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Abstract:

Empirical studies in macroeconomics and economic growth literature are highly dependent on the measure of the physical capital stock. The variables that contribute to explain the major problems in these areas of research always appear related, whether directly or indirectly, to capital stock and depreciation. The standard measurements of capital and depreciation are statistical measures based on assumptions about the average service life of capital goods, which are accumulated according to the perpetual inventory method. In this paper we propose an alternative method based on the equations that solve the dynamic optimization problem of the neoclassical firm. This method enables us to endogenously calculate the variables rate of depreciation and capital stock, yielding an economic estimation based on indicators of profitability such as the distributed profits and the Tobin's q ratio.

This represents a change of paradigm in measuring capital and depreciation, which we supply along with an application to the Spanish economy. The results show an economic depreciation rate (endogenous) that fluctuates around the statistical rate (exogenous), and two time profiles for the economic and statistical capital stocks that are markedly different. In the context of a growth accounting exercise we show how capital intensity and total factor productivity play different roles in explaining growth over the past fifty years, depending on whether we are using statistical or economic measures. Finally, we analyze the paradox of productivity and conclude that the absence of positive correlation between investment in information and communication technologies and the rate of growth of total factor productivity may be due to a combination of the delay effect associated with such investment and the under-estimation of the true economic depreciation.

Keywords: Capital, Depreciation, Growth, Slowdown, Total Factor Productivity.

JEL classification: C61, D92, E22, O47.

1 Introduction

Capital theory was subject to important convulsions during the years of the Cambridge controversies between Neokeynesian and Neoclassical schools. From the different dimensions of such clash of views, stand out the theory of economic growth, the theory of value and the theory of distribution between the factors of production. It is well-known that the different elements of confrontation are not independent to each other. However, our interest here is more on the consequences of the debate concerning the measure of capital, which involves the other debates about the role of concepts like production function, factor substitution, prices, shares and marginal products of factors at equilibrium, and so on. After some contributions from both sides, including the proposal of measuring capital in physical units as the labour hours, the neoclassical view ended up hegemonic in the literature on capital, placing at the very center of the subsequent modelization the concept of aggregate capital measured in terms of value.

According to Harcourt (1972) "*Aggregate production functions which invoke the concept of aggregate capital have been used not only in the pure theory of value, distribution, and growth, but also in the early post-war econometric studies of productivity growth over time, and of the possibilities of capital-labour substitution in economies and individual industries*". Associated to this dominant methodology, malleability, which allows abstracting from specificity and heterogeneity of capital goods, came to occupy a central position in the neoclassical scene. In addition, a kind of consensus was established among mainstream researchers who accepted the measure of aggregate capital at equilibrium under the assumptions of a perfectly competitive economy, perfect foresight, lack of uncertainty, and static expectations.

In this context, as detailed in Bitros (2010), Jorgenson (1963, 1974) placed the so-called proportionality theorem at the forefront of capital theory research. This theorem is now well known because of the hypothesis concerning depreciation in the process of getting a measure of capital. According to Jorgenson the rate of depreciation of physical capital is a constant proportion of the capital stock. Then, in case we proceed to measure capital directly at the aggregate level, this proportion does not change over time. Alternatively, if we differentiate among different types of capital assets, each one with an assumed constant useful life, the aggregate depreciation rate will change over time as long as the capital composition varies. In any case, this assumption of the theorem became a central feature of the algorithm designed to produce measures of capital stock through the perpetual inventory method.

Why is this methodological proposal so important? The neoclassical theory of capital is based on the hypothesis of an aggregate production function, which establishes a relationship between output, labor and capital. Of the three elements the most problematic to measure is capital. Among other reasons, this is because the theory claims for capital a measure in terms of value. With respect to production and labor there is a fairly broad consensus on how they should be quantified. A consequence of this general agreement is the existence of standardized records widely accepted by researchers and society as a whole. In contrast, talking about the physical capital input things are different. There are significant discrepancies both at the theoretical level and in practical application.

First of all, capital is a stock that provides services. This stock has its own law of motion with additions and deductions. On the one side capital formation is, more or less, a clear-cut concept which is well known and easily identifiable as gross investment. On the contrary, the meaning of capital deductions is not so clear, or at least entails a disparity of opinions about what is and what is not. People often distinguish between consumption of fixed capital, decline in productive capacity, efficiency losses, obsolescence, replacement, retirements, scrapping, and so on. Instead, we will refer to all of them as depreciation but we shall still consider a fundamental differentiation depending on its immediate cause. In this way, we could find deterioration and obsolescence. Deterioration, which may be physical (output decay) and economic (input decay), is an inherent characteristic of capital goods associated with the aging, use, and maintenance of equipment. Obsolescence, which may be technological and structural, comes from outside the productive assets suffering it, and appears associated with technical progress (mainly embodied but also disembodied), energy prices, patterns of international trade, regulatory programs linked to environmental policy, or changes in the output composition that affect the relative prices.

Secondly, there is the main issue of valuation. Despite how difficult it may be from an empirical point of view, the estimation of price and quantity indices for the capital goods that stand for gross investment is a feasible task since transactions are market observable transactions. Consequently, the measure of capital in terms of value, as the neoclassical theory claims, depends fundamentally on the possibility of obtaining a measure of depreciation in terms of value. That is, the value loss of capital due to any of the aforementioned causes of depreciation. This is a more difficult task because the economic transactions that have to do with capital depreciation activities are basically unobservable transactions. So what we really need is the shadow price that makes up the net present value condition, a condition stating that the market value of an asset is equal to the discounted stream of future benefits that are expected to yield. We agree with Bitros when he asks for a model with strong microeconomic foundations that treats depreciation as an endogenous variable, because the capital stock cannot be correctly measured in the absence of a theory that explains how investment and depreciation are determined.

Faced with this requirement, the Jorgenson's hypothesis of proportionality offers a weak theoretical answer based on the inaccurate assumption of an exogenously given constant depreciation rate. But the use of a constant to characterize such a complex process allows a huge simplification. In Jorgenson's framework only depreciation caused by output decay receives due attention, leading researchers to make flawed economic predictions in the short-run when economies experience important demand fluctuations as well as in periods of accelerated technological and structural change. However, from the point of view of empirical applications, and according to the OECD (2009) report, national and international organizations as well as private and government agencies continue to provide capital stock series on the basis of constant depreciation rates which are inversely proportional to the useful economic life of assets.[1]

The measure of capital stock obtained following the precepts of the OECD is a statistical measure of capital, not an economic measure in terms of value. This is because depreciation is estimated by adopting arbitrary assumptions about the functional form of three interconnected statistical functions: retirement, age-price, and age-efficiency functions, the parameters of which are taken from empirical studies. After some ups and downs, and the corresponding methodological revisions, the criterion that prevails today is the double-declining balance rate. This is a simple method to obtain a geometric pattern for depreciation, absolutely compatible with the above mentioned theorem of proportionality. Under the statistical measurement of depreciation, the sole loss in the value of assets taken into account is the one experienced as they age. Consequently, because it is ignored the crucial role of utilization, maintenance, and embodied technical progress, depreciation is considered rather a technical necessity than the result of economic decisions.

The purpose of this paper is to address the challenge posed by the neoclassical theory of

[1] This is the type of capital measures provided by the main databases in Europe (EU-KLEMS and AMECO), in the U. S. (BEA and BLS), as well as in Spain (IVIE-BBVA and BD.MORES).

capital and to achieve the goal of obtaining a true economic measure of capital stock at the aggregate level. To do this we intend to act in three areas simultaneously. First, we develop a dynamic optimization model that endogenizes the depreciation rate by adding it to the set of controls, together with the variables gross investment and employment. This optimal control problem may be read as a first step towards a more general theory explaining simultaneously the behavior of investment and depreciation. The key element here is the maintenance and repair expenditures, which combined with the adjustment costs will contribute to show that decisions about investment and depreciation, the major decisions involved in the dynamics of capital stock, are interrelated and are not independent of each other. Despite the similitude with those models populating the theoretical literature on endogenous depreciation, our model takes the advantage of going beyond the routinary study of the comparative dynamics focused on control variables. Here we explore the possibility of obtaining empirical measurements of the endogenous variables with the theoretical support of the model and the equations that characterize it.

Second, from the solution of the model we get two equations organized in an algorithmic form which are ready to be implemented. We apply this procedure to the Spanish data, getting the series of an economic measure of depreciation and the economic value of capital stock. This method yields an estimation based on profitability indicators such as firms' distributed profits and Tobin's q ratio. Although these ideas were initially raised in Escribá-Pérez and Ruiz-Tamarit (1995a,b), now we take an important step forward and develop the old ideas in a renewed and integrated theoretical-empirical framework. According to Triplett (1996) the market-based measure of economic depreciation and the corresponding economic value of capital stock are suitable for both the economic accounts of income and wealth and the production analysis and productivity measurement. Consequently, we get in this paper a measure of the capital stock that may represent simultaneously the net stock of capital in terms of wealth, and the productive capital stock from which it is immediate to deduce a proportional flow of capital services. Our economic measure of the capital stock can be used as indicator of the productive capacity and as a measure of capital input in studies of multifactor productivity. It can also be related to the added value to calculate capital-output ratios.

With these results in hand, we revisit the measures of depreciation and capital obtained under the rules of the proportionality theorem. Taking as reference the Spanish economy, we compare our series with those supplied by agencies that follow the recommendations of the OECD. In other words, we compare the economic measures of depreciation and capital stock with the more standard measures, which are statistical in nature, based on the assumption of constant useful lives, and accumulated according to the perpetual inventory method. The pictures of the Spanish economy during the period 1964-2011 differ substantially in their short-run evolutions, but it is possible to establish a close correspondence between the long-run profiles. In particular, we observe that the economic depreciation rate fluctuates around the statistical depreciation rate, which may be read as if the long-run statistical measurement of depreciation were a good approximation of the long-run economic depreciation.

Finally, in this paper we also include a review of the main macroeconomic indicators that describe the growth of the Spanish economy over the past fifty years. We reach to differentiate a certain number of subperiods, which will be characterized according to the particular evolution of the variables labor productivity, capital intensity, capital productivity and total factor productivity (TFP). Actually, we propose a double exercise of growth accounting on the basis that we manage two different series of the capital stock. The problem we face is the following: given that the rate of growth of capital stock is a central variable in the algebraic calculations of the growth of total factor productivity, and the depreciation rate is fundamental for the capital stock measurement, if the depreciation rate is not properly measured all the remaining computations will be wrong. That is, an under-estimation (resp. over-estimation) of depreciation produces an over-estimation (resp. under-estimation) of the growth of capital, which in turn leads to under-estimate (resp. over-estimate) total factor productivity growth.

As we will see later, given the two sets of measures it is possible to explain, in one way or another, the different productivity slowdowns experienced by the economy. In each case, and depending on whether the data source is statistical or economic, we must decide whether the main explanatory variable is the growth of TFP, the growth of capital intensity or just the rhythm of job creation. Another issue we address at the end of the paper is the role of information and communication technologies (ICT) in explaining the recent evolution of labor productivity and TFP. It is commonly accepted that investment in ICT and the rate of growth of TFP are highly correlated. However, when we use the statistical measures of depreciation and the capital stock it is difficult to recognize such a positive link, given the observed negative rates of growth of TFP during the boom of investment in ICT. Faced with this paradox of productivity, we try to find a response by inspecting the data under the double perspective of using the economic measures of depreciation and the capital stock, and accepting the possibility that the economy experiences the effects of investment in ICT with a delay. The two arguments are complementary to each other since the computer revolution implies a profound process of capital substitution, which takes time and causes a significant increase in economic depreciation.

The paper is organized as follows. Section 2 provides a theoretical model of optimizing behaviour for the representative competitive firm. In this context we derive an algorithmic procedure that allow us to calculate, in economic terms, the depreciation rate and the capital stock. Section 3 describes the standard procedure employed to obtain the traditional statistical measures of capital stock and the rate of depreciation. The remainder of the paper comes with an application to the Spanish data. Section 4 shows the *traditional* statistical and the *new* economic measures of capital and depreciation rate. Section 5 analyses the growth of total factor productivity undertaking an exercise of growth accounting which uses for comparison the measures obtained in the previous section. Finally, section 6 summarizes and concludes.

2 The economic measurement of capital

Let us consider the supply side of an economy consisting of a relatively large fixed number of identical firms. In this section we supply the theoretical model that shows the optimizing behavior of the individual price-taking firm in a competitive environment. However, given the representative agent assumption, the same model variables can be considered to represent the aggregate level and, moreover, that decisions are taken by the economy as a whole. The optimization problem that solves our agent is an intertemporal maximization problem that generalizes the standard problem in which the employment $L(t)$ and the investment $I^{G}(t)$ are controlled in order to maximize the present discounted value of cash-flow. The generalization consists in adding to the set of controls the depreciation rate of capital $\delta^{*}(t)$.[2] Output is

[2] We consider the Hayashi's (1982) model as the standard problem, which perfectly summarizes all previous literature corresponding to the neoclassical model with investment-related adjustment costs. The incorporation of the depreciation rate to the endogenous variables of the model is possible thanks to its link with maintenance and repair expenditures. See Escribá-Pérez and Ruiz-Tamarit (1996), Boucekkine and Ruiz-Tamarit (2003), and Kalyvitis (2006).

produced according to a neoclassical production function $Y(t) = A^*(t) F(L(t), K^*(t))$, where $A^*(t)$ is the exogenous technological level and $F(.)$ is an homogeneous of degree one function. This optimal control problem has a single state variable so it includes a dynamic constraint that expresses the accumulation process of capital stock $K^*(t)$. In general we can write

$$\max_{\{K^*, L, I^G, \delta^*\}} V(t_0) =$$

$$\int_{t_0}^{+\infty} \left(G\left(p(t), A^*(t), K^*(t), L(t), I^G(t), \delta^*(t)\right) - W(t) L(t) - p^k(t) I^G(t) \right) e^{-R(t)(t-t_0)} dt$$

$$s.t. \qquad \overset{\bullet}{K}^*(t) = I^G(t) - \delta^*(t) K^*(t), \tag{1}$$

$$K^*(t_0) = K_0^* > 0.$$

For the sake of simplicity we normalize to unity the price of output, $p(t) = 1$. The price of labor $W(t)$, the market price of capital goods $p^k(t)$ and the nominal interest rate $R(t)$ are given for the individual firm. Function $G(.)$ represents the value of net production after discounting investment-related adjustment costs $C\left(I^G, K^*\right)$ and the maintenance expenditures that contribute to determine the rate of depreciation $M(\delta^* K^*, K^*)$.These two functions are assumed homogenous of degree one in its corresponding determinants.[3] Under the above assumptions we can write $G\left(A^*, K^*, L, I^G, \delta^*\right) = A^* F(L, K^*) - \phi\left(\frac{I^G}{K^*}\right) K^* - m(\delta^*) K^*$, and we can also characterize the function by means of the sign of the first and second derivatives with respect to the controls and the state variable. That is, $G_L = A^* F_L > 0$, $G_{LL} = A^* F_{LL} < 0$, $G_{I^G} = -C_{I^G} = -\phi' < 0$, $G_{I^G I^G} = -C_{I^G I^G} = -\frac{\phi''}{K^*} < 0$, $G_{\delta^*} = -m' K^* > 0$, $G_{\delta^* \delta^*} = -m'' K^* < 0$. Moreover, given that $\phi(.)$ is strictly convex we get $C_{K^*} = \phi - \phi' \frac{I^G}{K^*} < 0$ and $C_{K^* K^*} = \phi'' \frac{\left(I^G\right)^2}{(K^*)^3} > 0$, consequently $G_{K^* K^*} = A^* F_{K^* K^*} - C_{K^* K^*} < 0$. Finally, we assume that the net marginal productivity of capital is positive, $G_{K^*} = A^* F_{K^*} - C_{K^*} - m > 0$.

When we add the dynamic constraint to the objective functional by introducing one more variable (multiplier) μ as expression of the shadow price of capital, we get the following Hamiltonian function written in current value

$$H^c = G\left(A^*(t), K^*(t), L(t), I^G(t), \delta^*(t)\right) - W(t) L(t) - p^k(t) I^G(t) + \mu(t) \left(I^G(t) - \delta^*(t) K^*(t)\right). \tag{2}$$

In addition, partly for technical reasons and partly for economic reasons we assume as constant and exogenously given the interest rate with which the cash-flow is discounted.[4] We can then apply in a simple way the Principle of Maximum, from which we get the necessary conditions for the control variables

$$H_L^c(.) = 0 = A^*(t) F_L(L(t), K^*(t)) - W(t), \tag{3}$$

[3] Actually, the maintenance cost function should be written as $M(D^*, K^*)$, where $D^* = \delta^* K^*$ represents the depreciation in level. Therefore, our homogeneity assumption involves the variables D^* and K^* instead of δ^* and K^*.

[4] It is well-known after Obstfeld (1990) and Marin-Solano and Navas (2009) that under a non-constant discount rate, either exogenous or endogenous, the Pontryagin's maximum principle cannot be applied directly because the corresponding intertemporally dependent preferences can create a time-consistency problem. Examples of how to proceed in case of a variable discount rate may be found in Palivos et al. (1997), Ayong and Schubert (2007), and Boucekkine et al. (2017).

$$H^c_{I^G}(.) = 0 = -\phi'\left(\frac{I^G(t)}{K^*(t)}\right) - p^k(t) + \mu(t), \tag{4}$$

$$H^c_{\delta^*}(.) = 0 = -m'(\delta^*(t))\,K^*(t) - \mu(t)\,K^*(t). \tag{5}$$

As well as the following Euler equation

$$\dot{\mu}(t) = R\mu(t) - H^c_{K^*}\left(A^*(t), K^*(t), L(t), I^G(t), \delta^*(t)\right), \tag{6}$$

where $H^c_{K^*}(.) = A^* F_{K^*} - \left(\phi - \phi' \frac{I^G}{K^*}\right) - m - \mu\delta^*$, the dynamic constraint

$$\dot{K}^*(t) = I^G(t) - \delta^*(t)\,K^*(t), \tag{7}$$

and the transversality condition

$$\lim_{t\to+\infty} \mu(t)\,K^*(t)\,e^{-R(t-t_0)} = 0. \tag{8}$$

The first order conditions (3)-(5) implicitly define a system of three control functions. After total differentiation, and according to the implicit function theorem, we get the results which allow us, with the help of the sign of the partial effects, to identify some of the main results from the neoclassical theory of factor demand[5]

$$L = L\left(\overset{+}{K^*}(t); \overset{+}{A^*}(t), \overset{-}{W}(t)\right), \tag{9}$$

$$I^G = I^G\left(\overset{+}{K^*}(t), \overset{+}{\mu}(t); \overset{-}{p^k}(t)\right), \tag{10}$$

$$\delta^* = \delta^*\left(\overset{-}{\mu}(t)\right). \tag{11}$$

Beyond these generic results and according to the main goal of our paper, we find that the differential equation (6) may be integrated forward solving for $\mu(t)$, under the non-explosivity condition $\lim_{t_F\to+\infty} \mu(t_F)\exp\left\{-\int_t^{t_F}(R+\delta^*(\tau))\,d\tau\right\} = 0$. The result we get may be put in terms of the marginal Tobin's q,

$$q^M(t) = \frac{\mu(t)}{p^k(t)} = \tag{12}$$

$$\frac{\int_t^{+\infty}\left(A^*(s)\,F_{K^*}(L(s), K^*(s)) - \left(\phi\left(\frac{I^G(s)}{K^*(s)}\right) - \frac{I^G(s)}{K^*(s)}\phi'\left(\frac{I^G(s)}{K^*(s)}\right)\right) - m(\delta^*(s))\right)e^{-\int_t^s(R+\delta^*(v))dv}ds}{p^k(t)}.$$

That is, the present value of the future stream of the net marginal productivity of capital, discounted by the sum of the constant interest rate plus the variable depreciation rate, and all that divided by the current market price of one unit of capital. In other words, the quotient

[5] These are not yet the usually pursued explicit labor demand function and the corresponding investment and depreciation functions. For that, we first would need to substitute (9)-(11) into (6) and (7). Then, we should have to solve the resulting dynamic system for K^* and μ depending on the exogenous variables and parameters. Finally, with the solution trajectories just found we could come back to the control functions and get the expected true factor demand functions depending only on the exogenous variables and parameters. However, this far-reaching goal is beyond the scope of the present paper.

between the shadow price of one unit of capital (the Hamiltonian multiplier μ) and its replacement cost (the market price of capital goods p^k). This is the variable which directly explains the flows of investment and depreciation that determine the dynamics of the capital stock.

Moreover, the property of homogeneity assumed on the production and cost functions as well as the first order conditions arising from our dynamic optimization problem, allow us to set the following linear ordinary differential equation in $X = \mu K^*$: $\dot{X} = RX - A^* F(L, K^*) + \phi\left(\frac{I^G}{K^*}\right) K^* + m(\delta^*) K^* + WL + p^k I^G$. It may be integrated forward solving for the product $\mu(t) K^*(t)$, under the transversality condition (8). The result we get may be put in terms of the observable average Tobin's q, the quotient between the market value of (all) the firm(s) and the economic value of the capital stock measured in nominal terms at its replacement cost,[6]

$$q^A(t) = \frac{\mu(t) K^*(t)}{p^k(t) K^*(t)} = \tag{13}$$

$$= \frac{\int_t^{+\infty} \left(G\left(A^*(s), K^*(s), L(s), I^G(s), \delta^*(s)\right) - W(s) L(s) - p^k(s) I^G(s)\right) e^{-R(s-t)} ds}{p^k(t) K^*(t)}.$$

Given (12) and (13), it is apparent the equality between marginal and average q. From now on, if we simplify notation calling the market value of the firm along the optimal equilibrium path as V_t^*, and rewrite it in discrete terms to make the relevant expressions computationally operative, we have on the one hand

$$q_t = \frac{V_t^*}{p_t^k K_t^*}. \tag{14}$$

On the other hand, the stock of capital is determined at each moment according to the first-order difference equation

$$K_t^* = I_t^G + (1 - \delta_t^*) K_{t-1}^*. \tag{15}$$

Given the capital stock of the previous period, by adding the flow of gross investment I_t^G and subtracting the depreciation flow $\delta_t^* K_{t-1}^*$, we obtain the stock of capital of the current period. Furthermore, under the assumption that financial markets are competitive, we can specify the market value of the firm V_t^* as the discounted present value of the infinite flow of distributed profits, B_t^*. The fact that both variables are in nominal terms imply that discount is made with the nominal interest rate, R_t,

$$V_t^* = \sum_{s=t}^{\infty} \frac{B_s^*}{(1 + R_s)^{s-t}}. \tag{16}$$

In this intertemporal context, it is important to know the way in which agents form their expectations about the future values of the variables. We will assume that economic agents have static expectations, so that in each period they consider that the current value will be repeatedly observed in the future. If we apply such an assumption to the terms of equation (16) we will find that, $\forall s \in [t, \infty[$, $B_s^* = B_t^* \left(1 + \pi_s^k\right)^{s-t}$ and $R_s = R_t$, being $\pi_s^k = \pi_t^k$ the inflation rate that follows from the price index corresponding to capital goods, p^k. We define the real

[6] Although we are expressing economic or market values, we can use, as in the literature, different names to refer to them. The valuation at replacement cost, in nominal terms or at current prices, is equivalent, and means that the capital goods are valued at the prices of the current period. This is opposed to valuation at historical prices, which means that the assets are valued at the prices at which they were originally purchased.

interest rate $r_t = R_t - \pi_t^k > 0$ and approximate the term $\frac{1+\pi_t^k}{1+R_t} = 1 + \pi_t^k - R_t$, taking the product $r_t R_t$ as negligible. Then, we can write

$$V_t^* = B_t^* \sum_{s=t}^{\infty} \left(\frac{1+\pi_t^k}{1+R_t} \right)^{s-t} = B_t^* \sum_{s=t}^{\infty} (1-r_t)^{s-t} = \frac{B_t^*}{r_t}. \tag{17}$$

Substituting this result in (14) we get

$$q_t = \frac{B_t^*}{r_t p_t^k K_t^*}. \tag{18}$$

Equations (18) and (15) give us an idea of how the process of capital accumulation is defined and also of its relation to the assessment that markets do of such process. In these equations we are considering the economic or market values of each of the variables; either the quantity-variables: distributed profits, flows of gross investment and depreciation, and the capital stock; or the price-variables: q ratio, interest rate and price of investment goods.

On the other hand, if we focus on the revenues of productive factors generated and distributed through the market mechanisms, it is obvious that the sum of the economic value of net distributed profits, B_t^*, and the flow of economic depreciation in nominal terms is equal to the distributed gross profits, $B_t^G = B_t^* + \delta_t^* p_t^k K_{t-1}^*$. Substituting in (18) we get

$$q_t r_t p_t^k K_t^* = B_t^G - \delta_t^* p_t^k K_{t-1}^*. \tag{19}$$

Therefore, if we know the value of all price variables as well as the value of the economic-accounting flows of gross investment and gross distributed profits, we can use the equations (15) and (19) to obtain the values of the depreciation rate and the capital stock. This dynamic system of two first-order difference equations allows us to express in each period the values of the endogenous variables K_t^* and δ_t^* as a function of the exogenous variables q_t, r_t, p_t^k, B_t^G, and I_t^G, given the predetermined value of K_{t-1}^*. We can write in closed-form the explicit solutions:

$$\delta_t^* = \frac{\frac{B_t^G}{q_t r_t p_t^k} - K_{t-1}^* - I_t^G}{\left(\frac{1}{q_t r_t} - 1 \right) K_{t-1}^*}, \tag{20}$$

$$K_t^* = \frac{K_{t-1}^* + I_t^G - \left(B_t^G / p_t^k \right)}{1 - q_t r_t}. \tag{21}$$

Thus, from a known K_0^* value we can use the above two equations sequentially to obtain the corresponding series of the capital stock K_t^* and the depreciation rate δ_t^*.

3 The statistical measurement of capital

In the previous section we have just offered an algorithmic procedure for calculating the depreciation rate and the capital stock based on the optimal decisions of economic agents. Such procedure is built on the basis that the depreciation of capital is endogenous and, consequently, we can expect that it represents a variable percentage of capital. However, from Jorgenson's (1963) pioneering work, a very different interpretative model known as the proportionality theorem has been hegemonic. This one states that the depreciation and replacement of capital goods is done at a constant rate, proportional to the corresponding capital stock. The hypothesis of proportionality as dominant paradigm concealed in the background all the previous discussion that for years had focused on the study of the endogenous determination of the optimal service life for different vintages and types of capital assets, which suddenly eclipsed. The corresponding constant rate of depreciation seemed to connect better with an exogenously predetermined (average) service life that converts depreciation in a mere technical necessity. As a result, a good approximation to the capital stock was considered to be the following weighted sum of current and lagged investment values,

$$K_t = \int_{-\infty}^{t} I_v^G e^{-\delta(t-v)} dv = \sum_{-\infty}^{v=t} I_v^G \left(\frac{1}{1+\delta} \right)^{t-v}. \tag{22}$$

This procedure, which adds at any moment the remaining depreciated quantity of every past investment, is known as the perpetual inventory method (PIM)[7] and gives us, when differentiating (22) with respect to the time index t, the most well-known equation of accumulation

$$K_t - K_{t-1} = I_t^G - \delta K_{t-1}. \tag{23}$$

This way of interpreting the phenomenon of depreciation, which spread quickly in the field of quantitative measurement, in theoretical and applied macroeconomics, as well as into the economic growth literature, was already challenged from the outset because it was based on too restrictive assumptions. The main problem of Jorgenson's theorem is that it focuses on age and ignores the role of economic variables such as the intensity of use, maintenance and repair, obsolescence caused by embodied technical progress, uncertainty, or the business cycle in determining the depreciation of capital assets. The problem was widely pointed out by Feldstein and Foot (1971), Eisner (1972), Feldstein and Rothschild (1974), Bitros and Kelejian (1974), Malcomson (1975), Nickell (1975), and Cowing and Smith (1977). These studies showed that depreciation varied considerably under the influence of conventional economic forces. Then, to be consequent with this fact, the dynamic process generating the capital stock should be represented as follows

$$K_t = K_{t-1} + I_t^G - D_t. \tag{24}$$

Total depreciation D_t may change considerably due not only to the variability generated by the own dynamics of the capital stock but also to the influence of the economic factors mentioned above. At this point we probably should abandon the strict assumption of a unique and constant rate of depreciation that applies to the aggregate capital stock, and implicitly define a variable rate of depreciation as

$$\delta_t = \frac{D_t}{K_{t-1}}. \tag{25}$$

This implicit depreciation rate reflects the variability of both the denominator and the numerator. But it may also occur that it shows some variability simply by the fact that the composition of the capital stock is changing. The latter is what happens when we replace the

[7] In the more basic view of the inventory method, where capital goods experience withdrawals without decay or obsolescence, the measurement of capital stock involves accumulating all past capital formation, and deducting the value of assets that have reached the end of their service lives.

assumption of a single constant rate with a multiplicity of constant rates, each one associated with a different type of capital asset.[8]

After a long period of theoretical discussion, the concrete application of the previous issues to the actual calculation of depreciation and capital stock has been carried out within the framework of methodological proposals from international organizations such as the OECD. The purpose of these proposals is twofold: to harmonize the uses and criteria of the different national statistical agencies and achieve the highest degree of homogeneity among the indicators calculated for the different countries.[9] While it is true that there are no fundamental problems with the quantification of the gross investment flow, since it measures the acquisition of new capital goods according to the explicit transactions made in the market, we cannot say the same about the depreciation flow for which reliable recorded data is not available. In the case of capital depreciation, the most common practice is to assign accounting values based on assumptions about the mathematical functional form of survival (retirement) profile, efficiency profile according to age, and age-price profile of an asset or cohort of assets.[10] Hence, the measure of the capital stock is a statistical measure because depreciation is estimated under arbitrary assumptions about the parameters that characterize those functions. Under the statistical measurement of depreciation, the sole loss in the value of assets taken into account is the one experienced as they age. Consequently, because it is ignored the role of utilization, maintenance and embodied technical progress, depreciation is treated rather as a technical necessity than as the result of economic decisions.

In this context, the accuracy in the implementation of the perpetual inventory method depends on the particular choice of the asset retirement distribution. A survival profile is required to model the retirement process, and a key parameter in this process is the average service life. Although questioned, it is usual to assume fixed service lives and one ad hoc pattern for retirements (one-hoss-shay, linear, or a bell-shaped function like Winfrey, Weibull and lognormal distributions). Moreover, the age-efficiency function is also assumed fixed, with several possible shapes (hyperbolic, linear, or geometric profiles). And finally, in coherence with the previous ones, the age-price function is also taken as fixed. It can be of the straight-line type, with prices falling by a constant amount each period, or of the geometric type, with prices falling each period by a constant rate.

In any case, what is pursued here is a measure of the depreciation flow, and different combinations of retirement patterns with age-efficiency patterns or with age-price patterns are admitted to achieve this goal. The functional form of these interconnected statistical functions is important to determine the functional form of the depreciation pattern, whose parameters are taken mainly from empirical studies (company accounts, statistical surveys and second-hand asset price records exploited using econometric methods). However, in the absence of conclusive econometric estimates and following the recommendations of the OECD, today most governmental and private statistical offices accept the geometric depreciation pattern as the most suitable approximation to the loss in the value of assets as they age. Very often, the method employed to estimate the depreciation rate is the well-known double-declining balance

[8] In this case, the endogeneity of the depreciation rate is not a substantial characteristic, because it is not a decision variable, but induced, because it depends on variables that are determined endogenously.

[9] A good description of the methodology used to estimate capital stock by the statistical agencies BEA and BLS may be found in Katz (2015).

[10] According to OECD (2009): the retirement pattern is a function representing the distribution of retirements around the average service life; the age-efficiency profile describes the time pattern of assets' productive efficiency as they age, i.e. the losses in productive capacity due to wear and tear; and the age-price profile is the index function of the price of assets, which declines with increasing age.

method, summarized in the expression $\delta_i = 2/\bar{T}_i$ where $\bar{T}_i$ is the average service life for assets of type i. According to this method, the measurement of depreciation is directly associated with the fixed service life of the different assets. Beyond the complex statistical forms of survival-retirement distributions and age-efficiency functions chosen ad hoc, this is the simplest statistical method available to obtain a measure of depreciation, which is absolutely compatible with the postulates of the proportionality theorem.

Consequently, after aggregation we can end up expressing the depreciation as in (25) and the dynamics of the capital stock with the following equation that generalizes the permanent inventory method,

$$K_t = I_t^G + (1 - \delta_t) K_{t-1}. \tag{26}$$

At this point we have equations (25) and (26) that allow us to perform a statistical measurement of depreciation and capital stock. Previously we have seen that equations (20) and (21) allow us to reach an economic measurement of depreciation and capital stock. In consequence, we have two different and apparently independent dynamic processes of capital accumulation. However, it is possible to establish some connections between them. Despite the two processes differ in the measure of depreciation, and the statistical depreciation flow implicitly defines a rate δ_t which does not have to coincide with the economic depreciation rate δ_t^*, both processes coincide in their measure of gross investment I_t^G.

On the other hand, we have to decide among the several paths that are obtained integrating the dynamic process of capital accumulation, and particularize specifying a boundary condition. We chose, as it is usual in these cases, the initial condition. Thus, in this context and in order to establish a reasonable comparison between the two capital trajectories, the statistical and the economic series, we assume that in the initial period both measures of the capital stock coincide, $K_0^* = K_0$.

Finally, it can be established another link, more conceptual than technical, between the two accumulation processes. Although we have remarked the short-run differences between the two measures of depreciation and capital stock, we have something new to tell about the long-run relationship. The assumption regarding depreciation as a fixed, or almost fixed, proportion of capital stock is inaccurate in the short-run, when the economy experiences important fluctuations due to the effects of continuous demand and supply shocks. However, after a while in which the economy has had time to absorb and adjust to the different monetary and real shocks, the statistical measurement of depreciation may be a good approximation of the economic depreciation. That is, in the long-run the average service life of assets and the perpetual inventory method can offer a satisfactory measure of depreciation and the capital stock.

4 Capital and depreciation in Spain

Given the algorithmic procedure for calculating the depreciation rate δ_t^* and the capital stock K_t^* based on the optimal decisions of economic agents, as shown in section 2, we next apply this procedure to the non-financial business sector data of the Spanish economy during the period 1964-2011.[11] We use as observables gross investment I_t^G, price of capital goods p_t^k, gross distributed profits B_t^G, q ratio q_t and the predetermined value of the initial capital stock K_0 for the period 1964-2011. These series of the variables considered in the system of equations (20)

[11] The non-financial business sector is defined as total activities in the economy excluding the financial intermediation sector, real estate and non-market services.

and (21) are available in Table A.1 of the Appendix. Likewise, in that appendix, the statistical sources of the used variables as well as the process followed for its elaboration are detailed.

Once the series of economic capital stock and depreciation rate are obtained, the next step is to analyze its evolution over the period compared with the corresponding series using statistical measures of depreciation and capital as described in section 3. Table 1 contains the detailed figures for the economic and statistical depreciation rates and stock of capital over the period 1964-2011. Figures 1 and 2 plot the evolution of the different measures of capital stocks and depreciation rates. While in Figure 1 the economic depreciation rate compared to the statistical depreciation rate is presented, in Figure 2 the two time profiles for the economic and statistical capital stocks for the non-financial business sector 1964-2011 are showed.

As can be seen in Table 1 we have established different subperiods in which the economic depreciation rate has values lower than the statistical depreciation rate (shaded in the table) and subperiods in which the behavior is the other. The first subperiods correspond to expansions in the Spanish economy and the latter to recessions.

During the expansionary period 1964-1974, the Spanish economy grew as a result of the stabilization plan to deal with the liberalization of the economy. In Figure 3 an enormous growth of capital stocks, $\overset{\wedge}{K^*} > \hat{K}$, can be observed as a consequence of the process of industrialization, the adoption of more intensive techniques in capital and strong importing of capital assets. The economic depreciation rate over this period is much less, with a value on average of 3.88%, than the statistical rate with a value of 6.25%, as a consequence of equipment maintenance expenditure. Furthermore, during these years, the q ratio is always higher than unity as can be observed in the Table A.1 in the Appendix.

The next period is a period of crisis that started in the mid-1970s and lasted almost a decade, through to the mid-1980s. The economic capital stock stagnates significantly with $\overset{\wedge}{K^*} < \hat{K}$, as a result of the massive depreciation of manufacturing equipment. That in turn is a direct consequence of the high vulnerability stemming from the energy crisis and major industrial restructuring process –traditional and/or heavy industry subject to the NICs competition– that occurred during this period. The economic deterioration, but primarily the structural obsolescence, could be at the heart of the significant rise in depreciation, which cannot be explained by the simple physical deterioration of the equipment from age and normal use, as seen by the statistical depreciation rate. During this period the economic depreciation rate is greater than the statistical rate with values on average of 10.59% and 6.86% respectively. The disparity of values that can be seen by comparing our new estimates with the more traditional ones, shows the inappropriateness of the perpetual inventory method for calculating the capital stock in periods of major economic upheaval.

In the new expansionary period from the mid-80s to the start of the 2000s, the two time profiles, for the economic and statistical capital stock, are similar and indicate strong growth for both series, greater in the case of economic capital. Associated with this general recovery, the q ratio continues to be more than unity (except between 1991 and 1993) reaching its highest value in the year 2000. In this year the turning point along the temporal line of evolution of capital stock, which shoots up to levels similar to those of the pre-crisis period is showed. Only during the recession of 1992-1993 is a slowdown in the trend over this long period observed. The economic depreciation rate has been quite volatile and smaller than the statistical rate in this period.

As of 2002, there is a slowdown in the rate of growth of economic capital stock, $\overset{\wedge}{K^*} < \hat{K}$, breaking away from the general trend and opening a large gap between K^* and K. The economic depreciation rate rises with a value on average of 10.36% above the statistical rate with a value of 7.52%. This could be the result the ICT investment boom that modifies production methods and increases the depreciation caused by economic deterioration and obsolescence. Moreover, the ICT investment makes non-residential investment fall sharply which can be seen in the official statistics in European countries in the early 2000s. The q ratio gradually decreases to values lower than 1. The last year in the sample marks the start of the recovery.

During the period 1964-2011 the performance of Spanish non-financial business sector differs substantially in its short-run evolutions, but it is possible to establish a close correspondence between the long-run profiles. In fact, as can be observed in the Figures 1 and 2, the economic depreciation rate fluctuates around the statistical depreciation rate and so does the economic capital stock around the statistic. So, it can be concluded that the long-run statistical depreciation rate is a good approximation of the long-run economic depreciation. In the same way that the statistical capital stock accumulated according to the perpetual inventory method is a satisfactory measure of the capital stock in the long-run, when the economies have had time to adjust to the shocks.

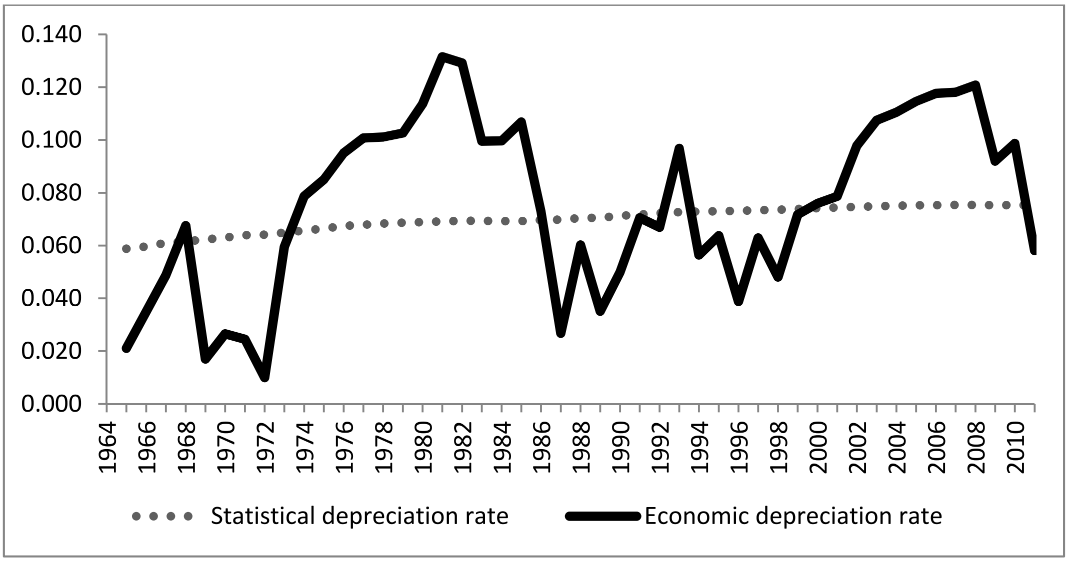

Figure 1. Economic and Statistical depreciation rate: Spanish non-financial business sector, 1965-2011.

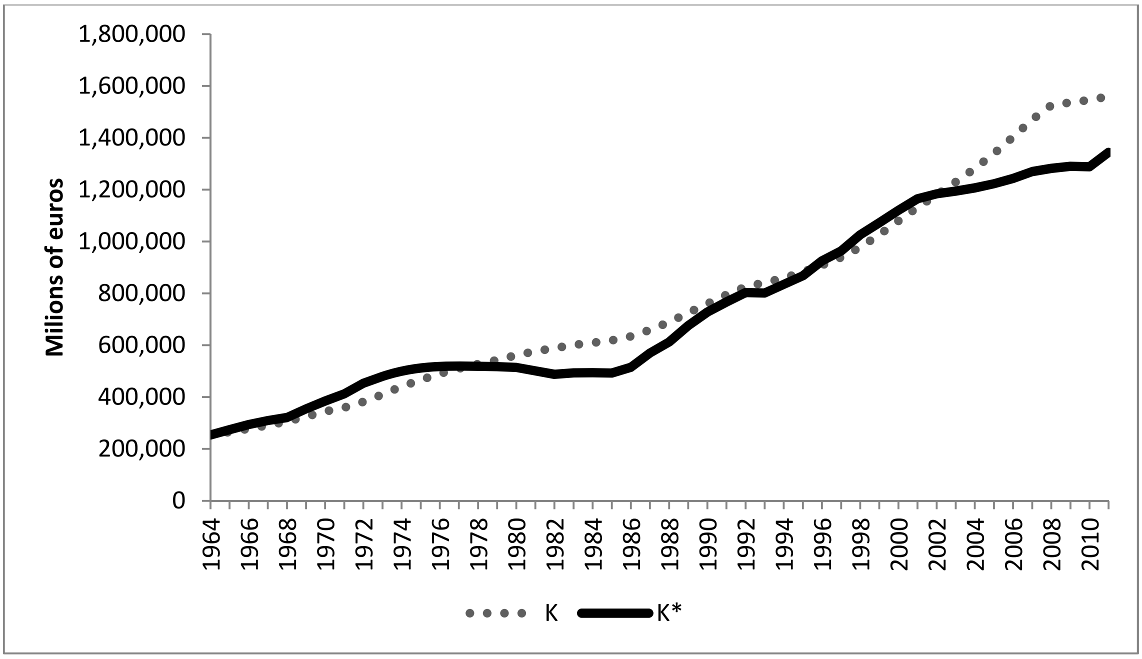

Figure 2. Economic and Statistical capital stock: Spanish non-financial business sector, 1964-2011.

Table 1. Economic and Statistical depreciation rates and capital stock (millions of euros of 2008) in Spanish non-financial business sector (1964-2011).

Year	Depreciation rate Economic δ^*	Depreciation rate Statistical δ	Capital Stock Economic K^*	Capital Stock Statistical K
1964			254,181	254,181
1965	0.0211	0.0588	274,421	264,832
1966	0.0349	0.0596	294,076	278,275
1967	0.0487	0.0610	309,549	291,098
1968	0.0676	0.0616	321,346	305,883
1969	0.0170	0.0623	354,306	325,236
1970	0.0266	0.0630	384,836	344,693
1971	0.0245	0.0639	412,411	359,668
1972	0.0100	0.0641	453,383	381,692
1973	0.0595	0.0649	479,626	410,142
1974	0.0787	0.0657	499,718	441,038
1975	0.0850	0.0664	511,839	466,365
1976	0.0950	0.0675	518,704	490,417
1977	0.1008	0.0678	519,251	509,968
1978	0.1011	0.0684	519,031	527,405
1979	0.1027	0.0687	517,559	542,993
1980	0.1136	0.0689	514,626	561,482
1981	0.1315	0.0691	500,956	576,681
1982	0.1292	0.0694	488,089	588,516
1983	0.0995	0.0693	493,167	601,376
1984	0.0997	0.0693	493,811	609,509
1985	0.1068	0.0692	492,606	618,855
1986	0.0729	0.0697	515,129	634,158
1987	0.0267	0.0698	570,127	658,682

Year	Depreciation rate Economic δ^*	Depreciation rate Statistical δ	Capital Stock Economic K^*	Capital Stock Statistical K
1988	0.0602	0.0704	612,356	688,898
1989	0.0351	0.0706	675,318	724,751
1990	0.0499	0.0713	728,277	759,779
1991	0.0705	0.0718	766,555	794,893
1992	0.0669	0.0723	803,245	825,419
1993	0.0968	0.0727	801,618	841,582
1994	0.0564	0.0729	834,929	858,780
1995	0.0637	0.0730	868,298	882,688
1996	0.0388	0.0732	925,454	908,963
1997	0.0629	0.0734	963,780	938,831
1998	0.0481	0.0736	1,025,848	978,137
1999	0.0717	0.0739	1,072,631	1,026,272
2000	0.0760	0.0741	1,120,565	1,079,612
2001	0.0786	0.0744	1,164,844	1,131,660
2002	0.0979	0.0747	1,183,795	1,180,095
2003	0.1075	0.0749	1,194,287	1,229,456
2004	0.1105	0.0751	1,206,816	1,281,636
2005	0.1146	0.0752	1,223,034	1,339,742
2006	0.1176	0.0753	1,243,424	1,403,097
2007	0.1181	0.0754	1,269,952	1,470,684
2008	0.1208	0.0754	1,282,322	1,525,592
2009	0.0920	0.0753	1,290,060	1,536,363
2010	0.0987	0.0753	1,288,119	1,546,047
2011	0.0581	0.0753	1,343,990	1,560,230

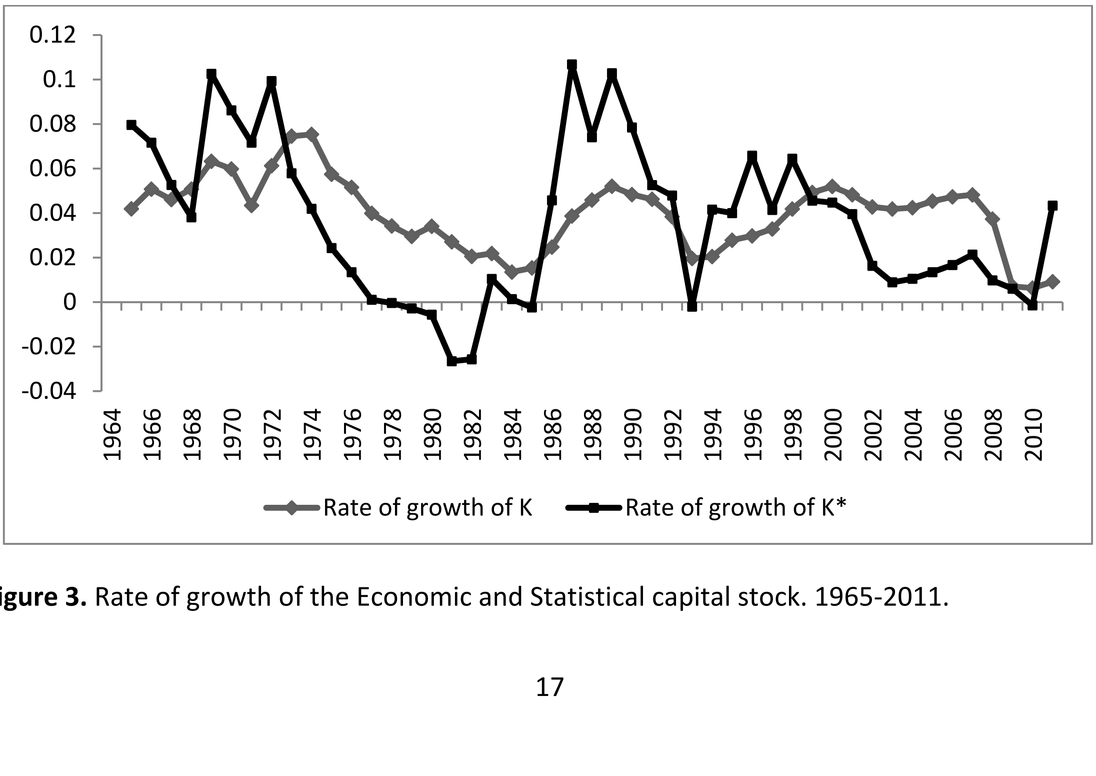

Figure 3. Rate of growth of the Economic and Statistical capital stock. 1965-2011.

5 Total factor productivity and growth in Spain

In this section we first remind the traditional results from a standard growth accounting exercise. We start with the definition of per capita income

$$y = \frac{Y}{N} = \frac{Y}{L}\frac{L}{H}\frac{H}{N}. \tag{27}$$

The involved variables are: income per capita y, output Y, population N, employment L, labor supply H, labor productivity $\frac{Y}{L}$, employment rate $\frac{L}{H}$, activity rate $\frac{H}{N}$. Calculating the rates of growth in the above expression we get

$$\hat{y} = \hat{Y} - \hat{N} = \overset{\wedge}{\left(\frac{Y}{L}\right)} + \overset{\wedge}{\left(\frac{L}{H}\right)} + \overset{\wedge}{\left(\frac{H}{N}\right)}. \tag{28}$$

Now, we consider that aggregate output is produced according to the production function $Y = AF(L, K)$, where A is a variable representing the technological level or the total factor productivity, and K is the physical capital stock. The rate of growth of per capita income is the sum of the rates of growth of the labor productivity, the employment rate and the activity rate. The employment and the activity rates are bounded between zero and one and, in the long-run, they are expected to be stationary. Consequently it is also expected that none of them will contribute to the long-run growth of the per capita income. Then, from a long-run perspective only the rate of growth of the labor productivity matters for the explanation of the economy's rate of growth.

As usual, we assume: *i*) the function $F(.)$ is homogeneous of degree one in all of its determinants taken simultaneously, and *ii*) factor prices are determined in competitive markets according to the marginalist theory of distribution, $\frac{W}{p} = w = \frac{\partial Y}{\partial L}$ and $\frac{C}{p} = c = \frac{\partial Y}{\partial K}$, where W represents the nominal wage and $C = p^k(r+\delta)$ the nominal rental price of capital. Therefore, we get $\Pi_K + \Pi_L = 1$, being $\Pi_K = \frac{cK}{Y}$ and $\Pi_L = \frac{wL}{Y}$. Finally, from the production function after substituting and rearranging terms we find

$$\overset{\wedge}{\left(\frac{Y}{L}\right)} = \hat{A} + \Pi_K \overset{\wedge}{\left(\frac{K}{L}\right)} = \hat{A} + c\frac{\overset{\wedge}{\left(\frac{K}{L}\right)}}{\left(\frac{Y}{K}\right)}, \tag{29}$$

$$\overset{\wedge}{\left(\frac{Y}{K}\right)} = \hat{A} - \Pi_L \overset{\wedge}{\left(\frac{K}{L}\right)} = \hat{A} - w\frac{\overset{\wedge}{\left(\frac{K}{L}\right)}}{\left(\frac{Y}{L}\right)}. \tag{30}$$

In (29) and (30) the term $\hat{A}$ is usually unknown, but it may be calculated as a residual in the following way: $\hat{A} = \hat{Y} - \Pi_L \hat{L} - \Pi_K \hat{K}$. These equations supply the rates of growth of every factor productivity taken separately. It has been mentioned the importance of the rate of growth of the labor productivity to understand the long-run growth of the economy. Now, equation (29) shows how this one depends on the rate of growth of total factor productivity as well as on a second term where the rate of growth of capital intensity is multiplied by the capital share.

Although these outcomes are well-known, they deserve a new inspection after the results shown in the previous section where we have supplied two different series of the capital stock, which is a central variable in the above algebraic calculations. If we use the economic measure of capital stock K^* instead of the statistical one K, we first observe that $Y = A^* F(L, K^*)$. This implies that any change in the measure of the capital stock will be completely absorbed by the residual, for any given figures of output and employment. Second, the above two expressions transform into

$$\overset{\wedge}{\left(\frac{Y}{L}\right)} = \overset{\wedge}{A^*} + \Pi_{K^*} \overset{\wedge}{\left(\frac{K^*}{L}\right)} = \overset{\wedge}{A^*} + c^* \frac{\overset{\wedge}{\left(\frac{K^*}{L}\right)}}{\left(\frac{Y}{K^*}\right)}, \tag{31}$$

$$\overset{\wedge}{\left(\frac{Y}{K^*}\right)} = \overset{\wedge}{A^*} - \Pi_{L^*} \overset{\wedge}{\left(\frac{K^*}{L}\right)} = \overset{\wedge}{A^*} - w^* \frac{\overset{\wedge}{\left(\frac{K^*}{L}\right)}}{\left(\frac{Y}{L}\right)}, \tag{32}$$

where $\Pi_{K^*} + \Pi_{L^*} = 1$, $\Pi_{K^*} = \frac{c^* K^*}{Y}$, $\Pi_{L^*} = \frac{w^* L}{Y}$, $w^* = A^* \frac{\partial F(L,K^*)}{\partial L}$, and $c^* = A^* \frac{\partial F(L,K^*)}{\partial K^*} = \frac{p^k}{p}(r + \delta^*)$.

In the standard growth accounting exercise there is only one capital stock involved, and the corresponding capital share is computed as a long-run constant, usually approached by its sample average value.[12] Instead, here we are managing two capital series and, hence, we face two different expressions for the capital share. This duality, which allows to write the rate of growth of the labor productivity twice in equations (29) and (31), introduces the additional requirement of deciding on the appropriate assumptions for the two capital shares. This is a necessary step previous to make operational the empirical measure of the relevant variables accounting for growth. By analogy we assume that both capital shares are treated as long-run constants given by their corresponding sample average values, $\bar{\Pi}_{K^*} \equiv \frac{1}{T}\sum_{t=1}^{T} \Pi_{K^*}(t)$ and $\bar{\Pi}_K \equiv \frac{1}{T}\sum_{t=1}^{T} \Pi_K(t)$. However, we have still to decide whether they are equal or different to each other. First, consider the case in which the two constants are equal and $\sum_{t=1}^{T} \frac{c(t)K(t) - c^*(t)K^*(t)}{Y(t)} = 0$. This result may arise because $cK = c^* K^* \; \forall t \in [1, T]$, which is too unrealistic, or because throughout the sample the positive values outweigh the negative ones.[13] The latter may be checked with the data from previous sections which give us the values $\bar{\Pi}_{K^*} = 0.2089$ and $\bar{\Pi}_K = 0.2085$. It is then accepted this assumption as an empirically proven fact, and we proceed to undertake the growth accounting exercise twofold on the basis that there exists two capital series.[14] Obviously, there

[12] See the seminal contributions of Solow (1957) and Denison (1962).

[13] Moreover, given that the economic rate of depreciation fluctuates around the statistical rate, in the long-run we expect to observe that $\bar{\delta}^* \equiv \frac{1}{T}\sum_{t=1}^{T} \delta^*(t) = \frac{1}{T}\sum_{t=1}^{T} \delta(t) \equiv \bar{\delta}$. Hence, we could substitute this common constant value into the user cost of capital getting the equality between the two measures $c^* = c = \bar{c} = \frac{p^k}{p}\left(r + \bar{\delta}\right)$, which is variable despite the constant value of the depreciation rate. Now, the constancy and equality of capital shares appear associated to the condition $\sum_{t=1}^{T} \frac{\bar{c}(t)(K(t) - K^*(t))}{Y(t)} = 0$. This result may arise because $K = K^* \; \forall t \in [1, T]$, which is basically false, or because throughout the sample the economic value of capital fluctuates around the statistical measure of capital, which is more or less the result we got in previous sections.

[14] Although the strong similarity of these figures confirm our insights, we have not used any of them to empirically obtain the decomposition of the rate of growth of labor productivity. Instead, we have taken the higher value $\bar{\Pi}_K = 0.3943$ from Spanish National Accounts, which is more in accordance with the estimated elasticity of output to capital. We would like to remark that in checking our hypothesis about the equality of the two constant capital shares, we have used a simplified definition of the user cost that ignores components such as the risk premium, taxation and so on.

is a link between the two exercises, represented when we connect the pair of equations (29) and (31) by the following expression

$$\hat{TFP^*} - \hat{TFP} = \left(\hat{\frac{A^*}{A}}\right) = \bar{\Pi}_K \left[\left(\hat{\frac{K}{L}}\right) - \left(\hat{\frac{K^*}{L}}\right) \right]. \tag{33}$$

We shall now study, with the help of the above accounting framework, the results for the Spanish economy. First of all, we divide the period 1964-2011 into six subperiods according to the particular evolution of the rate of growth of labor productivity. According to such a rule, we observe that the rate of growth of labor productivity is as high as 6% on average between 1964 and 1974. Along the next period, between 1975 and 1985, there is a first slowdown and the rate of growth experiences, on average, a lower value of 2.7%. During the period 1986-1991 there is a second slowdown and the rate of growth diminishes to 0.9% on average. In the two-year period 1992-1993 we observe an important recovery of the rate of growth of labor productivity, which takes the value 2.5%. The period 1994-2007 shows again a slowdown, the third, in which the average rate of growth is negative, -0.2%. Finally, in the period 2008-2011, corresponding to the first years of the Great Recession, there is an important recovery of the rate of growth of labor productivity, which reaches on average the value 2.7%.

Table 2.

Periods	$\hat{Y}$	$\hat{L}$	$\hat{\frac{Y}{L}}$	$\hat{K}$	$\hat{K^*}$	δ	δ^*
1964 – 1974	6.30	0.30	6.00	5.70	7.00	6.20	3.90
1975 – 1985	0.90	-1.9	2.70	3.10	-0.1	6.90	10.6
1986 – 1991	3.70	2.70	0.90	4.30	7.70	7.10	5.30
1992 – 1993	-0.5	-3.1	2.50	2.90	2.30	7.20	8.20
1994 – 2007	3.40	3.60	-0.2	4.10	3.40	7.40	8.30
2008 – 2011	-1.2	-3.9	2.70	1.50	1.40	7.50	9.20
1964 – 2011	2.93	0.59	2.34	3.95	3.66	6.85	7.43

Source: Own elaboration and official statistics (see Appendix).

Table 3.

Periods	$\frac{Y}{K}$	$\frac{Y}{K^*}$	$\hat{\frac{Y}{K}}$	$\hat{\frac{Y}{K^*}}$	$\hat{\frac{K}{L}}$	$\hat{\frac{K^*}{L}}$	$\hat{TFP}$	$\hat{TFP^*}$
1964 – 1974	0.761	0.689	0.60	-0.6	5.40	6.70	3.90	3.40
1975 – 1985	0.662	0.719	-2.2	1.00	5.10	1.80	0.80	2.10
1986 – 1991	0.607	0.674	-0.5	-3.7	1.50	4.80	0.30	-1.0
1992 – 1993	0.560	0.581	-3.3	-2.7	6.10	5.50	0.20	0.40
1994 – 2007	0.542	0.553	-0.6	0.10	0.50	-0.2	-0.4	-0.1
2008 – 2011	0.463	0.549	-2.6	-2.5	5.70	5.60	0.60	0.60
1964 – 2011	0.621	0.639	-1.0	-0.7	3.41	3.10	1.01	1.13

Source: Own elaboration and official statistics (see Appendix).

In the first subperiod, the strong increase of labor productivity is accompanied by a rise in employment, although minimal, of 0.3%. Nevertheless, the remaining subperiods with a significant growth in labor productivity, 1975-1985, 1992-1993 and 2008-2011, experienced major reductions in employment of -1.9%, -3.1% and -3.9%, respectively. Conversely, in the 1986-1991 and 1994-2007 subperiods, the strong rise in employment of 2.7% and 3.6%, respectively, are accompanied by weak growth or even negative growth in labor productivity. Thus, the pattern that characterizes the Spanish economy from the mid-70s up to the present day, is that of a persistent trade-off between the evolution of labor productivity and the evolution of employment.

The evolution of the rate of growth of labor productivity can be interpreted with respect to the evolution of its major components: *i*) the rate of growth of total factor productivity, and *ii*) the rate of growth of capital intensity. Nevertheless, the decomposition can be carried out using equations (29) or (31). This twofold decomposition depends on which of the different measures of capital stock, the statistical or the economic ones, we use to calculate the variables that appear in the above-mentioned equations. This is not a pure and simple zero-sum arithmetic exercise without any significance, but rather it conforms the basis for the interpretation and the explanation of the facts linked to the Spanish economic growth over the past 50 years.

From subperiod 1964-1974 to subperiod 1975-1985, labor productivity underwent a significant slowdown, with its rate of growth losing 3.3 percentage points. This important slowdown has been basically explained, using the standard statistical measures, by the sharp fall of the rate of growth of total factor productivity between these two subperiods (from 3.9% to 0.8%). Nevertheless, when the economic measures are used, we find an alternative explanation for the slowdown which is twofold: the fall in the growth of total factor productivity (from 3.4% to 2.1%) but also the strong reduction in the growth of capital intensity (from 6.7% to 1.8%). Behind the last one we can identify the impact of a higher and increasing depreciation rate in the second half of the seventies, which represents the effect of the obsolescence due to the rise in oil prices and the structural change in the output composition.

From the subperiod 1975-1985 to the subperiod 1986-1991, labor productivity underwent a new slowdown. We call this downfall, in which 1.8 percentage points were lost, the second slowdown to highlight a major difference with the experience of the U.S. economy where the rate of growth remained almost constant. It is in fact a continuation of the process initiated before. However, certain major differences may be found, with respect to the previous subperiod, in the evolution of variables underlying the rate of growth of the labor productivity. These differences do affect to the way we interpret the causes of this slowdown deepening. In fact, if we use the statistical measures, the slowdown can be explained almost exclusively with the fall in the rate of growth of capital intensity (from 5.1% to 1.5%), without even a slight effect on the rate of growth of total factor productivity between these two subperiods. Conversely, when we use the economic measures it appears a substantially different explanation of the labor productivity stagnation. Here we identify a double effect pushing in opposite directions: the reduction of the rate of growth of total factor productivity (from 2.1% to -1%) and the compensating rise in the rate of growth of capital intensity (from 1.8% to 4.8%). Behind the last one there is a lower depreciation rate, which is the result of two opposite forces in the economic deterioration: the strength of maintenance expenditures dominates on the impulse of a higher rate of productive capacity utilization.

From the subperiod 1986-1991 to the subperiod 1992-1993, labor productivity undergoes a relative acceleration, with its growth rate increasing by 1.4 percentage points. The return of labor productivity to the growth path during this short period of two years, accompanied by a sharp employment destruction, is primarily explained by the strong growth in capital intensity (from 1.5% to 6.1%) when we use the statistical measure of capital stock. Instead, when we use the economic measure of capital stock, the return to a significantly positive rate of growth of labor productivity is explained by the increase in the rate of growth of total factor productivity, which changes from a negative figure (-1%) to a moderately positive one (0.4%).

From subperiod 1992-1993 to subperiod 1994-2007, the Spanish economy once again undergoes a slowdown in the rate of growth of labor productivity, losing 2.7 percentage points. This loss was later recovered as it moved from the subperiod 1994-2007 to the subperiod 2008-2011, winning the rate of growth 2.9 percentage points. Even though the described movements occur in the opposite direction of each other, the explanation for both can be found in the evolution of the same underlying variable: the rhythm of creation and destruction of employment, and consequently in the fall and rise of the rate of growth of capital intensity. Furthermore, contrary to what we have seen earlier, there are no major differences in the explanation of these changes depending on whether we are operating with the statistical or economic measurement of the capital stock. The third slowdown experienced by the Spanish economy over the past fifty years is due to the change which led to a powerful wave of job creation. The rates of growth of total factor productivity obtained with any of the two capital stocks fall by approximately 0.5 percentage points, and the corresponding rates of growth of the capital intensity fall by just over 5.5 percentage points. The last movement comes associated with the onset of the Great Recession. In Spain, this meant a move to a drastic loss of employment which practically explains on its own the huge acceleration which can be attested to in labor productivity.[15] The rates of growth of total factor productivity obtained with either of the two capital stocks rise in parallel by 1 and 0.7 percentage points, respectively, and the corresponding rates of growth of capital intensity rise, respectively, by 6.2 and 5.8 percentage points.

From what we have just seen, the most striking result is the slowdown in the rate of growth of labor productivity in subperiod 1994-2007, which takes a negative average value of -0.2%. This result, although similar to what happened in the rest of the European Union, is at variance with what is observed in the United States since the beginning of the nineties, when there is a recovery of the labor productivity represented by an increased and high positive rate of growth. In the case of the U.S. economy there has been a huge discussion on the importance of information and communication technologies (ICT) in overcoming the previous productivity slowdown observed from the early seventies. In fact, since Solow first raised the idea of a *productivity paradox*,[16] a lot of contributions have attempted to provide an explanation, as it is shown in Brynjolfsson and Yang (1996), but it continues to be a controversial issue, as may be checked in Acemoglu et al. (2014). Nevertheless, many authors saw the above-mentioned increase in the rate of growth of labor productivity as the delayed resolution of the Solow paradox, associating such a recovery with the wave of investments in ICT deployed from mid-seventies.

Now, of course, we wonder what happens with this issue in the Spanish economy. In the case of Spain there is a less extensive literature and only much more recently has it become a subject for discussion. Hence, we are to consider the role that investment in ICT might have played in explaining the Spanish growth process during the final stages of the period under

[15] For the crisis period 2008-2012 Hospido and Moreno-Galbis (2015), using balance sheet information from a sample of Spanish manufacturing and services firms, points out that labor productivity also responds to the behavior of total factor productivity. The authors find a positive link between the latter and some composition effects associated to the proportion of temporary workers and to the weight of exporting firms facing international competition, which significantly contribute to the recent improvement in labor productivity.

[16] Solow (1987): "the fact that what everyone feels to have been a technological revolution ... has been accompanied everywhere ... by a slowingdown of productivity growth, not by a step up. You can see the computer age everywhere but in the productivity statistics".

inspection. First of all, Mas and Quesada (2005) reported that from 1964 to the mid-80s this type of investment was negligible, starting then a takeoff that after the short stop of 1991-1993 continued through until the telecom crisis at the beginning of the new millennium. Consequently, we can take for proved that between 1995 and 2000 there was a boom of investment in hardware and software in parallel with the sharp fall in hardware prices.

Looking at the statistics it seems that the expected positive effect of such a high investment in ICT is not present in the negative growth of labor productivity observed during the period 1994-2007. But we cannot be sure of that because the dynamics of the labor productivity in this period, as well as in the following period 2008-2011, is mainly driven by huge changes in employment. Consequently, the impact of large investments in ICT, if any, could be hidden behind the atypical behavior of the rate of growth of labor productivity in Spain.[17] Hence, we have to inspect the dynamic behavior of its two components: total factor productivity and capital intensity. But these ones depend on the measure of the capital stock that we use to calculate them.

There is the commonly accepted view according to which investment in ICT should be accompanied by high values of the rate of growth of total factor productivity. Nevertheless, this does not correspond with the real fact of the low values recorded for this rate when it is calculated using the statistical capital stock. In the subperiod 1994-2007 there was a sharp fall in the rate of growth of TFP in Spain with an annual average value of -0.4%. In any case, even if we use the economic measure of the capital stock we still get a negative value of -0.1%. This fall was also a general feature in Europe but not in the United States, where the rate of growth of TFP increased although less than expected by those who relied on the benefits of the *New Economy*. Extracting from Stiroh (1998) we find that it is not clear that more and better computers should accelerate total factor productivity. The computer revolution may be characterized by: *i*) a computer-producing sector which is subject to fundamental technological changes; and *ii*) the remaining computer-using sectors which, induced by falling prices, undertake a deep process of capital substitution. Although the first implies production function shifts that increase TFP, it is small and has no significant impact on the aggregate. The second, in turn, implies movements along the production function.

In what follows we shall focus on studying the relationship between investment in ICT and the rate of growth of TFP when we employ economic instead of statistical measures for both depreciation and capital. We pay special attention to the central role of depreciation in all this story because of its direct connection with any capital substitution process.

5.1 ICT investment, economic depreciation and TFP growth

As we have pointed out in this paper, the relationship between the statistical and the economic measures of capital stock is directly dependent on the relationship between the statistical and the economic measures of depreciation. The statistical depreciation tries to quantify the physical deterioration (*wear and tear*): depreciation caused by aging and the regular and constant use of capital goods. Instead, the economic depreciation includes in addition the economic deterioration and obsolescence: depreciation coming from the variable activity and uncertainty typical of business cycle, depreciation associated to the (lower) expenditure devoted to maintenance, and depreciation due to the technical progress embodied in new capital equipment and structural change.

[17] As De la Fuente (2009) remarks, the contribution of investment in ICT to productivity growth could be greater than that reported in previous works.

Our hypothesis here is that generalized investment in ICT and the introduction of new and improved ICT in most of the new investment capital goods exert an important indirect effect on the residual rate of growth of total factor productivity. Moreover, according to (33) $\overset{\wedge}{TFP^*} > \overset{\wedge}{TFP}$ if and only if $\overset{\wedge}{\frac{K}{L}} > \overset{\wedge}{\frac{K^*}{L}}$. In consequence, for a given rate of growth of labor productivity, the underlying explanatory contribution of the rate of growth of total factor productivity is conditioned on the measure of the capital stock that we use in calculations. And this measure highly depends on the magnitude of the corresponding measure of depreciation. Since the computer revolution implies additioning more productive equipment to the capital stock, we would expect an accompanying process of strong substitution of capital goods.[18] In other words, we expect a huge stream of economic deterioration and obsolescence not recorded in the statistical measures.

On the other hand, the expansion of investment in ICT that began around the year 1995 in Spain and rose the share of such items in total investment to more than 10% by 2004, do not provide a clear evidence to support our hypothesis during the second half of the nineties. This is probably due to the fact that it is too soon to observe its effects. If we consider that the acceleration of investment in ICT was gradual and, as De la Fuente (2009) sustains, it requires complementary investment in learning, human capital and restructuring production organization, there might be a certain lag before the process of input substitution takes off. [19] Hence, it is absolutely reasonable that the effect of investment in ICT on the rate of depreciation reveals with a certain delay. In Spain, using the economic measures of capital and depreciation, we can identify a second episode of higher and increasing depreciation rates in the statistics of the years 2002-2007. Consequently, we shall inspect the whole data corresponding to this time interval prior to the start of the Great Recession in 2008.

Table 4. Growth and Productivity in 2002-2007

$\hat{Y} = 2.9\%$	$\hat{L} = 3.7\%$		$\overset{\wedge}{\frac{Y}{L}} = -0.8\%$
$\hat{K} = 4.5\%$	$\overset{\wedge}{K^*} = 1.5\%$	$\delta = 7.5\%$	$\delta^* = 11.1\%$
$\frac{Y}{K} = 0.52$	$\frac{Y}{K^*} = 0.56$	$\overset{\wedge}{\frac{Y}{K}} = -1.5\%$	$\overset{\wedge}{\frac{Y}{K^*}} = 1.4\%$
$\overset{\wedge}{\frac{K}{L}} = 0.8\%$	$\overset{\wedge}{\frac{K^*}{L}} = -2.1\%$	$\overset{\wedge}{TFP} = -1.1\%$	$\overset{\wedge}{TFP^*} = 0.1\%$

Source: Own elaboration and official statistics (see Appendix).

As we can see from figures in Table 4, the negative rate of growth of labor productivity in period 2002-2007 may be explained almost exclusively with the high rate of growth of employment. Then, looking at the determinants of the rate of growth of labor productivity (those measured in economic terms with respect to those measured in statistical terms), we find a positive but not too high rate of growth of capital, an important depreciation rate beyond the average that represents an important economic deterioration and obsolescence,[20] and a rate of growth of TFP which is low but still positive.

[18] See Jorgenson and Stiroh (1999) and Whelan (2002).

[19] The idea that the measured consequences of investment in ICT need time to become visible in the macroeconomic aggregates has also been defended in Mas and Quesada (2006) and Martínez et al. (2008) when they study the so-called *Spanish productivity paradox*.

[20] Given that this period of strong economic growth represents an expansive phase of the business cycle, it is also expected a higher rate of productive capacity utilization. Therefore, a greater depreciation due to economic deterioration appears in our records combined with the greater depreciation caused by obsolescence.

This is just the accounting growth picture that according to Jorgenson and Stiroh (1999) may be summarized in the following way: "the story of the computer revolution is one of relatively swift price declines, huge investment in IT equipment, and rapid substitution of this equipment for other inputs. Perhaps surprisingly, this technological revolution has not been accompanied by technical change in the economic sense of the term"; that is, production function shifts and the corresponding growth of total factor productivity.

6 Conclusions

Most of the empirical studies in macroeconomics and the economic growth literature depend on the measure of the physical capital stock. The variables that explain the main phenomena and problems in these areas of economics always appear to be related, whether directly or indirectly, to capital stock and depreciation. The standard measurements of capital and depreciation are statistical measures based on assumptions about the average service life of capital goods, which are accumulated according to the perpetual inventory method.

In this paper we propose an alternative method based on the equations that solve the dynamic optimization problem of the neoclassical firm. In our model coexist an adjustment cost function with a maintenance cost function, which are linearly homogeneous. This method enables us to endogenously calculate the variables rate of depreciation and capital stock, yielding an economic estimation based on indicators of profitability such as the distributed profits and the Tobin's q ratio.

The above represents a change of paradigm in measuring capital and depreciation, from statistical terms to economic values, which we supply along with an application to the Spanish economy. The results show an economic depreciation rate (endogenous) that fluctuates around the statistical rate (exogenous), and two time profiles for the economic and statistical capital stocks that are markedly different to each other. These short-run differences are the result of a greater or lesser destruction of capital in different periods that are not recorded in the official statistics. Consequently, it is not unusual the existence of misleading economic interpretations in the short-run, based on the statistical depreciation that corresponds to a geometric pattern, and the capital stock that arises from the PIM methodology. Instead, in the long-run these two measures are good approximations of our endogenous measures in terms of economic value. This is because in the long term economies have had enough time to absorb and adjust to shocks, and the rule of double declining balance rate, built upon the average service life of assets, correctly captures the true depreciation.

In the context of a growth accounting exercise we show how capital intensity and total factor productivity play different roles in explaining growth over the past fifty years, depending on whether we use statistical or economic measures. While the slowdown in labor productivity experienced between 1975 and 1985 has been traditionally explained in terms of the fall in the rate of growth of TFP, our results point mainly to the increased obsolescence. We find a so high capital destruction that reduces the rate of growth of capital intensity by an amount that goes well beyond statistical measures. Nevertheless, we must conclude in the opposite direction when analyzing the new slowdown experienced in the period 1986-1991. Now maintenance and repair of equipment was much higher than the one implicit in the official statistics of depreciation. Consequently, while the traditional explanation was given in terms of the fall in the rate of growth of capital intensity, our results highlight the fall in the rate of growth of TFP, which was reduced well above the conventional estimates. Thus, we are ready to conclude that the

PIM is inappropriate to calculate the stock of capital in short periods of time and, therefore, to estimate the rate of growth of TFP.

In the last section we have tried to explain a non-trivial issue, which concerns the true impact of investment in ICT on labor productivity and TFP dynamics in Spain since the mid of the nineties. Beyond the punctual agreement with the literature studying the Spanish productivity paradox, which emphasizes the idea that the economy experiences the effects of investment in ICT with a delay, there are remarkable differences. Most of such literature concentrates on those secondary requirements that supposedly promote the diffusion of a technological revolution: lower adoption costs, higher rates of penetration and use of new ICT, better qualifications and training of labor force, newer organizational forms, less strict employment legislation, and a more friendly environment for firms and business. Instead, we focus on the relevance of depreciation, and the corresponding capital substitution process, as the main channel by which the materialization of any technical change happens.

7 Appendix: Data

The database of reference for most of the series used in the paper is the BD. MORES database.[21] The variables and their statistical sources, as well as the process followed for its elaboration when this has been necessary are detailed bellow

- Y_t: The Gross Value Added of non-financial business sector in constant euros in 2008. BD. MORES b.2008
- L_t: The number of employees of non-financial business sector. BD. MORES b.2008
- I_t^G: The Gross Fixed Capital Formation of non-financial business sector in constant euros in 2008. BD. MORES b.2008
- p_t^k: The non-financial business capital goods price. BD. MORES b.2008.
- K_0: The capital stock of non-financial business sector in constant euros in 2008 in the initial period. BD.MORES b.2008.
- R_t: The long term nominal interest rate. From 1964 to 1977 the nominal series by Escribá and Ruiz (1995b) and from this year until 2011 the nominal series by AMECO database are used.
- q_t: Tobin's q ratio. Series of q ratio values by Espitia (1987) from 1964 to 1982 are used. For the years 1983 to 1987 q ratio values by Alonso and Bentolila (1992) with information from the Central of Balances of the Bank of Spain are used. The q of firm series built by Ramirez, Rosell and Salas (2003), which the authors have annualized and provided to us, is used for the period 1988 to 2000. To lengthen the series until 2011, the growth rates of the q values by IMF (2015) report on prospects for the European and global economy have been used.

[21] The Spanish regional database BD.MORES b.2008 (see De Bustos et al., 2008) is compiled by the Budget General Directorate of the Spanish Ministry of Finance and Civil Service and is available on the following web page:
http://www.sepg.pap.minhafp.gob.es/sitios/sepg
/es-ES/Presupuestos/Documentacion/paginas/basesdatosestudiosregionales.aspx

- B_t^G: Gross distributed profits in current euros. These include series of dividends, interest payments as well as depreciation of fixed capital. These series have been developed as described below:

  Dividends: Series of dividends has been built from the CNE base 2010 by INE. Within the non-financial accounts of institutional sectors annual series *property´s income* (D.4) is used and within this only *distributed incomes of societies* (D.42) and *reinvested profits* (D.43) are used. These series for the time period 1999 to 2014 are available. The CNE base 95 provides data on about the *property's income* being without disaggregated and this has not been used. However, the CNE base 86 provide series from 1985 to 1997 of *dividends and other income distributed by societies* (R.44) for corporations and quasi-nonfinancial corporations. Based on this information we have built a homogenous series for the period 1985-2011 linking both series. From 1964 to 1985 the dividends series produced by Escribá and Ruiz (1995b) has been used. The initial values of the series (1964-1973) are respected and using the value of the dividends in 1985 according to CNE b.86 and using the growth tendency described by Escribá and Ruiz (1995b) to connect the values of 1973 and 1985.

  Interest payments: The series of interest payments has been built in a similar way to the series of dividends: from 1985 to 2014 using the information provided by the CNE base 2010 and base 86 about *interest* (R.41). From 1964 to 1985 interest series by Escribá and Ruiz (1995b) are used.

  Depreciation: Series of depreciation of non-financial business capital built using database BD. MORES b.2008.

Table A.1. Spanish non-financial business sector 1964-2011.

Year	I_t^G	p_t^k	R_t	q_t	B_t^G	Year	I_t^G	p_t^k	R_t	q_t	B_t^G
1964	21402	0.054	0.08	1.531		**1988**	76578	0.47	0.1175	1.34	41539
1965	25593	0.055	0.083	1.414	1671	**1989**	84457	0.495	0.137	1.26	46024
1966	29230	0.057	0.086	1.289	1618	**1990**	86677	0.525	0.1468	1.02	51350
1967	29797	0.06	0.089	1.159	1642	**1991**	89646	0.545	0.1138	0.91	56650
1968	32714	0.064	0.091	1.19	1934	**1992**	87974	0.557	0.1172	0.73	59482
1969	38415	0.067	0.096	1.467	2076	**1993**	76143	0.586	0.1021	0.58	59276
1970	39950	0.071	0.11	1.341	2512	**1994**	78521	0.605	0.0999	1.03	62357
1971	37006	0.075	0.113	1.416	3189	**1995**	86591	0.641	0.1127	1.00	63723
1972	45084	0.079	0.105	1.548	3189	**1996**	90848	0.655	0.0874	1.11	66338
1973	53212	0.086	0.107	1.411	3390	**1997**	96545	0.674	0.064	1.33	69404
1974	57844	0.102	0.127	1.079	4793	**1998**	108383	0.684	0.0483	1.64	70187
1975	54606	0.117	0.134	0.965	5894	**1999**	120379	0.705	0.0473	1.63	72345
1976	55509	0.135	0.124	0.705	7302	**2000**	129424	0.753	0.0553	1.70	85213
1977	52820	0.164	0.12	0.533	9202	**2001**	132406	0.776	0.0512	1.53	97003
1978	52298	0.19	0.1203	0.461	10566	**2002**	132945	0.808	0.0496	1.30	102529
1979	51811	0.213	0.1331	0.425	11914	**2003**	137730	0.84	0.0412	1.04	108548
1980	55887	0.242	0.1596	0.412	15438	**2004**	144514	0.877	0.041	1.09	119093
1981	54008	0.277	0.1581	0.44	19570	**2005**	154523	0.921	0.0339	1.15	134133
1982	51838	0.313	0.1599	0.678	23353	**2006**	164249	0.96	0.0378	1.21	151692
1983	53667	0.348	0.1691	0.93	26051	**2007**	173334	0.985	0.0431	1.21	170375
1984	49797	0.378	0.1652	0.84	30966	**2008**	165792	1	0.0437	1.00	190020
1985	51552	0.407	0.1337	0.98	32668	**2009**	125714	0.975	0.0398	0.70	172218
1986	58441	0.428	0.1136	1.32	33417	**2010**	125410	0.979	0.0425	0.79	162731
1987	68774	0.447	0.1281	1.43	36663	**2011**	130674	0.941	0.0544	0.87	173228

Note: The variable B_t^G is expressed in millions of current euros and I_t^G in millions of euros of 2008

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