g(N_K) + \theta(1 - g(N_K)))] \forall K \in \{A, B\}$. At $w_A = w_B = v/\theta$ the low-productivity workers are indifferent between working in the industry and working in the traditional sector. Hence, for any small change in the location of firms, the labor supply automatically matches the new labor demand at the same wage v/θ . However, if too many firms establish in region K , the industry exhausts the labor supply at wage v/θ . Therefore, the number of firms in a location equilibrium at wage v/θ is constrained upward; as shown in Appendix 1, this amounts to

$$N_K \leq \hat{N} \equiv \frac{2(b+d)I^2 \left(\frac{a}{v}\theta - b\right)}{(1-\theta)(2b+d)}. \quad (12)$$

In contrast, if few firms settle in region K , the labor demand falls below the labor supply at wage v/θ and none of the low-productivity workers find a job in the industry. The number of firms in a location equilibrium at wage v/θ is therefore constrained downward. As shown in Appendix 1, this constraint is

$$N_K \geq \hat{N} \equiv \frac{2I^2 \left(\frac{a}{v}\theta - b\right) (b+d) - \theta(1-\theta)(2b+d)I}{(1-\theta)^2 (2b+d)}. \quad (13)$$

Both constraints hold for the two regions: $\hat{N} \leq N_A \leq \hat{N}$ and $\hat{N} \leq N_B \leq \hat{N}$. Therefore, interior location equilibria with $w_A = w_B = v/\theta$ lie in the set

$$T = \left\{ \Delta N \text{ s.t. } 2\hat{N} - 1 \leq \Delta N \leq 2\hat{N} - 1 \text{ and } 1 - 2\hat{N} \geq \Delta N \geq 1 - 2\hat{N} \right\}.$$

Since, in the interior of this set, wages are equal to v/θ , any locational deviation from an equilibrium changes neither the costs nor the profits. So equilibria are stable in this set. One can show that $\hat{N}$ increases more with a/v than does $\hat{N}$. We denote $\hat{b}$ the value of a/v that solves $2\hat{N} - 1 = 0$. This yields the following Proposition.

Proposition 2 *Stable interior equilibria with wages equal to v/θ exist and satisfy $\Delta N \in T$. There are no stable interior equilibria at a wage strictly lower than v/θ .*

Figure 1 illustrates the Proposition. Stable interior equilibria where high productivity workers earn v/θ are represented by the black diamond on the

left of $\hat{b}$. Under these equilibria, the workers who qualify but who do not find the job that fits their abilities are indifferent between working in the traditional sector and working in the industry at the wage v . For any small change in the location of firms, labor supplies automatically matches the new labor demand at the same wage because the labor supply is locally infinitely elastic. Therefore, firms are indifferent between both locations. In these equilibria, the agglomeration and dispersion forces of labor markets become so weak that they do not influence the location of firms. Of course, once a new centripetal or centrifugal force is introduced in the model, these equilibria could degenerate.

3.3 Corner Equilibria

In this section we study the interior equilibria at which firms agglomerate in region K , leaving region L deserted. In region L no firm demands labor and no worker qualifies for a job in the industry. Thus the labor market in that region always clears but the market wage is undefined. In that case, we assume that the wage is the opportunity cost of labor in that region, i.e. the wage that a firm would face by settling in that region. Formally, let $N_L = \varepsilon > 0$ and $N_K = 1 - \varepsilon$. The market wage in region L is given by

$$\lim_{\varepsilon \rightarrow 0} w_L^*(\varepsilon)$$

where $w_L^*(\varepsilon)$ is continuous and solves the labor market clearing conditions: $L_K^D(w_K, w_L, \varepsilon) = L_K^S(w_K, \varepsilon)$ and $L_L^D(w_L, w_K, \varepsilon) = L_L^S(w_L, \varepsilon)$. Obviously, in the region where firms agglomerate, the market wage is well defined and is equal to $\lim_{\varepsilon \rightarrow 0} w_K^*(\varepsilon) = w_K^*$. This allows to derive the following proposition.

Proposition 3 *Stable corner equilibria exist if and only if $a/v < \hat{b}$ where*

$$\hat{b} \equiv \frac{(2b + d)(1 - \theta) \left(\theta + \frac{1 - \theta}{I} \right) + 2(b + d)bI}{2\theta(b + d)I}.$$

Proof. See Appendix 2. ■

Figure 1 illustrates the Proposition. Firms agglomerate in a single cluster for low product demand and / or high wage in the traditional sector. If $\hat{b} < a/v < \hat{b}$, firms hire high productivity workers at wage v/θ and some low productivity workers at the wage v . If $a/v \leq \hat{b}$, firms hire only high productivity workers at a wage lower than v/θ .

4 Comparative Statics

In this section we analyze how firms location responds to changes in the parameters. Quite naturally, we focus on the interior stable equilibria $X \in (0, 1)$. We derive the following proposition.

Proposition 4 *Suppose that $\hat{b} < a/v < \bar{b}$. Thus firms locate in asymmetric clusters ($X \in (0, 1)$). An increase in the global demand (a), in the product substitutability (d), in the uncertainty in the job match (I) or in the productivity of the unlucky qualified workers (θ) induce firms to locate more symmetrically. In contrast, an increase in the wage in the traditional sector (v) or in the sensitivity of the product demand to the output price (b) induce firms towards more agglomeration.*

Proof. See Appendix 3. ■

Changes in parameters have different effects on the labor demand curves and the labor supply curves according to the number of firms in each region. The direction of the (upward or downward) pressure on wages depends not only on the position but also on the slopes (or elasticities) of these curves. Since the firms' location decisions depend on the wage differential, the resulting changes in the location equilibrium depend on which of the supply and demand effects dominates. Fortunately, Proposition 6 shows that changes in the parameters (a, b, d, I, θ, v) lead to clear-cut effects. For the sake of clarity, we explain the impact of changes in these parameters by presenting the dominating effect behind these changes.

The effect of changes in the parameters (a, b, d) can be explained by the dominating force behind the changes in production levels. Indeed, for given equilibrium wages and location of firms, firms directly change their output only if the parameters of the consumer utility function (a, b, d) vary (see equations (7) and (8)). Output does not directly change if the parameters of labor supply (v, I, θ) vary. We first explain that larger a , d and b must increase production. We then explain that increases in production entice firms to locate more symmetrically.

It is obvious that the production in each firm increases when product demand (a) rises. Also, larger product substitutability (d) induces firms to produce more because it increases the effect of competition. Firms are less able to use their market power and to restrain production. Finally, a larger sensitivity of the product demand to the output price (b) leads to more production. Indeed, in this model with quadratic utility function of consumers, a decrease in b raises the firms' marginal revenues, which has a

positive impact on production. From equations (7) and (8), one can show that $dq_K/db < 0$.

The reason why increases in production entice firms to locate more symmetrically is the following one. Suppose that, in an initial equilibrium, each firm has one worker and that one firm locates in region A whereas 100 firms locate in region B . Suppose also that, because of a change in (a, b, d) , the output rises so that each firm wants to increase its production by doubling its labor force. In region B the increase in the labor force is equal to 100 workers whereas it is equal to one worker in region A . If firms do not relocate, wages must increase more in region B than in region A because the effort made by the 100th additional worker to qualify in region B is larger than that made by the single additional worker needed in region A ³. Since it is less costly to get additional workers in region A , firms in region B wish to relocate in region A . Suppose that one firm does so. Then the probability to have a good match doubles in region A whereas it diminishes only by 1% in region B . The incentive to qualify increases therefore much more in region A than it decreases in region B , which still attracts more firms in region A . Hence, an increase in production attracts firms toward the deserted region A .

The above comparative statics contrast with results of similar economic geography models where higher product demand (a) and substitutability (d) increase the likelihood of agglomeration. For instance, in Belleflamme, Picard and Thisse (2000) and in Ottaviano, Tabuchi and Thisse (2002), the dispersion force comes from the existence of transport cost and from the firms' desire to reduce competition. The agglomeration forces emanate either from the existence of Marshallian externalities or from (perfect) mobility of entrepreneurs between regions. In any case, the labor supply remains perfectly elastic in these models. Hence, higher product demands have no direct impact on wages. However, higher product demands reduce the relative size of transport cost to product price and thus reduce the dispersion force caused by firm competition. By contrast, our model does not include those dispersion and agglomeration forces. Indeed, here, the matching process induces agglomeration whereas dispersion is induced by the inelastic property of the labor supply. For higher product demand and substitutability, firms must provide larger wages in order to entice many workers to qualify for the

³Since $N_A \neq N_B$, the labor supply is different in both regions. Hence, a larger increase in the labor demand in region B could theoretically be associated with wages in region B increasing less than in region A . This could be the case if the labor supply is more elastic in region B than in region A . However, in our framework, the difference in the elasticities of labor supply is not sufficient to yield this effect.

industry skills. Because of such large wages, firms prefer dispersion in order to exploit the labor pool of both regions.

The effect of changes in the uncertainty in the job match (I) and in the productivity of the unlucky qualified workers (θ) is explained by the dominating force behind the changes in labor supply. That is, the unbalanced shift of labor supply curves. For given equilibrium wages and locations of firms, only the labor supply curves vary if the parameters (v, I) change. Labor supply in region K is given by the effort level and the average productivity in that region. Indeed, using (3) and (4), the labor supply can be written as

$$L_K^{S1} = (w_K \bar{\theta}_K - v) \bar{\theta}_K.$$

where $\bar{\theta}_K$ is the average productivity in region K :

$$\bar{\theta}_K \equiv \frac{N_K}{I} + \theta \left(1 - \frac{N_K}{I} \right).$$

Note that the labor supply increases with the average productivity in the region.

The average productivity $\bar{\theta}_K$ obviously decreases with the uncertainty in the job match (I). It decreases more in the region that counts the highest number of firms ($d\bar{\theta}_K/dI = -N_K(1-\theta)/I^2$). Hence, when the uncertainty in the job match (I) rises, labor supplies in both regions diminish, but the labor supply decreases more in the region where more firms agglomerate (say region B , $N_B > N_A$). Wages in that region B increase more than in the other region⁴. Firms quit region B where they were agglomerated and move to the deserted region. Hence, more uncertainty in the job match (I) induces firms to locate more symmetrically.

The average productivity rises with the productivity of the unlucky qualified workers (θ). It increases more in the region that counts the lowest number of firms ($d\bar{\theta}_K/d\theta = 1 - N_K/I$). Hence an increase in θ raises both labor supplies, but the labor supply rises less in the region where more firms agglomerate (say region B). Wages decrease less in that region B than in the other region. Firms quit region B for the deserted region. Hence, a higher productivity of the unlucky qualified workers (θ) induces firms to locate more symmetrically.

⁴Since $N_A \neq N_B$, the labor demand is different in both regions. Hence, a larger decrease in the labor supply in region B could theoretically be associated with wages in region B increasing less than in region A . This could be the case if the labor demand is more elastic in region B than in region A . However, in our framework, the difference in the elasticities of labor demand is not sufficient to yield this effect.

Moreover, when θ tends to 1, firms spread evenly across both regions. Indeed,

$$\lim_{\theta \rightarrow 1} X = \lim_{\theta \rightarrow 1} - \left(\frac{2I}{1-\theta} \right)^2 \left(\frac{a-bv}{a} \right) < 0.$$

When $\theta = 1$, unlucky workers receive the same wage as lucky workers. Uncertainty about productivity disappears and the average productivity $\bar{\theta}_K$ is equal to 1 in both regions. In contrast to the general case with $\theta < 1$, we have $L_K^{S1}(w_K) = (w_K - v)$ when $\theta = 1$. The position of the labor supply curve does not directly depend on the firms' location N_K . Moreover, both regions have identical labor supply curves. If firms did not locate in symmetric locations, the labor demand would be larger in the region where firms agglomerate. This would imply higher wages in that region and a relocation of firms towards the other region, until the symmetric location equilibrium is reached.

To end up, let us look at the effect of a change in the wage in the traditional sector (v). Unfortunately this effect cannot be explained only by the unbalanced shift of labor supplies as for parameters (I, θ) . The direction of the pressure on wages also depends on the differences in labor demand elasticities. The unbalanced shift of the labor supply curves would lead to more symmetric location equilibrium when v rises. Indeed, when v rises, labor supply in both regions decreases, but it diminishes more in the region where more firms agglomerate (say region B) because the average productivity is larger in that region. Thus, one would expect wages to increase more in region B than in the other region. Firms should quit the region where firms agglomerate. However, this is not true. The elasticity of labor demand indeed plays the dominant role in the location pattern of firms.

Since $N_B > N_A$, the labor demand is different in both regions. Indeed, using equations (7), (8) and the property that at an interior equilibrium, $w_A = w_B = w$, the demand for labor in region K can be written as $L_K^D = 2N_K(a - bw)(b + d)/(2b + d)$, or

$$w = \frac{a}{b} - \frac{2b + d}{2b(b + d)N_K} L_K^D.$$

The demand for labor is clearly more elastic in the region where firms agglomerate. Therefore, an equal decrease in the labor supply in both regions has more effect on wages in the deserted region. Wages increase more in region A than in region B . This effect through labor demand elasticity dominates the effect through the unbalanced shift in the labor supplies. Firms quit region A to further agglomerate in region B .

5 Welfare

In this section, we examine whether the decentralized location decision maximizes the consumer surplus, the profits and the welfare of workers. Unfortunately, it is not possible to derive algebraic expressions without making further assumptions. For this reason, we assume that goods are perfectly differentiated, that is, we set $d = 0$. As discussed in the previous section, although product differentiation contributes to the agglomeration, all the qualitative properties of the model are retained under perfect product differentiation.

The total consumer surplus is measured by the total consumption, $N_K q_K + N_L q_L$, the total profit by $N_K \Pi_K + N_L \Pi_L$, and the total welfare of workers by $V_K + V_L$ where V_K the utility of all workers residing in region K . The utility of workers in each region is equal to their utility derived from wages minus their educational cost. In each region, the last worker who qualifies exerts an effort $\tilde{e}_K$ and, because of the uniform distribution of effort, $\tilde{e}_K$ workers qualify for the industry. Thus, the educational cost is equal to $\int_0^{\tilde{e}_K} e_K de_K = (\tilde{e}_K)^2/2$. Furthermore, $\tilde{e}_K$ workers qualify for the industry in region K and get an expected utility $g(N_K)(w_K - v) + (1 - g(N_K))(\theta w_K - v)$ or $g(N_K)(w_K - v)$ according to the situation of the labor market. By the assumption of uniform distribution of effort, the expected utility is also equal to the number of workers $\tilde{e}_K$. Thus, the utility derived from wages is equal to $(\tilde{e}_K)^2$ and the utility of all workers in region K is $V_K = (\tilde{e}_K)^2/2$.

Under these definitions, we are able to derive the following proposition.

Proposition 5 (i) *Corner equilibria: when firms agglomerate in one region, the welfare of consumers, firms' owners and workers cannot be improved by re-locating some firms.*

(ii) *Interior equilibrium with wages equal to v/θ : the welfare of consumers and firms' owners is unchanged whereas workers are better off when agglomeration is increased.*

(iii) *Partially asymmetric stable equilibria: the welfare of consumers and firms' owners increases when agglomeration is increased.*

(iv) *Symmetric stable equilibria: workers, consumers and firms' owners may prefer more or less agglomeration according to the parameters of the model. They may have conflicting interests with respect to the equilibrium location of firms.*

In relation to Figure 1, the proposition suggests that there may exist too much dispersion particularly in industries with small product demand

and high wages in the alternative sector. The explanation of the proposition goes as follows. Welfare depends critically on the behavior of wages.

(i) When full agglomeration is a stable equilibrium, w_K decreases around $N_K = 1$ and is smaller than w_L (see Appendix 4). In fact, wages behave like in the left panel of Figure 2. Hence, any departure from full agglomeration raises wages in both regions. Costs therefore increase in all firms, which reduces profits, production and, as a result, consumer surplus.

INSERT FIGURE 2 HERE

Two opposite forces affect the welfare of workers when some firms leave the industrialized region K . On the one hand, as mentioned in the previous paragraph, wages increase in that region. On the other hand, the matching process worsens for a large number of workers. The net effect is a priori ambiguous. The welfare of workers is equal to $V_K = (\tilde{e}_K^3)^2 / 2 = [(w_K - v)g(N_K)]^2 / 2$. The derivative of V_K with respect to N_K is

$$V'_K = \tilde{e}_K^3 \frac{d\tilde{e}_K^3}{dN_K} = (\tilde{e}_K^3)^2 \left(\frac{w'_K}{w_K - v} + 1 \right) \quad (14)$$

where we applied the definition of $g(N_K)$ at $N_K = 1$ in the last equality. Since there are no job opportunities in the deserted region L , workers do not educate in that region: $\tilde{e}_L^2 = 0$. As a consequence, we have that $V'_L = \tilde{e}_L^2 * d\tilde{e}_L^2 / dN_L = 0$. In words, although a marginal re-location of firms in the deserted region L may increase the individual education and utility levels of qualified workers, it applies only to an infinitely small mass of workers and has no impact on total welfare. In sum, the total welfare of workers increases with N_K at the equilibrium if and only if $(V_K + V_L)' = V'_K > 0$. Therefore, as shown in (14), the total welfare of workers will increase as long as the elasticity of wages to firms' location is not too high. As shown in Appendix 4, this happens to be true in our model. Hence, agglomeration is a local maximum for the total welfare of workers.

(ii) In interior equilibrium with wages equal to v/θ , a re-location of firms from region K to region L does not alter wages in any region. Hence marginal costs, production, profits, and consumer surplus remain unchanged. There is however too much dispersion from the workers' point of view. Indeed, the welfare of workers is

$$V_K + V_L = \frac{1}{2} (\tilde{e}_K^2)^2 + \frac{1}{2} (\tilde{e}_L^2)^2 = \frac{1}{2} \left(v \frac{1 - \theta}{\theta} \right)^2 \left[(g(N_K))^2 + (g(1 - N_K))^2 \right]$$

This function is symmetric around $N_K = 1/2$ and convex in N_K . Thus, the welfare of workers increases with N_K if $N_K > 1/2$. In other words, the fact that agglomeration improves the matching process in one region is sufficient to induce workers to prefer more agglomeration. Note that this result holds for any value of the differentiation parameter d .

(iii) In Appendix 4, it is shown that under $d = 0$, wages in region K increase for low values of N_K and may eventually decrease for larger values. Partially asymmetric stable equilibria are depicted in the right panel of Figure 2. When agglomeration is increased, wages are reduced in both regions. Costs therefore decrease in all firms, which raises profits, production and, as a result, consumer surplus.

The impact of more clustering on the total welfare of workers is difficult to track. Indeed, an increase in agglomeration in the region K where firms are already more agglomerated have two opposite effects. On the one hand, the total welfare of workers may decrease because workers in both regions get lower wages and because workers in the less industrialized region get lower matching probabilities. On the other hand, the welfare of workers in the more industrialized region may rise because of a better matching. Still, we were not able to track the dominating effect using the above analytics. The impact of more clustering on the total welfare of workers remains thus undetermined.

(iv) At the symmetric location, changes in firms' location have no first order effects on welfare: $V'_K = (N_K q_K)' = \Pi'_K = 0 \forall K$ at $N_K = 1/2$. The welfare analysis must therefore focus on the second order effects, that is, on the concavity or convexity of the total welfare of workers, total consumer surplus and total profits. The total welfare of workers is given by $V = V_K + V_L$ where $V_K = (\tilde{e}_K^1)^2/2$. Agglomeration is preferred by workers if and only if the function V is convex, that is, if and only if $V''_K = (\tilde{e}_K^1)''^2 + \tilde{e}_K^1 \tilde{e}_K^{1''} > 0$. Similarly, the consumer surplus is convex at $N_K = 1/2$ if and only if $(N_K q_K)'' > 0$ at $N_K = 1/2$ and the total profit is convex if $\Pi''_K = (2N_K q_K^2/b)'' > 0$ at $N_K = 1/2$.

The analysis presents serious analytical problems. For the sake of clarity, we restrict the current presentation to the discussion of a specific example which illustrates the richness induced by the various parameter configurations.

The following picture gathers these results for the case where $d = 0$ and $I = b = 1$. The area above the bottom bold curve $\hat{b}$ represents the parameter configurations that generate stable symmetric equilibria ($a/v \geq \hat{b} \Leftrightarrow X \leq 0$). The areas below the three curves V''_K , Q''_K and Π''_K represents parameter con-

figurations for which the total welfare of workers, consumers and producers are convex at $N_K = 1/2$. The areas below these curve therefore show where agglomeration is preferred respectively by workers, consumers and producers. For parameter configurations like A and B, there exist no conflict of interest between workers, consumers and producers. At A, they all agree to depart from the symmetric equilibrium whereas, at B, they all agree on staying in symmetric clusters. There exist many intermediate situations in which the interests of workers, consumers and firms' owners do conflict. For instance, at C, consumers and firms' owner would favor agglomeration but workers would prefer symmetric clustering.

INSERT FIGURE 3 HERE

6 Conclusion

In this paper we build a model of imperfect competition with differentiated products, with competitive labor markets and with endogenous decisions of labor qualification. The location of firms is influenced by two opposite forces. On the one hand, when firms agglomerate in one region, labor demand increases in that region. Wages increase in the region where firms agglomerate and decrease in the region that is deserted. Since firms are attracted by low wages, this effect clearly induces them to spread evenly across regions. This is the dispersion force. On the other hand, if workers are uncertain about the possibility to get a good match with a firm that requires specific skills, they are more likely to invest in education if they live in a region with more firms. Indeed, the probability to find a job that fits their abilities and/or affinities increases in such a region. Hence, the supply of qualified labor increases with the number of firms in the region. Since wages are lower when the labor supply is higher, this creates an incentive for firms to agglomerate in one region. This is the agglomeration force. As a result of these two forces, firms may locate in a single cluster, symmetric clusters or even asymmetric clusters.

We show that if a parameter changes in a way that induces firms to produce more, then firms tend to locate more symmetrically. For instance, this is the case if the global demand increases, or if the product substitutability goes up or if the sensitivity of the product demand to the output price decreases. In relation with the parameters that are directly related to the supply of qualified labor, we show that an increase in the uncertainty in the

job match or in the productivity of the qualified workers who do not find a job that fits their abilities and/or affinities induces firms to locate more symmetrically. In contrast, an increase in the wage in the traditional sector induces firms towards more agglomeration.

From the welfare viewpoint, we show that there may exist too much dispersion particularly in industries with small product demand and high wages in the alternative sectors. Insufficient geographical dispersion is a reminiscent result in spatial economic theory using product differentiation (see Anderson et al., 1992; Thisse and Fujita, 2002). The originality of the present paper nevertheless lies on the one hand, in the analysis of the uncertainty in the job market as a possible cause of agglomeration and on the other hand, in the analysis of the possible conflicts of interest between the various economic agents with respect to the equilibrium outcome.

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Appendix 1: Interior Equilibria

Suppose that the wages w_A and w_B are larger than v/θ . The labor market equilibrium requires that $L_K^{S1} = L_K^D \forall K \in \{A, B\}$. That is, for all $K \in \{A, B\}$,

$$\tilde{e}_K^1 [g(N_K) + \theta(1 - g(N_K))] = 2(b + d) \left[-\frac{w_K}{2} + \frac{2a + d(N_A w_A + N_B w_B)}{2(2b + d)} \right] N_K.$$

Solving these equalities for w_A and w_B yields

$$w_K = D^{-1} \begin{pmatrix} 2a(b + d) N_K (m_L^2 + (b + d) N_L) \\ + v(2b + d) m_K (m_L^2 + (b + d) N_L) \\ - v(b + d) d N_L (m_K N_L - N_K m_L) \end{pmatrix}$$

for all $K \in \{A, B\}$, where $m_K = g(N_K) + \theta(1 - g(N_K))$ and $m_L = g(N_L) + \theta(1 - g(N_L))$ and where

$$D = (2b + d) m_K^2 m_L^2 + (b + d) m_K^2 N_L (2b + d N_K) \\ + (b + d) N_K m_L^2 (2b + d N_L) + 2(b + d)^2 N_K N_L b > 0$$

By (10), the profit differential is inversely related to the cost differential:

$$\Delta \Pi = K_1(\Delta N)(w_B - w_A) = -K_2(\Delta N) * \Delta N((\Delta N)^2 - X),$$

where

$$K_1(\Delta N) = (b + d) [(p_A - w_A) + (p_B - w_B)] > 0, \\ K_2(\Delta N) = \frac{1}{4} D^{-1} K_1(\Delta N) (\theta - 1)^2 \\ * (2a(b + d) I + (2b + d) v(1 - \theta)) I^{-3} > 0,$$

and

$$X \equiv \frac{2(a/v)I(b+d)[(1-\theta)^2-(2\theta I)^2]+(1-\theta)(2\theta I+(1-\theta))^2(2b+d)+8\theta I^3b(b+d)}{(1-\theta)^2[2(a/v)I(b+d)+(1-\theta)(2b+d)]}.$$

Suppose that the wages w_A and w_B are lower than v/θ . The labor market equilibrium requires that $L_K^{S3} = L_K^D \forall K \in \{A, B\}$. That is, for all $K \in \{A, B\}$,

$$\tilde{e}_K^3 g(N_K) = 2(b+d) \left[-\frac{w_K}{2} + \frac{2a + d(N_A w_A + N_B w_B)}{2(2b+d)} \right] N_K.$$

Solving these equalities for w_A and w_B and plugging the result in (10) gives

$$\begin{aligned} \Delta \Pi &= K_1(\Delta N)(w_B - w_A), \\ &= K_1(\Delta N) \Delta N \frac{8(b+d)I^2(a-vb)}{8b(b+d)^2I^4+2(b+d)(4b+d(1+(\Delta N)^2))I^2+(2b+d)(1-(\Delta N)^2)}, \\ &= \Delta N K_3(\Delta N), \end{aligned}$$

with $K_3(\Delta N) > 0$.

Finally, suppose that the wages w_A and w_B are equal to v/θ . The labor market equilibrium requires that $L_K^{S2} = L_K^D \forall K \in \{A, B\}$. That is, $L_K^D \in [\tilde{e}_K^2 g(N_K), \tilde{e}_K^2 (g(N_K) + \theta(1 - g(N_K)))] \forall K \in \{A, B\}$. This defines the following set of constraints for all regions K :

$$\begin{aligned} \tilde{e}_K^2 g(N_K) &\leq L_K^D, \\ \tilde{e}_K^2 (g(N_K) + \theta(1 - g(N_K))) &\geq L_K^D. \end{aligned}$$

At $w_A = w_B = v/\theta$ and $N_K > 0$, the first inequality is equivalent to

$$v \frac{1-\theta}{\theta} \left(\frac{N_K}{I} \right)^2 \leq \frac{2N_K(b+d)}{2b+d} \left[a - b \frac{v}{\theta} \right] \Leftrightarrow N_K \leq \hat{N} \equiv \frac{2(b+d)I^2(a-b)}{(1-\theta)(2b+d)} \forall K.$$

The second inequality is equivalent to

$$v \frac{1-\theta}{\theta} \left(\frac{N_K}{I} \right) \left(\theta + \frac{N_K}{I} (1-\theta) \right) \geq \frac{2N_K(b+d)}{2b+d} \left[a - b \frac{v}{\theta} \right] \forall K,$$

which yields

$$N_K \geq \hat{N} \equiv \frac{2aI\theta(b+d) - [(2b+d)(1-\theta)\theta + Ib(b+d)]}{(1-\theta)^2(2b+d)} I \forall K.$$

Therefore, interior location equilibria with $w_A = w_B = v/\theta$ lie in the set such that $\hat{N} \leq N_A \leq \hat{N}$ and $\hat{N} \leq N_B \leq \hat{N}$. This set is equivalent to $2\hat{N} - 1 \leq \Delta N \leq 2\hat{N} - 1$ and $1 - 2\hat{N} \geq \Delta N \geq 1 - 2\hat{N}$. Since, in the interior of this set, wages are equal to v/θ , any locational deviation from an equilibrium changes neither the costs nor the profits. So equilibria are stable in this set.

Appendix 2: Corner Equilibria

Before the proof of the proposition, we first establish three useful results. Note that all limits are taken for $\varepsilon \rightarrow 0$ (i.e. $N_L \rightarrow 0$). Thus, $\lim w_K^*(\varepsilon) \equiv \lim_{\varepsilon \rightarrow 0} w_K^*(\varepsilon) \forall K$.

Lemma (i) Wages in the deserted region L ($\lim w_L^*(\varepsilon)$) are lower or equal to v/θ . (ii) If they are strictly lower than v/θ , then wages in the industrialized region are strictly lower than in the deserted region: $\lim w_K^*(\varepsilon) < \lim w_L^*(\varepsilon) < v/\theta$. (iii) If either $\lim w_K^*(\varepsilon) \geq \lim w_L^*(\varepsilon)$, or $\lim w_L^*(\varepsilon) \geq v/\theta$ or $\lim w_K^*(\varepsilon) \geq v/\theta$, then $\lim w_L^*(\varepsilon) = v/\theta$.

Proof. (i) By contradiction, suppose $\lim w_L^*(\varepsilon) > v/\theta$. Then $w_L^*(\varepsilon) > v/\theta$ in the neighborhood of $\varepsilon = 0$ and the labor supply is L_L^{S1} in the deserted region L . Market clearing in that region requires

$$\begin{aligned} L_L^{S1}(w_L, \varepsilon) &= L_L^D(w_L, w_K, \varepsilon), \\ &[w_L(\theta + \frac{\varepsilon}{I}(1-\theta)) - v](\theta + \frac{\varepsilon}{I}(1-\theta)) = \\ &2\varepsilon(b+d) \left[-\frac{w_L}{2} + \frac{2a+d(\varepsilon w_L + (1-\varepsilon)w_K)}{2(2b+d)} \right]. \end{aligned}$$

Solving for w_L and taking the limit gives

$$\lim w_L^*(\varepsilon) = \frac{v}{\theta},$$

which contradicts $\lim w_L^*(\varepsilon) > v/\theta$. Hence, $\lim w_L^*(\varepsilon) \leq v/\theta$.

(ii) If $\lim w_L^*(\varepsilon) < v/\theta$ then, by continuity, $w_L^*(\varepsilon) < v/\theta$ in the neighborhood of $\varepsilon = 0$. In this case, the labor supply is L_L^{S3} in the deserted region L . Market clearing in that region requires

$$\begin{aligned} L_L^{S3}(w_L, \varepsilon) &= L_L^D(w_L, w_K, \varepsilon), \\ \frac{\varepsilon^2}{I^2}(w_L - v) &= 2\varepsilon(b+d) \left[-\frac{w_L}{2} + \frac{2a+d(\varepsilon w_L + (1-\varepsilon)w_K)}{2(2b+d)} \right]. \end{aligned}$$

Solving for w_L and taking the limit gives

$$\lim w_L^*(\varepsilon) = \frac{2a + d \lim w_K^*(\varepsilon)}{(2b+d)} = \frac{2a + d w_K^*}{(2b+d)}.$$

Since by A1, $w_K^* < a/b$, it is easy to check that $\lim w_L^*(\varepsilon) > \lim w_K^*(\varepsilon)$ and thus, $\lim w_K^*(\varepsilon) < \lim w_L^*(\varepsilon) < v/\theta$.

(iii) Part (iii) is trivially obtained from (i) and reverting the implication in (ii). ■

This lemma implies the following property of agglomeration equilibria:

Corollary *If agglomeration is a stable corner equilibrium, then either*

(i) *labor market in region K is in regime 3: $\lim w_K^*(\varepsilon) < \lim w_L^*(\varepsilon) \Leftrightarrow \lim w_K^*(\varepsilon) < v/\theta$, or,*

(ii) *labor markets in both regions are in regime 2: $\lim w_K^*(\varepsilon) = \lim w_L^*(\varepsilon) \Leftrightarrow \lim w_K^*(\varepsilon) = v/\theta$.*

Proof. (i) Suppose by contradiction that $\lim w_K^*(\varepsilon) \geq \lim w_L^*(\varepsilon)$ and $\lim w_K^*(\varepsilon) < v/\theta$. Then, by property (iii) of the Lemma, we have that $\lim w_L^*(\varepsilon) = v/\theta$, and hence $\lim w_K^*(\varepsilon) \geq v/\theta$, which is a contradiction. Similarly, suppose that $\lim w_K^*(\varepsilon) < \lim w_L^*(\varepsilon)$ and $\lim w_K^*(\varepsilon) \geq v/\theta$. Then, by property (iii) of the Lemma, $\lim w_L^*(\varepsilon) = v/\theta$, and hence $\lim w_K^*(\varepsilon) < v/\theta$, which also yields a contradiction.

(ii) Part (ii) directly follows from property (iii) of the Lemma. For instance, if $\lim w_K^*(\varepsilon) = \lim w_L^*(\varepsilon)$, by property (iii) of the Lemma, $\lim w_L^*(\varepsilon) = v/\theta$ and therefore $\lim w_K^*(\varepsilon) = v/\theta$. The reverse implication follows a similar argument. ■

Finally, Proposition 3 states that stable corner equilibria exist if and only if $a/v < \hat{b}$. To have agglomeration in region K , wages in that region must be lower, or equal to wages in the other region: $\lim w_K^*(\varepsilon) \leq \lim w_L^*(\varepsilon)$. We can re-write the proposition more precisely as

Proposition (i) *When labor market in region K is in regime 3, agglomeration is a stable corner equilibrium if and only if $a/v < \hat{b}$. (ii) When labor markets in both regions are in regime 2, agglomeration is a stable corner equilibrium if and only if $\hat{b} < a/v < \hat{b}$, where*

$$\hat{b} \equiv \frac{(2b+d)(1-\theta) + 2b(b+d)I^2}{2\theta(b+d)I^2}$$

Hence, corner equilibria are stable if $a/v < \hat{b}$.

Proof. (i) We show that $\lim w_K^*(\varepsilon) < v/\theta \Leftrightarrow a/v < \hat{b}$. If $\lim w_K^*(\varepsilon) < v/\theta$, then the labor supply in region K is L_K^{S3} and labor market equilibrium requires

$$L_K^{S3}(w_K, \varepsilon) = L_K^D(w_K, w_L, \varepsilon),$$

$$\frac{(1-\varepsilon)^2}{I^2} (w_K - v) = 2(1-\varepsilon)(b+d) \left[-\frac{w_K}{2} + \frac{2a + d((1-\varepsilon)w_K + \varepsilon w_L)}{2(2b+d)} \right].$$

This gives

$$\lim w_K^*(\varepsilon) = \frac{2a(b+d)I^2 + v(2b+d)}{2b(b+d)I^2 + (2b+d)}.$$

One can readily show that the condition $\lim w_L^*(\varepsilon) < v/\theta$ is equivalent to $a/v < \hat{b}$.

(ii) We now show that $\lim w_K^*(\varepsilon) = v/\theta \Leftrightarrow \hat{b} < a/v < \hat{b}$. When $\lim w_K^*(\varepsilon) = \lim w_L^*(\varepsilon) = v/\theta$, the location equilibrium at $N_K = 1$ is stable if and only if the slope of $\Delta\Pi(\Delta N)$ is negative at $N_K = 1$. A similar analysis has already been done for interior equilibria with $w_K = w_L = v/\theta$ in Proposition 4. In that analysis, the labor market clearing condition in region K implies that $N_K \in [\hat{N}, \hat{N}]$. Therefore, agglomeration with $N_K = 1$ and $\lim w_L^*(\varepsilon) = \lim w_K^*(\varepsilon) = v/\theta$ implies that $1 \in [\hat{N}, \hat{N}]$. By (12) and (13), one can show that this is equivalent to $\hat{b} < a/v < \hat{b}$. ■

Appendix 3: Comparative Statics

We first prove that (i) $X_a < 0$, $X_v > 0$, $X_b > 0$ and $X_d < 0$; then we prove that (ii) $X_I < 0$ and (iii) $X_\theta < 0$.

(i) Let us first define $z = a/v$. Then, using the definition of X , (equation (11)) one can show that

$$\begin{aligned} X_z &= -8I^2(b+d)\theta \frac{2b(b+d)I^2 + (2b+d)(1-\theta)(1-\theta+2\theta I)}{(1-\theta)^2[2zI(b+d) + (1-\theta)(2b+d)]^2} < 0, \\ X_b &= 8\theta I^2 \frac{[2(b+d)^2I^2 + d(1-\theta)(2\theta I - \theta + 1)]z + I(2b^2 + 2bd + d^2)(1-\theta)}{(1-\theta)^2[2zI(b+d) + (1-\theta)(2b+d)]^2} > 0, \\ X_d &= \frac{-8[I(2z\theta - b) + z(1-\theta)]\theta I^2 b}{[2zI(b+d) + (1-\theta)(2b+d)]^2(1-\theta)} < 0. \end{aligned}$$

(ii) To show that $X_I < 0$, let us define $A = 2b + d$ and $B = b + d$. We can write

$$X = \frac{N}{D} = \frac{2zIB((1-\theta)^2 - (2\theta I)^2) + (1-\theta)(2\theta I + (1-\theta))^2 A + 8\theta I^3 bB}{(1-\theta)^2(2zIB + (1-\theta)A)}, \quad (15)$$

where N and D are the numerator and denominator of this expression. After simplification we can write

$$X < 1 \Leftrightarrow P(I) \equiv 2B(\theta z - b)I^2 - (1-\theta)\theta AI - (1-\theta)^2 A > 0 \quad (16)$$

This polynomial $P(I)$ has a negative and a positive root since $P(-\infty) > 0$, $P(0) < 0$ and $P(\infty) > 0$. Let I_1 be the positive root of this polynomial. Thus we have that

$$X < 1 \Leftrightarrow I > I_1 \quad (17)$$

Since I_1 is the largest root, it is larger than half the sum of roots:

$$I_1 \geq \frac{(1-\theta)\theta A}{4B(\theta z - b)} \quad (18)$$

Then, we compute

$$\begin{aligned} X_I = & -(K(I))^{-1} (8B^2 z (\theta z - b) I^3 + 2BA(1-\theta)(2\theta z - 3b) I^2 \\ & - 2A^2 \theta (1-\theta)^2 I - A^2 (1-\theta)^3) \end{aligned} \quad (19)$$

where $K(I) = (\theta - 1)^2 (2zIB + A(1-\theta))^2 / 4\theta > 0$.

Multiplying the inequality (16) by $A(1-\theta)$ gives

$$-A^2(1-\theta)^3 > -2AB(1-\theta)(\theta z - b)I^2 + A^2\theta(1-\theta)^2 I.$$

Substituting this in the last term of (19) yields the inequality

$$-K(I) X_I > I(2zIB + A(1-\theta))(4IB(\theta z - b) - A\theta(1-\theta))$$

The right hand side is positive by (17-18). Hence $X_I < 0$.

(iii) To show that $X_\theta < 0$, we differentiate equation (15):

$$X_\theta = \frac{N_\theta D - D_\theta N}{D^2} = \frac{N_\theta - D_\theta X}{D}.$$

We note that $D_\theta = -3(1-\theta)^2 A - 4IzB(1-\theta) < 0$. Since $X \in [0, 1]$, $N_\theta - D_\theta X \leq N_\theta - D_\theta$. Thus a sufficient condition for $X_\theta < 0$ is $N_\theta - D_\theta < 0$. We have

$$N_\theta - D_\theta = 4I [-4zB\theta I^2 + 2BbI^2 + \theta A(2-3\theta)I + A(1-3\theta)(1-\theta)].$$

From (16), we know that

$$4B\theta z I^2 > 4BI^2 b + 2(1-\theta)\theta AI + 2(1-\theta)^2 A$$

Hence, substituting $4Bz\theta I^2$ in $N_\theta - D_\theta$ yields

$$N_\theta - D_\theta < 4I [-2I^2 b B - A - A\theta^2(I-1)] < 0$$

which proves the result.

Appendix 4: Proposition 5

The proofs of parts (ii) and (iv) in Proposition 5 are already presented in the text. We need to demonstrate parts (i) and (iii).

Part (i) : Welfare under Agglomeration

In the text, we explain that, at agglomeration equilibrium in a region K , total production and profit cannot be improved by re-locating some firms because such a re-location would raise wages, that is, $w_L > w_K$ and $w'_K > 0$ at $N_K = 1$. Furthermore, we argue that the welfare of workers cannot be improved because $w'_K / (w_K - v) + 1 > 0$ at $N_K = 1$. We now formally show each of these elements when $d = 0$.

By part (i) of the lemma in Appendix 2 (proof of proposition 3), we know that, $w_L \leq v/\theta$ when firms agglomerate in region K . By part (ii) of the same lemma, we also know that, if $w_L < v/\theta$, then $w_K < w_L < v/\theta$ when firms agglomerate in region K . As a result, wages can take the three following values: first, $w_K = w_L = v/\theta$, second, $w_K < w_L = v/\theta$, and finally, $w_K < w_L < v/\theta$. The first case corresponds to interior equilibria in the set T where wages are equal to v/θ . This is studied in part (ii) of Proposition 5. In the last two cases, we naturally have that $w_K < w_L$. Moreover, since $w_K < v/\theta$, the labor market clearing in region K implies that $L_K^S = L_K^D$. Using $d = 0$, we get

$$g_K^2 (w_K - v) = bN_K \left(\frac{a}{b} - w_K \right)$$

The wage is explicitly solved as

$$w_K = \frac{vg_K^2 + aN_K}{g_K^2 + bN_K}$$

with $g_K = N_K/I$. The derivative of this expression with respect to N_K is

$$w'_K = -\frac{a - vb}{(bI^2 + N_K)^2} I^2 < 0$$

Hence, when some re-locate to the deserted region L , wages increase in the region K where firms were agglomerated. Furthermore, at $N_K = 1$,

$$\frac{w'_K}{w_K - v} + 1 = \frac{bI^4}{w_K - v} \frac{a - vb}{(bI^2 + 1)^2} > 0$$

Therefore, the elasticity of wages to firms' location is not too high and the total welfare of workers increases.

Part (iii) Welfare under stable asymmetric interior equilibrium

In this section, we show that, under stable asymmetric interior equilibrium, the wage schedule must be bell-shaped and wages must decrease for large values of N_K .

When goods are totally differentiated ($d = 0$), it is readily checked that wages at an interior location equilibrium, defined at the beginning of Appendix 1, take the following simple expression:

$$w_K = \frac{I \{N_K [aI + v(1 - \theta)] + v\theta I\}}{(1 - \theta)^2 N_K^2 + I [bI + 2\theta(1 - \theta)] N_K + \theta^2 I^2} \quad \forall K \in \{A, B\}$$

The derivative of wages is

$$\frac{dw_K}{dN_K} = \frac{-(1-\theta)^2 [aI + v(1-\theta)] N_K^2 - 2v\theta I (1-\theta)^2 N_K + \theta I^2 [(a\theta - vb)I - \theta v(1-\theta)]}{\{(1-\theta)^2 N_K^2 + I [bI + 2\theta(1-\theta)] N_K + \theta^2 I^2\}^2} I$$

The numerator of this expression is quadratic. It has two roots in N_K , one of which being negative. Wages are computed assuming an interior location equilibrium, that is under the hypothesis $a/v > \hat{b}$. Under $d = 0$, $\hat{b}$ is

$$\hat{b} \equiv \frac{bI^2 + \theta(1 - \theta)I + (1 - \theta)^2}{I^2\theta}.$$

Under this condition, it is readily checked that dw_K/dN_K is positive at $N_K = 0$. Thus wages start by increasing around $N_K = 0$ and may eventually decrease for larger values of N_K . When partially asymmetric equilibria exist, the wage schedule must be bell-shaped and wages must decrease for large values of N_K .

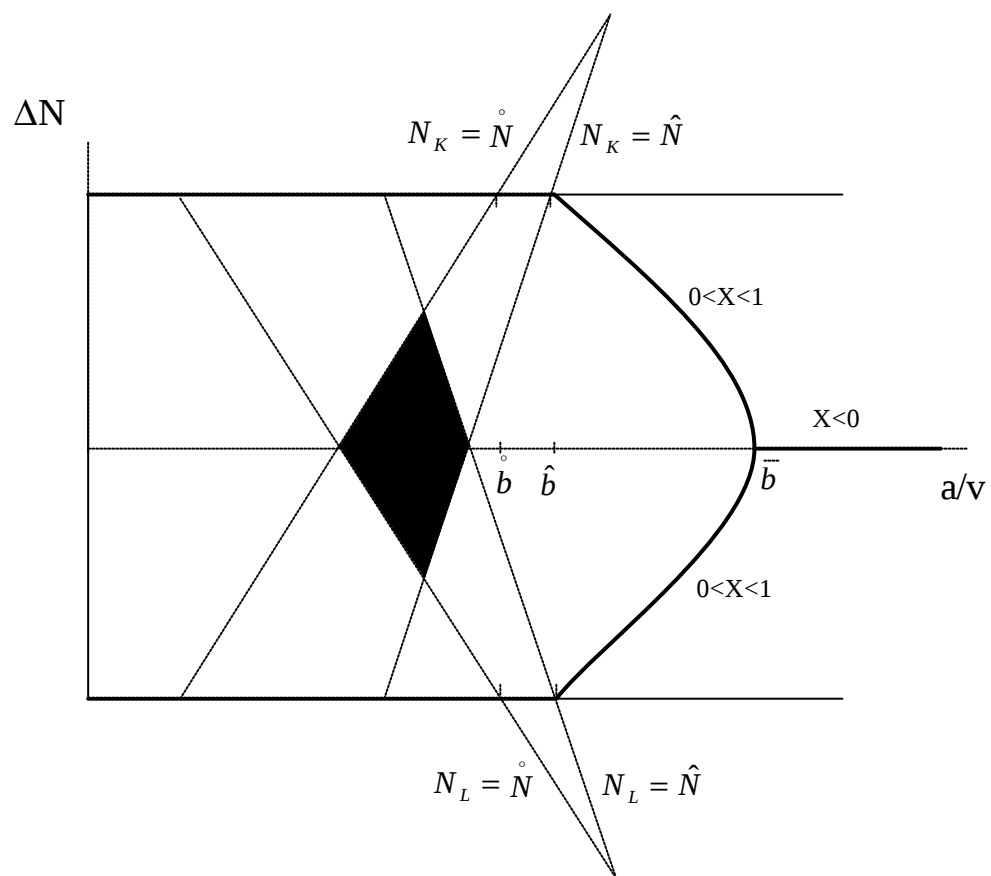


Figure 1: Stable Location Equilibria

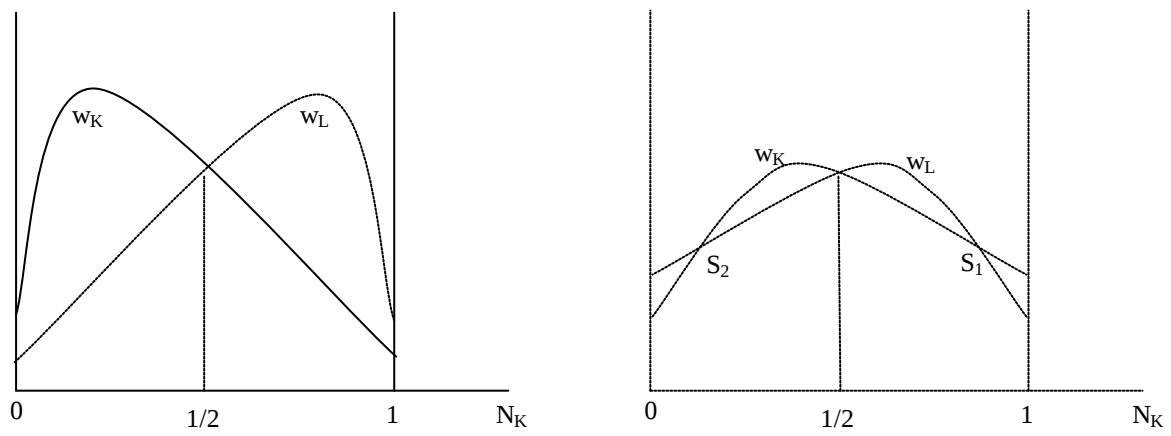


Figure 2: Full agglomeration and partial agglomeration.

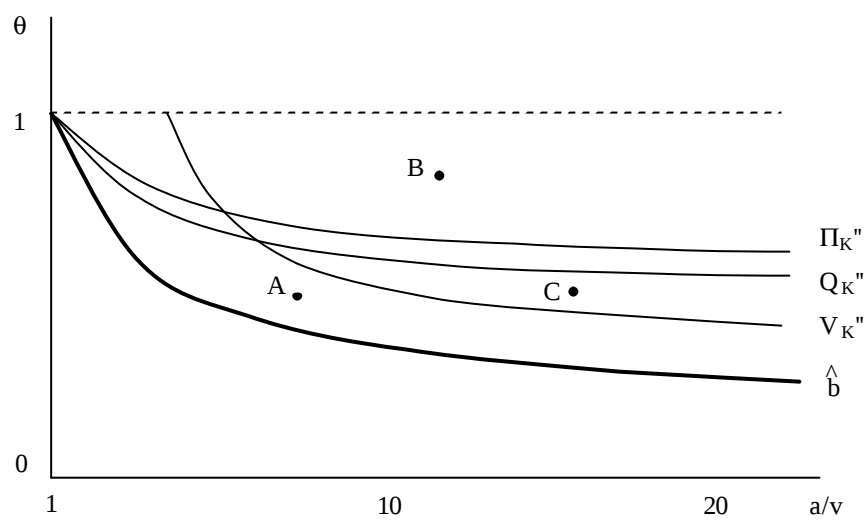


Figure 3: Welfare of workers, consumers and firms owners ($d=0$, $b=I=1$)