

CHAPTER 2

ON-WAFER MICROWAVE MEASUREMENT METHODS

2.1 Introduction

Because of the reduction of the transistor size, the operating frequency of CMOS circuits has reached microwaves. Moreover, in the last few years, CMOS technology has demonstrated to be a real contender for the realization of RF circuits. It should however be noted that although analog designers develop monolithic microwave circuits (MMICs), they are not familiar with all the concepts involved in the device measurements at microwave frequencies. The aim of this chapter is to introduce these concepts. The different formalisms describing the behavior of microwave devices will first be introduced. A great care is taken when defining the scattering and *pseudo scattering* matrices, in order to avoid errors and confusion. Then several algorithms utilized to calibrate Vector Network Analyzers (VNAs) are described. The equations of the general Thru-Attenuator-Network (TAN) self-calibration procedure is also described in details. Finally, all the concepts discussed in this chapter are used to introduce the different aspects of the on-wafer measurement.

2.2 Waves and scattering parameters

2.2.1 Introduction

The behavior of a linear n-port is usually expressed in terms of impedance (Z-parameters), or admittance (Y-parameters) matrices. These parameters are obtained by measuring the currents or voltages at each ports of the device under test (DuT) when one port is connected to a generator, and the others are connected to open or short circuits.

At microwave, it is not possible to realize perfect open and short circuits. Practically, these circuits are small capacitances and inductances that can induce significant effects at high frequencies. Then it is not possible to measure Z- or Y-parameters directly. In fact, it is more suitable to do measurements with respect to reference impedances. A generator is connected in turn to each port of the DuT, and known loads are connected to the others. Scattering parameters, which are the combination of the currents and voltages at the input of the DuT, are measured. The definition and properties of these parameters are presented in section 2.2.4.

In the next sections, concepts used for high frequency measurement will be presented. We first define two kinds of waves, taking into account that each port of the device is connected to a reference impedance. Then, based on these waves, the scattering and transfer matrices, which are fundamental formalisms for the analysis of microwave devices, will be discussed. The relation between these matrices and the immittance matrices (Y- and Z- matrices) will also be given, as relations allowing changing the reference impedance of scattering matrices.

2.2.2 Power waves

First introduced by BELEVITCH [1], and extensively discussed by KUROKAWA [2], the power waves are defined by

$$\mathbf{a}_i = \frac{\mathbf{v}_i + Z_i \mathbf{i}_i}{2\sqrt{\Re(Z_i)}} \quad (2.1)$$

$$\mathbf{b}_i = \frac{\mathbf{v}_i - Z_i^* \mathbf{i}_i}{2\sqrt{\Re(Z_i)}} \quad (2.2)$$

where Z_i is the impedance of the generator connected to the i th port of a n-port device, while $\mathbf{v}_i$ and $\mathbf{i}_i$ are the voltage and the current at that port, respectively (Figure 2.1a). The power waves have been defined to express the amount of power going from a source to a load.

Indeed, let us consider the simple 2-ports shown in Figure 2.1b. The input of the 2-ports is connected to a generator with impedance Z_1 , and to a load Z_2

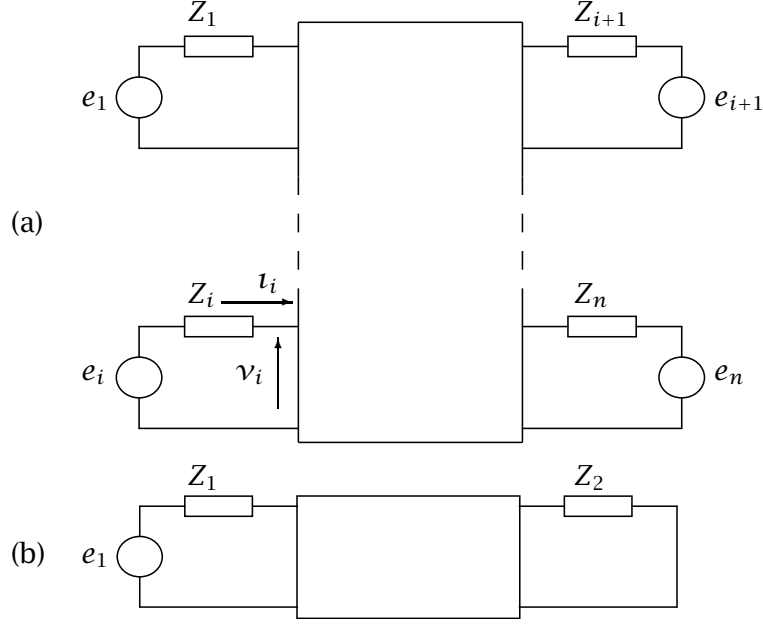


Figure 2.1: Schematic view of a linear n-ports (top), and a 2-ports with only one generator (bottom).

at the output. The real part of the impedances is assumed positive. The power waves are given by

$$\mathbf{a}_1 = \frac{v_1 + Z_1 i_1}{2\sqrt{\Re(Z_1)}} = \frac{e_1}{2\sqrt{\Re(Z_1)}} \quad (2.3)$$

$$\mathbf{b}_1 = \frac{v_1 - Z_1^* i_1}{2\sqrt{\Re(Z_1)}} \quad (2.4)$$

$$\mathbf{a}_2 = \frac{v_2 + Z_2 i_2}{2\sqrt{\Re(Z_2)}} = 0 \quad (2.5)$$

$$\mathbf{b}_2 = \frac{v_2 - Z_2^* i_2}{2\sqrt{\Re(Z_2)}} = \frac{-2\Re(Z_2) i_2}{2\sqrt{\Re(Z_2)}} \quad (2.6)$$

By using the power waves it is possible to compute the power delivered by the generator to the 2-ports or by the 2-ports to the load. The total net real power delivered by the generator to the 2-ports can be written as

$$P = \Re(v_1 i_1^*) = |\mathbf{a}_1|^2 - |\mathbf{b}_1|^2 \quad (2.7)$$

When the input impedance is equal to Z_1^* , conjugate matching is achieved at the input, the quantity $\mathbf{b}_1$ vanishes. Consequently, the power delivered to the load is equal to $|\mathbf{a}|^2$ and corresponds to the maximum available power (P_a). The

term $|\mathbf{b}_1|^2$ represents the reflected power ($P_{reflect}$). The power delivered to the load is given by

$$P_L = \Re(\nu_2(-\iota_2^*)) = |\mathbf{b}_2|^2 \quad (2.8)$$

where $|\mathbf{b}_2|^2$ is the power delivered to the load by the 2-ports.

2.2.3 Pseudo waves

Another set of parameters will now be introduced. The *pseudo* waves are defined, according to MARKS and WILLIAMS [3], by

$$a_i = \frac{\sqrt{\Re(Z_i)}}{2|Z_i|}(\nu_i + Z_i\iota_i) \quad (2.9)$$

$$b_i = \frac{\sqrt{\Re(Z_i)}}{2|Z_i|}(\nu_i - Z_i\iota_i) \quad (2.10)$$

where Z_i is the reference impedance equal to the impedance of the generator. To avoid any confusion between pseudo waves and power waves defined by Equations (2.1) and (2.2), different typographies are used for power (**a**, **b**) and *pseudo* waves (*a*, *b*). These latter are mathematical artifact usefull to simplify the computation, as it will be shown in section 2.2.4 and 2.2.5. In spite of power waves, the pseudo waves are not suitable to express power transfer of circuits. For instance, let us consider a linear 2-ports (Figure 2.1b). The power delivered by the generator to the 2-ports is given by

$$P = \Re(\nu_1 \iota_1^*) = |a_1|^2 - |b_1|^2 + 2\Im(a_1 b_1^*) \frac{\Im(Z_1)}{\Re(Z_1)} \quad (2.11)$$

This relation is much more complicated than the one obtained when using the power waves (Equation (2.7)).

On the other hand, pseudo waves allow us to simplify matching network problems. It is interesting to note that when the input impedance is equal to that of the generator, the term b_i vanish. Reflectionless matching is reached, then the reflected wave is equal to zero. In that case, the current is given by

$$\iota_i = \nu_i / Z_i$$

then

$$b_i = \frac{\sqrt{\Re(Z_i)}}{2|Z_i|}(\nu_i - Z_i \iota_i) = 0$$

Furthermore, if the generator impedance is real, the *pseudo* waves are equal to the power waves.

2.2.4 Scattering Matrices

The scattering matrix defines the relation between the “a” ($\mathbf{a}$, a) and the “b” ($\mathbf{b}$, b) waves for linear circuits. In the case of linear n-ports, the scattering matrix is given by

$$\mathbf{b} = \mathbf{S} \mathbf{a} \quad (2.12)$$

where $\mathbf{a}$ and $\mathbf{b}$ are the vectors of “a” and “b” waves, while $\mathbf{S}$ is the scattering matrix. When the power waves are used, the vectors and the matrix are given by

$$\mathbf{b} = \begin{bmatrix} \mathbf{b}_1 \\ \vdots \\ \mathbf{b}_n \end{bmatrix}, \mathbf{a} = \begin{bmatrix} \mathbf{a}_1 \\ \vdots \\ \mathbf{a}_n \end{bmatrix}, \mathbf{S} = \begin{bmatrix} s_{11} & \cdots & s_{1n} \\ \vdots & \ddots & \vdots \\ s_{n1} & \cdots & s_{nn} \end{bmatrix} \quad (2.13)$$

and they are given by the following relations if the *pseudo* waves are considered

$$\mathbf{b} = \begin{bmatrix} b_1 \\ \vdots \\ b_n \end{bmatrix}, \mathbf{a} = \begin{bmatrix} a_1 \\ \vdots \\ a_n \end{bmatrix}, \mathbf{S} = \begin{bmatrix} s_{11} & \cdots & s_{1n} \\ \vdots & \ddots & \vdots \\ s_{n1} & \cdots & s_{nn} \end{bmatrix} \quad (2.14)$$

Two different scattering matrices can be defined according to the wave (power waves or *pseudo* waves) definition adopted. To avoid confusion, the scattering matrix, defined by using the *pseudo* waves, will be called *pseudo scattering matrix*. The *pseudo scattering matrix* is used mainly in the calibration process, as it will be presented in section 2.3.2. On the other hand, the scattering matrix is used in circuit design, because it allows us to express directly the relationships between input and output powers for circuits. Indeed, the elements of the scattering matrix (S-parameters) are defined as ratio of power waves. As discussed in section 2.1, power waves are proportional to the net real power, which goes

from one access to another. Consequently, the scattering parameters simply represent the power gain of devices or circuits. In the case of a 2-ports, the transducer gain (G_t), defined as the ratio between the power delivered to the load and the available input power, is given by

$$G_t = \frac{|\mathbf{b}_2|^2}{|\mathbf{a}_1|^2} = |s_{21}|^2 \quad (2.15)$$

In addition to this gain, there are other useful relation measuring the power flow from a 2-ports:

$$G = \frac{\text{Power delivered to the load}}{\text{Power into the 2-ports}} = \frac{|s_{21}|^2}{1 - |s_{11}|^2} \quad (2.16)$$

$$G_a = \frac{\text{Power available at the output}}{\text{Power available from the generator}} = \frac{|s_{21}|^2}{1 - |s_{22}|^2} \quad (2.17)$$

Usually, measured S-parameters are known with reference to generator impedances equal to 50 Ohms. This is far from conjugate matching condition. Then in order to maximize the transducer gain for circuit design, using S-parameters, it is necessary to develop expressions allowing us to change independently the reference impedances of each access. These expressions are discussed in section 2.2.7.

Scattering and *pseudo scattering* matrices have different behaviors. When complex reference impedances are used, misleading results are obtained when confusing these matrices. When considering real reference impedances, scattering and *pseudo scattering* matrices provide the same results, as the power and *pseudo* waves are equal.

In the following sections, two simple examples are used to show how is it important to avoid confusing these matrices. The scattering and *pseudo scattering* matrices of a transmission line and a thru line are compared to each other. In both cases, the generator impedances and characteristic impedance of the line (Z_c) are assumed to be equal.

2.2.4.1 The transmission line

Let's considered a transmission line with known characteristic impedance (Z_c), propagation constant (γ) and length (l). The generator impedances are equal to the characteristic impedance of the line. As a consequence, the currents and voltages at distance (l) from the line input can be expressed by

$$v(l) = v(0)e^{-\gamma l}, \quad i(l) = i(0)e^{-\gamma l} \quad (2.18)$$

where $v(0)$ and $i(0)$ are the voltage and current at the input of the line, respectively. In that configuration, the power waves are given by

$$a_1 = \frac{v_1 + Z_c i_1}{2\sqrt{\Re(Z_c)}} \quad (2.19)$$

$$a_2 = \frac{v_2 + Z_c i_2}{2\sqrt{\Re(Z_c)}} = \frac{v_1 e^{-\gamma l} + Z_c (-i_1) e^{-\gamma l}}{2\sqrt{\Re(Z_c)}} \quad (2.20)$$

$$b_1 = \frac{v_1 - Z_c^* i_1}{2\sqrt{\Re(Z_c)}} \quad (2.21)$$

$$b_2 = \frac{v_2 - Z_c^* i_2}{2\sqrt{\Re(Z_c)}} = \frac{v_1 e^{-\gamma l} - Z_c^* (-i_1) e^{-\gamma l}}{2\sqrt{\Re(Z_c)}} \quad (2.22)$$

and the pseudo waves by

$$a_1 = \frac{\sqrt{\Re(Z_c)}}{|Z_c|} (v_1 + Z_c i_1) \quad (2.23)$$

$$a_2 = \frac{\sqrt{\Re(Z_c)}}{|Z_c|} (v_2 + Z_c i_2) = \frac{\sqrt{\Re(Z_c)}}{|Z_c|} (v_1 e^{-\gamma l} + Z_c (-i_1) e^{-\gamma l}) \quad (2.24)$$

$$b_1 = \frac{\sqrt{\Re(Z_c)}}{|Z_c|} (v_1 - Z_c i_1) \quad (2.25)$$

$$b_2 = \frac{\sqrt{\Re(Z_c)}}{|Z_c|} (v_2 - Z_c i_2) = \frac{\sqrt{\Re(Z_c)}}{|Z_c|} (v_1 e^{-\gamma l} - Z_c (-i_1) e^{-\gamma l}) \quad (2.26)$$

By applying the definition of the (pseudo-)scattering parameters

$$s_{11} = \left. \frac{b_1}{a_1} \right|_{a_2=0} \quad s_{12} = \left. \frac{b_1}{a_2} \right|_{a_1=0} \quad s_{21} = \left. \frac{b_2}{a_1} \right|_{a_2=0} \quad s_{22} = \left. \frac{b_2}{a_2} \right|_{a_1=0} \quad (2.27)$$

The scattering and *pseudo scattering* matrices are obtained.

The scattering matrix is given by

$$S = \frac{1}{Z_c} \begin{bmatrix} \Im(Z_c) & \Re(Z_c)e^{-\gamma l} \\ \Re(Z_c)e^{-\gamma l} & \Im(Z_c) \end{bmatrix} \quad (2.28)$$

and the *pseudo scattering* matrix is given by

$$S = \begin{bmatrix} 0 & e^{-\gamma l} \\ e^{-\gamma l} & 0 \end{bmatrix} \quad (2.29)$$

In spite of what it is found sometimes in the literature, the scattering matrix of a matched transmission line has non-zero reflection at its ports if its characteristic impedance is complex ($s_{11} \neq 0$, $s_{22} \neq 0$ if Z_c is complex). Sometimes, when transmission lines are investigated, the *pseudo scattering* matrix is confused with the scattering matrix, as in [4].

2.2.4.2 The thru line

The thru line is a virtual element considering that for a 2-ports, the two reference planes are merged. It can be seen as a zero length transmission line. Thus, the scattering and *pseudo scattering* matrices of the thru line are obtained using the expression previously calculated for transmission lines, imposing $l = 0$. Thus, the scattering matrix of the thru line is given by

$$S = \frac{1}{Z_c} \begin{bmatrix} \Im(Z_c) & \Re(Z_c) \\ \Re(Z_c) & \Im(Z_c) \end{bmatrix} \quad (2.30)$$

where Z_c being the impedance of the generators. The *pseudo scattering* matrix of the thru line is given by

$$S = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad (2.31)$$

It is important to note that, when complex generator impedances are used, even a thru line as a non-trivial scattering matrix, contrary to its *pseudo scattering* matrix which is very simple.

2.2.5 Transfer Matrix

When several circuits are connected together in series, it is useful to have a formalism that allows us to easily obtain the parameters of the total circuit based on the parameters of each of the sub-circuits. When waves are used, this formalism is the transfer matrix. This matrix represents the relationship between the a and b *pseudo* waves at the input of the circuit and those at the output. The transfer matrix can also be defined using the power waves but does not have any interesting properties, as it will be shown at the end of this section. In the case of 2-ports networks, the transfer matrix, denoted T , satisfies the following equation

$$\begin{bmatrix} b_1 \\ a_1 \end{bmatrix} = T \begin{bmatrix} a_2 \\ b_2 \end{bmatrix} \quad (2.32)$$

The relation between the S and T matrices can be easily obtained. If the *pseudo scattering* matrix is rearranged properly

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = S \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \quad (2.33)$$

The transfer matrix is given by

$$T = \frac{1}{s_{21}} \begin{bmatrix} -\Delta_s & s_{11} \\ -s_{22} & 1 \end{bmatrix} \quad (2.34)$$

where $\Delta_s = s_{11}s_{22} - s_{12}s_{21}$. The whole transfer matrix of a network is simply the product of the single transfer matrix of each circuit, but only if the reference impedances at each junction are the same.

The transfer matrix defines waves in a forward direction, from an input port to an output port (T_{for}). A matrix linking the waves from the output port to the input port can also be defined (*reverse direction*; T_{rev}), but that transfer matrix is not equal to the inverse of T_{for} . The transfer matrices are defined by

$$\begin{bmatrix} b_1 \\ a_1 \end{bmatrix} = T_{for} \begin{bmatrix} a_2 \\ b_2 \end{bmatrix} \quad (2.35)$$

$$\begin{bmatrix} b_2 \\ a_2 \end{bmatrix} = T_{rev} \begin{bmatrix} a_1 \\ b_1 \end{bmatrix} \quad (2.36)$$

T_{for}^{-1} expresses the following relation between the “a” and “b” waves:

$$\begin{bmatrix} a_2 \\ b_2 \end{bmatrix} = T_{for}^{-1} \begin{bmatrix} b_1 \\ a_1 \end{bmatrix} \quad (2.37)$$

In order to obtain the expression for T_{rev} , the element of the matrix T_{for}^{-1} must be rearranged. By doing so, T_{rev} can be written as

$$T_{rev} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} T_{for}^{-1} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad (2.38)$$

Transfer matrices are extremely useful, and are intensively used in the calibration of a vector network analyzer (VNA), as shown later in section 2.3.2. As introduced at the beginning of this section, there is no interest in defining transfer matrices with power waves. These transfer matrices cannot be multiplied, if the reference impedances at each junction are the same, to obtain the whole transfer matrix of a circuit.

Indeed, when considering two cascaded 2-ports having the same complex reference impedance at the junction (Z_{ref}) (Figure 2.2). If the transfer matrices can be used, the $b_{out,1}$ wave of the first sub-circuit at the junction must be equal to the $a_{in,2}$ wave of the second sub-circuit. But these waves are defined by

$$b_{out,1} = \frac{v_j - Z_{ref}^* i_j}{2\sqrt{\Re(Z_{ref})}}, \quad a_{in,2} = \frac{v_j - Z_{ref} i_j}{2\sqrt{\Re(Z_{ref})}} \quad (2.39)$$

they can be equal only if the reference impedance is real. This is an restrictive condition.

2.2.6 Immittance Matrices

The impedance and the admittance matrices, denoted Z and Y respectively, relate the column vectors v and i , whose elements are the voltages and currents

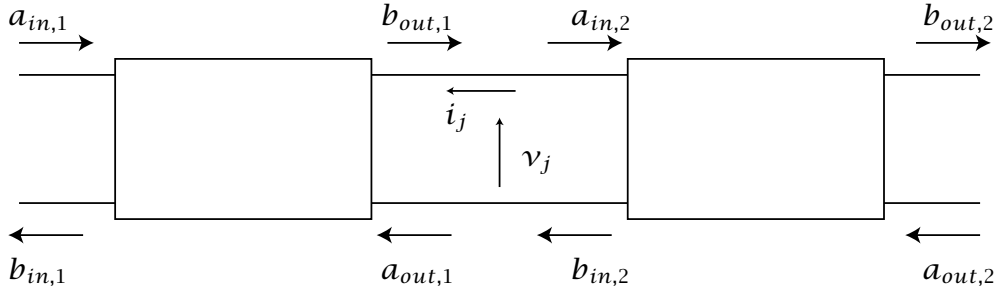


Figure 2.2: Schematic view of two 2-ports connected in series

at each of the ports of a circuit:

$$\mathbf{v} = \mathbf{Z} \mathbf{i} \quad (2.40)$$

$$\mathbf{i} = \mathbf{Y} \mathbf{v} \quad (2.41)$$

where

$$\mathbf{v} = \begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix}, \mathbf{i} = \begin{bmatrix} i_1 \\ \vdots \\ i_n \end{bmatrix} \quad (2.42)$$

and

$$\mathbf{Z} = \begin{bmatrix} z_{11} & \cdots & z_{1n} \\ \vdots & \ddots & \vdots \\ z_{n1} & \cdots & z_{nn} \end{bmatrix}, \mathbf{Y} = \begin{bmatrix} y_{11} & \cdots & y_{1n} \\ \vdots & \ddots & \vdots \\ y_{n1} & \cdots & y_{nn} \end{bmatrix} \quad (2.43)$$

Contrarily to S and T , Z and Y are independent of the impedances of the generators. Also, these representations are much more practical for modeling small integrated devices, like transistors. Indeed, as it will be presented in the next chapter, transistors can be modeled using lumped elements. That kind of model are easily written down by using the immittance parameters.

Scattering matrix For the scattering matrix, the $\mathbf{a}$ and $\mathbf{b}$ vectors, defined by Equations (2.1) can (2.2) can be rewritten by

$$\mathbf{a} = \mathbf{F}(\mathbf{v} + \mathbf{G}\mathbf{i}) \quad (2.44)$$

$$\mathbf{b} = \mathbf{F}(\mathbf{v} - \mathbf{G}^*\mathbf{i}) \quad (2.45)$$

where

$$\mathbf{F} = \begin{bmatrix} \frac{1}{2\sqrt{\Re(Z_1)}} & 0 & \dots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & \frac{1}{2\sqrt{\Re(Z_n)}} \end{bmatrix} \quad (2.46)$$

$$\mathbf{G} = \begin{bmatrix} Z_1 & 0 & \dots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & Z_n \end{bmatrix} \quad (2.47)$$

The symbol $*$ indicates the complex conjugate transposed matrix. After some calculation, the following relation is obtained

$$\mathbf{Z} = \mathbf{F}^{-1}(\mathbf{I} - \mathbf{S})^{-1}(\mathbf{S}\mathbf{G} + \mathbf{G}^*)\mathbf{F} \quad (2.48)$$

where $\mathbf{I}$ is for the unity matrix.

Pseudo scattering matrix For the *pseudo scattering* matrix, the relations are obtained in a similar way, using the definition of the *pseudo* waves (Equations (2.9) and (2.10)):

$$\mathbf{a} = \mathbf{U}(\mathbf{v} + \mathbf{G}\mathbf{i}) \quad (2.49)$$

$$\mathbf{b} = \mathbf{U}(\mathbf{v} - \mathbf{G}\mathbf{i}) \quad (2.50)$$

where $\mathbf{U}$ is defined by

$$\mathbf{U} = \begin{bmatrix} \frac{\sqrt{\Re(Z_1)}}{|Z_1|} & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \frac{\sqrt{\Re(Z_n)}}{|Z_n|} \end{bmatrix} \quad (2.51)$$

The final relation is given by

$$\mathbf{Z} = (\mathbf{I} - \mathbf{U}^{-1}\mathbf{S}\mathbf{U})^{-1}(\mathbf{I} + \mathbf{U}^{-1}\mathbf{S}\mathbf{U})\mathbf{G} \quad (2.52)$$

2.2.7 Change of reference impedance

It is possible to develop expressions allowing changing the generator impedances of the (pseudo-)scattering matrix. These relations are presented in the following sections.

2.2.7.1 Scattering Matrix

Considering

$$S = F(Z - G^*)(Z + G)^{-1}F^{-1} \quad (2.53)$$

Z can be expressed as a function of a new scattering matrix (S') having new generator impedances (Z'_i).

$$Z = F'^{-1}(I - S')^{-1}(S'G' + G'^*)F' \quad (2.54)$$

Combining the Equations (2.54) and (2.53), and using Γ defined by

$$\Gamma = (G' - G)(G' + G^*)^{-1} \quad (2.55)$$

S' can be rewritten in the form

$$S' = F'F^{-1}(I - S)^{-1}(S - \Gamma^*)(I - \Gamma^*)(I - \Gamma)(I - S\Gamma)^{-1}(I - S)FF'^{-1} \quad (2.56)$$

With

$$(I - S)^{-1}(S - \Gamma^*)(I - \Gamma^*) = (I - \Gamma^*)(S - \Gamma^*)(I - S)^{-1} \quad (2.57)$$

and

$$(I - \Gamma)(I - S\Gamma)^{-1}(I - S) = (I - S)(I - S\Gamma)^{-1}(I - \Gamma) \quad (2.58)$$

S' becomes

$$S' = A^{-1}(S - \Gamma^*)(I - \Gamma S)^{-1}A^* \quad (2.59)$$

where A is a diagonal matrix defined by

$$A = F'^{-1}F(I - \Gamma^*) \quad (2.60)$$

Calculation of the i th diagonal component A_i shows that

$$A_i = \frac{1 - r_i^*}{|1 - r_i|} \sqrt{|1 - r_i r_i^*|} \quad (2.61)$$

where r_i is the i th diagonal component of Γ and is given by

$$r_i = \frac{Z'_i - Z_i}{Z'_i + Z_i^*} \quad (2.62)$$

From this latter, r_i can be interpreted as the power wave reflection coefficient of Z'_i with respect to Z_i^* .

2.2.7.2 Pseudo scattering matrix

The general equation of the change of reference impedances of the *pseudo scattering* matrix can be obtained in a similar way as for the scattering matrix. However there is a more simple method, based the transfer matrix properties. If the transfer matrix T_{dut} can be defined, a change of its reference impedances is equivalent to multiply that matrix by two other matrices corresponding to thru lines with different reference impedances. The new transfer matrix is given by

$$T'_{dut} = T_{left} T_{dut} T_{right} \quad (2.63)$$

where T_{left} and T_{right} are transfer matrices of thru lines with different generator impedances at input and output ports. Since T'_{dut} is known, the new *pseudo scattering* matrix is deduced using the relations presented in section 2.2.5.

In the case of a 2-ports, T_{left} and T_{right} are given by

$$T_{left} = \frac{\sqrt{\Re(Z_{new,1}/Z_{old,1})}}{2|Z_{new,1}/Z_{old,1}|} \begin{bmatrix} \frac{Z_{new,1} + Z_{old,1}}{Z_{old,1}} & \frac{Z_{old,1} - Z_{new,1}}{Z_{old,1}} \\ \frac{Z_{old,1} - Z_{new,1}}{Z_{old,1}} & \frac{Z_{new,1} + Z_{old,1}}{Z_{old,1}} \end{bmatrix} \quad (2.64)$$

$$T_{right} = \frac{\sqrt{\Re(Z_{old,2}/Z_{new,2})}}{2|Z_{old,2}/Z_{new,2}|} \begin{bmatrix} \frac{Z_{old,2} + Z_{new,2}}{Z_{new,2}} & \frac{Z_{new,2} - Z_{old,2}}{Z_{new,2}} \\ \frac{Z_{new,2} - Z_{old,2}}{Z_{new,2}} & \frac{Z_{old,2} + Z_{new,2}}{Z_{new,2}} \end{bmatrix} \quad (2.65)$$

where $Z_{old,1}$ and $Z_{old,2}$ are the old input and output reference impedances, $Z_{new,1}$ and $Z_{new,2}$ being the new ones.

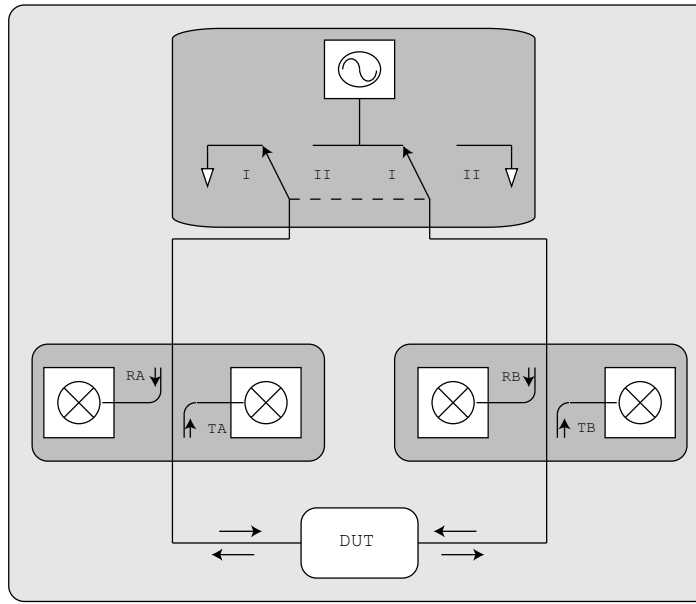


Figure 2.3: Principle of operation of a VNA

2.2.8 Conclusion

In the previous section, general expressions for scattering parameters have been extensively discussed. Moreover, relation among different parameters have been formalized in term of a general matrix approach. Simplified expressions, when a 2-ports circuits is used, can be found in appendix A.

2.3 The Vector Network Analyzer

2.3.1 Introduction

Vector network analyzers (VNA) are able to measure the amplitude as well as the phase of the scattering parameters of the device under test (DuT). The schematic representation of a VNA is shown in Figure 2.3. The VNA is made up of a source delivering a RF signal. The ports are measured in turn, when the source is switched from one port to another. When one port is connected to the generator, the other is switched to a matched load. The switch is in the “I” state, when the first port is connected to the generator, and the other to the matched load.

The delivered signal is then split in two parts: the first part, denoted RA or RB, is an image of the incident wave. The second part propagates toward the

DuT. Directional couplers are used to recover the reflected part of the signal (TA) and a part of the signal that came out of the DuT (TB).

The measured signals (R_i and T_i) are first down converted, and then they are compared to a known reference using synchronous detection. Thus, their magnitude and their phase are known. The ratio between these quantities, which correspond to the power waves, allows us to determine the S-parameters using the following relations

$$s_{11} = \frac{TA}{RA} \quad s_{12} = \frac{TA}{RB} \quad s_{21} = \frac{TB}{RA} \quad s_{22} = \frac{TB}{RB} \quad (2.66)$$

However, measured S-parameters are not the real S-parameters of the DuT. Indeed, couplers are not ideal, and the reference plane of the VNA can be far from the edge of the DuT. The DuT is usually connected to the VNA through long coaxial cables. As a consequence, the measured S-parameters must be corrected. This operation is done by using some specific techniques to calibrate the VNA. Some of them are presented in the next section.

2.3.2 Calibration

The algorithms described here are generally called self calibration methods, as they only need a partial knowledge of the characteristics of the calibration standards [5]. The other methods will not be presented, as they do not apply for on-wafer measurements. We will first discuss error models within the frame of the transfer matrix approach. They are used to determine the number of error coefficients that must be extracted by calibration. Also, equations allowing us to obtain the wanted S-parameters from the raw measurements are given, assuming that the error coefficients are known.

Then the general Thru-Attenuator-Network (TAN) method used to extract these coefficients is explained. Some derived methods called Thru-Reflect-Line (TRL) and Thru-Reflect-Match (TRM) are described, as they are practical applications of the TAN method.

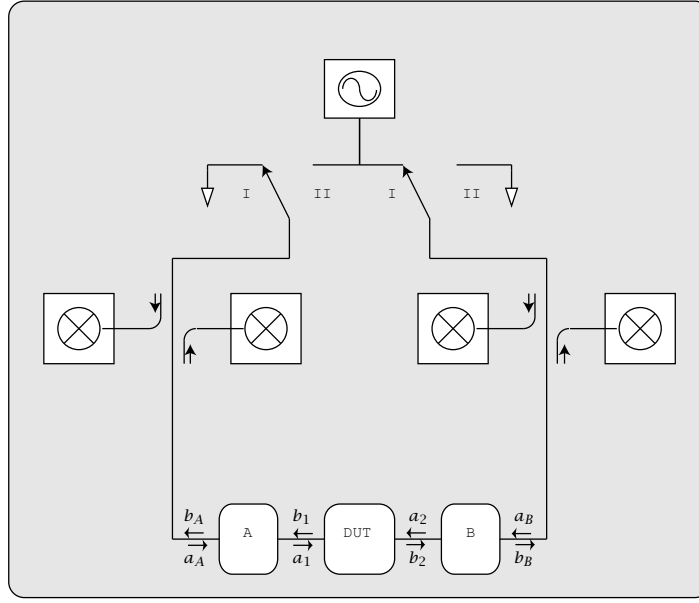


Figure 2.4: VNA with the 2 error 2-ports model

2.3.2.1 The transfer function formalism

Any calibration procedure requires an error model. In the case discussed in this chapter, the error terms are considered as the elements of two passive 2-ports network located before and after the device under test, as shown in Figure 2.4. Except the two 2-ports A and B, the component of the VNA are considered as ideal. This error model neglects direct coupling between the two ports. The transfer matrices of the 3 2-ports are defined by

$$\begin{bmatrix} b_A \\ a_A \end{bmatrix} = \mathbf{T}_A \begin{bmatrix} b_1 \\ a_1 \end{bmatrix} \quad (2.67)$$

$$\begin{bmatrix} b_1 \\ a_1 \end{bmatrix} = \mathbf{T}_{DUT} \begin{bmatrix} a_2 \\ b_2 \end{bmatrix} \quad (2.68)$$

$$\begin{bmatrix} b_B \\ a_B \end{bmatrix} = \mathbf{T}_B \begin{bmatrix} b_2 \\ a_2 \end{bmatrix} \quad (2.69)$$

Based on the setup shown in Figure 2.4 we can define the two following matrices which contain the measured a and b waves for each position of the switch (I or

II).

$$\mathbf{M}_A = \begin{bmatrix} b_{A_I} & b_{A_{II}} \\ a_{A_I} & a_{A_{II}} \end{bmatrix}, \quad \mathbf{M}_B = \begin{bmatrix} a_{B_I} & a_{B_{II}} \\ b_{B_I} & b_{B_{II}} \end{bmatrix} \quad (2.70)$$

The relationship between these two matrices is given by

$$\mathbf{M}_A = \mathbf{T}_A \mathbf{T}_{DuT} \mathbf{T}_{B^{-1}} \mathbf{M}_B \quad (2.71)$$

where $\mathbf{T}_{DuT}$ is the true transfer matrix of the device under test.

Equation (2.71) allows some level of indetermination. Indeed, if a scalar coefficient multiplies both $\mathbf{T}_A$ and $\mathbf{T}_B$, the results of equation (2.71) will be unchanged. In order to get rid off the indetermination, the normalized transfer matrix is introduced

$$\mathbf{T}_x = t_x \mathbf{T}_x \quad (2.72)$$

$$\mathbf{T}_x = \begin{bmatrix} t_{x11} & t_{x12} \\ t_{x21} & 1 \end{bmatrix} \quad (2.73)$$

where we assumed $\mathbf{T}_{x22}$ equal to 1. It allows to define the normalized transfer matrix for devices with a zero forward transmission ($s_{21} = 0$), since $\mathbf{T}_{x22} = 1/s_{21}$ (Equation (2.34)) The reverse normalized transfer matrix is given by

$$\mathbf{T}_{x^{-1}} = \frac{1}{t_x} \mathbf{T}_{x^{-1}} \quad (2.74)$$

Applying the definition of the normalized transfer matrix to Equation (2.71) we have

$$\mathbf{M}_A = \frac{t_A}{t_B} \mathbf{T}_A \mathbf{T}_{DuT} \mathbf{T}_{B^{-1}} \mathbf{M}_B \quad (2.75)$$

Equation (2.75) shows that only the ratio between t_A and t_B is important, and not their values. Now, using the measured waves ($\mathbf{M}_A, \mathbf{M}_B$), the transfer matrix of the measurement can be obtain by evaluating

$$\mathbf{M} = \mathbf{M}_A \mathbf{M}_B^{-1} = \frac{t_A}{t_B} \mathbf{T}_A \mathbf{T}_{DuT} \mathbf{T}_{B^{-1}} \quad (2.76)$$

Equation (2.76) holds when the device under test has a non zero forward transmission $s_{21} \neq 0$.

Indeed, if the DuT has a zero forward transmission, and when the VNA switch is on position I , $a_B = 0$ because the port B is not connected to the generator, and $b_2 = 0$ because no power are transmit through the DuT. Since the 2-ports B is considered as a passive element, there is no signal which comes out of the 2-ports. Then $b_B = 0$. Thus the elements of the first column of $\mathbf{M}_B$ are equal to zero, and by consequence $\mathbf{M}_B^{-1}$ is not defined. To avoid this limitation, Equation (2.71) can be rewritten

$$\mathbf{M}_A \tilde{\mathbf{M}}_B = \frac{t_A}{t_B} \mathbf{T}_A \mathbf{T}'_{DuT} \mathbf{T}_B^{-1} \quad (2.77)$$

where $\mathbf{T}'_{DuT} = \det(\mathbf{M}_B) \mathbf{T}_{DuT}$. The symbol $\sim$ indicates the transposed adjoint matrix. Equations (2.76-2.77) are extremely important in the determination of the error coefficients, as it will be shown in the next section.

When the two error matrices $\mathbf{T}_A$ and $\mathbf{T}_B$ and the ratio $\frac{t_A}{t_B}$ are known, it is possible to recover the *pseudo scattering* matrix of the device under test using the following relations proposed by HEUERMANN and SCHIEK in [6].

$$\mathbf{C} = \tilde{\mathbf{T}}_A \mathbf{M}_A \tilde{\mathbf{M}}_B \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \frac{t_B}{t_A} \mathbf{T}_B \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} \quad (2.78)$$

$$s_{11} = \frac{c_{12}}{c_{22}} \quad (2.79)$$

$$s_{12} = \frac{\det(\mathbf{M}_A) \det(\mathbf{T}_B)}{c_{22}} \quad (2.80)$$

$$s_{21} = \frac{\det(\mathbf{M}_B) \det(\mathbf{T}_A)}{c_{22}} \quad (2.81)$$

$$s_{22} = -\frac{c_{21}}{c_{22}} \quad (2.82)$$

2.3.2.2 General TAN self calibration procedure

The transfer matrix $\mathbf{T}_A$, $\mathbf{T}_B$ and $\frac{t_A}{t_B}$ are determined by using the TAN self calibration procedure. The TAN method consists in two steps: the determination of the true characteristics of the standards used, which are partially known, and then the calibration of the system. This method requires the use of 3 standards. Their *pseudo scattering* and transfer matrices are given by the following

equations.

$$S_\tau = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, S_\alpha = \begin{bmatrix} 0 & s_{\alpha r} \\ s_{\alpha f} & 0 \end{bmatrix}, S_\nu = \begin{bmatrix} r_\nu & s_{\nu r} \\ s_{\nu f} & r_\nu \end{bmatrix} \quad (2.83)$$

$$T_\tau = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, T_\alpha = \begin{bmatrix} s_{\alpha r} & 0 \\ 0 & 1/s_{\alpha f} \end{bmatrix}, T_\nu = \frac{1}{s_{\nu f}} \begin{bmatrix} -\Delta s_\nu & r_\nu \\ -r_\nu & 1 \end{bmatrix} \quad (2.84)$$

where index τ stands for the thru, index α for the attenuator and index ν for the network. No assumptions are made on the non zero coefficient of the 3 standards.

The consequences of these assumptions are the following [3][5]:

- The expression of T_τ forces to have identical reference impedances at both ports. Indeed, as presented in section 2.2.5, the transfer matrix equal to the identity matrix corresponds to a thru line with the same generator impedance at both ports. Also, the reference planes are defined in the middle of the measured thru line.
- The expression of T_α determines the value of the reference impedance. It will be equal to an impedance Z_L if the input impedance of the attenuator is equal to Z_L when Z_L is connected at its output. If the attenuator is obtained by connecting two identical one-port loads, then the reference impedance is set equal to the load impedance to ensure $S_{\alpha 11} = S_{\alpha 22} = 0$.

The measurement equations of the standards can be easily obtained. Applying the Equation (2.76) to the thru yields

$$M_\tau = M_{A\tau} M_{B\tau}^{-1} = \frac{t_A}{t_B} T_A T_{B^{-1}} \quad (2.85)$$

As no assumption have been made on the values of $s_{\alpha r, f}$ and $s_{\nu r, f}$, zero transmission is possible for the attenuator and the reflect. To allow a possible zero transmission for the α and ν standards, Equation (2.77) is used

$$M'_\alpha = M_{A\tau} \tilde{M}_{B\tau} = \frac{t_A}{t_B} T_A T'_\alpha T_{B^{-1}} \quad (2.86)$$

$$M'_\nu = M_{A\nu} \tilde{M}_{B\nu} = \frac{t_A}{t_B} T_A T'_\nu T_{B^{-1}} \quad (2.87)$$

where T'_α and T'_ν are called *pseudo* transmission matrix [7], and are equal to

$$T'_\alpha = \det(M_{B\alpha})T_\alpha = \begin{bmatrix} s'_{\alpha r} & 0 \\ 0 & 1/s'_{\alpha f} \end{bmatrix} \quad (2.88)$$

$$T'_\nu = \det(M_{B\nu})T_\nu = \frac{1}{s'_{\nu f}} \begin{bmatrix} -\Delta s'_\nu & r'_\nu \\ -r'_\nu & 1 \end{bmatrix} \quad (2.89)$$

The measurement equations being defined, the unknown coefficients of the standards defined by Equation (2.84), can be determined.

Determination of the standards Combining Equations (2.86) and (2.87), with (2.85) to eliminate the $T_{B^{-1}}$ leads to the following equations:

$$M'_\alpha M_\tau^{-1} = T_A T'_\alpha T_A^{-1} \quad (2.90)$$

$$M'_\nu M_\tau^{-1} = T_A T'_\nu T_A^{-1} \quad (2.91)$$

Multiplying these two equations together leads to a third useful relation:

$$M'_\alpha M_\tau^{-1} M'_\nu M_\tau^{-1} = T_A T'_\alpha T'_\nu T_A^{-1} \quad (2.92)$$

Equations (2.90) through (2.92) are related to the measurements by a similarity transform defined by the matrix T_A . The properties of the similar matrices can be exploited (i.e. $M'_\alpha M_\tau^{-1}$ and T'_α). They have equal determinants and traces. Using these properties, five new equations can be derived [8]:

$$\theta_{\alpha\tau} = \text{trace}(T'_\alpha) = \text{trace}(M'_\alpha M_\tau^{-1}) \quad (2.93)$$

$$\delta_{\alpha\tau} = \det(T'_\alpha) = \det(M'_\alpha M_\tau^{-1}) \quad (2.94)$$

$$\theta_{\nu\tau} = \text{trace}(T'_\nu) = \text{trace}(M'_\nu M_\tau^{-1}) \quad (2.95)$$

$$\delta_{\nu\tau} = \det(T'_\nu) = \det(M'_\nu M_\tau^{-1}) \quad (2.96)$$

$$\theta_{\alpha\nu\tau} = \text{trace}(T'_\alpha T'_\nu) = \text{trace}(M'_\alpha M_\tau^{-1} M'_\nu M_\tau^{-1}) \quad (2.97)$$

The α -standard Solving Equation (2.93) and (2.94) allows us to determine the parameters of the attenuator:

$$s'_{\alpha r} = \frac{\theta_{\alpha\tau} \pm \sqrt{\theta_{\alpha\tau}^2 - 4\delta_{\alpha\tau}}}{2} \quad (2.98)$$

$$s'_{\alpha f} = \frac{1}{\theta_{\alpha\tau} - s'_{\alpha r}} \quad (2.99)$$

The indetermination on the value of $s'_{\alpha r}$ can be lifted if the behavior of the attenuator is known. For instance, if there are significant losses, the right choice is the root which leads to $s_{\alpha r} < 1$ and $s_{\alpha f} < 1$.

The ν -standard Equations (2.95)-(2.97) yield

$$s'_{\nu f} = \frac{1 - s'_{\alpha r} s'_{\alpha f}}{s'_{\alpha f} (\theta_{\alpha\nu\tau} - s'_{\alpha r} \theta_{\nu\tau})} \quad (2.100)$$

$$s'_{\nu r} = s'_{\nu f} \delta_{\nu\tau} \quad (2.101)$$

$$r_\nu = \pm \sqrt{s'_{\nu f} (s'_{\nu r} - \theta_{\nu\tau}) + 1} \quad (2.102)$$

The indetermination of the last equation can be easily resolved using a priori information on the phase of the reflection coefficient r_ν .

Determination of the error coefficients The method shown below has been proposed by HEUERMANN and SCHIEK [6], and the equations of the system have been corrected by GILLON in [9]. The measurement Equation (2.85) of the thru line can be rewritten as

$$\mathbf{T}_B \frac{t_B}{t_A} = \mathbf{M}_\tau^{-1} \mathbf{T}_A \quad (2.103)$$

When substituting this equation in both the measurement equations of the attenuator and the reflect (Equations (2.90) and (2.91)), it yields to

$$\mathbf{M}'_\alpha \mathbf{M}'_\tau \mathbf{T}_A = \mathbf{T}_A \mathbf{T}'_\alpha \quad (2.104)$$

$$\mathbf{M}'_\nu \mathbf{M}'_\tau \mathbf{T}_A = \mathbf{T}_A \mathbf{T}'_\nu \quad (2.105)$$

As $\mathbf{T}_{A22} = 1$, only three equations are needed to determined the unknown coefficients of $\mathbf{T}_A$, but eight are available. According to HEUERMANN and SCHIEK,

the following system of equations provides good results:

$$\begin{bmatrix} \mathbf{T}_{A11} \\ \mathbf{T}_{A12} \\ \mathbf{T}_{A21} \end{bmatrix} = \begin{bmatrix} \mathbf{M}_{\alpha\tau_{11}} + \mathbf{M}_{\alpha\tau_{21}} - s'_{\alpha r} & 0 & \mathbf{M}_{\alpha\tau_{12}} + \mathbf{M}_{\alpha\tau_{22}} - s'_{\alpha r} \\ 0 & \mathbf{M}_{\alpha\tau_{11}} + \mathbf{M}_{\alpha\tau_{21}} - 1/s'_{\alpha f} & 0 \\ -r_v/s'_{vf} & \mathbf{M}_{\alpha\tau_{11}} + \mathbf{M}_{\alpha\tau_{21}} + s'_{vf} & -r_v s'_{vf} \end{bmatrix}^{-1} \times \begin{bmatrix} 0 \\ s'_{\alpha f} - \mathbf{M}_{\alpha\tau_{12}} - \mathbf{M}_{\alpha\tau_{22}} \\ s'_{vf} - \mathbf{M}_{v\tau_{12}} - \mathbf{N}_{v\tau_{22}} \end{bmatrix} \quad (2.106)$$

where the following notations were used

$$\mathbf{M}_{\alpha\tau} = \mathbf{M}'_{\alpha} \mathbf{M}_{\tau}^{-1} \quad \mathbf{M}_{v\tau} = \mathbf{M}'_v \mathbf{M}_{\tau}^{-1} \quad (2.107)$$

The values of $\frac{t_B}{t_A} \mathbf{T}_B$ are obtained using Equation (2.103).

2.3.2.3 Practical applications

The algorithm presented above is general, and can be used with various attenuators or α -standards. Usually, attenuators are made by transmission lines or two perfectly matched loads. In this case, the general TAN procedure is simply called TRL (Thru-Line-Reflect) or TRM (Thru-Reflect-Match), respectively. These two specific cases of the general TAN algorithm are briefly presented hereafter.

Thru-Reflect-Line This method relies on the measurement of two different lines with different lengths and a reflective element. The shortest line is considered as a thru (τ -standard). The long line is considered as an attenuator (α -standard). Then its *pseudo-scattering* matrix is given by

$$\mathbf{S}_{\alpha} = \begin{bmatrix} 0 & e^{-\gamma L} \\ e^{-\gamma L} & 0 \end{bmatrix} \quad (2.108)$$

where L is the difference between the physical length of the two lines. The line propagation constant is directly extracted by the algorithm. It is then possible to move the reference planes along the line, and hence to compensate the offset.

Because of the influence of the line length, this method can be applied over a narrow frequency range only. Indeed, when the losses are not too high, the accuracy of the method collapses at every frequency where L is a multiple of half of the wavelength on the line. On the other hand, the method works fine within the following limits [9] :

$$\frac{\pi}{9} \leq \Im(\gamma L) - n\pi \leq \frac{8\pi}{9} \quad \text{where } n \in \mathbb{N} \quad (2.109)$$

Another limitation of the TRL algorithm is that the reference impedance becomes the characteristic impedance of the calibration lines. And this impedance is not known.

The determination of this reference impedance is then critical as it is discussed later in this chapter, when the on-wafer measurement techniques are described.

Thru-Reflect-Match This method uses two perfectly matched loads as an attenuator to set the value of the reference impedance. These two loads will act as infinite transmission line, resolving the frequency limitation of the TRL [10]. The *pseudo* scattering matrix of the attenuator is given by

$$S_{\alpha} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \quad (2.110)$$

The TRM algorithm does not allow to determine a propagation constant. It is then impossible to shift the reference plane along a line.

2.4 On-wafer measurement

2.4.1 Introduction

The On-wafer measurement term refers to the measurement of integrated devices directly on the wafer, immediately after fabrication, before slicing and encapsulation. It is the most reliable method to characterize integrated devices, as there is minimal parasitics around it. In order to make on-wafer measure-

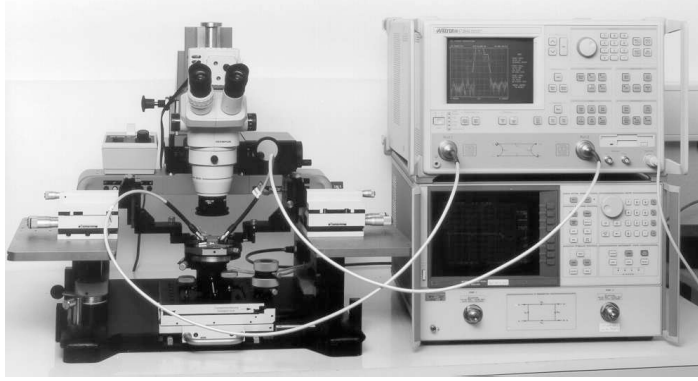


Figure 2.5: On-wafer probing station connected to a VNA

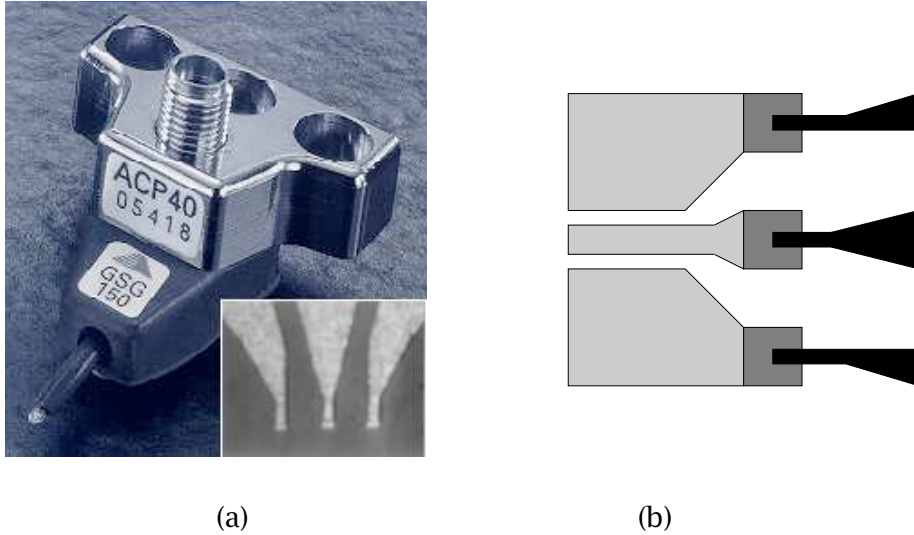


Figure 2.6: (a) On-wafer CPW probes (The inset shows a microscopic view of the probe tips), (b) Artist's view of a probe contacting a CPW structure

ment, the wafer is put on a specific probe station, where mechanical arms allow to connect tips to the device. The probes are connected to the VNA through coaxial cables, as shown in Figure 2.5. As a matter of fact they are transitions from coaxial cables to coplanar wave guide (CPW), as shown in Figure 2.6. In the following sections, techniques used for the measurement of silicon integrated devices are presented. Moreover useful guidelines to increase the accuracy of the RF measurements are discussed.

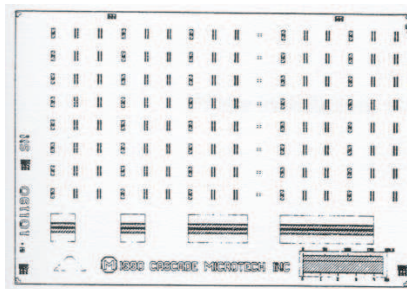


Figure 2.7: Calibration substrate used for the TRM

2.4.2 Measurement of silicon CMOS devices

2.4.2.1 Limitations due to CMOS technology

In order to perform accurate on-wafer measurement, an on-wafer calibration is required. It must be done by using standards fabricated on the wafer. The most attractive calibration procedure seems to be the TRM, because it is not frequency limited. This algorithm requires the measurement of *two perfectly matched load* (one for each port). In CMOS technologies, it is not possible to achieve the required match between loads. Indeed, variations close to 10% of the DC values of resistances made on a CMOS process has been observed many times during this work. For information, commercial calibration substrate, made on alumina, tolerate variations less than 0.1% on the load for an accurate calibration up to 40GHz. TRL method must then be used, as TRM cannot be implemented on silicon-wafer. However, as it is frequency dependent, we have to know the reference impedance. Practically, in order to minimize the frequency limitation of the TRL, two successive calibrations must be performed. This technique, usually called *Two steps calibration* is described hereafter.

2.4.2.2 Two steps calibration

TRM The first calibration consists of a TRM calibration using standards made on an alumina substrate (Figure 2.7). These standards are certified, and allow us making an accurate calibration up to 40GHz. After this calibration, the reference planes move at the end of the probe tips, and the reference impedance is equal to 50 Ω .

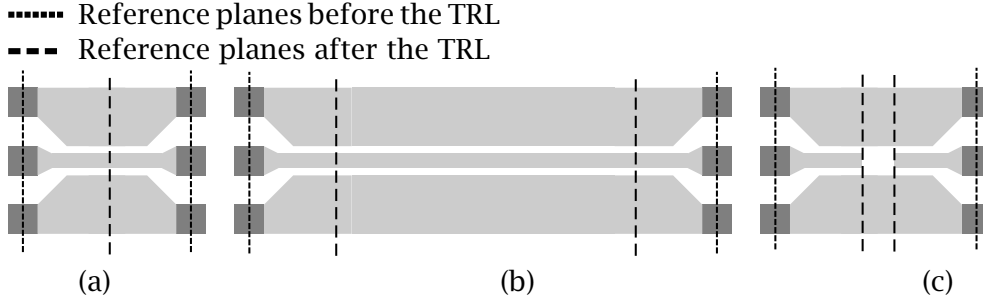


Figure 2.8: Layout of calibration standard for a TRL calibration on a silicon-wafer ((a) Thru line, (b) Line, (c) reflect)

TRL The next step consist of a TRL calibration in order to determine parasitics introduced by the pads made on the silicon-wafer, as shown in Figure (2.6b). The standards usually utilized for TRL calibration are two CPW lines, and a reflective device. The latter can be an open or a short circuit. The CPW is designed to have a characteristic impedance of nearly 50Ω (for a perfect dielectric substrate). Then the line length is chosen in order to have the most accurate measurement in the frequency band of interest according to Equation (2.109). If the validity of the calibration must be extended to nearly the whole frequency band, multiple lines have to be used. Figure 2.8 shows typical standards necessary for a TRL calibration. After the calibration, the reference planes have been shifted to the middle of the thru line. It is now necessary to determine the reference impedance of the TRL. In the next paragraphs, three different methods are described. The first one, called “Load Method”, is probably the most accurate. Five standards are required to use this method with the TRL. The second method, called “Open method”, requires only 3 standards. The last one, called “LL method” requires the measurements of 2 transmission lines realized on the substrate.

1. **Load Method** This method, proposed by GILLON in [11], is based on the relationship between the load impedance and its reflective coefficient as described in [3]

$$Z_{load} = Z_{ref} \frac{\Gamma_{load} + 1}{\Gamma_{load} - 1} \quad (2.111)$$

Based on this equation, it is possible to extract the reference impedance

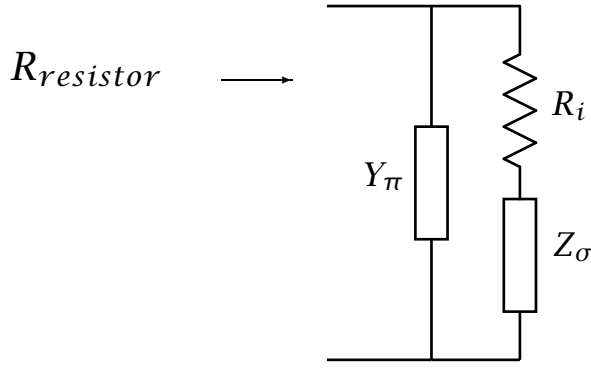


Figure 2.9: Equivalent circuit of the on-wafer resistor showing the intrinsic resistor R_i , the parallel and series parasitics, Y_π and Z_σ .

(Z_{ref}). The latter is equal to the ratio between the absolute load impedance (Z_{load}) and the normalized load impedance (z_{load}) obtained from the scattering parameters. In this method, an integrated resistor is used. However, in order to extract correct values for absolute and normalized impedances, an accurate equivalent circuit for the resistor is required. According to [12], the equivalent circuit of a resistor must include parasitic elements as shown in Figure 2.9. The resistor impedance can then be modeled by the following equation:

$$Z_{resistor} = \frac{R_i + Z_\sigma}{1 + Y_\pi (R_i + Z_\sigma)} \quad (2.112)$$

where R_i is the intrinsic resistance of the device. The DC and RF values of R_i are considered as equal. The reference impedance is obtained doing the ratio between the absolute value R_i and its relative value r_i . R_i and r_i are deduced from DC and RF measurement respectively.

It is then necessary to determine the effect of the parasitics surrounding R_i and r_i in order to obtain the most accurate results. To determine R_i and r_i three structures are measured, the integrated resistor, an open and a short circuit (Figure 2.10).

Open and short circuit impedances are deduced from Equation (2.112), imposing successively $R_i = \infty$ and $R_i = 0$.

$$Z_{open} = \frac{1}{Y_\pi} \quad Z_{short} = \frac{Z_\sigma}{1 + Y_\pi Z_\sigma} \quad (2.113)$$

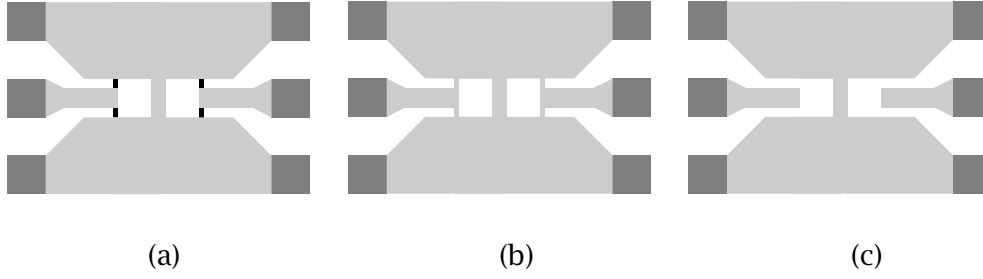


Figure 2.10: Standards required to extract the reference impedance using the load method. (a) resistor, (b): short circuit, (c): open circuit

Combining Equations (2.112) and (2.113) allows us to determine the intrinsic resistance value.

The absolute value of the intrinsic resistance is deduced from DC measurement of resistor, short and open circuit by

$$R_i = \frac{Z_{resistor_{DC}} Z_{open_{DC}}}{Z_{open_{DC}} - Z_{resistor_{DC}}} - \frac{Z_{short_{DC}} Z_{open_{DC}}}{Z_{open_{DC}} - Z_{short_{DC}}} \quad (2.114)$$

The value of the normalized intrinsic resistance is given by

$$r_i = \frac{Z_{resistor_{AC}} Z_{open_{AC}}}{Z_{open_{AC}} - Z_{resistor_{AC}}} - \frac{Z_{short_{AC}} Z_{open_{AC}}}{Z_{open_{AC}} - Z_{short_{AC}}} \quad (2.115)$$

where $Z_{resistor_{AC}}$, $Z_{open_{AC}}$ and $Z_{short_{AC}}$ are the relative values of the resistor, open and short circuit impedance. They are deduced from the S-parameters after the TRL calibration. The reference impedance is directly computed from the ratio R_i/r_i

$$Z_{ref} = \frac{\left(\frac{Z_{resistor_{DC}} Z_{open_{DC}}}{Z_{open_{DC}} - Z_{resistor_{DC}}} - \frac{Z_{short_{DC}} Z_{open_{DC}}}{Z_{open_{DC}} - Z_{short_{DC}}} \right)}{\left(\frac{Z_{resistor_{AC}} Z_{open_{AC}}}{Z_{open_{AC}} - Z_{resistor_{AC}}} - \frac{Z_{short_{AC}} Z_{open_{AC}}}{Z_{open_{AC}} - Z_{short_{AC}}} \right)} \quad (2.116)$$

As previously explained, this method requires 3 specific standards, a load, a short and an open circuit. In order to ensure accurate results, it is important that these structures are extremely similar to ensure that the parasitics Y_π and Z_σ are the same for the three measured structures.

Very good results have been obtained when using this method. However, when the difference between the measurements of the load and the short or

between the load and the open is small, the extracted reference impedance is not accurate. Indeed, if the difference $Z_{resistor} - Z_{short}$ is small, variations in the contact resistance during the measurement of the short and the resistor induce a significant variation of the total resistance. Then $Z_{resistor} - Z_{short}$ cannot be considered as equal to R_i . Furthermore, if $Z_{open} - Z_{resistor}$ is small, the values of r_i will become extremely sensitive to the measurement noise, as $Z_{open} - Z_{resistor}$ is the denominator of the first term of Equation (2.115).

2. **Open method** This method was proposed by BRACALE in [13]. as the previous method, it is based on the comparison between absolute and normalized value of a load. However, in this case, the load is the admittance corresponding to the transmission of an open circuit located in the middle of a thru line. The structure of this open circuit is presented in Figure 2.8c. The first step is the determination of the absolute value of the admittance (Y_{open}). In order to extract that value, measurements of a thru line and an open are used before doing the TRL. The equivalent circuits for these two elements are shown in Figure 2.11. In the open equivalent circuit (Figure 2.11b), Y_{open} is surrounded by different impedances corresponding to half thru lines. In order to determine the value of Y_{open} , the two impedances z_1 and z_2 must be known and de-embedded from the measurements. To determine their values, the measurement of the thru line is used. By considering the equivalent circuit of the thru line (Figure 2.11a), z_1 and z_2 are given by

$$z_1 = Z_{11Thru} - Z_{12Thru} \quad (2.117)$$

$$z_2 = Z_{22Thru} - Z_{12Thru} \quad (2.118)$$

Now that the values of z_1 and z_2 have been determined, they are de-embedded from the measurement using impedance matrices. The reference planes are shifted from the point (1) to the point (2) as shown in Figure 2.11b. Then Y_{open} is extracted using the admittance matrix. Now, the normalized value y_{open} has to be determined. It is extracted from the measurement of the open circuit after the TRL calibration using the following relation, presented

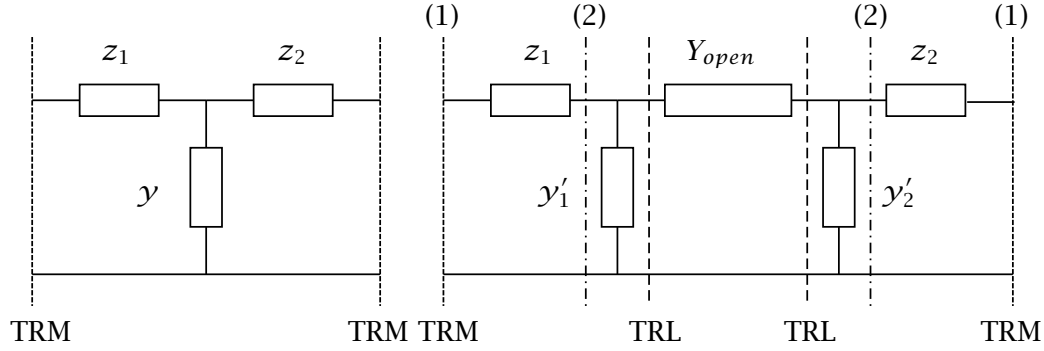


Figure 2.11: T-model of the through line (a), and the model of the open (b) used for the determination of the reference impedance using the open method

in Appendix A:

$$y_{open} = \frac{-s_{21}}{(1 + s_{11})(1 + s_{22}) + s_{12}s_{21}}$$

Then, the reference impedance is given by

$$Z_{ref} = \frac{y_{open}}{Y_{open}} \quad (2.119)$$

This method requires the use of a thru line and an open circuit. The open circuit must have a significant transmission between the two ports.

According to [14], the dimensions of the structures have to be optimized by using electromagnetic simulations to ensure that the corresponding equivalent circuit are corrects.

3. **LL method** This method, developed during this work, requires only two lines, a through and a line during the TRL calibration. The two lines are supposed symmetrical and reciprocal. This means that for both lines $s_{11} = s_{22}$ and $s_{12} = s_{21}$. Also, the thru line is considered as a transmission line. The method discussed in this paragraph is in two steps. First the thru line is characterized. Then the TRL is performed, and the reference impedance is deduced from the coefficient of the error matrix T_A .

Thru Line : The first step of this method consists on the determination of both the characteristic impedance (Z_{ct}) and the propagation con-

stant (γ_t) of the thru line. These parameters are the result of the *homogenization* of the thru line. In other words the line is considered as uniform. These parameters are not the characteristic impedance and the propagation constant of the uniform transmission line, which is the attenuator used for the TRL. To extract quickly Z_{c_t} and γ_t , the even and odd modes of the line are studied [15]. The even mode is defined when $\mathbf{a}_1 = \mathbf{a}_2$, and, because of the symmetry, $\mathbf{b}_1 = \mathbf{b}_2$. In this configuration, we can consider that a magnetic wall is located in the middle of the line. The reflection coefficient s_e seen when the even mode is measured is given by

$$s_e = \frac{\mathbf{b}_1}{\mathbf{a}_1} = s_{11} + s_{12} = \frac{Z_{c_t} \coth(\gamma_t l_t / 2) - Z_r}{Z_{c_t} \coth(\gamma_t l_t / 2) + Z_r} \quad (2.120)$$

where Z_r is the reference impedance of the TRM calibration ($Z_r = 50\Omega$). The odd mode is defined when $\mathbf{a}_1 = -\mathbf{a}_2$, and, because of the symmetry, $\mathbf{b}_1 = -\mathbf{b}_2$. In this case, short circuit is located in the middle of the line. The reflection coefficient s_o is given by

$$s_o = \frac{\mathbf{b}_1}{\mathbf{a}_1} = s_{11} - s_{12} = \frac{Z_{c_t} \tanh(\gamma_t l_t / 2) - Z_r}{Z_{c_t} \tanh(\gamma_t l_t / 2) + Z_r} \quad (2.121)$$

When combining the last two equations, it is possible to extract Z_{c_t} and γ_t :

$$Z_{c_t} = Z_r \sqrt{\frac{(1 + s_{11} + s_{12})(1 + s_{11} - s_{12})}{(1 - s_{11} - s_{12})(1 - s_{11} + s_{12})}} \quad (2.122)$$

$$\gamma_t = \frac{1}{l/2} \tanh^{-1} \left(\sqrt{\frac{(1 - s_{11} - s_{12})(1 + s_{11} - s_{12})}{(1 + s_{11} + s_{12})(1 - s_{11} + s_{12})}} \right) \quad (2.123)$$

TRL : The thru line being characterized, the TRL can be done. The parameters of the thru line will allow us to extract the wanted reference impedance using the normalized transfer matrix $\mathbf{T}_A$

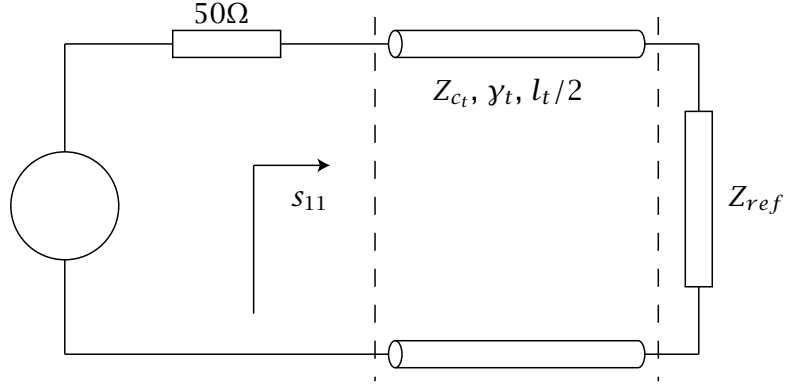


Figure 2.12: Schematic representation of the reflection coefficient s_{11} of T_A when the TRL is used

defined in section 2.3.2.

The transfer matrix T_A can be considered as the normalized transfer matrix of the half thru line with a reference impedance $Z_r = 50\Omega$ at the input and the reference impedance of the TRL (Z_{ref}) at the output.

The element of the normalized transfer matrix can be expressed as a function of the S-parameters:

$$T_A = \begin{bmatrix} -\Delta_s & s_{11} \\ -s_{22} & 1 \end{bmatrix} \quad (2.124)$$

The reflection coefficient s_{11} can be easily extracted from the normalized transfer matrix. When considering the equivalent circuit shown in Figure 2.12. The reference impedance Z_{ref} can be easily deduced from the line theory

$$Z_{ref} = Z_c = \frac{50(1 + s_{11}) - Z_{ct}(1 - s_{11})}{50(1 + s_{11}) + Z_{ct}(1 - s_{11})} e^{\gamma_t l_t} \quad (2.125)$$

This new method has been tested and compared to the *Open method* on several experimental technologies when the *Load method* cannot be used. A comparison is shown in Figure 2.13.

Similar values are extracted for both methods over the whole

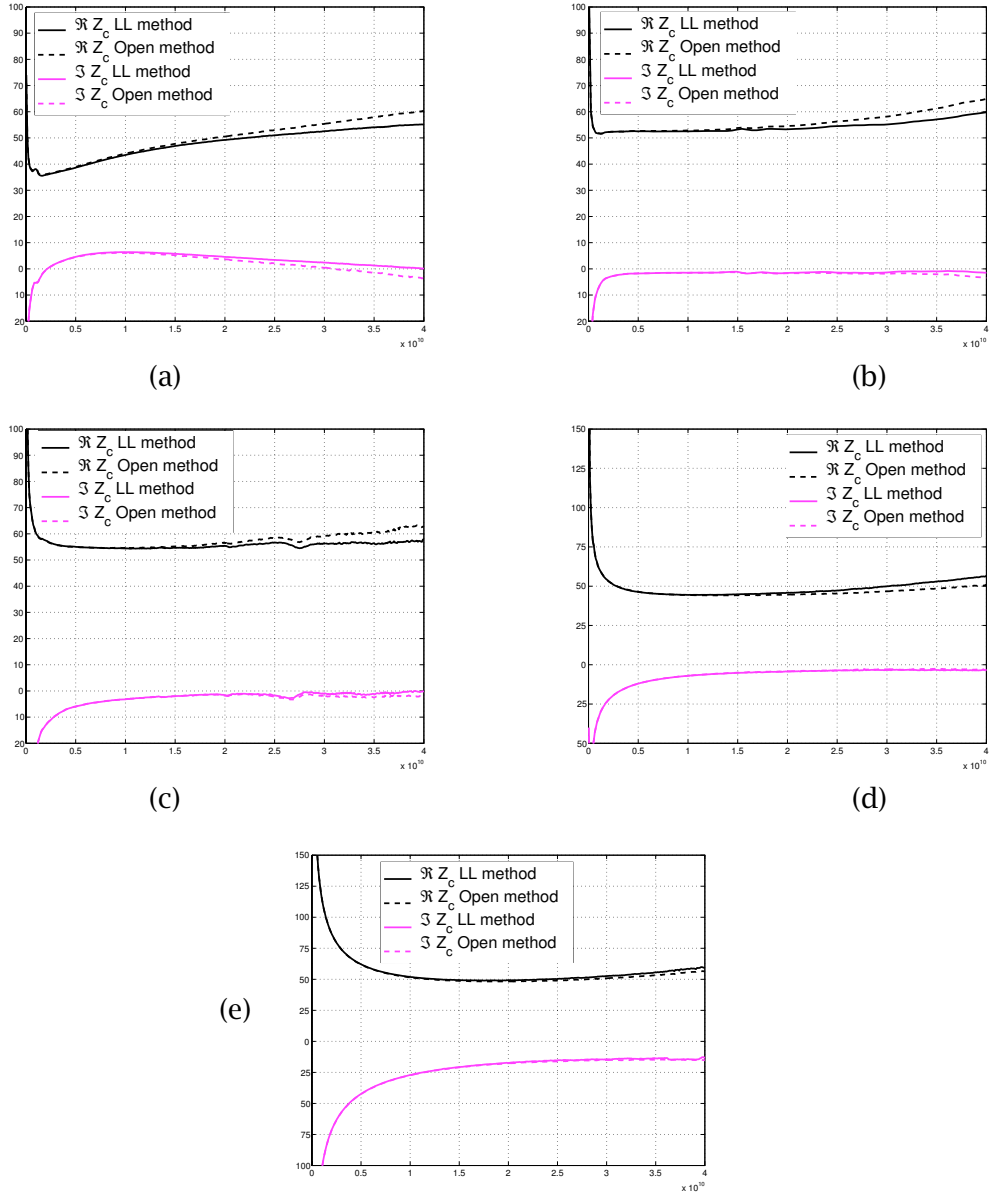


Figure 2.13: Comparison of the characteristic impedance of a transmission line using the *Open method* and the new *LL method* for various lines (a: CPW made on a medium resistivity substrate ($\rho \approx 20\Omega cm$), b: CPW made on a high resistivity substrate ($\rho > 1k\Omega$), c: CPW made on SOS, d: Thin Film Microstrip Line (TFMS) on top of a SOI silicon wafer, e: strip line on top of a SOI silicon wafer)

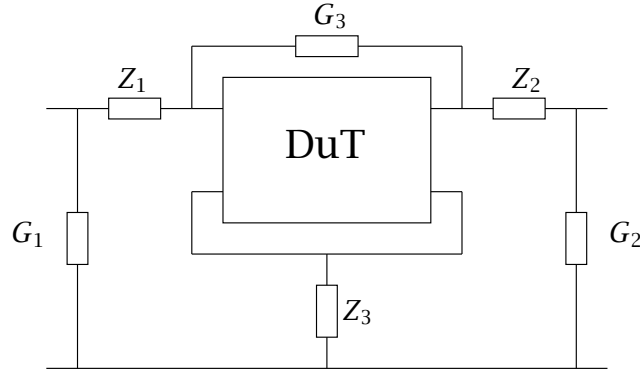


Figure 2.14: Equivalent circuit model for the immittance correction method.

frequency band. This method allows us to extract the TRL reference impedance with the same accuracy than with the open method, using two standards in place of three.

2.4.2.3 Alternative to the two step calibration

When the measured devices have small dimensions, like integrated submicron transistors, it is possible to use another method than the on-wafer TRL. This method, called *immittance corrections*, was first proposed by VAN WIJNEN for bipolar devices [16] and then enhanced by FRASER [17] and by CHO [18]. Its validity has been recently demonstrated by VANDAMME [19] for deep submicron MOSFETs up to 40GHz. The immittance method is based on an equivalent circuit describing the DuT and the probing structure. Figure 2.14 shows the equivalent circuit proposed in [19]. In order to extract the different parasitic elements, specific de-embedding structures are measured (open and short circuits,...). All these structures are designed in order to have same parasitics. The different open and short circuits utilized for the calibration are assumed ideal. Because of the latter assumption this algorithm cannot be applied to large devices at high operation frequencies.

2.4.2.4 Discussion

The calibration algorithm we have to apply depends on the measurement goal. When the aim of RF measurements is to characterize a technology, requiring the study of a large amount of active and passive devices, the two-step calibration is

preferred. Indeed, on a single die, we design only one set of standard as they can be used to measure multiple devices. Moreover, if the transmission line used in the TRL is a CPW, its study can provide information about the underlying substrate, such as its effective resistivity [20].

Unfortunately the accuracy of this method is quite poor when the length of the line is close to the half of the wavelength (section 2.3.2.3), and the determination of the characteristic impedance of the line can be difficult.

The first problem can be partially solved using more than one line, in order to have a better accuracy over a wider frequency range. The second problem is solved by using one of the three methods presented.

During this work, the “Load Method” has been intensively used, as assumptions made when deriving this method are less restrictive than those applied to the others.

When only few small devices, like submicron transistors, must be characterized, the immittance correction should be considered. This calibration method is simpler, and it requires less area on the die in comparison with the two-step calibration method.

This is especially true when the frequency of interest is around few GHz, because the TRL requires a line of few thousands of microns to be accurate. Also, the immittance correction technique has real advantages for industrial applications, when a huge number of devices have to be measured. Indeed, the distance among measurement pads can be kept constant for all the measured structures. This allows us to perform easily measurement using automatic probing station.

2.4.3 Optimization of the measurement procedure

2.4.3.1 Introduction

In the previous section, the principle of the on-wafer measurement technique has been presented. By performing accurate measurement an appropriate procedure is required. This is because it is important to minimize errors introduced by the user and the environment, in order to have a good accuracy. The next sections provide some “rules of thumb” concerning the on-wafer measurement

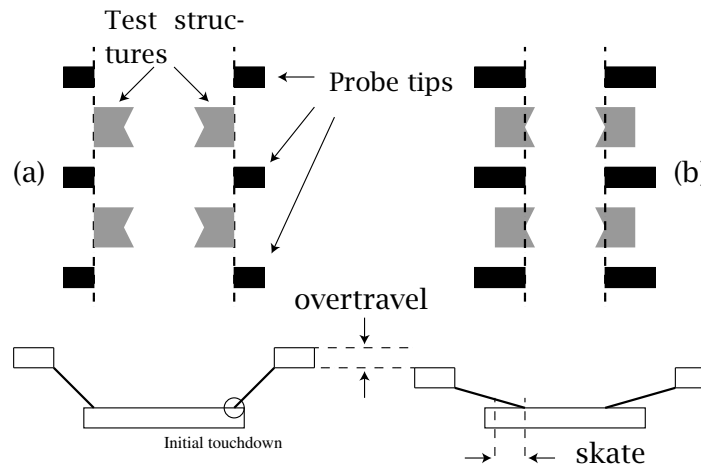


Figure 2.15: Test structure used to determine the optimal overtravel

of integrated devices.

2.4.3.2 TRM Calibration

The TRM calibration is achieved by using standards made on alumina substrate (Figure 2.7). Several matched loads, thru lines, open and short circuits, as well as CPW lines are realized on the substrate. All metal strips are made of gold, to ensure low losses for passive elements and high quality contacts between the probes and the standards. In order to make a correct calibration, repeatable measurements must be performed. Consequently, it is important to ensure a good contact between probes and standards. Specific test structures made of gold and realized on the calibration substrate (Figure 2.15), can be used to determine the vertical movement of the probes ensuring good contacts. The length of this vertical movement is called the overtravel. When probes are pushed on a surface, their extremities move in the horizontal direction. The distance covered by the probes is called skate. Its measure is used to determine the overtravel. More exactly, the user must lower the probes until they reach the initial touchdown at the beginning of the test structures (Figure 2.15a). Then the probes are lowered until they reach the end of the test structures, represented by the dashed lines in Figure 2.15b. The total vertical movement made by the probes, necessary to have good contact, is then determined.

Except from the errors introduced by the manipulation of the probes, mea-

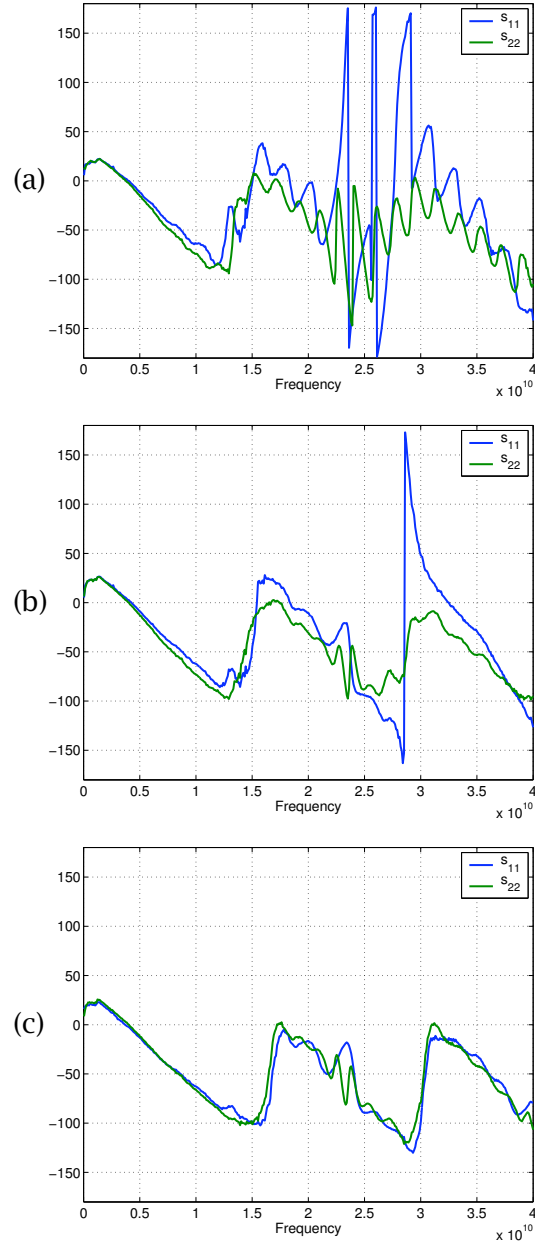


Figure 2.16: Phases of s_{11} and s_{22} measured after a TMR calibration using an averaging of 20 (a), 150 (b) and 300 (c)

measurements are also influenced by white noise, which follows a Gaussian law. In order to reduce the noise influence, the averaging process of the VNA can be adapted. In order to quantify the influence of that parameter, several calibrations were made using averaging from 20 to 500 measurements for all standards. After each calibration, a transmission line made on the calibration substrate was measured. The quality criteria utilized are the symmetry ($s_{11} \approx s_{22}$), and the reciprocity of the line ($s_{21} \approx s_{12}$). More specifically the phase difference between s_{11} and s_{22} , which is the most sensitive element, has been considered. Figure 2.16 shows the phases of s_{11} and s_{22} for the measured line, for an averaging of 20, 150 and 300 measurements during the calibration. The averaging used for the line measurement was made on 20 measurements. The increase of the averaging allows us to have nearly the same phase between the s_{ii} parameters.

We did not observed significant improvement when increasing the averaging above 300 for the measurement of the calibration standards.

2.4.3.3 On-wafer TRL Calibration

The TRL calibration using the standards realized on silicon-wafers has been studied following the same guide lines than that for the TRM calibration. The on-wafer measurements were made after a TRM calibration using an averaging of 300. First, the influence of the contacts has been studied. Indeed, good contacts between probes and pads are more difficult to achieve when the pads are made on aluminum. This is the case in silicon CMOS technologies. An optimal overtravel has to be determined for the on-wafer measurement. Moreover, it must be evaluated for all processes that are measured, as the quality of the aluminum pads is not certified. To illustrate the problem, Figure 2.17 shows the measurement of the s_{ii} parameters of a load and a short circuit before and after optimization of the overtravel. The measured structures were made on a commercial SOS process from PEREGRINE. The first two measurements (a-b) were obtained by using a classical overtravel. It was the overtravel used successfully to measure components made on a classical SOI technology.

Clearly, the measurements were wrong as the load was not symmetrical,

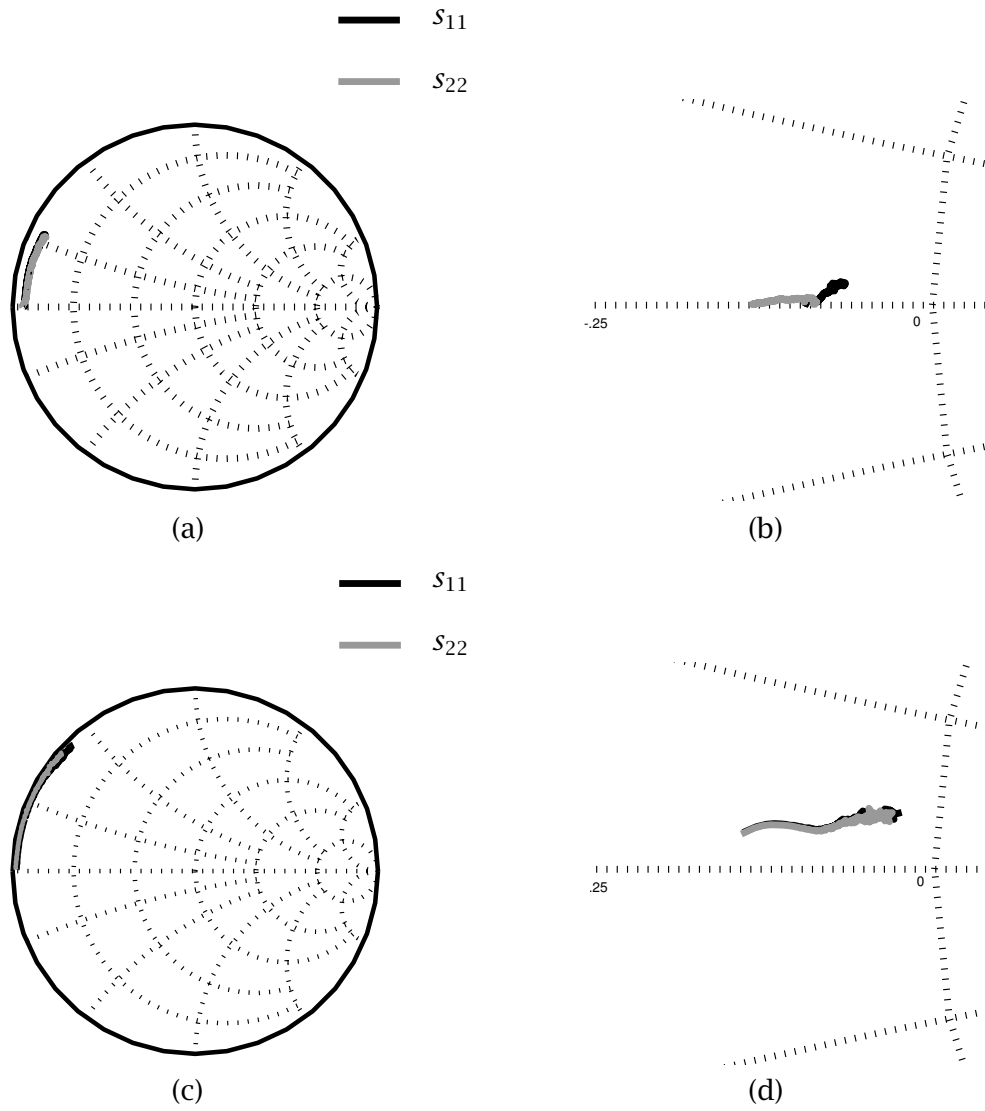


Figure 2.17: Measured s_{ii} parameters of a short circuit (a) and a load (b) before the optimization of the overtravel, and after the optimization (c-d)

and the short circuit shows an anomalous high impedance; the S-parameters were not close to the left border of the Smith chart. The overtravel should be modified.

After the optimization of the overtravel, more realistic measurements have been observed for the load and the short circuits.

The study of the influence of the averaging shows that no significant improvement in the measurement are observed when the averaging is raised above 25.

2.4.3.4 Influence of the power

When active devices or circuits are measured, it is necessary to know approximately the input and output power used. Indeed, the input power must not be too high to avoid non-linear behavior of the DuT. On the other hand, the output power must be kept under a threshold defined by the VNA. Above this threshold, gain compression occurs, and the measurements accuracy decreases. When MOSFET transistors are measured, the input power ensuring a linear behavior has to be checked. If the input power is too high, an attenuator has to be used between the VNA and the transistor. Usually, the output power is lower than the acceptable limit, because even if MOSFETs have huge power gain, most of the input power is reflected. The output power stays then under the 10 dBm limit of the VNA. However, when matched circuits are measured, the output power can be very high. It is then necessary to use an attenuator between the circuit output and the VNA.

2.4.3.5 Control of the quality of the calibration

In good experimental conditions such as constant temperature, ..., the calibration of the VNA is usually correct up to 48 hours. However, it is sometimes difficult to know exactly when it is necessary to repeat the calibration. A simple method can be used to check the evolution of the VNA parameters as a function of the time. The measurement of an open circuit obtained by raising the probes above the wafer can be observed periodically, and compared to its measurement stored when the calibration has been just done. In most cases, the differences

between the stored measurement and the new one first appear at the end of the frequency band. After a while, the difference appears at lower frequency. Practically, when differences appear below 30 GHz, it is necessary to restart TRM calibration of the VNA.

2.5 Conclusion

In this chapter we have introduced scattering and *pseudo* scattering parameters. They are essential instruments required to investigate the behavior of RF on-wafer devices. Some properties of these parameters have been extensively described as well as the relation between scattering and immittance parameters. The differences between scattering and *pseudo scattering* matrices have been explained.

The general TAN calibration technique, which is the basis of on-wafer measurement, has been described in details. Specific aspects of the measurement of on-wafer silicon based devices, such as the determination of the reference impedance for the TRL algorithm, has been intensively described. Moreover, a new simple method, using two transmission line, developed to solve this problem, has been presented (section 2.4.2.2). Also, some practical aspects of measurement procedures have been introduced, some encountered problems have been highlighted, and some useful guidelines presented, in order to increase the accuracy of the measurements.

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