DYNAMIC PROGRAMMING IN INVENTORY MANAGEMENT: A REVIEW

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Dynamic Programming in Inventory Management: A Review

Inventory Management (IM) research has significantly advanced, with increasingly complex and realistic inventory problems being investigated. Dynamic Programming (DP) plays an increasingly vital role to find viable solutions to these problems, particularly in balancing the trade-off between good-quality solutions and computational efficiency. While previous literature reviews have primarily focused on either IM or DP independently, or have studied very specific IM-DP settings, we are the first to provide a comprehensive overview of the interaction between IM and DP. By analyzing 278 relevant articles, we explore how DP methodologies are applied across various inventory problems, identifying key trends, challenges, and research gaps. We introduce a unique typology to classify these articles based on the type and structure of the inventory problem and DP methodology employed. Additionally, our review provides insights into practical applications of DP in IM, and examines the current and potential future integration of sustainability and technology topics in this area.

Keywords: Dynamic Programming, Inventory Management, Review, Typology

1. Introduction

Inventory Management (IM) plays a crucial role within firms and supply chains, and highly impacts firms' competitiveness and financial performance, as evidenced in sectors such as retailing (Gallino and Moreno 2014, Aryapadi et al. 2020) and healthcare systems (Ahmadi et al. 2022), among others. In addition, IM can significantly impact environmental and social objectives, as well as supply chain resilience, which are becoming increasingly important. Examples of cases of IM that deal with these challenges include: waste mitigation for perishable products (Chernonog 2020), remanufacturing (Zhou and Yu 2011), humanitarian logistics (Kawase and Iryo 2023), and shortages and disruptions caused by events such as the COVID-19 outbreak (Alicke et al. 2021) or geopolitical conflicts (Srai et al. 2023). These trends explain the wide attention for IM by researchers and practitioners, and call for new approaches to deal with the increasing complexity in IM.

IM is typically defined as a supply chain function that involves the systematic coordination and control of procurement, storage, and distribution of goods. Its objective is to optimize overall operational efficiency by ensuring that optimal order quantities are shipped in a timely matter. Various ordering decisions and policies (Clark and Scarf 1960, Ahmadi et al. 2022) have been developed for IM problems. Different supply-chain decisions related to, for instance, production and distribution (Coelho et al. 2014), the consideration of uncertainty (Giri 2011), and the inclusion of market factors such as pricing and auctions (Federgruen and Heching 1999) have been integrated within IM problems. To model such complex real-life problems, and to optimize IM decisions, the majority of the IM research in operations literature has relied on optimization models. As a result, a multitude of optimization techniques have been proposed, including linear and integer programming, heuristics, metaheuristics, and Dynamic Programming (DP) (Barbosa-Póvoa et al. 2018).

DP formulations are particularly well-suited for (multi-period) IM problems, owing to their inherent structural properties of problem partition and recursion. The development of DP theory by Bellman (1954) was notably motivated by the study of scenarios including IM. Since then, the use of DP in IM problems has increased over time. In addition, the curse of dimensionality led to the exploration of various other techniques such as state-space reduction, the Clark-Scarf method and novel DP-derived approaches, such as approximate and Bayesian DP (see e.g., Clark and Scarf 1960, Woeginger 2000, Powell 2016).

New inventory problems have spurred the evolution of DP methods for solving the increasingly complex IM problems. For instance, Adida and Perakis (2010) highlight the importance of DP in aggregating time-varying data in inventory contexts, providing closed-form solutions that enable efficient decision adjustments amid uncertainties. As such, DP is essential in the search for more efficient solutions for the real-world intricacies of IM. Therefore, the purpose of this article is to provide a comprehensive and systematic literature review on DP approaches applied in IM. This article highlights the evolution of DP to deal with increasingly complex and realistic IM problems. We examine both problem- and methodology-based aspects to showcase DP's strengths, limitations, and future research directions in the IM context.

Numerous literature reviews exist on IM or on DP separately, however, to the best of our knowledge, this review is the first to explore and focus on the interaction of DP and IM. Our review contributes in several ways. First, we classify different inventory problems in the IM-DP literature. As IM problems are becoming increasingly complex (integrating several supply chain levels such as production and distribution), we aim to show how DP is applied to tackle the wide range of IM problems. This will support researchers in understanding how DP can help address the complexity of specific IM problems. Second, by identifying how DP-derived approaches in IM have evolved (e.g., ranging from conventional DP to approximate, Bayesian, and hybrid DP), we can identify trends in DP applications for IM problems and uncover how distinct forms of DP are developed to solve IM problems. Third, by analyzing problem assumptions (e.g., the planning horizon, single/multi-period, product and sourcing strategies), objectives, policies, constraints, types of DP optimization models (i.e., deterministic, stochastic, and robust) and solution methods, we provide a clear insight into how DP can be used to solve IM problems. Finally, our analysis investigates how DP can be deployed to real-life IM applications, and highlights the integration of contemporary topics, such as sustainability considerations.

In particular, we aim to answer the following research questions:

RQ1 Which **IM problems** (and IM problem characteristics) have been addressed by DP in the literature, and how have these IM problems evolved over time. How have these problems been formulated in terms of objective function, policies, and constraints?

RQ2 Which **DP models and solution methods** have been applied to different IM problems, and what methodological progress has been made to deal with increasingly complex IM problems?

RQ3 What are the **practical applications** of DP in IM problems, showcased by the literature?

RQ4 How have recent **sustainability challenges in IM** been addressed by DP?

RQ5 What **gaps** and limitations can be highlighted as topics for future research?

This literature review can stimulate further use of DP in IM problems, lead to additional efforts to improve the computational performance of DP, and prompt applications of DP in new and more complex IM problems. In what follows, we first position our work relative to already existing reviews.

2. Position among literature reviews

We conduct an initial exploration for related literature-review articles on IM (search 1) and DP (search 2). To do so, we perform a broad search involving title, abstract, and keywords in the *Scopus* database by entering keywords **inventor*** (for search 1) and **dynamic program*** (for search 2), respectively. The search covers "review" articles on *Scopus* from 1950 up to October 2023.

Search 1 results in 448 articles that have explored IM. A manual selection process indicates that 215 articles do not qualify as literature reviews and 163 articles are not essentially related to IM. As our interest is to show the level of DP coverage in existing IM reviews, we focus on reviews that cover optimization. Therefore, we further reduce the 70 remaining IM literature reviews to 40 relevant reviews.

Search 2 initially provides a sample of 278 articles that is narrowed down to 13. This elimination stems from the fact that 94 of the review articles are not reviews and 171, although encompassing cases in operations research (OR), do not relate to operations management (OM) or management sciences, covering, for instance, topics in water management and energy optimization.

Table A1 in Appendix summarizes the key details of the reviews resulting from both searches. From Search 1, we find that articles with a low occurrence of DP generally do not center their analysis on this methodology. Those with a moderate focus on DP (i.e., DP is mentioned between

5 and 15 times) explore mainly specific inventory contexts, such as multiple suppliers (Svoboda et al. 2021), transportation modes (Engebrethsen and Dauzère-Pérès 2019), perishable products (Chaudhary et al. 2018), guaranteed-service models (Eruguz et al. 2016), lateral transshipment (Paterson et al. 2011), production systems (Goyal and Gunasekaran 1990), and deep reinforcement learning (Boute et al. 2022). In contrast, only three articles that intensively mention DP (i.e., more than 15 times) focus primarily on broader areas such as rolling horizons in OR (Chand et al. 2002), control theory in supply chain models (Sarimveis et al. 2008), and asymptotic analysis in inventory systems (Goldberg et al. 2021). While these articles mainly focus on the drawbacks imposed by DP in IM, we study the suitability and evolution of DP for distinct IM problems.

From Search 2, we find that articles that mention inventory in DP either focus on investigating DP alongside several types of optimal control methodologies (Powell 2019, Jackson et al. 2020), or on specific applications of DP formulations such as machine scheduling and revenue management (de Souza et al. 2022, Koulamas and Kyparisis 2023, Strauss et al. 2018, Rempel and Cai 2021), or on specific DP approaches such as approximate DP (Powell 2016). Hence, this indicates the absence of a literature review that focuses on DP applications in IM, a gap we aim to fill.

3. Review methodology

In this section, we first discuss how articles were selected, and then provide a typology of the collected articles.

3.1. Article selection

To select the articles relevant for this IM-DP review, we first conducted a title search using the term **inventor*** and simultaneously performed a broader search including titles, abstracts, and keywords with **dynamic program***. Our search was conducted in this manner because our primary focus is IM, while DP should at least be mentioned in the abstract. This resulted in 705 articles ranging from 1977 to October 2023. We further refined our selection to include only articles from top-tier journals (i.e., 4*, 4, and 3 journals), as ranked by the Chartered Association of Business Schools (ABS) list in their Academic Journal Guide 2024[1] . This list was selected for its established quality standards, academic rigor, and internationally recognized ranking benchmark (Krammer et al. 2024). It has also been widely employed in article selection for reputable literature reviews in management disciplines (see e.g., Mellahi and Harris 2016, Rajeev et al. 2017, Krammer et al. 2024). Furthermore, we focused on 3-4* journals due to their high citation metrics, strong reputation compared to other journals within the field, and the rigorous review process, which ensures the quality and relevance of published research (Chartered Association of Business Schools

[1]available at https://charteredabs.org/insights/news/academic-journal-guide-2024-available-now

2024). Focusing exclusively on articles within OR, management science, and OM, we identified 318 articles from 22 journals (see Table A2 in Appendix for an overview of the selected journals and their abbreviations). A manual screening was then conducted to include articles primarily focusing on DP-derived techniques for inventory, excluding those using DP merely as a benchmark or lacking DP-based formulations. This led to a final selection of 278 relevant articles (see Figure 1).

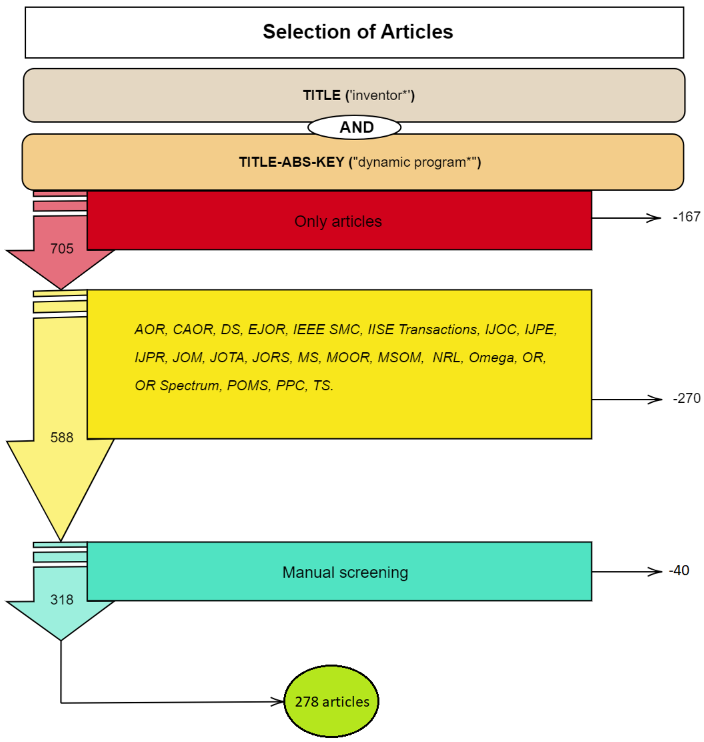


Figure 1 Framework for selection of articles.

3.2. Typology of articles

To address our research questions, we developed a typology framework, presented in this section, to systematically categorize the collected studies. This classification provides a structured approach to discuss the state of the art in the IM-DP literature, revealing common patterns and identifying research gaps. Table 1 presents our typology, outlining the groups, main categories and subcategories, associated entry names, and the number of articles per (sub)category.

To establish our unique typology, we conducted a detailed review of each collected article, extracting key information that led to the following main categories: (a) the type and structure of the IM problem (see **IM problem structure classification** in Section 3.2.1), (b) the applied DP approach (see **DP-based classification** in Section 3.2.2), (c) the model, including decisions, constraints, and policies considered (see **Decisions and policies** in Section 3.2.3), and (d) the exact and heuristic solution methods (see **Solution Methods** in Section 3.2.4).

While some of our identified (sub)categories align with existing classifications in the IM literature (e.g., those introduced by Silver 1981, Svoboda et al. 2021), we also developed new (sub)categories specifically tailored to the IM-DP field to ensure the completeness of our typology. By integrating both IM and DP categorizations, developing novel categories, and incorporating a high level of detail in the (sub)categories, our framework provides new insights into the IM-DP literature. This structured approach also facilitates a more comprehensive study of the interaction between these two research streams. For instance, it allows to study which IM problems are solved by which DP approaches and which solutions methodologies are most suited.

Group	Category	Sub-category	Elements	Entry	**Model-based DP**		
					Deterministic	Stochastic	Robust
Problem structure	*Inventory problem*		Pure Inventory	I	10	133	6
			Inventory-Production	IP	10	28	2
			Inventory-Scheduling	IS	5	5	0
			Inventory-Distribution	ID	2	36	0
			Inventory-Routing	IR	4	7	0
			Inventory-Pricing	IPr	2	26	0
			Integration	In	2	6	0
	Number of suppliers		Single supplier	SS	34	192	7
			Multiple supplier	MS	1	46	0
	Product analysis	Product diversity	Single item	SI	24	197	7
			Multiple item	MI	12	45	0
		Customer perspective	Backlogs	BL	11	151	4
			Lost sales	LS	4	110	2
	Time	Period information	Single-period	SP	4	39	0
			Multi-period	MP	31	229	7
		Planning horizon	Finite horizon	FH	34	193	7
			Infinite horizon	IH	1	88	0
		Replenishment fashion	Periodic-review	PR	25	192	3
			Continuous-review	CR	9	29	4
		Lead time		LT	5	71	1
Decisions and Policies	*Objectives*		Cost-related	C	28	153	4
			Profit-related	P	6	67	3
			Service-related	S	0	3	0
			Asset-related	A	1	8	0
			Other	Z	0	11	0
	Policies		Optimal	O	26	218	7
			Near/sub-optimal	H	11	112	1
	Constraints		Inventory capacity	IC	13	44	1
			Production capacity	PC	10	23	0
			Supply capacity	SC	0	20	0
			Order size	OS	0	9	0
			Product lifetime	L	8	21	0
			Service level	SL	0	12	0
			Budget	B	0	10	0
			Other	Z_1	9	22	0
Solution methods			Exact	E	30	433	11
			Heuristic	H	13	177	1
Structure-based DP			Dynamic programming	DP	30	214	7
			Approximate dynamic programming	ADP	2	26	0
			Bayesian dynamic programming	BDP	0	16	0
			Hybrid dynamic programming	HDP	3	8	0

Table 1: Typology and notation for DP-based formulations in inventory problems.

In Tables A4-A6 in Appendix, we provide the full list of selected articles, and classify each article according to the presented typology. Note that the numbers in certain tables and figures (e.g., Tables 1 - 3 and Figures 4 - 5), represent how many times certain (sub)categories appear in the selected articles. Thus, their sum is not necessarily equal to the number of collected articles, as not all categories or elements appear in all articles or some articles simultaneously cover multiple subcategories or elements.

In what follows, we first describe the main categories used to classify the different collected articles.

3.2.1. IM Problem Structure

IM-problem classification Articles in the IM literature tend to be associated with several supply chain management decisions. When summarizing the information derived from the collected articles, seven classes have surfaced, namely *pure inventory (I)*, *inventory-production (IP)*, *inventory-scheduling (IS)*, *inventory-distribution (ID)*, *inventory-routing (IR)*, *inventory-pricing (IPr)*, and *integration (In)*. Figure 2 illustrates the flowchart that was used to assign articles to these classes.

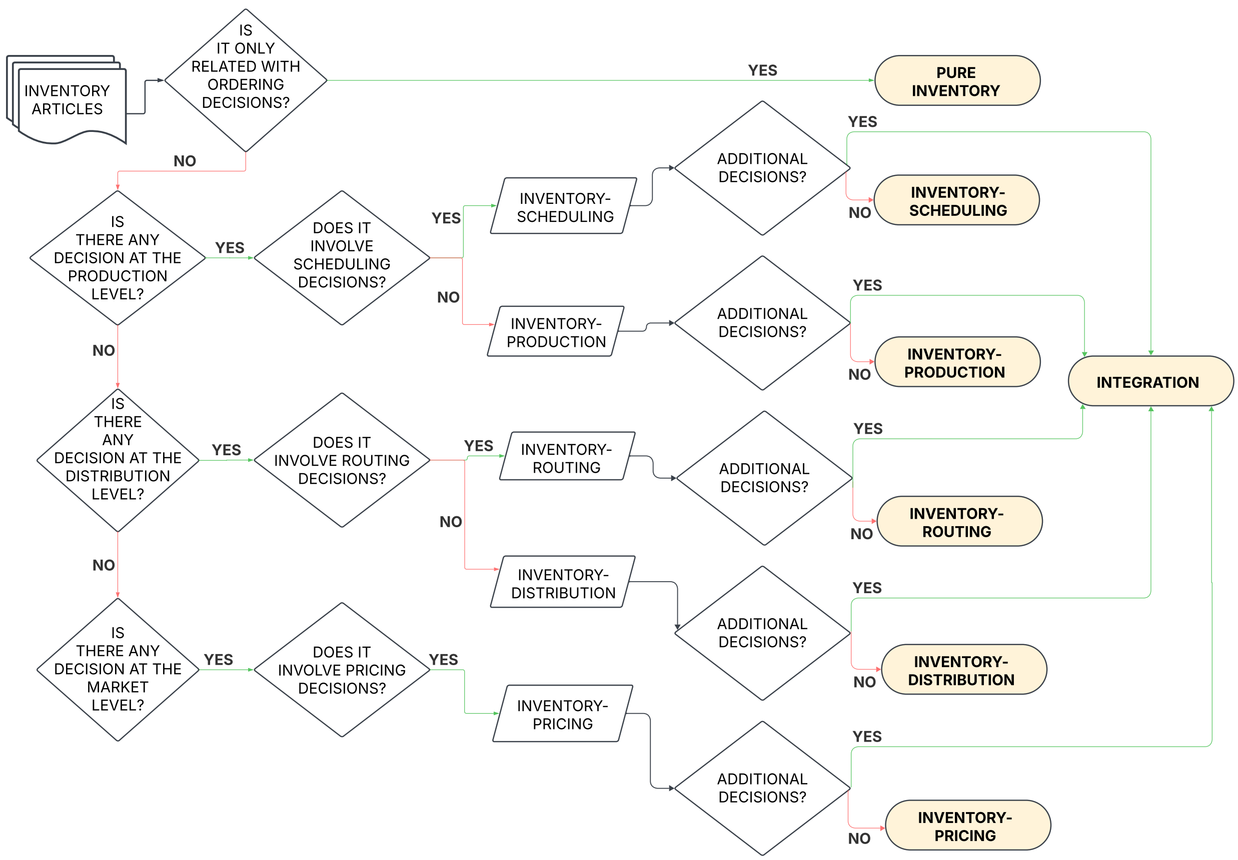


Figure 2 Flow chart for classification of inventory problems.

In the context of decision making, the *pure inventory* classification is defined by actions linked to procurement (ordering) and replenishment policies. The *inventory-production* classification aligns optimal decisions in terms of quantities that factories must produce alongside procurement or stocking analysis, whereas the *inventory-scheduling* classification adjoins scheduling elements such as optimal sequencing, delivery times, and maintenance. *Inventory-distribution* coordinates optimal ordering or stocking decisions with the distribution process that might be associated with inventory allocation, balancing, positioning, and transportation. *Inventory-routing* seeks optimal routing decisions in conjunction with optimal distribution of inventory quantities along customers in the supply chain. At the market level, certain studies combine inventory decisions with optimal pricing strategies by sellers, classified within the *inventory-pricing* classification. Studies that entail three or more types of decisions are placed under the *integration* classification (e.g., inventory-production-distribution).

IM Problem characteristics classification Furthermore, we encompass three extra categories: the number of suppliers, product analysis, and time considerations. Several of these align with those proposed by Svoboda et al. (2021) and Silver (1981), which is not surprising, given that IM problems are typically characterized by these factors. Details of these categories and their subcategories, as well as their name entries, are provided in Table 1.

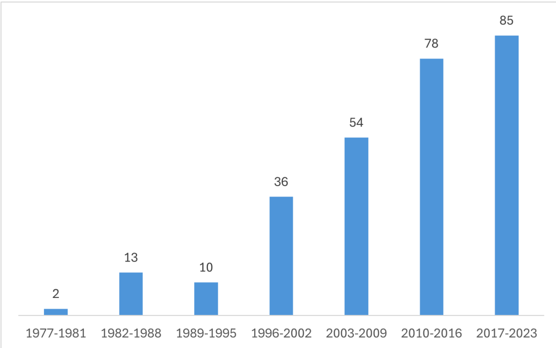
3.2.2. DP-based classification We categorize the collected papers based on their DP formulation along two axes: *Structure-based DP classification*, which examines the structure of the Bellman equation and the resulting DP formulations (see last rows in Table 1) and *Model-based DP classification*, which considers the nature of uncertainties in the DP model (see last columns Table 1).

The structure-based DP classification covers conventional DP, approximate DP (ADP), Bayesian DP (BDP), and hybrid DP (HDP). Conventional DP decomposes the main problem into sub-problems, optimally solving and storing their solutions for reuse. ADP modifies the objective function's structure using approximation methods, yielding efficient but potentially sub-optimal solutions. BDP incorporates Bayesian probability theory into the objective function. HDP combines DP with other OR methodologies (e.g., branch-and-bound). The formulations and specific properties of these structure-based DP classes are discussed in detail in the WebAppendix.

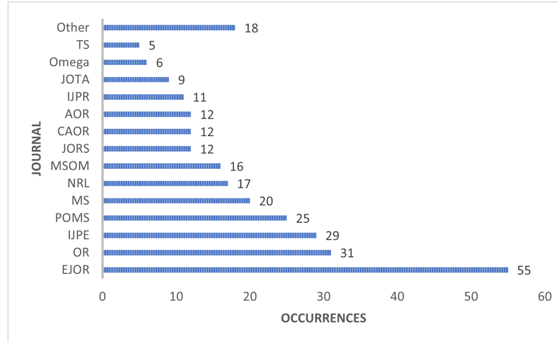
The model-based DP classification categorizes DP according to three modeling-based approaches (in line with previous literature reviews, e.g., Chand et al. (2002) and Silver (1981)): deterministic, stochastic, and robust. In contrast to deterministic DP models, stochastic models operate in the presence of random variables or parameters. Robust DP accounts for both deterministic and stochastic behavior, typically when assumptions on probability distributions are unknown and extended to uncertainty sets. Note that for IM problems with uncertainty, especially in stochastic models, we also categorize the type of uncertainties (see Table 3).

3.2.3. Decisions and policies

We categorize the collected articles along three categories related to decisions and policies: objectives, policies, and constraints. We distinguish four types of *objective functions*: Cost-related (C), Profit-related (P), Service-related (S), and Asset-related (A). In particular, the fourth type of objective refers to financial functions such as investments and net present value (NPV). Additional



(a) Publications distributed over time.



(b) Publications distribution over top-ranked journals in ABS.

Figure 3 Distribution of collected IM-DP articles over time and journals.

Note: The category Other corresponds to journals with less than 5 occurrences (i.e., IJOC, MOOR, PPC, OR Spectrum, JOM, DS, and IEEE SMC). The list of abbreviations can be found in Table A2 in Appendix.

objectives that rarely surface or that do not belong to the main categories are classified as Other (Z). Within the category *Policies*, Optimal (O) and Suboptimal (H) policies are distinguished. Sub-optimal policies are generally employed to improve computational efficiency or handle more complex real-life problems such as a multi-item, multi-stage inventory, production problem with limited production capacity, postponement, random and seasonally fluctuating demand (Aviv and Federgruen 2001). For the different *Constraints* identified (and their entry names), we refer to Table 1.

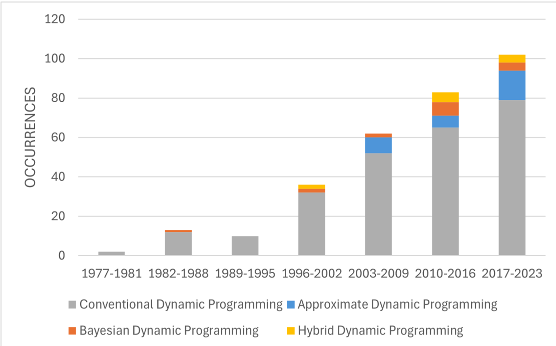
3.2.4. Solution methods The analysis of the collected articles based on the solution methods (exact and heuristic) aims to understand their application in solving specific DP formulations in IM, and to explore recent advancements that offer more practical and efficient tools for researchers in solving various IM problems. The classifications of exact and heuristic methods for different IM problems, for deterministic, stochastic, and robust models are provided in Tables A7 - A9. The list of solution methods with their abbreviations is provided in Table A3 in Appendix.

4. Research demographics

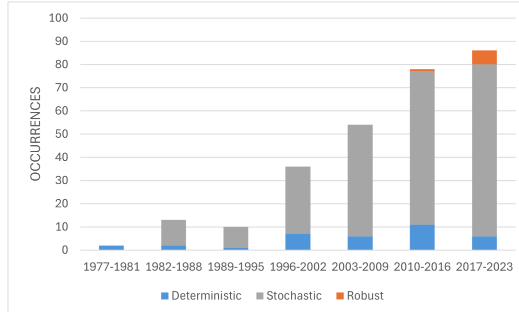
This section looks at the 278 selected articles over time and across journals. It also discusses industrial applications.

4.1. Distribution over time and across journals

Figure 3(a) shows an increasing use of DP for inventory problems, especially in more recent years. Figure 3(b) shows that the collected articles are distributed across 22 high-ranked journals in the ABS list, with 11 of these journals (each containing more than 10 articles) accounting for over 86 % (240 articles) of the selected publications. The journal distribution is, in itself, relevant since we can comprehend the different domains in which IM is explored due to the tendency of some journals to investigate specific topics (e.g., transportation, finance, accounting, engineering, marketing).



(a) Structure-based DP formulations distributed per time period.



(b) Model-based DP formulations in inventory over time.

Figure 4 Distribution over time of DP in IM-DP literature.

4.2. Analysis of DP-based classifications

Figure 4(a) demonstrates that, despite conventional DP still being the predominant formulation, the other approaches have gained momentum over the last two decades. Figure 4(b) shows the evolution of model-based DP formulations, in which the prevalence and growth of stochastic DP can be observed. These figures confirm that the use of DP has clearly evolved over time and that more advanced DP formulations and approaches have been developed to deal with more complex and realistic IM problems.

4.3. Analysis of IM-based classification

From Table 2, the prevalence of pure inventory problems is evident, comprising the majority of the selected articles. Additionally, the literature also extensively studies inventory-production, inventory-distribution, and inventory-pricing problems, while other types of IM problems receive limited attention. As shown in Figure 5, during the past two decades, there has been a notable trend in the literature toward integrating various aspects of supply chains, including production, routing, and distribution, among others. This evolution recognizes the heightened complexity introduced by these interconnected elements to inventory problems, leading authors to explore more realistic scenarios through DP-derived methodologies.

Type of inventory problem	DP	Approximate DP	Bayesian DP	Hybrid DP	TOTAL
Pure Inventory	130	8	14	7	149
Inventory-Production	38	3	1	0	40
Inventory-Scheduling	10	3	0	0	10
Inventory-Distribution	37	7	1	0	37
Inventory-Routing	8	7	0	1	11
Inventory-Pricing	26	1	1	1	28
Integration	7	1	0	1	8
TOTAL	256	30	17	10	

Table 2: Assignment of DP-based approaches to inventory problems under different types of modeling.

Table 2 shows that conventional DP is used for all classes of inventory problems, especially pure inventory, inventory-production, inventory-scheduling, and inventory-distribution, where it has been successful in obtaining optimal policies. Alternative methodologies are more prominent in inventory-routing and inventory-distribution problems, likely due to the higher complexity of such problems, for which computational efficiency is often prioritized over solution optimality.

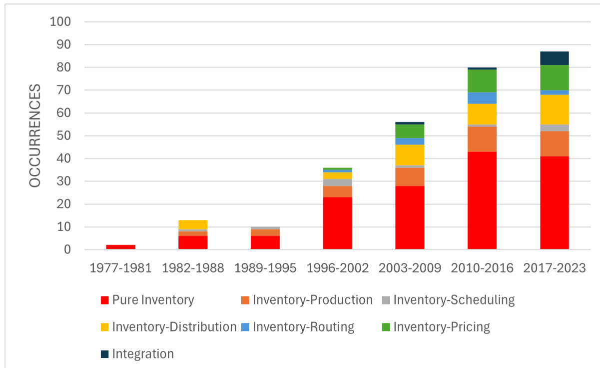


Figure 5 Distribution over time of inventory problems in IM-DP literature.

4.4. Industrial applications

Among the collected articles, 50 are connected to, or inspired by, specific industrial settings, with 30 of them analyzing real-world data. These articles span approximately 13 different industries (see Figure 6). The vehicle industry (including automobile and rentals), commerce (including retail activities), and chemical industry (composed by several sub-categories such as gas, petrochemical, and pharmaceutical), are the most common. These segments are followed by electronics (11.54%) and general service (9.62%) sectors, which mainly consists of airline booking. The remaining industries include healthcare (e.g., blood banks), clothing, manufacturing (e.g., appliances and spare parts), food (e.g., seafood and pork), humanitarian (e.g., resource allocation through disasters), consumer goods, military (e.g., resource allocation among military settlements), and finance. In Tables A4-A6 in Appendix we indicate which articles include industrial applications and which ones use real-world data.

The majority of these articles formulate the problem using either conventional DP (32), or first providing formulations with conventional DP and then with ADP (13), while HDP and BDP are used in 3 and 2 articles, respectively. This may imply that for several companies, small and medium-sized instances are sufficient, and that they also prioritize the benefit of working with optimal solutions, albeit their computational time might be longer.

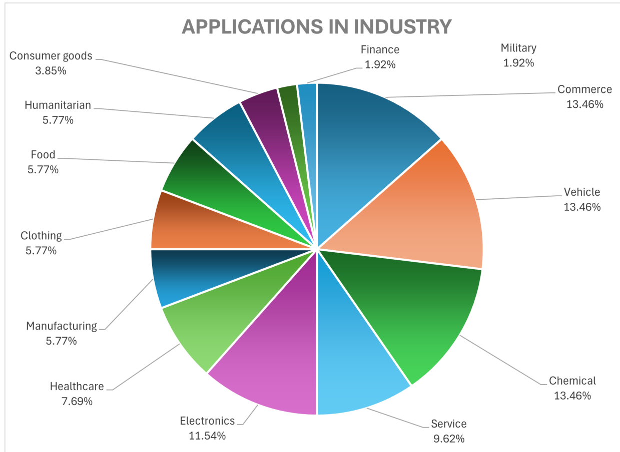


Figure 6 DP-based formulations applied to industry in IM.

5. Deterministic models

Table A4 in Appendix provides the list of articles with deterministic IM-DP models, categorized based on our typology. These models mainly cover pure inventory, inventory-production, and inventory-scheduling problems. Most articles consider a finite time horizon, reducing the complexity of the DP model. Majority of articles employ conventional DP to examine single-item, single-supplier, multi-period environments under periodic review, with the objective to minimize costs.

5.1. Problem structure

The deterministic approach to **pure inventory** problems focuses on classic replenishment strategies. Most of these articles model a single supplier, single-item, multi-period, and finite horizon scenario. Some articles, however, introduce complexity by considering various demand behaviors, such as constant, linear time-dependent, non-stationary or a multivariate function of price and time. Additionally, incorporating backorders (see e.g., Tai et al. 2016) increases complexity, as the computation of states grows with the number of backlogs allowed. A few articles use DP to determine both optimal order quantities and replenishment periods under periodic-review systems (see e.g., Rosenblatt 1985). Joint replenishment problems in multi-item settings aim at bundling products based on common order cycles and common order quantities (Chakravarty 1981, Rosenblatt 1985, Jing and Mu 2019). Although these models do not consider shortage costs, other elements can be integrated in order to make the formulations more realistic, as is the case in Jing and Mu (2019), where the authors formulate a model based on a two-product scenario that considers stock deterioration rates, age-dependent holding costs, and one-way substitution.

In **inventory-production** problems, the studies predominantly tackle Lot-Sizing Problems (LSP), categorized into Capacitated (CLSP) and Economic (ELSP) LSPs, with lost sales (Jing and

Chao 2021, Sandbothe and Thompson 1993), backorders (Grubbström and Huynh 2006, Hsu and Lowe 2001, Ou 2012), and full or partial outsourcing (Chu and Chu 2008). The articles focus mainly on systems involving a single supplier, single-item, multi-period, and finite horizon. Periodic- or continuous-review replenishment is considered in the articles, with the majority represented by the former. Incorporating lead times increases the complexity of the model, therefore, requiring the design of an efficient state space, as in the work of Grubbström and Huynh (2006), which considers both the lead time for the production of an item and the maximum lead time of all items.

For **inventory-scheduling** problems encompass single- and multi-machine environments. For instance, Yeung et al. (2011) introduce a two-machine flow shop scheduling model to represent supply chain scheduling for a short-life product, while Boysen et al. (2008) examine the assembly line sequencing problem of an Original Equipment Manufacturer (OEM) that supplies components from a 3rd Party Logistics provider (3PL). The 3PL consolidates these parts and stocks them in a consignment warehouse. As IS problems typically include multi-item and lead-time complexities, higher-dimensional formulations are likely to arise. However, by using reducing transformations (e.g., Laplace transforms) in the models and relying on a one-period lead time assumption (Segerstedt 1996), conventional DP can still be used without major drawbacks.

In **integration** problems, Chen and Chen (2004) consider single-item inventory-scheduling-pricing problem with backlogs allowed and aimed to develop decision support systems for enterprise resource planning, integrating shortage management with scheduling and pricing functions. Li et al. (2017a) consider multi-item inventory-scheduling-distribution problem with pre-specified delivery time windows and splittable and non-splittable order delivery. Both papers assume a periodic review policy.

5.2. Decisions and Policies

In environments associated with pure **inventory**, **inventory-production**, **inventory-scheduling**,**inventory-distribution**, and **inventory-routing**, **objectives** that are *cost-related* functions, focusing on total, average, and discounted total cost, prevail. These models incorporate various cost assumptions such as fixed costs per period (see e.g., Suzanne et al. 2020), linear (see e.g., Jing and Mu 2019, Chu and Chu 2008), non-linear cost functions such as total costs dependent on exponentially deteriorating items (Hariga 1997) and concave functions (see e.g., Gutiérrez et al. 2003). The concave cost assumption is rather convenient when modeling LSPs, as they can be translated into concave network flow problems, from which optimality characteristics on the stationary point can be derived (Hwang and Van Den Heuvel 2012). Fewer studies in **inventory-pricing**, **inventory-routing** and **integration** focus on *profit-related* functions, such as total profit (Chen and Chen 2004, Chen and Wei 2012) and total/unit net profit (Toriello et al. 2010, Yu et al. 2013).

These objective functions demonstrate smooth concavity, even when price is a decision variable, facilitating the analysis of solutions for Bellman equation. An *asset-related* objective is represented by a two-dimensional net present value function dependent on cumulative demand and cumulative production, as considered in Grubbström and Huynh (2006) for an **inventory-production system**.

In the assessment of **policies**, most articles involving **inventory**, **inventory-production**, **inventory-scheduling**, **inventory-pricing** and **integration** incorporate *optimal structures*, leveraging assumptions concerning monotonicity and convexity. Notably, particular inventory policies emerge, predominantly encompassing lot-sizing (e.g., Jing and Mu 2019). Some policies also include a cycle-based approach for joint replenishment problem (Rosenblatt 1985). Additional types of policies involve those obtained recursively without a closed-form solution (e.g., Chen and Wei 2012) or based on thresholds (e.g., Gutiérrez et al. 2003). Conversely, *suboptimal* policies are used mainly for **inventory-distribution** and **inventory-routing** problems, particularly when DP solutions are limited by time periods or perform poorly computationally. These suboptimal policies often involve EOQ-based formulations, zero inventory ordering, load-truck, and cycle-related policies (see e.g., Chakravarty 1981). Additionally, approximation policies are also employed for these problems.

For **inventory** and **inventory-production** environments, **constraints** are mainly incorporated by *inventory capacity*, *production capacity* and *life-time* constraints. For the latter, the life-time is mainly dependent on perishability, which is either a fixed or continuous time-dependent function. *Production capacity* is mostly related to the manufacturer's production bottleneck in terms of number of units to produce. *Other* constraints include delivery due date and cargo carriers' limited sizes for **inventory-scheduling** problems, vehicle, vessels, and port capacity, especially for **inventory-distribution** and **inventory-routing** problems.

5.3. Models and solution methods

In this subsection, we discuss the models and solution methods employed for specific IM problems. The classification is provided in Table A7 and the list of abbreviations for the solution methods is provided in Table A3. Conventional DP with classical exact approaches, such as Forward Induction (FI) and Backward Induction (BI) dominate other types of approaches. In terms of heuristics, the main solution method is based on custom heuristics.

For **inventory** and **inventory-production** problems, conventional DP with exact solution methods is used, in particular *FI* and *BI*, especially for single-item settings, provided that the number of periods is limited (see e.g., Zhu et al. 2011). Exceptions are observed in Jing and Mu (2019), where an $O(T^5)$ FI algorithm is applied to a perishable two-item inventory-production problem with one-way substitution, and in Grubbström and Huynh (2006), where BI and an efficiently designed state space are implemented for a capacitated multi-item inventory-production problem with non-zero lead times. Other types of exact solution methods are used for a dynamic LSP such as *Wagner-Whitin* (for more information refer to Wagner and Whitin 1958) with discounted total costs (Chand and Tang 1985), and an $O(T^4)$ *Florian-Klein*-based algorithm with concave production and holding costs under stationary production capacity (Akbalik et al. 2015).The development of *heuristics* is rather limited, with applications to only pure inventory problems. For instance, Hariga (1997) develops optimal and near-optimal solution methods for problems considering exponentially deteriorating products. Chakravarty (1981) uses heuristics based on FI of the DP formulation for cycle-based items grouping, while Buchanan (1980) combines a myopic method with BI for an inventory problem with no-shortage and linearly increasing demand.

Even though **inventory-scheduling** problems are complex due to their multi-item nature, conventional DP has been mostly applied for a non-extensive number of periods, as the solutions mainly conveyed theoretical results. These solutions are obtained using *FI* (Segerstedt 1996, Boysen et al. 2008), *BI* (Yeung et al. 2011), and *Wagner-Within algorithms* (Potamianos and Orman 1996, Potamianos et al. 1997). Boysen et al. (2008) also employ a *Bounded DP* procedure, which uses a heuristic to obtain a global upper bound, which reduces the number of states in the DP approach originally developed to find the optimal solution.

For **inventory-distribution**, Palak et al. (2018) develop efficient heuristic induction DP algorithms under zero-inventory ordering and less-than-truckload properties. A heuristic DP-based algorithm with BI is based on the solution of a simpler problem (Bertazzi and Speranza 1999).

Inventory-routing problems are modeled using HDP and ADP. For HDP, a DP-based FI and branch-price-and-cut algorithms are combined to tighten the formulation (Engineer et al. 2012). ADP in Papageorgiou et al. (2015) integrates *lookahead* approximations to facilitate decomposition and solvability, especially for large instances. Toriello et al. (2010) address a large-period problem by solving single-period subproblems using ADP techniques, in particular Approximate Value Iteration (AVI) with a forward pass and data fitting.

Inventory-pricing and **integration** have been mainly modeled through conventional DP with FI solution algorithms. For HDP, due to the analytical intractability of the complex multiple-decision-maker problem considered in Yu et al. (2013), a *hybrid* algorithm combining DP, a Genetic Algorithm (GA), and analytical methods is proposed. The approach outperforms the benchmark GA algorithm in computational efficiency.

6. Stochastic models

6.1. Uncertainty

In stochastic inventory models, uncertainty is vital to accurately represent real-world challenges. In Table 3 we categorize uncertainties into five groups (i.e., demand, supply, production, price, and lead time uncertainties) and show the number of stochastic IM-DP models for each category. The table also shows interactions among two types of uncertainties, mostly when demand uncertainty is considered with another type. Although not evidenced in Table 3, seven articles enclose triple interactions simultaneously mainly related to demand, production and supply uncertainties.

Our analysis confirms **demand-related uncertainty** as the dominant type, in line with the historical role of demand behavior in inventory theory (see e.g., Zhao and Lian 2011, Bensoussan and Guo 2015, Ozyoruk et al. 2022, Gavirneni et al. 1998). Three main categories emerge. The first encompasses *specific probability distributions*, including the Poisson (e.g., Mayorga and Ahn 2011, Van Wijk et al. 2019), Normal and Gamma distributions, among others. The martingale distribution, another example in this category, characterizes demand behavior based on forecast updates and is defined by a steady-state conditional expectation from a given point onwards (see e.g., Iida and Zipkin 2006). The second category involves *general distributions* for non-specific cases (see e.g., Li et al. 2017b). Although some demand information is available, its probability distribution may be *unknown*, or the demand random variables may be *unobservable* - as in the third category - due to real-world complexities such as inventory inaccuracy and censored demand (see e.g., Kamath and Pakkala 2002, Kunnumkal and Topaloglu 2008b).

Within **supply-related uncertainties**, we distinguish mainly between supply capacity uncertainty and supply disruptions. *Supply-capacity* uncertainty arises when supply capacity is either not coordinated by specific probability distributions (see e.g., Liu et al. 2015) or follows a predefined distribution. For instance, Wang et al. (2010) assume that supply capacity follows a Markovian distribution, and DP is defined as a solution method for the Markov chain formulation. As for *supply disruptions*, uncertainties relate to supply availability (Yeo and Yuan 2011) and random yield, which is defined as the inconsistency in delivering the total procured amount, due to disruptions faced by the supplier (see e.g., Gao 2015).

Production-related uncertainties such as *production rates*, *machine availabilities*, *product returns* are concentrated mainly in inventory, inventory-production, and inventory-scheduling problems. Some studies focus on specific probability distributions. For example, Kiesmüller and Scherer (2003) consider normally-distributed product returns, while Mayorga and Ahn (2011) consider that processing times of the in-house capacity are exponentially distributed and that an additional capacity option can be utilized according to a Poisson process. Uncertainty in *failure and repair*

	Demand-related	Supplier-related	Production-related	Price/cost-related	Lead time
Demand-related	166	20	24	7	16
Supplier-related	20	3	1	0	2
Production-related	24	1	4	1	3
Price/cost-related	7	0	1	0	1
Lead time	16	2	3	1	0
Total	233	26	33	9	22

Table 3: Interactions among two types of uncertainty in articles regarding stochastic DP.

time is generally included in inventory-scheduling and inventory-production problems, and usually modeled using an exponential distribution (see e.g., Lawrence and Schaefer 1984, Smith and Schaefer 1985). **Lead-time uncertainty** is mostly represented by the lead-time of the replenishment, which is introduced mainly in inventory problems. Some articles include known distributions such as uniform (Gijsbrechts et al. 2022) and exponential (e.g., Song et al. 2014, Olsson 2009). Some other studies resort to more general formulations, such as Srinivasan and Duff (1989), who consider a free-form distribution for procurement lead time. **Price/cost-related uncertainty** is generally modeled as a Markovian process (Kouvelis et al. 2018) or a Brownian motion (Liu et al. 2018), but also using general distributions (Kunnumkal and Topaloglu 2008b).

6.2. Problem structure

Although the stochastic **pure inventory** problems traditionally focus on ordering and replenishment aspects (Reyes-Aldasoro et al. 1999, Bensoussan et al. 2010, Chen et al. 2014, Hosseinifard et al. 2022), recent extensions have also explored assortment planning, financial considerations, marketing strategies, and sales operations. Assortment planning, incorporating product substitution to address potential shortages, has been demonstrated by Honhon et al. (2010). Financial elements have been addressed by considering short-term loans to finance inventory levels and budgetary analysis (Gong et al. 2014, Chen and Rossi 2021), managing risks through financial hedging strategies (Kouvelis et al. 2018), incorporating corporate income taxes into inventory control models (Pang and Xiao 2022), and considering inventory management of deposits at the Federal Reserve Banks (Nair and Anderson 2008). Marketing and promotion strategies have been implemented by linking free-shipping offers (Zhou et al. 2009), and handling random deal offerings from suppliers (Gurnani 1996). Studies on sales operations encompass various environments such as e-commerce (Lee et al. 2006), a non-transactional website to feature company's products online with offline ordering (Huang and Van Mieghem 2014), excess inventory acquisition by secondary markets (Angelus 2011) and volatile markets (Kouvelis et al. 2023). Pure inventory problems have also been studied in contexts involving perishability, deteriorating items, end-of-life products (see e.g., Chen and Lin 2002, Li et al. 2016, Pourakbar and Dekker 2012), substitution (see e.g., Li et al.

2017b, Luo et al. 2023), unreliable suppliers (see e.g., DeCroix 2013, Özekici and Parlar 1999, Yeo and Yuan 2011), among others.

The majority of pure inventory problems are still addressed considering characteristics related to single-item/single-supplier models (see e.g., Parlar and Rempala 1992, Song and Lau 2004, Wang and Mersereau 2017). Nevertheless, some articles also study more realistic problems considering factors such as invisible and censored demand (e.g., Bensoussan et al. 2016) and age-dependent demand (see e.g., Chen and Sapra 2021). Multiple sourcing is also observed, primarily characterized by dual sourcing, where trade-offs arise between fixed and variable costs (Fox et al. 2006), as well as shorter and longer lead times (DeCroix 2013, Gijsbrechts et al. 2022). Other examples of dual sourcing include regular and emergency supply modes, with emergency orders placed through an electronic marketplace (Lee et al. 2006). A very limited number of articles consider multi-item and multiple-supplier problems (DeCroix 2013, Puranam et al. 2017, Bi et al. 2020). In addition to these characteristics, backorders or lost sales are often considered, despite adding complexity. Although regular analyses are embodied in the majority of articles, some distinct features of these characteristics are also evaluated via DP, such as unobserved backorders (see e.g., Bensoussan et al. 2010), unobserved lost sales (see e.g., Bensoussan and Guo 2015) and partial backorders (see e.g., Lin et al. 2020). Regarding time characteristics, most articles integrate multi-period, finite-horizon formulations under a periodic-review fashion. Yet, some studies also show that convergence can be reached for the infinite-horizon version of some problems (e.g., Parker and Kapuscinski 2004). Regarding lead time considerations, the majority of articles covers short lead times, some of them also extend their formulations to accommodate longer and stochastic lead times (see e.g., Gijsbrechts et al. 2022).

The range of sub-problems within **inventory-production** problems is diverse, including categories such as make-to-stock production environments (see e.g., Mayorga and Ahn 2011), lot-sizing problems (see e.g., Minoux 2018, Wang et al. 2014), maintenance and repair centers (see e.g., Wang and Zhu 2021, Smith and Schaefer 1985), queuing systems (see e.g., Zhao and Lian 2011, Veatch and Wein 1994), serial production and inventory systems with n stages (see e.g., Klosterhalfen et al. 2013), and labour-intensive manufacturing environments with capacity targets defined by the number of workers (Pac et al. 2009). In general, the articles are largely represented by single-item, single supplier, multi-period, and backlog penalties. Although this seems to provide less burdensome models, some articles include additional complexity, by considering, for instance, the value of full and partial information between the manufacturer and the retailers (Wang and Zhang 2021). Regarding single-period problems, it is observed that the multi-stage characteristics of DP are mainly represented by the number of spare parts in the system (see e.g., Smith and Schaefer 1985), rather than time-related stages.

A few articles in our research introduce **inventory-scheduling** problems, focusing on job scheduling (Liu 1989), scheduling, allocation, inventory replenishment problem of drone and electric vehicle battery swap stations (Asadi and Nurre Pinkley 2022), inventory control and maintenance of reparable/spare parts (Khademi and Eksioglu 2018, Schaefer 1983), and scheduling of workers to tasks across various shifts (Valeva et al. 2021). Most articles assume a single-supplier and finite-horizon setting, incorporating either lost sales or no backlog/lost sales assumptions, along with periodic or continuous review policies. One exception is the study of Khademi and Eksioglu (2018), which examines a multi-item lost sales problem under periodic review with lead times and an infinite horizon.

Inventory-pricing decisions addressed via DP seem to have drawn the attention of several researchers over the last two decades. Although a pattern cannot be immediately identified regarding the type of problems within inventory-pricing, it is noticeable that the majority of articles still focus on demand uncertainty, which is price-sensitive, under additive form (see e.g., Yang et al. 2014a) with some other cases including a multiplicative form (Feng et al. 2014). In addition to that, several distinct angles have been studied by researchers in this type of problems such as considerations on substitutable products (see e.g., Fang et al. 2021, Dong et al. 2009), perishable items (see e.g., Zhou et al. 2023, Fang et al. 2021), quantity discounts (see e.g., Lu et al. 2014), returns and expediting (see e.g., Zhu 2012), online reviews (Yang et al. 2014a) partial ordering through auctions (Van Ryzin and Vulcano 2004), models with reference price (Güler et al. 2015), demand cancellations (You 2003), customer negotiation strategies (see e.g., Kuo et al. 2011), and sponsored search advertising with dynamic pricing (Tunuguntla et al. 2019).

Inventory-pricing problems are mainly represented by single-supplier and single-item environments considering multi-period scenarios with finite horizon and periodic review. For multi-item environments, two-product scenarios are mainly addressed due to solution feasibility for the models (Zhu and Thonemann 2009, Dong et al. 2009, Fang et al. 2021, Xu et al. 2020). For instance, Xu et al. (2020) consider a joint pricing and inventory management problem of a company that sells a product and a product-centric service. Lead times are typically not considered, except as possible extensions to proposed models. In such cases, solutions may involve heuristics, as discussed in Zhu and Thonemann (2009), or the models may be simplified - for example, by assuming deterministic demand, as presented in Lu et al. (2014).

Inventory-distribution problems mainly analyze inventory and distribution interactions via inventory transshipment (see e.g., Archibald 2007), inventory expediting and allocation (Shen et al. 2022, Topaloglu and Kunnumkal 2006), and inventory balancing (see e.g., Zipkin 1984). Some studies are applied or motivated by real-life examples in humanitarian logistics, such as providing funding for treatment (Natarajan and Swaminathan 2017), supplying refugee operations (McCoy and Brandeau 2011), or relocating mobile inventory in disaster relief scenarios (Zhang et al. 2022). Other examples include on-demand bike rental services (Benjaafar et al. 2022), a brewery (Chen et al. 2022c), a catalog sales industry (Bitran and Mondschein 1996), or a modal shift transport from rail to road (Dong and Transchel 2020). Most problems consider single-product, single/multiple supplier and transshipment as an additional sourcing strategy to reduce the expected stockouts among different stocking points. The majority of problems include either backordering, lost sales or both, along with periodic-review replenishment in multi-period formulations (see e.g., Bitran and Mondschein 1996, Benjaafar et al. 2022). Lead-time considerations receive moderate attention, with Olsson (2009) being the only study addressing uncertain lead times modeled with an exponential distribution.

Inventory-routing (IR) problems examine inventory and routing decisions for specific contexts, such as bike-sharing (Brinkmann et al. 2019), vendor managed inventory for air products (Kleywegt et al. 2004), the gas industry (Berman and Larson 2001), and military operations (McKenna et al. 2020). Routes may be closed, open, or probabilistic, in some cases incorporating routed pooling (Berman and Larson 2001). McKenna et al. (2020) account for uncertainties in deliveries due to combat risks, mechanical failures, and weather conditions. Most problems consider single-item and single-supplier scenarios. Exceptions include the multi-item setting for a general IR problem, as in Adelman (2004), and for a problem with dynamic relocation of bikes across stations (Brinkmann et al. 2019). Other common assumptions are periodic review and multiple periods.

Regarding **integration**, articles explore various combinations of decision processes. For inventory-distribution-pricing, Li and Mizuno (2022) analyze pricing and inventory policies within dual-channel distribution systems, considering different power structures such as manufacturer/retailer Stackelberg and vertical Nash. In inventory-routing-distribution, Turan et al. (2017) examine transshipment policies aimed at balancing inventories across multiple locations while optimizing the associated routes. Inventory-production-distribution scenarios are explored by Bhatnagar and Lin (2019) and Lin et al. (2023), focusing on make-to-stock (MTS) manufacturing systems combined with transshipment to balance inventories throughout the supply chain, and Malladi et al. (2020), who focus on multi-location settings with transportable modular production capacity.

Finally, García-Alvarado et al. (2015) investigate replenishment remanufacturing and manufacturing quantities and carbon management decisions within a product recovery system that operates under a cap-and-trade system, considering limited production and inventory capacity. Multiple supplier, single item and backorders studies have been prevalent, whereas elements such as multiple items and lead times have been scarcely analyzed.

6.3. Decisions and policies

For pure **inventory** problems, similar to the other categories of IM problems (except for inventory-pricing), DP mainly formulates *cost-related* objective functions. These correspond mainly to discounted and average costs for finite- and infinite-horizon problems (Li 2015). The majority of articles also prove convexity for such functions. Similarly, *profit-related* functions are also modeled in terms of total expected discounted profit (Li et al. 2015) and average profit (Chen and Moinzadeh 2018) functions. Some other functions involve expected net present value (Nair and Anderson 2008), expected inventory investment (Srinivasan and Duff 1989), expected utility (see e.g., Liu et al. 2015) and fuzzy-related functions (Liu 1999).

The majority of policies are *optimal*. They are mainly represented by classical forms, such as order-up-to (Chiang 2001, Sharma et al. 2020), base-stock (Huang and Van Mieghem 2014) and (s, S) (Özekici and Parlar 1999) policies. Some articles also include their variants, such as the equity-level-dependent base-stock policy for an expected capital maximization problem of dynamic inventory control under a financial constraint (Gong et al. 2014) , or signal-based supply forecast dependent (s, S) policy for a forecasting, contracting, SC risk management problem (Gao 2015). The optimal policies in dual sourcing can be supplier dependent, as in Fox et al. (2006) who derive the optimal policy that can take one of the three forms: base-stock policy for the high variable cost supplier, (s,S) policy for the low variable cost supplier; and a hybrid policy for buying from both suppliers. Nevertheless, due to the versatility of inventory scenarios, some policies can also be adapted to specific environments, as is the case for the $X - Y$ band policy, derived in Chan and Song (2003), Yang et al. (2014b) for periodic review inventory problems under capacity constraints, or the (SL, SU) policy for a periodic-review inventory system with fixed order sizes and a standing order (Chiang 2007) and convex ordering costs (Henig et al. 1997). *Suboptimal* policies have been fairly explored, assuming mainly the form of myopic ones (see e.g., Li et al. 2016), such as a myopic base-stock (Xu 2011) and lookahead policies (Kouvelis et al. 2023). Specific problem-related suboptimal policies are developed, such as the disposal saturation policy for a multi-echelon inventory system with secondary market sales (Angelus 2011), the cash-constrained modified (s, S) policy Chen and Rossi (2021), and the (s, t, S) policy for a multi-period periodic review inventory model with free shipping offers Zhou et al. (2009).

Regarding constraints, *inventory* capacity is typically limited by a predetermined value and is assumed to be fixed over different periods (Li et al. 2015). *Production* capacity is often considered as fixed with a few articles assuming variable or stochastic capacities. *Supply* capacity has been adopted under different forms; however, most are still related to fixed capacities across different periods, with some exceptions involving random supply capacity (see e.g., Liu et al. 2015). *Product lifetime* constraints are often linked to perishable items (see e.g., Puranam et al. 2017, for a blood bank inventory replenishment problem with multiple supply sources), shelf life extensions (Li et al. 2017b), age-differentiated demand (Chen et al. 2021), and stochastic deterioration rates (Chen and Lin 2002). Obsolescence effects are also included and, in some cases related to EOQ-modified policies (see e.g., Song and Lau 2004). *Order size* constraints typically specify a minimum order quantity under contractual agreements. For some articles, such constraint is also subject to flexible contracts, in which the initial order size can be modified over different periods (see e.g., Sharma et al. 2020). *Service-level* constraints focus on customer service levels such as the probability of no-stock-out (e.g., Özen et al. 2012) and business-to-business (B2B) agreements, including penalties for unmet service levels (Hosseinifard et al. 2022). *Budget constraints* relate to capital-constrained models, directly influencing and limiting the ordering levels for inventory under capital/cash constraints (see e.g., Chen and Rossi 2021) and external financing through loans (Bi et al. 2020, Gong et al. 2014). *Other* constraints include, for instance, no-ordering allowed in certain periods (Sethi and Cheng 1997), and availability of transportation (Arifoğlu and Özekici 2011).

For **inventory-production** (IP) problems, *cost-related* objective functions are the most frequently modeled type. IP problems are predominantly characterized by *optimal* policies, represented by various forms such as base-stock (see e.g., Özer and Wei 2004), X-Y band (see e.g., Feng et al. 2011), switching curves (see e.g., Flapper et al. 2012), order-up-to (see e.g., Pac et al. 2009, Wang et al. 2014), (Q, R)-based (see e.g., Wang and Zhang 2021), produce-down/up-to policy (Liu et al. 2020a), and maintenance-related policies (Jensen et al. 1991, Smith and Schaefer 1985). Approximate, myopic/threshold policies are the most used *suboptimal* policies. The majority of constrained IP problems are characterized by *inventory* and *production* capacities. *Budget* constraints are associated with IP problems involving system failure and repair (Smith and Schaefer 1985, Lawrence and Schaefer 1984). A *service-level* constraint, a modified fill rate measured by the maximum average backorders, is considered in a serial IP problem in Klosterhalfen et al. (2013). Only few articles incorporate multiple constraints. For instance, Song et al. (2014) address a multiple-supplier, multi-item, infinite-horizon IP problem under supply, production, and demand uncertainties, considering both supply and production capacity constraints. Yang et al. (2022)

examine a blood IP environment that integrates various constraints such as lifetime, production, supply and disposal ones, along with age-based inventories, substitution, and random demand and supply.

Cost- and profit-related functions dominate the **inventory-scheduling** (IS) environment. For one specific case, Schaefer (1983) also considers functions associated with the job-completion rate. IS problems are characterized by both *optimal* and *suboptimal* policies, such as an order-up-to-based type optimal policy (Asadi and Nurre Pinkley 2022), lookahead and myopic policies (Valeva et al. 2021). Constraints are mainly represented by capacity-related factors (*inventory, supply and production*), while constraints on holding costs and required job-completion rates are incorporated in Schaefer (1983).

Regarding **inventory-pricing** problems (IPr), research has mainly contributed to the literature of *profit-related* functions, more specifically distributed among discounted expected profit (see e.g., Chen et al. 2006), expected long-run average profit (see e.g., Chen et al. 2022a) and expected total profit (see e.g., Qin et al. 2022). Exceptions are identified in Chew et al. (2009) and Dong et al. (2009) where revenue functions are considered. In terms of policies, most policies are *optimal*, with base-stock list-price (BSLP) usually being obtained for different IPr problems, with general demand (Bensoussan et al. 2019, Shen et al. 2018), dynamic pricing and demand learning (Liu et al. 2020b), a generalized additive model for demand distributions (Feng et al. 2014), a data-driven setting (Qin et al. 2022), and returns and expediting (Zhu 2012). Furthermore, additional types of policies have also been obtained such as (s, S, P) (see e.g., Chen et al. 2006, 2022a), base-stock reserve-price auction (Van Ryzin and Vulcano 2004), price reference BSLP (Wang et al. 2021a), and an online rating-dependent BSLP (Yang and Zhang 2022). Few studies derive *suboptimal* policies and are problem-specific, such as Chew et al. (2009) who consider perishable products with lifetime larger than two periods, and Yang and Zhang (2022) who propose a dynamic look-ahead heuristic policy for the joint pricing and IM problem considering online reviews. Few studies incorporate constraints, with *inventory, supply, and lifetime* constraints being the most commonly considered (Güler et al. 2015, Huh et al. 2011, Yang et al. 2014a, Zhu 2012).

For **inventory-distribution** problems, most models incorporate *cost-related* objective functions, while fewer studies include *profit-, revenue-* or service level-related functions. Regarding *service-related* functions, Pishchulov and Richter (2009) investigate the maximization of a customer fill rate function, while McCoy and Brandeau (2011) aim to minimize the disaster response penalty subject to budget constraints. Natarajan and Swaminathan (2017) aim to minimize the number of disease-adjusted life periods lost in a humanitarian health setting with funding constraints. For ID problems, *suboptimal* policies have been more recurrently designed, especially for those involving multiple locations and multiple suppliers or transshipment. For instance, Archibald et al. (2009) develop an index heuristic based on a pairwise decomposition for transshipment decisions, while Meissner and Senicheva (2018) develop reactive, proactive and lookahead transhipment policies. Additionally, commonly studied policies in the literature, such as hold-back and complete pooling (Van Wijk et al. 2019), as well as myopic policies, are often implemented. In particular, for a problem with lateral transshipments, Glazebrook et al. (2015) propose a myopic policy which approximates the future costs of a decision assuming no-transshipment. Other policies include base-stock (McCoy and Brandeau 2011), (s, S)-based (Federgruen and Zipkin 1984a), (R, Q) based (Paterson et al. 2012), and order-up-to (Nambiar et al. 2021) policies. Over half of the ID studies consider various constraints.

Inventory-routing problems incorporate predominantly *cost-based* objective functions and consider limited *inventory, vehicle capacity, and fleet size constraints.* Some sector-specific studies (Brinkmann et al. 2019, McKenna et al. 2020) explore *non-cost objectives*, such as expected number of unsatisfied future demand, or the expected total discounted reward. Due to the complexity of IR problems, obtaining optimal policies with conventional DP is typically feasible only for small instances, if at all. Thus, *suboptimal* policies are derived by approximating value functions via ADP and/or through heuristic approaches. These policies remain problem-specific, and include order-up-to (Bouma and Teunter 2016), Fill, Fill, Dump (Berman and Larson 2001), dynamic lookahead (Brinkmann et al. 2019), price directed operating policies (Adelman 2004).

For problems in **integration**, the objective is generally *cost* minimization. Most studies derive both *suboptimal and optimal* policies, except for two studies that obtain either optimal (Li and Mizuno 2022) or suboptimal policies (Turan et al. 2017). Specific policies are obtained such as order-up-to, base-stock (Bhatnagar and Lin 2019) and BSLP policy (Li and Mizuno 2022). Various constraints are considered, such as *inventory* (García-Alvarado et al. 2015, Lin et al. 2023), *production* (García-Alvarado et al. 2015, Malladi et al. 2020), *fleet size* (Turan et al. 2017, Malladi et al. 2020), *routing-related* (Turan et al. 2017), and *environmental* constraints for a closed-loop system under cap-and-trade mechanism (García-Alvarado et al. 2015).

6.4. Models and solution methods

Structural properties (SP), derived analytically, using mostly (backward) induction, first-order conditions, and bounding schemes, appear in all types of IM problems, especially pure inventory and inventory-pricing, with the exception of inventory routing, for which the optimal solutions are rather intractable. Deriving structural properties reduces the complexity of the problem, facilitates the derivation of optimal policies, and enables the development of more efficient algorithms for solving problems. In particular, for the majority of cases, authors are able to ascertain the characteristics of the optimal policy by analyzing the convexity/concavity/quasi-convexity/monotonicity of the Bellman equation, as well as its monotonicity with respect to certain state-variables and parameters. For instance, Benjaafar et al. (2018) derive structural properties based on convexity of the Bellman equation and conditional monotonicity properties for the order-up-to-level to obtain a generalized (s, S) policy which is optimal for an inventory problem with concave ordering costs. Moreover, Li et al. (2017b) prove properties such as quasi-convexity, monotonicity and supermodularity, and show that the single-period solution (newsvendor-based) serves as a bound for the solution in subsequent periods of the multi-period formulation.

Some articles also derive closed-form solutions for optimal policies via e.g., the Lagrangian method, first order conditions and KKT conditions (see e.g., Zhu and Thonemann 2009, Zhou et al. 2023, Kouvelis et al. 2023).

Previous literature (see e.g., Bellman and Dreyfus 2015, Puterman 2014) indicates that certain methods are commonly used to solve stochastic conventional DP models, such as *backward induction* (BI), *value iteration* (VI) and *policy iteration* (PI). These solution methods have rather straightforward solution mechanisms. BI is the most recurrent one in IM-DP literature, since the majority of problems are modeled with finite planning horizons, facilitating the computations when compared, for example, with forward induction (FI) algorithms. Various other solution methods frequently appear. In what follows, we discuss models and solution methods applied in each category of inventory problems.

In the context of pure **inventory**, conventional DP is mostly used, often solved through BI (Huang and Van Mieghem 2014), and VI (Arifoğlu and Özekici 2010, 2011). VI and PI are also applied together as in Tao et al. (2017) where PI has been used to a two-control policy limit and VI has been applied when seeking an optimal stationary policy. State-space reduction techniques (SSR), which simplify the problem by reducing the number of states or dimensions, are usually used to explore structural properties. For instance, Zhang et al. (2012) and DeCroix (2013) apply this method to perform transformations in the original multidimensional problem into a single-dimensional one and, to simplify the computation of optimal policies by reducing the size of assembly systems, respectively. Similarly, state-space relaxation that modifies the DP formulation to obtain a simpler version of the problem, with the optimal solution being a lower bound for the original problem, is often combined with other solution methods, such as filtering and augmenting procedures (Rossi et al. 2011). Decomposition used to decompose a problem into subproblems is another technique to facilitate derivation of the optimal policy as in Cheaitou et al. (2009). Numerical search, simulation, and enumeration are other techniques used Özen et al. (2012), Hosseinifard et al. (2022). Bounding scheme is used as a method to reduce the policy search space within the limits of lower and upper bounds or to derive heuristics from the solution bounds. Furthermore, such algorithms can be useful for analyzing infinite-horizon problems, as demonstrated by Parker and Kapuscinski (2004), who prove that bounds for the finite-horizon version can be extended to the infinite-horizon version in a capacitated two-echelon system, showing that the finite-horizon value function and the optimal policy converge to their infinite-horizon counterparts. For heuristics, several different types have been employed, however they are mainly represented by myopic (Gavirneni et al. 1998) or custom heuristics (DeCroix 2013). Specifically for myopic, they are mainly represented by single-period solutions (Xu et al. 2011) or by known heuristics in the literature, such as a myopic base-stock policy (Xu 2011). Some other types of heuristics are also employed efficiently, even if their occurrence is limited. For instance, Chen et al. (2021) rely on an adaptive approximation that embodies quadratic approximation and multimodularity in a non-stationary manner.

Regarding *ADP* approaches, some often encountered methods in the literature include approximate Linear Programming (ALP) (Meissner and Strauss 2012), approximate policy iteration (API) (Khademi and Eksioglu 2018) and rollout algorithms associated with lookahead policies (Kouvelis et al. 2023). Other non-conventional methods are also identified. For instance, Gijsbrechts et al. (2022) employ deep reinforcement learning to derive an efficient actor-critic method that combines both value-based and policy-based methods for dual sourcing inventory problems with stochastic lead times. The authors prove that the method can outperform other commonly used methods in the literature. To approximate a partially observed MDP, Mersereau (2013) rely on an adaptive grid interpolation method from the artificial intelligence literature. This method constructed via BI, interpolates the DP value functions on an irregular grid of a Bayesian inventory record.

BDPs are mainly addressed by exact methodologies, among which we can highlight analytical insights for SP, BI (Kadiyala et al. 2020), SSR (Azoury 1985), VI (see e.g., Bensoussan et al. 2008), and bounding scheme (Chen 2010). In particular, Chen (2010) points out the relevance of bounds for IM problems under BDP modeling, since BDP can rapidly be affected by the curse of dimensionality, making tractability challenging with classic solution methods in more complex settings. Moreover, since computing integrals for Bayesian updates is required, bounding schemes generally require less computational effort. Other techniques are less frequently used, such as the ones that use unnormalized probabilities to linearize the information updating process in Bayesian formulations and simplify the Bellman equation or a nonlinear DP (Bensoussan and Guo 2015, Bensoussan et al. 2010). Myopic and custom heuristics appear to be more frequently applied, although the use of data-driven algorithms is also observed (Luo et al. 2023).

For *HDP*, different types of solution methods are employed in both exact and heuristic realms. For instance, when citing exact solution methods, Liu (1999) devises a combination of fuzzy optimization and dynamic programming, and proves by induction that the optimal policy is a bounded critical number policy. Regarding heuristics, some of the articles rely on structures that depend, for instance, on Silver-Meal heuristics and simulation (Gutierrez-Alcoba et al. 2017), BI and DP-based heuristics (Chen and Moinzadeh 2018) and neural networks (Reyes-Aldasoro et al. 1999, Qi et al. 2023). For instance, a labeling algorithm that combines conventional DP and neural networks to capture the dynamics of the optimal solution in future periods is used in Qi et al. (2023).

Conventional DP has also been predominantly used to model **inventory-production** problems. Mostly implemented exact solution methods to extract optimal policies for IP problems are BI (see e.g., Minoux 2018), often combined with enumeration and decomposition (Jensen et al. 1991, Wang 2012), or numerical methods via e.g., numerical search, experiments, simulation or Matlab (Pac et al. 2009, Wang et al. 2014, Kiesmüller and Scherer 2003, Kapuściński and Tayur 1998, Liu et al. 2020a). Other methods, less frequently used, are VI (see e.g., Mayorga and Ahn 2011, Veatch and Wein 1994) and PI (Song et al. 2017). In particular, in Veatch and Wein (1994), VI and the coordinate search algorithm were used to evaluate candidate policies and find the best base-stock levels for work-in-progress and finished goods for a given type of policy. Fewer studies incorporate heuristic approaches ranging from custom heuristics (Mayorga and Ahn 2011, Wang et al. 2014), binary search (Feng et al. 2011) to rollout algorithms (Raman and Kim 2002). For instance, Mayorga and Ahn (2011) provide a series of heuristics based on production/ rationing and capacity decisions to provide more easy-to-implement solutions. Raman and Kim (2002) reduce a quite complex DP formulation via approximations that lead to a two-period formulation with lookahead at one-forecast revision. For *ADP*, methods such as AVI and reinforcement learning for a complex real-life problem of a blood center are used (Yang et al. 2022) . For a complex multi-factory, multi-product inventory system, Çimen and Kirkbride (2017) show that policies obtained via temporal difference learning algorithms perform well with respect to the optimal policies or policies derived from deterministic approximation of the problem. Derivative and recursive algorithm for an IP problem of a recovery system (Kiesmüller and Scherer 2003) and a control algorithm for IM and revenue management problems (Secomandi 2008) are also studied. In particular, Secomandi (2008) incorporates effects of changes in the environment observed over time in the control policy by resolving the MP approximation of an MDP formulation and revise the decision-rule instantiation with the newly obtained solution of the resolved MP. The only study on *BDP* considers both pure inventory and IP problems (Chen and Plambeck 2008). In particular, for a capacitated IP problem under continuous replenishment, the authors characterize the optimal base stock policy and derive maximum likelihood estimators (MLE) of the demand rate, as Bayesian optimization is intractable.

For **inventory-scheduling** problems, most articles provide optimal DP formulations and use exact methods such as BI to derive optimal policies. For complex and large-size problems, authors obtain near-optimal policies via marginal analysis Schaefer (1983) or use ADP to obtain approximations. In particular, data-driven methodologies have been used for an ADP formulation, such as a regression-based approximation (Asadi and Nurre Pinkley 2022) and a rollout algorithm based on a lookahead policy (Valeva et al. 2021). Khademi and Eksioglu (2018) employ an API algorithm based on the relaxation of the MILP-derived formulation of DP and uses a Monte-Carlo simulation to estimate the expected values.

Conventional DP has been employed in almost all articles entailing **inventory-pricing** (IPr). Dimensionality reduction techniques are used to simplify the derivation of the optimal policy. For instance, Yang and Zhang (2022) perform a sample path analysis to reduce a two-dimensional DP formulation to a one-dimensional model, Liu et al. (2020b) applies the reduction technique to the Bayesian IPr model with the multiplicative demand, and unobserved lost sales. Heuristic approaches have also been considered under myopic and custom heuristics, as well as lookahead heuristic approaches (see e.g., Yang and Zhang 2022). A data-driven approximation of the Bellman equation is proposed in Qin et al. (2022) to deal with the complexities associated with the lack of information regarding demand distribution. An online learning algorithm for the joint pricing and IM problem with fixed ordering costs is applied in Chen et al. (2022a), where historical data are used to estimate the unknown parameters of the demand function. A similar scenario with an unknown demand distribution and dynamic pricing strategies is addressed via BDP in Liu et al. (2020b). The authors prove analytically the optimality of the BSLP policy under Bayesian updates via derivative analysis and apply a dimensionality reduction technique to decompose the value function.

Conventional DP is also used for the majority of **inventory-distribution**(ID) problems. Exact solution methods, such as VI, PI, and BI (Olsson 2009, Archibald et al. 2010, Agrawal et al. 2004) are applied, although only to few cases. Given the complexity of the ID problems, the optimal policy is either extracted for small instances or not explicitly obtained. In such cases, researchers mainly employ heuristics or ADP formulations to derive approximate policies. Heuristic approaches for DP formulations include successive approximation methods (Hu et al. 2005, Bitran and Mondschein 1996), a bisection method for multi-item problems with transshipment and two depots (Archibald et al. 1997), pairwise decomposition and simulation to estimate the expected costs rate for heuristic policies (Archibald et al. 2009), inventory separation and linear approximation of the profit-to-go function(Li et al. 2022), and (quasi) myopic algorithms (see e.g., Glazebrook et al. 2015), among others. Problems that have been modeled via ADP resort to solutions that mostly involve Lagrangian relaxation and decomposition (see e.g., Kunnumkal and Topaloglu 2011, 2008a, Topaloglu and Kunnumkal 2006) or ALP (see e.g., Topaloglu and Kunnumkal 2006, Wang et al. 2021b). In a two-period ID problem, generalized Bayesian updating was used to forecast demand of multiple retail stores (see Agrawal and Smith (2013)). An optimal approximation of the excess inventory, based on the Central Limit Theorem, was developed which reduces the state space of the system in the DP formulation.

Inventory-routing problems are modeled using both DP and ADP. Berman and Larson (2001) and Bouma and Teunter (2016) provide DP formulations and use BI to derive optimal policies. Due to the intractability of exact solutions for large instances, standard exact solution methods - such as Linear Programming (LP) for DP, and ALP (Adelman 2004) - are only applied to small-scale problems. Inexact ADP approaches include value function approximation (VFA) with decomposition, the Gauss-Seidel policy evaluation algorithm (Kleywegt et al. 2004), lookahead with non-parametric VFA (Brinkmann et al. 2019), API using least squares temporal differencing (McKenna et al. 2020), and hybrid algorithms combining local search, simulation, and parametric VFA (Kleywegt et al. 2004).

Conventional DP is typically used to formulate the **integration problems** and VI have been mostly used to derive the optimal policy. Closed-form solutions for optimal policies are obtained in Li and Mizuno (2022). Heuristics include decomposition-based methods (Lin et al. 2023, Bhatnagar and Lin 2019), genetic algorithm (García-Alvarado et al. 2015), and one-step lookahead method (Bhatnagar and Lin 2019). Hybrid DP heuristics, combining variable neighborhood search with DP solved via BI, is developed in (Turan et al. 2017). ADP techniques such as AVI, one-step rollout algorithms and decomposition based approaches are considered in Malladi et al. (2020).

7. Robust models

A summary of the collected articles that include robust IM-DP modeling, categorized according to our typology, can be found in Table A6 in Appendix.

7.1. Uncertainty

For the robust IM models, demand-related uncertainty occurs in all articles. This can be justified by the significant amount of previous research executed for stochastic environments under demand uncertainty, which can be seen as a baseline for more complex models, that entail unknown sets, such as in robust models, instead of known parameters or distributions. In Minoux (2018), the uncertainty is defined through a general state-dependent type of uncertainty, which can take the form of price- or demand- related uncertainty for a specific inventory problem. Turgay et al. (2015) incorporate production-related uncertainty in the form of production rates. Turgay et al. (2018) consider general uncertainty sets on the transition probabilities, which, in inventory and production environments, can represent demand or processing rates.

7.2. Problem structure

Pure **inventory** problems are represented by classical approaches of replenishment problems. Usually these articles focus on different classes of specification for uncertainty, as seen, for instance, in Xin and Goldberg (2021) and Xin and Goldberg (2022), where the authors try to mitigate the issues associated with assumptions for uncertainty sets. Similar approaches have been adopted by Turgay et al. (2018) and Chen et al. (2023). Regarding **inventory-production**, the models focus more exclusively on replenishment opportunities and production rationing decisions (Turgay et al. 2015), and lot-sizing aspects(Minoux 2018). The articles on robust DP cover only single-item, single-supplier, finite-horizon models and. In some cases, more complex strategies are designed, by incorporating backlogs (Qiu et al. 2017, see e.g.,), lost sales Turgay et al. (2015), Chen et al. (2023) or deterministic lead times (Chen et al. 2023). As robust modeling primarily addresses uncertainty issues that are difficult to deal with using stochastic approaches, the scenarios considered are already complex enough without incorporating multiple suppliers or infinite planning horizon. For instance, Chen et al. (2023) note the computational intractability of robust models under long horizons, necessitating heuristic solutions for extended periods, which explains the absence of articles focusing on infinite horizons.

7.3. Decisions and policies

For pure **inventory** problems, the articles mainly cover *cost-related* functions, represented by expected discounted costs (Chen et al. 2023). *Profit* is also investigated under the form of expected discounted profit functions (Minoux 2018). **Inventory-production** problems focus solely on profit functions under total expected profit (Turgay et al. 2015) and expected discounted profit (Minoux 2018). Regarding *policies*, the majority of articles rely on the structure of classical policies, such as base-stock and (s, S). In particular, Xin and Goldberg (2021) explore the dynamic aspect of policy optimality, known as time consistency, which refers to the stability of optimal policies over different time periods, and show the optimality of base-stock policies in this case. An extension can be seen in Xin and Goldberg (2022) where they prove the optimality of a time- and state-dependent base-stock policy for the scenario with martingale demand. For **inventory-production**, Turgay et al. (2015) show the effects of robustness on admission and production policies. *Constraints* are represented by inventory capacity and ramping for the capacitated lost-sizing problem (Minoux 2018).

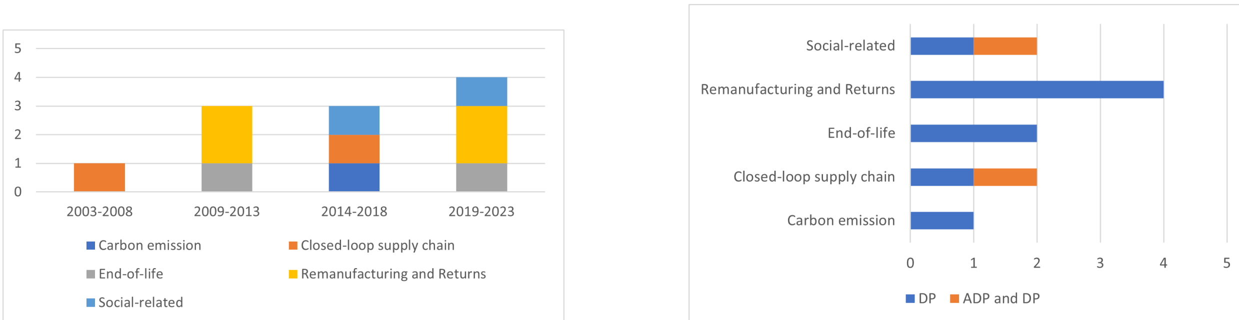
7.4. Models and solution methods

Both **inventory** and **inventory-production** problems are addressed via conventional DP and the majority of these are solved via BI. Structural properties are also derived. Some particular studies solve formulations through other types of methods, such as dual LP and second-order cone programming (Qiu et al. 2017). Just one article (Chen et al. 2023) includes a lookahead heuristic that is computationally efficient for large problem instances with lost sales. Minoux (2018) examines the performance of the robust DP with respect to the stochastic DP and shows that the former outperforms the latter under low risk levels.

8. Sustainability aspects in IM-DP

Sustainability has significantly evolved over the past three decades (Figure 7(a)), shaping corporate practices. In IM, it involves optimizing stock to minimize environmental impact and promote social responsibility. This includes reducing inventory, recycling, recovering materials, and cutting carbon emissions.

In our analysis, we investigate whether articles address sustainability topics and explore potential areas for further research. Although sustainability has received considerable attention within the OR community, our findings show that only 11 out of our selected articles have used (deterministic and stochastic) DP in IM to tackle these issues. From Figure 7, we observe that DP-derived methodologies have been mainly applied to problems related to products that undergo remanufacturing and returns (4 articles), end-of-life aspects associated with obsolescence (2 articles), closed-loop supply chains (2 articles), with one of them being associated with cash reuse and recycling, carbon emissions (1 article), and social aspects (2 articles). For the latter, the authors focused on the development of the workforce through human learning and training (Valeva et al. 2021), and humanitarian aspects (Natarajan and Swaminathan 2017). Since these topics have not been widely addressed by DP, they present a clear opportunity for further research. Figure 7(b) visualizes that these sustainability topics in IM have been mainly addressed through conventional DP and in a limited number of cases (for workforce and product returns) by ADP. Conventional DP has been solved by a variety of methods such as BI, FI, VI, bounding schemes, numerical solution, among others. For ADP, lookahead approximations have been used in a workforce development scenario to enable solutions for a large number of periods.



(a) Evolution of different sustainability topics approached by DP over time.

(b) Use of different DP methodologies over different sustainability topics.

Figure 7 Distributions of DP-based formulations over sustainability topics.

In the context of problem structure, the articles are divided among inventory, inventory-production, inventory-distribution, inventory-scheduling, and integration problems. Due to the limited number of articles, a pattern is hard to establish. However, we observe that inventory and inventory-production problems seem to comprise sustainable topics related to remanufacturing (Chou et al. 2020), returns (Flapper et al. 2012), end-of-life (Pourakbar and Dekker 2012), and closed-loop supply chains (Zhu et al. 2011). The decisions for such problems rely, in general, on the balance between whether to produce/order new products or remanufacture given product returns in order to reduce waste. In some cases, disposal decisions are also considered. Carbon emissions have also been considered as one of the main drivers for pure inventory problems (García-Alvarado et al. 2015). Regarding the social pillar, two articles are identified in inventory-distribution and inventory-scheduling. For the former, Natarajan and Swaminathan (2017) focus on humanitarian operations, while for the latter, Valeva et al. (2021) investigate the performance of workers based on their learning process and training.

For the decisions and policies classification, the articles focus on cost- and profit-based functions, even when carbon emissions are considered (see e.g., García-Alvarado et al. 2015). Few articles incorporate constraints, mainly associated with the shelf life of perishable items, obsolescence, and carbon emissions. Regarding policies, although some authors demonstrate the optimality of known policies such as (s,S) and base-stock-policies(see e.g., Flapper et al. 2012, Ozyoruk et al. 2022), most focus on solving the DP formulations without explicitly obtaining a policy.

9. Conclusion

9.1. Discussion and concluding remarks

The application of DP in IM has been pivotal in advancing both fields. Applying DP-derived methodologies allows to find viable solutions to increasingly complex inventory problems. In turn, the complexity of these inventory problems drives the continuous development of DP. To the best of our knowledge, this is the first literature review that studies the intersection between both IM and DP topics, analyzing 278 articles.

This article makes several significant contributions. First, we develop a classification typology which allows to distnguish different IM problems, related to the distinct decisions and function in the supply chain, and their characteristics, such as the number of suppliers and products, uncertainty, and multi-period planning. This enables an examination of which IM problems and characteristics are modeled with DP, as well as the identification of the most commonly used combinations of characteristics for specific models. Our analysis reveals the increasing complexity of IM problems and how DP-derived methodologies address these challenges. Second, we classify DP methodologies, examine their applications and evolution in IM, identifying emerging trends. Third, we offer insights into the optimization aspects in the IM-DP literature (objective functions, policies, and constraints) within deterministic, stochastic, and robust modeling frameworks. Fourth, we discuss the DP solution methods for IM problems, guiding the reader on suitable models-solution method combinations for each type of IM problem. Finally, future literature reviews could build upon our work by using the proposed typology framework as a foundation for exploring alternative OR methodologies in IM. Moreover, our study could encourage researchers to conduct comparative literature reviews on the application of several methodologies to specific supply chain functions integrated in IM.

Our analysis enables us to answer our research questions. In particular, the analysis conducted to answer RQs 1-2 highlights the evolving complexity of IM problems and the suitability of DP in addressing these advanced challenges. Regarding RQ1, a growing group of researchers has focused on joint decisions, particularly in areas such as inventory-distribution, inventory-production, and inventory-pricing. While most of the research focuses on simpler scenarios (e.g., single suppliers, single item, backlogs, finite horizons, no lead time), more complex settings with multiple suppliers, multiple items, lost sales, infinite horizons, and lead time-remain under-explored, despite efforts to maintain manageable computational burdens. For instance, for scenarios involving censored demand, information asymmetry, and partially observed demand, BDP emerged as a precise tool to model drastic gaps in demand that cannot be captured by specific or general distribution probability functions. Similarly, in inventory-scheduling, inventory-distribution, and inventory-routing domains, where complexities such as transportation modes, transshipment, and various machine environments arise, ADP and HDP are instrumental in obtaining sub-optimal, yet high-quality solutions for longer periods. Future research should focus more on the development of novel DP formulations and efficient solution methods for these more realistic and complex scenarios.

Regarding RQ2, our insights show that conventional DP has often been used, because many decision-making problems assume properties such as convexity, modularity, and monotonicity. The use of alternative DP approaches, such as ADP, BDP, and HDP, have moderately increased over recent years, offering particularities that mitigate computational burden of complex problems in IM. Exact solution methods remain dominant, which is expected given the prevalence of conventional DP models in the literature. Structural properties (the most common approach for stochastic problems), forward induction (the most common method for deterministic problems), backward induction, derivative analysis, state-space reduction, and value iteration (for stochastic problems) have been broadly employed to more classical inventory problems, such as pure inventory and inventory-production. In terms of heuristics methods, a variety of algorithms have been applied, primary custom heuristics and approximate methodologies for problems such as inventory, inventory-distribution, and inventory-routing. Some methods such as bounded dynamic programming, matheuristics, and data-driven methods, among others, despite being scarcely applied, can provide powerful solutions for certain problems. The advancement of methods in OR, including mathematical programming, fuzzy theory, approximation methods, and machine learning techniques, has significantly enhanced the algorithmic efficiency of DP for inventory problems. However, despite their advantages, the use of these methods remains inferior to conventional DP, presenting a promising avenue for future research to address complex IM problems. Regarding RQ3, only a limited number of IM-DP articles focus on industrial applications, and even fewer rely on real data. Our findings show that DP has been applied to different industry sectors, such as commerce, automobile, clothing and pharmaceutical industries and, unexpectedly, their solutions have been mainly conveyed by conventional DP. Conversely, models such as ADP and HDP, which are highly practical, have not been widely implemented to study real company cases. These methods could potentially serve as alternative solutions for more complex inventory problems that arise in practice.

Regarding RQ4, sustainability aspects are underrepresented, despite their increasing relevance, and are distributed over pure inventory, inventory-production, inventory-scheduling, inventory-distribution, and integration problems. In this context, certain sustainability metrics can be either integrated and explicitly modeled into objective functions (e.g., aggregation of carbon emission in cost functions) or practical constraints (e.g., obsolescence). A range of solution methods - including structural properties, derivative analysis, numerical solutions, backward induction, and value iteration - has been applied to such problems. For more complex settings, ADP models with linear and lookahead approximations are able to maintain tractability over extended planning horizons. Overall, these advancements demonstrate the potential of DP methodologies to integrate economic, environmental and social objectives, thereby enhancing the sustainability and efficiency of IM. This presents opportunities for further research.

RQ5 was concerned with gaps and limitations, and future research avenues, which we discuss in more detail in the next section.

9.2. Future research agenda

This section covers the gaps and future research topics that could potentially be explored with the aid of DP-based applications in IM. We distinguish between six main topics, related to (a) sustainability, (b) closed-loop supply chains and reverse logistics, (c) applied studies, (d) problem and decision-making characteristics, (e) solution methods and (f) data analytics and artificial intelligence.

- **Sustainability aspects:** Despite the increasing interest in sustainability, our analysis showed that the main objectives analyzed in IM-DP studies are economic. Only few DP articles consider environmental objectives, such as energy-based, emission-based or social based objective functions. It is known that these types of functions usually require multi-criteria optimization, for which conventional DP might not be suitable due to the curse of dimensionality. However, metaheuristics have been widely employed in such type of optimization and, therefore, future research could explore the effects of HDP based on metaheuristic to obtain more tractable formulations regarding these dimensionality issues. Furthermore, since metaheuristics enable such tractability, investigating more extensively IM problems that integrate functions such as distribution or routing (since they affect directly, for instance, environmental aspects of sustainability) could also be another topic to further explore. As our review focuses on the intersection of IM and DP, two broad topics in the literature, our article selection process relied on IM- and DP-related keywords and top-tier journals in OM, OR, and MS. However, this approach may have resulted in a limited number of studies identified on the topic of sustainability. Future research could therefore broaden the scope by incorporating sustainability-related keywords and extending the search to other journals, particularly those centered on more context-specific scenarios, such as energy storage or water resource optimization, in order to identify a wider range of relevant sustainability-related studies that may intersect with other themes discussed in this article.

- **Closed-loop supply chains and reverse logistics:** Remanufacturing operations frequently intersect with inventory-production problems, primarily focusing on analyzing inventory levels of remanufactured products post-process. However, these analyses can be extended in several directions. Firstly, while remanufacturing positively impacts sustainability, it also entails significant energy consumption. Therefore, a fruitful study could explore minimizing energy consumption, potentially under government subsidies, to optimize remanufactured product inventory levels using DP-derived methodologies. States could be represented by both inventory and subsidy levels. Secondly, quality constraints are crucial for retaining customers, but achieving homogeneity in remanufactured products is challenging due to stochastic factors. DP-derived methodologies combined with data-driven algorithms could calibrate quality levels to meet a given threshold while reducing standard deviation. Lastly, while inventory problems typically focus on order or production quantities, incorporating reverse logistics can lead to policies that determine the optimal balance between remanufactured and new products, or hybrid approaches.

- **Applied studies:** The conducted review shows that conventional DP has been predominantly used to model real-life inventory problems. However, with the rise of computational capabilities and DP-derived methodologies, ADP and HDP have gained traction, opening unexplored avenues in modeling more complex applied IM problems. Both ADP and HDP benefit from data-driven methodologies, which can expedite convergence processes for ADP iterations and provide more accurate samplings or improved forecasting for uncertainties in the modified Bellman equation in HDP.

- **Problem and decision-making characteristics:** Future research could address the under-representation of service- and asset- related objective functions in IM-DP literature. Although, conventional DP provides theoretical insights for certain service level functions, practical approaches in generating efficient solutions for non-linear service level agreements are limited. HDP approaches could enhance modeling by integrating data-driven techniques, such as SoftMax functions in neural networks, to improve tractability. Similar advancements could be made regarding objectives related to unsatisfied customers, net present value, conditional value-at-risk, among others, which remain underexplored, despite their practical relevance for industry.

The majority of problems have been characterized by single-item and single-supplier settings. As mentioned by Svoboda et al. (2021), problems that extend these characteristics to address multiple items and suppliers simultaneously are either inexistent or less evidenced in the IM literature, despite their practical relevance. Although devising state-space reduction techniques relying on variable transformation does not seem to be an easy task in such case, state-trimming techniques via dominance rules and bounding schemes or DP-based algorithms that perform heuristically under such conditions can be explored (e.g., Bounded Dynamic Programming). Also, as shown in Benjaafar et al. (2022), Stochastic Dual DP seems to be a quite efficient method in terms of piece-wise linear approximations for the Bellman equation. As they can be dynamically updated under a certain portion of the state-space, it could be promising when dividing the complex structure of a multi-item, multiple suppliers IM formulation with DP.

The simultaneous analysis of uncertainties, particularly price and lead-time, remains scarce. Future research could focus on reducing complexity of such interaction through state-space reduction and aggregation. For instance, in cases with interdependent relationships between random variables, dimension reduction through variable transformation techniques could simplify the structure of the Bellman equation. Another manner to address this challenge could be to employ BDP and robust optimization to explore how properties of these uncertainties can be extended to uncertainty sets and to analyze patterns of these sets using Bayesian probability theory.

- **DP models and solution methods:** Conventional DP with classical solution methods effectively evaluates policy performance over shorter periods, but developing more efficient techniques for longer horizons remains an open challenge. Over the past decade, machine learning and data-driven approaches have enhanced algorithmic performance in OR. Future research could explore novel algorithms integrating these techniques with well-established DP methods. For instance, combining neural networks with value and policy iteration could accelerate convergence and improve the accuracy of state transition probability estimation.

Most ADP models have focused on inventory-distribution problems, considering linear or piecewise value function approximations. While ADP effectively addresses the curse of dimensionality for these problems, the reviewed articles have not explored other efficient approximation schemes in detail. Future research could compare various approximation schemes such as temporal differences, deep Q-networks, and actor-critic methods, particularly in less-explored areas such as inventory-production or inventory-pricing. This would help researchers determine correlations between approximation schemes and specific IM problems. Additionally, hierarchical ADP combined with custom solutions methodologies could be useful for problems requiring decisions at multiple levels.

BDP presents a particularly challenging formulation due to the complexities in computing integrals for Bayesian updates, and the underestimation of uncertainty, among others. Future studies should address these challenges by applying appropriate techniques such as low-dimensional formulations and discretization over the beliefs associated with uncertainties, which could be combined with value or policy iteration for obtaining theoretical results for a small state-space and approximation grid for the beliefs. Approximating Bayesian Bellman equation and replicating ADP algorithms in BDP could help develop novel heuristic methods for tackling burdensome BDP formulations. Decomposition methods combined with relaxation methods could also be a useful direction for problems with high-dimensional formulations.

Combining DP with other OR approaches, HDP offers vast possibilities for further investigation. Most HDP studies focus on heuristics for complex inventory-routing and inventory-distribution problems, but this approach could be expanded to other IM problems involving multiple decision types such as integration, which have been underexplored in the IM-DP literature. Our review shows that some commonly used methodologies in OR such as Bounded Dynamic Programming and matheuristics have been effective used with HDP for IM problems (such as inventory-scheduling and inventory-routing problems, respectively) but not widely. Additionally, data-driven approaches could enhance the performance of these heuristics. For example, machine learning techniques such as neural networks can help identify promising states for pruning, reducing the action space in DP. Furthermore, machine learning can aid in tuning the parameters of heuristics and metaheuristics, offering a more adaptive alternative to the fixed values often used in the literature.

DP modeling under robust optimization in IM seems to have received little attention in research compared to deterministic and stochastic DP models, yet there are potential research avenues. For instance, Turgay et al. (2015) model inventory-production systems with uncertain demand and production rates. Despite the complexity of the problem, due to the presence of uncertainty sets for two types of variables, the authors have managed to obtain the optimal policy via structural properties. This approach could be extended to integrate other supply chain functions such as routing, distribution, and pricing in IM.

- **Data analytics and artificial intelligence:** The most challenging factors in inventory problems stem from asymmetric information or censored demand environments, failure and deterioration rates, quality levels of remanufactured inventory, production capacity, and supplier availability. These factors rarely belong to a class of well-distributed probability functions. Thus far, only a few studies with BDP and robust DP have attempted to address some of these challenges. With the advent of data analytics and AI, however, it is possible to track the behavior of these factors and understand patterns for a large set of data. For instance, an interesting direction is to use reinforcement learning as a mechanism to predict uncertainty behavior of suppliers' lead-times based on previous data regarding their disruptions. Once the output can be approximated by a given probability distribution, DP is suitable to minimize the long-term costs and predict the amount of orders that should be considered under these disruption scenarios. Another potential direction is the use of AI to select the best parameters for DP-based heuristics that are employed in inventory systems that change dynamically over periods (e.g., seasonality). Therefore, it is desirable to expand studies that combine DP-based methodologies with AI and data analytics for inventory problems, aiming to obtain solutions with a small margin of error or with the desired accuracy.

Appendix

Authors	Focus	Occurrence of DP (x) Low ($x < 5$)	Medium ($5 \leq x < 15$)	High ($x \geq 15$)
Search 1				
Silver (1981)	Inventory management	✓		
Goyal and Gunasekaran (1990)	Inventory and production integration		✓	
Chand et al. (2002)	Forecast and rolling horizons			✓
Ortega and Lin (2004)	Control theory in inventory-production	✓		
Sarimveis et al. (2008)	Dynamic modeling in supply chains			✓
Sasikumar and Kannan (2008)	Inventory management in reverse supply chains	✓		
Andersson et al. (2010)	Inventory and routing integration	✓		
Akçalı and Çetinkaya (2011)	Inventory-production in closed-loop supply chains	✓		
Krishnamoorthy et al. (2011)	Inventory with positive service time	✓		
Paterson et al. (2011)	Inventory with lateral transshipments		✓	
Simchi-Levi et al. (2012)	Stochastic multi-echelon inventory	✓		
Govindan (2013)	Vendor-managed inventory	✓		
Glock et al. (2014)	Lot-sizing	✓		
Khanlarzade et al. (2014)	Perishable inventory	✓		
Ozguven and Ozbay (2014)	Inventory with emergency orders	✓		
Balcik et al. (2016)	Inventory in humanitarian supply chains	✓		
Eruguz et al. (2016)	Guaranteed-service for multi-echelon inventory		✓	
Iraqi et al. (2016)	Spare parts inventory	✓		
Bounou et al. (2017)	Spare parts inventory	✓		
Kawale and Sanas (2017)	Inventory under trade credit	✓		
Zemzam et al. (2017)	Inventory management with robust control theory	✓		
Chaudhary et al. (2018)	Perishable inventory		✓	
Momeni and Azizi (2018)	Ordering in manufacturing environments	✓		
Uzsoy et al. (2018)	Inventory in semiconductor supply chains	✓		
Alfares and Ghaithan (2019)	EOQ and EPQ with variable holding costs	✓		
Choi et al. (2019)	Queuing inventory systems	✓		
Engebrethsen and Dauzère-Pérès (2019)	Transportation mode selection in inventory		✓	
Mosca et al. (2019)	Inventory integrated with transportation	✓		
Jackson et al. (2020)	Inventory control and optimal control parameters	✓		
Lukinskiy et al. (2020)	Inventory control for low-demand items	✓		
Song et al. (2020)	Integration between inventory and capacity	✓		
Becerra et al. (2021)	Sustainable inventory management	✓		
Glock and Grosse (2021)	Controllable rates in inventory systems	✓		
Goldberg et al. (2021)	Asymptotic analysis in inventory management			✓
Svoboda et al. (2021)	Multiple sourcing in inventory models		✓	
Boute et al. (2022)	Deep reinforcement learning for inventory		✓	
Goltsos et al. (2022)	Inventory and forecasting	✓		
Erkip (2023)	Data-driven systems in inventory	✓		
Fagerholt et al. (2023)	Maritime inventory-routing	✓		
Vidal (2023)	Deterministic and Stochastic inventory models in production	✓		
Search 2				
		Occurrence of inventory (y) Low ($y < 5$)	Medium ($5 \leq y < 15$)	High ($y \geq 15$)
Powell (2016)	Approximate DP	✓		
Strauss et al. (2018)	Revenue management		✓	
Powell (2019)	Stochastic optimization		✓	
Jackson et al. (2020)	Inventory control and optimal control parameters			✓
Rempel and Cai (2021)	Applications in military OR		✓	
de Souza et al. (2022)	Machine scheduling	✓		
Koulamas and Kyparisis (2023)	Machine scheduling	✓		

Table A1: Literature reviews related to IM and DP.

Journal	**Abbreviation**
Annals of Operations Research	AOR
Computers and Operations Research	CAOR
Decision Sciences	DS
European Journal of Operational Research	EJOR
IEEE Transactions on Systems, Man, and Cybernetics Systems	IEEE SMC
IISE Transactions	IISE
INFORMS Journal on Computing	IJOC
International Journal of Production Economics	IJPE
International Journal of Production Research	IJPR
Journal of Optimization Theory and Applications	JOTA
Journal of Operations Management	JOM
Journal of the Operational Research Society	JORS
Management Science	MS
Manufacturing and Service Operations Management	MSOM
Mathematics of Operations Research	MOOR
Naval Research Logistics	NRL
Omega	Omega
Operations Research	OR
OR Spectrum	OR Spectrum
Planning and Production Control	PPC
Production and Operations Management	POMS
Transportation Science	TS

Table A2: List of journals and their abbreviations.

Abbreviation	Solution method
AA	Adaptive approximation
ALP	Approximate linear programming
API	Approximate Policy iteration
AVI	Approximate value iteration
BI	Backward induction
BM	Bisection method
BS	Bounding scheme
C	Custom
CA	Control algorithm
CSD	Clark-Scarf decomposition
CT	Convergence Theorem
D	Derivative
DD	Data-driven
DE	Decomposition
DLP	Dual Linear Programming
DPB	DP-based heuristic
E	Enumeration
ECA	E-convex approximation
FI	Forward induction
FK	Florian-Klein
FPTAS	Fully Polynomial Time Approximation scheme
G	Greedy heuristic
GA	Genetic algorithm
HJM	Hooke-Jeeves Method
IPA	Infinitesimal Perturbation Analysis
IPM	Interior-point method
IS	Iterative Solution
LP	Linear Programming
LS	Local Search
MA	Marginal analysis
MH	Metaheuristics
MIP	Mixed integer programming
MLE	Maximum likelihood expectation
MVT	Middle value theorem
MY	Myopic
NA	Not specified/not obtained
NVB	Newsvendor-based
OL	Online learning
PA	Power approximation
PI	Policy iteration
QL	Q-learning
R	Relaxation
RL	Reinforcement Learning
RO	Rollout
S	Simulation
SA	Successive approximation
SAN	Stationary analysis
SMH	Silver-Meal Heuristic
SP	Structural properties
SSR	State-space reduction
TD	Temporal difference
VI	Value iteration
WW	Wagner-Whitin

Table A3: List of solution methods identified in the literature.

Article	Inventory problem	Number of suppliers	Product diversity	Time	Objectives	Policy	Constraint	DP-based approaches	Solution methods
	Structure				Decision and Policies				
Jing and Mu (2019)	I	SS	MI	MP,FH,CR	C	O	L	DP	E: FI
Chakravarty (1981)	I	SS	MI	SP,FH,PR	C	H		DP	H: FI
Rosenblatt (1985)	I	SS	MI	SP, FH, PR	C	O		DP	E: FI
Buchanan (1980)	I	SS	SI	MP, FH, CR	C	H		DP	H: BI, MY
Gutiérrez et al. (2003)	I	SS	SI	MP, FH, PR	C	O	IC	DP	E: FI
Zhu et al. (2011)[3,5]	I	SS	SI	MP, FH, PR	C	O		DP	E: FI
Chand and Tang (1985)	I	SS	SI	MP, FH, PR	C	O		DP	E: WW
Hariga (1997)	I	SS	SI, BO	MP, FH, CR	C	O,H	L	DP	E: FI H:C
Tai et al. (2016)	I	SS	SI, BO	MP, FH, PR	P	O	L	DP	E: FI
Hwang and Van Den Heuvel (2012)	I	SS	SI, BO	MP, FH, CR	C	O	IC	HDP	E: BI
Grubbström and Huynh (2006)	IP	SS	MI, BO	MP, FH, PR, LT	A	O	PC, Z_1	DP	E: BI, SSR
Chu and Chu (2008)	IP	SS	SI	MP, FH, CR	C	O	IC	DP	E: FI
Akbalik et al. (2015)	IP	SS	SI	MP, FH, CR	C	O	IC, PC	DP	E: FI
Suzanne et al. (2020)	IP	SS	SI	MP, FH, PR	C	O	IC	DP	E: FI
Önal et al. (2012)	IP	SS	SI	MP, FH, PR	C	O	IC,	DP	E: BI
Hsu (2000)	IP	SS	SI	MP, FH, PR	C	O	L, DC	DP	E: FI
Ou (2012)	IP	SS	SI, BO	MP, FH, CR	C	O	DC, PC	DP	E: BI
Hsu and Lowe (2001)	IP	SS	SI, BO	MP, FH, PR	C	O	PC, DC	DP	E: BI, FI
Sandbothe and Thompson (1993)	IP	SS	SI, LS	MP, FH, PR	C	O	IC, PC	DP	E: FI
Jing and Chao (2021)	IP	SS	SI, LS	MP, FH, PR	C	O	IC, L, DC	DP	E: FI
Yeung et al. (2011)	IS	SS	MI	MP, FH, LT	P	O	Z_1	DP	E: BI
Potamianos and Orman (1996)[3,5]	IS	SS	MI	MP, FH, PR, LT	C	O	PC	DP	E: WW
Potamianos et al. (1997)[3,5]	IS	SS	MI	MP, FH, PR, LT	C	O	PC, DC	DP	E: WW
Boysen et al. (2008)	IS	SS	MI	MP, FH, CR	C	O, H	PC, Z_1	DP	E: FI, H: FI, BDP
Segerstedt (1996)	IS	SS	MI, BO	MP, FH, PR, LT	C	O	PC, IC	DP	E: FI
Bertazzi and Speranza (1999)	ID	SS	MI	MP, FH, PR	C	H	Z_1	DP	H: BI
Palak et al. (2018)	ID	SS	SI	MP, FH, PR	C	H	L, Z_1	DP	H: BI
Toriello et al. (2010)	IR	MS	SI	MP, FH, PR	P	H	IC, Z_1	ADP	H: MIP
Papageorgiou et al. (2015)	IR	SS	SI, LS	MP, FH, PR	C	H	IC, Z_1	ADP	H: C
Engineer et al. (2012)[3,5]	IR	SS	SI	MP, FH, PR	C	H	IC	HDP	H: C
Adelman (2003)[3,5]	IR	SS	SI, MI, BO, LS	MP, IH, CR	C	H	IC, Z_1	DP,ADP	E: NA H:ALP
Chen and Wei (2012)	IPr	SS	SI	MP, FH, PR	P	O	L	DP	E: FI
Yu et al. (2013)	IPr	SS	SI, BO	SP, FH, PR	P	H	PC	HDP	H: C
Li et al. (2017a)	In	SS	MI	SP, FH, PR	C	O	Z_1	DP	E: FI
Chen and Chen (2004)	In	SS	SI, BO	MP, FH, PR	P	O	L	DP	E: FI, SSR

Table A4: Characterization of articles that entail deterministic models.

[3]Industrial applications.

[4]Sustainability aspects.

[5]Real-world data.

Article	Inventory problem	Number of suppliers	Product diversity	Time	Objectives	Policy	Constraint	DP-based approaches	Solution methods
	Structure				Decision and Policies				
DeCroix (2013)	I	MS	MI,BO	MP,IH,PR	C	O,H		DP	E: BI H: C
Bi et al. (2020)	I	MS	MI, BO, LS	SP, MP, FH, PR	P	O	B, L	DP	E: D, SP
Puranam et al. (2017)[3],[5]	I	MS	MI, LS	MP, FH, PR	C	O, H	L	DP	E: NA, SP H: VI
Lee et al. (2006)	I	MS	SI, BO	MP, FH, PR, LT	C	O, H	Z_1	DP	E: SP, D H: NS, E
Xu (2009b)	I	MS	SI, BO	MP, FH, IH, PR, LT	C	O	IC	DP	E: C, SA, SP
Chiang (2001)[3]	I	MS	SI, BO	MP, FH, IH, PR, LT	C	O		DP	E: C, SP
Sethi et al. (2003)	I	MS	SI, BO	MP, FH, IH, PR, LT	C	O		DP	E: SP
Chiang (2003)	I	MS	SI, BO	MP, IH, PR, LT	C	O		DP	E: SP, BS
Yeo and Yuan (2012)	I	MS	SI, BO	SP, MP, FH, IH, PR, LT	C	O	SC	DP	E: C, D, SP
Fox et al. (2006)	I	MS	SI, BO, LS	MP, FH, IH, PR	C	O		DP	E: SP
Zhang et al. (2012)	I	MS	SI, LS	MP, FH, IH, PR	C	O, H	OS, SC	DP	E: SSR, SP H: C
Zhou et al. (2011)[3],[5]	I	MS	SI, LS	SP, MP, FH, PR, LT	C	O	L	DP	E: BI H: C
Honhon et al. (2010)	I	SS	MI	SP, FH, PR	P	O, H		DP	E: C H: C
Beyer et al. (2001)	I	SS	MI, BO	MP, FH, IH, PR	C	O	IC	DP	E: SP
Frank et al. (2009)	I	SS	MI, BO	MP, FH, PR	C	O	L	DP	E: SP, SSR, NVB, DE
Chou et al. (2020)[3]	I	SS	MI, BO	MP, FH, PR	C	O		DP	E: BS
Inderfurth (1991)	I	SS	MI, BO	MP, FH, PR, LT	C	O	SL	DP	E: BI
M'Hallah et al. (2020)	I	SS	MI, BO	MP, IH, PR, LT	C	O		DP	E: FI
Agrawal and Smith (2019)[3],[5]	I	SS	MI, BO, LS	MP, FH, IH, PR	P	O		DP	E: SSR, LP, DLP
Chen et al. (2022b)[3],[5]	I	SS	MI, BO, LS	MP, FH, PR	P	O	L	DP	E: SP
Pourakbar and Dekker (2012)[4]	I	SS	MI, LS	MP, FH	C	O	L	DP	E: BI, NS, SP
Sharma et al. (2020)	I	SS	MI, LS	MP, FH, PR	C	O, H	IC, OS	DP	E: SP H: C
Li et al. (2017b)[4]	I	SS	MI, LS	MP, FH, PR	C	O	L	DP	E: SP, BI, D
Banerjee et al. (1985)	I	SS	SI	MP, FH, IH, CR	C	O		DP	E: NS
Nair and Anderson (2008)	I	SS	SI	MP, FH, PR	A	O, H	Z	DP	E: BI H: C
Graves and Willems (2008)	I	SS	SI	MP, FH, PR, LT	C	H	Z_1	DP	H: C
David et al. (1997)	I	SS	SI	MP, IH, CR	C	O	L	DP	E: BI, D, SP
Parlar and Rempala (1992)	I	SS	SI	SP, MP, PR	C	O		DP	E: BI, SP
Chen (1998)	I	SS	SI, BO	MP, FH, CR	A	O	L	DP	E: FI
Chen and Lin (2002)	I	SS	SI, BO	MP, FH, CR	C	O	L	DP	E: FI
Song and Lau (2004)	I	SS	SI, BO	MP, FH, CR, PR	C	O, H	L	DP	E: BI H: C
Sethi and Cheng (1997)	I	SS	SI, BO	MP, FH, IH, PR	C	O	IC, SL, Z_1	DP	E: SP
Yang et al. (2014b)	I	SS	SI, BO	MP, FH, IH, PR	C	O	OS	DP	E: SP
Henig et al. (1997)	I	SS	SI, BO	MP, FH, IH, PR	C	O	Z_1	DP	E: SP, HJM
Beyer et al. (1998)	I	SS	SI, BO	MP, FH, IH, PR	C	O		DP	E: SP
Flynn and Garstka (1990)	I	SS	SI, BO	MP, FH, IH, PR	C	O, H		DP	E: SP H: MY
Xue et al. (2017)	I	SS	SI, BO	MP, FH, IH, PR	P	O	IC	DP	E: SP
Parker and Kapuscinski (2004)	I	SS	SI, BO	MP, FH, IH, PR, LT	C	O	PC	DP	E: BS, D, DE, SP
Li (2013)[4]	I	SS	SI, BO	MP, FH, IH, PR, LT	C	O		DP	E: SP
Ehrhardt (1984)	I	SS	SI, BO	MP, FH, IH, PR, LT	C	O, H		DP	E: SP H: PA
Li et al. (2009)	I	SS	SI, BO	MP, FH, IH, PR, LT	C	O, H		DP	E: SP, BI H: MY
Li et al. (2013)	I	SS	SI, BO	MP, FH, PR	A	O	B	DP	E: SP, BI, D, MY
Guan and Liu (2010)	I	SS	SI, BO	MP, FH, PR	C	O	IC, PC	DP	E: BI, D, SP
Hosseinifard et al. (2022)	I	SS	SI, BO	MP, FH, PR	C	O	SL	DP	E: BI, NS, S
Rossi et al. (2011)	I	SS	SI, BO	MP, FH, PR	C	O	SL	DP	E: C, SSR
Khang and Fujiwara (2000)[3]	I	SS	SI, BO	MP, FH, PR	C	O		DP	E: D, SP
Güllü et al. (1999)	I	SS	SI, BO	MP, FH, PR	C	O		DP	E: SP, BI
Özen et al. (2012)	I	SS	SI, BO	MP, FH, PR	C	O, H	IC, SC, SL	DP	E: SP, BI, E H: BI, MY, R
Srinivasan and Duff (1989)	I	SS	SI, BO	MP, FH, PR, LT	A	O	SL	DP	E: BI
Angelus (2011)	I	SS	SI, BO	MP, FH, PR, LT	A	O, H		DP	E: SP, BI, SSR H: CSD
Huang and Van Mieghem (2014)[3]	I	SS	SI, BO	MP, FH, PR, LT	C	O		DP	E: BI
Xu (2009a)	I	SS	SI, BO	MP, FH, PR, LT	C	O, H	OC	DP	E: BM, SP H: MY
Xu (2011)	I	SS	SI, BO	MP, FH, PR, LT	C	O, H	SC, OS	DP	E: BI, BM H: MY
Cheaitou et al. (2009)	I	SS	SI, BO	MP, FH, PR, LT	P	O	OC	DP	E: DE, SP, D, NS

Zhao and Lian (2011)	I	SS	SI, BO	MP, IH, CR	C	O		DP	E: C, SP
Beyer and Sethi (1997)	I	SS	SI, BO	MP, IH, CR	C	O		DP	E: SP
Song and Dinwoodie (2008)	I	SS	SI, BO	MP, IH, CR, LT	C	O	OS	DP	E: SP, VI
Song and Zipkin (1996)	I	SS	SI, BO	MP, IH, LT (-FH)	C	O	PC	DP	E: SP, SSR
Beyer and Sethi (1999)	I	SS	SI, BO	MP, IH, PR	C	O	IC, SC	DP	E: SP
Van Foreest and Kilic (2023)	I	SS	SI, BO	MP, IH, PR	C	O		DP	E: DPB, NS
Özekici and Parlar (1999)	I	SS	SI, BO	MP, IH, PR	C	O		DP	E: SP, SAN
Ahmed et al. (2007)	I	SS	SI, BO	SP, MP, FH, CR	Z	O		DP	E: SP, BI
Yeo and Yuan (2011)	I	SS	SI, BO	SP, MP, FH, IH, PR	C	O		DP	E: C, D, SP
Arifoğlu and Özekici (2010)	I	SS	SI, BO	SP, MP, FH, IH, PR	C	O	SC	DP	E: SP, D, VI
Li and Ryan (2012)	I	SS	SI, BO	SP, MP, FH, IH, PR, LT	C	O		DP	E: SP, D, BI, NA
Reyman (1989)	I	SS	SI, BO	SP, FH, PR	C	O		DP	E: BI, SR
Zhou et al. (2009)	I	SS	SI, BO	SP, MP, FH, IH, PR	C	O, H	OS	DP	E: SP, VI, LP H: C, LP
Arifoğlu and Özekici (2011)	I	SS	SI, BO	SP, MP, FH, IH, PR	C	O	SC, Z_1	DP	E: SP, VI
Gavirneni et al. (1998)	I	SS	SI, BO	SP, MP, FH, IH, PR	C	O, H		DP	E: BI H: MY
Bao et al. (2018)	I	SS	SI, BO	SP, MP, FH, PR	C	O		DP	E: DE, SSR, BI, D
Bhaskaran et al. (2010)	I	SS	SI, BO, LS	MP, FH, CR	C	O		DP	E: SP, D
Chiang (2007)	I	SS	SI, BO, LS	MP, FH, IH, PR	C	O	OS	DP	E: CT, SP
Li et al. (2015)	I	SS	SI, BO, LS	MP, FH, IH, PR	P	O, H	IC	DP	E: D, SP H: NVB
Chan and Song (2003)	I	SS	SI, BO, LS	MP, FH, PR	C	O	IC, SC	DP	E: C
Ozyoruk et al. (2022)[4]	I	SS	SI, BO, LS	MP, FH, PR	C	O	L, SC	DP	E: BI, SP
Sakaguchi (2009)	I	SS	SI, BO, LS	MP, FH, PR	C	O		DP	E: C, SP
Xiang et al. (2023)	I	SS	SI, BO, LS	MP, FH, PR	C	O		DP	E: VI
Kunnumkal and Topaloglu (2008b)	I	SS	SI, BO, LS	MP, FH, PR	C	O, H		DP	E: SP H: C
Pang and Xiao (2022)	I	SS	SI, BO, LS	MP, FH, PR	P	O		DP	E: BI, D, SP
Gökbayrak and Kayış (2023)[4]	I	SS	SI, BO, LS	MP, FH, PR	P	O		DP	E: SP
Halman et al. (2009)	I	SS	SI, BO, LS	MP, FH, PR, LT	C	H	IC	DP	H: FPTAS
Fisher et al. (2001)[3],[5]	I	SS	SI, BO, LS	MP, FH, PR, LT	C	H	L	DP	H: NVB
Riezebos and Gaalman (2009)	I	SS	SI, BO, LS	MP, FH, PR, LT	C	H		DP	H: MY
Benjaafar et al. (2018)	I	SS	SI, BO, LS	MP, FH, PR, LT	C	O		DP	E: SP, BI
Chiang (2013)	I	SS	SI, BO, LS	MP, IH, PR, LT	C	O		DP	E: SP
Chiang (2008)	I	SS	SI, BO, LS	MP, IH, PR, LT	C	O, H		DP	E: C, D H: C
Miller (1986)	I	SS	SI, BO, LS	SP, FH, PR	C	O		DP	E: SSR, BI
Gurnani (1996)	I	SS	SI, BO, LS	SP, MP, FH, PR	C	O		DP	E: D
Liu et al. (2018)[3]	I	SS	SI, LS	MP, FH, CR	P	O, H	L	DP	E: SP, BI, NS H: G, NVB
Caliskan-Demirag et al. (2013)	I	SS	SI, LS	MP, FH, IH, PR	C	O, H		DP	E: BS, SP H: C
Gong et al. (2014)	I	SS	SI, LS	MP, FH, PR	A	O	B	DP	E: SP, D, BI
Kouvelis et al. (2018)	I	SS	SI, LS	MP, FH, PR	A	O		DP	E: SP, D, BI
Anderson and Sibdari (2012)[3],[5]	I	SS	SI, LS	MP, FH, PR	P	O		DP	E: SP, BI
Gao (2015)	I	SS	SI, LS	MP, FH, PR	P	O		DP	E: SP, BI
Li et al. (2016)	I	SS	SI, LS	MP, FH, PR	P	O, H	L	DP	E: SP, NS H: MY
Xu et al. (2011)	I	SS	SI, LS	MP, FH, PR	P	O, H		DP	E:D, SP H: MY
Chen et al. (2014)	I	SS	SI, LS	MP, FH, PR, LT	C	H		DP	H: BI, ECA
Chen and Sapra (2021)	I	SS	SI, LS	MP, FH, PR, LT	C	O, H	L	DP	E: SP, D, BI, NS H: C
Lee and Hersh (1993)[3]	I	SS	SI, LS	SP, MP, FH	P	O	IC	DP	E: BI
Tao et al. (2017)	I	SS	SI, LS	SP, MP, FH, PR	P	O		DP	E: D, PI, SP, VI
Feng and Xiao (2001)[3]	I	SS	SI, LS	SP, MP, FH, CR	P	O	IC	DP	E: D, SP
Li (2015)	I	SS	SI, LS	SP, MP, FH, IH, PR, LT	C	O		DP	E: VI, SA, D, SP
Liu et al. (2015)	I	SS	SI, LS	SP, MP, FH, PR	Z	O	SC	DP	E: SP, D, BI
Chen and Moinzadeh (2018)	I	SS, MS	SI, BO, LS	MP, FH, CR, LT	P	O, H		DP	E: D, S H: BI, C
Wang et al. (2010)	I	MS	SI, LS	MP, FH	C	O, H	SC	BDP	E: D, SP, VI H: MY
Wang and Mersereau (2017)	I	SS	SI, BO	MP, FH, PR	C	O, H		BDP	E: BI, BS, SP H: MY, RO, S
Kamath and Pakkala (2002)	I	SS	SI, BO	MP, FH, PR	P	O		BDP	E: BS, C
Azoury (1985)	I	SS	SI, BO	MP, FH, PR, LT	C	O		BDP	E: SSR, SP, BI
Bensoussan et al. (2010)	I	SS	SI, BO	MP, IH, PR	C	O		BDP	E: SP, SSR, VI
Lariviere and Porteus (1999)	I	SS	SI, LS	MP, FH, IH, PR	P	O, H		BDP	E: SP, BI, SSR H: MY
Chen (2010)	I	SS	SI, LS	MP, FH, PR	C	O, H		BDP	E: BS, SP H: BS, D
Kadiyala et al. (2020)	I	SS	SI, LS	MP, FH, PR	P	O		BDP	E: BI, D
Bensoussan et al. (2008)	I	SS	SI, LS	MP, IH, CR	C	O		BDP	E: SP, VI
Bensoussan and Guo (2015)	I	SS	SI, LS	MP, IH, PR	C	O		BDP	E: SP, SSR
Luo et al. (2023)[3]	I		SI, LS	SP, MP, FH	R	O, H	IC	BDP	E: MY, NA H: DD, MY

Farahvash and Altiok (2011)	I	MS	SI, BO	MP, FH, PR, LT	C	H		HDP	H: BI, S
Qi et al. (2023)[3,5]	I	SS	SI, BO	MP, FH, PR, LT	C	H		HDP	H: DD, SP
Liu (1999)	I	SS	SI, BO, LS	MP, FH, CR	Z	O		HDP	E: SP, BI
Reyes-Aldasoro et al. (1999)[3,5]	I	SS	SI, LS	MP, FH, PR, LT	C	H	SL	HDP	H: DD
Bensoussan et al. (2016)	I	SS	SI, LS	MP, IH, PR	P	O, H	SL	BDP, ADP	E: SP, SSR H: AVI, MY
Mersereau (2013)	I	SS	SI, LS	SP, MP, FH, PR	C	O, H	SL	BDP, ADP	E: BI H: BI, NS, MY
Kouvelis et al. (2023)[3]	I	SS	MI	MP, FH, IH, PR	P	O, H	SC, Z_1	DP, ADP	E: SP, D, BI H: RO
Iida and Zipkin (2006)	I	SS	SI, BO	MP, FH, PR, LT	C	O, H		DP, ADP	E: BS H: C, MY
Lin et al. (2020)	I	SS	SI, BO, LS	MP, IH, PR, LT	C	O, H	IC, OS	DP, ADP	E: NA H: ALP, NS
Chen et al. (2021)[3]	I	SS	SI, LS	MP, FH, PR	C	O, H	L	DP, ADP	E: SP, NA H: AA
Gijsbrechts et al. (2022)	I	SS, MS	SI, BO, LS	MP, IH, PR, LT	C	O, H		DP, ADP	E: NS H: DD
Chen and Rossi (2021)	I	MS	SI, LS	MP, FH, PR	A	O, H	B	DP, HDP	E: SP, BI, NA H: LP, C
Gutierrez-Alcoba et al. (2017)	I	SS	SI, BO	SP, MP, FH, PR	C	O, H	L	DP, HDP	E: BI H: SM, S
Mayorga and Ahn (2011)	IP	MS	SI, BO	MP, FH, IH	C	O, H	PC	DP	E: SP, VI H: C
Song et al. (2014)	IP	MS	SI, BO	MP, IH, CR, LT	C	O	PC, SC	DP	E: SP, VI
Song et al. (2017)	IP	MS	SI, BO	MP, IH, CR, LT	C	O, H		DP	E: PI, SP H: C
Li and Wang (2017)	IP	MS	SI, MI, BO	SP, MP, IH, CR, LT	C	O, H	PC, OS	DP	E: MVT H: DPB
Aviv and Federgruen (2001)[3,5]	IP	SS	MI, BO	MP, FH, IH, PR	C	H	PC	DP	H: R, VI, C, S
Liu et al. (2020a)	IP	SS	MI, BO	MP, FH, IH, PR, LT	P	O		DP	E: SP, BI
Raman and Kim (2002)[3,5]	IP	SS	MI, BO	MP, FH, PR	C	O, H	PC	DP	E: NA H: RO
Karamatsoukis and Kyriakidis (2009)	IP	SS	SI	MP, FH, CR	C	O	IC	DP	E: FI, SP
Wang (2012)	IP	SS	SI	MP, FH, PR, LT	C	O		DP	E: BI, E
Wang and Zhu (2021)	IP	SS	SI	MP, IH, PR	C	O	IC	DP	E: E, SP, VI
Kapuściński and Tayur (1998)	IP	SS	SI, BO	MP, FH, IH, PR	C	O	PC	DP	E: SP, MY, S, IPA
Özer and Wei (2004)	IP	SS	SI, BO	MP, FH, IH, PR, LT	C	O	PC	DP	E: BI, SP
Feng et al. (2011)	IP	SS	SI, BO	MP, FH, PR	C	O, H	PC	DP	E: SP, NA H: IS
Klosterhalfen et al. (2013)	IP	SS	SI, BO	MP, FH, PR	C	O, H	SL	DP	E: NA H: BI, DE, IS
Veatch and Wein (1994)	IP	SS	SI, BO	MP, IH, CR	C	O, H		DP	E: C
Pac et al. (2009)	IP	SS	SI, BO	SP, MP, FH, IH, PR	C	O	PC	DP	E: D, NS, SP
Wang et al. (2014)	IP	SS	SI, BO	SP, MP, FH, PR, LT	C	O, H	IC, PC	DP	E: BI, D, NS, SP H: C
Flapper et al. (2012)[4]	IP	SS	SI, BO, LS	MP, IH, LT	C	O		DP	E: SP, VI
Wang and Zhang (2021)	IP	SS	SI, LS	MP, IH, CR	C	O	PC	DP	E: SP, VI
Jensen et al. (1991)	IP	SS	SI, MI, LS	MP, FH, CR	C	O	IC	DP	E: BI, DE, E
Lawrence and Schaefer (1984)	IP	SS	SI, MI, LS	SP, FH	C	O	B	DP	E: E, NA
Smith and Schaefer (1985)	IP	SS	SI, MI, LS	SP, FH	C, Z	O, H	B	DP	E: NA H: MA
Kiesmüller and Scherer (2003)[4]	IP	MS	SI, BO	MP, FH, PR, LT	C	O, H		DP, ADP	E: SP, D, NS H: LP
Yang et al. (2022)[3,5]	IP	SS	MI, LS	MP, FH, PR	C	O, H	IC, PC, SC, L, Z_1	DP, ADP	E: NA H: AVI, RL
Çimen and Kirkbride (2017)	IP	SS	MI, LS	MP, IH, PR	C	O, H	PC, IC	DP, ADP	E: VI H: TD, QL, S
Schaefer (1983)	IS	SS	MI, LS	MP, FH, CR	Z	O, H	B, SL	DP	E: FI H: MA
Liu (1989)	IS	SS	SI	MP, FH, PR	C	O	IC, Z_1	DP	E: BI
Asadi and Nurre Pinkley (2022)[3]	IS	MS	SI	MP, FH, CR	P	O, H	IC, SC, Z	DP, ADP	E: SP, BI H: DD
Valeva et al. (2021)[4]	IS	SS	MI, LS	MP, FH, PR	P	O, H	Z, PC	DP, ADP	E: NA H: RO
Khademi and Eksioglu (2018)[3,5]	IS	SS	MI, LS	MP, IH, PR, LT	C	O, H	IC, Z	DP, ADP	E: NA H: API, BS, VI
Glazebrook et al. (2015)	ID	MS	MI, BO, LS	MP, FH, PR	C	H		DP	H: MY
Chou et al. (2013)[3]	ID	MS	MI, LS	SP, MP, FH, PR	P	O		DP	E: SP, D
Van Wijk et al. (2019)	ID	MS	MI, LS, BO	MP, IH, CR, LT	C	O	PC	DP	E: SP
Zhang et al. (2022)[3]	ID	MS	SI	MP, FH	C	O, H		DP	E:, BI, SP H: C, MY
Xiao et al. (2009)[3]	ID	MS	SI	MP, FH,	R, P	O		DP	E: BI, SP
Li et al. (2022)	ID	MS	SI	MP, FH, PR	P	O, H		DP	E: SP, NS H: C
Hu et al. (2005)[3]	ID	MS	SI, BO	MP, FH, PR	C	O, H		DP	E: NA H: MY, SA
Shen et al. (2022)	ID	MS	SI, BO	MP, FH, PR, LT	C	O, H	SL	DP	E: SP, C, BI, SSR, CSD H: MY
Paterson et al. (2012)	ID	MS	SI, BO	MP, IH, CR, LT	C	O, H	IC	DP	E: VI H: MY
Dong and Transchel (2020)[3,5]	ID	MS	SI, BO	MP, IH, PR, LT	C	O		DP	E: SP, PI, BM, E
Abouee-Mehrizi et al. (2015)	ID	MS	SI, LS	MP, FH, PR	C	O	IC	DP	E: SP, BI, D
Olsson (2009)	ID	MS	SI, LS	MP, IH, CR, LT	C	O		DP	E: PI
Archibald (2007)	ID	MS	SI, LS	SP(MP epochs), FH, PR	C	O, H	IC	DP	E: SP, NA, H: MY
Archibald et al. (2009)[3]	ID	MS	SI, LS	MP, FH, PR	C	O, H		DP	E: NA H: DE
Archibald et al. (1997)[3]	ID	MS	SI, MI, LS	MP, IH, PR	C	O, H	IC	DP	E: SP, VI H: BM
Bitran and Mondschein (1996)[3,5]	ID	SS	BO LS	MP, FH, PR	P	H	B	DP	H: C, SA
Chen et al. (2022c)[3,5]	ID	SS	MI	MP, FH, PR	R	H	IC	DP	H: MY
Ahn et al. (2018)[3]	ID	SS	MI, BO, LS	MP, FH	C	O, H		DP	E: NA, RO

McCoy and Brandeau (2011)[3]	ID	SS	MI, LS	SP, MP, FH, PR	S	O, H	B	DP	E: D H: NS, S
Natarajan and Swaminathan (2017)[3],[5]	ID	SS	SI , BO, LS	MP, FH, PR	Z	O, H	B	DP	E: SP, NA H: C
Pishchulov and Richter (2009)	ID	SS	SI, BO	MP, FH, PR	P, S	O	SC	DP	E: BI
Zipkin (1984)	ID	SS	SI, BO	MP, FH, PR, LT	C	H		DP	H: MY, R
Federgruen and Zipkin (1984b)	ID	SS	SI, BO	MP, FH, PR, LT	C	H		DP	H: MY, R, SSR, BI
Federgruen and Zipkin (1984a)	ID	SS	SI, BO	MP, IH, PR, LT	C	H		DP	H: R, MY
Nambiar et al. (2021)	ID	SS	SI, LS	MP, FH, PR	C	O, H		DP	E: BI, H: R, LR
Agrawal et al. (2004)	ID	SS	SI, LS	SP(MP), FH, PR, LT	C, Z	O, H	Z_1	DP	E: SP, BI H: S, MY
Love (1985)[3]	ID	SS	SI, LS	SP, MP, FH, IH, PR	C	O, H	IC	DP	E: NA H: C
Agrawal and Smith (2013)footnotemark[3],[5]	ID	SS	SI, LS	MP, FH, PR	P	O	IC	BDP	E: SP, D, SSR
Archibald et al. (2010)[3]	ID	MS	SI	MP, FH, PR, LT	C	O, H		DP, ADP	E: VI, NS H: S
Wang et al. (2021b)footnotemark[3],[5]	ID	MS	SI, BO	MP, IH, PR, LT	C	O, H		DP, ADP	E: NS, SP H: ALP, IPM, NVB, NS, S
Benjaafar et al. (2022)[3],[5]	ID	MS	SI, LS	MP, FH, IH, PR	C	O, H		DP, ADP	E: SP, BI, NA H: C
Meissner and Senicheva (2018)	ID	MS	SI, LS	MP, FH, PR	P	O, H		DP, ADP	E: DP, BI H: ALP, C
Kunnumkal and Topaloglu (2008a)	ID	SS	SI, BO	MP, FH, PR, LT	C	O, H		DP, ADP	E: NA H: LR, DE, BI, NS
Kunnumkal and Topaloglu (2011)	ID	SS	SI, BO	MP, FH, PR, LT	C	O, H	SC	DP, ADP	E: NA H: LR, DE
Topaloglu and Kunnumkal (2006)	ID	SS	SI, LS	MP, FH, PR	P	O, H	IC, PC	DP, ADP	E: NA H: ALP, BDE, LR
Berman and Larson (2001)[3]	IR	SS	SI	SP, FH	C	O	IC, Z_1	DP	E: SP, BI
Bouma and Teunter (2016)	IR	SS	SI, BO	SP, FH	C	O		DP	E; BI, NS
Brinkmann et al. (2019)[3],[5]	IR	MS	SI, LS	MP, FH, PR, LT	S	O (DP), H(ADP)	IC, Z_1	DP, ADP	BI, RO
Adelman (2004)	IR	SS	MI, BO, LS	MP, IH, PR	C	O, H	IC, Z_1	DP, ADP	E: LP, NA H: ALP, E
McKenna et al. (2020)[3],[5]	IR	SS	SI	MP, IH, PR	Z	O, H	IC, Z_1	DP, ADP	E: NA H: API, TD
Kleywegt et al. (2004)[3]	IR	SS	SI, BO, LS	MP, IH, PR	P	O (DP), H (ADP)	IC, Z_1	DP, ADP	E: NA H: NS, DE, LS
Bertazzi et al. (2015)	IR	SS	SI, LS	MP, FH, PR	C	O, H	IC, Z	DP, HDP	E: BI H: MH
Adida and Perakis (2010)	IPr	SS	MI	MP, FH, PR	P	O	IC	DP	E: BI, R
Fang et al. (2021)footnotemark[3],[5]	IPr	SS	MI, BO	MP, FH	P	O		DP	E: SP, BI, LR, IS
Zhu and Thonemann (2009)	IPr	SS	MI, BO	MP, FH, IH, PR	P	O		DP	E: BI, D
Xu et al. (2020)	IPr	SS	MI, BO	MP, FH, PR	P	O		DP	E: D, IS, SP
Dong et al. (2009)	IPr	SS	MI, LS	MP, FH	P	O, H		DP	E: SP, D H: C
Xu et al. (2016)	IPr	SS	MI, LS	MP, FH, PR	P	O, H	L	DP	E: BI, SP H: C
Zhu (2012)	IPr	SS	SI, BO	MP, FH, IH, PR	P	O	SC	DP	E: BI, D, SP
Yang and Zhang (2022)[3]	IPr	SS	SI, BO	MP, FH, IH, PR	P	O, H		DP	E: SSR, SP H: MY, RO
Güler et al. (2015)	IPr	SS	SI, BO	MP, FH, IH, PR	P	O	IC	DP	E: SP, BI, DE
Chen et al. (2022a)	IPr	SS	SI, BO	MP, FH, PR	P	H		DP	H: OL
Yang et al. (2014a)	IPr	SS	SI, BO	MP, FH, PR	P	O	SC	DP	E: BI, SP
Feng et al. (2014)footnotemark[3],[5]	IPr	SS	SI, BO	MP, FH, PR	P	O		DP	E: SP, D, BI
Qin et al. (2022)	IPr	SS	SI, BO	MP, FH, PR	P	O, H		DP	E: BI H: BI, DD
Lu et al. (2014)	IPr	SS	SI, BO	MP, FH, PR, IH	P	O		DP	E: SP, BI, D
Van Ryzin and Vulcano (2004)	IPr	SS	SI, BO	MP, IH, PR	P	O		DP	E: SP
Wang et al. (2021a)	IPr	SS	SI, BO	SP, MP, FH, PR	P	O		DP	E: D, SP
Hu et al. (2019)	IPr	SS	SI, BO	SP, MP, FH, PR	P	O, H		DP	E: SP, BI H: C
Cheng and Sethi (1999)	IPr	SS	SI, BO, LS	MP, FH, IH, PR	P	O, H		DP	E: SP, BI H: MY
Bensoussan et al. (2019)	IPr	SS	SI, BO, LS	MP, FH, IH, PR	P	O		DP	E: D, SP, BI
Shen et al. (2018)	IPr	SS	SI, BO, LS	MP, FH, PR	P	O		DP	E: BI, D, SP
Zhou et al. (2023)	IPr	SS	SI, BO, LS	MP, FH, PR	P	O		DP	E: D
Chen et al. (2006)	IPr	SS	SI, BO, LS	SP, MP, FH, PR	P	O		DP	E: SP, D
You (2003)	IPr	SS	SI, LS	MP, FH	P	O		DP	E: SP
Chew et al. (2009)	IPr	SS	SI, LS	MP, FH, PR	P	O, H		DP	E: D, SP, BI H: C, MY
Liu et al. (2020b)	IPr	SS	SI, BO, LS	MP, FH, PR	P	O		BDP	E: D, SP, SSR, MY
García-Alvarado et al. (2015)[4]	In	MS	SI, BO, LS	MP, FH, PR, LT	C	O, H	IC, PC, Z_1	DP	E: VI H: GA
Bhatnagar and Lin (2019)	In	MS	SI, BO, LS	MP, IH, CR	C	O, H		DP	E: SP, VI H: PI, DE, BM, RO
Lin et al. (2023)[3],[5]	In	MS	SI, MI, BO, LS	MP, FH, PR	C	O, H	IC	DP	E: VI H: DE
Li and Mizuno (2022)	In	SS	SI, BO	MP, FH, PR	P	O		DP	E: SP, BI, DE
Turan et al. (2017)	In		MI	MP, FH, PR, LT	P	H	Z_1	HDP	H: LS, BI
Malladi et al. (2020)	In	SS	SI, BO	MP, FH, PR	C	O, H	PC, Z_1	DP, ADP	E: BS, VI H: AVI, MY, RO, DE
Li and Yu (2014)	I, ID	SS, MS	SI, BO, LS	MP, FH, PR, LT	C, P	O	L, SC	DP	E: SP
Minoux (2018)	I, IP	SS	SI, LS (IP: BO)	MP, FH, PR	P	O	IC IP (IC, PC)	DP	E: BI
Secomandi (2008)	I, IP	SS	MI, LS	MP, FH, PR, LT	P	H		ADP	H: CA

Chen and Plambeck (2008)	I, IP	SS	MI, LS	MP, IH, PR, IP(CR, LT)	P	O, H	IP (PC)	BDP	E: SP, BI, NS (SP) H: MY (MLE)
Huh et al. (2011)	I, IPr	SS	SI, BO, LS	MP, IH, PR, LT	C, P(for IPr)	O	IC	DP	E: SP

Table A5: Characterization of articles that entail stochastic models.

Article	Inventory problem	Number of suppliers	Product diversity	Time	Objectives	Policy	Constraint	DP-based approaches	Solution methods
	Structure				Decision and Policies				
Turgay et al. (2018)[3]	I	SS	SI	MP,FH,CR	P	O*		DP	E: SP
Xin and Goldberg (2022)	I	SS	SI,BO	MP,FH,CR	C	O		DP	E: BI
Xin and Goldberg (2021)	I	SS	SI,BO	MP,FH,CR	C	O		DP	E: D, SP
Qiu et al. (2017)	I	SS	SI,BO	MP,FH,PR	C	O		DP	E: DLP, BI , SP
Chen et al. (2023)	I	SS	SI,BO,LS	MP,FH,PR,LT	C	O,H		DP	E: BI, H: RO
Turgay et al. (2015)	IP	SS	SI,LS	MP,FH,CR	P	O		DP	E: SP
Minoux (2018)	I, IP	SS	SI	MP,FH,PR	P	O	IC	DP	E: BI

Table A6: Characterization of articles that entail robust models.

Inventory Problem	Solution Type	DP	ADP	HDP	SUM
I	Exact	[FI (6), WW (1)]		[BI (1)]	8
	Heuristic	[BI (1), C(1), FI (1), MY (1)]			4
IP	Exact	[BI (4), FI (7), SSR (1)]			12
	Heuristic				0
IS	Exact	[BI (1), FI (2), WW (2)]			5
	Heuristic	[BDP (1), FI (1)]			2
ID	Exact				0
	Heuristic	[BI (2)]			2
IR	Exact	[NA (1)]			1
	Heuristic		[ALP (1), C (1), MIP (1)]	[C (1)]	4
IPr	Exact	[FI (1)]			1
	Heuristic			[C(1)]	1
In	Exact	[FI (2), SSR (1)]			3
	Heuristic				0
SUM	Exact	29	0	1	30
	Heuristic	8	3	1	13

Table A7: Solution methods applied to DP deterministic

Inventory Problem	Solution Type	DP	ADP	BDP	HDP	SUM
I	Exact	[BI (40), BM (2), BS (5), C (10), CT (1), D (28), DE (4), DLP (1), DPB (2), E (1), FI (3), HJM (1), IS (1), LS (2), MY (1), NA (5) NS (9), NVB (1), PI (1), SA (2), SAN (1), SP (77), SSR (9), VI (7)]		[BI (6), BS (3), C (1), D (2) , MY (1), NA (1), NS(1) SP (9), SSR (4), VI (3)]	[BI (1), SP (1)]	247
	Heuristic	[BI (3), C (13), CSD (1), E (1), ECA (1), FPTAS (1), G (1), IS (1), LP (1), MY (9), NS (1), NVB (3), PA (1), R (1), VI (1)]	[AA (1), ALP (1), AVI (1), BI (1), C (1), CA (1), MY (3), NA (2), RO (1)]	[BS (1), D (1), DD (1), MY (5), RO (1), S (1)]	[BI (1), DD (2), S (2), SP (1)]	67
IP	Exact	[BI (6), C (1), D (3), DE (1), E (4), FI (1), IPA (1), MVT (1), MY (1), NA (6), NS (3), PI (1), S (1), SP (15), VI (6)]		[SP (1)]		51
	Heuristic	[BI (1), C (4), DE (1), DPB (1), IS (2), MA (1), R (1), RO (1), S (1), VI (1)]	[AVI (1), CA(1), LP (1), RL (1), S (1), TD (1), QL (1)]	MLE (1)		22
IS	Exact	[BI (2), FI (1), NA (2), SP (1)]				6
	Heuristic	[MA (1)]	[API (1), BS (1), DD (1), RO (1), VI (1)]			6
ID	Exact	[BI (9), BM (1), C (1), CSD (1), D (3), DP (1), E (1), NA (10), NS (3), PI (2), SP (15), SSR (1), VI (3)]		[D (1), SP (1), SSR (1)]		54
	Heuristic	[BI (1), BM (1), C (5), DE (1), LR (1), MY (11), NS (1), R (4), RO (1), S (2), SA (2), SSR (1)]	[ALP (3), BDE (1), BI (1), C (2), DE (2), IPM (1), LR (3), NS (2), NVB (1), S (2)]			49
IR	Exact	[BI (4), LP (1), NA (3), NS (1), SP (1)]				10
	Heuristic		[ALP (1), API (1), DE (1), E (1), LS (1), MH (1), NS (1), RO (1), TD (1)]		[LS (1)]	10
IPr	Exact	[BI (15), D (12), DE (1), IS (2), LR (1), R (1), SP (20), SSR (1)]		[D (1), MY (1), SP (1), SSR (1)]		56
	Heuristic	[BI (1), C (4), DD (1), MY (3), OL (1), RO (1)]				11
In	Exact	[BI (1), BS (1), DE (1), SP (2), VI (4)]				9
	Heuristic	[BM (1), DE (2), GA (1), PI (1), RO (1)]	[AVI (1), DD (1), MY (1), RO (1)]		[BI (1), LS (1)]	12
SUM	Exact	392	0	39	2	433
	Heuristic	102	55	11	9	177

Table A8: Solution methods applied to DP stochastic models

Inventory Problem	Solution Type	DP	SUM
I	Exact	[BI (4), D (1), DLP (1), SP (3)]	9
	Heuristic	[RO (1)]	1
IP	Exact	[BI (1),SP (1)]	2
	Heuristic		0
SUM	Exact	11	11
	Heuristic	1	1

Table A9: Solution methods applied to DP robust models

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Appendix B: Online Supplementary Material

Notions on Dynamic Programming-based Structures

Since this article is dedicated to DP-based formulations in inventory problems, it is logical to introduce some notions on DP and the branches we explore, once several fundamental terminologies are replicated throughout the text and it is crucial to establish a link between those and inventory characteristics. In order to accomplish this, we divide this section into the four main types of DP detected in the methodology section: DP, ADP, BDP and HDP.

B.1. Dynamic Programming

Dynamic programming, as defined in Bellman (1954), is an implicit enumeration problem-solving method, which divides a complex problem into simpler structures, allowing for sub-optimal solutions that converge to the optimal value for the initial problem. For a problem to be eligible as a DP formulation, the following specific identifiable elements must be present:

- Stage (t): Stages correspond to the order in which sub-problems are solved and also represent the distance from the targeted solution for the global problem. Since DP is suitable for multi-stage environments, it can be directly associated with the planning horizon. Generally, for inventory problems, they are equivalent to time periods, which are also valuable when distinguishing discrete-time and continuous-time settings. Furthermore, forward and backward approaches can be identified by stages.

- State (x_t): Variable that represents the dynamic element within the DP structure and is influenced by decisions made in previous stages. Depending on the problem, one or multiple states might affect the dynamic nature of the problem. They are also useful when indicating whether the main variables are deterministic or stochastic. Mathematically, a general dynamic optimization problem is defined by:

$$\min_{u_t \in U_t} \sum_{t=0}^{T} g_t(x_t, u_t) \tag{1}$$

$$x_{t+1} = f_t(x_t, u_t), \quad t = 0, \ldots, T-1 \tag{2}$$

$$x_0 \in \mathbb{R} \tag{3}$$

where Eq. 1 defines the general cumulative objective function that depends on the state variable x_t and $u_t \in U_t$, which represents a decision made internally to alter the current state. Eq. 2 designates the motion of the system and, therefore, the behavior of the state variable. Eq. 3 corresponds to the initial state of the system. By considering the simplest scenario in IM, x_t could denote the inventory level in period t and u_t, the ordering decision at the end of the same period.

• Functional equation: Also known as Bellman equation, it describes the sequence of value functions that satisfies the set of equations Eq.(1) - (3). A general structure is given as follows:

$$V_t(x_t) = \min_{u_t \in U_t} \{g_t(x_t, u_t) + V_{t-1}(x_{t-1})\}, \quad t = 1, \ldots, T \tag{4}$$

$$V_0(x_0) = c, \quad c \in \mathbb{R} \tag{5}$$

Note that a recursive relationship is employed to address optimal solutions for preceding sub-problems, aligning with Bellman's Principle of Optimality. This principle suggests that the optimal solution at each stage can be constructed based on previously determined optimal values from less complex sub-problems. As is customary with recursions, an initial value is necessary to initiate the computation (in Eq. 5, the initial value is denoted by c). In the realm of IM, functional equations can, for instance, encapsulate the dynamics of cost functions.

• Policy (π): It is defined as a set that contains the decisions that have been made since the first stage up to the last and is mathematically described as $\pi_k = \{u_{1k}, u_{2k}, \ldots, u_{Tk}\}$, where k defines the cardinality of the set. Clearly, among the k policies, there exists one that leads to the optimal value for the objective function, which is characterized as an optimal policy and is defined by:

$$\pi^* = \underset{x_t \in X_t}{\arg\min}\, V^*(x_T). \tag{6}$$

Therefore, policies in inventory could be related to the processing of orders at each stage and, consequently, the optimal policy is that which minimizes the cost throughout the planning horizon.

The presented structure embodies necessary conditions directly related to the essence of DP, encapsulated by divisibility and recursion. Notably, dynamic programming offers the dual advantage of serving as a modeling framework through functional equations and a solution methodology. It can potentially yield closed-loop formulas, a desirable outcome in the fields of MS and OR, providing valuable insights into optimal policies. Additionally, due to its versatile framework, DP includes embedded tools for analyzing structural properties in both unconstrained and constrained optimization. On the other hand, it grapples with the curse of dimensionality, manifested through an exponential growth in state space complexity, rendering it intractable (Truong 2014). To address this problem, researchers employ various strategies, including integration with other optimization methods and the proposition of state space reduction techniques (discussed in the following sections).

As one can notice, the aforementioned equations are within a class of discrete models. Although we will not scrutinize all the possible branches of DP in this article, one must comprehend that some analogies are quite straightforward for the continuous DP problem, once the integral operator replaces the summation operator over discrete time, and similarly, the difference equation is substituted by a differential one. Clearly, this increases the complexity of the optimization problem and, as expected, the same will occur to the analogous version of Bellman equation, which is notably known as Hamilton-Jacobi-Bellman equation and demands a much more complex structure. For further information, refer to Bellman (1954).

Additionally, the general DP that has been addressed comprises a class of problems that belong to **deterministic** approaches. Nevertheless, several problems also consider the presence of noises or random variables and how they modify the dynamic system. Therefore, borrowing the notations from the previous model, a general **stochastic** DP model is of the form:

$$\min_{u_t \in U_t} \mathbb{E}\left[\sum_{t=0}^{T} g_t(x_t, u_t, \eta_{t+1})\right] \tag{7}$$

$$x_{t+1} = f_t(x_t, u_t, \eta_{n+1}), \quad t = 0, \ldots, T-1 \tag{8}$$

$$\mu(u_t) \subset \mathcal{M}_{\sqcup} := \mu(\eta_0, \ldots, \eta_t) \tag{9}$$

$$x_0 \in \mathbb{R} \tag{10}$$

and therefore, the stochastic version of Bellman equation is given by:

$$V_t(x_t) = \min_{u_t \in U_t} \mathbb{E}[g_t(x_t, u_t, \eta_{t+1}) + V_{t-1}(x_{t-1})], \quad t = 1, \ldots, T \tag{11}$$

$$V_0(x_0) = c, \quad c \in \mathbb{R} \tag{12}$$

In the model above, η_t is defined as a random, independent and exogenous variable that model the system dynamics, and according to Eq. 9, the choice of u_t is also contingent on the effect of such variable. In the IM scenario, there are several possibilities that η_t can replace, such as demand, lead time, system failures, among others.

Stochastic DP models are appropriate in occasions where the random variable has a well-defined distribution and/or its parameters are known in advance. Inventory problems, as we will see in the following sections, are majorly approximated through classic probability distributions or essential parameters. This is an advantage once the likelihood of obtaining closed-form expressions for known elements is higher than for those that display unknown characteristics. However, by including random variables in the system, an additional type of curse of dimensionality surfaces, which is the main issue faced by researchers when modeling via stochastic DP (Ross 2014).

In addition to widely used deterministic and stochastic models, there exists a third class of approach that seeks to address aspects beyond their scope. These are known as **robust** models, distinguishing themselves from stochastic models by not relying on known transition probabilities, namely ambiguity. To accommodate this modeling challenge, uncertainty sets are introduced to estimate uncertain parameters, giving rise to an equivalent of the Bellman equation seen below, which incorporates non-parametrized information inherent to the problem.

$$V_t(x_t) = \max_{u_t \in U_t} \left\{ \min_{p \in \mathcal{P}(x_t, u_t)} \mathbb{E}^p \left[g_t(x_t, u_t) + V_{t-1}(x_{t-1}, h_t)\right] \right\}, \quad t = 1, \ldots, T \tag{13}$$

In this context, h_t denotes the available historical information without defined parameters and $u_t := u_t(x_t, h_t)$. The max-min formulation is attributed to the fact that such models can also be interpreted as a game between two adversaries in which once a decision maker selects a given policy, the adversary chooses a set of given measures that consists of certain probabilities $p \in \mathcal{P}(\S_\sqcup, \sqcap_\sqcup)$ that minimize the reward (Iyengar 2005). This type of formulation, although more complex than the ones previously seen, contributes for more realistic scenarios, in which full information is rarely provided to the decision maker. Analogies can be made, for example, with models where demand depends on information regarding past sales.

B.2. Approximate Dynamic Programming

Despite its versatility and potential results, DP tends to reside in a more theoretical domain, largely due to the curse of dimensionality inherent in the Bellman recursive equation. Deterministic problems may face up to two types of curses related to the evolution of state and decision spaces over time. Stochastic and robust models introduce an additional curse in computing expected succeeding value functions, which may explode given a large outcome space (Powell 2007).

Recent research has focused on finding alternatives to efficiently address or entirely overcome DP's dimensionality issues, particularly through methods such as Approximate Dynamic Programming (ADP).

According to Powell (2016), ADP is defined as an MDP-based modeling structure that aims to weaken the consequences of the three curses of dimensionality by essentially replacing the original DP value function by an statistical approximation function denoted by $\hat{V}_t(x_t)$. In addition to that, for a given initial state x_0, a specific sample path $\theta \in \Theta$ is followed as an approximation of the random information/noise that interferes with the dynamic state equation. This process is generally carried out in an iterative manner through n steps. Therefore, the analogous of the Bellman equation for such can be defined as follows:

$$\hat{V}_t^i(x_t^i) = \min_{u_t \in U_t} \mathbb{E}\left[g(x_t^i, u_t) + \hat{V}_{t-1}^{i-1}(x_{t-1}^{i-1})\right], \quad t = 1, \ldots, T \quad i = 1, \ldots, n \tag{14}$$

Several methods, including lookup tables, linear programming approximations, and Monte-Carlo, can be employed to estimate the sample path and approximate the value function. The choice of method directly impacts the quality of the solution and is closely related to the type of iterative formulation and the resulting structural properties.

B.3. Bayesian Dynamic Programming

Stochastic DP relies on estimations rather than complete information about probability distributions and parameters, often neglecting forecast considerations for stochastic variables and their impact on future distribution functions (Adida and Perakis 2010). To address this, Bayesian DP has been developed, wherein transition probabilities, reward functions, and dynamic updates follow Bayesian probability theory. This approach is advantageous for incorporating prior information based on observed data, particularly useful in inventory theory for handling censored demand or unobserved lost sales problems, challenging to convey in stochastic DP formulations. Although mathematical formulations for optimization problems and the Bellman optimality equation have been discussed previously, we omit them in this subsection due to their complexity for this setting. For a more in-depth analysis, refer to Rieder (1975).

B.4. Hybrid Dynamic Programming

A myriad of methods have been developed to enhance DP formulations and mitigate issues related to the curse of dimensionality. As discussed in section 3.2, ADP serves as a viable alternative, yielding fruitful results. However, some other alternatives might guarantee similar performances by having completely different structures.

Combining established OR methods with DP offers a promising approach. DPs are versatile and can inspire the design of bounds and heuristics. Integrating exact and heuristic approaches with DP can lead to mutual improvement and even develop more exact algorithms with a smaller computational complexity. This has been explored in the OR literature as an alternative to increment the power of DP methodologies and simplify the burden associated with the Bellman equation (e.g. Bautista and Pereira (2009)).

Although these hybrid strategies are already known but treated within their particularities, we have decided to bundle them in a classification that has been coined under the name "Hybrid Dynamic Programming", once they are designed by the synthesis between DP and other well-structured OR methodologies.

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