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BONUS-MALUS SCALES IN SEGMENTED TARIFFS: GILDE & SUNDT'S WORK REVISITED

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Abstract

In a recent paper, the authors have shown how to compute analytically the Bayesian relativities for a Bonus-Malus scale superimposed on a segmented tariff. However, the percentages associated with the levels of such a scale may exhibit an abrupt rise or drop from one level to another. For commercial reasons, it is sometimes preferable that each level inflicts the same relative penalty. In this paper we will show how to obtain such linear relativities for a given Bonus-Malus System. The model allows for *a priori* ratemaking. It is applied to a real-life portfolio.

Key words and phrases: Bonus-Malus System, Markov Chain, Stationary distribution, *A priori* ratemaking, Linear scale.

1 Introduction

One of the main tasks of the actuary is to design a tariff structure that will fairly distribute the burden of claims among policyholders. The classification variables introduced to divide risks into homogeneous cells are called *a priori* variables (as their values can be determined before the policyholder starts to drive). In motor third-party liability insurance, the a priori variables include age, gender and occupation of the policyholder, the type and use of the car, place of residence and sometimes even the number of cars in the household, marital status, smoking behaviour or colour of the vehicle.

At this stage however, there are many important factors that cannot be taken into account, such as annual mileage, speed of reflexes, aggressiveness in traffic or knowledge of the highway code. Consequently, tariff cells are still quite heterogeneous despite the use of many *a priori* variables. Even though features are usually impossible to measure and to incorporate in a price list, it is reasonable to believe that they are revealed by the number of claims reported by the policyholders. Hence the adjustment of the premium from the individual claims experience in order to restore fairness among policyholders.

Bonus-Malus systems (BMS, in short) penalizing insureds responsible for one or more accidents by way of premium surcharges (or *mali*), and rewarding claims-free policyholders by awarding them discounts (or *boni*), are in force in many countries. Within such systems, the premium charged to the policyholder is obtained by applying the relative premium corresponding to his current level in the BMS (this level depends on his claims history) to a base premium depending on his a priori characteristics incorporated into the price list. This *a posteriori* ratemaking is a very efficient way of classifying policyholders into cells according to their risk. For a thorough presentation of the techniques relating to BMS, see Lemaire (1995).

In a recent paper (Pitrebois, Denuit & Walhin (2003b)), the authors have shown how to compute the relativities relating to the levels of the BM scale, starting from the simulation procedure pioneered by Taylor (1997). However, these relativities may exhibit a rather irregular pattern, and this may be undesirable for commercial purposes. It may therefore be interesting to smoothen this scale in order to obtain relativities which are increasing according to the level (the higher the policyholder is in the scale, the higher the premium to be paid).

The paper is further organized as follows. Section 2 describes the *a priori* tariff that is used in the paper and introduces the model for *a posteriori* ratemaking. Section 3 shows the link between Markov models and Bonus-Malus systems. Section 4 shows how to adapt Gilde and Sundt's formulae in a setting containing covariates. Section 5 provides the numerical examples. Section 6 shows how to account for a special bonus rule in the scale.

A few words on the notation and terminology used troughout the paper. We shall denote a point of the real n-dimensional space $\mathbb{R}^n$ by a bold letter $\boldsymbol{x}$; the ith component of $\boldsymbol{x}$ is x_i, $i = 1, 2, \ldots, n$. All the vectors are tacitly assumed to be column vectors. The matrices are denoted by a capital letter in boldface, for instance $\boldsymbol{M}$; $\boldsymbol{M}^t$ denotes the transposition of $\boldsymbol{M}$. The vector of ones, that is $(1, 1, \ldots, 1)^t$, will be denoted by $\boldsymbol{e}$. The identity matrix (with entries 1 on the main diagonal and 0 elsewhere) is denoted by $\boldsymbol{I}$. Finally, we shall denote by $\mathbb{N}$ the set $\{0, 1, 2, \ldots\}$ of the non-negative integers and by $\mathbb{R}^+$ the half-positive real line $[0, +\infty)$.

Variable	Level	$\widehat{\beta}_j$
Intercept		-2.1975
Gender*Age	Female $18-30$ + Male $25-30$	0.2351
	Male $18-24$	0.6235
	Female >30 + Male >30	0
Type of district	Rural	-0.1809
	Urban	0
Split of payment	Yes	0.4677
	No	0
Use of vehicle	Professional use	0.2150
	Leisure and commuting	0

Table 2.1: *A priori* segmentation.

2 Modelling Claim Frequencies

2.1 Poisson regression for setting the *a priori* tariff

As it is not our aim in this paper to discuss in detail how the *a priori* tariff has been obtained, the interested reader is referred to Pitrebois et al. (2003a) for more details hereon. We shall work within a Poisson regression framework. The a priori characteristics of the policyholder are summarized in a vector of dummies, indicating each level of the categorical covariates listed in Table 2.1 (other explanatory variables have been excluded from the model since they were considered to be insignificant, and the interaction Gender*Age has been created). At this stage, the number of claims N_i, for a driver with characteristics $\boldsymbol{x}_i$ in a period of length d_i is assumed to be Poisson distributed with mean

$$\lambda_i = d_i \exp(\boldsymbol{\beta}^t \boldsymbol{x}_i) \quad , \quad i = 1, 2, \ldots, n.$$

A statistical analysis of our reference portfolio provides the significant covariates. The point estimates $\widehat{\boldsymbol{\beta}}$ of the regression coefficients $\boldsymbol{\beta}$ (obtained via maximum likelihood) are given in Table 2.1.

For example, the expected annual claims frequency for a male driver aged 35, living in the suburbs, paying an advance premium and making professional use of his car is estimated by

$$\exp(-2.1975 + 0 - 0.1809 + 0 + 0.2150) = 0.1149.$$

2.2 Residual heterogeneity justifying *a posteriori* ratemaking

Even with the *a priori* segmentation proposed above, some heterogeneity remains within the defined classes. This is due to the unobservable variables such as speed of reflexes, aggressiveness in traffic, drug abuse, ... This residual heterogeneity is taken into account with a random effect Θ_i. More precisely, we now assume that N_i obeys a mixture of Poisson distributions (where the random parameter expresses the residual heterogeneity), that is

$$\mathbb{P}[N_i = k|\Theta_i = \theta] = \exp(-\lambda_i\theta)\frac{(\lambda_i\theta)^k}{k!} \quad , \quad k = 0, 1, 2, \ldots$$

The Θ_i's are assumed to be independent and to have common gamma density function

$$u(\theta) = \frac{1}{\Gamma(a)} a^a \theta^{a-1} \exp(-a\theta) \quad , \quad \theta > 0.$$

The estimator $\widehat{\boldsymbol{\beta}}$ obtained under the Poisson assumption (i.e. by maximizing the Poisson log-likelihood) for the N_i's remains consistent in the Poisson mixture model. Therefore, we shall adhere to the estimations displayed in Table 2.1. Nevertheless, we now have to estimate a. For our portfolio with the *a priori* tariff described in the previous section, the estimation of the parameter a is $\hat{a} = 1.2401$.

For the sake of comparison, we shall also provide the estimate for a in case of there not being any *a priori* segmentation : $\hat{a} = 0.8888$. In this case, the average claims frequency is $\hat{\lambda} = 0.1474$.

3 Markov Models for BM Scales

3.1 Modelling of the trajectory of policyholders in the scale

Let us denote by $s + 1$ the number of levels in our BMS. The index $\ell = 0, \ldots, s$ will denote a given level within the scale.

In most of the commercial BM systems, the knowledge of the current level and of the number of claims during the current period suffice to determine the next level in the scale: these BM systems are Markovian, which is a good mathematical property ensuring easy calculations. Even when the BMS is not Markovian due to special transition rules, it is often possible to supplement the scale with fictitious levels in order for the memoryless property to be satisfied. The reader interested in setting relativities for the BMS with fictitious classes is referred to Centeno et al. (2002) and Pitrebois et al. (2003c) and also to Section 6 where an example is given.

3.2 Transient distributions

Let $p_{\ell_1\ell_2}(\vartheta)$ be the probability of moving from level ℓ_1 to level ℓ_2 for a policyholder with mean frequency ϑ. Further, $\boldsymbol{M}(\vartheta)$ is the one-step transition matrix, i.e. $\boldsymbol{M}(\vartheta) = \{p_{\ell_1\ell_2}(\vartheta)\}$, $\ell_1, \ell_2 = 0, 1, \ldots, s$. Taking the νth power of $\boldsymbol{M}(\vartheta)$ yields the ν-step transition matrix whose element $(\ell_1\ell_2)$, denoted as $p^{(\nu)}_{\ell_1\ell_2}(\vartheta)$, is the probability of moving from level ℓ_1 to level ℓ_2 in ν transitions.

3.3 Stationary distribution

All BM systems in practical use have a "best" level. A policy in this level remains in the same level after a claims-free period. In what follows, we shall restrict our attention to such non-periodic bonus rules. The transition matrix $\boldsymbol{M}(\vartheta)$ associated with such a BMS is regular, i.e. there is an integer $\xi_0 \geq 1$ so that all entries of $\{\boldsymbol{M}(\vartheta)\}^{\xi_0}$ are strictly positive. Consequently, the Markov chain describing the trajectory of a policyholder with expected claim frequency ϑ accross the levels is ergodic and thus possesses a stationary distribution $\boldsymbol{\pi}(\vartheta) = (\pi_0(\vartheta), \pi_1(\vartheta), \ldots, \pi_s(\vartheta))^t$; $\pi_\ell(\vartheta)$ is the stationary probability for a policyholder with mean frequency ϑ to be in level ℓ i.e.

$$\pi_{\ell_2}(\vartheta) = \lim_{\nu \to +\infty} p^{(\nu)}_{\ell_1 \ell_2}(\vartheta).$$

Note that $\boldsymbol{\pi}(\vartheta)$ does not depend on the starting level.

Let us now recall how to compute the $\pi_\ell(\vartheta)$'s. The vector $\boldsymbol{\pi}(\vartheta)$ is the solution of the system

$$\begin{cases} \boldsymbol{\pi}^t(\vartheta) = \boldsymbol{\pi}^t(\vartheta)\boldsymbol{M}(\vartheta), \\ \boldsymbol{\pi}^t(\vartheta)\boldsymbol{e} = 1. \end{cases}$$

Let $\boldsymbol{E}$ be the $(s+1)\times(s+1)$ matrix all of whose entries are 1, i.e. consisting of $s+1$ column vectors $\boldsymbol{e}$. Then, it can be shown that

$$\boldsymbol{\pi}^t(\vartheta) = \boldsymbol{e}^t\big(\boldsymbol{I} - \boldsymbol{M}(\vartheta) + \boldsymbol{E}\big)^{-1},$$

which provides a direct method to get $\boldsymbol{\pi}(\vartheta)$. For a derivation of the latter representation of $\boldsymbol{\pi}(\vartheta)$, see e.g. Rolski et al. (1999).

Let L be the BM level occupied by a randomly picked policyholder who has a stationary position in level ℓ. We have

$$\mathbb{P}[L = \ell] = \sum_k w_k \int_0^\infty \pi_\ell(\lambda_k\theta)u(\theta)d\theta, \quad \ell = 0, 1, \ldots, s$$

where w_k is the weight of the kth risk class whose annual expected claims frequency is λ_k. We have $w_k = \mathbb{P}[\Lambda = \lambda_k]$, where Λ represents the *a priori* claims frequency for a policyholder picked at random.

4 Setting the Relativities

4.1 Norberg's relativities in segmented tariffs

According to Norberg's criterion, the relativities r_ℓ are obtained by minimizing the squared difference between the true relative premium Θ and the relative premium r_L applicable to the policyholder when a stationary position has been reached. The aim is thus to minimize $\mathbb{E}\left[(\Theta - r_L)^2\right]$. The solution is given by :

$$r_\ell = \frac{\sum\limits_k w_k \int\limits_0^\infty \theta\pi_\ell(\lambda_k\theta)u(\theta)d\theta}{\sum\limits_k w_k \int\limits_0^\infty \pi_\ell(\lambda_k\theta)u(\theta)d\theta}.$$

This formula extends Norberg (1976). Further details about the derivation of this formula are given in Pitrebois et al. (2003b).

We also immediately verify that

$$\mathbb{E}[r_L] = \sum_{j=0}^{s} r_\ell \mathbb{P}[L = \ell] = 1$$

which ensures that the BMS is financially balanced at steady state.

4.2 Linearization of the relativities

The obtained relativities may have a chaotic behaviour and one may want to smoothen them. A linear scale of the form $r_\ell^{lin} = \alpha + \beta\ell, \ell = 0, 1, ..., s$ could then be desirable. The Norberg's criterion becomes a constrained minimization :

$$\min \mathbb{E}\Big[(\Theta - r_L^{lin})^2\Big] = \min \mathbb{E}\Big[(\Theta - \alpha - \beta L)^2\Big].$$

From the theory of linear regression we know that the solution of this optimization problem is given by :

$$\beta = \frac{\mathbb{C}ov[L, \Theta]}{\mathbb{V}ar[L]} \text{ and } \alpha = \mathbb{E}[\Theta] - \frac{\mathbb{C}ov[L, \Theta]}{\mathbb{V}ar[L]}\mathbb{E}[L].$$

The linear relative premium scale is thus of the form

$$r_\ell^{lin} = 1 + \frac{\mathbb{C}ov[L, \Theta]}{\mathbb{V}ar[L]}(\ell - \mathbb{E}[L]),$$

where

$$\begin{aligned}
\mathbb{C}ov[L, \Theta] &= \mathbb{E}\Big[\mathbb{C}ov[L, \Theta|\Theta]\Big] + \mathbb{C}ov\Big[\mathbb{E}[L|\Theta], \mathbb{E}[\Theta|\Theta]\Big] \\
&= \mathbb{C}ov\Big[\mathbb{E}[L|\Theta], \Theta\Big] \\
&= \sum_k w_k \sum_{\ell=0}^{s} \ell \mathbb{E}[\Theta \pi_\ell(\lambda_k \Theta)] - \mathbb{E}[L]\mathbb{E}[\Theta] \\
&= \sum_k w_k \sum_{\ell=0}^{s} \ell \int_\theta \theta \pi_\ell(\lambda_k \theta) u(\theta) d\theta - \mathbb{E}[L], \\
\mathbb{P}[L = \ell] &= \sum_k w_k \int_\theta \pi_\ell(\lambda_k \theta) u(\theta) d\theta, \\
\mathbb{E}[L] &= \sum_{\ell=0}^{s} \ell \mathbb{P}[L = \ell], \\
\mathbb{V}ar[L] &= \sum_{\ell=0}^{s} (\ell - \mathbb{E}L)^2 \mathbb{P}[L = \ell].
\end{aligned}$$

Starting level	Level occupied if				
	0	1	2	3	≥4
	claims are reported				
8	7	8	8	8	8
7	6	8	8	8	8
6	5	8	8	8	8
5	4	7	8	8	8
4	3	6	8	8	8
3	2	5	7	8	8
2	1	4	6	8	8
1	0	3	5	7	8
0	0	2	4	6	8

Table 5.1: Transition rules for the BMS -1/+2.

Level ℓ	$\Pr[L=\ell]$	$\mathbb{E}[\Lambda \mid L=\ell]$
8	2.9%	19.3%
7	2.4%	18.3%
6	2.3%	17.4%
5	2.3%	16.8%
4	3.2%	16.0%
3	3.4%	15.7%
2	8.1%	14.9%
1	6.8%	14.8%
0	68.7%	14.0%

Table 5.2: Distribution of L and average *a priori* claims frequency in level ℓ, $\mathbb{E}[\Lambda|L=\ell]$with *a priori* ratemaking for the BMS -1/+2.

5 Numerical Results

Let us consider the soft experience rating systems defined in Taylor (1997). There are 9 bonus-malus levels, level 6 being the starting point. A higher level indicates a higher premium. If no claims have been reported by the policyholder he will move one level down. If a number of claims, $n_t > 0$, has been reported during year t, the policyholder will move $2n_t$ levels up. The transition rules are described in Table 5.1.

Table 5.2 displays the proportion (at stationary regime) of policyholders in each level and the average *a priori* expected claims frequency in each level. These two quantities are the same for both the non-constrained scale and the linear scale.

Table 5.3 allows to compare the relativities of the two scales. As we can see, the values are very close to each other. The biggest difference may be observed at level 1, the transition from level 0 to level 1 is higher for the non-constrained scale than it is for the linear scale. The constant step between two levels in the linear scale is equal to 27%.

Relativities		
Level ℓ	General scale	Linear scale
8	285.6%	286.2%
7	255.8%	259.2%
6	226.4%	232.2%
5	205.5%	205.1%
4	174.4%	178.1%
3	160.6%	151.1%
2	122.3%	124.1%
1	115.9%	97.1%
0	68.5%	70.1%

Table 5.3: Relativities r_ℓ and r_ℓ^{lin} with *a priori* ratemaking for the BMS -1/+2.

Relativities		
Level j	General scale	Linear scale
8	353.7%	342.7%
7	306.7%	307.2%
6	262.3%	271.7%
5	231.1%	236.2%
4	189.2%	200.7%
3	170.2%	165.2%
2	122.8%	129.7%
1	114.6%	94.3%
0	58.0%	58.8%

Table 5.4: Relativities r_ℓ and r_ℓ^{lin} without *a priori* ratemaking for the BMS -1/+2.

The values of the expected errors $Q_1 = \mathbb{E}\left[(\Theta - r_L)^2\right]$ and $Q_2 = \mathbb{E}\left[(\Theta - r_L^{lin})^2\right]$ are respectively given by

$$\begin{aligned} Q_1 &= 0.48375 \\ Q_2 &= 0.48684 \end{aligned}$$

so that in fact the additional linear restriction does not really deteriorate the fit.

Table 5.4 displays the relativities without *a priori* segmentation. As can be observed, the scale without *a priori* segmentation is more elastic, which is logical since it has to take into account the full heterogeneity. The mean square errors are now given by

$$\begin{aligned} Q_1 &= 0.52582 \\ Q_2 &= 0.53005 \end{aligned}$$

so that again the additional linear restriction does not really deteriorate the fit. Obviously, Q_1 and Q_2 are higher than before (when a priori risk classification was in force). This was expected as Θ is now more variable.

6 Belgian Scale

The Belgian BMS consists of 23 levels. The transition rules are the following :

1. each year a one-level bonus is given.
2. each claim is penalized by five levels.
3. the maximal bonus is level 0.
4. the maximal malus is level 22.
5. a policyholder with four consecutive claims-free years may not exceed level 14 (special bonus rule).

Because of the special bonus rule the Belgian scale is not Markovian. Fortunately, it is possible to introduce fictitious levels in order to fulfill the memoryless property. Specifically, Lemaire (1995) proposed to split the levels 16 to 21 into sublevels, depending on the number of consecutive years without accident. This authorizes to take account of the special bonus rule. A level $\ell.i$ is to be understood as level ℓ and i consecutive years without accidents. Let n_ℓ be the number of sublevels to be associated to bonus level ℓ. The transition rules are completely defined in Table 6.1 and the different values for n_ℓ are given in Table 6.2. We have taken some liberty with the notations by using the value 0 for the subscript i and by not using a subscript when $n_\ell = 1$.

We shall now demonstrate how to set up the relativities for the Belgian scale. In addition to the constraint of building a linear scale, we add a new constraint : the artificial states must have the same relativity. The minimization problem then becomes :

$$\min \mathbb{E}\Big[(\Theta - r_L^{lin})^2\Big] = \min \mathbb{E}\Big[(\Theta - \alpha - \beta L)^2\Big]$$

so that

$$r_\ell = r_{\ell.1} = r_{\ell.2} = r_{\ell.n_\ell} \quad , \quad \ell = 0, 1, \ldots, s.$$

The solution to this problem is the same as in Section 4. We merely have to replace π_ℓ by $\sum_{i=1}^{n_\ell} \pi_{\ell.i}$. The results are displayed in Tables 6.3 and 6.4. The constant step between two levels of the linear scale is equal to 9.2%.

Now let us compare the values of the mean square errors:

$$\begin{aligned} Q_1 &= 0.36876 \\ Q_2 &= 0.37376 \end{aligned}$$

The impact of the additional linear constraints on the relativities is moderate. The major advantage of the linear scale is that it gives a system that is commercially more acceptable. In fact, the general scale is not increasing between levels 13 to 15, in contrast to the linear scale. In this example the difference between the non-constrained scale and the linear one is bigger than in the previous example. The linear scale is less elastic than the general one.

Starting level	Level occupied if 0 claims are reported	1	2	3	4	≥5
22	21.1	22	22	22	22	22
21.0	20.1	22	22	22	22	22
21.1	20.2	22	22	22	22	22
20.0	19.1	22	22	22	22	22
20.1	19.2	22	22	22	22	22
20.2	19.3	22	22	22	22	22
19.0	18.1	22	22	22	22	22
19.1	18.2	22	22	22	22	22
19.2	18.3	22	22	22	22	22
19.3	14	22	22	22	22	22
18.0	17	22	22	22	22	22
18.1	17.2	22	22	22	22	22
18.2	17.3	22	22	22	22	22
18.3	14	22	22	22	22	22
17	16	21.0	22	22	22	22
17.2	16.3	21.0	22	22	22	22
17.3	14	21.0	22	22	22	22
16	15	20.0	22	22	22	22
16.3	14	20.0	22	22	22	22
15	14	19.0	22	22	22	22
14	13	18.0	22	22	22	22
13	12	17	22	22	22	22
12	11	16	21.0	22	22	22
11	10	15	20.0	22	22	22
10	9	14	19.0	22	22	22
9	8	13	18.0	22	22	22
8	7	12	17	22	22	22
7	6	11	16	21.0	22	22
6	5	10	15	20.0	22	22
5	4	9	14	19.0	22	22
4	3	8	13	18.0	22	22
3	2	7	12	17	22	22
2	1	6	11	16	21.0	22
1	0	5	10	15	20.0	22
0	0	4	9	14	19.0	22

Table 6.1: Transition rules of the Belgian BMS

ℓ	n_ℓ
22	1
21	2
20	3
19	4
18	4
17	3
16	2
15	1
14	1
13	1
12	1
11	1
10	1
9	1
8	1
7	1
6	1
5	1
4	1
3	1
2	1
1	1
0	1

Table 6.2: Sublevels of the Belgian BMS

Level ℓ	$\Pr[L = \ell]$	$\mathbb{E}[\Lambda \mid L = \ell]$
22	4.0%	18.7%
21	2.8%	17.9%
20	2.1%	17.4%
19	1.7%	17.0%
18	0.9%	16.4%
17	1.0%	16.3%
16	1.0%	16.1%
15	1.0%	15.9%
14	2.2%	16.2%
13	2.0%	15.9%
12	1.8%	15.7%
11	1.8%	15.5%
10	1.7%	15.3%
9	1.8%	15.1%
8	2.0%	14.9%
7	2.1%	14.8%
6	2.1%	14.7%
5	2.1%	14.6%
4	4.7%	14.2%
3	4.2%	14.2%
2	3.8%	14.1%
1	3.4%	14.1%
0	49.7%	13.7%

Table 6.3: Distribution of L and average *a priori* claims frequency in level ℓ, $\mathbb{E}[\Lambda|L = \ell]$ with *a priori* ratemaking for the Belgian BMS

Relativities		
Level ℓ	General scale	Linear scale
22	267.5%	254.7%
21	246.1%	245.5%
20	228.8%	236.3%
19	214.4%	227.1%
18	193.6%	218.0%
17	188.8%	208.8%
16	182.1%	199.6%
15	175.0%	190.4%
14	186.7%	181.2%
13	175.2%	172.0%
12	164.7%	162.9%
11	155.4%	153.7%
10	147.2%	144.5%
9	137.0%	135.3%
8	127.2%	126.1%
7	120.0%	116.9%
6	114.3%	107.8%
5	109.6%	98.6%
4	88.2%	89.4%
3	85.3%	80.2%
2	82.7%	71.0%
1	80.1%	61.8%
0	49.9%	52.7%

Table 6.4: Relativities r_ℓ and r_ℓ^{lin} with *a priori* ratemaking for the Belgian BMS

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